The Character of Expiratory Flow in the Lung

by

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Submitted to the Department of Mechanical engineering in January, 1988, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

ABSTRACT

The primary objective of this study is to predict the expiratory pressure-flow relationship in a human lung. Experiments and analysis of the flow in a single bifurcation provides a means to numerically synthesize the pressure-flow character for a complete lung, as well as providing insight to the fluid dynamic character of expiratory flow.

A novel experimental technique was developed to measure the pressure drop across a single bifurcation within a multi-generation airway model. Energy considerations allowed the pressure drop to be separated into components owing to kinetic energy fluxes and viscous energy dissipation. Experiments were first conducted in an idealized symmetric, planar airway spanning the physiologically important Reynolds number range from 50 to 8000. Sensitivity of the results was determined for typical pulmonary conditions;

- Non-planar geometries
- L/D ratio variations
- Non-circular airway shape
- Asymmetric flow conditions
- Turbulent/Laminar flow regimes.

The numerical simulations of the human lung generated excellent agreement with available physiological data. Different morphometric models were investigated, demonstrating the importance of the asymmetric geometry of the lung.

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1. Introduction

1.1 Objective

The primary objective of this work is to obtain a relationship between pressure drop and flow rate for expiratory flow from a human lung. Specifically, a relationship is sought which describes the pressure drop across a single bifurcation which can be considered to be imbedded inside the airway network. A related objective is to gain insight to the nature and character of expiratory pulmonary fluid dynamics.

The results of this study are intended to make it possible to synthesize the flow resistance of an entire airway system based on predictions for each of the individual bifurcations within the system. Accurate predictions require that the relationship be sufficiently general to allow for the "non-ideal" characteristics of a real human lung. Pulmonary parameters are known to change with position in the lung and with the evolving state of lung inflation during expiration.

The number of potential combinations of these parameters which are encountered in the lung make it impractical to experimentally investigate each one. A dimensional analysis of pulmonary flows to describe steady flow across a bifurcation (Jaffrin and Kessic, 1972) identified the relevant dimensionless groupings as the Reynolds number in each branch, and the geometric scaling parameters. This study will attempt only to establish a relationship for a typical bifurcation
within the lung over the range of Reynolds numbers in the parent or tracheal branch from 100 to 10,000. The sensitivity of that relationship to the expected range of "non-ideal" geometry and deviations from symmetry will also be examined.

1.2 MOTIVATION

The major motivation to study the expiratory pressure-flow relationship of a bifurcation is the desire to improve the diagnostic utility of the maximal expiratory flow volume (MEFV) pulmonary function test. Computer models exist to predict the results of the MEFV as a diagnostic aid (Elad et al, 1987, Lambert et al, 1982), but these suffer from the lack of accurate pulmonary pressure-flow relationships.

The few correlations for expiratory flow resistance found in the literature (Reynolds, 1980, Reynolds and Lee, 1979, Reynolds, 1982, Hardin and Yu 1980) agree only to a factor of about three. While this variability could result from real intra-study differences in the system geometry, the range is surprisingly large especially in view of the statistical averaging that would result from multiple-generation networks. Further skepticism is based on the seemingly unrealistic limits approached by the correlations at both high and low Reynolds numbers.

A second, but equally compelling motivation to investigate the expiratory pressure-flow relationships is the profound lack of fundamental understanding of expiratory flows, even during normal breathing. While several models of inspiratory
resistance exist (Pedley et al 1971, Pedley et al 1970a and b, Jaffrin and Kessic 1972), no equivalent extensive study of expiratory flow resistance has been conducted. Similarities are expected between inspiration and expiration, but differences between the two flows, as discussed below, preclude a direct comparison.

1.3 Approach

The approach used in this study is primarily experimental. Analytical treatments of the problem are limited by the complexity associated with a fully three dimensional flow extending from low ($\sim 10$) to moderately high ($\sim 10,000$) Reynolds numbers. Some progress can be made, however, in the use of simple analogous flows. These are included to serve as both theoretical support for the results, and to aid in developing physical insight.

A novel experimental approach, called the subtraction technique, was developed to provide the pressure difference across a bifurcation. This new method avoids the restrictive assumption made by most previous investigators that the form for the loss in a single bifurcation is the same for that found for the airway as a whole. It also avoids the substantial errors involved in the direct measurement of the pressure drop across a single generation due to the considerable variation in pressure over the cross-section. A series of "companion" experiments, including flow visualization and hot wire anomometry were used to establish transition from laminar to
turbulent flow.

Experiments were conducted on a network of bifurcations modeled after an "idealized bifurcation" presented by Pedley (1977). The network from which casts were made was designed and fabricated in two pieces on a numerically controlled milling machine by Drs J. Hammersley and D.E. Olson. (Univ. of Mich.) Figure 1.3-1 presents the dimensions of the idealized bronchial bifurcation, which can be further characterized as having:

- An ratio of the total daughter tube area to parent tube area of about 1.2, based on a mean diameter ratio of .75

- A variable ratio of the radius of curvature of the tube center-line to the tube radius, with a mean of about 1/7. The tube curvature is greatest in the daughter tube at the bifurcation, and gradually straightens once the branching angle is reached.

- A changing cross-sectional shape which is circular in the parent and daughter tubes, blended with progressively elliptical to dumbell shapes in the junction of the bifurcation.

- A branching angle, with a sharp flow divider, which varies from ~60° in the large airways to 100° in the smaller airways with a average value of about 70°.

- Tube length/diameter ratio of 3.5 between bifurcations.

- Smooth walls as a result of a thin mucas layer.

The idealized bifurcation was chosen rather than a cast from a real lung to maximize experimental control, and to minimize the number of unknowns. Effort was directed first to understanding the most simple planar symmetric flow conditions and then extended to determine the sensitivity of the results to typical variations in upstream boundary conditions, flow
asymmetries and changes in bifurcation geometry.

Applicability of the results found in the constructed idealized models is investigated by comparing predicted and measured values of the pressure-flow relationships in rigid human lung casts.
2. Background

2.1 Forced Expiration

Despite the present lack of detailed understanding, forced expiration has long been recognized as a valuable tool in assessing pulmonary function. Hutchinson (1846) recognized the technique as indicator of pulmonary vital capacity and pulmonary dysfunction over a century ago. It was not until the insightful work of Hyatt et al, in 1958, however, that forced expiration results were described in the more useful form of maximum expiratory flow vs. lung inflation. Forced expiration tests are currently used (Hyatt) to indicate the advent and progression of such important pulmonary disorders as:

- Emphysema
- Bronchitis
- Cystic Fibrosis
- Asthma

The utility of the MEFV function test is further enhanced because it can be generated from a relatively simple clinical evaluation (Hyatt 1983). The MEFV curves are generated from the flow vs. time traces obtained from a forced vital capacity, FVC, maneuver.

To obtain the FVC traces, a subject inhales maximally (to total lung capacity) and then exhales into a flow meter as forcefully and completely as possible to residual volume. The
total expired volume (total lung volume minus residual volume) is the expired lung vital capacity. Typically a Spirometer with an attached recording device is used to record the flow rate verses time. The FVC curves are integrated and re-plotting as flow vs. expired lung volume to obtain the Maximal Expiratory Flow Volume (MEFV) forced expiration curve.

The MEFV curve, see Figure 2.1-1 (Hyatt, 1983), consists of two characteristic regions; one which is effort dependent and another which is effort independent. The effort dependent region occurs during the initial rapid increase in flow, lasting until about 80% of vital capacity. The effort independent region, or flow limited region, occurs with the decay in flow rate below approximately 75% of vital capacity. Curves corresponding to various degrees of effort are also shown in Figure 2.1-1, clearly demonstrating the effect of effort.

The utility of the MEFV curves in detecting pulmonary dysfunction is primarily a result of the flow limited portion of expiration. During flow limitation for a given gas, the flow is only a function of the mechanical properties of the lung and the geometry of the lung.

2.2 Flow Limitation

The flow limiting nature of expiratory flow has been described by analogy to the flow limitation which occurs in thin walled collapsible tubes. Shapiro (1977a), in a summary of the physiological and medical aspects of flow in collapsible
tubes, describes the effect this way, "The (extremely compliant) airways are easily collapsed by an in-wardly acting transmural pressure; the resulting decreased cross-sectional area makes for increased air speeds; this in turn produces an increased pressure drop associated with the Bernoulli dynamic pressure and with frictional resistance."

Efforts to model forced expiration have proven extremely difficult because of the coupled effects of the airway structure and fluid dynamics. Additionally, information on the mechanical properties of the airways and the detailed flow behavior in a bifurcation is lacking.

To understand how these interactions and the effect of friction within the lung affects flow limitation, it is useful to examine the equations for the thin-walled compliant tubes. A one dimensional model has been developed by Shapiro (1977b) to predict the structural/fluid dynamic interactions in a thin walled, massless straight compliant tube with varying wall properties and rest area. The interactions are shown to depend on the following:

- The nature of the vessel structure in the form of a "tube law" which describes the vessel area as a function of transmural pressure (internal-external pressure) and vessel stiffness, $K_p$.
- The external pressure acting on the vessel, and both the end pressure boundary conditions.
- Frictional dissipation within the flow.
- Changes in rest area along the vessel.

The nature of the interactions are summarized in the
The differential equation below;

\[
\frac{dU}{(1-S^2)} = \frac{-dAo}{Ao} + \frac{d(P+\zeta gZ)}{\zeta C^2} + \frac{2S^2f}{Dh} \frac{dx}{\zeta C^2} + \frac{1}{\zeta C^2} \left[ \frac{dKp}{dx} + \frac{dp'}{dx} \right] dx
\]

where \(U\) is the mean flow speed, \(S\) is the ratio of \(U\) to \(C\), the local tube wave speed, \(P\) is the static pressure, \(p'\) is the transmural pressure normalized to the tube stiffness, \(Kp\), \(Ao\) is the local rest area, \(\zeta\) is the fluid density, \(g\) is gravitational acceleration, \(Z\) is elevation, \(f\) is some friction factor which can depend on the local Reynolds number and geometry, \(Dh\) is the hydraulic diameter defined as \(4\times\text{area}/\text{perimeter}\), and \(x\) is the distance down the tube.

Flow limitation occurs when the axial velocity of the fluid matches the local wave speed of the tube. When flow limitation, or "choking" occurs, the flow will not be increased by reductions in downstream pressure. This phenomena is exactly the same as that which is found in trans-sonic compressible gas flow and in the flow over a weir or "waterfall".

Much work remains before equation 2.2.1 can be applied with more confidence to expiratory flow. At best, only estimates can now be applied to each of the terms in the equation. This work will concentrate on the frictional losses alone, and not attempt to describe the remaining terms.

The effect of frictional dissipation on flow limitation can be seen from equation 2.2.1 to be unique. Friction which always acts to remove energy from the flow, drives the flow
toward a "choked" condition, increasing/decreasing the flow speed to meet the local tube wave speed. For sub-critical flows, $S<1$, the decrease in transmural pressure caused by friction reduces the tube area, which, in turn, accelerates the flow toward the tube wave speed. In super-critical flows, the decrease in transmural pressure results in the opposite effect, a decrease in the flow speed toward the wave speed. It is expected then that friction will play an important role in determining not only the required expiratory effort but also the location of the flow limiting or "choked" point.

2.3 Work To-date

Relatively few studies have examined the frictional dissipation in an airway network during expiratory flow. Pedley (1977) has made the observation that "it is as if everyone doing a model experiment on inspiration ran out of time or energy when it came time to repeating the measurements for expiration".

The few studies which are available consist mainly of experimental model studies in airway networks which are either idealized planar geometric models or pulmonary cast models. Scherer (1972) developed a theoretical model to describe expiratory flow through a single bifurcation. The assumptions required by Scherer's analysis, such as restricting the flow to a two dimensional form limits the utility of the model.

shows high variability in both the form and magnitude of results. Figure 2.3-1 graphically presents the inferred results for the frictional pressure loss in a standardized bifurcation based on the correlations published for the results of the 4 different model studies. The standardized bifurcation chosen as the basis of comparison is a typical pulmonary bifurcation as put forth by Pedley (1977), and graphically depicted in Figure 1.3-1. The results for the pressure loss are normalized by the corresponding fully developed Poiseuille flow pressure loss through an equivalent length tube.

The magnitude of the results do not agree to better than within a factor of about 3. The nature of these results have some puzzling attributes which appear to be inconsistent with intuition. First, at low Reynolds number the results are 1.5 to 3.5 higher than would be expected based on asymptotic limits of fully developed Poiseuille flow. Second, with the exception of results of Hardin (1982), little similarity exists with the form of the pressure losses found in flows which are expected to be generally similar. These similarities tend to indicate that a Re^{-0.5} dependence is likely to occur, and will be discussed in more detail in the conceptual framework section.

The inadequacy of these relationships is also seen when the pressure flow relationship found by D.B. Reynolds (1980) is incorporated into expiration models (Elad et al 1985, Lambert & Wilson 1982). Elad et al (1985) found that the dependence of maximal flow on viscosity, especially at low lung volumes, appears to be underestimated in comparison with physiological
data from experiments conducted by Schilder et al (1963), Staats et al (1980), and Wood and Bryan (1969). The Lambert & Wilson model was able to produce a better viscosity dependence agreement with experiments, but required that the pressure losses be 3.5 times the corresponding laminar flow losses at very low Reynolds numbers.

A possible reason for these discrepancies may lie in the method used in previous studies for extracting local pressure-flow relations from global measurements. In all previous experiments, the form of the pressure drop across a single bifurcation was assumed to be that found for the model as whole. Curve fitting was then used to determine the coefficients for a single bifurcation. Imposition of this form for the relationship, which is actually a weighted average over the entire network, may mask the true form of the relation for a single bifurcation.
3. Conceptual Framework

3.1 Bifurcation Flow Characteristics

The complexity of the expiratory flow within a bifurcation is currently beyond the capabilities of theoretical analysis. The form of the Navier-Stokes equation which describes the flow cannot be solved analytically, and numerical treatments of the full three dimensional flow are beyond the practical limits of current computers. Current computational capabilities are typified by the work of Wille (1984), which consumed about 2 months of processor time to solve the similar steady inspiratory flow problem at a Reynolds number of only 10.

The remaining alternative to understand the flow is to use the results of model experiments in bifurcations to develop an understanding of the nature and character of the flow. Similarities can then be drawn with the flow characteristics in similar, but less complicated flows. Generally, the less complicated flows are better understood, and the proper analogies taken together can form a conceptual framework in which to interpret expiratory flows.

Flow visualization and hot-wire velocity mapping studies performed in airway models and in single bifurcations (West and Hugh-Jones 1959, Schroter and Sudlow 1969, Patra and Afify 1983, Chang and Menon 1985, Jan 1986 ) have lead to a number of consistent observations on expiratory flows;

- The most notable is that a four cell secondary flow pattern occurs downstream of the junction for tracheal
Reynolds numbers from 50 to several thousand. This flow pattern is shown schematically in Figure 3.1-1.

- The secondary flow pattern is developed either within the junction or just downstream of it.
- Flow patterns are qualitatively similar for the range of inlet conditions tested; fully developed parabolic inlet flows, blunt inlet flows, and flows with secondary flows similar to those found in the outlet.
- The magnitude of the secondary flow velocity is between approximately 1/5 and 1/2 of that in the bulk axial flow.
- Axial velocity profiles just downstream of the junction contain a dip which is rapidly transformed into a peak in the plane of the bifurcation, but remains flat in the plane normal to the bifurcation.
- Distances greater than about one diameter downstream of the junction, the axial velocities are "blunt", and are characterized by a relatively thin boundary layer.
- For the case of a long straight parent branch, the secondary flows generated in the bifurcation are convected downstream through the equivalent of many branch lengths.

Inspection of these flow characteristics, and the geometry of a bifurcation lead to the consideration of several flow types as possible analogies to steady expiratory flow. The following are the flow problems for which possible analogies apply, and are well understood based on experimental and theoretical investigations.

- Fully developed steady laminar flow in a straight tube.
- Fully developed steady turbulent flow in a straight tube.
- Steady entry flow in a straight tube.
- Steady flow in a convergent 2-D channel.
- Fully developed steady flow in a curved tube.
- Kinetic energy dissipation.
3.2 Fully Developed Laminar Straight Tube Flow

Fully developed straight tube flow was one of the earliest analogies drawn to describe pulmonary flows (Rohrer 1915). The analogy stems from geometric similarities given that airways, in the most elementary form, can be considered a series of straight tubes. The analogy neglects the effects of curvature on the secondary flows and that each tube section is typically too short for a fully developed flow to develop. Schlichting (1979) gives the entrance length required to establish a fully developed Poiseuille profile to be;

3.2.1 \[ Le = 0.03 \times Re \times Dia \]

A fully developed flow will therefore not evolve within a bifurcation which has a typical length of 3.5*Dia. (Pedley 1977) until the Reynolds number drops below about 100. For Reynolds numbers significantly below 100, though, an increasing length of the tube will be fully developed. The frictional dissipation in a fully developed straight tube flow therefore serves as a low asymptotic limit for the flow in a bifurcation.

Derivation of the fully developed pipe flow equations can be found in most text books on fluid mechanics. The problem represents the Navier-Stokes equation in its most simple form;

3.2.2 \[ \frac{dP}{dx} = \mu \frac{du}{dr^2} \]

where \( u \) is the axial velocity, \( r \) is the radial position, \( x \) the
axial position, \( p \) the pressure, and \( \mu \) is the fluid viscosity.

Solving 3.2.2 for the pressure drop across a tube of length \( L \) gives;

\[
\text{3.2.3} \quad \Delta P = 32\mu UL/D^2
\]

where \( U \) is the bulk averaged velocity defined as the flow rate divided by area, and \( D \) is the tube diameter. Equation 3.2.3 can be written in another form which introduces a characteristic pressure \( P^* \), and the Reynolds number

\[
\text{3.2.4} \quad \Delta P = 32L/D P^* \text{Re}
\]

where \( P^* \) is given by;

\[
\text{3.2.5} \quad P^* = \frac{\mu^2}{\zeta D^2} = \frac{\zeta \text{Nu}^2}{D^2}
\]

where \( \text{Nu} \) is the kinematic viscosity.

A scaling argument which provides more insight to the nature of the pressure drop can be used to arrive at the same result to within a constant factor. While this is not necessary due to the susceptibility of the problem to rigorous analysis, it helps to establish a comparison for other more complicated effects.

From a force balance on a section of tube with a length \( L \), shown in Figure 3.2-1, the pressure force acting on the tube cross-sectional area, can be seen to be balanced by the wall shear stress acting on the wall;
3.2.6 \((P_1-P_2)\pi D^2 = \tau w(\pi DL)\)

where \(\tau w\) is the wall shear stress, and for a Newtonian fluid is given by;

3.2.7 \(\tau w = \frac{du}{dr}\) | \text{wall}

which can be approximated by representing the velocity gradient at the wall as the ratio of a characteristic velocity, the bulk velocity, by a characteristic length, the tube diameter. Combining the approximation with equations 3.2.7 and 3.2.6 gives the pressure drop as;

3.2.8 \(\Delta \pi = P - P = \text{Const.}\times\mu L U / D^2\)

Comparing equation 3.2.8 and 3.2.3 shows that the two forms agree if the unknown constant in equation 3.2.8 is taken to be 32.

3.3 Fully Developed Turbulent Pipe Flow

Laminar flow in a smooth straight tube will remain despite the presence of slight vibrations and other perturbations encountered in "real world flows" for as long as the flow is stable. Instabilities occur in steady flows when the inertial effects dominate the viscous effects. Stability is typically characterized by the ratio of these two effects in the form of the Reynolds number. A Reynolds number near 2300 for steady pipe flows is the value at which the flow becomes unstable, and therefore subject to turbulence.
Turbulent flows are expected to exist in expiration both because of the high Reynolds numbers encountered, and the significant flow disturbances which are present. As will be discussed later, the conditions for transition are complicated by spatial accelerations and secondary flows, both of which are expected to delay transition to higher Reynolds numbers.

No theory based on first principles is yet available to describe turbulent flows, and, the small scale of turbulence renders computer models impractical for a complete solution of all but the simplest of flows. Experimental results have, though, lead to a detailed understanding of structure of turbulence.

The pressure loss in turbulent smooth-walled pipe flows has been well documented (Schlichting 1979), and can be explained with the scaling arguments outlined above for laminar flows. Referring to Figure 3.3-1, unlike laminar flows, turbulent flows are characterized by a relatively blunt velocity profile with a thin boundary layer. The small scale eddies efficiently transfer momentum radially to smooth the velocity in the core. Imposition of a no-slip boundary at the wall, generates a region of laminar flow with high shear close to the wall. Estimation of the shear stress in the laminar sublayer which acts on the wall is then given by;

\[ \tau_w = \mu \frac{du}{dr} \bigg|_{wall} \propto \mu \frac{u(\delta)}{\delta} \]

where \( \delta \) is the laminar sub-layer thickness, and \( u(\delta) \) is the
velocity at the boundary layer edge, and are given by;

3.3.2 \[ \delta \propto \mu/v^* \]

3.3.3 \[ u(\delta) \propto v^* \]

where \( v^* \) is the characteristic velocity for turbulent flows;

3.3.4 \[ v^* \equiv \sqrt{\tau w/\zeta} \]

Experimental measurements of velocity distributions and wall (see Schlichting) shear stress give \( v^* \) to be;

3.3.5 \[ v^* = 0.1823 \left[ \frac{7}{U \mu/r} \right]^{1/8} \]

combining 3.2.4 and 3.3.1-3.3.5 gives the familiar Blasius friction relation for smooth walled turbulent pipe flows;

3.3.6 \[ \frac{\Delta P}{P} = 0.3146(\frac{\tau w}{\zeta})^{1/4}(L/D)/Re \]

where \( Re \) is the tube Reynolds number \( Re \equiv U\mu/\zeta D \). Using the \( p^* \) notation, equation 3.3.6 can be written more concisely as;

3.3.7 \[ \frac{\Delta P}{P} = 0.157 P^{1.75} (L/D) Re \]

For the case where surface roughness extends into the flow past the laminar sublayer, this relation no longer holds. A transitional region exists when the mean roughness height, \( K_s \), is between the laminar sublayer, \( yv^*/\nu = 5 \) to about \( yv^*/\nu = 70 \). For \( K_s \) above \( yv^*/\nu = 70 \) the flow is considered fully rough, and the increased velocity gradient at the wall generates the
pressure drop given by the expression;

\[ 3.3.8 \quad \Delta P = \frac{\mu P}{2} \frac{L}{D} \text{Re}^2 \left[ 2 \log \left( \frac{r}{K_s} \right) + 1.74 \right]^{-2} \]

Schroter and Sudlow (1969) have noted that bronchial walls are coated with a liquid mucus layer which is "microscopically" thin, and therefore hydraulically smooth. Macroscopic corrugations in the trachea and main bronchi are present due to cartilidge rings. The effective Ks generated by these corrugations is relatively low due to their wide spacing. These corrugations are therefore unlikely to generate fully rough flow. Consequently it is more probable that 3.3.7 will apply in the lung, rather than 3.3.8.

3.4 Steady Entrance Flow

The entrance type phenomena expected in a bifurcation are similar to those for the steady entrance flow problem in a straight tube, but more complicated. The entrance phenomena are a result of the effects of viscosity diffusing in-ward from the wall to adjust the inlet velocity profile to its fully developed final form. The extent of the difference between the inlet profile and the fully developed profile will therefore determine the magnitude of entrance effects on the flow.

The entrance analogy in expiratory flows can be seen by considering a lung as being roughly modelled by the geometry shown in Figure 3.4-1. The area changes experienced as the
flow proceeds toward the "trachea" can be approximated as a series of straight tube sections, \( L/D = 3.5 \), with a constant area ratio contraction, \( \frac{A_n}{A_{n+1}} = 1.2 \), joining the tubes. The viscous effects start to propagate in from the wall as the flow proceeds in each straight section. In the contraction section, the flow accelerates, and the viscous boundary layer thins to maintain continuity. Once in the straight section, the boundary layer continues to grow, but starts from a non-zero value.

Rivas and Shapiro (1956) in an analysis of ASME test nozzles accounted for the non-zero initial boundary layer thickness in the straight tube section downstream of a bellmouth entrance with an "equivalent length" concept. The equivalent length, \( L_{eq} \), is the length of straight tubing required to develop a boundary layer thickness equivalent to that found in the inlet of the straight section downstream of the bellmouth. In general, the equivalent length is dependent on the flow rate, and bellmouth geometry. The loss downstream of the entrance can then be calculated as the difference in a tube of the actual length plus \( L_{eq} \), and a tube of length \( L_{eq} \).

Applying entrance flow theory with the added correction of an equivalent length to expiratory flow is difficult. The difficulty arises due to the presence of secondary flows, nonuniform inlet velocity profiles, and the flow details required to calculate \( L_{eq} \). Despite these problems, entrance flow analogies have been applied with some success (Pedley 1977) to the study of inspiratory flows, and are expected to
provide some understanding of expiratory flows.

A sizeable amount of literature has been devoted to the study of flow in the entrance of a circular pipe. Boussinesq is the first to be cited as having made a theoretical investigation of the problem in 1891. Since that time many experimental studies have been done, and correspond well to theoretical predictions. Most current analyses of the problem are numerical, but the more simple boundary layer analysis of Boussinesq provides insight to the flow character for large Reynolds numbers.

Investigations (Shapiro et al 1954) have shown that for the thin boundary layers near the entrance, the falling pressure gradient has negligible effect on the boundary layer development. Flat plate, Blasius type, boundary layer growth therefore occurs, which gives the boundary layer thickness as;

\[ \delta(x) = 1.72D \sqrt{\frac{x}{D \text{Re}}} \]

where \( D \) is the tube diameter, \( x \) the distance along the tube axis, and \( \text{Re} \) is the tube Reynolds number. In the first approximation, the core velocity can be taken to be constant, which allows the wall shear rate to be written as;

\[ \tau_x(x) = \mu \frac{du}{dr} \bigg|_{\text{wall}} \propto \mu \frac{U}{\delta(x)} = \mu U \left[ \frac{\text{Re}}{xD} \right]^{\frac{1}{2}} \]

which can be integrated to give the loss over a length of pipe \( L \);
3.4.3 \[ \Delta P = \text{const.} \frac{\mu^2}{\zeta D^2} \left[ \frac{3}{\text{Re} \frac{L}{D}} \right]^\frac{1}{2} \]

where the published experiments place the constant as 4.87. Note that the typically reported value is 6.87, which is higher because it includes the effects of the changing velocity profile. Incorporating the equivalent entrance length and \( P^* \) into equation 3.4.3 gives;

3.4.4 \[ \Delta P = 4.87 \left[ \left( \frac{L}{D} + \text{Leq}/D \right)^{\frac{1}{2}} - \frac{\text{Leq}}{D} \right] \text{Re}^{1.5} \]

The validity of this expression for straight tubes is confined to intermediate values of Reynolds number. On the low end of the range, the thin boundary layer approximation fails as the entrance length approaches the tube length, and Poiseuille flow results. For cases in which the product of the Reynolds number and L/D ratio exceeds about 10E5, turbulence is present in the boundary layer and the growth pattern of the boundary layer is altered.

This boundary layer transition to turbulence may arise in expiratory flows. Assuming an \( \text{Leq} \) near 1, with an L/D ratio of 3.5, the transition would occur on the order \( \text{Re} \approx 10,000 \).

3.5 Steady flow in a convergent 2-D channel.

Another geometry that can be used to model both a single bifurcation and the entire airway is a convergent channel. This representation is not very accurate, but provides a means to approximate the effect of a continuing balance between a
boundary layer growth and thinning. The growth of the boundary layer results from the diffusion of viscosity as the fluid passes through the network, while the boundary layer thinning results from the increasing velocities imposed by the progressively smaller flow area.

Schlichting (1979) has summarized the exact solution to Navier-Stokes equation for the 2-D channel. The equations can be considerably simplified for this case in which the flow is everywhere radial, and the flow profile depends only on the azimuthal angle, \( \theta \). The geometry is shown in Figure 3.5-1. Defining a local Reynolds number as:

\[ \text{Re} = \frac{U_o h}{\text{Nu}} \]

where \( U_o \) is the local centerline velocity, and \( h \) is the local channel height. Normalizing the velocity by \( U_o \), and the radial distance by \( h \), allows the N-S equations to be written in terms of a single non-linear differential equation;

\[ \frac{\text{Re}}{2} \frac{d^2 \hat{U}}{d \theta^2} + \frac{4}{2\alpha} \hat{U} - \frac{K}{2\alpha} \hat{U}^2 = -K \frac{2\alpha}{\text{Re}} \]

with the boundary condition;

\[ \hat{U}(\pm \alpha) = 0 \ldots \text{No slip at the walls.} \]

\[ \frac{d\hat{U}(0)}{d\theta} = 0 \ldots \text{Centerline symmetry} \]

with the definitions;

\[ \hat{U} = \hat{U}(\theta) = U(\theta, r)/U_o(r) \]
3.5.6 \[ \alpha = \tan \left( \frac{\pi h}{r} \right) \approx \frac{\pi h}{r} \]

The convergence angle, \( \alpha \), which is appropriate for use in the lung is difficult to define because of the difficulty in converting from a three-dimensional to a two-dimensional geometry. One approach to define an appropriate value of \( \alpha \) for the lung is to write the area of the lung in the exponential form;

3.5.7 \[ A = A_0 \exp \left[ \frac{L}{D} \right] \]

which can be linearized about a typical bifurcation with an L/D of 3.5 and an area ratio of 1.2, giving \( \alpha \) as approximately 0.052 (representing an included divergence angle of about 6°).

The constant \( K \) in equation 3.5.2 is the imposed wall pressure gradient, in the form;

3.5.8 \[ K = \frac{3}{\zeta \left( \frac{\delta P}{\delta r} \right)} \]

Equation 3.5.2 can be solved numerically, or analytically for the extremes in Reynolds numbers. The numerical solution to 3.5.2 is presented in Figure 3.5-2 for the dimensionless velocity profile for a range of Reynolds numbers and the typical pulmonary value of \( \alpha \). The profile is seen to progress from a Poiseuille type profile at low Reynolds numbers to a profile characterized by a blunt core and thin boundary layer at higher Reynolds numbers.

The pressure flow characteristics of the geometry also
follow the same progression from a Poiseuille form to a boundary layer form. The boundary layer form can be seen most easily by solving 3.5.2 for the high Reynolds number case and evaluating the wall friction. The high Reynolds number limit solution to 3.5.2 is;

$$3.5.9 \quad \hat{u} = 3 \tanh^2 \left[ \frac{1}{2} \left( \frac{\text{Re} \, \alpha}{2} \right) \left( 1 - \frac{\theta}{\alpha} \right) + \tanh \frac{1}{\sqrt{2/3}} \right]$$

which generates a shear stress at the wall corresponding to the pressure drop;

$$3.5.10 \quad \frac{\Delta P}{\Delta P_{\text{pois.}}} = \frac{2}{\sqrt{3}} \alpha \cos(\alpha) \left[ \text{Re} \, \hat{u}(0) \right]^{1/2} \frac{1}{\text{Re} \gg 1}$$

Using the value $\alpha = 0.052$, corresponding to the lung, and numerically evaluating $\hat{u}(0)$ to be approximately 1.15, gives the asymptotic frictional relationship to be;

$$3.5.11 \quad \frac{\Delta P}{\Delta P_{\text{pois.}}} \approx \left[ 0.06 \sqrt{\text{Re}} \right] \frac{1}{\text{Re} \gg 1}$$

A relationship which can be applied to lower Reynolds numbers must be obtained from the direct solution to 3.5.2.

3.6 Flow in a Curved Tube

The generation of secondary flows downstream of a bifurcation during expiration can also be described by analogy to curved tube flow. Figure 3.6-1 shows the secondary flow
pattern established by the centrifugal forces generated by the core flow as the flow proceeds around the bend in a curved tube. The resulting pressure gradient across the tube cross-section causes fluid in the core to move outward where the slowing moving fluid at the wall has insufficient axial velocity to generate the same pressure gradient, and is driven inward to preserve continuity.

The four cell structure found in the parent tube of a bifurcation, shown in Figure 3.1-1, can be generated by approximating the bifurcation geometry as two curved tubes oriented back to back. This geometry is depicted in Figure 3.6-2, and shows that for symmetric flows a balanced four cell pattern is expected.

A basic difference which must be noted between curved tube and flow in a bifurcation is that in a bifurcation both the geometry and area change as the flow proceeds along the bend. Despite these differences the analysis of the driving factors which influence curved tube flow should provide insight as to the nature of secondary flows in expiratory flow. This hypothesis is supported by the work of Jan (1986) which indicated similarities between the velocity profiles in quasi-steady oscillatory flow in a bifurcation those predicted with curved tube flow theory.

For curved tube flow the natural coordinate system is a toroidal one, as shown in Figure 3.6-3. The orthogonal unit vectors are in the $r$, $\alpha$, and $s$ directions where $r$ is measured radially outward from the center of the cross-section, $\alpha$ is the
angle between the radius vector and the plane of symmetry, and 
s is measured along the tube axis. In this coordinate system 
the steady non-dimensional Navier-Stokes equation take the form 
given in Equation 12 of Berger et al. The dimensional 
parameters ( denoted with a ') are nondimensionalized by;

\[ r = r'/a \quad s = Rc0/a \quad p = p'/\left(\text{cU}^2\right) \]

3.6.1

\[ u = u'/U \quad v = v'/U \quad w = w'/U \]

Berger, Talbot and Yao (1983) outlined a simplification of 
these equations for the case of gradual curvature, \( \delta r \equiv r/Rc \ll 1 \), by neglecting all terms of order \( \delta r^1 \) or higher. The 
centrifugal force terms which are necessary to drive the 
secondary motions were retained in the analysis by rescaling 
the axial velocity as;

\[ 3.6.2 \quad u = u'/ \left[ U\delta r \right]^{1/2} \]

and for convenience the axial distance is rescaled as

\[ 3.6.3 \quad z = s\delta r \]

Introducing the Dean number, \( Dn \), which is the ratio of the 
square root of the inertia and centrifugal force product to 
the viscous force;

\[ 3.6.4 \quad Dn \equiv \text{Re} \delta r \]

the resulting equation, along with the continuity condition for 
fully developed flow is given in equation 13 of Berger et al. 
This equation was obtained by recognizing that the velocities
no longer vary with axial position. Inspection of this equation indicates that the axial pressure gradient is linear.

A stream function is introduced and, following cross-differentiation to eliminate the pressure, equations 15 and 16 of Berger et al are obtained.

For small values of Dn the equations can be solved by expanding the solution in powers of Dn about the Poiseuille straight tube profile. The result, when expressed in terms of the axial pressure difference is given by Dean (1927, 1928);

\[ \frac{\Delta P}{\Delta P_{\text{Poiseul}}} = \left[ 1 + 0.0306(K/576) - 0.0110(K/576)^4 + \ldots \right] \]

where \( \Delta P_{\text{Poiseul}} \) is the Poiseuille pressure drop in a straight tube with the same flow rate, and \( K \) is the original form of the Dean number defined by Dean as;

\[ K \equiv 2 \frac{a}{Rc} \left[ \frac{W_{\text{max}}}{\nu^2} \right]^2 \]

where \( W_{\text{max}} \) is the maximum velocity in a straight pipe of the same radius resulting from the same imposed pressure gradient. The relationship between \( D_n \) and \( K \) depends on the ratio of the flux in the straight tube at a specified pressure gradient, \( Q_s \), and the flux in the curved tube with the same imposed pressure gradient, \( Q_c \);

\[ D_n = \left( \frac{Q_s}{Q_c} \right)^{1/2} \left[ \frac{1}{2} K \right] \]
where the ratio $Q_c/Q_s$ is (Berger et al):

$$
\frac{Q_s}{Q_c} = 1 - 0.0306 \left( \frac{K}{576} \right) + 0.0120 \left( \frac{K}{576} \right)^2 + O \left( \frac{K}{576} \right)^3
$$

Dean's series expansion solution is valid up to $K \approx 576$. Figure 3.6-4 presents the results of Dean's solution for the pressure loss in a curved tube normalized to that in a Poiseuille flow with the same bulk flow rate versus $D_n$. The graph was generated by calculating the flow ratio based on a value of $K$, and then finding the $D_n$ corresponding to $K$ and the flow ratio. It is interesting to note that for $D_n < 10$, effects of curvature on the pressure gradient are almost negligible. Even at the high limit of applicability for Dean's solution, the effects are small, generating only a 2% increase in the flow resistance at $K = 576$, corresponding to $D_n \approx 16$.

The nature of the flow and the scaling of Dean's solution for the secondary flow can be understood by considering the forces acting on the fluid. The axial velocity establishes a pressure gradient which is balanced by the shear stress at the wall induced by the secondary flows. The resulting pressure balance can be written as:

$$
\frac{2 \sigma U}{R_c} \sim \mu \frac{V_{sec}}{r}
$$

which gives the secondary velocity scaling found by Dean:

$$
\frac{V_{sec}}{U} \sim \delta r \frac{Re}{U}
$$
At higher values of Dean number, numerical studies have been utilized to solve equation 3.6.7. The most expansive of these numerical studies is by Collins and Dennis (1975) who were able to extend Dean's solution up to $Dn=5000$. The numerical studies suggest that the structure of the curved tube flow at high Dean numbers is that of an inviscid rotational core surrounded by a thin boundary layer at the tube wall.

An understanding of the nature and flow characteristics which typify the high Dean number curved tube flow can be gained by investigating the asymptotic limits of the governing equations. Pedley (1980) formed the asymptotic limits by scaling equation 3.6.7 with the asymptotic form for the streamfunction, $\phi$, Dean number, $Dn$, and boundary layer thickness, $\delta$;

\begin{align*}
3.6.11a & \quad Dn \sim U \sim \sqrt{K} \\
3.6.11b & \quad \phi \sim \sqrt{K}/Dn \\
3.6.11c & \quad \delta \sim \sqrt{K}
\end{align*}

Note; Pedley used a dean number, $D$, defined as $\sqrt{K}$, and normalized velocities by $Nu/a$ rather than $U$. These differences in notation account for in the somewhat unusual notation presented in 3.6.14 to preserve the form of Pedley's analysis.

For the core flow, the viscous effects are negligible, and the pressure gradient established by the centrifugal forces
are balanced by the inertial terms: equation 3.6.7a therefore gives;

\[ 1 = \alpha + \beta \]

In the boundary layer, there is a balance between the inertia, viscous and the centrifugal force terms gives;

\[ 4\tau + \beta - 2\alpha = 2(\beta - \alpha) + 3\tau = \tau \]

Solving the three equations for the three unknowns gives;

\[ \beta = \tau = \frac{1}{3} \alpha = 1/3 \]

which can be inverted to give the parameters in terms of Dn;

\[ 3.6.15a \quad \sqrt{K} \sim Dn = Dn \]

\[ 3.6.15b \quad \phi \sim \sqrt{K} /Dn = Dn \]

\[ 3.6.15c \quad \tau \sim \sqrt{K} = Dn \]

The pressure drop is therefore seen to go as;

\[ 3.6.16 \quad \text{Del} P = \tau w = \mu \frac{L}{D} \frac{du}{dr} \text{wall} \sim \mu \frac{L}{D} \frac{*L}{D} \frac{\tau}{D} = \frac{U}{L} \frac{Dn}{D} \]

In contrast to the secondary velocity scaling at low Dean numbers, equation 3.6.15 allows the secondary velocity scaling to be written directly as;
giving the ratio of secondary velocity to that in the core as a constant, unlike the low dean case in which the ratio increases with Dean number.

These asymptotic forms have been verified by numerous experimental and numerical studies and are summarized by Berger et al (1983). Results of the experimental studies for the frictional losses are shown in Figure 3.6-5. The experimental data is seen to be correlated well by the Hasson correlation below curvature dependent branch points which are identified as transitions to turbulence. The correlation is valid down to a Dn of about 20 where Dean's solution asymptotes to Poiseuille flow. The Hasson correlation is:

\[ \text{Del P} = 32 \frac{L}{D} \left( 0.556 + 0.0969 \frac{\text{Dn}}{\text{Re}} \right)^{\frac{1}{2}} \]

Ito (1959), experimentally documented the frictional loss in turbulent flows for a curved tube. The transition to turbulence was found to be retarded by the action of the secondary flows. The exact reason for the postponement of transition is still not clear, but it is generally believed that the action of secondary flows to thin the boundary layer is similar to the effect of a favorable pressure gradient. For the range of experiments \( 1/15 > \delta r > 1/9000 \), Ito reports the critical Reynolds number to be;
3.6.19 \( (Re)_{\text{crit}} = 2 \times 10^4 \delta r \)

The experiments were correlated by a Blasius type expression with an additional curvature effect;

\[
3.6.20 \quad \Delta P = \frac{1}{2} \frac{L}{D} \delta r \left[ 0.029 + 0.034 \delta r \frac{\delta r}{D_n} \right]^{-3/8} \frac{\delta r}{D_n}^{-1/4} Re^2
\]

validated over the experimental range:

\[
1.5
300 > D_n \delta r = Re \delta r^2 > 0.034
1.5
\]

For larger values of \( D_n \delta r \), the expression becomes:

\[
3.6.21 \quad \Delta P = \frac{1}{2} \frac{L}{D} \delta r \left[ \frac{\delta r}{D_n} \right]^{1/5} \frac{\delta r}{D_n}^2 Re
\]

While the typical curvature encountered in the lung, \( \sim 1/7 \), is below the range of Ito's experiments, it can be used to estimate a critical reynolds number of about 10,000. This places the \( Re \delta r^2 \) product at 200 at transition, indicating that 3.6.20 is the most likely form applicable to pulmonary conditions.

3.7 Kinetic Energy Dissipation

The case of expiratory flow through a bifurcation which is embedded within an airway can be considered fully developed in the sense that the entering velocity profile will be similar to
the exiting velocity profile. This must be the case for a fully developed symmetric flow within a "large" airway system because the inlet of a bifurcation will be the outlet of another bifurcation.

The most striking feature of this situation is that a total of eight vortices in the two inlet flows are converted within the bifurcation to only four in the outlet. The mechanism for this conversion is not clear, although it can most likely be considered to fall between two extreme alternatives:

- Each of the eight inlet vortices are dissipated through viscous action, and four more are generated through the conversion of potential to kinetic energy.

- The kinetic energy of the inlet vortices are conserved by the enhancement of the two outer vortices of each daughter tube by the two inner vortices feeding them through the secondary flow.

The dissipation in the first case would be similar in nature to fully rough turbulent flows in which large eddies are generated and then dissipated due to viscous action. The generation and dissipation of the secondary flow vorticies will not occur randomly as in turbulent flow, but will occur once at each bifurcation. Another difference from turbulent flow is the orientation of the secondary velocities. In bifurcations, the vorticies are oriented along the axis of flow, which is not always the situation in turbulent flows (Schlichting, 1979).

An estimation of the magnitude of the pressure loss that
would result if the energy in the inlet vorticies were
dissipated is the magnitude of kinetic energy;

3.7.1 \[ \text{Del } P \propto \frac{1}{2} \gamma \nu \text{Vsec}^2 \]

where Vsec is the average secondary velocity in the inlet
flow. The magnitude of the secondary flows was given in
section 3.1 to be between 1/5 and 1/2 of the bulk flow, U.
Using 1/4 as a typical value, the pressure loss across a single
generation can be approximated as;

3.7.2 \[ \text{Del } P \approx \left(\frac{1}{32}\right) P^* \text{Re}^2 \]

Conservation of the kinetic energy in the second case
would result in no induced pressure drop.

3.8 Definition of Pressure Drop

Before estimates for the form of the pressure drop in
bifurcating flow can be made based on the effects discussed
above, the pressure drop must first be defined. The
difficulty in defining the pressure drop comes because the
pressure across a leg of a bifurcation is not uniform. The
strong secondary flows and tube curvature create a pressure
variation within the tube cross section which are a substantial
fraction of those across the bifurcation (Pedley 1980).

The nature of the flow does not allow many simplifying
assumptions such as inviscid or low Reynolds number flow to be
made to help in defining pressure losses. The pressure loss due to the bulk behavior of the fluid is desired, which allows integral techniques to be used without loss of information.

The two integral approaches which are candidates for the analysis are momentum and energy balances. Momentum balances are difficult to implement in the analysis because the walls of a bifurcation are inherently curved, and as a result, introduce unknown wall tensile forces. Energy balances are well suited to the analysis, "accounting" for changes in kinetic energy flux by the rate at which work is done on the bifurcation by pressure and viscous forces.

The steady-state energy equation for an incompressible fluid can be written in integral form as:

\[ 3.8.1 \int_{0}^{I} \rho u \, da - \int_{I}^{0} \rho u \, da + \int_{\text{Vol}} \Phi \, d\text{Vol} = \int_{I}^{0} \frac{V^2}{2} \phi u \, dA - \int_{0}^{I} \frac{V^2}{2} \phi u \, dA \]

Where the subscript I represents integration over the region of inflow and 0 the region of outflow, \( dA \) is a differential area, \( \text{Vol} \) is the control volume, \( u \) is the axial velocity component, \( \rho \) is the static pressure, \( \Phi \) is the viscous dissipation function, (see Potter and Foss pp 192), and \( V \) is magnitude of the velocity;

\[ 3.8.2 \quad V^2 = u^2 + V_{\text{sec}}^2 \]

where \( V_{\text{sec}} \) is the magnitude of the secondary velocity;

\[ 3.8.3 \quad V_{\text{sec}}^2 = v^2 + w^2 \]

where \( v \) and \( w \) are the secondary velocity components.
The integral of the viscous dissipation function over the control volume represents the total energy dissipation by viscosity, and for later reference is defined as:

\[ D \equiv \int \frac{\Phi}{\text{Vol}} \ dVol \]

The bulk properties which are measured experimentally are the flow rate, \( Q \), and average static pressure, \( R \), defined as:

\[ R \equiv \frac{1}{A} \int p \ dA \]

\[ Q \equiv \int u \ dA \]

The average pressure term can be introduced into 3.8.1 by rewriting the local static pressure as the sum of the average pressure, \( R \), and a perturbation, \( p' \). This substitution allows the pressure work terms to be written:

\[ \int p u \ dA = \int [R + p'] u \ dA = Q [ R + P_c ] \]

where \( P_c \) is a correction to the average pressure which accounts for the non-uniform pressure profile, and is defined as:

\[ P_c \equiv \frac{1}{Q} \int p'u \ dA \]

The kinetic energy flux terms in 3.8.1 can also be written in terms of bulk properties by introducing the secondary
velocity correction factor, $f_s$, and the axial velocity shape factor, $f_a$;

\[ f_s = \frac{1}{A} \int \left[ \left( \frac{w}{U} \right)^2 + \left( \frac{v}{U} \right)^2 \right] \left( \frac{u}{U} \right) \, dA \]

\[ f_a = \frac{1}{A} \int \left[ \frac{u}{U} \right] \, dA \]

where $U$ is the average velocity, defined as $U = \frac{Q}{A}$.

Introduction of these terms and dividing through by the flow rate, $Q$, allows the steady-state energy equation to be written in the form:

\[ R = \frac{D}{Q} \int \frac{\zeta Q}{A} \left[ \frac{f}{s} + \frac{f}{a} \right] \, dA + \frac{P_c}{Q} \]

The terms on the right hand side of 3.8.10 represent the relative contributions of viscosity, kinetic energy changes, and pressure uniformity correction to the drop in the average static pressure respectively, or;

\[ \text{Del} \, R = \text{Del} \, P_v + \text{Del} \, P_k + \text{Del} \, P_c \]

where the pressure loss terms are defined as:

\[ \text{Del} \, P_k = \frac{\zeta Q^2 (f + f) / A^2}{a \, s} \]

\[ \text{Del} \, P_v = \frac{D}{Q} \]

\[ \text{Del} \, P_c = \frac{P_c}{Q} \]
3.8.15 \[ \Delta p_c = p_c \]

3.9 Summary of Pressure Loss Correlations

To compare the order of magnitude of each of the above effects, the pressure drop in an idealized bifurcation was calculated. The pressure drop was calculated assuming that theoretical pressure loss form for a particular flow condition applied to each branch in the bifurcation. Results can then be compared to the experimentally determined pressure losses and any similarities in either form or magnitude can serve as an indicator of flow similarities.

Table 3.9-1, below, summarizes the expressions developed for the cases of interest, presented in normalized form. The normalizing factor is the loss if the flow were everywhere fully developed Poiseuille flow. This normalization was chosen because it is the anticipated asymptotic lower limit for the experimental results, and indicates the extent to which the flow resistance is enhanced over Poiseuille flow.

Figure 3.9-1 presents the results for the normalized loss in a branch of a bifurcation vs tracheal Reynolds number. For comparison, the factor of three band of results found by the previous investigators presented in Figure 2.3-1 is also shown. The magnitude and form of the results are seen to be most similar to curved tube, 2-D convergent channel, and entrance flow theory.
### Table 3.9-1

<table>
<thead>
<tr>
<th>Case</th>
<th>Del P$<em>v$/Del P$</em>{pois}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curved tube flow Re &gt; 60</td>
<td>0.556 + 0.096$Re\delta r$</td>
</tr>
<tr>
<td>Entrance flow Re &gt;&gt; 1</td>
<td>$0.152 \left[ \frac{\sqrt{D}}{(L+Leq) - Leq} \right]^{\frac{1}{2}} Re^{\frac{1}{2}}$</td>
</tr>
<tr>
<td>2-D Channel flow Re &gt;&gt; 1</td>
<td>$\frac{2}{\sqrt{3}} \frac{\ln(An-1/An)}{L/D} Re^{\frac{1}{2}}$</td>
</tr>
<tr>
<td>Turbulent pipe flow Smooth walls, Re &gt; 2000</td>
<td>$4.9E^{-3} Re^{3/4}$</td>
</tr>
<tr>
<td>Turbulent curved tube flow Re &gt; 0.034/$\delta r^2$</td>
<td>$\frac{1}{10} \frac{4}{5} 4.9E^{-3} \delta r Re$</td>
</tr>
<tr>
<td>Kinetic Energy Diss.</td>
<td>$9.76E^{-4} Re$</td>
</tr>
</tbody>
</table>
4. Experimental Methods

4.1 Subtraction Method

The experimental approach developed to quantify the pressure drop across a single bifurcation embedded within an airway system is termed the "subtraction method". Development of this new approach was deemed necessary because of the restrictive assumptions imposed by the methods of previous investigations.

The strong secondary velocities and wall curvature typical of expiratory flows establish "substantial" cross-sectional pressure variations. Experiments by Jaffrin and Hennessey (1972) and Akhavan (1980), found that in oscillatory flows the cross-sectional pressures can vary from 20% to 100% of the pressure difference across a bifurcation. While no similar study exists for expiratory flows, large cross-sectional pressure variations can be expected to make a direct measurement of pressure loss across a bifurcation difficult.

Previous investigators have typically avoided this problem by basing the loss in a single bifurcation on the measured pressure-flow curve for an entire airway network. Errors in measuring pressures, upstream in plenums and downstream in plenums or straight tube sections, can be expected to be small in comparison to the overall pressure loss. Unfortunately, this method requires that some algebraic form for the loss in a single bifurcation, typically that found for the entire network be assumed, without substantiation. Since the character of the
entire network represents a weighted average over the entire airway, the overall relationship may not simultaneously reflect the relationship in each bifurcation. In addition, entrance effects may be erroneously included, and it will be difficult at best to distinguish the existence of different flow regimes.

The subtraction technique was also developed with the intention of extending the model experiments to include asymmetric effects. Basing the loss in a single bifurcation on systemic characteristics becomes extremely difficult with flow asymmetries.

The subtraction method for symmetric flow conditions is shown schematically in Figure 4.1-1. The method requires an airway system which consists of a set of matched upstream airways and a single test bifurcation. The upstream airways are required to produce a boundary condition for the flow into the test bifurcation which can be considered "typical" of the inlet flows to a bifurcation embedded in the lung. The scheme to determine the pressure drop of the test bifurcation alone is as follows;

- The pressure-flow relationship for the identical upstream airway network (peripheral to the shaded zone in Figure 4.4-1) is measured over the desired range of flow rates.
- Two identical upstream airway networks are then attached to the daughter branches of the test bifurcation.
- The pressure-flow relationship of the new combined airway system, consisting of the upstream airway networks and the test bifurcation, is measured. The range of flow rates should be about double the range used to test the upstream airways.
The symmetry condition ensures that the flow rate through each of the upstream airway systems is equal to half of the total.

The pressure drop due to the test bifurcation can then be calculated by taking the difference in pressure drops between the combined airway at a specified flow rate, and the pressure drop for the upstream airways at half the flow rate.

The subtraction method requires that the pressure-flow relationship of the upstream airways be fully documented, and it assumes that the addition of the test bifurcation does not alter this relationship. This assumption must be justified in light of the experimental measurements of the pressures in a finite pipe bend, Ito (1960), demonstrating that the effect of a pipe bend propagates on the order of a pipe diameter upstream of the bend. Figure 4.1-2 presents Ito's results for the pressure distribution in a finite pipe bend with straight upstream and downstream tangents.

The pipe bends used by Ito were severely curved, with a radius ratio $\delta r=1/3.7$. As a result, a very strong pressure gradient is established at the bend entrance which generates secondary flows. The subtraction technique is not expected to be significantly effected by this effect for several reasons;

- The idealized bifurcations which make-up the airways are separated by straight sections of about a diameter in length.
- The blunt velocity profiles typical in expiratory flows offer a much thinner boundary layer with the low velocities that are most affected by adverse pressure gradients.
- The radius ratio in the test bifurcation is more gradual than used by Ito, $1/7$ vs $1/3.7$. 
Finally, it can be seen from Ito's results that while the variations in pressure can be substantial, the mean pressure variation is significantly lower than the mean pressure loss across the bend.

Subtraction of two values obtained in separate experiments may result in significant error if the overall experimental signal/noise ratio is low. The signal/noise ratio for the subtraction experiments is given by the difference in signals from two experiments divided by the sum of the noise in each experiment. High signal/noise ratio therefore require not only that the experimental equipment be capable of low noise levels, but also that the experiments be designed to provide significant differences between signals.

4.2 Downstream Boundary Condition

The downstream boundary condition imposed on the terminal airway is an important consideration in determining the signal differences between experiments. Selection of the downstream boundary condition is therefore an important consideration, while assumed not to have an influence on the pressure-flow relationship of the airways upstream of the exit.

Three downstream boundary conditions were evaluated, a long straight tube, a conical diffuser, and exhausting into a reservoir. Exhausting into a reservoir was chosen for two principle reasons. The first is that it introduces no additional unknown frictional dissipation sources to the system. The second is that in evaluating the kinetic energy flux exhausted into the reservoir, a better understanding of
the flow nature might be obtained.

This selection was made despite the introduction of slight errors, and that some advantages are present with the other options. Error are introduced with the assumption that the effect of the uniform pressure plenum does not propagate upstream into the "tracheal" branch, as discussed in the previous section, and that a pressure difference between the imposed hydrostatic head in the tank and the jet pressure may exist. This error is proportional to the kinetic energy flux and streamline curvature, and is therefore expected to be small because the curvature in the exiting stream is only significant at low Reynolds numbers, where the corresponding kinetic energy flux is small.

The long straight tube has the advantage that it establishes a Poiseuille flow profile, allowing a side pressure tap to measure the average cross-sectional pressure, and generating a known kinetic energy flux. A conical diffuser theoretically would allow recovery of the kinetic energy head, leaving systemic pressure losses as a result of friction alone and therefore maximizing the relative difference between experimental signals.

4.3 Subtraction Method Analysis

Analysis of the subtraction technique demonstrates that it is a viable method to measure the area-averaged static pressure difference at a given flow rate across a single bifurcation. It remains to separate that pressure difference into a
component attributable to friction and one attributable to the convective acceleration. To make this distinction, the energy based definition of pressure drop and its two components developed in 3.7 can be applied to the airway systems defined in Figure 4.1-1.

Consider first the airway system consisting of the two upstream airways and the test bifurcation. The total static pressure loss of the combined system, \((\Delta R)_c\), at a flow rate \(Q\), can be written as the sum of the loss in the test bifurcation and the loss upstream of the test bifurcation;

\[
4.3.1 \quad (\Delta R)_c = (\Delta R)_{I-E} + (\Delta R)_{R-I}
\]

Now consider the system which is comprised of just the two upstream airways. \((\Delta R)_{R-I}\), can be seen to be the same as the total pressure drop for just one of the upstream airways, \((\Delta R)_U\), at \(\frac{1}{2}\) the flow rate. Rewriting 4.3.1 to give the pressure loss in the test bifurcation, it can be seen to be given by the difference in the driving pressure required for a flow rate of \(Q\) for the combined airway system and the driving pressure required for a flow rate of \(Q/2\) for just one upstream airway.

\[
4.3.2 \quad (\Delta R) = (\Delta R)_{I-E} + (\Delta R)_{U} - (\Delta R)_C Q - (\Delta R)_U Q/2
\]

Recalling equation 3.7.12, the static pressure loss across the test bifurcation can be written as the sum of the viscous and kinetic energy losses plus a correction term for the non-
uniform cross-sectional pressure distribution;

4.3.3 \( (\Delta R)_{I-E} = \Delta P_v + \Delta P_k + \Delta P_c \)

which can be combined with 4.3.2 to write the viscous pressure loss in the test bifurcation as;

4.3.4

\[
\Delta P_v = \left[ \frac{(\Delta R)}{C} \right] - \left[ \frac{(\Delta R)}{Q} \right] - \Delta P_k - \Delta P_c
\]

The kinetic energy loss, \( \Delta P_k \), can be evaluated with 3.7.13, written for the case of symmetric flow conditions as;

4.3.5

\[
\Delta P_k = \frac{1}{2} \left[ \frac{P \cdot \text{Re} \cdot (fa+fs)}{E} \right]^* - \frac{1}{2} \left[ \frac{P \cdot \text{Re} \cdot (fa+fs)}{E} \right]^I
\]

where it is understood that the subscript I denotes evaluation of the terms at the inlet of the test bifurcation and E for the outlet, or exit. Applying the principle of continuity, and recalling fs and fa are functions of the local flow conditions, equation 4.3.5 can be written;

4.3.6

\[
\Delta P_k = \frac{1}{2} \left[ \frac{P \cdot \text{Re}}{E} \right]^* \left[ (fa+fs) - \frac{4}{E} (fa+fs) \right]^I
\]

where \( \epsilon \) is defined as the ratio of the parent to daughter tube diameters.

Equations 4.3.6 and 4.3.4 combine to give the viscous
pressure loss in the test bifurcation. Application of the expressions requires the difference in driving pressures from two different experiments, and a functional dependence of the \((fa+fs)\) sum and the pressure correction term on the exiting flow rate or Reynolds number. The correction factors will be explored in greater detail in Chapter 5.

4.4 Quasi-steady Test Apparatus

The quasi-steady experimental system developed to measure the total static pressure-flow characteristics of an airway system is shown in Figure 4.4-1. The method is considered quasi-steady because a slowly varying hydrostatic head is used to generate the flows to cover the desired range of Reynolds numbers.

The quasi-steady flow is driven by the hydrostatic head and resisted by fluid acceleration and friction within the test airway. The desired range of flow rates will be shown to depend on the characteristic pressure \(P^*\); a function of fluid viscosity, density and airway size. \(P^*\) determines the scale of the pressure loss at a given flow, and can be set to amplify the pressures into an easier range to measure with conventional pressure transducers.

The driving pressure head is generated in a circular constant diameter fluid filled reservoir. Continuity principles therefore dictate the flow rate through the airway will be proportional to the rate at which the reservoir level varies. For sufficiently low flow rates, pressure gradients
within the reservoir will only be due to hydrostatic forces, generating a pressure at the bottom of the reservoir which is proportional to the fluid level in the reservoir. The instantaneous flow rate in the system can therefore be determined by the time derivative of the measured hydrostatic head at the base of the reservoir.

Hydrostatic conditions also exist in both the plenum and exhaust tank for sufficiently low flow rates. The level in the exhaust tank is maintained by a weir. Flow over a weir is a well documented phenomena, (see for example Sabersky, pp 107), and is given by;

\[ Q = \frac{2}{3} \frac{h}{w} C_d (2g)^{1.5} \]

where \( Q \) is the flow rate, \( w \) is the width of the weir, \( g \) is gravity, \( C_d \) an experimentally determined discharge coefficient (typically ~0.62) and \( h \) is the fluid height above the weir level.

The driving pressure across the airway system at any time is given by the pressure difference between the reservoir plenum and the exhaust tank. For quasi-steady hydrostatic conditions, pressure taps with fluid filled lines leading to a differential transducer measures the driving pressure. The pressure-flow relationship for the airway system over a range of flow conditions can therefore be determined by recording two pressure signals over time.

The variation in downstream level with flow rate can be
made infinitesimally small relative to the change in the 
plenum level by increasing the weir width. In this limit, the 
experiment would be simplified because only one pressure 
signal would be required to monitor both the flow rate and 
driving pressure. Practical size limitations and the desired 
accuracy prohibit this experimental simplification of 
eliminating one pressure measurement for these experiments.

4.5 Experimental Scaling

Analysis of the quasi-steady experiments is necessary to 
establish the experimental scale for which the quasi-steady 
approximation can be supported, and determine the required 
fluid properties. The governing equations for an approximate 
one-dimmensional model analysis are;

**Continuity**

\[ \frac{dH}{dt} = \frac{dD}{dt} \]

where \( Q_r \) is the flow in the reservoir, \( A_r \) the reservoir area, 
\( H \) the reservoir height, \( Q_{\text{weir}} \) is the flow over the weir, \( A_e \) is 
the area of the exhaust tank, \( D \) is the level of the exhaust 
tank over the weir level, and \( Q_t \) is the flow rate in the 
"tracheal" tube.

**Weir flow**

\[ Q_{\text{weir}} = k \, D_w^{1.5} \]

where \( k \) is given from 4.4.1 as \( k = \frac{2}{3} \, C_d \sqrt{2gL} \)

**Motion along a streamline (Euler)**
where \( \delta \) is the partial differential operator, \( V \) is the velocity magnitude, \( Z \) is the height in the gravity field, \( g \), \( Fr \) is a frictional loss per unit mass, and \( s \) is the distance along the streamline. Integrating 4.5.3 along a hypothetical streamline from the free surface in the reservoir to the exit of the "trachea", gives;

\[
4.5.4 \int_{r}^{t} \frac{\delta V}{\delta t} ds + \frac{1}{\zeta} \left[ \frac{V_{2}^{2} - V_{1}^{2}}{2} + \frac{1}{\zeta} \left[ \frac{P_{t} - P_{atm}}{t} \right] - gH + \int_{r}^{t} Fr \, ds \right]
\]

where the subscript \( t \) refers to the tracheal values, and \( r \) refers to the values on the free surface of the reservoir.

For large exhaust tank areas, a hydrostatic head can be assumed to exist, and using a gage pressure referenced to the tracheal pressure if the exhaust tank were at the weir level, the pressure term can be written;

\[
4.5.5 \frac{1}{\zeta} \left[ \frac{P_{t} - P_{atm}}{t} \right] = gDw
\]

Defining a characteristic length, \( L^* \):

\[
4.5.6 L^* = \int \frac{At}{A} \, ds
\]

where \( A \) is the cross-sectional flow area, such that \( V = Q/A \), equation 4.5.4 can be written;
4.5.7 \[ L* \frac{dQ}{At \ dt} + \frac{Q^2}{A^2} \left[ 1 - \left(\frac{At}{Ar}\right)^2 \right] + gDw + \int_{t}^{t} \frac{Fr \ ds}{r} = gH \]

and introducing continuity to give the relation between tank height and flow:

4.5.8 \[ Q = -Ar \frac{dH}{dt} \quad \text{and} \quad \frac{dQ}{dt} = -Ar \frac{d^2H}{dt^2} \]

and writing the frictional loss in the form which it is typically available:

4.5.9 \[ \Delta P_{fr} = \int_{r}^{t} Fr \ ds = P^* \text{Fn(Re)} \]

where Fn(Re) represents some function of the tracheal Reynolds number in the airway.

Together these expressions give 4.5.7 in a form which demonstrates the balance of the temporal, convective and frictional terms to the driving head:

4.5.10 \[ -\zeta L \frac{dH^2}{At \ dt^2} + \frac{\zeta}{At} \left[ \frac{Ar}{At} \right]^2 \left[ \frac{dH}{dt} \right]^2 \left[ 1 - \left(\frac{At}{Ar}\right)^2 \right] + P \text{Fn(Re)} = \zeta g(H-Dw) \]

which is a one dimensional non-linear differential equation, with a coupling term Dw, given by the weir equation and continuity. Assuming that the exiting kinetic energy is dissipated before flowing over the weir, the weir height is
Equations 4.5.10 and 4.5.11 can be solved numerically given an approximate frictional relation. For the purposes of ensuring that quasi-steady conditions exist, only an order of magnitude analysis of the equations is needed, however. The analysis needs to demonstrate that the pressure drops associated with the temporal terms are small in comparison to both the spatial convective acceleration terms and the frictional terms.

From the results of Hardin and Yu (1980), the frictional loss experienced in a 3 generation airway model, based on the "idealized bifurcation" in Figure 1.3-1, can be used as an order of magnitude estimate. The relationship is;

\[
4.5.12 \quad \frac{\Delta P}{P_{fr}} = \frac{1.6}{25.3 + 1.24 \times Re^2}
\]

Giving a low limit functional dependence \( F_n \) as approximately;

\[
4.5.13 \quad F_n \approx Re^{1.6} \text{ or } 10 \times Re^{1.6}
\]

where the tracheal Reynolds number is given by;

\[
4.5.14 \quad Re = \frac{4 \times Ar}{\pi \times D \times Nu} \left[ \frac{-dH}{dt} \right]
\]

The order of magnitude analysis of 4.5.11 indicates that
the exhaust tank level $D_w$, can be considered small as compared to the reservoir head, $H$, as long as the tank width is large compared to the tracheal diameter. To estimate the order of magnitude of the temporal terms, 4.5.10 can be written;

$$4.5.15 \quad -\frac{\zeta L}{At \, dt^2} = -\frac{\zeta L}{4 \, D \, dt} \sim \frac{\zeta L}{D \, T^*}$$

where $T^*$ is a characteristic time, and can be estimated as the time it takes for the tank level to drop a distance which flushes the airway system;

$$4.5.16 \quad T^* \sim L \left[ \frac{dH}{dt} \right]^{-1} \left[ \frac{Nu \, At}{D \, Ar} \right]^{-1} \left[ \frac{At}{Nu \, Re} \right] \frac{D}{L}$$

giving the inertial loss in 4.5.15 as;

$$4.5.17 \quad \frac{\zeta L}{At \, dt^2} \sim P \left[ \frac{At}{Ar} \right]^2 \frac{Re}{P}$$

Substituting 4.5.17, 4.5.14, and 4.5.13 gives the order of magnitude of each term in 4.5.10;

$$4.5.18 \quad P \left[ \frac{At}{Ar} \right]^2 \frac{Re}{P} \left[ 1 - \frac{[At/Ar]}{Re} \right]^2 \frac{2}{10} \, P \, Re \, P$$

Temporal Acceleration
Convective Acceleration
Frictional Losses
Driving Head

Comparison of terms shows that the temporal acceleration terms will be small in comparison to convective accelerations if;
4.5.19 \frac{At}{Ar} \ll 1

and comparison of the frictional losses to the temporal losses indicates that the quasi-steady approximation requires the potentially more restrictive condition that;

4.5.20 \frac{0.4}{Re} \left[ \frac{At}{Ar} \right] \ll 10

For the maximum Reynolds number under investigation, Re=10,000, this condition reduces to the same of 4.5.19, indicating that the quasi-steady approximation is valid for a test apparatus with a sufficiently large reservoir tank.

The scale of the experiment and the range of fluid properties required to generate measurable pressure drops and remain quasi-steady can therefore be estimated from equation 4.5.10 by neglecting the temporal terms. The second order differential equation therefore reduces to the approximate algebraic relation;

4.5.21 \frac{H - Dw}{H^*} = \frac{2}{\Re} + 20 \Re$

where $H^*$ is defined as;

4.5.22 \frac{H^*}{\frac{P^*}{\frac{\zeta g}{g}}} = \frac{\text{Nu}^2}{g}

For the maximum Reynolds number of 10,000, the convective and frictional losses in 4.5.18 are on the same order, and
4.5.21 gives the required driving head to be;

4.5.23 \( H \approx 10^8 H^* \)

At the lower range of Reynolds numbers, near 100, the reservoir height can similarly be approximated by;

4.5.24 \( H \approx 10^4 H^* \)

Imposing practical limits on the experiments, such as a reservoir height of approximately 100 cm, and minimum measured pressures of about 1% full scale, or 1 cm, and tracheal diameters about 1 in, (2.54 cm) the fluid properties required are for a fluid with "substantially" higher density than air, because buoyancy forces have been neglected, and kinematic viscosities in the range;

4.5.25 \( .05 \leq \text{Nu} \leq .5 \)

Other practical considerations such as cost, availability and safety limit the possible choices. The selection of water-glycerin mixtures stand-out as an excellent choice. Figure 4.5-1 presents the kinematic viscosity of water-glycerin as a function of water content. The desired kinematic viscosity range falls between 20% and 70% water content by weight.

4.6 Data Collection

A Digital Equipment corp. MINC-23 digital lab computer was
used to digitize and record the pressure signals. The MINC is equipped with a -5 to +5 volt 16 bit A-D converter, with 4 reserved bits for configuration information, giving a digitization level of 2.44 mvolts. The differential pressure was measured with a Valydyne differential pressure transducer (model DP 103-10) driven with a Valydyne carrier-demodulator (model CD 15). The hydrostatic reservoir pressure head was measured with a Honeywell Microswitch pressure transducer, (model 143PC), and passed through an analog signal conditioner. The analog signal conditioner is based on two 751 op-amps, and provides a signal offset and amplification. Both the signal conditioner and the carrier demodulator allowed the output of each transducer to cover the full -5 to +5 volt scale of the A-D converter to minimize digitization error.

Before an experimental run, the kinematic viscosity of the water-glycerin mixture is measured with a Cannon-Fenske (Tube #200) drip type viscometer. The simple drip viscometer was calibrated with a Haake (Model RV12) viscometer at a shear rate of 346 1/sec to insure accurate results. A typical mixture was used to generate a temperature viscosity correlation, shown in Figure 4.6-1. For the range of fluid temperatures experienced, the viscosity measured with the Cannon-Fenske viscometer (Tube #200), partially submerged in the exhaust tank for temperature equilibration, and the viscosity interpolated based on the exhaust tank temperature were in agreement to within 1%.

An experiment is begun by filling the reservoir to the
level required to generate the maximum desired Reynolds number, see 4.5.21. The transducer outputs are checked and scaled if necessary, and the flow gate is removed. The time for the tank to empty is recorded, and the low transducer output is checked, and also scaled to utilize the full -5 to +5 voltage range. The reservoir is then refilled, stopping at about 10 different tank levels to record and re-calibrate the pressure transducers. The re-calibration is necessary not only because the offsets and amplification may have been changed, but also to maintain the desired accuracy in tank and differential head levels as a result of potential changes in transducer amplification with ambient temperature changes.

Experience demonstrated that slight non-linearities, on the order of 1 to 2%, in the pressure signals introduced errors up to about 5% in the results. The errors appeared primarily as slight oscillations when pressure differences were normalized by the square of the calculated flow rate. The digital signal processing easily allowed the non-linearities to be corrected by fitting the transducer calibration curve to higher order curves. A second order curve provided enough correction for the small non-linearities in the differential height reading, while a third order curve was necessary for the reservoir level transducer. Figure 4.6-2 presents the typical differences between the tank levels measured with a manometer, and those predicted with the non-linear transducer calibrations. The errors are less than the digitation errors represented by ±2mvolts≈±0.03cm, and errors in reading the
manometer, ±0.05cm.

Once the tank is full after finishing the transducer calibrations, the experiment is ready to begin. The data acquisition is started as the flow gate was removed. The data acquisition rate was fixed to give 500 sets of voltage readings over the time required for the tank to empty. The data sets, along with the calibration values and experimental conditions were then stored on floppy disks for later transmission and analysis.

4.7 Data Reduction

The data sets obtained in each experiment were reduced to obtain the non-dimensional pressure-flow relationship of the test airway. The digital scheme implemented with a fortran code first converts the data from a set of voltages to reservoir level vs time and non-dimensional differential pressure vs time by multiplying by the appropriate combination of the stored values of the transducer calibrations, kinematic viscosity, tracheal diameter, and data acquisition rate.

The data near the start of the experiment are discarded until the flow becomes quasi-steady. The time scale for quasi-steady state to be established is also given by $T^*$ defined in 4.5.16, which can also be interpreted as the time necessary for the fluid initially in the network to be convected out.

The curve fitting errors outlined above are approximately constant throughout the signal range. Percentage errors are
therefore greater at the lower signal levels. To minimize these errors, the experiments are allowed to run until the tank level reaches zero. Any non-zero asymptotic reading in the differential pressure signal is assumed to be an offset, and is subtracted from the results to improve the low pressure reading. Offsets greater than one or two digitation levels are taken as an indication that the calibration is no longer valid, and both the experiment and calibration are repeated. Data below some critical tank level, typically 1 cm is used, or 1% full scale, are also discarded due to the high percentage errors expected.

The height vs time data is converted to a Reynolds number vs time form by evaluating Equation 4.5.14 at each point. Calculation of the time derivative of the reservoir height data proved to be the most sensitive step in scheme. Digitation errors and experimental noise resulted in significant errors in calculating the derivative with simple finite difference schemes. To minimize these errors, a curve fitting technique was developed to calculate the slope at a point. A second order curve was fit to a specified number of points before and after each data point. The curve fit was then analytically differentiated and then evaluated at the time corresponding to the data point. Typically 10 points on either side of the time being evaluated were used, corresponding to 21 points out of the 500 total points involved in each curve fit.

The numbers of points used in the averaging represents a
compromise between competing factors. Point to point noise increases as the number of points in the averaging decreases, but inaccuracies in the curve fit appear when averaged over too wide a range. Additionally, as the number of points included in the curve fit are increased, data at the ends of the curves is lost.

The dimensionless pressure vs time, and the Reynolds number vs time data are then assembled parametrically to generate the dimensionless pressure-flow relationship for the airway. The data are stored in this form for later use in the implementation of the subtraction technique.

4.8 Technique Verification

Verification of both the quasi-steady experimental approach and the subtraction technique was deemed necessary to validate the subtraction technique. For this purpose, flow from a reservoir through a bellmouth entry into a straight tube was selected because it is both well understood (Rivas and Shapiro 1958), and can be implemented on the same scale as the airway system.

Two sets of experiments with different water-glycerin mixtures, and therefore Reynolds number ranges, were conducted in the geometry depicted in Figure 4.8-1. The same bellmouth entrance was placed on the inlet of two different 1 inch diameter tubes, one with a length of 6.3 inches, and the other 9.8 inches.

Figure 4.8-2 presents the plot of reservoir height verses time for the short entrance at two different mixtures with
kinematic viscosities of 0.125 and 0.475 cm$^2$/s. Note that in both cases sufficient time is allowed for the zero level asymptotic limit to be achieved.

The asymptotic region of the differential pressure signal is amplified in Figure 4.8-3. The amplification shows that a small offset exists. The offset is about equal to the digitation level, so can be subtracted from the differential signal to improve the accuracy of the low flow results.

The noise in the signals is difficult to see, but can be noted to be small in comparison to the signal. The noise over the bulk of the signal can be more clearly seen by taking the difference between the actual signal and the mean signal. Applying the curve fitting technique at each point as previously described, but in addition to evaluating the derivative, the mean height value can be approximated by the curve fit value. The difference between the curve fit values and the actual values of the height data is shown in Figure 4.8-4. The noise levels are seen to be primarily a result of digitation errors, ±0.03cm, which remain approximately constant over the range of reservoir levels.

Figure 4.8-5 presents the calculated results for Reynolds number vs time for two experiments with the short entrance. Parametrically combining the dimensionless pressure data and Reynolds number data gives the dimensionless pressure-flow relationship. The combined experimental relationships are shown in Figure 4.8-6 for both experiments. For comparison, the theoretical prediction for the geometry is also shown. The
close agreement, to within 1-2%, demonstrates the validity of the quasi-steady technique. The validity of the non-dimensionilization is demonstrated by the overlapping region of results which correspond to substantially different flow rates.

To test the validity of the subtraction technique, the tests were repeated for the longer entrance, and the additional loss in the longer tube due to the additional 3.5 inches calculated. The combined results for the longer tube at the two viscosities is also shown in Figure 4.8-6. Once again, the results fall within 1-2% of theory, and the overlapping region confirms the non-dimensionilization.

The tubes are both the same diameter, allowing the additional pressure drop due to the additional length to be calculated from the simple difference in the pressure flow relationship for the two geometries. The results are summarized in Figure 4.8-6, showing the losses in each experiment and the difference which represents the loss in the additional length of tubing. The loss in the additional length of tubing alone is shown in Figure 4.8-7, along with a published empirical correlation, normalized to the characteristic pressure, $P^*$. The error between the results and theory are seen to be within 10%.

Important characteristics of the results are:

- Experimental results fall within about 10% of the theoretical prediction over the Reynolds range from 100 to 10,000

- The accuracy of the results is achieved despite a difference in signals ranging from about 10 to 15%. This can therefore be seen as a severe test of the subtraction technique because signal differences
expected in the airways are larger.

* The Reynolds number and $P*$ properly scale the results.

The small error between theory and experiment confirms the analysis which indicated that the temporal accelerations are a negligible portion of the total loss. Further confirmation can be found by calculating the temporal terms based on the actual experimental results. The pressure drop was calculated to be no more than .03 cm H$_2$O, which places its magnitude below the system resolution level.

### 4.9 Airway Models

The dimensions of the symmetric airway model built to typify symmetric airways of the lung are given in Table 4.9-1. The model consists of symmetric bifurcations modeled after the "typical" bifurcation in Figure 1.3-1. The bifurcations are coupled with rotary junctions which allow generation to generation orientations to vary, as well as allowing the addition or removal of generations to the airway network.

<table>
<thead>
<tr>
<th>Generation Number</th>
<th>Diameter cm</th>
<th>Length cm</th>
<th>Number of Branches</th>
<th>Flow Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.44</td>
<td>4.31</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>1.85</td>
<td>6.47</td>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>1.39</td>
<td>4.85</td>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>1.08</td>
<td>3.78</td>
<td>8</td>
<td>12.5</td>
</tr>
<tr>
<td>4*</td>
<td>1.00</td>
<td>2.00</td>
<td>16</td>
<td>6.25</td>
</tr>
</tbody>
</table>

* Note Generation 4 was not used in all experiments
Construction of the bifurcations which make-up the airways was achieved with casting techniques and a planar, two piece, three generation airway model. The two piece airway model was designed and built by Drs J. Hammersley and D.E. Olson (Univ of Mich.). Each half of the airway was cut in clear plexiglas using a numerically controlled milling machine, with a series of ball-end end mills. The circular cross-section of the branches, which are smoothly blended at the bifurcation junctions, are formed by the two halves and the branching geometry is formed by the tool path.

The geometric scaling for the airway is that given by Pedley (1977), with two exceptions. The availability of only nominal size end mills restricts the values of the tube diameters. The resulting daughter-parent tube diameter ratios vary slightly from one generation to another and from the value of 0.78 given by Pedley. The diameter ratio's are 0.75 for the first two generations, and 0.778 for the third.

The second difference is that the radius of the tube curvature is fixed, and does not taper from the bifurcation to the point at which the branching angle is met. A constant radius of curvature is used instead, representing a typical value of the curvature radius to parent tube radius of 7. This difference is significant and advantageous from an experimental standpoint because it generates a constant diameter straight section of tubing, L/D ≈ 1, between each bifurcation. The straight section isolates the airways, as described in the subtraction technique discussion, and allows
the circular collars forming the rotary junctions to be cut on a lathe.

Construction of the individual bifurcations began by making a wax cast of the planar airway formed by the void left by the two halves of the plexiglas model. The wax had a 3\% shrink ratio when cooled (Aqua-wax # 261-8105, Gesswiein Corp, Bridgeport, CT), allowing the airway to easily be removed when the plexiglas model is separated. A wax bifurcation is cut out of the airway at the mid-section between bifurcation junctions. Aluminum collars are placed on the straight section, L/D \approx \frac{1}{2}, at the ends of the daughter and parent tubes. The collars and wax bifurcation are then placed in a specially built two piece mold, and a casting compound (Emmerson and Cumming, Stycast #2850 FT with catalyst #11) is poured into the mold. The mold is parted, the casting is removed and placed in boiling water to remove the wax.

Figure 4.9-1 shows the cross-sectional view of the final bifurcation. Smooth transitions between bifurcations are assured by the concentric collars which are indexed to the tube center-line by the \frac{1}{2} diameter straight sections at the ends of the parent and daughter branches. The concentricity also allows the bifurcations to be rotated while maintaining the smooth transition.
5. Estimation of Correction Factors

5.1 Axial Velocity Profile Factor

Factors were introduced into the steady one-dimensional energy equation in Chapter 3 to include the effects of the three-dimensionality in the flow. The axial velocity profile factor, $f_a$, is defined in equation 3.7.10, and can be written in the equivalent form;

$$ f_a = \frac{1}{U^2Q} \int u \, dA = \int \left[ \frac{u}{U} \right]^3 \frac{dA}{A} $$

Inspection of 5.1.1 shows that $f_a$ represents the ratio of actual kinetic energy flux to the flux that would occur at the same flow rate with a blunt velocity profile. The value of $f_a$ can therefore be considered an indication of the "bluntness" the velocity profile, decreasing to 1 as the profile becomes flat.

Evaluation of $f_a$, and its Reynolds number dependence, requires detailed measurements of the velocity profile. The laborious nature of velocity mapping techniques and the need to evaluate $f_a$ at every flow rate tested places limits on the practical number of experiments for which $f_a$ could be evaluated. Alternately, the dependence of $f_a$ on more easily measured bulk experimental quantities can be investigated, and correlations and/or theoretical predictions developed to calculate $f_a$. The resulting measurements or estimates of $f_a$
based on bulk properties can be used, if the errors introduced are within an acceptable range.

Two methods have been investigated to estimate the value of $fa$. The first method utilized 5.1.1 to develop an approximate relation between $fa$ and the exiting momentum flux, which is a measurable experimental quantity. The relation between $fa$ and the momentum flux can be seen by writing the axial velocity, $u$, as the sum of the average velocity, $U$, and a perturbation, $\hat{u}$. Substitution into 5.1.1 gives:

$$5.1.2 \quad fa = 3\beta a - 2 + \int \left[ \frac{\hat{u}}{U} \right]^3 \frac{dA}{A}$$

where $\beta a$ is the normalized axial momentum flux, defined as the ratio of the actual momentum flux to the flux that would occur at the same flow rate with a blunt profile. The normalized flux, $\beta a$, is given by:

$$5.1.3 \quad \beta a = \int \left[ \frac{u}{U} \right]^2 \frac{dA}{A} = 1 + \int \left[ \frac{\hat{u}}{U} \right]^2 \frac{dA}{A}$$

The exiting momentum flux, $M$, can therefore be written in terms of $\beta a$ as:

$$5.1.4 \quad M = \beta a \left[ \frac{\xi U^2}{2} \right]$$

Application of the momentum equation to the control volume shown in Figure 5.1-1, relates the momentum flux to a experimentally measurable quantity, a restraining force. The momentum flux, $M$, is balanced by the force created by the
 exiting fluid as it impinges on a stationary plate which turns
the flow 90°. The momentum correction factor, βa, can be
determined along with the flow rate and driving pressure by
measuring the restraining force in addition to the two
pressures already identified.

Relating βa to fa requires that the integral quantity in
5.1.2 be known. Inspection of the integral term indicates that
it should be small in comparison to the other terms in 5.1.2,
which are on the order ~1. The velocity perturbation, G, is
small in comparison to U, and the average value of G when
integrated over the cross-section is defined to be zero. The
cube of G/U integrated over the area can therefore be expected
to be small in comparison to 1 because the values are small,
and the values are both positive and negative over the cross-
section. The axial velocity correction factor can therefore be
approximated in terms of the momentum flux and Reynolds
number, two experimentally determined quantities as;

\[
5.1.5 \quad fa = 3βa - 2 = \frac{M}{P*Re^2} - 2
\]

The second method developed to determine fa based on bulk
properties utilizes published velocity profiles measured in
expiratory flows. The results for the velocity profiles
allows direct integration of 5.1.1 numerically, and possible
correlations of fa with experimental conditions to be
developed. Only a limited amount of experimental data on the
velocity profiles in expiratory flows is available to evaluate
the integrals. The published velocity profiles available, see Figures 5.1-2a to g, were scaled, and the integrals evaluated with finite difference approximations. The errors introduced by the scaling process and the integration approximation are difficult to quantify, but are estimated to be on the order of 10%.

The results for a range of expiratory flow conditions were found to correlate well with the tracheal Reynolds number. Additionally, the results provided confirmation of the approximation in arriving at 5.1.5. The resulting values of $\beta_a$ and $\alpha_a$, and the range of experimental conditions and Reynolds numbers at which the values are evaluated are summarized in Table 5.1-1.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Configuration</th>
<th>Tracheal Reynolds #</th>
<th>$\beta_a$</th>
<th>$\alpha_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 1986</td>
<td>Quasi-steady, single bifurcation</td>
<td>200</td>
<td>1.15</td>
<td>1.47</td>
</tr>
<tr>
<td>Chang and Menon 1985</td>
<td>Asymmetric human upper airway cast</td>
<td>1060</td>
<td>1.11</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5712</td>
<td>1.07</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6000</td>
<td>1.03</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6811</td>
<td>1.02</td>
<td>1.08</td>
</tr>
<tr>
<td>Hardin and Yu 1980</td>
<td>Symmetric idealized pulmonary model</td>
<td>2760</td>
<td>1.06</td>
<td>1.17</td>
</tr>
<tr>
<td>Reynolds 1982</td>
<td>Asymmetric human airway cast</td>
<td>2260</td>
<td>1.09</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8000</td>
<td>1.04</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12200</td>
<td>1.04</td>
<td>1.13</td>
</tr>
</tbody>
</table>
The cross-plot of $\beta_a$ and $f_a$, along with the predicted relation is shown in Figure 5.1-3. The small deviations of the experimentally determined relation between $f_a$ and $\beta_a$ from the prediction are within the approximate error band of $\pm 10\%$. Included in the Figure are also theoretical predictions of the relation between $f_a$, and $\beta_a$ for the self-similar velocity profiles established in laminar and fully developed turbulent flow in a long straight tube. These comparisons can be considered to cover the profile extremes expected in expiratory flows and further validates the assumptions used to generate 5.1.5.

Figure 5.1-4 presents the cross-plot of $f_a$ and tracheal Reynolds number. The plot shows $f_a$ decreasing with increasing Reynolds number, confirming the observations that velocity profiles become more blunt as the Reynolds number increases. A least squares curve fit gives the approximate relation;

\[
5.1.6 \quad f_a = 1.05 + \frac{6.54}{\sqrt{Re}}
\]

The behavior of $f_a$ can be explained in terms of the observed expiratory flow characteristics. The axial velocity profile is typified by a blunt core and thin boundary layer, similar to both entrance type flows and high Dean number curved tube flows. In both cases, the boundary layer is seen to thin as the flow increases for a fixed geometry.

Estimation of $f_a$ values and a confirmation of the boundary
layer analogy can be generated by considering the effect of a thinning boundary layer of exiting kinetic energy flux. The flow profile can be approximated as shown in Figure 5.1-5, a blunt core with velocity $U_0$, and a linearly decreasing velocity in the boundary layer. Integration of the velocity profile for a given value of the normalized boundary layer thickness, $\delta' = \delta/a$, gives the average flow velocity and exiting kinetic energy flux to be:

$$U = U_0 \left( 1 - \delta' + \frac{2}{3} \delta' \right)$$  

$$\int u \, dA = U_0 \left[ 1 - \frac{3}{2} \delta' + \frac{3}{5} \delta'^2 \right]$$

which can be combined to give $f_A$ as:

$$f_A = \frac{\left[ 1 - 1.5 \delta' + 0.6 \delta' \right]^2}{\left[ 1 - \delta' + 0.333 \delta'^2 \right]^3}$$

For entrance flows, the boundary layer thickness can be approximated by the Blasius flat plate boundary layer growth (Equation 3.4.1) with free stream velocity $U_0$, as long as $\delta' \ll 1$. Figure 5.1-6 shows the prediction of $f_A$ for the Blasius relation at tube lengths of 1.75, and 3.5 diameters. The relation is seen to be relatively insensitive to length over the narrow range, and provides an excellent prediction of $f_A$ values found in expiratory flows.

For high Dean number curved tube flows, the boundary layer was
shown in 3.5 to be:

\[ 5.1.10 \quad \delta' = C \left( \frac{\text{Re} \, \delta r}{x} \right)^{-\frac{1}{2}} \]

where \( C \) is an unknown constant near 1, and the Reynolds number is based on the average tube velocity. To evaluate \( C \), the high Dean values of \( f_a \) calculated by Olson (1971) can be used to compare the results of 5.1.9 with the boundary layer thickness given by 5.1.10. Olson's experimental values of \( f_a \) are well fit for Dean numbers greater than 200 if \( C \) is taken to be approximately equal to 2. Applying 5.1.10 to the typical lung geometry with \( \delta r = 1/7 \), values of \( f_a \) based on the curved tube boundary layers are also shown in Figure 5.1-6 to predict the form and magnitude of the expiratory values of \( f_a \).

In a similar fashion, the results for \( f_a \) based on the velocity profiles calculated for the 2-D convergent channel are also shown in Figure 5.1-6. The ability of the curved tube, entrance and 2-D convergent channel profiles to so closely predict the results for the expiratory data strongly supports the observation that expiratory flow can be characterized by a blunt profile and thin boundary layer. The consistent predictions for \( f_a \) for a wide range of geometries also supports the use of the correlation in 5.1.6 to predict \( f_a \) for the idealized bifurcation. Errors in using this form for \( f_a \) are therefore not likely to be large, and can be estimated from the variation in the results to be within about \( \pm 5\% \).

The error in determining \( f_a \) experimentally from
measurements of exiting momentum flux can be estimated to be significantly higher. Achieving ±5% error overall and allowing only ±2% error in the Reynolds number measurement, requires that the momentum flux be measurable to within accuracies of about ±1%.

This accuracy is highly unlikely due to the difficulties associated with measuring forces, and the noises introduced by turbulence and other fluid dynamic effects in the exhaust tank. High accuracies require that the diverter plate be large to insure that no axial momentum "leak" around the edges. Small pressure variations within the exhaust tank as a result of the fluid motions therefore can exert large forces when working over the large surface of the diverter plate. In view of these uncertainties in the direct measurement of \( \beta a \), the correlation with Reynolds number, 5.1.6, will be used to reduce the experimental results.

5.2 Secondary Velocity Correction factor

The secondary velocity correction factor, \( f_s \), represents the kinetic energy flux contained in the secondary velocities of the exiting flows. The magnitude of \( f_s \) is the ratio of kinetic energy flux contained in the secondary velocities, normalized to the axial kinetic energy flux that would occur with the same flow rate and a blunt axial velocity profile.

The value of \( f_s \) is defined in equation 3.9.7 and can be written as;
5.2.1 \[ f_s = \int \left[ \frac{V_{sec}}{U} \right]^2 \frac{u}{U} dA \]

where \( V_{sec} \) is the magnitude of the secondary velocity.

Evaluation of \( f_s \) is more difficult than for \( f_a \) because of the lack of experimental data on secondary flow profiles in expiratory flow. The secondary velocity profiles have been visually studied, but are difficult experiments to quantify because the secondary velocities are typically a fraction of the local velocity magnitude. A number of studies have been conducted on inspiratory profiles, but only two studies have been published which present measurements on secondary flows in expiration.

Chang and Menon (1985) measured the secondary velocities in a model of the human upper airways, and Jan (1986) measured the secondary velocity profiles in a single model bifurcation in oscillatory flow. The secondary velocity profiles are shown in Figure 5.2-1a to c. These published results can be integrated numerically similarly to the method described for \( f_a \). An alternate technique is to approximate the integral in 5.2.1 as:

5.2.2 \[ f_s = \int \left[ \frac{V_{sec}}{U} \right]^2 \frac{u}{U} dA \approx \left( \frac{V_{sec}}{U} \right)^2 \left( \frac{U}{A} \right)^2 \]

The results for the numerical integrals are in good agreement with the estimate based on the average value of the normalized secondary velocity magnitude. The results for the integrals and estimates are summarized in Table 5.2-1.
Table 5.2-1

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Tracheal Reynolds #</th>
<th>Integrated fs</th>
<th>Vsec/U (ave)</th>
<th>Estimated fs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chang &amp; Menon</td>
<td>1060</td>
<td>.04</td>
<td>0.215</td>
<td>.046</td>
</tr>
<tr>
<td>(Station 3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chang &amp; Menon</td>
<td>1060</td>
<td>.04</td>
<td>0.214</td>
<td>.046</td>
</tr>
<tr>
<td>(Station 2s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan</td>
<td>166</td>
<td>.028</td>
<td>0.18</td>
<td>.032</td>
</tr>
</tbody>
</table>

The magnitude of the secondary velocities are most likely established by the curved tube type phenomena. The secondary velocity scaling for curve tube flow was shown to be linear with $U$ for high Dean numbers. This scaling explains the constant $V_{sec}/U$, as seen in Table 5.2-1, and allow $fs$ to be estimated as a constant for high flow rates.

5.2.3 $fs \approx 0.04$

This approximation is not valid at lower flow rates, where the scaling changes, and the secondary velocities are reduced relative to $U$. But, as seen in table 5.2-1, the scaling is approximately valid down to a Reynolds number of about 200 for a typical bifurcation. At these lower reynolds numbers, the kinetic energy flux becomes small in comparison to other terms in the energy equation, which minimizes any error in approximating $fs$ as a constant.
5.3 Pressure Correction

The pressure correction term introduced in equation 3.7.7 accounts for the effect of non-uniform pressure and axial velocity profiles in using a simple area averaged pressure in the energy equation. The correction becomes necessary in the subtraction experiments because average pressures and not velocity weighted pressures are specified with the hydrostatic heads.

Evaluation of the pressure correction term is difficult due to the complete lack of experimental measurements of static pressure distributions in expiratory flows. Estimates of the correction term of sufficient accuracy can be made, however, as the correction terms are likely to be small and errors in the estimate are therefore only second order system errors.

To estimate the order of $P_c$, the pressure perturbations can be considered to be a result of the curvature in the flow streamlines. The Euler equation allows the pressure gradients normal to the streamlines to be written;

\[
\frac{dP}{d\eta} = \frac{\zeta V^2}{R_s}
\]

where $V$ is the velocity magnitude, $R_s$ is the radius of curvature in the streamline, and $\eta$ is in the direction perpendicular to the streamline. The magnitude of the pressure variations across the tube can approximated by integrating the pressure gradient and using an average radius of curvature,
giving;

5.3.2 \[ p' \approx C \frac{a \zeta}{R_s} \]

where \( C \) is an unknown constant, near 1, and \( a \) is the tube radius.

The radius of curvature for a streamline can be written in the vector equation form (Hildebrant);

5.3.3 \[ R_s = \left( \frac{V}{A \times V} \right)^3 \]

where the acceleration vector \( A \) is a result of the velocity vector \( V \) changing with position in a steady flow. Evaluating 5.3.3 in the torroidal coordinate system, allows the curvature radius to be written in terms of the tube radius, and the velocity components;

5.3.4 \[ R_s = r \left[ 1 + (v/w)^2 \right]^{1/2} \left[ \frac{u^2 + v^2 + w^2}{v^2 + w^2} \right] \]

For expiratory flows, the radial and tangential secondary velocity components, \( v \) & \( w \), are on the same scale, and the square of the axial velocity is typically large compared to the square of the secondary velocities \( (v^2 + w^2) \), which allows an average radius of curvature to be approximated as;

5.3.5 \[ R_s \approx \sqrt{2} a \left[ \frac{V}{V_{sec}} \right]^2 \]

combining with 5.3.2 gives;

5.3.6 \[ p' \approx \sqrt{2} C \zeta V_{sec} = \frac{1}{2} \frac{u^2}{C} \sqrt{2} C \left( \frac{V_{sec}/U}{U} \right)^2 \]

Experimental support for this scaling can be demonstrated
by measurements of the pressure variations along the tube wall. While this is not equivalent to establishing the true pressure variations, it should reflect the magnitude. Figure 5.3-1 presents the pressure difference measured in two pressure taps at the top and side of the daughter branch of a bifurcation with planar symmetric upstream airways. The pressure difference wall at these wall positions, separated by $90^\circ$ was measured instead of the hydrostatic pressure difference, otherwise the experiment was conducted as described in Chapter 3. These two points were chosen, as they should reflect the maximum pressure variation along the wall given the expiratory flow profiles previously presented. The experimental results are fit reasonable well by the curve;

5.3.7 \[ \text{Del P} \approx 0.06 \rho_{\text{Re}} = 0.12 \frac{\rho}{\text{cU}} \]

confirming the $\rho_{\text{Re}}$ and Reynolds number scaling and allows the constant $C$ in 5.3.6 to be approximately evaluated as;

5.3.8 \[ C = \frac{2}{\sqrt{2}} \cdot 0.12 \cdot \sqrt{\frac{\text{Vsec}}{\text{U}}} = 3.67 \]

where an additional factor of 2 is introduced to reflect that the pressure perturbation along the wall will be about half of the maximum. The results confirm that the constant $C$ is of order 1.

To determine the pressure correction term, a more sophisticated analysis is needed because each of the factors in the expression giving Del Pc can be both positive and negative. So that while the scale of the term can be found relatively easily, the actual value may be much lower, or possibly even
negative.

To evaluate the pressure term, the vortex cells can be considered to have a low pressure in the center of curvature, \( P_L \), increasing outward toward the wall and the center of the tube. The average pressure in the cell can therefore be written as;

\[
P = P + \frac{1}{2} \zeta U^2 \left( \int_0^r \left[ \frac{\text{Vsec}}{U} \right]^2 \frac{1}{\varepsilon} \, d\varepsilon \right) \frac{dA}{A}
\]

where \( \varepsilon \) is a dummy variable of integration, \( r \) is the radial distance from the center of the vortex cell, and \( C \) is the correction constant evaluated previously. The pressure perturbation can therefore be written as;

\[
p' = p - P = \frac{1}{2} \zeta U^2 \left( \int_0^r \left[ \frac{\text{Vsec}}{U} \right]^2 \frac{1}{\varepsilon} \, d\varepsilon \right) \frac{dA}{A}
\]

which allows \( P_c \) to be written as;

\[
P_c = \frac{\frac{1}{2} \zeta U^2}{\int_0^r \left[ \frac{\text{Vsec}}{U} \right]^2 \frac{1}{\varepsilon} \, d\varepsilon}
\]

Numerical evaluation of the right hand side of equation 5.3.11 was used to give the approximate pressure correction term. The integrals were difficult to perform because it required estimating the center of the vortex cell. The available data is sparse to evaluate the integrals, and is the same as the data to evaluate secondary velocity correction factor. The results give a value of \( .06 \), which as expected is
small, and on the order of the secondary velocity correction factor.

5.4 Correction factor Implementation

The frictional pressure drop in the test bifurcation was written in equation 4.3.4 as the difference between two experimental values, minus the sum of two correction factors. The correction factors can be combined into one expression by writing the pressure correction term as $f_s=0.04$, $P_c/\gamma/cU^2=.06$, to give;

\[
\text{Del } P_k + \text{Del } P_c = \frac{1}{2} \left( P \text{ Re} \right)^2 \left[ (fa+0.1) - \frac{4}{E} (fa+0.1) \right]
\]

With the aid of 5.2.3 and 5.1.6, the sum of the correction factors can be combined into one correction factor, $f$, which can be written in terms of the local reynolds number;

\[
f = fa + 0.1 = 1.15 + \frac{6.54}{\sqrt{Re}}
\]

this allows the correction factors to be implemented with a reasonable degree of accuracy, an estimated ±5%, in terms of only one measurement, the tracheal Reynolds numbers. The form of 5.4.2 as well as the constant were found to be relatively insensitive to the geometrical details, and can be expected to apply with about the same level of accuracy to real pulmonary flows, as well as those in idealized bifurcations.
6. Symmetric Airway Results

6.1 Planar System Results

The planar system results covered the Reynolds number range from about 50 to 10,000 with two sets of the quasi-steady experiments. The fluid mixture used in the first set of experiments had a high glycerin content with a kinematic viscosity of about 0.45 cm²/sec, resulting in tracheal Reynolds numbers from about 50 to 2000. The fluid mixture used in the second set of experiments had a lower glycerin content, yielding a kinematic viscosity of about 0.125 cm²/sec, and tracheal Reynolds numbers from about 300 to 10,000. The normalized results from the two sets of experiments were superimposed to cover the entire Reynolds number range, overlapping from Reynolds numbers of about 300 to 2,000.

Typical experimental tank height verses time results for the low and high viscosity mixtures for both the four generation, and the three generation "upstream" airway network are shown in Figure 6.1-1. The curves were then differentiated and scaled, as described in chapter 3, to generate the tracheal Reynolds number verses normalized pressure difference. The noise is similar to that found for the entrance test, and is equivalent to the level of digitation noise associated with the data acquisition equipment.

Each experiment was conducted at least twice, and the data reduced and analyzed separately. The normalized results were checked, and found generally to vary by no more than about 1 to
2%. Larger variations were taken as an indication of experimental error, and the experiments were repeated. This level of error can be expected primarily as the result of digitation noise in calculating the Reynolds number.

The normalized pressure-flow results were then synthesized into two overlapping experimental ranges. The overlap of the two ranges, as with the entrance test, confirm both the theory and conclusion that the experiments scale with $P*$ and Reynolds number.

The static pressure difference for the single test bifurcation is given by subtraction of the experimental results according to 4.3.2. The subtraction is graphically depicted in Figure 6.1-2, where the results for the 3 generation experiment were re-scaled to represent the total upstream loss, normalized to the test bifurcation diameter $P*$, and the corresponding Reynolds number in the trachea of the test bifurcation.

The static pressure loss in the test bifurcation can be seen to represent about 30% of the magnitude of the system losses. This is contrasted with the entrance flow experimental results, in which ±5% accuracy was achieved with experiments differing by only about 10%. Errors in the experimental measurements of the test bifurcation can therefore be conservatively estimated to be within ±5%.

The frictional component of the pressure loss in the test bifurcation can be computed by subtracting the corrected kinetic energy and pressure correction terms as given in 5.4.3. Figure 6.1-3 presents the total static loss, kinetic energy and
pressure correction component, and resulting frictional component. The components are all normalized by the nominal kinetic energy, given by convention as:

6.1.1 Nominal Kinetic Energy \( \equiv \frac{1}{2} \gamma U^2 = \frac{1}{2} \rho \cdot \text{Re}^2 \)

Figure 6.1-3 shows, as would be expected, that the frictional losses dominate for the lower Reynolds numbers. Additionally, that the kinetic energy losses become more important and eventually dominate at the higher Reynolds numbers. The transition, where the contribution of frictional loss and changes in kinetic energy are about equal, occurs near a Reynolds number of 5000.

To better understand the nature of the frictional loss in the bifurcation, the frictional loss can be re-normalized to the loss that would occur if the flow were everywhere Poiseuille-like. This loss is the expected lower asymptotic form, and the normalization demonstrates the extent to which the flow is affected by the presence of the bifurcations. The re-normalized results are shown in Figure 6.1-4.

The frictional loss is seen to approach the expected Poiseuille-like form for Reynolds numbers of order 100. The loss smoothly increases with Reynolds numbers, and approaches nearly a 10-fold increase over the Poiseuille form at a Reynolds number of about 10,000.

6.2 Embedded Bifurcation Assumption

Inclusion of the upstream airways was intended to
establish inlet boundary conditions for the test bifurcation which could be considered "typical" of an embedded bifurcation. The embedded bifurcation assumption is equivalent to assuming that the loss in any bifurcation within an idealized airway of infinite extent is only a function of the local flow rate. While it is impossible to experimentally verify this assumption, it can be approximately tested by comparing the pressure-flow results found in bifurcations at different generations in the airway.

To test the assumption, the planar experiments conducted in the four generation system described above were repeated in a three generation system. The three generation system was created by removing the terminal generations from the upstream airways. The terminal branches were removed rather than the "tracheal" bifurcation so that the same test bifurcation could be used for both experiments, thus reducing errors introduced by slight model differences.

Figure 6.1-4 also presents the results for the frictional pressure loss found for the test bifurcation with two generations upstream of the test bifurcation (the three generation system), and with three generations upstream. The results are seen to be unaffected by the change in generation, differing by less than the estimated experimental error of ±5%.

6.3 Effect of Upstream Airway Orientation

The bifurcations in the lung are not oriented in a planar
manner, but at a variety of relative angles, generating a fully three-dimensional branching network. To determine the effect of three-dimensionality on the pressure difference across a bifurcation, the bifurcations in the upstream airway were systematically rotated $90^\circ$ from the planar orientation.

Three sets of experiments with increasing levels of symmetric "three-dimensionality" were conducted to estimate the effect of bifurcation orientation on the pressure loss. Configurations tested are shown in Figure 6.3-1, and were selected so as to preserve the symmetry of the airway system. System symmetry allows the preceding analysis to be used in reducing the experimental results, maintaining equal flows down each branch of the system.

Normalized results for the frictional pressure loss in the test bifurcation for the three non-planar configurations tested are presented in Figure 6.1-4. The results for the frictional loss were obtained by assuming that the velocity profiles are unaffected by the change in the orientation of the upstream bifurcations. This assumption allows the correction factors previously developed to be used.

Support for making this approximation was obtained from the experimental results of Chang and Menon (1985). The factors calculated from Chang and Menon's work were measured in non-planar pulmonary casts, and were found to fall on the same correlations as the results for correction factors found from the symmetric planar systems.

Similarity in the results for the frictional losses in the
test bifurcation despite the variety of upstream bifurcation orientations indicates that any three-dimensional effects on the pressure-flow relationship in the test bifurcation are negligible. The results consistently fall within a band of about ±5%, and show no observable trends. The ±5% band can be considered negligible as it is of the same magnitude as the estimated error in the system results.

6.4 Effect of Additional Parent Tube Length

Typical length-to-diameter ratios encountered in the human lung are not fixed, as in the idealized model at 3.5, but vary from about 2 in the upper airways, to up to about 5 in the peripheral airways (Nikiforov and Schlesinger 1985). The effect of the length to diameter ratio on the frictional loss in a bifurcation can be estimated by studying the loss in a straight tube segment downstream of a bifurcation.

A complete assessment of the effects of length to diameter ratios on the pressure-flow relationship is likely to be more complicated, as changes in kinetic energy fluxes for a given flow rate can also be expected to occur. Additionally, possible cascading effects of length-to-diameter ratios are also neglected.

This experiment was conducted by adding a straight tube section of length 3.5 diameters to a planar three generation airway. Once again, symmetry is preserved, allowing the same previously described analysis to reduce the experimental results. A simplifying difference in applying the subtraction
technique is that the additional loss in the added length of tubing does not include a change in velocity or kinetic energy. Because the flow area is constant and the correction factors are assumed to depend only on Reynolds numbers, no kinetic energy change is assumed to occur.

The pressure flow relations for the airway with and without the additional straight section, along with the difference due to the additional tube section is shown in Figure 6.4-1. The additional pressure loss, normalized by the Poiseuille loss at the same flow is shown in Figure 6.4-2, along with the normalized loss in a planar bifurcation.

Comparison of the results for the two systems demonstrates the striking similarity. Differences in the results are negligible, as they approximately fall within the ±5% band found in the orientation experiments, assumed to represent the experimental error. The similarity in results is an indication that for the length-to-diameter ratios typically encountered in the lung, the pressure-flow relationship can be considered to be linearly proportional to tube length.

A slight increase in the measured loss in the straight tube section at high Reynolds numbers may be an exception. While the magnitude of the deviation is not large, the trend indicated by the results may lead to significant differences at Reynolds numbers higher than ~10,000. This deviation could, though, reflect an increased error in the results for the straight tube. Figure 6.4-1 shows that the signal represents only about 5% of the signal at the higher Reynolds
numbers, increasing the potential errors.

6.5 Comparison of Results

A summary of all of the experimental results for the pressure-flow relationships for symmetric flow conditions in the idealized test bifurcation are shown in Figure 6.5-1. The results can all be represented by a single band which represents a prediction to within ±5%. The band of results are seen to be well represented by the curved tube theoretical prediction, with the bifurcation curvature ratios of 1/7, given by the simple relation:

\[
\text{Del } P = \left[ 0.556 + 0.057 \text{ Re} \right]^\frac{1}{2}
\]

which is valid down to a Reynolds number of about 60, below which, the frictional loss Del P is given identically by the Poiseuille loss. The factor K is introduced in the Poiseuille loss term to account for the difference in daughter/parent tube Reynolds numbers encountered in a bifurcation, allowing the use of just tracheal Reynolds number in the expression. The factor K is defined by applying the Poiseuille loss to each leg in the bifurcation, giving:

\[
K = \frac{1}{2} \left[ 1 + \frac{3}{2} (Dp/Dd)^3 \right] = 1.092
\]

where Dp/Dd is the parent tube - daughter tube diameter ratio, which is a constant 1.333 for the idealized bifurcation.

Applying this relationship to each branch of a bifurcation
alters the numerical constants in 6.5.1 slightly because of the Reynolds number dependence. The first term remains unchanged because it is linear, but the second term must be raised to 0.067 to account for the lower velocity in the daughter branch. Therefore, to apply the results to each branch of a bifurcation, basing on the local Reynolds number, and not the parent Reynolds number the form to use is;

\[
6.5.3 \quad \frac{\Delta P}{P_{\text{Poис}}} = \left[ 0.556 + 0.067 \text{Re} \right]^{\frac{1}{2}}
\]

Application of the entrance flow and 2-D convergent channel flow pressure-flow predictions are also shown in Figure 6.5-1. Two empirical entrance-flow predictions are included for comparison, one with no equivalent length assumed upstream of the bifurcation junction, and the other with an equivalent of one diameter assumed upstream of the bifurcation junction. The prediction with no assumed upstream equivalent length is high, but can be corrected with the addition of the assumed constant equivalent length.

The entrance theory with \( \text{Leq}=D \) predicts all of the results for the test bifurcation well, but fails to predict the results for the straight tube downstream of the bifurcation. The straight tube section does not contain a junction, and therefore no boundary layer thinning and re-growth occurs. The loss should therefore be given by the application of the entrance theory over the length of the additional tube, accounting for the parent tube length and the addition of the
previously identified one diameter equivalent length. This was not found to be the case, while entrance theory can predict the results for the straight tube section, it requires that no equivalent length be used.

The apparently unpredictable nature of the equivalent length factor in the entrance theory suggests that the better correlation to the experimental results is that provided by curved tube theory. The similarity in the form of both entrance theory and curved tube theory, though, strongly suggests that while neither analogy may strictly apply, the Reynolds number to the $\frac{1}{2}$ power is the characteristic form for the pressure-flow behavior of a bifurcation.

The 2-D convergent channel predictions, though, consistently predict the trend in the experimental results. The L/D dependence is not linear, but varies inversely with L/D for sufficiently high Reynolds Numbers. For lower Reynolds numbers this inverse dependence is not dominate, due to the Poiseuille-like form assumed by the flow. The drop in the prediction due to the additional length is therefore not as substantial as it could be for very high Reynolds numbers, and results in a prediction close to both the equivalent curved tube prediction and experimental results.

6.6 Transition to Turbulence

The range over which the experiments in the idealized bifurcation were conducted varied from, 50 to 10,000. This range contains the Reynolds number values which other
investigators have reported for the transition from laminar to turbulent flow.

By analogy to the sudden increase in pipe flow resistance that accompanies the transition to turbulence, many investigators have used changes in the slope of the Log-Log plots of the dimensionless pressure-flow behavior of expiratory flows as an indication of transition to turbulence. Hardin et al. (1980) reported such a transition at a parent Reynolds number of 4500, D.B. Reynolds (1982) noted two transitions at Reynolds numbers of 1800 and 4300, and Isabey & Chang (1981) noted that the transition occurs between Reynolds numbers of 1,500 and 4000. Each investigator noted that the transitions were not sudden, but gradual, and attributed the gradual nature of the transition to the averaging that occurs in measuring losses over an entire system.

Other expiration investigators have used flow visualization and hot-wire anemometry to detect transitions to turbulence. Chang and Menon (1985) used hot wire anemometry to measure velocity profiles, and while not directly investigating transitions to turbulence, noted that the flow was not fully laminar at a Reynolds number of only 1060. West & Hugh-Jones (1959) used die traces in water and noted that "wavering", preceding fully turbulent flow, occurred in the main bronchus and trachea at a Reynolds number of 1550.

Inspection of the experimental results for frictional pressure drop in the test bifurcation, presented in Figure 6.1-4, gives no suggestion of a transition to turbulence. The
results increase smoothly from the Poiseuille form to the fully developed curved tube form with no apparent discontinuity characteristic of a transition to turbulence. Possible averaging in a complete airway which may mask a sudden transition should not occur with the experimental subtraction technique which determines the results for a single bifurcation.

To further investigate the possible transition to turbulence in the test bifurcation, two sets of experiments were conducted. Hot wire anemometry and die-in-water traces established that transition to turbulence occurs in the Reynolds number range from about 1000 to 1500.

Hot-wire measurements were made by placing the probe in the center of an extended "trachea", positioned at the axial location corresponding to the exit of the test bifurcation. The probe position along the axis of the trachea of a three generation planar airway, at a distance of 1.75 diameters from the bifurcation center. A.C. components of the hot wire traces recorded on a storage oscilloscope are shown in Figure 6.6-1 for six different Reynolds numbers. Orientation and position in the cross-section were varied, and not found to significantly effect the results. The width of the probe and its support prohibited measurements close to the tube wall, where velocity fluctuations are expected to be lower. The oscilloscope magnification, Reynolds number and average peak velocity fluctuations relative to the bulk flow are summarized below in Table 6.6.1.
Table 6.6.1

<table>
<thead>
<tr>
<th>Trace #</th>
<th>Reynolds #</th>
<th>Y scale mv/div.</th>
<th>X scale sec/div.</th>
<th>G/U</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>713</td>
<td>20</td>
<td>.2</td>
<td>0.008</td>
</tr>
<tr>
<td>2</td>
<td>1037</td>
<td>20</td>
<td>.2</td>
<td>0.033</td>
</tr>
<tr>
<td>3</td>
<td>1548</td>
<td>20</td>
<td>.2</td>
<td>0.047</td>
</tr>
<tr>
<td>4</td>
<td>1585</td>
<td>20</td>
<td>.2</td>
<td>0.048</td>
</tr>
<tr>
<td>5</td>
<td>1697</td>
<td>20</td>
<td>.2</td>
<td>0.057</td>
</tr>
<tr>
<td>6</td>
<td>2286</td>
<td>50</td>
<td>.1</td>
<td>0.071</td>
</tr>
</tbody>
</table>

The traces show that small random fluctuations typical of turbulence are present at Reynolds numbers as low as 700, below which only the high frequency measurement noise is present. The fluctuations grow to represent about 7% of the mean velocity at a Reynolds number of about 2300, above which the magnitude of the fluctuations were found to remain roughly constant. Selecting a single value at which turbulence begins is difficult, but the fluctuations appear to grow rapidly until a Reynolds number of about 1000, which will be considered the transition value. This transition value is consistent with the other hot wire results of Chang and Menon with reported turbulence at a Reynolds number of 1060.

Flow visualization experiments were used to identify the transition from laminar to turbulent flows with an India ink dye tracer in water. The original two piece planar airway formed by two clear plexiglas halves was used in the experiments to allow the flow to be easily viewed. The airway
model was placed in a large water filled container with the trachea connected to a hose leading to a drain. India ink dye was injected at various positions within the airway by maneuvering a catheter into position, and injecting the dye from a syringe source.

The flow was driven by the hydrostatic head in the supply tank, and controlled with a variable resistance valve downstream of the airway. A simple bucket and stopwatch approach was used to measure the flow rates.

The flow is observed to remain laminar, although clearly not poiseuille-like as secondary flows disperse the dye, up to a Reynolds number of about 1400. At a Reynolds number of about 1500 slight "wavering", as described by West & Hugh-Jones in the dye traces occurs at the exit of the trachea. As the Reynolds number increases, the wavering grows stronger and propagates upstream toward the bifurcation junction. The intensity of the dye wavering, or velocity fluctuations continue to increase until the dye breaks up into a fully turbulent flow at a Reynolds number of about 2300.

The dye fluctuations are also seen to have propagated into the daughter branch at the same tracheal Reynolds number of 2300, which corresponds to a local Reynolds number of about 1500. The strongly turbulent region also propagates upstream toward the bifurcation as the Reynolds number increases. At a Reynolds number of 3400 in the trachea, or 2300 in the daughter tube, the strongly turbulent region is seen to enter the daughter tube.
This pattern continues to progress upstream as the Reynolds number increases. "Wavering" starts at each branch at a local Reynolds number of 1400, breaking up into strong turbulence at a local Reynolds number of about 2300. Thus establishing a transitional Reynolds number of about 1500, in agreement with the results of West & Hugh-Jones, but slightly higher than the transitional value found with hot-wire techniques.

Discrepancies between the transitional Reynolds number value between the hot-wire technique and flow visualization techniques are not large, but appear to be consistent. A possible explanation for the difference is that the velocity fluctuations are a small fraction of the bulk velocity at the start of transition, and build to more significant levels with increasing Reynolds numbers. The fluctuations are therefore not be easily visualized until they are large enough to grow in the time before the fluid leaves the bifurcation.

These results are in general agreement with the results of the previous investigators with the exception of the noted transition at or near a Reynolds number of 4500 by D.B. Reynolds and Hardin & Yu. An explanation for this discrepancy lies in the manner in which the transition at 4500 was identified. Hardin & Yu and Reynolds assumed that a change in the dimensionless static pressure-flow relation signified the transition to turbulence.

The transition identified by these investigators and others can be explained in terms of the pressure - flow results. At a
Reynolds number of about 5000, the results show that the
dominate factor contributing to the static pressure loss
changes from a viscous losses to an inertial losses. The
transition noted by these investigators may therefore not be a
turbulent transition, but a regime transition, supporting the
results found in this investigation.

6.7 Effect of Airway Collapse

The idealized bifurcation proposed by Pedley, and used as the basis for this study presumes a circular cross-section for the airways. While this geometry is generally valid for neutrally and positively inflated airways (Hyatt & Flath 1966, Winter et al 1982), its validity for negative transmural pressure is not clear. Non-circular airways are likely to effect the characteristics of the pressure-flow relationship, and are expected to be most important during expiratory flows where negative transmural pressures are present.

The cross-sectional geometry of the airways is a complicated function of tissue properties, tissue tethering, and lung inflation. Figure 6.7-1 presents a cross-section of a typical airway proposed by Hughes et al (1974), showing the forces acting to form the airway. Internal airway pressure and tension in the lung parenchyma act to distend the airway, while alveolar pressure acts to collapse the airway. Tension in the airway wall tissue acts to resist distention under inflation, and bending stiffness resists airway collapse under negative transmural pressures.
Studies of human tracheas and upper bronchial airways (Griscom & Wohl 1983, Jones et al 1975, Khosla 1985) have demonstrated consistent crescent collapse patterns. The results of Khosla is typical, showing the crescent shape collapse for increasingly negative transmural pressures. These collapse patterns are not expected to be consistent throughout the airways because of the additional asymmetric cartilage tissue support in the upper bronchial airways.

Collapse patterns in all but the upper airways have not been documented well because of the experimental difficulties. Casting techniques are difficult to implement because the pressure distributions which establish the airway collapse have not been achieved with static conditions necessary for cast setting. Optical observations are also difficult to implement in all but the largest airways. The small airway dimensions under collapsed conditions, and the distortions caused by hemispherical optics typical of commercially available bronchoscopes limit their usefulness.

Stereo X-ray-tantalum dust techniques have been successfully implemented by Hughes et al (1974) to study intrapulmonary airways between 4.8 and 9 mm I.D.. The stereo views of the airways were compared, and cross-sectional geometries were constructed. The results obtained by Hughes et al for a nominal 4.8 mm airway at two different lung volumes and a variety of transpulmonary pressures are shown in Figure 6.7-2. In contrast to the upper airway collapse, the results show nearly concentric collapse, except at the low
lung volumes and high (10 cm) collapsing transpulmonary pressures. The tedious nature of this approach has limited the available airway testing, leaving the exact nature of airway collapse still unclear.

Without more extensive physiological information the effect of airway collapse on the cross-section geometry and the resulting influence on the pressure-flow relationship can only be approximate. To determine the extent to which "reasonable" airway geometries during collapse may effect the pressure flow relationship, a simple semi-circular cross-section geometry was chosen because,

- The crescent type collapse is typical of the trachea and upper airways.
- It can be easily constructed with casting techniques and the two piece airway model.
- It is similar to the previously tested circular geometry.

The subtraction technique was implemented with the semi-circular geometry with the two airway systems consisting of semi-circular airways. The results for the static pressure drop across the test bifurcation as a function of flow rate were obtained using the same method as before. To further reduce the data in non-dimensional form and into a kinetic energy and viscous loss component, some additional assumptions were made.

First, the characteristic length used to calculate the Reynolds number and characteristic pressure $P^*$, was chosen to be the hydraulic diameter of the cross-section.
Secondly, the correction factors developed and used for the circular cross-section are also applied to the semi-circular cross-section. Additionally, the frictional results are normalized by a Poiseuille-like form, which has a longer L/D ratio due to the decrease in diameter without a corresponding change in length.

The results for the frictional losses, normalized to the Poiseuille form are shown in Figure 6.7-3. Along with the experimental results are the predictions for curved tube type correlation losses, entrance type losses, and the loss in a 2-D channel. The results are based on the hydraulic diameter, and have employed a curvature ratio of 1/11.45, and a L/D ratio of 5.72.

Despite the significant change in cross-sectional geometry, the results for the frictional loss falls within ±5% of the empirical curved tube correlation over most of the tested Reynolds number range. Both the entrance and convergent channel results are lower due to the weaker L/D dependence. At Reynolds numbers below about 300, the results start to deviate more significantly, but are still within 10% at the lowest Reynolds number tested of 200. This deviation at the lower Reynolds numbers can be expected based on the results for flows in straight tubes with non-circular cross-sections. Schlichting (1979) shows that the hydraulic diameter properly reduces the frictional results for a variety of cross-sections.
in the turbulent regime, but typically under-predicts the results in the laminar regime by about 10%.

The ability of the analogy to curve tube flow is striking given that the semi-circular model changed not only the shape of the test bifurcation, but also the L/D and radius ratio. While the tests are not conclusive, it provides substantial support to use the simple curved tube correlation for the lung in most any state. The underprediction of the curved tube theory at the low Reynolds number range is not expected to be significant in determining the overall pressure drop given that the errors are relatively low, and that the magnitude of the loss is small in comparison to the higher Reynolds number regions.
7. Flow Partition Experiments

7.1 Background

The actual flow conditions in the lung are not symmetric, as modeled here, but include asymmetric geometries and flows. These types of asymmetries have been found and documented throughout the lung (Horsfield 1971, Weibel 1963, Snyder et al. 1981) but their effect on the pressure-flow characteristics of the lung have not been elucidated.

Asymmetric flow partitioning is expected to have a significant influence on the pressure-flow relationship within the bifurcation. The most obvious effect of asymmetric flows is that the pressure drop from each daughter branch in a bifurcation to the parent branch is likely to be different. In addition, the mixing of two streams of unequal velocities in the parent branch is likely to generate increased frictional dissipation over the symmetric flow case.

The lack of published information may be a result of the deficiencies in the systemic approaches alluded to above. The subtraction technique, however, offers a systematic means to study the effects of asymmetric flow partitioning in a single bifurcation.

The expected flow asymmetries in the upper airways have been documented by Horsfield. In order to calculate the flows in each branch of the lung, Horsfield assumed the flow to be proportional to the number of terminal bronchioles, and therefore acini, which are subtended by the branch. The flow
ratios vary from unity to over 4. The average flow ratio is substantially higher than unity, at a value of 2.1, with a one sigma standard variation of 1.1. These flow ratios do not represent the velocity difference in the two joining streams because the daughter tube area's are not included. This effect will be discussed further in the next chapter.

The approach used to understand the effect of flow asymmetries on the pressure-flow relationship is parametric, utilizing the symmetric bifurcation previously studied. The parametric approach and the subtraction technique, utilized in the studies of symmetric bifurcations, are likely to introduce some errors in modeling the flow. The subtraction technique, as has been described before, introduces potential errors such as neglecting cascading effects. Further errors introduced by the parametric approach arise because flow asymmetries are not likely to occur alone, but in conjunction with other asymmetries.

Olson (1971) presented a typical pulmonary bifurcation which is not symmetric, as has been modeled, but asymmetric. Olson's asymmetric bifurcation model is shown in Figure 7.1-3, with daughter tube diameters which are not equal. The differences in resulting flow area's will interact with the flow asymmetries to determine velocities in the daughter branches and therefore kinetic energy fluxes. While it is important to understand the limitations imposed by the parametric approach, insights developed by controlled variation of parameters are expected to improve the
understanding of pulmonary flows.

7.2 Experimental Procedure

The conceptual approach in applying the subtraction technique to the asymmetric flow problem is similar to that for the symmetric flow case. The experimental approach must be changed to establish the asymmetric flow, and to isolate the pressure drops from each of the daughter branches to the parent branch. The experimental apparatus developed which achieves both the pressure isolation and asymmetric flow partitioning is shown in Figure 7.2-1.

The asymmetric flow in the test bifurcation is established by specifying the flow in one of the daughter branches, while the other is allowed to vary with the changing hydrostatic head. Various flow ratios are therefore generated as the flow rate in the hydrostatically driven branch changes with the tank level. The prescribed flow, which is contained in a separate circuit to isolate the fluid, is generated by a centrifugal pump (Gould model #3642 size #1), metered with a flow restricting valve, and measured with a Rotometer (Fischer and Potter, model 10A 1027A). Upstream boundary conditions for each daughter branch of the test bifurcation are maintained as before with airways distal to both daughter branches.

The experimental techniques developed to measure the pressure flow characteristics of an airway for the symmetric flow studies are essentially unchanged in this case.
Differentiation of the tank level to measures flow rate, and a differential pressure transducer between the reservoir and exhaust tank measures the driving pressure. Subtraction of the loss in the upstream airway, which is an airway with symmetric flow conditions, from the loss in the hydrostatic branch gives the loss in the hydrostatic leg of the test bifurcation. The pressure difference in the constant flow rate branch is not measured.

Four valves are positioned to allow the system to be filled and run with the same pump. The valve positions in the run and the fill mode are summarized in Table 7.2-1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Valve Status</th>
<th>Gate Status</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#1</td>
<td>#2</td>
</tr>
<tr>
<td>Fill</td>
<td>Open</td>
<td>Closed</td>
</tr>
<tr>
<td>Run</td>
<td>Closed</td>
<td>Variable</td>
</tr>
</tbody>
</table>

Near steady flow rates in the "constant" flow rate branch are maintained despite the change in operating pressures resulting from the change in pressure with hydrostatic level. Maintaining a constant inlet pump head, and minimizing the pump outlet pressure variations gives the near constant flow rate. Constant inlet pressures are maintained by supplying the pump from the exhaust tank which has its level maintained with an overflow weir. An added benefit of using the exhaust
tank as a supply to the pump is that it minimizes air entrainment in the system which would result if the aerated fluid in the storage tank were used.

Near constant downstream pressures are maintained by using a high pressure (several hundred inches of water) centrifugal pump and metering the flow with a high resistance valve (#2). Pump flow studies with varying exit pressures typical of those expected in an experiment (0 to 50 cmH2O), showed that the flow rate varied by less than 1.5%.

This potential error is combined with two other additional sources of error to increase the expected error level in the flow partition experiments. Vibrations introduced to the system by the pump and errors in reading the constant flow source flow meter are expected to add to the approximately ±5% error found in the symmetric flow experiments.

7.3 Data Reduction

The subtraction technique, as outlined above, can be used to give the total static pressure difference across one leg of a bifurcation during asymmetric flow conditions. The analysis previously developed for the symmetric flow conditions to decompose the static pressure loss into frictional and kinetic energy components can not be used, as it is no longer applicable.

While the previously developed analysis does not apply directly, the energy based methodology used to develop the
decomposition procedure can be used for the asymmetric flow conditions as well. The control volume selected to analyze the flow is shown in Figure 7.3-1. The selection of the control volume is such that;

- The flow enters/exits the control volume normal to the control surface.
- The control volume includes one daughter branch and the parent branch of the bifurcation.
- The inlet to the control volume from the second daughter branch is the same area as the branch itself.

Using subscripts 1,2 and 3 to refer to the daughter branch with the variable flow, the second daughter branch with the fixed flow, and parent branch respectively, allows the continuity equation and energy equation (3.7.1) to be written as;

7.3.1 Continuity ... \( A \bar{U} + A \bar{U} = A \bar{U} \)

\[
\begin{align*}
&11 \quad 11 \\
&22 \quad 22 \\
&33 \quad 33 
\end{align*}
\]

7.3.2 Energy Equation ...

\[
\int_{V} \Phi \, dVol = \int_{V} \left[ -Q \left[ P + \frac{1}{2} f \left( \frac{1}{2} \frac{P}{Re} \right)^2 \right] - Q \left[ P + \frac{1}{2} f \left( \frac{1}{2} \frac{P}{Re} \right)^2 \right] + Q \left[ P + \frac{1}{2} f \left( \frac{1}{2} \frac{P}{Re} \right)^2 \right] \right] 
\]

Where the correction factors and the pressure correction term have been included into the factor \( f \), defined in 5.4.2. Note that the correction factors were developed for the symmetric flow conditions, but it was found (see Chapter 5) that the same correction factors applied to asymmetric as well as symmetric flows.

Following the definition of frictional pressure drops, the integral of the dissipation function can be written as;
\[
\int \phi \, d\text{Vol} = Q \text{Del PV}_1 + Q \text{Del PV}_3
\]

where the subscripts on the viscous pressure drops, Del PV, refer to the leg of the bifurcation in which the dissipation occurs.

Before 7.3.3 can be used, the pressure at leg 2 must be estimated. The pressure, P_2, can be written in terms of P_1 and two pressure differences;

\[
P = P_2 - \left[ \left( P - P'_1 \right) - \left( P'_2 - P \right) + \left( P_2 - P'_1 \right) \right]
\]

The pressure loss in daughter branch #1, P_1-P'_1, and branch #2, P_2-P'_2, are due to frictional losses alone because the area of the daughter branch remains constant between bifurcation junctions. The pressure loss in the variable flow daughter branch can therefore be written as Del PV_1, to be consistent with 7.3.4.

The difference in pressures at the base of each leg, P'_1-P_2, is more difficult to assess, but can be approximated by considering the flow patterns in the junction. The flow geometry in the vicinity of the junction can be approximated as flow in two curved tubes. The pressure differences are a result of gradients established by the centrifugal forces, given to a first approximation by the Euler normal equation, equation 5.3.1. Integrating the Euler equation across the cross-section and weighting the pressure by the differential area, gives the average pressure difference between the
centerline of the tube and the bifurcation carina as;

\[ \text{Del } P = \frac{\zeta}{\frac{4}{3} \pi a^2} \left[ \int_{0}^{a} \frac{U^2}{Rc} \left( \frac{r}{a} \right)^2 \left( 2 \sqrt{a^2 - r^2} \right) \text{dr} \right] = \frac{4U^2 a}{3\pi Rc} \]

where \( a \) is the daughter tube radius, \( Rc \) is the radius of curvature, and the ratio of the two for the test bifurcation is \( .75 \times (1/7) \). The change in the radius of curvature across the cross section and the pressure variations due to the secondary flows have been neglected, and the axial velocity has been approximated by the average axial velocity, \( U \). The pressure difference across the junction between the two legs can be approximated with 7.3.6 as;

\[ P' - P \approx \frac{4 a}{\pi Rc} \left[ \frac{U}{U} - \frac{2}{2} \right] = \frac{4 a}{3\pi Rc} \left( \frac{R^2 - 1}{(1 + \Gamma)^2} \right) \]

where \( \Gamma \) is defined as the flow ratio, and for the symmetric geometry of the ideal bifurcation is given by;

\[ \Gamma = \frac{\bar{U}}{\bar{U}} = \frac{Q}{Q_1} = \frac{Q_2}{Q} \]

Combining the expression for \( P_e \) and the continuity equation into the energy equation gives;

\[ \text{Del } P_v + \text{Del } P_v = \text{Del } R - \text{Del } P_k + \frac{4 a}{3 \pi Rc} \left( \frac{A_1}{A_2} \right)^2 \left( \frac{R^2 - 1}{(1 + \Gamma)^2} \right) \]

where the static pressure drop, \( \text{Del } R \), and the kinetic energy pressure drop, \( \text{Del } P_k \), are given by;
7.3.10 \( \text{Del } R = P_1 - P_3 \)

7.3.11 \( \text{Del } P_k = \frac{1}{2} P_3 \text{Re} \left( f - \left[ \frac{A_3}{A_1} \right]^2 \right)^3 \left( \frac{f}{1 + \Gamma} + f^2 \right) \)

where \( \text{Del } R \), \( \Gamma \), and the Reynolds number are experimentally measured quantities.

7.4 Results

The experiments were conducted at seven different fixed flow rates. The fixed flows varied from 0 to 902 cc/sec, which corresponds to a Reynolds number range in the daughter tube from 0 to 4390.

The normalized results for the total static pressure loss in the test leg of the bifurcation at 6 of the 7 different fixed flow rates in the second bifurcation leg are shown in Figure 7.4-1. The pressure loss is normalized to the kinetic energy flux, \( \frac{1}{2} P \text{Re}^2 \), in the parent tube of the test bifurcation, and is plotted against the flow ratio, \( \Gamma \). (The experiment with 0 fixed flow is not shown because \( \Gamma \) is infinite)

The normalized pressure loss is seen to have a maximum of about +.9, near a flow ratio of .8, and increases with decreasing values of the fixed daughter tube Reynolds number. The pressure loss is seen to decreases as \( \Gamma \) both increases and decreases away from 0.8. Negative pressure losses occur at
low $\Gamma$, indicating that the presence of the fixed flow rate can cause a pressure increase along the direction of flow in the test leg of the bifurcation.

This apparent paradox can be explained by the pressure changes caused by the fluid accelerations. For conditions where the fixed flow rate is substantially higher than the variable flow in the test leg of the bifurcation, some pressure recovery occurs as the fixed flow slows upon entering the larger parent branch. This negative pressure loss can be accounted for by subtracting the predicted pressure loss due to kinetic energy changes.

Figure 7.4-2 presents the decomposition of the static pressure loss, $\Delta P$, into the kinetic energy component, $\Delta P_k$, and the frictional component, $\Delta P_v$, for a typical experimental run. The negative pressure drops are shown to be predicted by the changes in kinetic energy, and leave as a remainder a positive pressure loss which is attributable to the affects of friction plus the curvature pressure correction.

Further data reduction requires partitioning the viscous dissipation between the daughter branch and the parent branch. The viscous pressure loss in the daughter branch can be approximated by the loss that would occur in the leg if the flows were symmetric. This assumption is valid if the effects of the flow asymmetry do not propagate upstream. The results of the symmetric experiments give this loss in the daughter branch as;
7.4.1  \[ \text{D} \text{el} \ \text{Pv} = 32 \ \text{P} \ \text{Re} \left[ \frac{0.556 + 0.067 \sqrt{\text{Re}}}{1} \right] \frac{L}{D} \]

The viscous pressure loss in the trachea was assumed to be the sum of two components; the loss that would occur if the flow were symmetric, and an excess component. The excess loss for the 6 different experiments is shown in Figure 7.4-3. The loss is seen to be the same for each experiment, to within about \pm 15\%, and is given by;

7.4.2  \[ \text{D} \text{el Pe} = \frac{2}{3} \text{P} \ \text{Re} \left[ \frac{1 - \Gamma}{1 + \Gamma} \right]^2 \]

This loss represents the excess kinetic energy flux due to the differences in velocity of the two mixing streams. A simple momentum analysis of a mixing flow in the straight section of the parent tube is outlined in Figure 7.4-4, and gives the potential pressure recovery in the parent branch owing to the velocity equilibration as;

7.4.3  \[ \frac{P}{3} - \frac{P}{3} = \zeta \left[ \frac{U}{31} \frac{2}{3} + \frac{U}{32} \frac{2}{3} \right] - \left[ \frac{\frac{U}{31} + \frac{U}{32}}{2} \right]^2 \]

where \( U_{31} \) and \( U_{32} \) refer to the velocity that the fluid in leg 1 and 2 attain when in the parent branch. Using continuity, 7.4.3 can be written as;

7.4.4  \[ \frac{P}{3} - \frac{P}{3} = \zeta \frac{U}{3} \left[ \frac{U_{31} - U_{32}}{U_{31} + U_{32}} \right]^2 = \frac{2}{3} \zeta \frac{U}{3} \left[ \frac{\Gamma - 1}{\Gamma + 1} \right]^2 \]
which is identical to the excess loss over and above estimated frictional losses found experimentally. The similarity of the losses indicates that the available kinetic energy in the mixing streams due to the bulk velocity differences is not recovered as the flows mix to a uniform velocity.

The 15% error represented by the results in Figure 7.4-3 is not reflective of the true predictive capability of the experimental correlations, as it is a cumulative error applied to only one component of the pressure loss. The excess loss was computed by subtracting a friction prediction which has an accuracy of about ±5%, and a kinetic energy prediction which employed a number of assumptions. The total loss is a sum of the three components, all of which are approximately of the same order. For typical flow ratios of about 2.1 (or $1/2.1=.476$), the additional asymmetric pressure loss is approximately equal to the change in kinetic, which results in an accuracy of about ±5% in predicting the static pressure loss.

The form used to describe the additional pressure difference created by the asymmetric effects has the distinct advantage of reducing to the symmetric form automatically when the flow ratio is one. Computational models can therefore include the asymmetric pressure loss term, and apply to symmetric conditions with no loss of accuracy.
8. Implications for Pulmonary Flows

8.1 Introduction

A primary objective of this work is to predict the relationship between pressure and flow rate which exists during expiration from a human lung. As a first step, experiments were designed, developed and executed to describe the pressure-flow relationship in the structural elements which make-up the lung, a single bifurcation. The previous chapters have described these efforts and presented the experimental results.

The next step is to use these results to predict the pressure-flow performance of a complete lung. The entire airway network can be simulated by numerically assembling the bifurcations into a complete system, and the overall performance can be calculated. Intrinsic to this approach is the assumption that the experiments properly simulated the flow conditions in the lung, and that the lung geometry and flow distribution are known well enough to permit accurate numerical modeling.

To test the accuracy of these assumptions, simple airway networks geometries will be tested first, then, more complicated physiological models before simulating a complete human lung. There are a number of reasons for working with the simplified models first:

- Geometries and flow distributions of the simplified networks are known, and do not have to be approximated.
- Experimental results for a range of simplified geometries are available in the literature, and can serve as a check
on the numerical prediction over a wide range of flow rates.

- The quality of the fit between the numerical predictions and the observed experimental results can serve as an indicator of the error in predicting the performance of a complete human airway.

The first set of simplified airways which will be tested are idealized models constructed to simulate only a few generations of the airway network. These models are smooth walled, branch symmetrically, and have equal flows in branches of the same order. Because the terminal branches can represent a significant portion of the pressure drop of the entire airway, care is taken to represent the pressure-flow relation of these airways accurately. The numerical model uses entrance flow correlations (Rivas & Shapiro 1956) for these branches, and the experimentally determined curved tube type correlation elsewhere for the frictional dissipation.

A number of physiological airway models are also available in the literature for comparison of the measured and predicted pressure-flow relationships. These models are less well characterized in terms of both geometry and flow distribution, but the pressure-flow character of the airway is well documented. The models are not smooth walled, branch asymmetrically, and sometimes branch trichotomously rather than dichotomously. Numerical simulation of these models, as well as a real human airway, require that the equations written previously for symmetric branching be modified for daughter branches of unequal size.

For comparison purposes, Figure 8.1-1 presents the
estimated frictional pressure drop across a single bifurcation based on the measured systemic performance by several different investigators and the results from this study. Inspection of the figure shows that this study appears to under-predict the frictional terms. One might therefore expect that the systemic predictions based on the results from this study will also be low. If this is not the case, and comparison of the systemic prediction and actual experimental results are close, it would indicate that basing the performance of a single bifurcation of the systemic performance can introduce significant errors.

8.2 Pressure Drop correlations

The correlations for the pressure drop components previously presented strictly apply only to symmetrically branched bifurcations. Real airways, however, typically branch asymmetrically, which requires that these expressions be modified. Asymmetric geometric conditions have not been tested experimentally, so the applicability of the asymmetric forms must be assumed to be at least approximately accurate.

The geometry of the bifurcation is assumed to be that given in Figure 8.2-1. It is further assumed that the radius ratio of a bifurcation branch is constant at the same value as the experimental test bifurcations, \( r = 1/7 \). The viscous pressure loss correlation which applies to each branch of the bifurcation therefore remains unchanged.

Kinetic Energy Flux
Application of the energy equation to determine the pressure drop due to the changing kinetic energy flux in a symmetric bifurcation with symmetric flow conditions is outlined in 3.7. The expression was modified in 5.4 to include the energy in the secondary flows, and the pressure work due to pressure variations generated by the secondary flows. The form of the equation was further modified to account for the case of asymmetric flow conditions in 7.3. The case of physiological interest, asymmetric geometry and flows, can be seen to be a simple extension of the equation, and is given by:

\[
\text{8.2.1} \quad \Delta P = \frac{1}{2} P \frac{\text{Re}}{\text{KE}} * 2 \left[ f - \left( \frac{A}{A^3} \right)^2 \right] = \frac{f}{1 + \Gamma} + \frac{2}{1} \Phi^2
\]

Where \( \Phi \) is defined as the daughter tube area ratio;

\[
\text{8.2.2} \quad \Phi = \frac{A}{A}
\]

Inspection of 8.2.1 shows that it reduces to the previously developed expressions when the ratios are set equal to 1; symmetric geometry with \( \Phi = 1 \), and symmetric flow conditions with \( \Gamma = 1 \).

**Mixing Loss**

The experimental investigation of additional viscous dissipation resulting from asymmetric flow conditions was reported in chapter 7. The results of the experiments indicated that the pressure loss can be predicted by assuming
the excess momentum in the two streams is lost as the
velocities equilibrate. Applying the momentum equation to the
straight section of the parent branch of the bifurcation in
8.2-1 gives;

\[ \text{8.2.3} \quad \Delta P = \frac{1}{2} \rho \frac{V^2}{R} \frac{2}{m} \left( \frac{\Gamma - \Phi}{\Phi} \right)^2 \]

Inspection of 8.2.3 shows that it reduces to the proper
form for the two simplifying cases. For the case of a
symmetric bifurcation, \( \Phi = 1 \), and the earlier result is
obtained. No mixing loss occurs when the velocities of the
two daughter branch streams are equal;

\[ \text{8.2.4} \quad \frac{V}{Q} = \frac{1}{A} = \frac{1}{2} \]

or when;

\[ \text{8.2.5} \quad \frac{\Gamma}{Q} = \frac{1}{A} = \frac{2}{\Phi} \]

8.3 Prediction of Model Experiments

Two sets of model experiments are available in the
literature which present measurements of the static pressure
drop across a set of symmetric bifurcations. Hardin and Yu
(1980) developed a smooth walled symmetric lung model from
blown glass tubing. D.B. Reynolds (1980) conducted two sets of
experiments in rigid "Y" connectors, one with L/D ratios of 5,
the other with L/D ratios of 9.

Prediction of Hardin & Yu's Results

The bifurcations used in the study are very similar to
those in this study, and it could therefore be expected that the results should also be similar. The reported frictional pressure drops, though, differ by about a factor of 2.

Figure 8.3-1 presents the range of Hardin and Yu's experimental results, and the numerical prediction for the normalized total static pressure drop. The agreement is seen to be good, to within about 10% over the Reynolds range from 1000 to 10,000, and is best in the higher Reynolds number range. The agreement is good over the entire range, despite Hardin and Yu's contention that a transition to turbulence occurs at a Reynolds number of about 4500.

The agreement between the measured and predicted static pressure drop is much better than would be expected given the difference in calculated frictional pressure drop. The agreement is better than would be expected for two primary reasons. The first is that Hardin and Yu assumed that the kinetic energy correction factors were unity, under-predicting by about 15%. The second reason is that Hardin and Yu did not include entrance effects in the peripheral airways which were a significant fraction of the total loss in their system, and the averaging resulted in an overprediction of the true dissipation results for a bifurcation.

**Prediction of D.B. Reynolds Results**

The experimental results and prediction are excellent agreement for both airway models, as shown in Figure 8.3-2. The agreement supports the linear L/D dependence found in
this study, given the much longer L/D ratio's tested by Reynolds, and the fit for both L/D ratios tested.

The close agreement in the value of the static pressure loss could be expected given the close agreement in frictional losses shown in Figure 8.1-1. Unlike the case for Hardin and Yu, frictional resistances in airways with constant area tubes drop off rapidly away from the trachea. Reynolds data therefore more closely reflects the frictional relations in the internal bifurcations.

8.4 Prediction of Physiological Model Experiments

Two sets of experiments were published which present the pressure-flow relationship for a more physiologically accurate airway network than the idealized models. The first was reported by Isabey and Chang (1981) in a model of the central airways of the human lung (Zavala Lung Model, Meditech, Watertown, Ma). The second was reported by Reynolds and Lee (1979) in a cast of a human lung below the right intermediate bronchus.

Zavala Lung Model

A drawing of the Zavala lung model is shown in Figure 8.4-1. The details of the Zavala model geometry were reported by Slutsky et al (1980), presenting the branching pattern, including trichotomies, and the branch lengths and diameters. For the purposes of simulating the flow in the cast, the model was reconfigured into the "dichotomous" representation shown in
Figure 8.4-2. Table 8.4-1 presents the branching pattern and dimensions of the dichotomous model.

Note that the dimensions reported for the dichotomous model are different from those given by Slutsky et al, in part due to the translation of trichotomies. Inspection of a male cast of the Zavala lung model also generated some dimensional differences because the diameters reported by Slutsky et al were the major axes for elliptical sections, and not a hydraulic or area averaged diameter.

**Prediction of Isabey & Chang's Results**

Despite the differences in geometry, the agreement between the experimental results of Isabey and Chang and the numerical prediction is good. The comparison in Figure 8.4-3 presents the experimental results and the numerical prediction based on two different flow distribution assumptions. The first assumption is that the flow is distributed according to the fraction of alveoli subtended by the branch (Horsfield et al, 1971). The second flow distribution is believed to better represent the actual experimental conditions by generating a uniform pressure in the peripheral airways exposed to the plenum.

Figure 8.4-4 presents the pressure drop components, and as expected, the static pressure drop is dominated by viscous dissipation at low Reynolds numbers. At a Reynolds number of about 2200, the kinetic energy component starts to dominate. Mixing losses are an order of magnitude lower than the other
Table 8.4-1

"Dichotomous" Zavala Lung Model

"Internal" Branches...

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<th>Length (cm)</th>
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Table 8.4-1 (Continued)
"Dichotomous" Zavala Lung Model

"Terminal" Branches

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components.

The prediction is seen to fall below the experiments at high Reynolds numbers. The reason for this is assumed to be the roughness of the Zavala model, which, as discussed before, is not expected to be important in a real lung. The overprediction at low Reynolds number is not great, only about 10%, and may be attributable to inaccuracies in the geometry modeling.

**Generation of Uniform Pressure Boundary Condition**

Given that the Zavala lung model was not supplied by alveoli, but by a uniform pressure plenum, the flow distribution calculated by Horsfield et al will not be correct. The more correct flow distribution is the one which generates uniform pressures in the peripheral branches. Unfortunately, the coupled non-linear equations which characterize the pressure-flow relation are written to calculate a pressure drop given a flow rate. The equations and calculational scheme cannot easily be inverted to generate the required flow rates in an airway to produce a uniform upstream pressure boundary condition.

To determine this more correct flow distribution, an iterative approach was used. Starting with the Horsfield et al flow distribution, the problem was linearized by forming a resistance matrix and calculating the change in pressure in branch $i$, given a change in flow rate in branch $j$. 
8.4.1 \[ R_{ij} = \frac{\Delta P}{\Delta Q_{ij}} \]

After inverting the resistance matrix and multiplying by a uniform pressure vector, the flow vector is given. This process had to be repeated a number of times because the problem is not linear, and the solution only represents an estimate of the correct flow distribution. After a number of iterations the flow vector converged, and remained constant. A graphical presentation of the resistance matrix is shown in Figure 8.4-5, showing that the matrix is primarily diagonal as would be expected, but that it also contains off-diagonal negative values which is why a full matrix approach is needed to converge to a solution.

The non-linear resistances result in the flow distribution being a function of flow rate. The flow distribution is shown in Figure 8.4-6 for a number of different flow rates, as well as the initial assumed flow distribution. The flow distribution is seen to vary markedly from the initial assumed form, but the pressure-flow results in Figure 8.4-3 show that the systemic performance reacts by varying only about 10%. This finding will be important when the full lung is tested. Given that the respiratory tree has upwards of 230,000 terminal branches it becomes prohibitive to form and invert a resistance matrix. The 10% overprediction with the Horsfield et al flow distribution can then be taken as an indication of the error introduced by inaccurate flow assumptions.
Prediction of Reynolds & Lee Lung Cast Results

To model the lung portion used by Reynolds and Lee, the asymmetric lung model 2 proposed by Horsfield et al (1971) was used. The model uses a statistical average representation for the central airways, terminating in bronchopulmonary segments (BPS). Each BPS is modeled as an asymmetric airway, where a branch of order N bifurcates into two branches, one of order N-1, and one of order N-M, where M varies between 3 and 5. The dimensions of the branches for each BPS are different to provide the best current pulmonary model. The Horsfield et al Model 2 is schematically represented in Figure 8.4-7.

The experimental results of Reynolds and Lee, along with the numerical prediction for the pressure drop up to the right intermediate bronchus are shown in Figure 8.4-8. The agreement between the two is excellent, with the numerical prediction falling within the experimental error band over the entire range from \(100 < \text{Re} < 10,000\). The curvature of the prediction though, tends to be slightly less than the experimental results.

The flow distribution was assumed to be that given by Horsfield et al. No attempt was made to change the flow distribution to generate a uniform pressure boundary condition because of the large number of terminal branches. Based on the Zavala lung model results, the error in the flow distribution is likely to introduce no more than 10% error in the pressure flow relation.
8.5 Lung Simulations

The numerical predictions based on the experimental results of this study have shown excellent agreement with published results for systemic pressure drops in a wide variety of airway models. With this agreement, we can move on with confidence to predict the pressure flow characteristics of a complete human lung.

Two lung models will be investigated numerically because they are both widely used, and represent extremes in approaches to modeling the geometry of the lung. The symmetric lung model of Weibel (1963) is the most used pulmonary model due to its simplicity, and the asymmetric Horsefield et al lung model 2 provides the most accurate representation of a lung, but its complexity has limited its use. Comparison of the pulmonary predictions based on these two lung models should give an indication of the effects of asymmetry, and the sensitivity of pulmonary predictions on the details of the lung geometry.

Figure 8.5-1 presents the results for the pressure-flow relationships of the two pulmonary models. The results are cast in two common forms, pressure normalized to the kinetic energy flux, $\frac{\rho V^2}{2}$, and dimensional airway resistance, pressure/flow rate, verses the Reynolds number in the trachea. The agreement between the two lung models is surprisingly close; at worst, not more than 20% apart. A major reason for the difference is that the lung models give different tracheal diameters, resulting in different flow rates for the
same tracheal Reynolds number.

Upon closer inspection of the results from the two lung models the tracheal diameter difference is seen to account for most of the difference at low Reynolds numbers. If the diameters are set equal, the Weibel model results then fall below the asymmetric model prediction at high Reynolds numbers. The reason for this is seen in Figure 8.5-2 which presents the pressure drop components for the Horsfield asymmetric model. In the higher Reynolds number range the pressure loss associated with the mixing due to the model asymmetry represents about 20% of the total static pressure loss. The symmetry of the Weibel model precludes this type of loss, and results in the static pressure prediction falling below the prediction of the asymmetric model.

Pressure Distributions

The distribution of pressure within the lung is important to determine the relative contribution of each generation to the overall pressure-flow characteristics. A difficulty arises in trying to present pressures and pressure gradients in the asymmetric lung model because it will depend on the path chosen. Unlike the symmetric airway model, the static pressure and viscous dissipation pressure drops will not be the same in branches which are the same number of generations distal to the trachea.

Additionally, the number of generations between the trachea and alveoli will also vary depending on the path chosen with
the asymmetric lung model. Figure 8.5-3 presents the percentage of paths through each of the BPSs with a given number of branches between the trachea and alveoli for the horsfield lung model 2. Shown are the percentage of paths with a certain number of total generations following the shortest and longest possible path for each BPS. Note that the asymmetric lung model will have many more generations into the lung than the symmetric Weible model which consists of 18 generations of conducting airways.

To present the data averaged quantities will be used. The average used for the asymmetric model is a simple arithmetic mean of the values at a specified number of generations into the lung. No flow percentage weighting is used. Figure 8.5-4 presents a comparison of the static pressures verses generations into the lung for a range of flow rates. Because of the averaging problem, the averaging static pressure in the asymmetric lung model is actually an integration of the average static pressure drop in each generation.

Comparison of the results from the two models shows generally good agreement. The gradient of static pressure is seen to extend more deeply into the lung for low flow rates, demonstrating the increased importance of frictional resistance. At higher flow rates the inertial pressure difference in the larger airways dominates. An interesting difference exists between the results of the two models; the pressure gradients are seen to remain at significant levels further into the lung for the asymmetric lung model.
Viscous Pressure Drops

The reason for the more significant static pressure gradients deep inside the lung with the asymmetric model is the difference in frictional characteristics. Figure 8.5-5 presents the pressure drop associated with viscous energy dissipation, including the curved tube type friction and mixing losses, vs generation into the lung for the two models. Surprisingly, the asymmetric lung model shows that two local maximums for the frictional pressure drops can occur. The first is similar in nature to the one predicted by the symmetric lung model near generation 5. The second occurs much more deeply in the lung, near generation 22.

To ensure that this second maximum does indeed occur, and is not a manifestation of the averaging process, the average viscous pressure drop ± 1 standard deviation for four different flow rates are shown in Figure 8.5-6. The behavior of the entire band of results indicates that this second maximum occurs, and it is therefore not likely to be a aberration generated by the averaging process.

Note that the second maximum is most pronounced at lower flow rates, presumably that as the flow rates are increased with the higher Reynolds numbers near the trachea, the terms which behave as $Re^{1.5}$ (curved tube type friction) and $Re^2$ (mixing loss terms) become dominate.

To better understand the reason for the second maximum, the results were recast to present the frictional pressure drop vs
diameter, Figure 8.5-7. The results once again indicate that the second local maximum exists, and did not result from a peculiarity associated with the distribution of average airway diameter with generation. Figure 8.5-8 presents the average diameter and length to diameter ratios for both lung models. Apart from the difference in number of generations, both lung models demonstrate the same general dimensional characteristics with generation into the lung.

The reason for the second maximum must therefore be based on the interaction fluid dynamic phenomena and the asymmetry of the lung geometry. To analyze how a local maximum can occur in the asymmetric lung geometry, consider a single bifurcation with unequal flows in each daughter branch. Neglecting mixing losses, which are not important deep into the lung, the viscous pressure drops are simply the curved tube type result found for symmetric flow conditions. The ratio, $\Phi$, of the viscous pressure loss in the parent branch, of order $N$, to the daughter branch, of order $N-1$, is given by:

$$8.4.2 \quad \Phi = \left[ \frac{1 + \Gamma}{2} \right] 1.5 \left[ \frac{D_{n-1}/D_n}{0.743} \right] 3.5 \frac{[L/D]^n}{[L/D]^{n-1}} \quad \text{Re} \gg 60$$

$$8.4.3 \quad \Phi = \left[ \frac{1 + \Gamma}{2} \right] \left[ \frac{D_{n-1}/D_n}{0.794} \right]^3 \frac{[L/D]^n}{[L/D]^{n-1}} \quad \text{Re} \leq 60$$

The ratio is constructed from three terms; a flow ratio factor, diameter ratio factor and L/D factor, which are nominally near unity. The product of these ratios will
determine the factors for the Horsfield model 2 verses generations into the lung. The results are shown in Figure 8.5-9, and indicate that the local maximum is primarily due to the product of the flow ratio factor and diameter ratio factor reaching a minimum below 1 near generation 22. The minimum is not likely to occur with the symmetric flow model because the flow ratio factor is identically unity, and the diameter ratio remains above 1.

Comparison of Different Flow assumptions

For comparison purposes, Figure 8.5-10 presents the prediction for the viscous flow resistance verses number of generations into the Weibel lung model for three different flow assumptions. The figure presents the results of Pedley et al (1970) for inspiratory flows, the results of this study for expiratory flows and for a baseline comparison the resistance assuming Poiseuille flow.

The results show that the three flow assumptions produce significantly different viscous pressure profiles above about the 10th generation. Below the 10th generation, the Reynolds number drops low enough that the flow is essentially Poiseuille like. The difference in profile between the inspiratory and expiratory is due primarily to the difference in L/D dependence. Both inspiration and expiration depend on Reynolds number to the 1.5 power, but Pedley et al found a square root dependence on L/D where this study found a linear dependence.
8.6 Physiological Data

The final comparison which can be drawn between the numerical pulmonary predictions and published data is with experiments conducted on living humans. The available data is sparse presumably due to the difficulties associated with simultaneously monitoring alveolar and tracheal (not mouth) pressures to determine the pressure-flow character of the intrathoracic airway. The data also tend to be incomplete, with such critical parameters as lung volume, tracheal diameter not reported, and contain great variability.

Most of the data is not available at the lung volume, 75% TLC, for which the airway models are based. To correct for lung volume, the dimensions of the lung can be approximated to change with lung volume to the 1/3 power. This approximation was found to be valid for the lower airways by Hughes et al (1972), but can only be considered a rough approximation for the trachea and bronchial airways which are supported with cartilage rings.

To determine the lung volume correction for the numerical prediction, the dimensional airway resistance can be written in terms of the dimensionless Moody type friction factor for the airway;

\[ 8.6.1 \quad \text{Res} = \frac{8 \cdot \zeta \cdot f'}{(\pi \cdot D_t^2)^2} \cdot Q \]

where \( D_t \) is the tracheal diameter, and the friction factor is;
The airway resistance at any lung volume can therefore be expressed in terms of the calculated resistance at 75% TLC and the same flow rate by;

\[
f' = \frac{\Delta P}{\frac{1}{2} \rho V^2}
\]

\[
8.6.2
\]

where \( f'' \) is friction factor calculated at the new Reynolds number due to the change in tracheal diameter for a fixed flow rate.

Writing the lung volume, \( LV \), in terms of the lung residual volume, \( RV \), and vital capacity, \( VC \), the lung volume at 75% TLC is;

\[
8.6.4 \quad Vol_{75\% TLC} = 0.75( RV + VC )
\]

The actual lung volume can also be rewritten as;

\[
8.6.5 \quad LV = RV + ( LV - RV )
\]

Taking the ratio of 8.6.4 to 8.6.5, and dividing through by \( VC \) gives;

\[
8.6.6 \quad \frac{Vol_{75\% TLC}}{LV} = \frac{0.75 ( RV/VC + 1 )}{RV/VC + %VC}
\]

Where \%VC lung volume in terms of the percentage of lung vital capacity, and the ratio of residual volume to vital capacity is approximately constant at a value of 0.29 for the studies below.

Figures 8.6-1 to 8.6-3 presents the numerical prediction

The predictions fall within the experimental observations, but typically tend to be near the high end of the data. While this tendency is not very significant, it is possibly, as with the comparison with the Weibel model, due to the relatively small tracheal diameter of 1.6 cm. The agreement appears to be equally good over the entire flow range tested, from .5 to 2 L/sec. The lung volume correction, while only a rough approximation, appears to predict the character of the results well.

A similar approach can be utilized to address the simulation of pulmonary flows with gasses other than air. These tests have proven useful in some clinical settings to augment the pressure drops at low flow rates, making measurements easier and more reliable with conventional pressure transducers. The dimensional groupings which characterize the flows are the Reynolds number and characteristic pressure, P*. Increasing both Nu and ζ, increase P*, while decreasing Reynolds number for a fixed flow rate. The effect on the dimensional resistance at a fixed flow rate can be expressed similarly to equation 8.6-3;

\[
Res_{\text{Gas}} = Res_{\text{air}} \left( \frac{f_{\text{air}}}{f_{\text{gas}}} \right)
\]

where the friction factors are calculated at the Reynolds
number given the flow rate, airway diameter, gas viscosity, and density.
IX. Conclusions

A new experimental technique was developed to measure the pressure-flow character of a "typical" bifurcation embedded within a lung. The technique was tested on a section of the entrance region in a straight tube flow, a well understood flow. Experimental errors of less than 5% demonstrated the validity of the technique, and can serve as an estimate of the error in measuring the static pressure difference across a bifurcation.

Experiments utilizing a range of water-glycerin mixtures allowed the physiologically important Reynolds number range to be examined;

$$50 \leq \text{Re} \leq 8,000$$

Analysis of the results indicate that the character of the flow within the bifurcation is similar to flows typified by mechanisms which thin the boundary layer.

- Curved tube flow
- Entrance flow in a tube
- Flow in a two dimensional convergent channel

The boundary layer control/growth similarity is drawn because of the experimental results indicating;

- Lack of sensitivity to core flow details
  - Turbulence was observed at a Reynolds number of about 1000, but no change in the frictional pressure flow character accompanied the transition.
  - The orientation upstream bifurcations had no effect on the pressure-flow results in the test bifurcation.
- Velocity profiles were characterized by a kinetic energy correction factor, $f_a$, which was shown to be
similar in form and magnitude to flows with boundary layer growth.

\[ f_a = 1.05 + 6.54/\sqrt{Re} \]

- The form and magnitude of the pressure drop due to viscous dissipation is also similar to the boundary layer flows;

\[ \Delta P_{\text{v}} = \Delta \left( 0.556 + 0.67 \sqrt{Re} \right) \]

The curved tube flow analogy was found to apply best for the range of "perturbation" experiments performed, although the 2D convergent channel flow also closely predicted the observed behavior.

- Approximately linear in L/D
- Relatively insensitive to shape with:
  - the use of a hydraulic diameter
  - the corresponding change in curvature ratio

The preference for the curved tube analogy over the 2D channel analogy also results from the similarity in the observed secondary flow patterns. The existence of the secondary flows, and the similar scaling with curved tube flows presents a strong case for the influence of curvature. The linear dependence in the frictional term was also demonstrated, both in the "perturbation" experiments, and by the ability to predict the D.B Reynolds (1980) experiments in which L/D's varied from 5 to 9. While the curved tube flow analogy may be preferred slightly over the 2D channel analogy, each is likely to be of importance, acting together, and not as isolated effects.

The effects of asymmetric flows on the pressure-flow results can be approximated as an additional loss
corresponding to the loss in the excess momentum of the unequal velocities.

\[ \Delta P = \frac{P}{m} \left( \frac{\Gamma - 1}{\Gamma + 1} \right)^2 \]

Numerical model predictions of idealized lung models were in excellent agreement with published experimental results
- Geometric pulmonary models
- Physiological models and casts

Numerical simulations of the complete human lung also demonstrated excellent agreement with the available physiological data. The use of an asymmetric lung model was shown to be important, explaining the observed increased pulmonary dependence of gas viscosity at low flow rates. The distribution of the viscous pressure losses demonstrated that this increase is due primarily to increased resistance in the lower airways. This finding helps to explain the utility of the MEFV curve in diagnosing lower airway pulmonary disorders.

The dimensional scaling shows that this effect occurs at low Reynolds numbers, not necessarily low flow rates, which highlights the utility of using gasses other than air to generate measurable pressures at low Reynolds numbers flows.

With the ability of these results to predict pressures resulting from specified flows, they can be applied to the collapsible tube flow pulmonary models with more confidence. Additionally, the deeper understanding of expiratory flows in general may prove useful in a wider context, with possible applications to mass and thermal transfer processes.
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Figure 1.3-1  Idealized dimensions of a pulmonary bifurcation, after Pedley (1971)
Figure 2.1-1  A typical maximal expiratory flow volume (MEFV) curve, and the effect of expiratory effort, after Bates et al (1971)
Previous Investigations

Normalized Frictional Loss in Each Leg of a Bifurcation

Legend
B-B Reynolds (1980)
C-C Reynolds (1982)
D-D Reynolds & Lee (1979)

Figure 2.3-1 Comparison of published frictional pressure losses for the idealized bifurcation.
Figure 3.1-1 Typical four cell secondary flow pattern in the parent branch downstream of a bifurcation.

Figure 3.2-1 Velocity profile in a fully developed laminar tube flow, and the resulting force balance acting on a section of the tube.

Figure 3.3-1 Velocity profile in fully developed turbulent pipe flow.
Figure 3.4-1  Stepwise contracted tube pulmonary model, with constant L/D ratio's of 3.5, and area ratio's of 1.2.

Figure 3.5-1  Representation of laminar flow in a 2-Dimensional convergent channel.
Convergent Channel Flow

Velocity Profiles

Figure 3.5-2  Computed velocity profiles in a 2-D convergent channel flow for a range of flow conditions.
Figure 3.6-1  Secondary flow pattern in fully developed curved tube flow.

Figure 3.6-2  Generation of the four cell secondary flow pattern typical of expiratory flows from modeling a bifurcation as two curved tubes oriented back to back.
Figure 3.6-3  Toroidal coordinate system used to analyze curved tube flow.
Friction Loss in a Curved Tube
Dean Solution

Figure 3.6-4 Prediction of the pressure loss in a curved tube, normalized by the Poiseuille loss at the same flow rate from the theory of Dean (1927).
Friction ratios for various curvature ratios. ○, White (1929); ●, Adler (1934); △, Itô (1959) (κ = 2D (2aW₀/ν)(a/R)¹/²). From Van Dyke (1978).

**Figure 3.6-5** Experimental results for the pressure loss in a curved tube, normalized by the Poiseuille loss that would occur at the same flow rate, after Berger et al (1983)
Figure 3.9-1 Pressure loss correlations for curved tubes, entrance flow, 2-D convergent channel flow, and turbulent flows in straight and curved tubes.
Subtraction Technique Schematic

Plenum $P_I$

Static Pressure Drop

$$\Delta P_S = (P_U - P_O) = (P_I - P_O) - (P_I - P_u)$$

Figure 4.1-1 Schematic representation of the subtraction method for symmetric flow conditions.
Figure 4.1-2 Pressure distributions owing to a 90° pipe bend between two long straight tubes, after Ito (1960)
Quasi-Steady Test Apparatus

\[ Q = A_R \frac{d(P/\rho g)}{dt} \]

Figure 4.4-1 Schematic of the quasi-steady experimental test apparatus and pressure measurement locations.
Kinematic Viscosity of Glycerin
Data from C.R.C., @ 20c

Figure 4.5-1  Kinematic viscosity of water-glycerin mixtures as a percentage of water content, from C.R.C.
Viscosity - Temperature
Measured with Haake Viscometer

Figure 4.6-1 Measured kinematic viscosity for a 21% by water glycerin-water mixture as a function of temperature using Haake viscometer, and C.R.C. value.
Calibration Errors

cm H20

Figure 4.6-2  Typical errors between tank level calibration manometer readings and predicted levels based on the curve fits and output voltages.
Figure 4.8-1  Entrance test experimental bellmouth and straight tube geometry.
Reservoir Tank Level vs. Time
Short Entrance Test

Figure 4.8-2  Tank level vs time measurements for the short entrance test (L/D = 6.3) at kinematic viscosities of 0.475 and 0.125 cm²/sec
Differential Height Signal
Typical Asymptotic Region Data

Figure 4.8-3  Asymptotic region of the differential pressure signal, showing a small offset error on the order of the digitation errors.
Error in Curve Fit
Curve Fit Value – Data Value

Figure 4.8-4  Error between the curve fit and the actual measured tank level, note that the magnitude of the error are typically around the digitation level, and not systematic.
Calculated Reynolds Number vs Time
Short Entrance Experiments

Figure 4.8-5  Calculated Reynolds numbers vs time for the short tube entrance test with kinematic viscosities of 0.125 and 0.475 cm²/sec.
**Quasi-Steady Entrance Experiments**

Static pressure loss normalized by \( P^* = \rho \nu^2 / D^2 \), vs Reynolds number

**Figure 4.8-6** Combined dimensionless pressure vs Reynolds number for both entrance tests and the subtraction of the two, and theoretical predictions.
Entrance Theory & Experimental Results

Static pressure loss normalized by $P^* - \rho u^2 / D^2$, vs Reynolds number
For the entrance section of a tube from L/D=6.3 to L/D=9.8

Figure 4.8-7 Results for the loss in the additional length of tubing and theory
Model Bifurcation Construction

Figure 4.9-1  Cross-section of a typical bifurcation showing the cast section and aluminum coupling collars.
Momentum Analysis

Figure 5.1-1  Momentum analysis of an exiting jet impinging on a flat plate and turning 90°.
Figure 5.1-2 A) Jan, Re = 200

Figure 5.1-2 B) Station 2S, Re = 1060

Figure 5.1-2 Axial velocity profiles used to calculate the axial correction factor $f_a$. A) Jan, Re=200; B) Chang & Menon, Re=1060; C) Chang & Menon, Re=5712; D) Chang & Menon, Re=6000; E) Chang & Menon, Re=6811; F) Hardin & Yu, Re=2760; G) D.B. Reynolds, Re=2260, Re=8000 and Re=12200
Figure 5.1-2 C)
Station 9, Re = 5712

Figure 5.1-2 D)
Station 4, Re = 6000

Figure 5.1-2 E)
Station 5, Re = 6811
Figure 5.1-2 F)
Hardin & YU
Re = 2760

○ IN PLANE OF BIFURCATION
□ NORMAL TO PLANE OF BIFURCATION

VELOCITY U, m/s

Re₀ = 2760

Figure 5.1-2 G)
D.B. Reynolds
Re = 2260
Re = 8000
Re = 12200
**Fa vs Ba**

**Experiments and Estimation**

![Graph](image)

**Figure 5.1-3** Cross plot of $fa$ and $Ba$ as calculated from the numerical integrals of published velocity profiles, and approximate relation.
Figure 5.1-4 Results of the numerical integrals of $fa$ vs tracheal Reynolds number, and curve fit.
Boundary Layer Approximation

Figure 5.1-5  Approximate flow profile used to estimate the mean flow rate and fa factor based on boundary layer thickness.
Fa Comparison
Experiment and Theory

Flow Type

- Entrance, L/D = 3.5
- 2-D Channel, alpha = 0.05
- Curved Tube, Curvature Ratio = 1/7
- Entrance, L/D = 1.75

Experimental Data

Figure 5.1-6 Prediction of fa based on the boundary layer approximation for an entrance prediction, at L/D=1.75 and 3.5, curved tube flow, δr=1/7, and 2-D convergent channel flow, α=0.05.
Figure 5.2-1 A) Secondary velocity profiles used to numerically evaluate the secondary velocity correction factor, $f_s$; A) Chang & Menon Re=1060, @station 3, B) Chang & Menon Re=1060 @station 2s, C) Jan Re=200.
Figure 5.3-1  Normalized pressure difference measured between the top and side of bifurcation leg during expiration vs local Reynolds number.
Figure 6.1-1  Experimental results for the reservoir level vs time for the experiments, Gen's 4-2, Nu=.475; Gen's 4-2, Nu=.125; Gen's 4-1, Nu=.475; Gen's 4-1, Nu=.125.
Planar Bifurcation Results
Static pressure loss/P*tr vs tracheal Reynolds number

Legend
A-A 4 Gen. Network, \( \nu=0.476 \text{ cm}^2/\text{sec} \)
B-B 4 Gen. Network, \( \nu=0.125 \text{ cm}^2/\text{sec} \)
C-C 3 Gen. Network, \( \nu=0.497 \text{ cm}^2/\text{sec} \)
D-D 3 Gen. Network, \( \nu=0.125 \text{ cm}^2/\text{sec} \)
E-E Drop Across Test Bifurcation

Note: P* and Reynolds number refer to tracheal values in the test bifurcation.

Figure 6.1-2 Subtraction of the three generation "upstream" airways from the four generation airway to give the normalized static pressure loss in the test bifurcation.
Figure 6.1-3  Decomposition of the total static pressure loss across a bifurcation into a kinetic energy and frictional component normalized to $\frac{\Delta P}{\frac{1}{2} \rho v^2} \cdot \Delta P_s$.

$\Delta P_k$

$\Delta P_v$

$\Delta P_s$
Symmetric Bifurcation Results

Frictional pressure loss/Poiseuille loss vs Reynolds Number

Legend

A-A 3 Planar Upstream Airways
B-B 2 Planar Upstream Airways
C-C 3 Upstream Airways, Config. A
D-D 3 Upstream Airways, Config. B
E-E 3 Upstream Airways, Config. C

Figure 6.1-4 Frictional pressure loss across a single bifurcation normalized to the Poiseuille loss for the symmetric flow experiments.
Figure 6.3-1 Configurations for the nonplanar symmetric flow experiments.
**Results for Additional Parent Length**

Figure 6.4-1  Subtraction of the three generation "upstream" airways from the three generation airway with the additional tracheal length of $L/D = 3.5$.  

![Graph showing the relationship between Airway with Additional Parent Tube Length, Airway Alone, and Loss in Additional Parent Tube Length vs. Reynolds Number.](image)
Pressure Loss in Additional Parent Tube Length

![Graph showing experimental results and prediction form for frictional pressure loss.]

**Experimental Results**

**Prediction Form**

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*Figure 6.4-2* Frictional pressure loss in a straight tube, L/D = 3.5, downstream of a bifurcation, normalized to the Poiseuille loss in the equivalent length of tubing.
Symmetric Bifurcation Results

![Graph showing Symmetric Bifurcation Results]

Experimental Results
 +/- 5% Band

Reynolds Number

**Prediction Form**
- Curved Tube
- Entrance
- 2D Channel

**Figure 6.5-1** Comparison of curved tube, entrance theory and 2-D convergent channel flow to the band of symmetric flow results.
Figure 6.6-1 Photographs of the oscilloscope traces from the hot wire probe, not transition to turbulence appears to occur near Re=1000.
"Schematic representation of bronchial lumen, bronchial wall, peribronchial space, limiting membrane (Im), and surrounding alveolar tissue. Intrabronchial (Pib), extrabronchial (Pexb), alveolar (Palv), and pleural (Ppl) pressures are indicated as well as elastic recoil pressure of bronchus [Pel(br)] and alveoli locally [Pel(local)]. Bronchial narrowing during expiratory flow (indicated by arrow) is illustrated by lighter shading of bronchial wall."

**Figure 6.7-1** Schematic representation of the bronchial support structure and pressure forces, after Hughes (1974)
Stereoscopic measurements of bronchial cross-sections at various transmural pressures, and two lung inflations.
Semi-Circular Airway Results

Experimental Results
$\pm 5\%$ Band

Prediction Form

Figure 6.7-3 Normalized frictional loss in the semi-circular test bifurcation
Figure 7.1-3  Most typical asymmetric bifurcation model in the middle airways, after Olson (1971)
Quasi-Steady Test Apparatus
(Modified for the Flow Partition Experiments)

\[ Q = A_R \frac{d(P/\rho g)}{dt} \]

Water/Glycerin

Reservoir

Gate

Weir

Collection Tank

Figure 7.2-1 Experimental apparatus for the flow partition experiments. Note separate fluid circuits.
Control volume selected to analyze the pressure loss in a bifurcation with asymmetric flow conditions.
**Static Pressure Drop**

Across the Variable Flow Branch with Asymmetric Flow

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**Figure 7.4-1** Results for the static pressure loss across the variable flow leg of a bifurcation, for six different fixed flow rates in the other leg vs flow ratio, \( \Gamma \).
Decomposition of the static pressure loss across a bifurcation into a kinetic energy component vs tracheal Reynolds number. (fixed daughter Reynolds # of 1506)
Figure 7.4-3 Results for the excess pressure loss due to the mixing of two streams vs $\Gamma$ (flow ratio)
Momentum Analysis
(For Equal Daughter Tube Diameters)

\[ P' - P = \varphi \left( \frac{1}{2}U_{31}^2 + \frac{1}{2}U_{32}^2 \right) - \left( \frac{1}{2}U_{31} + \frac{1}{2}U_{32} \right)^2 \]

\[ = \frac{2}{3} \varphi \left( \frac{U_{31} - U_{32}}{U_{31} + U_{32}} \right)^2 = \frac{1}{2} \varphi \left( \frac{r - 1}{r + 1} \right)^2 \]

Figure 7.4-4  Momentum analysis of the maximum pressure recovery due to the unification of two unequal flows.
Comparison of Investigators Results
Viscous Pressure Loss in an
Idealized Bifurcation

Figure 8.1-1 Comparison of the results for the friction pressure loss in an idealized bifurcation for this and previous investigations.
Figure 8.2-1  Geometry of a typical asymmetric bifurcation used for analysis.
Pressure-Flow Results
Experiment vs. Prediction
(Idealized 3 Gen. Airway Network)

Figure 8.3-1  Comparison of numerical prediction, and experimental results of Hardin & Yu, for the static pressure drop across an idealized three generation pulmonary model.
Pressure-Flow Results
Experiment vs Prediction
("Y" Connectors, L/D=5)

Pressure-Flow Results
Experiment vs Prediction
("Y" Connectors, L/D=9)

Figure 8.3-2 Comparison of numerical prediction, and experimental results of D.B Reynolds, for the static pressure drop across three generations of rigid "Y" connectors.
Zavala Lung Cast
TRACHEOBRONCHIAL TREE

Figure 8.4-1 Depiction of the Zavala lung model, courtesy of Medi-tech, Watertown Ma.
Zavala Lung Model
"Dichotomous" Branching Pattern

Figure 8.4-2  Dichotomous branching pattern approximation for the Zavala lung model.
Zavala Lung Cast Results
Experiment vs Prediction

Model Prediction
Horsfield Flow Distribution
Uniform Pressure Flow Dist

Experimental Results

Figure 8.4-3 Comparison of numerical prediction, and experimental results of Isabey and Chang, for the static pressure drop across the Zavala lung model.
Zavala Lung Cast Results
Pressure Drop Components

Figure 8.4-4  Pressure drop components which comprise the calculated static pressure drop across the Zavala lung model for uniform pressure upstream boundary conditions.
Zavala Lung Cast
Resistance Matrix

Perspective View

$R_{ij} = \frac{D_{Pi}}{D_{Qj}}$

$17 < \text{BPS} \# < 33$

Horizontal View Along Diagonal

Resistance Matrix
$R_{ij} = \frac{D_{Pi}}{D_{Qj}}$

Figure 8.4-5  Calculated Resistance matrix for the Zavala lung model.
Flow Distribution Results
Zavala Lung Cast

Figure 8.4-6  A comparison of the flow distributions for the Zavala lung model.
Figure 8.4-7  Asymmetric branching pattern for the lung according to the Horsfield et al (1971) lung Model 2.
Pressure--Flow Results  
Experiment vs. Prediction  
(Human Lung Cast Below R.I.B.)

**Figure 8.4-8** Comparison of numerical prediction, and experimental results of Reynolds and Lee, for the static pressure drop across a pulmonary cast below the right intermediate bronchus.
Pulmonary Pressure–Flow Prediction
Resistance and Friction Factors

Figure 8.5-1 Comparison of the overall pressure-flow characteristics for the Weibel and Horsfield lung models.
Pulmonary Pressure Drop Components
Horsfield Asymmetric Pulmonary Model

Figure 8.5-2  Pressure drop components which comprise the calculated static pressure drop across the Horsfield lung
Horsfield & Cumming Model 2
Distribution of Number of Generations

% of B.P.S. Paths

Number of Generations into the Lung

Figure 8.5-3  Percentage of "long" and "short" paths terminating at a given number of bifurcations into the lung.
Figure 8.5-4  Static pressure vs generation into the lung for both the Weibel and Horsfield lung models for several flow rates.
Weibel Lung Model Results
Viscous Pressure Drop vs. Generation

Horsfield & Cumming Results
Viscous Pressure Drops vs Generation

Figure 8.5-5  Normalized viscous pressure drop vs generation into the lung for both the Weibel and Horsfield lung models for several flow rates.
Horsfield & Cumming Model
Viscous Pressure Loss Distribution

Flow Rate = 125cc/Sec

Flow Rate = 250cc/Sec

Flow Rate = 500cc/Sec

Flow Rate = 1 L/Sec

Figure 8.5-6 The variation of normalized viscous pressure drop vs generation into the lung for Horsfield lung models for several flow rates.
Horsfield & Cumming Model 2 Results
Viscous Pressure Drops vs Diameter

Average Viscous Pressure Drop/KE

Flow Rate
- .125 L/S
- .25 L/S
- .5 L/S
- 1 L/S
- 2 L/S

Average Airway Diameter cm

Figure 8.5-7 Normalized viscous pressure drop vs average airway diameter for the Horsfield lung model at several flow rates.
Comparison of Lung Models

Figure 8.5-8  Average and variation of the diameter and L/D ratio vs generations into the lung for both the Weibel and HOrsfiels lung models.
Viscous Pressure Drop Factors vs. Generations into the Lung (Low Reynolds # Form)

Viscous Pressure Drop Factors vs. Generations into the Lung (Large Reynolds # Form)

Figure 8.5-9  High and low Reynolds numbers Ratio factors vs generation for the horsfield et al lung model.
Weibel Lung Model Comparison
Expiration, Inspiration and Poiseuille Flows

Flow Rate = 0.125 L/S

Flow Rate = 0.25 L/S

Flow Rate = 0.5 L/S

Flow Rate = 1.0 L/S

* Expiration  ◇ Inspiration  * Poiseuille

Figure 8.5-10 Comparison of the effect of flow types; inspiration, expiration and Poiseuille flows.
Figure 8.6-1  Comparison of the numerical prediction for pulmonary resistance based on the horsfield lung model and the experiments of Blide et al, and Vincent et al.
Airway Resistance Comparison
Flow Rate = 1 L/sec

Experiment vs. Numerical Prediction

Figure 8.6-2 Comparison of the numerical prediction for pulmonary resistance based on the horsfield lung model and the experiments of Hyatt and Wilcox.
Airway Resistance Comparison @ 50% Vital Capacity

Figure 8.6-3 Comparison of the numerical prediction for pulmonary resistance based on the horsfield lung model and the experiments of Ferris et al.