I. INTRODUCTION

The internal structure of a nucleon probed in inclusive scattering can be expressed in terms of four structure functions: two unpolarized structure functions ($F_1$ and $F_2$) and two polarized structure functions ($g_1$ and $g_2$). Within the quark-parton model $F_1$, $F_2$, and $g_1$ depend on unpolarized and polarized quark distributions. In contrast, $g_2$ has no direct link to quark distributions but is related to the interaction between quarks and gluons inside the nucleon. This makes the $g_2$ structure function ideal for the study of quark-gluon correlations.

The measurements of nucleon polarized structure functions in deep inelastic scattering (DIS) have been instrumental in advancing our understanding of quantum chromodynamics (QCD) (for a recent review of nucleon spin structure measurements, see [1,2]). In DIS, the incident electron interacts with the nucleon constituents by exchanging a virtual photon of four-momentum squared $q^2 = -Q^2$ and energy $\nu$. At very large values of $Q^2$, the lepton-nucleon interaction can be described by the incoherent sum of quasielastic scattering from asymptotically free quarks, with a momentum fraction $x = Q^2/(2M\nu)$ of the parent nucleon’s momentum ($M$ is the mass of the nucleon). Most of the former polarized structure function measurements were performed using nucleon targets polarized longitudinally with respect to the lepton spin. In this case the helicity-dependent cross section difference is dominated by the $g_1$ spin structure function, and as a result, this structure function is known with high precision in most kinematic regions.

In the quark-parton model, the contributions to the structure functions due to electron scattering off the asymptotically free quarks inside the nucleon are independent of $Q^2$, up to...
corrections due to gluon radiation and vacuum polarization. At high \( Q^2 \) these corrections can be accurately calculated using perturbative QCD. As \( Q^2 \) decreases, quark-gluon and quark-quark correlations make increasingly important contributions to the structure functions. In the \( g_1 \) structure function these correlation terms are suppressed by factors of \( (1/Q)^\tau \) with respect to the asymptotically free contributions. In the case of the second spin structure function, \( g_2 \), the nonperturbative multiparton correlation effects contribute at the same order in \( Q^2 \) as asymptotically free effects.

The moments of structure functions provide especially powerful tools to study fundamental properties of the nucleon because they can be compared to rigorous theoretical results such as sum rules and lattice QCD calculations. The operator product expansion (OPE) of QCD [3,4] can be used to relate the hadronic matrix elements of current operators to the moments of structure functions. In the OPE, the moments are expanded in a series ordered by \( 1/Q^\tau \). In this expansion \( \tau = 2, 3, 4 \ldots \) is known as the twist (dimension minus spin) of the operator. The twist-2 contributions to the moments correspond to scattering off asymptotically free quarks, where the higher twist contributions arise due to multiparton correlations.

The Cornwall-Norton (CN) moments of \( g_1 \) and \( g_2 \) are defined by the equation

\[
\Gamma_{1,2}^{(n)}(Q^2) \equiv \int_0^1 dx x^{n-1} g_{1,2}(x, Q^2). \tag{1}
\]

In addition, at high \( Q^2 \), the twist-3 reduced matrix element \( d_2 \) can be related to the second moment of a certain combination of \( g_1 \) and \( g_2 \):

\[
d_2(Q^2) = \int_0^1 dx x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)] = 3 \int_0^1 dx x^2 [g_2(x, Q^2) - g_{WW}^2(x, Q^2)]. \tag{2}
\]

Furthermore, the leading twist contributions to \( g_2 \) can be calculated using measured values of \( g_1 \) in the Wandzura-Wilczek relation,

\[
g_{WW}^2(x, Q^2) = -g_1(x, Q^2) + \int_0^1 dy y g_1(y, Q^2). \tag{3}
\]

Hence, it is possible to cleanly isolate the twist-3 contribution in a measurement of \( g_2 \) by subtracting the leading twist part.

II. THE EXPERIMENT

The measurement of \( g_2 \) requires a longitudinally polarized electron beam scattering off a longitudinally and also transversely polarized nucleon according to the following formula:

\[
g_2 = \frac{M Q^2 v^2}{4\alpha_e^2} \int_0^1 \frac{d\theta}{2E' E + E'} \left[ E + E' \cos \theta - E' \sin \theta \Delta \sigma_{\perp} - \Delta \sigma_{\parallel} \right], \tag{4}
\]

where \( \Delta \sigma_{\parallel} \) and \( \Delta \sigma_{\perp} \) are the polarized cross section differences corresponding to longitudinal and transverse target polarization directions, respectively. Their contributions to \( g_2 \) are weighted by three kinematical variables: the electron incident energy \( E \), the scattered electron energy \( E' \), and angle \( \theta \). The variable \( \alpha_e \) is the electromagnetic constant. As can be seen in Eq. (4), the transverse polarized cross section difference is the dominant contribution to \( g_2 \). In the present paper we report results from Jefferson Lab (JLab) Experiment E01-012 of the \( g_2 \) structure function measured in the nucleon resonance region at intermediate \( Q^2 \), using a polarized \( ^3 \)He target as an effective polarized neutron target. We formed polarized cross-section differences from inclusive scattering of longitudinally polarized electrons off a longitudinally or transversely polarized \( ^3 \)He target at a scattering angle of 25° for three incident beam energies, 3.028, 4.018, and 5.009 GeV, and at 32° for an incident beam energy of 5.009 GeV. The beam polarization varied between 71.4% and 84.9% during the experiment depending on the incident beam energy and the running status of the other experimental halls. The total relative uncertainty on the beam polarization was 3.4%. The target spin could be set to three directions with respect to the beam helicity: two longitudinal configurations with target spin direction at 0 and 180° and one transverse configuration at 90°. The average target polarization was \((38.0 \pm 2.0)%\) absolute. We used the two Hall A high resolution spectrometers in standard configuration for electron detection [12]. The structure function \( g_{2,\, ^3 \text{He}} \) was extracted at constant beam energies and scattering angles. However, the integrations to form moments require the structure function values at a constant \( Q^2 \). Therefore our \( g_{2,\, ^3 \text{He}} \) results were
interpolated to extract $g_2^{\text{He}}$ values at four constant $Q^2$ values of 1.2, 1.8, 2.4, and 3.0 (GeV/c)^2. The results from $g_2^{\text{He}}$ were reported in a previous publication [13,14] along with the details of the experimental setup and the systematic uncertainties relevant to both structure functions. Figure 1 presents the results on $g_2^{\text{He}}$ from E01-012 at the four $Q^2$ values. Also shown are the $g_2^{\text{He}}$ curves generated from the next-to-leading order (NLO) parton distribution functions of Refs. [8–11] using Eq. (3), including target mass corrections (TMCs) from the formalism of Ref. [15].

III. THE TWIST-3 REDUCED MATRIX ELEMENT $d_2$

The $g_2^{\text{He}}$ results at the four $Q^2$ values were used to evaluate the resonance region contribution to $d_2(Q^2)$ for $^3\text{He}$ of Eq. (2). The DIS contribution at each $Q^2$ value was evaluated from Eq. (3) with the already published E01-012 results of $g_2^{\text{He}}$ [13]. The $x$ region covered by our data corresponds to a range in the invariant mass of $1.080 \leq W \leq 1.905$ GeV at the given value of $Q^2$. Then $d_2(Q^2)$ for the neutron was extracted from $d_2^{\text{He}}(Q^2)$ using the method described in Ref. [16]:

$$d_2^2 = \frac{1}{p_n} d_2^{\text{He}} - 2 \frac{P_p}{p_n} d_2^p,$$  

where $p_n$ and $p_p$ correspond to the effective polarization of the neutron and proton inside $^3\text{He}$ [17]. This neutron extraction method is expected to be accurate at the 5% level [16]. The resonance and DIS contributions of $d_2^p$ were calculated using data from JLab experiment EG1b [18] for the proton spin structure function $g_1^p$ and the Hall B model [19] for $g_2^p$. A conservative uncertainty of 100% on $g_2^p$ was taken into account in our systematics uncertainties.

We extracted the inelastic contribution to $d_2^2$ at our four $Q^2$ values by adding the resonance and the DIS contributions (see Table I, where the results were multiplied by 10 for listing in the table). Including the $Q^2$ evolution from Ref. [20], we performed the weighted average and obtained $d_2^{\text{He}} = 0.00034 \pm 0.00045 \pm 0.00107$ for $(Q^2) = 2.4$ (GeV/c)^2, as shown in Fig. 2. The elastic contribution, shown separately, was evaluated using elastic form factors from Refs. [21,22] following the formalism of Ref. [23]. Uncertainties of 5%, 1%, 14%, and 2.5% were assigned to the proton and neutron form factors $G_1^p, G_2^p, G_1^n, G_2^n$, and $G_1^l, G_2^l$, respectively. JLab experiments E94-010 [24] and RSS [25] reported only the resonance contribution to $d_2^2$ and it can be seen that these data are in very good agreement with the MAID model [26]. Since $d_2(Q^2)$ is weighted by $x^2$, one would expect it to be dominated by the contribution coming from the resonance region, which sits at higher $x$ compared to the DIS region. Our data show the inelastic contribution to $d_2(Q^2)$ becoming very small by $Q^2 = 2$ (GeV/c)^2, as also indicated by the MAID model. JLab E99-117 [6,7] evaluated $d_2(Q^2)$ at $(Q^2) = 5$ (GeV/c)^2 including the previous data from SLAC experiment E155x [27]. The result shows $d_2(Q^2)$ to be large and positive, about 1.5$\sigma$ away from the lattice QCD prediction [29]. The trend of the experimental inelastic contributions at $Q^2 \leq 2.4$ (GeV/c)^2 and the falloff of the elastic contribution appear to be in agreement with the lattice QCD prediction at 5 GeV/c^2.

IV. THE BURKHARDT-COTTINGHAM SUM

The Burkhardt-Cottingham (BC) sum rule [31] is a superconvergence relation derived from a dispersion relation in which the virtual Compton helicity amplitude $S_2$ falls off to zero more rapidly than $\frac{1}{v}$ as $v \to \infty$. The sum rule is expressed as follows:

$$\Gamma_2(Q^2) \equiv \int_0^1 dx g_2(x, Q^2) = 0,$$  

and is predicted to be valid at all $Q^2$. It should be noted that the validity of the sum rule has been questioned [32,33]. Furthermore, the BC sum rule cannot be extracted from the OPE due to the nonexistent $n = 0$ expansion of $g_2$ moments. The data for $\Gamma_2(Q^2)$ at 5 (GeV/c)^2 from the SLAC E155x experiment showed that the BC sum rule is satisfied within a large uncertainty for deuteron. However, they found a violation of almost 3$\sigma$ for the more precise proton measurement.

We separate the full $\Gamma_2(Q^2)$ integral into DIS, resonance and elastic components as follows:

$$\Gamma_2(Q^2) = \Gamma_2^{\text{DIS}}(Q^2) + \Gamma_2^{\text{Res}}(Q^2) + \Gamma_2^{\text{EL}}(Q^2) = \int_0^{x_{\text{max}}(Q^2)} dx g_2(x, Q^2) + \int_{x_{\text{min}}(Q^2)}^{x_{\text{p}}(Q^2)} dx g_2(x, Q^2) + \int_{x_{\text{p}}(Q^2)}^1 dx g_2(x, Q^2).$$  

The variables $x_{\text{min}}(Q^2)$ and $x_{\text{p}}(Q^2)$ are the $x$ values corresponding to the invariant mass $W = 1.905$ GeV and to $W$ at pion threshold, respectively, at the given value of $Q^2$. We measured the $\Gamma_2^{\text{Res}}$ part in our experiment. The elastic contribution, $\Gamma_2^{\text{EL}}$, was evaluated using the method as described in the previous section. The quasielastic contribution to the $^3\text{He}$ BC sum was extracted from

$$\Gamma_2^{\text{He}, Q^2} = (p_n \Gamma_2^{n, EL} + 2p_p \Gamma_2^{p, EL})/f$$  

FIG. 2. (Color online) Result on inelastic contribution to the neutron $x^2$-weighted moment $d_2^2(Q^2)$ from E01-012. The elastic contribution is displayed by the brown band. The inner (outer) error bar represents statistical (total) uncertainty. The grey band represents the experimental systematics uncertainties. To be compared with the resonance contribution, we plotted the MAID model [26]. Also plotted are the total $d_2^2$ from SLAC E155x [27] and JLab E99-117 [6,7] combined, and, the lattice QCD prediction [28].
TABLE I. E01-012 results given at the scale of 10^{-5}.

<table>
<thead>
<tr>
<th>$Q^2 (\text{GeV}/c)^2$</th>
<th>Resonance ($10^{-5}$)</th>
<th>DIS ($10^{-5}$)</th>
<th>Elastic or QE ($10^{-5}$)</th>
<th>Total ($10^{-5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_2^u$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>186 ± 136 ± 156</td>
<td>−2 ± 6 ± 3</td>
<td>−2342 ± 204</td>
<td>−2158 ± 136 ± 257</td>
</tr>
<tr>
<td>1.8</td>
<td>−32 ± 177 ± 107</td>
<td>1 ± 9 ± 5</td>
<td>−1075 ± 96</td>
<td>−1105 ± 177 ± 144</td>
</tr>
<tr>
<td>2.4</td>
<td>−55 ± 118 ± 101</td>
<td>3 ± 7 ± 4</td>
<td>−468 ± 40</td>
<td>−520 ± 118 ± 109</td>
</tr>
<tr>
<td>3.0</td>
<td>80 ± 88 ± 137</td>
<td>13 ± 6 ± 2</td>
<td>−211 ± 16</td>
<td>−117 ± 88 ± 138</td>
</tr>
<tr>
<td>$\Gamma_2^{3\text{He}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>582 ± 245 ± 115</td>
<td>−162 ± 72 ± 41</td>
<td>−558 ± 31</td>
<td>−139 ± 255 ± 126</td>
</tr>
<tr>
<td>1.8</td>
<td>180 ± 182 ± 82</td>
<td>−114 ± 67 ± 36</td>
<td>−219 ± 12</td>
<td>−153 ± 194 ± 90</td>
</tr>
<tr>
<td>2.4</td>
<td>68 ± 94 ± 33</td>
<td>−55 ± 38 ± 18</td>
<td>−90 ± 5</td>
<td>−77 ± 101 ± 37</td>
</tr>
<tr>
<td>3.0</td>
<td>127 ± 68 ± 23</td>
<td>−3 ± 24 ± 7</td>
<td>−40 ± 2</td>
<td>84 ± 72 ± 24</td>
</tr>
<tr>
<td>$\Gamma_2^n$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>634 ± 285 ± 153</td>
<td>−26 ± 84 ± 50</td>
<td>−1165 ± 58</td>
<td>−558 ± 297 ± 171</td>
</tr>
<tr>
<td>1.8</td>
<td>114 ± 212 ± 141</td>
<td>12 ± 78 ± 43</td>
<td>−532 ± 27</td>
<td>−407 ± 226 ± 150</td>
</tr>
<tr>
<td>2.4</td>
<td>−9 ± 109 ± 98</td>
<td>21 ± 44 ± 24</td>
<td>−253 ± 13</td>
<td>−241 ± 118 ± 102</td>
</tr>
<tr>
<td>3.0</td>
<td>78 ± 79 ± 76</td>
<td>65 ± 28 ± 10</td>
<td>−128 ± 7</td>
<td>15 ± 84 ± 77</td>
</tr>
</tbody>
</table>

There is not enough experimental data currently available to evaluate $\Gamma_2^{\text{DIS}}$ in the $Q^2$ range covered by our experiment. Therefore, it is not possible to evaluate the full $\Gamma_2(Q^2)$ integral to test the BC sum rule without assumptions. Previously, JLab Hall A experiment E94-010 evaluated the BC sum, using the $\Gamma_2^{\text{WW}}$ part for the unmeasured DIS region, at six $Q^2$ values from 0.1 to 0.9 (GeV/c)^2. The same method was used here: $\Gamma_2^{\text{WW}}$ for $^3\text{He}$ is calculated using our $g_1^{3\text{He}}$ data [13]. The extraction of the neutron $\Gamma_2$ integrals were done using the same method as described by Eq. (5), using $g_2^p$ data from [18] and $g_2^n$ from Hall B model [19] to evaluate the proton $\Gamma_2^{\text{WW}}$ and $\Gamma_2^{\text{Res}}$ respectively. Figure 3 shows $\Gamma_2^{\text{Res}}$ and the extrapolated BC sum for $^3\text{He}$ and the neutron compared to the same quantities from the previous experiments E94-010 [24,30] and RSS [25]. It should be noted that RSS extracted their neutron result from the deuteron and the agreement with our data demonstrates that the nuclear corrections for deuteron and $^3\text{He}$ are well understood. All results are in good agreement with the BC sum rule for $^3\text{He}$ and within 2σ from the neutron BC sum rule, as shown on the bottom panel of Fig. 3 and in Table I (the results were multiplied by 10^5 for listing in the table).

V. CONCLUSION

In summary, we have measured the inelastic contribution to the neutron $d_2(Q^2)$ matrix element at $Q^2 = 2.4$ (GeV/c)^2 and found it to be very small, in agreement with the lattice QCD calculation. We also formed the $^3\text{He}$ and neutron $\Gamma_2$ moments over the $Q^2$ range of 1.2 to 3.0 (GeV/c)^2. Our data show both moments to be small and to gradually decrease in magnitude with $Q^2$. The BC sum for $^3\text{He}$ and the neutron was then evaluated from our data in the resonance region, adding the elastic contribution from elastic form factors and using $g_2^{\text{WW}}$ for the low x unmeasured part of the integral. Our data confirmed the validity of the BC sum rule at the 1.5σ level.

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