Miniature, High-Speed Imaging Transform Spectrometers and Advanced Sampling Algorithms

by

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Abstract

This thesis describes the development of miniature imaging Fourier transform spectrometers and irregular sampling techniques. An imaging spectrometer is a device that takes a series of images that include information from a variety of wavelengths of light at high spectral resolutions. When this device is combined with different types of input light for absorption, fluorescence or Raman scattering experiments, it can be used to differentiate between materials that are indistinguishable to the naked eye in a nondestructive, non-contact manner.

A variety of small devices that can fit in the palm of the hand or at the end of a robotic endoscope (up to 15 mm in diameter) are designed. The device could be set to several spectral analysis modes with resolutions up to 1.6\ cm^{-1} and is capable of producing continuous video. Such a device requires the design of custom optical, electrical, mechanical, and software systems. Small linear Lorentz force voice coil actuators, fast piezoelectric actuators, flexures, and miniature precision position and tip-tilt measurement systems are implemented and characterized. Feedback, feed-forward, and high speed hybrid-actuator control systems are also used for fast step positioning at up to 2 kHz.

It is also desirable to obtain spectra quickly, therefore methods for under-sampling and computing spectra with a small number of data points is explored. An adaptive sampling algorithm is devised and shown to outperform compressive sensing techniques for systems with larger occupancies (in the range of 10 % or more). For systems with less than perfect occupancy, adaptive sampling techniques can reduce the acquisition time for a spectrum by up to ten fold. Additional recursive update algorithms as well as uniformly sampled and non-uniformly sampling digital filtering algorithms were also explored in order to produce color video at 45 frames per second or faster from raw black and white data taken at 2010 frames per second without any physical filters.

Finally, several applications for the imaging spectrometer are explored including
the measurement of light sources like lasers and LEDs. The blood oxygen levels inside
the human finger are also explored along with the response of dissimilar fluorophores
under a microscope. Raman scattering spectroscopy is also conducted with the device
to characterize materials like diamond. The work conducted with these instruments
may lead to portable commercial devices in the future for a wide variety of appli-
cations including biology, environmental science, process monitoring, and material
identification.

Thesis Supervisor: Ian W. Hunter
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Contents

1 Introduction .................................................. 27
  1.1 Hyperspectral Imaging ..................................... 31
  1.2 Goals and Methodology .................................... 36

2 Scaling and Comparison .................................. 41
  2.1 Fourier Transform Spectrometers ....................... 41
    2.1.1 Advantages and Disadvantages ...................... 43
    2.1.2 Interference Equations .............................. 46
  2.2 Design Scaling ........................................... 47
  2.3 Comparison of Design Topologies ....................... 49
    2.3.1 Single-Pass Michelson Interferometer .............. 53
    2.3.2 Double-Pass Michelson Interferometer ............. 56
    2.3.3 Common Path Sagnac ................................. 57
    2.3.4 Electro-Optic Modulation ........................... 59
    2.3.5 Spectral Resolution Comparisons ................... 60
    2.3.6 Image Sensor Format Comparisons ................... 64
  2.4 Interferometer Ray Tracing ............................. 67

3 Design and Manufacturing ................................ 73
  3.1 Manufacturing Techniques ............................... 74
    3.1.1 Electric Discharge Machining ...................... 75
    3.1.2 Aluminum Anodization ............................... 78
    3.1.3 Thin Optical Components .......................... 81
  3.2 Lorentz Force Voice Coil Motor Design ................ 84
    3.2.1 Magnet Structure .................................. 87
    3.2.2 Flexure Design ..................................... 88
    3.2.3 Flexure Damping ................................... 92
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>Piezoelectric Actuator Design</td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>3.3.1 Series Actuation</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>3.3.2 Parallel Actuation</td>
<td>99</td>
</tr>
<tr>
<td>3.4</td>
<td>Software and Electronics</td>
<td>101</td>
</tr>
<tr>
<td></td>
<td>3.4.1 Electronics Design and Manufacturing</td>
<td>105</td>
</tr>
<tr>
<td></td>
<td>3.4.2 Microcontroller Architecture</td>
<td>106</td>
</tr>
<tr>
<td></td>
<td>3.4.3 FPGA Design Architecture</td>
<td>109</td>
</tr>
<tr>
<td></td>
<td>3.4.4 User Interface Design</td>
<td>111</td>
</tr>
<tr>
<td>4</td>
<td>Implementations</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>4.1 Prototype Design</td>
<td>116</td>
</tr>
<tr>
<td></td>
<td>4.2 Multi-Axis Design</td>
<td>121</td>
</tr>
<tr>
<td></td>
<td>4.3 Endoscope Hyper-Spectral Imaging Design</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>4.4 High Speed Design</td>
<td>131</td>
</tr>
<tr>
<td></td>
<td>4.5 Handheld Prototype Design</td>
<td>138</td>
</tr>
<tr>
<td></td>
<td>4.6 Implementation Comparison</td>
<td>142</td>
</tr>
<tr>
<td>5</td>
<td>Measurement and Control</td>
<td>145</td>
</tr>
<tr>
<td></td>
<td>5.1 Position Measurement</td>
<td>147</td>
</tr>
<tr>
<td></td>
<td>5.1.1 Two-Signal Analog Encoder</td>
<td>148</td>
</tr>
<tr>
<td></td>
<td>5.1.2 Four-Signal Analog Encoder</td>
<td>152</td>
</tr>
<tr>
<td></td>
<td>5.1.3 Phase Calculation</td>
<td>155</td>
</tr>
<tr>
<td></td>
<td>5.1.4 Phase Observer</td>
<td>156</td>
</tr>
<tr>
<td></td>
<td>5.2 Positioning System Characterization</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>5.2.1 Stroke</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>5.2.2 Temperature</td>
<td>164</td>
</tr>
<tr>
<td></td>
<td>5.3 Dynamic Modeling and System Identification</td>
<td>168</td>
</tr>
<tr>
<td></td>
<td>5.3.1 Lorentz Force Voice Coil Characterization</td>
<td>169</td>
</tr>
<tr>
<td></td>
<td>5.3.2 Piezoelectric Actuator Characterization</td>
<td>173</td>
</tr>
<tr>
<td></td>
<td>5.4 Feedback Controller Architecture</td>
<td>180</td>
</tr>
<tr>
<td></td>
<td>5.4.1 Translation Control</td>
<td>181</td>
</tr>
<tr>
<td></td>
<td>5.4.2 Multi-Axis Tip and Tilt Control</td>
<td>183</td>
</tr>
<tr>
<td></td>
<td>5.4.3 Fast Positioning Hybrid Control</td>
<td>184</td>
</tr>
<tr>
<td></td>
<td>5.4.4 Disturbance Rejection</td>
<td>187</td>
</tr>
<tr>
<td></td>
<td>5.5 Full Inversion Feed-Forward</td>
<td>188</td>
</tr>
</tbody>
</table>
5.5.1 Position Control ........................................ 190
5.5.2 Fast Positioning Hybrid Control ....................... 194
5.6 Controller and Imaging Coordination .................... 196
   5.6.1 Video Rate Coordination .......................... 196
   5.6.2 High Speed Camera Coordination ................. 197
   5.6.3 Long Integration Coordination .................... 199
5.7 Blur Considerations .................................... 200
   5.7.1 Timing and Step Size Effect ....................... 200
   5.7.2 Controller and Solver Effect ....................... 203
   5.7.3 Human Disturbances ............................... 204

6 Imaging Fourier Transform Applications .................. 207
   6.1 Intensity Measurements .............................. 208
      6.1.1 Input Light and Filters ......................... 208
      6.1.2 Compensation Curves ............................ 211
   6.2 White Light Interferometry ......................... 212
   6.3 Color Measurement .................................. 213
      6.3.1 Narrowband Laser Light ......................... 213
      6.3.2 Broadband LED Light ........................... 215
   6.4 Blood Oxygen Levels ................................ 218
   6.5 Fluorescence Applications ......................... 221
   6.6 Raman Applications ................................ 229
      6.6.1 Dispersive Measurements ....................... 232
      6.6.2 Diamond Characterization ....................... 234

7 Irregular Sampling Theory .................................. 237
   7.1 Generalized Solution Method ......................... 239
   7.2 Non-Baseband Sampling Methods ....................... 242
      7.2.1 Under-Sampling ................................ 243
      7.2.2 Optimal Under-Sampling ....................... 245
   7.3 Adaptive Under-Sampling ............................ 247
      7.3.1 Algorithm ....................................... 248
      7.3.2 Convergence of Simplified Algorithm .......... 250
      7.3.3 Full Simulation Performance ................. 252
      7.3.4 Experimental Results ......................... 258
7.4 Recursive Spectral Update

7.4.1 Algorithm

7.4.2 Noise and Forgetting-Factor Considerations

7.4.3 Experimental Results

7.5 Digital Filtering

7.5.1 Uniform Sampling Experimental Results

7.5.2 Non-Uniform IIR Digital Filtering

7.5.3 Non-Uniform FIR Digital Filtering

7.5.4 Non-Uniform Sampling Experimental Results

8 Conclusions and Recommendations

8.1 Project Goals

8.2 Future Work

Bibliography

A Hardware Design

A.1 Mechanical Parts Lists

A.2 Electronic Schematics

A.2.1 Power and Programming

A.2.2 Base Board

A.2.3 Sensor and Illumination Boards

A.2.4 Actuator Boards

A.2.5 Portable Power

A.2.6 Field Programmable Gate Array

B Software Design

B.1 Microcontroller Code Structure

B.2 User Interface Design

B.2.1 Main Window

B.2.2 Observer Window

B.2.3 Video and Image Grabber Windows

B.2.4 System ID and Feed-Forward Generator Window

B.2.5 Adaptive and Recursive Sampling Windows
List of Figures

1.1 Hyperspectral Imaging Applications ................. 29
1.2 Spectral Data Cube ................................ 32
1.3 Hyperspectral Imaging Spectrometer Design Prior Art .... 34
1.4 Interferometer Manufacturing Techniques Prior Art .......... 36

2.1 Imaging Spectrometer Field of View .................. 50
2.2 Comparison of Different Imaging Spectrometer Topologies .... 52
2.3 Optimization of Dimensions as a Function of OPD with Idealized Parameters ........ 62
2.4 Optimization of Dimensions with Vignette and Diffraction Limits as a Function of OPD with Idealized Parameters .......... 63
2.5 Optimization of Dimensions as a Function of OPD with Realistic Parameters .......... 64
2.6 Optimization of Dimensions as a Function of Camera Format with Realistic Parameters .......... 66
2.7 Ray Tracing Simulation and OPD Variation ............. 70

3.1 Optical Layout of the Imaging Fourier Transform Spectrometer .... 74
3.2 Electric Discharge Machining Manufacturing ............ 76
3.3 Aluminum Anodization Process ....................... 79
3.4 Sputter Coating of Thin Mirrors ....................... 81
3.5 Sputter Coating of Thin Mirrors for Scratch Resistance .... 82
3.6 Sputter Coating of Mirror Levels ...................... 83
3.7 Lorentz Force Voice Coil Motor and Flexure Design .......... 85
3.8 Simulations of the Magnetic Field Distribution of Motor Designs .... 87
3.9 Motor Magnet Designs, Simulation Specifications and Design Examples .... 88
3.10 Translation Flexure Design for Linear Motor ............ 88
3.11 Simulated Motor Specifications and Dynamics ..................................... 90
3.12 Two Double-Compound Flexure Design for the Linear Motor ................. 91
3.13 Damping of the Lorentz Force Voice Coil Motor Resonance ................. 93
3.14 Series Piezoelectric Tip-Tilt-Translate Actuator Design ....................... 97
3.15 Series Piezoelectric Tip-Tilt-Translate Actuator Simulation .................. 98
3.16 Series Piezoelectric Tip-Tilt-Translate Actuator Simulation for High Speed Implementation .................................................. 98
3.17 Piezoelectric Actuator Designs for High Speed Implementation .......... 100
3.18 Parallel Piezoelectric Tip-Tilt-Translate Actuator Simulation for High Speed Implementation .................................................. 101
3.19 Resonant Mode Analysis for High Speed Implementation ..................... 101
3.20 Electronics Architecture and Signal Pipelines .................................. 102
3.21 Power and Programming Circuit Layouts and Implementations ............. 105
3.22 Microcontroller State Diagram ...................................................... 107
3.23 Field Programmable Gate Array Imaging Sensor Data Handling ........... 110
3.24 User Interface Main Window ......................................................... 112

4.1 Development of Initial Prototype Actuators, Layouts, and Manufacturing Techniques .................................................. 116
4.2 Exploded View of Prototype Design ............................................... 117
4.3 Layout and Optical Path of Prototype Design ................................... 118
4.4 Circuit Board Layout and Implementation of Prototype Design ............. 119
4.5 Prototype Design Implementation and Test Electronics ....................... 120
4.6 Exploded View of Multi-Axis Design ............................................... 121
4.7 Layout and Optical Path of Multi-Axis Design .................................. 123
4.8 Circuit Board Layout and Implementation of Multi-Axis Design ............. 123
4.9 Internal Layout and Build Process of the Multi-Axis Design ................. 124
4.10 Multi-Axis Design Implementation and Test Electronics ..................... 126
4.11 Mechanical and Circuit Board Design for a Robotic Endoscope Without Hyper-Spectral Imaging .................................................. 127
4.12 Exploded View of Endoscope Hyperspectral Imaging Design ............... 128
4.13 Layout and Optical Path of Endoscope Hyperspectral Imaging Design .... 129
4.14 Circuit Board Layout and Implementation of Endoscope Hyperspectral Imaging Design .................................................. 130
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.15</td>
<td>Endoscope Hyperspectral Imaging Design Implementation and Test Electronics</td>
<td>131</td>
</tr>
<tr>
<td>4.16</td>
<td>Exploded View of High Speed Design</td>
<td>132</td>
</tr>
<tr>
<td>4.17</td>
<td>Implementation of Two Double-Compound Flexure Design for the Linear Motor</td>
<td>133</td>
</tr>
<tr>
<td>4.18</td>
<td>Layout and Optical Path of High Speed Design</td>
<td>134</td>
</tr>
<tr>
<td>4.19</td>
<td>Circuit Board Layout and Implementation of High Speed Design</td>
<td>135</td>
</tr>
<tr>
<td>4.20</td>
<td>High Speed Design Implementation and Test Electronics</td>
<td>136</td>
</tr>
<tr>
<td>4.21</td>
<td>Solid Model of Handheld Prototype</td>
<td>138</td>
</tr>
<tr>
<td>4.22</td>
<td>Layout of Handheld Prototype Assembly</td>
<td>139</td>
</tr>
<tr>
<td>4.23</td>
<td>Comparison of the Handheld Prototype</td>
<td>140</td>
</tr>
<tr>
<td>4.24</td>
<td>Electronics for the Handheld Prototype</td>
<td>141</td>
</tr>
<tr>
<td>5.1</td>
<td>Velocity Control versus Position Control for Spectrometer Sampling</td>
<td>146</td>
</tr>
<tr>
<td>5.2</td>
<td>Two-Signal Analog Encoder for Translation and Tilt</td>
<td>149</td>
</tr>
<tr>
<td>5.3</td>
<td>Two-Signal Analog Encoder Sensing</td>
<td>150</td>
</tr>
<tr>
<td>5.4</td>
<td>Four-Signal Analog Encoder for Translation, Tip and Tilt</td>
<td>152</td>
</tr>
<tr>
<td>5.5</td>
<td>Factors Affecting Encoder Sensitivity and Resolution and Encoder Mirror Surfaces</td>
<td>154</td>
</tr>
<tr>
<td>5.6</td>
<td>Direct and Iterative Phase Calculation Methods</td>
<td>156</td>
</tr>
<tr>
<td>5.7</td>
<td>Phase Observer Algorithm</td>
<td>157</td>
</tr>
<tr>
<td>5.8</td>
<td>Phase Observer Results for Translation, Tip and Tilt</td>
<td>158</td>
</tr>
<tr>
<td>5.9</td>
<td>Phase Observer Measurements of the Translation, Tip and Tilt Piezoelectric Actuators</td>
<td>159</td>
</tr>
<tr>
<td>5.10</td>
<td>Coordinate Rotation for Tip and Tilt Actuators</td>
<td>159</td>
</tr>
<tr>
<td>5.11</td>
<td>Encoder Reading Variations for Long Travel</td>
<td>161</td>
</tr>
<tr>
<td>5.12</td>
<td>Long Travel and Tilt Phase for Prototype Design</td>
<td>161</td>
</tr>
<tr>
<td>5.13</td>
<td>Long Travel, Tip and Tilt Phase for Multi-Axis Design</td>
<td>162</td>
</tr>
<tr>
<td>5.14</td>
<td>Images of Long Travel for High Speed Design</td>
<td>162</td>
</tr>
<tr>
<td>5.15</td>
<td>Temperature as a Function of Position for Prototype and Multi-Axis Versions</td>
<td>165</td>
</tr>
<tr>
<td>5.16</td>
<td>Temperature as a Function of Position for Endoscope and High Speed Versions</td>
<td>165</td>
</tr>
<tr>
<td>5.17</td>
<td>Temperature as a Function of Steady State Input Power for Different Implementations</td>
<td>166</td>
</tr>
</tbody>
</table>
5.18 Stochastic System Identification of Actuator System ................................ 168
5.19 Modeling for the Lorentz Force Linear Voice Coil .................................. 170
5.20 Step Responses for Different Motor Implementations ................................. 171
5.21 Motor Impulse Response, Mean Squared Coherence and Frequency Re-
   sponse ............................................................................................................ 172
5.22 Piezoelectric Actuator Step Responses for the Multi-Axis Version .......... 175
5.23 Translation Piezoelectric Actuator Impulse Responses and Step Re-
   sponses of Different Amplifiers for the Multi-Axis Version ....................... 175
5.24 Translation Piezoelectric Actuator Frequency Responses of Different
   Amplifiers for the Multi-Axis Version ........................................................ 176
5.25 Series Piezoelectric Actuator Step Responses and Frequency Responses
   for the High Speed Version ......................................................................... 177
5.26 Step Responses for Parallel Piezoelectric Actuator for the High Speed
   Version ........................................................................................................ 178
5.27 Parallel Piezoelectric Actuator Impulse Response, Mean Squared Co-
   herence and Frequency Response for the High Speed Version ................. 178
5.28 Piezoelectric Actuator Hysteresis for Translation, Tip and Tilt ............. 179
5.29 Feedback Controller Transfer Function Architecture ............................... 180
5.30 Feedback Controller Diagram for Translation, Tip-Tilt, and Fast Hy-
    brid Translation Controllers ........................................................................ 181
5.31 Closed Loop Feedback Control and Input Shaping for Translation .......... 182
5.32 Closed Loop Feedback Control on Tip and Tilt Angles .............................. 184
5.33 Closed Loop Feedback Control on Piezoelectric Actuator Translation .... 185
5.34 Input Shaping and Results for High Speed Hybrid Translation Control 186
5.35 Input Shaping and Results for Multi-Step High Speed Hybrid Trans-
    lation Control .............................................................................................. 187
5.36 Simulated and Experimental Rejection of External Disturbances ............ 188
5.37 Full Inversion Feed-Forward Control Diagram ......................................... 189
5.38 Design of Full Inversion Feed-Forward Parameters in Frequency Domain191
5.39 Impulse Response and Simulated Results of Full Inversion Feed-Forward
    Control ........................................................................................................ 192
5.40 Comparison of Full Inversion Feed Forward to Input Shaping Results
    for Position Control .................................................................................... 193
5.41 Comparison of Full Inversion Feed Forward to Input Shaping Results for Hybrid Control ......................................................... 194
5.42 Comparison of Full Inversion Feed Forward to Input Shaping Results for Multi-Step Hybrid Control ........................................... 195
5.43 Controller and Imaging Coordination for Video Rate Camera ............. 197
5.44 Controller and Imaging Coordination for High Speed Camera ............. 198
5.45 Controller and Imaging Coordination for Long Integration Mode .......... 199
5.46 Timing and Step Size Effect on Signal Error .............................. 201
5.47 Error Statistics of Timing and Step Size Effects ........................... 202
5.48 Spectrum Calculation Accuracy as a Function of Controller Type and Solution Type ............................................................. 203
5.49 Effect of Human Motion on Measured Spectrum ............................ 205
5.50 Error In Measured Spectrum due to Human Motion as a Function of Frame Rate .............................................................. 205

6.1 Interferometric Measurements of Laser, LED, and Fluorescent Light .... 208
6.2 Measurements of White Sources and Fluorescent Dye ..................... 209
6.3 Measurement of Filters and Colored Glass ................................. 210
6.4 Spectral Compensation Curves for LUPA 300 .................................. 211
6.5 White Light Interferometer Measurements ................................. 212
6.6 Conversion of Raw IFTS Laser Data at 78 fps to Color Image ............. 214
6.7 Conversion of Raw IFTS Laser Data at 2010 fps to Color Image .......... 215
6.8 Spectrum of LED Array .................................................... 216
6.9 Processing LED Array Data to Full Color Image .......................... 217
6.10 Processing LED Array Data to Color by Peak Wavelength ............... 217
6.11 Blood Oxygen Measurement Input Design .................................. 219
6.13 Fluorescence Measurement Input Design .................................. 222
6.14 Fluorescence Measurement Intensity Compensation ....................... 224
6.15 Fluorescence Measurement of Two Different Fluorophores ............... 225
6.16 Fluorescence Measurement of Two Similar Fluorophores ................. 227
6.17 Distribution of Fluorescence for Two Similar Fluorophores .............. 228
6.18 Elastic and Inelastic Scattering for Fluorescence and Raman Measure- ments ................................................................. 229
6.19 Input Design for Raman Scattering Measurements ........................ 231
6.20 Input Design for Dispersive Raman Measurements of Fluorescent Diamond ........................................... 232
6.21 Dispersive Raman Measurements of Polyethylene and Polystyrene ......................................................... 233
6.22 Raman Measurement Intensity Compensation .......................................................................................... 234
6.23 Raman Measurement of Fluorescent Diamond ......................................................................................... 235

7.1 Simulation of Under-Sampled Spectral Data .............................................................................................. 243
7.2 Measurements of Under-Sampled Spectral Data ......................................................................................... 244
7.3 Simulations and Experiments for Obtaining the Cost Function for Optimal Sampling .............................. 246
7.4 Adaptive Sampling Algorithm .................................................................................................................. 249
7.5 Simplified Adaptive Sampling Algorithm Convergence Simulation ......................................................... 251
7.6 Adaptive Sampling Algorithm Simulations ................................................................................................. 253
7.7 Simulations of Adaptive and Random Sampling Algorithm ....................................................................... 254
7.8 Convergence and Performance Comparison for the Full Adaptive Sampling Algorithm .......................... 256
7.9 Simulation of the Effect of Noise on the Adaptive Sampling Algorithm ..................................................... 257
7.10 Simulation for Adaptive Sampling of the Benzene Raman Spectrum ......................................................... 258
7.11 Measurement of the Adaptive Sampling Algorithm on Laser Inputs ......................................................... 259
7.12 Simulation of the Recursive Algorithm Response to Different Forgetting Factors .................................. 263
7.13 Simulation of the Recursive Algorithm Peak Response and Noise Reduction ........................................... 264
7.14 Experimental Measurements of the Recursive Update Algorithm ............................................................. 265
7.15 Experimental Measurements of Peak Magnitude and Noise for the Recursive Update Algorithm ............ 265
7.16 Schematic of the System Used to Test the Digital Filtering Algorithms ..................................................... 267
7.17 Digital Filtering Process for Converting Black and White Data to Color Data ......................................... 267
7.18 Digital IIR Filter Designs .......................................................................................................................... 269
7.19 Digital FIR Filter Designs .......................................................................................................................... 270
7.20 Measured Data Showing the Raw Input Data and the Signal Processed by the Digital Filter Algorithm .................................................................................................................. 271
7.21 Black and White as well as Digital Filter Processed Color Images ............................................................ 272
7.22 Uniform and Non-uniform FIR Digital Filter Schemes ............................................................................. 275
7.23 Simulation of Non-uniform FIR Digital Filtered Data .......... 276
7.24 Measured Non-Uniform Sampling Intervals .................... 277
7.25 Comparison of the Uniform and Non-Uniform FIR Filter Results . 278
7.26 Effect of Filter Length on Color Separation and Frame Rate .... 278
7.27 Filter Rejection Ratio and Frame Rate as a Function of Filter Length 279

8.1 IFTS Implementations ........................................ 284
A.1 Schematic of Power Board ..................................... A7
A.2 Schematic of 20 V Boost Power Board ........................ A8
A.3 Schematic of 60 V Boost Power Board ....................... A8
A.4 Schematic of Programmer Adapter Board .................... A9
A.5 Schematic of Base Board Microcontroller .................. A10
A.6 Schematic of Base Board Video Pipeline .................... A11
A.7 Schematic of Base Board Connections ....................... A12
A.8 Schematic of Laser & Light Board ........................... A13
A.9 Schematic of Photodiode Board ............................... A14
A.10 Schematic of Camera Board ................................. A15
A.11 Schematic of High Speed Camera Board ................... A16
A.12 Schematic of Motor Power Board ............................ A19
A.13 Schematic of 35 V Piezoelectric Amplifier Board .......... A20
A.14 Schematic of 60 V Piezoelectric Amplifier Board .......... A20
A.15 Schematic of Portable Power Board Battery Charger ....... A21
A.16 Schematic of Portable Power Board Buck and Boost Converters . A22
A.17 Schematic of FPGA Board Microcontroller ................ A23
A.18 Schematic of FPGA Board Spartan 6 Chip ................ A23
A.19 Schematic of FPGA Board Connections .................... A24
A.20 Schematic of Button Board .................................. A24

B.1 User Interface for Main Window Options ..................... A32
B.2 User Interface for Observer Calculations Window .......... A35
B.3 User Interfaces for Video Options and Image Grabber Windows . A36
B.4 Fixed Pattern Noise Removal .................................. A37
B.5 User Interface for System Identification Window ............ A38
B.6 User Interface for Adaptive Sampling Window ............... A39
B.7 User Interface for Recursive Sampling Window ............... A41
# List of Tables

2.1 Single-Pass Michelson Interferometer Dimensions Table .......................... 54
2.2 Double-Pass Michelson Interferometer Dimensions Table ....................... 56
2.3 Common Path Sagnac Interferometer Dimensions Table ........................ 59
2.4 Electro-Optic Modulator Interferometer Dimensions Table .................... 60
3.1 Series Piezoelectric Actuator Command Table ........................................ 97
3.2 Parallel Piezoelectric Actuator Command Table ..................................... 100
4.1 Summary of the Specifications of Different Implementations ................. 143
5.1 Summary of the Temperature and Power Characteristics of Different Designs .......................................................... 167
6.1 LED Calibration Plate Wavelengths ......................................................... 216
7.1 Spectral Sampling Techniques as a Function of Prior Knowledge .............. 238
7.2 Sampling parameters for Normal and Under-Sampling Measurements ....... 244
A.1 Prototype Design EDM Parts List .......................................................... A2
A.2 Multi-Axis Design EDM Parts List ........................................................ A3
A.3 Endoscope Design EDM Parts List ......................................................... A4
A.4 High Speed Design EDM Parts List ....................................................... A5
B.1 Microcontroller Register and Command Map ......................................... A26
B.2 Spectral Acquisition Function Commands .............................................. A30
# Nomenclature

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADC</td>
<td>Analog to Digital Converter</td>
</tr>
<tr>
<td>AVI</td>
<td>Audio Video Interleave File Type</td>
</tr>
<tr>
<td>BBO</td>
<td>Beta Barium Borate</td>
</tr>
<tr>
<td>BGA</td>
<td>Ball Grid Array</td>
</tr>
<tr>
<td>BIN</td>
<td>Binary File Type</td>
</tr>
<tr>
<td>CARS</td>
<td>Coherent Anti-Stokes Raman Scattering Spectroscopy</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge-Coupled Device</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary Metal Oxide Semiconductor</td>
</tr>
<tr>
<td>DAC</td>
<td>Digital to Analog Converter</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DMA</td>
<td>Dynamic Memory Access</td>
</tr>
<tr>
<td>DPSS</td>
<td>Diode Pumped Solid State</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processing Unit</td>
</tr>
<tr>
<td>EDM</td>
<td>Electric Discharge Machining</td>
</tr>
<tr>
<td>EEPROM</td>
<td>Electrically Erasable Programmable Read-Only Memory</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FIFF</td>
<td>Full Inversion Feed-Forward</td>
</tr>
<tr>
<td>FIFO</td>
<td>First-In First-Out</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite Impulse Response</td>
</tr>
<tr>
<td>FTIR</td>
<td>Fourier Transform Infra-Red</td>
</tr>
<tr>
<td>FOS</td>
<td>Fast Orthogonal Search</td>
</tr>
<tr>
<td>FOV</td>
<td>Field of View Angle</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field-Programmable Gate Array</td>
</tr>
<tr>
<td>FPN</td>
<td>Fixed Pattern Noise</td>
</tr>
<tr>
<td>FWHM</td>
<td>Full Width Half Maximum</td>
</tr>
<tr>
<td>HeNe</td>
<td>Helium Neon Laser</td>
</tr>
<tr>
<td>I2C</td>
<td>Inter-Integrated Circuit Protocol</td>
</tr>
<tr>
<td>I2S</td>
<td>Integrated Interchip Sound Protocol</td>
</tr>
<tr>
<td>IFTS</td>
<td>Imaging Fourier Transform Spectrometer</td>
</tr>
<tr>
<td>IIR</td>
<td>Infinite Impulse Response</td>
</tr>
<tr>
<td>IR</td>
<td>Infrared</td>
</tr>
<tr>
<td>ISO</td>
<td>International Organization for Standardization for Measuring Film Speed</td>
</tr>
<tr>
<td>JTAG</td>
<td>Joint Test Action Group Protocol</td>
</tr>
<tr>
<td>KTP</td>
<td>Potassium Titanyl Phosphate Crystal</td>
</tr>
<tr>
<td>LED</td>
<td>Light Emitting Diode</td>
</tr>
<tr>
<td>LIGA</td>
<td>Lithography, Electroplating and Molding</td>
</tr>
<tr>
<td>LPF</td>
<td>Low Pass Filter</td>
</tr>
<tr>
<td>LS</td>
<td>Least Squares</td>
</tr>
<tr>
<td>LVDS</td>
<td>Low Voltage Differential Signal Protocol</td>
</tr>
<tr>
<td>MEMS</td>
<td>Micro-electro-mechanical System</td>
</tr>
<tr>
<td>NdFeB</td>
<td>Neodymium Iron Boron Magnet</td>
</tr>
<tr>
<td>NDR</td>
<td>Non-Destructive Readout</td>
</tr>
<tr>
<td>Nd:YAG</td>
<td>Neodymium-doped Yttrium Aluminum Vanadate Crystal</td>
</tr>
<tr>
<td>NIR</td>
<td>Near Infrared</td>
</tr>
<tr>
<td>OD</td>
<td>Outer Diameter</td>
</tr>
<tr>
<td>OPD</td>
<td>Optical Path Difference</td>
</tr>
<tr>
<td>PCI</td>
<td>Peripheral Component Interconnect</td>
</tr>
<tr>
<td>PD</td>
<td>Proportional, Derivative Controller</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional, Integral, Derivative Controller</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase-Locked Loop</td>
</tr>
<tr>
<td>QCIF</td>
<td>Quarter Common Intermediate Image Format</td>
</tr>
<tr>
<td>RAM</td>
<td>Random Access Memory</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Full Form</td>
</tr>
<tr>
<td>--------------</td>
<td>-----------</td>
</tr>
<tr>
<td>RGB</td>
<td>Red Green Blue Colorspace</td>
</tr>
<tr>
<td>SCSI</td>
<td>Small Computer System Interface</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning Electron Microscope</td>
</tr>
<tr>
<td>SLR</td>
<td>Single Lens Reflex Camera</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SPI</td>
<td>Serial Peripheral Interface</td>
</tr>
<tr>
<td>SWD</td>
<td>Single Wire Debug Protocol</td>
</tr>
<tr>
<td>TMR</td>
<td>Tetramethylrhodamine</td>
</tr>
<tr>
<td>UI</td>
<td>User Interface</td>
</tr>
<tr>
<td>USART</td>
<td>Universal Asynchronous Receiver</td>
</tr>
<tr>
<td>Transmitter Protocol</td>
<td>USB</td>
</tr>
<tr>
<td>UV</td>
<td>Ultraviolet</td>
</tr>
<tr>
<td>VCSEL</td>
<td>Vertical-Cavity Surface-Emitting Laser</td>
</tr>
<tr>
<td>VGA</td>
<td>Video Graphics Array Image Format</td>
</tr>
<tr>
<td>XML</td>
<td>Extensible Markup Language File Type</td>
</tr>
<tr>
<td>YUV</td>
<td>Luma and Chrominance Colorspace</td>
</tr>
<tr>
<td>ZPD</td>
<td>Zero Path Difference</td>
</tr>
</tbody>
</table>
# List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\textbf{A}$, $\textbf{A}_m$</td>
<td>Spectral content vector</td>
</tr>
<tr>
<td>$\hat{\textbf{A}}$</td>
<td>Spectral content vector estimate</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Entrance aperture area</td>
</tr>
<tr>
<td>$\textbf{A}_d$</td>
<td>Continuous domain $\textbf{A}$ matrix</td>
</tr>
<tr>
<td>$\textbf{A}_L$</td>
<td>Intermediate variable for spherical lens intersection calculation</td>
</tr>
<tr>
<td>$A_S$</td>
<td>Spectrometer surface area</td>
</tr>
<tr>
<td>$A_T$</td>
<td>Total imaging spectrometer area</td>
</tr>
<tr>
<td>$a(z)$</td>
<td>Position sensor signal at sensor 1</td>
</tr>
<tr>
<td>$a_j$</td>
<td>Digital filter feedback coefficients</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>Low pass filter constant</td>
</tr>
<tr>
<td>$B_{ADC}$</td>
<td>Bits on the ADC</td>
</tr>
<tr>
<td>$\textbf{B}_d$</td>
<td>Continuous domain $\textbf{B}$ matrix</td>
</tr>
<tr>
<td>$\textbf{B}_L$</td>
<td>Intermediate variable for spherical lens intersection calculation</td>
</tr>
<tr>
<td>$b(z)$</td>
<td>Position sensor signal at sensor 2</td>
</tr>
<tr>
<td>$b_1$, $b_2$</td>
<td>Damping at damper 1 or 2</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Flexure width</td>
</tr>
<tr>
<td>$b_t$</td>
<td>Digital filter feed-forward coefficients</td>
</tr>
<tr>
<td>$b^m_1$</td>
<td>Master filter coefficients</td>
</tr>
<tr>
<td>$b_p$</td>
<td>Piezoelectric system damping</td>
</tr>
<tr>
<td>$b_t$</td>
<td>Total damping</td>
</tr>
<tr>
<td>$C(s)$</td>
<td>Controller transfer function</td>
</tr>
<tr>
<td>$\textbf{C}_d$</td>
<td>Continuous domain $\textbf{C}$ matrix</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Hysteretic unit capacitance</td>
</tr>
<tr>
<td>$\textbf{C}_L$</td>
<td>Intermediate variable for spherical lens intersection calculation</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Heat capacity</td>
</tr>
<tr>
<td>$C_{pm}$</td>
<td>Piezoelectric actuator capacitance</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Constant related to the probability of success</td>
</tr>
</tbody>
</table>

$c$ | Speed of light in vacuum |
$c(z)$ | Position sensor signal at sensor 3 |
$c_i$ | Constants for aspheric lens |
$D(s)$ | Disturbance input |
$\textbf{D}_d$ | Continuous domain $\textbf{D}$ matrix |
$\textbf{D}_L$ | Intermediate variable for spherical lens intersection calculation |
$d, d_x, d_y$ | Distance between two sensors along the $x$ or $y$ axis |
$d(z)$ | Position sensor signal at sensor 4 |
$d_a$ | Translation of Sagnac interferometer |
$d_f$ | Flexure thickness |
$d_{OPD}$ | Optical path difference |
$d_R$ | Interferometer translation |
$d_{shear}$ | Optical shear distance in an interferometer |
$E$ | Young's modulus |
$E_0$, $E_1$, $E_2$ | Electric field |
$E_c$ | $\text{Êtendue}$ of an optical system |
$F$ | Force |
$F_{\text{max}}$ | Maximum force |
$F_s$ | Sampling frequency |
$f(\Delta z)$ | Noise ratio |
$G$ | Recursive update input matrix with $\Lambda$ half exponential |
$G(s)$ | Plant transfer function |
$G_F$ | Area under the curve $\tilde{F}(t)$
GL Area under the curve \( \tilde{L}(t) \)

g Size of group of data

\( g(\Delta z) \) Data acquisition time

H Recursive update input matrix with \( \Lambda \) exponential

\( \hat{h} \) Impulse response

\( h(\Delta z) \) Solution time

\( h_t \) Thermal convection coefficient

I Optical intensity

I Identity matrix

\( I_1, I_2 \) Intensity at wavelength 1 or 2

\( I_{1o}, I_{2o} \) Initial intensity at wavelength 1 or 2

i Counting variable

J Spectrum magnitude

j Counting variable

K Recursive update gain

\( K_0 \) Voltage to position constant

\( K_1, K_2, K_3 \) Relative scaling constants

\( K_e, K_f \) Motor constants

\( K_m \) Current to position constant

\( K_x, K_y, K_z \) Predicted stiffness in x, y, or z-direction

\( K_\alpha, K_\beta, K_\gamma \) Predicted torsional stiffness about the x, y, or z-axis

k Wave vector

\( k_1, k_2 \) Stiffness at spring 1 or 2

\( k_t \) Constants for aspheric lens

\( k_p \) Piezoelectric system stiffness

\( k_t \) Total stiffness

L Inductance

\( L(s) \) Low pass filter transfer function

\( L(t) \) Time domain response after \( L(s) \) block

\( \tilde{L}(t) \) Impulse response of \( L(s) \)

\( L_b \) Optical path through interferometer

\( L_c \) Focal length of lens

\( L_f \) Flexure length

\( L_S \) Saganc interferometer side length

M Counting variable

m Counting variable

\( m_1, m_2 \) Mass 1 or 2

\( m_p \) Piezoelectric system mass

\( m_t \) Total mass

N Counting variable

\( N(t) \) Nonlinear rate filter output

\( N_0 \) Initial number of samples collected

\( N_{frames} \) Number of frames to acquire

\( N_{int} \) Number of images to integrate per frame

\( N_m \) Number of bins used for buffering the limits

\( N_{occupied} \) Number of occupied spectral bins

\( N_{total} \) Total number of spectral bins

n Counting variable

\( n_f \) Refractive index

\( \delta n \) Index difference between the ordinary and extraordinary axes

P Power

\( P, P_m(n) \) Matrix containing frequency and position information

\( P_{avg} \) Average power

P Pixel size

\( Q_a \) Total number of feedback filter coefficients

\( Q_b \) Total number of feed-forward filter coefficients

q Charge

\( q_{bi} \) Hysteretic unit blocking charge

\( q_{max} \) Maximum deflection before yield

\( R \) Electrical resistance

\( R_0 \) Electrical resistance at room temperature

Rc Resolving power of a spectrometer

\( R_t \) Intensity ratio

\( R_T \) Thermal resistance

r Spherical lens radius

r Position vector

S(\( \omega \)) Power spectral density of the electric field

\( SaO_2 \) Blood oxygen saturation

s Laplace domain

T Toeplitz matrix of input autocorrelation
$T_{\text{avg}}$ Average temperature  
$T_C$ Camera thickness  
$T_L$ Lens thickness not taken up by the focal length  
$T_M$ Mirror thickness  
$T_p$ Polarizer thickness  
$T_{pm}$ Piezo transformer ratio  
$T_r$ Rise time  
$\Delta T$ Change in temperature  
$t$ Time  
$t_L$ Lens thickness  
$V$ Voltage  
$V_i$ Hysteretic unit voltage  
$V_{\text{max}}$ Maximum voltage  
$V_T$ Total imaging spectrometer volume  
$\Delta V$ Difference between the maximum and minimum voltage  
$v$ Speed of light in a medium  
$v_i$ Hysteretic unit breakaway voltage  
$v_t$ Piezoelectric system back electromotive force  
$w$ Digital filter state variable  
$w_L$ Intermediate variable for spherical lens intersection calculation  
$x$ Tip axis  
$x()$ Input variable  
$x_a$ Aperture height  
$x_b$ Beam height  
$x_{bt}$ Beam height with tolerance  
$x_c$ Camera format height  
$x_{cz}$ Total camera length in the x-direction  
$x_{cy}$ Total camera length in the y-direction  
$x_e$ Entrance height  
$x_L$ X-axis position in lens frame of reference  
$x_m$ Distance from beam splitter to Sagane mirror  
$x_r$ X-axis position of incoming ray in lens frame of reference  
$x_{r0}$ X-axis position of incoming ray in main coordinate system  
$\Delta z$ Difference in displacement in x-direction  
$Y, y(n)$ Interferogram intensity vector  
$Y_g$ Recursive update group intensity vector  
$y$ Tilt axis  
$y()$ Output variable  
$y_L$ Y-axis position in lens frame of reference  
$y_r$ Y-axis position of incoming ray in lens frame of reference  
$y_{r0}$ Y-axis position of incoming ray in main coordinate system  
$\Delta y$ Difference in displacement in y-direction  
$z, z_{ab}$ Measured position or optical path difference  
$z_0$ Rest position of the actuator  
$z_1, z_2$ Deflection at mass 1 or 2  
$z_{des}$ Desired position  
$z_{max}, z_{min}$ Maximum or minimum position  
$\Delta z$ Position step size  
$\overline{\Delta z}$ Average position step size  
$\alpha$ Temperature coefficient  
$\alpha_L$ Ray angle in lens frame of reference  
$\alpha_{01}, \alpha_{02}$ Absorption coefficient of oxyhemoglobin at wavelength 1 or 2  
$\alpha_r$ Ray angle of incoming ray in lens frame of reference  
$\alpha_{r0}$ Ray angle of incoming ray in main coordinate system  
$\alpha_{r1}, \alpha_{r2}$ Absorption coefficient of deoxyhemoglobin at wavelength 1 or 2  
$\Gamma$ Occupancy ratio  
$\Gamma_{\text{guess}}$ Occupancy ratio estimate from the adaptive algorithm  
$\Gamma_{\text{rand}}$ Occupancy ratio for random sampling  
$\Gamma_{\text{true}}$ True underlying occupancy ratio  
$\Delta, \Delta_x, \Delta_u$ Projected offset between two signals along an axis x or y
\( \delta_b \) Beam splitter to mirror tolerance
\( \delta_c \) Lens to beam splitter tolerance
\( \delta_e \) Entrance tolerance
\( \delta_L \) Spectral lower bound
\( \delta_t \) Beam splitter side tolerance
\( \varepsilon \) Electric field magnitude
\( \varsigma \) Damping ratio
\( \eta_c \) Spectrometer efficiency
\( \theta, \theta_y \) Tilt angle
\( \theta_{T3} \) Angle of flexure at translation
actuator T3
\( \lambda \) Wavelength of light
\( \lambda_{\text{max}}, \lambda_{\text{min}} \) Maximum or minimum wavelength
\( \Delta \lambda \) Wavelength resolution
\( \nu \) Wavenumber of light
\( \nu_{\text{max}}, \nu_{\text{min}} \) Maximum or minimum wavenumber
\( \Delta \nu \) Wavenumber resolution
\( \Delta \nu_{\text{desired}} \) Desired wavenumber resolution
\( \rho \) Density
\( \sigma_{\text{max}} \) Yield stress
\( \varsigma \) Spectrum phase
\( \tau \) Time delay
\( \tau_t \) Thermal time constant
\( \Phi \) Recursive update input-output matrix
product
\( \Phi_c \) Solid angle
\( \phi, \phi_{ab} \) Phase difference between two signals
\( \phi_{\text{FOV}} \) Field of view angle
\( \phi_L \) Intermediate variable for spherical lens
intersection calculation
\( \phi_R \) Light entrance angle
\( \phi_{R_{\text{max}}} \) Maximum light entrance angle
\( \phi_{S1} \) Sagnac interferometer angle
\( \phi_{S2} \) Sagnac interferometer complementary
angle
\( \phi_x, \phi_y \) Phase angle difference along the x or
y axis
\( \varphi_{xx} \) Autocorrelation of x
\( \varphi_{xy} \) Cross-correlation of x and y
\( \varphi(\tau) \) Autocorrelation of the electric field
\( \Psi \) Recursive update input-input matrix
product
\( \omega \) Frequency of light
\( \omega_n \) Natural frequency
Chapter 1

Introduction

An imaging spectrometer is a device that takes a series of images that include information from a variety of wavelengths of light at high spectral resolutions. When this device is combined with different types of input light, such as ambient light for absorption measurements or laser light for fluorescence or Raman measurements, it can be used to differentiate between materials that are indistinguishable to the naked eye in a nondestructive, non-contact manner. Imaging spectrometers are useful instruments that have been utilized in many critical scientific applications from space probes to biological imaging. These instruments are also slowly gaining momentum for research and commercial applications from investigating air pollution to health care monitoring. Despite the usefulness of these instruments, these devices are currently large and inaccessible to the general population. Current instruments tend to be complex and require mounting to optical tables to reject external vibrations. As technology continues to develop in areas from electronics to manufacturing, new opportunities begin to open up in developing smaller, lower cost and more efficient instruments.

Miniaturization and commercialization of single-point measurement spectrometers is currently a fast-growing field, with several low cost instruments (such as those...
available at Ocean Optics), emerging techniques, and startups in the area. In contrast to these single-point instruments, an imaging spectrometer can monitor many points at the same time and be much more easily positioned relative to the object of interest, vastly improving usability. The ability to switch between regular and hyperspectral imaging makes it possible to align the spectrometer with respect to the sample without an additional alignment camera. Faster imaging spectrometer systems can be incorporated with image stabilization and object tracking algorithms to account for human or object motion, something that would be difficult to do with a non-imaging system. Being able to select the wavelength resolution on the fly and to obtain high spectral resolution images (with resolutions of 1.6 cm\(^{-1}\) for Raman spectroscopy to 70 cm\(^{-1}\) for low resolution spectroscopy) is also desirable.

In order to make these devices more accessible for more applications, this work focuses on the development of miniature, portable imaging spectrometers that have the relatively high spectral resolutions necessary for making scientific measurements. The spectrometer can also be mounted inside other packages, such as an endoscope, smartphone, scanner gun, microscope, or telescope. A spectrometer that can be small enough to fit in the palm of the hand or at the end of an endoscope would enable a host of new applications. Some of these applications are shown in Figure 1.1.

One of the most challenging problems in health care today is fast, on-site diagnosis of disease. The keys to improving diagnostics are the development of new technologies and increasing patient compliance by making the diagnostic techniques minimally invasive, fast, and accessible. One of the applications for this work is the development of a small spectroscopy system to aid in the identification of diseases.

Non-invasive diagnostics using small spectrometers have many applications in medicine and biology. A small, fast, and high spectral resolution imaging system can be applied to many areas including the analysis of the concentration of blood an-
Figure 1.1: There are many applications for hyperspectral imaging including cancer detection, blood analyte analysis, agriculture tracking, process monitoring, metal alloy classification, geology identification and gemology verification. The images are compiled from [24, 78, 115, 137, 149]

analytes such as glucose [135, 164], triglyceride, cholesterol, or even blood oxygen levels. Optical biopsy of tissues is also an important application area. The instrument could be used to investigate potential skin cancer [3], to conduct human breath analysis [165] and to look for cancer margins during surgeries or during ablation therapy.

Current methods in endoscopy, as shown in Figure 1.1, use miniature scanning mirrors or fiber optic bundles [34]. Endoscopic spectrometers can be used for a variety of diagnostic applications, including identifying flat, small cancerous lesions [10, 83, 117, 133] and ulcerated tissue in the gastrointestinal tract and other body lumens. An endoscopic spectrometer [102, 112] can also be used to inspect crawl spaces, packages, pipes, and holes.
A miniature portable imaging spectrometer has many other applications outside of biology including environmental science, process monitoring, and security. Applications can include food safety checks, fruit and vegetable freshness tests, quality verifications, and identification of chemical compounds. Figure 1.1 shows how an imaging spectrometer can be used to identify the compound distributions inside a pill. Such a device can be used for identifying purity of substances like diamonds and other crystalline materials as well as alloys, plastics, powders, and liquids. Air pollution detection, agriculture monitoring, lead paint identification, letter and package screening [70] and contaminant monitoring could also be conducted. For instance, the spectrometer could be mounted on an aircraft or drone for crop monitoring and localized pesticide/herbicide distribution, or on robots for investigating hazardous environments such as contaminated, high-pressure, and extreme temperature environments.

The same spectrometer topology can be designed to operate over a variety of wavelengths from ultraviolet (UV), visible, near infrared (NIR), and infrared (IR) [69]. Specialized versions of the device can also be used for FTIR (Fourier transform infrared) spectroscopy, white light interferometry and optical coherence tomography [16]. Some topologies can also be used for optical pulse width measurements as well as imaging.

This work discusses the design decisions as well as mathematical methods for creating a fast miniature imaging transform spectrometer. A transform spectrometer, one example being a Fourier transform spectrometer, takes several data points at different autocorrelations of the input light and then reconstructs the spectral data. To provide stable handheld operation, the device may use active position measurement sensors, a high-speed actuator, and a control system to stabilize internal scanning mirrors and other moving components. The systems include small linear Lorentz-
force motors, piezoelectric actuators, flexures, precise position measurement systems, and external disturbance rejection control systems.

The nature of most Fourier transform spectrometers requires that a large number of images be taken to reconstruct the spectra. One of the contributions of this work is the development of algorithms that drastically decrease the number of images necessary to reconstruct the spectral information therefore requiring as little as 10\% of the number of images that a normal Fourier transform spectrometer requires. Thus, these devices may operate about ten times faster than a conventional Fourier-transform spectrometer for applications where high spectral resolutions limit acquisition speed. Novel algorithms for minimizing the number of required samples for video-rate spectral readings at high image resolutions are also presented. In order to provide context for the work, prior art is discussed.

1.1 Hyperspectral Imaging

Hyperspectral imaging is often thought of in terms of a data cube, as shown in Figure 1.2a. The image is contained in the $x$ and $y$ axes while the spectral information is contained in the $\lambda$ axis. Ideally, each pixel would have its own spectroscopic sensor to obtain data for a full spectral data cube at the same time. One possible future implementation could be a full, high resolution array of standing wave sensors [100, 151], each capable of measuring the Fourier transform spectrum of each pixel in the array. Since this type of sensor has not yet been developed, obtaining a full, high resolution data cube requires scanning at least one of the three axes of the data cube in one fashion or another. Alternative dispersive coded aperture or snapshot imaging systems could be used to obtain a full cube without moving elements but at a loss of spatial and/or spectral resolution.

For high speed operation, an imaging sensor is preferable because it is limited only
Figure 1.2: a) The spectral data cube includes the full image in the spatial domain as well as spectral data in the \( \lambda \) domain. b) A leap-frog design can also be implemented where only some parts of the image have full spectral data in the \( \lambda \) domain while other areas only have partial spectral information.

by the electronic data transfer rate (tens of megahertz or gigahertz) while the scanning axis is limited by the data frame rate or mechanical scanning rate. It is desirable to keep the axes that are least correlated with the other two (the spectral axis) on the slowest scanning rate. This is because if some motion happens to occur during the scanning, an imaging spectrometer can use image tracking to compensate. A system that mechanically scans one of the spatial axes would not be able to compensate in the same way.

During scanning, a full data cube is desirable and can be obtained with some spectral designs while others only provide partial data from the cube. One such example would be the “leap-frog” design shown in Figure 1.2b where only some spatial data points have full spectral data. Some designs, such as Sagnac interferometers only provide high resolutions for some parts of the image and low resolutions for other parts of the image.

There are many possible designs for an imaging spectrometer \([20, 46, 47, 48, 49, 166, 167, 168]\) and some of them are shown in Figure 1.3. There are several classes these designs break down into: dispersive systems, filter based systems, resonator systems, input variation systems, and Fourier transform systems.
Many small spectrometers in the prior art contain imaging sensors but cannot conduct imaging without line scanning. Most of these small instruments contain dispersive elements so they measure the spectra of a single pixel or a line of pixels in the full image. Dispersive methods do not take advantage of spectral multiplexing and cannot benefit from the irregular sampling techniques outlined in this work. The first row of spectrometers in Figure 1.3 contains Offner spectrometers that use dispersive or diffractive elements and require line scanning. These designs typically have a fixed spectral resolution. This topology is common for satellites and aerial scanning. Other raster scanning [66, 88] and push-broom designs also use dispersive topologies.

Dispersive elements, mirrors, and lens arrays can also be used to project an array of smaller, lower spatial resolution images on a large image sensor. These systems tend to have either no motiving parts or only small moving mirror arrays. Some of these designs either have low spectral or spatial resolution [138]. Included in this class of imaging spectrometers are coded aperture designs and snapshot designs [17, 18, 19, 160].

Another class of imaging interferometers uses filters to separate different wavelengths. This can include Bayer mask systems and rotating filter wheel systems. Filtering can also be conducted with Fabry-Perot cavities [124], tunable Bragg filters[109], and acousto-optic [103] or electro-optic tunable filters. Tunable input systems are an interesting way of creating an imaging spectrometer from a simple camera. The sources, such as LED (light emitting diode) arrays, laser arrays, tunable lasers [165], or optical parametric oscillators, project several wavelengths at different times and the camera is used to capture the full images. This type of system is limited to absorption or transmission experiments and cannot be used for spectral analysis in fluorescence or Raman experiments as those interactions are nonlinear.

One of the many advantages that Fourier transform spectrometers have over dis-
Figure 1.3: Several different hyperspectral imaging techniques are shown including line scan techniques, multi-wavelength input lasers, Fabry-Perot designs, coded aperture designs, multi-image snapshot designs, Fourier transform designs, and Sagnac designs. The images are compiled from [2, 5, 19, 54, 96, 128, 144, 148, 165].

persive systems, tunable input designs and filter based topologies is that the spectral information is multiplex leading to high throughput efficiency. Traditional Fourier transforms require that at least two points are sampled in the space domain for every one point in the spectral domain. This would generally require at least twice as many samples as other dispersive or filter based system. However, due to the spectral multiplexing, Fourier transform systems work well with under-sampling techniques and phase correction techniques. These can be used to greatly reduce the number of samples needed. If there is low spectral occupancy, then Fourier transform systems
require fewer samples than traditional dispersive or filter based designs for the same spectral resolution.

As with dispersive systems, many Fourier transform designs can utilize raster scanning [26, 44, 89, 100, 108, 140, 142, 151, 163, 171, 177], line scanning [116, 118, 162], or snapshot designs [96]. There are also several single-point topologies that use imaging sensors where each pixel is at a different optical delay through a Michelson interferometer [75, 92]. It is also common to scan the image and deliver the measured light through an optical fiber to the small spectrometer [175]. These systems may have miniature probes but the full system may be much larger. It is also possible to create an interferometer with the fiber system components [29].

Fourier transform imaging spectrometers can be broken down into two different categories: spatial and temporal [1]. Spatial systems include shift interferometers such as Wollaston prism [14, 15, 72], Savart plate, and Sagnac designs [6, 76, 95, 96]. These topologies create a lateral shift of two copies of the same image and recombine the image after the last imaging lens. Temporal systems include moving mirror Michelson interferometers [25, 90, 119, 166], moving sensor Michelson interferometers [71, 99] and electro-optic modulator (EOM) polarization delay [30] designs. These systems shift two copies of the same image in time by delaying one image with respect to the other. Generally, spatial designs have lower spectral resolutions, are easier to control and contain fewer off-axis artifacts that need to be corrected in software [1]. The Michelson topology is particularly interesting due to its high spectral resolution and many possible applications for measuring spectral information, short temporal pulses (picosecond and femtosecond pulses), optical coherence and even three dimensional coherence [110].

One of the important challenges in designing a small imaging spectrometer with high spectral resolution is the choice of manufacturing techniques. Several techniques
Several possible manufacturing techniques have been explored in the prior art including LIGA, MEMS and glass assemblies. These images are compiled from [44, 105, 163]. Techniques such as micro-electromechanical system (MEMS) and lithography, electroplating and molding (LIGA) can be used but these tend to have cantilevered features that would be susceptible to external vibrations. The manufacturing techniques themselves also tend to be restricted to very small scales and are generally expensive. Another manufacturing technique would be an assembly that is created entirely from glass. Although this concept is more immune to vibrations, it is also difficult to implement without high accuracy glass polishing tools. This work seeks to find a more flexible manufacturing technique for mounting optics and creating small actuators. Although the methods used for prototyping may not extend to high volume manufacturing, the designs themselves should be simple to implement with lower cost methods.

### 1.2 Goals and Methodology

Fourier transform spectrometers can also provide high, selectable wavelength resolution by changing the stroke of the measurement. The spectral resolutions needed can vary greatly. For example, gas phase Raman spectroscopy of adjacent rotational transitions are on the order of 0.5 cm$^{-1}$ apart. Vibrational states can be resolved with high spectral resolutions near 2 cm$^{-1}$ and with moderate resolutions near 10 cm$^{-1}$ [21]. In Fourier transform topology spectrometers, spectral information is multiplexed...
thereby leading to a high throughput efficiency, which also leads to the possibility of greatly under-sampling the spectra. This in turn leads to higher speeds. Based on these reasons, this class of designs was selected and optimized for the imaging Fourier transform spectrometer (IFTS) design described in this work.

Even though there has been a focus on miniaturizing single-point Fourier transform spectrometers, there has not been a focused effort to miniaturize imaging Fourier transform spectrometers. One of the reasons is that these spectrometers have generally been developed and used in scientific laboratories where size or portability is not a major concern and efficient light collection or other factors are more important. For space-borne applications, component robustness is critical in order to survive launch. Components used for imaging infrared spectroscopy also tend to be larger and more complex [43, 123]. Hence, spectrometers for these applications are an order of magnitude larger than the designs proposed in this work. Another reason for a lack of focus on small imaging spectrometers has been that long stroke interferometers, especially for the Michelson topology, are difficult to build and to keep in alignment over time, leading instead to a focus on lower spectral resolution Sagnac and Fabry-Perot topologies.

There are several goals for this work that make the designs unique, allow the devices to be useful in practice, and lead to novel contributions:

- **Create an imaging spectrometer:** When the user is not looking at a uniform object, an imaging system is necessary to determine the properties of the entire system. An imaging spectrometer can improve measurement accuracy because they are more intuitive to align to an object of interest. It also provides more information than a single point spectrometer as an imaging interferometer can track movements across the full image and can provide hundreds of spectra at a time.
• **Create a small device:** This would allow the instrument to be portable and incorporated in several new application areas such as at the tip of an endoscope, on small unmanned aerial vehicles, or on a desktop. Smaller devices also have lower power requirements which can be run using smaller electrical components. In large quantities (and by utilizing the proper mass-manufacturing techniques), these devices can also be lower cost as they would require less material and smaller optics. This goal requires the design of unique actuators, positioning systems, optical components and mechanical components.

• **Create a device that is optical-table-free:** A system that can be optical-table free would be very useful for handheld applications or for situations where there may be vibrations. An optical-table-free system requires a combination of stable mechanical design, good disturbance rejection control capability, high speed acquisition, and image tracking techniques.

• **Create a device with high spectral resolution:** By aiming for higher spectral resolutions, the number of applications for the device increases. Not only can it be used to characterize broadband light for everyday applications, but can also be used as an accurate Raman spectrometer. This requires the analysis of different topologies to find a design that scales well.

• **Create a device that is capable of getting spectra quickly:** High speed data acquisition improves the usability of the device and can be utilized on applications where phenomenon happen rapidly or decay quickly over time (such as fluorescence). This creates unique mechanical and electrical design demands as well as interesting controls challenges.

• **Develop mathematical techniques for getting spectra quickly:** The measurement speed can be further improved by using advanced mathematical techniques for spectral sampling and data processing. These methods should
take fewer samples to achieve the same desired spectral resolution and be able to produce video-rate spectra or video-rate false color spectral images to help the user make real-time decisions. This creates unique challenges for designing algorithms as well as unusual controls challenges.

These goals drive the overall direction of the research and the methods used to achieve these goals are outlined in this work [81]. This thesis first outlines different topologies and scaling laws which guide the design choices for the IFTS in Chapter 2. Next it covers the detailed design and manufacturing methods for the different components in Chapter 3. Several detailed implementations for a prototype, a multi-axis design, a robotic endoscope concept and a high speed video design are described in Chapter 4. The position measurement methods are delineated in Chapter 5 along with characterization of the different actuators. A number of control methods are discussed for the several actuator topologies along with a full model inversion feed-forward design. Coordinating video acquisition with mirror movements is also discussed. Next, results for different experiments are exhibited in Chapter 6. Examples include measuring the blood oxygen content of the human finger, assessing different fluorescent beads, and identifying materials using Raman spectroscopy. Advanced sampling and solution methods are discussed in Chapter 7 including an adaptive sampling algorithm, recursive update method, and digital filtering procedures for obtaining video rate spectra. Lastly, this work concludes with a discussion of the results and methods in which different components could be improved for implementation in a commercial device.
Chapter 2

Scaling and Comparison

Many Fourier transform spectrometers have been developed over the years and it is important to incorporate the lessons from previous designs in the design of a new, miniature imaging spectrometer. This chapter covers the history of Fourier transform spectrometers as well as some of the pros and cons of the topology. Correction algorithms and design factors are also discussed. A quantitative method for comparing some of the designs is also developed and a detailed ray tracing program is created to accurately determine the non-ideal aspects in the specific design and formulate methods for correcting those factors.

2.1 Fourier Transform Spectrometers

Optical spectroscopy is built on a long history of discoveries and innovations. The word “spectrum” was first used by Sir Isaac Newton between 1666 and 1672 to describe how white light is decomposed into a variety of colors when shown through a prism. The wave nature of light was first postulated by Christiaan Huygens in 1678 and was later published in 1690. Joseph von Fraunhofer’s work on dispersion and diffraction lead to the creation of more accurate scientific spectrometers in the 1800’s.
while Gustav Kirchhoff and Robert Bunsen provided the theoretical and experimental basis for modern spectroscopy [51].

In 1803, Thomas Young used a double slit experiment to demonstrate interference in optics. Experiments conducted by Albert A. Michelson and Edward W. Morely in 1887 led to the invention of the Michelson interferometer that was later used for Fourier transform spectroscopy [51]. The theoretical basis for Fourier transform spectroscopy was laid out independently by Norbert Wiener for deterministic functions and Aleksandr Khinchin for stationary stochastic processes. The Wiener-Khinchin theorem states that the autocorrelation function of a stationary random process has a spectral decomposition that is the same as the power spectrum of the process.

The two major nonlinear processes that can be studied with Fourier transform spectroscopy are fluorescence and Raman scattering. The process of fluorescence in some materials has been known since the 1500’s but the term fluorescence was coined by George Gabriel Stokes in a footnote in 1852 to describe the optical properties of fluorite [51]. The Raman effect was first discovered by Sir Chandrasekhara Venkata Raman in 1928 and the discovery that light passing through a transparent material can sometimes change wavelength eventually won him the Nobel prize in physics in 1930 [21]. Despite the fact that Raman scattering was a very important phenomenon, technical challenges from weak intensities to fluorescence interference to poor light collection prevented this method from being used readily for chemical analysis [114]. In 1986, Fourier transform Raman spectroscopy was re-introduced with charge-coupled devices (CCDs), small computers, and near infrared lasers which sparked rapid growth in the area [33, 114, 177].
2.1.1 Advantages and Disadvantages

Several methods for utilizing Fourier transform spectra for infrared spectroscopy and Raman spectroscopy have been analyzed and developed over the years. There are several key parameters that can be used to compare different designs. The first key parameter is the étendue which characterizes how spread out the light is. The étendue is the product of the entrance aperture and the solid angle that the source subtends as seen from the aperture, \( E_c = A_c \Phi_c = A_c \pi \sin^2(\phi_{\text{max}}) \). Another important aspect is resolving power which is defined as \( R_c = \lambda/\Delta \lambda \). The efficiency of the spectrometer is defined as the product of the resolving power and the étendue. If pinholes and slits are not used, Fourier transform spectrometers have better efficiencies, \( \eta_c = R_c E_c \) [21], than dispersive spectrometers based on the higher étendue due to large input aperture areas and sometimes better resolving power due to high spectral resolutions \( \Delta \lambda \).

There are several sources of noise in the sensors used in spectrometers that can limit the system performance. Sources include detector thermal (dark) noise, optical shot noise, input light modulation noise, and electronic amplification noise. Noise from light modulation can be controlled or compensated using additional sensors. In some applications, thermal noise can be reduced by cooling the sensor which can reduce the dark noise by as much as 100 times if cooled near -25 °C. Thermal noise and shot noise can sometimes be mitigated using multiplexing techniques in time or using multiple measurements. If the system noise is dominated by shot noise, then the multichannel gain in the signal to noise ratio (SNR) is equal to \( \sqrt{M} \). This can be used effectively in imaging spectrometers through binning if \( M \) pixels are measuring the same target point with the same information [21].

There are many clear advantages and disadvantages to using Fourier transform spectroscopy with respect to monochromator and dispersive spectrometers [7, 21].
• Fellgett’s advantage (multiplex advantage): Some sources treat Fellgett’s advantage differently than the multiplex advantage. In these sources, Fellgett’s advantage refers to the ability to use multiple sensors and the multiplex advantage refers to multiplexing on a single sensor[7]. If the comparison was restricted to the use of a single sensor or pixel, Fourier transform spectroscopy has a clear advantage. In Fourier transform spectroscopy, all the wavelengths are collected simultaneously on the same sensor. Monochromators and scanning dispersive spectrometers using a single sensor must collect these wavelengths one at a time. It is also important to point out that not all Fourier transform spectrometers can benefit from this type of multiplex advantage. In particular, designs that use separate sensors to measure different lags cannot benefit from this type of advantage as these designs use multiple sensors [100, 116, 151].

When comparing multiple sensor (or single sensor) Fourier transform spectrometers to multiple sensor dispersive spectrometers, there are some advantages to dispersive designs. For example, for two systems with equivalent overall integration times, a multiplexed dispersive system would measure larger signal values, while Fourier transform systems would sense weaker oscillations on a larger constant offset and the signal variations from different offsets may potentially be less than the quantization levels of the sensor [7].

For infrared wavelengths for FTIR and Raman applications, thermal noise is a limiting factor and Fourier transform designs can use Fellgett’s advantage effectively. For visible and ultraviolet wavelengths, on the other hand, the advantage is unclear. There is an advantage only in parts of the spectrum that are at least twice as big as the average power in the measured spectrum. If the spectrum is dominated by strong lines, as in the case of the Rayleigh line in Raman spectroscopy, the SNR of the Fourier transform design can be worse.
than dispersive spectrometers [21]. This can potentially be mitigated by heavily filtering the Rayleigh line in Fourier transform Raman spectrometers. In Fourier transform multiplexed systems, the noise in a measurement is spread across the spectrum. This allows the SNR to be high for peaks but correspondingly poor for absorption lines.

- Jacquinot’s advantage (throughput advantage): Fourier transform spectrometers have better throughput than monochromators and dispersive spectrometers if no pinholes or slits are used. Depending on the grating, some dispersive spectrometer designs can have low diffraction efficiency. Fourier transform spectrometers can obtain the same SNR as a dispersive design in less time.

- Connes’ advantage (wavelength accuracy advantage): Fourier transform spectrometers often have an internal reference laser. In the case when a helium neon (HeNe) laser is used, the internal reference is stable over time and results in better wavelength accuracy for the measured spectrum.

- Stray light sensitivity: Fourier transform designs are less sensitive to stray light than dispersive designs.

- Spectrum discontinuities: Fourier transform spectrometers do not have breaks in the measured spectrum. In grating or filter-based systems, discontinuities can occur if a pixel is missing or if filters are changed. If one data point is not present or incorrect in a Fourier transform spectrometer, this would be represented as noise that is spread out across the spectrum.

- Electronic filtering techniques: There are some advanced methods in dispersive spectrometers that can be used to decrease the SNR [66] that cannot be used in Fourier transform spectrometers. Fourier transform spectrometers, however, can use a variety of under-sampling techniques to decrease the sampling time.
2.1.2 Interference Equations

The interference equations for a Fourier transform spectrometer are based on measuring the interference of electric fields. If the electric field is of a single wavelength, the field can be represented as $E_0 = \varepsilon e^{i(kr - \omega t - \phi)}$ where $\varepsilon$ is the magnitude of the field, $r$ is the position vector, $k$ is the wave vector, $\omega$ is the frequency of light and $\phi$ is the phase shift. Generally, the electric field can be composed of a continuum of wavelengths. When an electric field $E_0$ enters a spectrometer, it is broken into two separate beams, delayed and recombined. The two beams, in this case for a Michelson system, that are recombined in the interferometer, $E_1 = \frac{1}{4} E_0(t)$ and $E_2 = \frac{1}{4} E_0(t + \tau)$ have different time delays $\tau$ resulting from spatial delays. Since the sensors being used can only measure intensity and not electric field, the signal available at the sensor is,

$$I = \langle |E_1 + E_2|^2 \rangle = \langle |E_1|^2 + |E_2|^2 + E_1 \ast E_2 + E_1 \ast E_2 \rangle. \quad (2.1)$$

The brackets $\langle \rangle$ represent a time average and $E_1^\ast$ represents the complex conjugate of $E_1$. The first two terms on the right become constants while the last two terms are the autocorrelation,

$$\varphi(\tau) = \langle E_0(t) \ast E_0(t + \tau) \rangle. \quad (2.2)$$

By taking the Fourier transform of the autocorrelation, it is possible to obtain the power spectral density or spectrum of the light passing through the interferometer,

$$S(\omega) = \int_{-\infty}^{\infty} \varphi(\tau) e^{-i\omega \tau} d\tau. \quad (2.3)$$

Equations 2.2 and 2.3 form the basis of the Wiener-Khinchin theorem.

Several techniques exist for spectral correction in Fourier transform spectroscopy. One common problem is phase error which can come from positioning errors. Asym-
metric interferogram sampling is also a common problem. There are many methods for phase correction [134] including the Mertz method and the symmetrization-convolution method. The latter approach produces lower errors if several iterations are used [32]. Traditional Fourier transform techniques normally sample both sides of the interferogram, allowing for an increase of \( \sqrt{2} \) in the SNR on all spectral channels. With phase correction methods, only one side of the interferogram is needed to produce accurate spectra. This would make Fourier transform spectrometers more competitive in terms of number of the samples required. Based on the many advantages of the Fourier transform topology, it was chosen for use in the miniature imaging spectrometer designs.

2.2 Design Scaling

Scaling is an important part of the instrument design process. Aside from small devices being more portable, smaller systems also have several other advantages in terms of temperature equalization, speed and bandwidth, power requirements, and vibration rejection.

The temperature equalization properties of an optical instrument are important due to the change in part dimensions over time. This may be alleviated by using materials like Invar or Zerodur [11]. For a given material, however, the smaller the instrument, the better the overall temperature equalization properties. For example, the thermal expansion of a component scales with the length of the component. Therefore, smaller instruments will have less position deviation as a function of temperature.

The thermal time constant of the system determines the time it takes for the device to equalize. The shorter the equalization time, the sooner the instrument positions stabilize and the sooner it can be used. The thermal time constant scales as
\[ \tau_t = \frac{\rho C_p V_T}{h_t A_S} \]

where \( \rho \) is the density, \( C_p \) is the thermal capacitance, \( h_t \) is the convection coefficient, \( V_T \) is the total volume and \( A_S \) is the surface area. Fast thermal time constants can be obtained if the volume is low and the surface area is high. For the same shape, the volume to surface area ratio scales with the characteristic length of the instrument. A smaller instrument will also have a lower ratio and therefore a faster thermal time constant. It is important to note, however, that if the instrument has a good reference laser, the expansion effect over time can be compensated.

The speed or bandwidth of the instrument actuators comes from the power system and from the resonant frequency of the actuator. It is generally preferable to reduce the total power consumption. High power electrical components also tend to occupy more space. Because power provided to a motor scales as \( P = k_t z \dot{z} \) where \( k_t \) is the stiffness of the actuator flexure, \( z \) is the translation and \( \dot{z} \) is the velocity, a lower power would require a lower stiffness. The position and velocity of the actuator are variables that determine the frame rate and resolution of the spectrometer. The actuator bandwidth scales with the resonant frequency \( \omega_n = \sqrt{k_t/m_t} \). If the stiffness of the actuator is fixed by power requirements, then a motor with less moving mass will have a larger bandwidth and therefore be faster to position.

Mechanical vibrations can also be reduced by reducing instrument size. The mechanical vibrations can come from human motion or from 1/f noise. Both of these disturbances have smaller magnitudes at higher frequencies (not including impulses). Mechanical vibrations not associated with actuator dynamics also scale as a function of the bending modes of the system. The lowest bending mode is a function of the size and shape of the instrument, the stiffness of the material and the total mass. A lower total mass and the use of fewer or shorter cantilevered features will help increase the frequency of the bending modes so that they are not significantly activated by external 1/f noise. One way to reduce cantilevered features (such as unsupported
mirrors and lenses) is to build a full enclosure around the optical components and contact the mirrors and lenses at as many points as possible, rather than gluing them standing up on a flat surface. A full enclosure would also help reduce external light leakage for sensitive measurements.

There are also downsides to smaller spectrometer designs. One example in particular would be a darker image due to a smaller spectrometer input aperture. This can potentially require longer integration times for some classes of experiments. However in cases where additional optics are used, the design of the spectrometer itself may not be the limiting factor in the system. One example would be if the spectrometer input diameter is the same size or larger than the output diameter of the microscope objective being used in the system.

Based on this simple analysis, it is clear that making the instrument smaller would provide many benefits. For instruments that have high frame rates and high spectral resolutions, the power requirements can already be significant. By reducing the actuator size and instrument size, the total power requirements necessary for moving the actuator mass and rejecting external disturbances decreases and helps with reducing the overall size of the electrical components.

2.3 Comparison of Design Topologies

There are several different topologies for Fourier transform imaging spectrometers. The optimal dimensions of some of these topologies can be compared to determine the appropriate design that has the smallest area or volume at the desired spectral resolution. Four main topologies are explored, the single-pass Michelson, the double-pass Michelson, a common path Sagnac, and an electro-optic modulator design. It is important to note that the Saganc interferometer is a shear-based interferometer and is well-known to scale relatively poorly in terms of spectral resolution [1]. It
is illustrated in this section only as a reference comparison. Here, a method for quantitative comparison of the designs based on optimization of the dimensions is presented. The equations governing the optical path difference, the field of view, the area and volume of these spectrometers are discussed and compared in order to help choose a topology with the smallest volume that meets the desired specifications. Additional topologies can also be compared using the same methodology.

The fundamental variables that control imaging are shown in Figure 2.1. For space minimization, lenses rather than parabolic mirrors [79] were selected for focusing. There are several cases of interest including the case were the entrance height $x_e$ is larger than the aperture height $x_a$ and vice versa.

From this figure, the field of view $\phi_{FOV}$ and maximum light entrance angle $\phi_{R_{max}}$ are related to the entrance height $x_e$ and maximum path length through the interfer-
For a defined camera format $x_e$ the focal length of the imaging lens $L_c$ can be solved using,

$$L_c = \frac{x_c}{x_e} L_b.$$  \hspace{1cm} (2.5)

From Figure 2.1a, it is clear that the beam height $x_b$ must be either as large as the entrance height or as large as the aperture height. In most cases, it is possible to define $x_e = x_b$ if there are no constraints on the size of the aperture.

If there are constraints on the size of the aperture, then several other constraints can be defined. The aperture size can be governed by a diffraction limit constraint where $x_a = 1.22 \frac{\lambda_{\text{min}}}{P_x} L_c$ provides the maximum light input with diffraction limited imaging. In this equation $\lambda_{\text{min}}$ is the minimum wavelength of interest and $P_x$ is the size of a pixel on the imaging sensor. In the case where the diffraction limit constraint is enforced and the entrance height is smaller than the aperture size, then $x_b = x_a$. If no vignetting at maximum path length difference is desired, then $x_e = \frac{1}{2} x_a$ can be defined.

These conditions can be applied to the topologies shown in Figure 2.2. A single-pass Michelson system is the most common type of interferometer where the optical path difference (OPD) scales with two times the mirror movement $d_R$. A double-pass Michelson interferometer doubles the resolution with OPD scaling as four times the mirror movement. The common path Sagnac is a shear based interferometer. The path lengths of the light are the same no matter which arm the light travels through. Therefore, the OPD is controlled by the shear of the image. An electro-optic modulator interferometer can have no moving parts and the resolution is controlled by the
Figure 2.2: Several imaging spectrometer topologies are compared including a) single-pass Michelson, b) double-pass Michelson, c) common path Sagnac, and d) electro-optic modulator style.

difference between rays traveling down the ordinary and extraordinary polarization axes in the EOM material.

The optimal dimensions, in particular the beam width $x_b$, can be determined from a set of realistic or ideal input parameters and by calculating and optimizing either the total optical path length $L_c + L_b$, the total optics area $A_T$, or the total optics volume $V_T$ using,

$$\frac{d(L_c + L_b)}{dx_b} = 0,$$

(2.6)
The area and volume listed here are only for the optical components and do not include the area or volume occupied by the actuator, electronics and enclosure.

In some of the designs, it is not mathematically possible to optimize on the optical path length. It is feasible to optimize on total area or total volume for all of the systems presented in the following sections. Generally, optimizing on area or volume produce very similar results.

### 2.3.1 Single-Pass Michelson Interferometer

The single-pass Michelson interferometer has a simple geometry that can be broken down into equations for the longest path traveled by a ray through the interferometer $L_b$ and the several other geometric parameters that determine the area and volume occupied by the optical components. The angle between the input arm and the output arm is fixed at 90 degrees. Other angles were explored and 90 degrees was found to produce the smallest interferometer areas and volumes. For convenience, the size of the beam splitter $x_{bt}$ is defined as the size of the beam $x_b$ plus extra tolerance on each side $\delta_t$,

$$x_{bt} = x_b + 2\delta_t. \quad (2.9)$$

Based on this definition, the longest path that a ray of light can travel through the interferometer portion can be approximated as,

$$L_b = T_L + \delta_c + \delta_e + 2\delta_b + 2d_R + 2x_{bt}, \quad (2.10)$$
Table 2.1: Single-pass Michelson interferometer dimensions table

<table>
<thead>
<tr>
<th></th>
<th>Length</th>
<th>Width</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Center</td>
<td>$x_{bt}$</td>
<td>$x_{bt}$</td>
<td>$x_{bt}$</td>
</tr>
<tr>
<td>Input Arm</td>
<td>$\delta_e$</td>
<td>$x_{bt}$</td>
<td>$x_{bt}$</td>
</tr>
<tr>
<td>Fixed Arm</td>
<td>$\delta_b + T_M$</td>
<td>$x_{bt}$</td>
<td>$x_{bt}$</td>
</tr>
<tr>
<td>Moving Arm</td>
<td>$\delta_b + d_R + T_M$</td>
<td>$x_{bt}$</td>
<td>$x_{bt}$</td>
</tr>
<tr>
<td>Output Arm</td>
<td>$\delta_c + T_L$</td>
<td>$x_{bt}$</td>
<td>$x_{bt}$</td>
</tr>
<tr>
<td>Lens and Camera</td>
<td>$L_c + T_C$</td>
<td>$x_{cx}$</td>
<td>$x_{cy}$ *</td>
</tr>
</tbody>
</table>

*Changes to $\max(x_{cx}, x_a)$ and $\max(x_{cy}, x_a)$ for a constraint on $x_a$

which is a function of the lens thickness not included in the focal length $T_L$, the
tolerance between the lens and the beam splitter $\delta_c$, the tolerance between the beam
splitter and the entrance $\delta_e$, the tolerance between the mirror and the beam splitter
$\delta_b$, the total stroke $d_R$ and the beam splitter size $x_{bt}$. Although corner cubes could
simplify angular alignment and could be used to replace the flat mirrors, corner cubes
tend to add extra lines to the image. Corner cubes require relative planar alignment,
increase the size of the overall system design and increase the path length of the
light. Corner cubes are also heavier therefore leading to lower bandwidths, are more
difficult to obtain/manufacture at smaller sizes and are harder to modify.

Clever design and folding of optical paths allows a designer to efficiently pack the
dimensions of an interferometer. Therefore, the calculation of the area or volume
occupied by the interferometer is not as straightforward as drawing a box around
the designs shown in Figure 2.2. Assuming that the optical paths from each arm
of the interferometer do not cross each other, the area and volume occupied by the
interferometer components can be calculated from the length, width, and height listed
in Table 2.3.1. Each of these arms can be folded in any dimension and the occupied
volume should not change.

The interferometer width and height are defined by the size of the beam splitter
which must be larger than the aperture height and the entrance height. The lens
and camera section of the interferometer has a length and height of $x_{cx}$ and $x_{cy}$ which are the real outside dimensions of an imaging camera chip. This assumes that $x_{cx} \geq x_a$ and this remains true for most of the cases of interest. If the diffraction limit constraint is enforced, the length and width values are changed to $\max(x_{cx}, x_a)$ and $\max(x_{cy}, x_a)$. The optical path difference as a function of mirror travel for this design is,

$$d_{OPD} \approx 2d_R.$$ (2.11)

It is important to point out that the delay across the image is not constant for this topology. If a ray enters at an angle $\phi_R$ through the center of the interferometer and the mirrors have a relative displacement of $d_R$, the relative delay between the ray that goes down the fixed arm with respect to the ray that goes down the moving arm is,

$$d_{OPD} \approx 2d_R \cos(\phi_R) - 2d_R \tan(\phi_R) \sin(\phi_R).$$ (2.12)

This equation assumes that a very thin plate beam splitter is used and there is a negligible effect from diffraction through the beam splitter glass. A thin lens approximation is also used here. The first term in this equation accounts for the delay from the rays going down the two different paths. The second term accounts for the shear between the two beams. When the two beams travel different distances, they also separate or shear by $d_{shear} = 2d_R \tan(\phi_R)$ in their projection on the imaging lens. These two rays are then recombined by the lens leading to an additional path difference that can be estimated as $d_{shear} \sin(\phi_R)$. Based on this equation, a field of view of 3 ° (comparable to the high speed version) will have a maximum OPD variation of 0.41% and a field of view of 6 ° (comparable to the multi-axis version) will have a maximum OPD variation of 1.6 %. These variations are fairly low across the image and can be corrected using the LS solution method in Section 7.1.

A more complex simulation is necessary for looking at real optical components,
such as thick or aspheric lenses, or for looking at interactions of beams that approach the interferometer at different angles or positions. This is discussed further in Section 2.4.

2.3.2 Double-Pass Michelson Interferometer

The equations for a double-pass Michelson interferometer are very similar. The longest path that a ray of light can travel through the interferometer is,

\[ L_b = T_L + \delta_c + \delta_e + 2\delta_b + 4d_R + 6x_{bt}. \]  \hspace{1cm} (2.13)

The corner cubes are approximated to have similar dimensions to the beam splitter with the same tolerances \( \delta_t \). The total area and volume can be calculated from the dimensions listed in the table. For the area calculations, \( \delta_{bt} = \delta_b + T_m \). Note that the area and volume occupied by the triangular mirrors in the fixed and moving arms are simplified in Table 2.3.2 by drawing a simple rectangular bounding box around the mirrors and a more accurate calculation of the area could be used if the designer is able to find a way to use the extra area.

<table>
<thead>
<tr>
<th></th>
<th>Length</th>
<th>Width</th>
<th>Height</th>
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</thead>
<tbody>
<tr>
<td>Center</td>
<td>( x_{bt} )</td>
<td>( x_{bt} )</td>
<td>( x_{bt} )</td>
</tr>
<tr>
<td>Input Arm</td>
<td>( \delta_e )</td>
<td>( x_{bt} )</td>
<td>( x_{bt} )</td>
</tr>
<tr>
<td>Fixed Arm</td>
<td>( \delta_{bt} + x_{bt} )</td>
<td>( 2x_{bt} )</td>
<td>( x_{bt} )</td>
</tr>
<tr>
<td>Moving Arm</td>
<td>( \delta_{bt} + d_R + x_{bt} )</td>
<td>( 2x_{bt} )</td>
<td>( x_{bt} )</td>
</tr>
<tr>
<td>Output Arm</td>
<td>( \delta_c + T_L )</td>
<td>( x_{bt} )</td>
<td>( x_{bt} )</td>
</tr>
<tr>
<td>Lens and Camera</td>
<td>( L_c + T_C )</td>
<td>( x_{cx} ) *</td>
<td>( x_{cy} ) *</td>
</tr>
</tbody>
</table>

*Changes to \( \max(x_{cx}, x_a) \) and \( \max(x_{cy}, x_a) \) for a constraint on \( x_a \)

56
The resolution of the double-pass Michelson interferometer is,

\[ d_{OPD} \approx 4d_R. \]  

(2.14)

Although this design has a higher resolution as a function of mirror translation, it does not scale as well as the single-pass Michelson interferometer design. The additional corner cubes add more total area and total volume to the design and the longer path length \( L_b \) contribute to a larger focal length \( L_c \) which will further increase the size of the interferometer for a desired resolution. Therefore, the size of a double-pass imaging Michelson interferometer will always be bigger than the size of a single-pass imaging Michelson interferometer for the same desired spectral resolution. These lessons can be applied to several other spectrometer designs. Longer optical paths inside the interferometer generally lead to larger designs due to the imaging requirements.

### 2.3.3 Common Path Sagnac

Common path Sagnac interferometers and Savart plates tend to be used as shear interferometers where the optical delay comes mainly from the lateral displacement or shear generated by the two paths through the interferometer [98, 97, 8]. For common path interferometers, there is no difference between the two paths taken through the interferometer and the OPD generated through shear is generally small, on the order of several hundred micrometers. The field of view of these interferometers are also typically limited to around 6°. The shear generated by a common path Sagnac is a function of the offset of the interferometer mirror \( d_a = \frac{d_R}{\sqrt{2}} \). The shear can then be estimated from the geometric conditions as \( d_{shear} = \sqrt{2}d_a = d_R \) [97] leading to an
estimate for the optical path difference,

\[ d_{OPD} \approx d_R \sin(\phi_R). \]  \hfill (2.15)

This equation shows that if the input light angle is zero, there is no optical path
difference, which means the center of the image will always have a OPD of zero. The
common path Saganc imaging interferometer can be used by either translating the
mirrors along the \( d_R \) axis or by rotating the entire assembly to scan an image through
different input angles. For the second usage case where the assembly is rotated, the
interferometer can be easily operated without worrying about the alignment through-
out the scanning process.

For the scaling equations, it is convenient to define \( \phi_{S1} = 67.5^\circ \) as angle of the mir-
ror relative to the input beam travel direction and \( \phi_{S2} = 22.5^\circ \) as the complementary
angle. It is also convenient to define, \( x_m = \delta_b + x_{bt} + \frac{3}{2} \tan(\phi_{S2}) \) as the distance from
the beam splitter to the center of the mirror. This distance includes the beam splitter
to mirror tolerance \( \delta_b \) and must be larger than the beam splitter size \( x_{bt} \) in order to
avoid interference between the beam splitter and the common path. Note that the
area and volume occupied by the common path interferometer portion is simplified
by drawing a simple rectangular bounding box around the mirrors. A slightly more
accurate calculation of the area could be used but will not change the relative scaling
appreciably.

The longest path that a ray of light can travel through the interferometer is then,

\[ L_b = T_L + \delta_c + \delta_e + 2 (x_{bt} + x_m) + \sqrt{2 \left( \frac{x_{bt}}{2} + x_m \right)}. \]  \hfill (2.16)

For a common path interferometer, this is the path length regardless of the displace-
ment \( d_R \) or \( d_q \). The area and volume of the interferometer can be calculated from
Table 2.3: Common path Sagnac interferometer dimensions table

<table>
<thead>
<tr>
<th></th>
<th>Length</th>
<th>Width</th>
<th>Height</th>
</tr>
</thead>
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<tr>
<td>Sagnac</td>
<td>$L_S$</td>
<td>$L_S + d_a$</td>
<td>$x_{bl}$</td>
</tr>
<tr>
<td>Input Arm</td>
<td>$\delta_e$</td>
<td>$x_{bt}$</td>
<td>$x_{bt}$</td>
</tr>
<tr>
<td>Output Arm</td>
<td>$\delta_c + T_L$</td>
<td>$x_{bt}$</td>
<td>$x_{bt}$</td>
</tr>
<tr>
<td>Lens and Camera</td>
<td>$L_c + T_C$</td>
<td>$x_{cx}^*$</td>
<td>$x_{cy}^*$</td>
</tr>
</tbody>
</table>

*Changes to $\max(x_{cx}, x_a)$ and $\max(x_{cy}, x_a)$ for a constraint on $x_a$

Table 2.3.3. It is convenient to define the Sagnac side length as,

$$L_S = (2 + \tan(\phi_{s2}) x_{bt} + \delta_b + T_M / \cos(\phi_{s1})).$$ (2.17)

Due to the small OPD and field of view, these interferometers are not generally used for high resolution imaging spectroscopy but can be used effectively for lower spectral resolution applications.

### 2.3.4 Electro-Optic Modulation

Electro-optic modulation is an interesting method for obtaining an optical path difference without using any moving parts. The most basic design is shown in Figure 2.2d. For practical implementations, several polarizer and EOM stages are needed [30]. The optical delay of this system comes from the difference between the index of the EOM material along the extraordinary axis versus the ordinary axis for $\phi_R = 0$,

$$d_{OPD} \approx \Delta n d_R,$$ (2.18)

with an EOM material thickness of $d_R$. The index of refraction in the material can be changed by altering the voltage on the EOM. Large values for EOM materials are in the range of $\Delta n = 0.124$ for Beta Barium Borate (BBO). The index difference for
Table 2.4: Electro-optic modulator interferometer dimensions table

<table>
<thead>
<tr>
<th></th>
<th>Length</th>
<th>Width</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOM</td>
<td>$T_L + 2T_p + \delta_c + \delta_e + 2\delta_b + d_R$</td>
<td>$x_{bt}$</td>
<td>$x_{bt}$</td>
</tr>
<tr>
<td>Lens and Camera</td>
<td>$L_c + T_C$</td>
<td>$x_{cx}$</td>
<td>$x_{cy}$</td>
</tr>
</tbody>
</table>

*Changes to max($x_{cx}$, $x_a$) and max($x_{cy}$, $x_a$) for a constraint on $x_a$*

Liquid crystal is in the range of $\Delta n = 0.22$ [172].

The maximum path length of a beam of light with an entrance angle of $\phi_R = 0$ is approximately,

$$L_b = T_L + 2T_p + \delta_c + \delta_e + 2\delta_b + n_I(\lambda)d_R.$$  \hspace{1cm} (2.19)

The path length through the crystal is governed by the crystal length $d_R$ and the maximum index of refraction $n_I(\lambda)$. For BBO, this is between 1.65 and 1.82 and is a function of wavelength. The area and volume can be similarly calculated, this time without incorporating the refractive index in the length of the EOM section as shown in Table 2.3.4.

Because the EOM design does not include a beam splitter and has no moving parts, it seems ideal for many compact, lower spectral resolution applications. Because the refractive index tends to vary as a function of wavelength, however, small artifacts may be created in the image if different wavelengths are measured at the same time. Spectral correction algorithms may also be necessary. Additional optics used to correct these non-ideal aspects may lead to spectrometer sizes that are larger than the ones predicted with this model.

### 2.3.5 Spectral Resolution Comparisons

The models described earlier were used to determine the smallest spectrometer design for a given OPD specification. For these simulations, a small 1/6" format camera was used with a height of $x_c = 3$ mm and a total size of $x_{cx} = x_{cy} = 6$ mm. The total
volume of each topology was minimized to determine the optimal beam height $x_b$ for a desired OPD. First, idealized parameters were simulated in Figure 2.3 where all the thickness and tolerances are set to zero. This simulation also has no constraints set on $x_a$.

From these graphs, it is clear that the simple single-pass Michelson topology has the lowest volume for the largest OPD. For OPD less than 500 $\mu m$ the minimum volume can be achieved with the EOM design. The double-pass Michelson interferometer has a much larger size for the same optical path difference even though the optimal beam height and view angle are much smaller. The common path Sagnac interferometers have much larger areas and have very short OPD.

Since these solutions are not constrained, these designs can be made diffraction limited by making the aperture size equal to the $x_a$ value. In the case where $x_b$ is smaller than $x_a$, the design is already diffraction limited. The field of view is the largest for the EOM design for low OPD followed by the single-pass Michelson. The double-pass Michelson and the common path Sagnac designs have the lowest fields of view. For higher spectral resolutions, the EOM designs have smaller fields of view.

Constrained solutions are also simulated in Figure 2.4 with no vignetting and with diffraction limited imaging. Due to the constraints, the overall dimensions are all larger. The beam size $x_b$ is always larger than or equal to the aperture size $x_a$. In some cases the aperture size $x_a$ is larger than $x_{cr}$ and $x_{cy}$ which further increases the total area and volume. This can be seen in the area and volume of the EOM and double-pass Michelson designs when the line curves up slightly for $x_b > x_{cr}$ where $x_{cr} = 6 \, mm$. The smallest topology in the constrained case is also the single-pass Michelson design.

Realistic parameters can also be simulated in Figure 2.5. The tolerances are generally 0.5 $mm$ between components and 0.25 $mm$ extra on each side of the beam
Figure 2.3: Optimization of different dimensions as a function of OPD is shown with idealized parameter values. The dimensions include a) heights $x_b$ and $x_a$, b) lengths $L_b$ and $L_c$, c) total area, d) total volume, and e) view angle. The parameters used for optimization are shown in the bottom right corner.

splitter. The component thicknesses are from components that are used in the different implementations listed in Chapter 3. In general, the use of the realistic thicknesses and tolerances makes the overall dimensions much larger but do not largely change
Figure 2.4: Optimization of different dimensions with vignette and diffraction limit constraints as a function of OPD is shown with idealized parameter values. The dimensions include a) heights $x_b$ and $x_a$, b) lengths $L_b$ and $L_c$, c) total area, d) total volume, and e) view angle. The parameters used for optimization are shown in the top bottom corner.

the relative comparison between the topologies. The single-pass Michelson design has the smallest volume for large OPD.
Figure 2.5: Optimization of different dimensions as a function of OPD is shown with realistic parameter values. The dimensions include a) heights $x_b$ and $x_a$, b) lengths $L_b$ and $L_c$, c) total area, d) total volume, and e) view angle. The parameters used for optimization are shown in the bottom right corner.

2.3.6 Image Sensor Format Comparisons

The variation of the spectrometer parameters can also be explored when the camera size is changed. For these simulations, the optical path difference was chosen as
6 mm. Since the common path Sagnac and EOM designs are not competitive for this set of dimensions, they are not included. Between the single-pass and double-pass Michelson interferometer designs, Figure 2.6 shows that single-pass design scales the best across a full range of camera sizes.

The lessons learned from the scaling simulations can be applied directly to a realistic implementation. A 1/6" format camera has a size of $x_c \approx 3$ mm and a 1/2" format camera has a size of $x_c \approx 6.4$ mm. For the smaller format camera, the optimal beam size is 3 mm. The closest beam splitter size that is readily available is 5 mm. For the larger format camera, the optimal beam size is 5.6 mm. The beam splitter size that is readily available that is larger than this size plus the tolerance is 10 mm. Going to the larger size generally allows for better mounting and allows for additional optics to be used for position measurement. By using these parameters, the other dimensions can be calculated and component can be selected. For example, this information can then be used to choose a lens with the proper focal length and utilized to determine how to fold the optical paths to keep the enclosed volume low. The optical étendue of these designs with 6 mm OPD is typically in the range of 0.1 to 0.32 mm$^2$ sr. These values are much larger than typical dispersive spectrometers due to the larger input apertures.

The comparison of the design topologies discussed here encompass only the areas and volumes of the optical components. There are, however, clear scaling contributions from actuators, enclosures, and electronics. A single-pass Michelson topology is clearly better for the high spectral resolutions desired. The additional advantage that it has over a double-pass topology is that there are fewer components such as mirrors and would therefore require fewer mounting features. A double-pass Michelson interferometer, however, requires less mirror translation $d_R$ for the same OPD therefore requiring a smaller actuator. But, the larger triangular mirror in the double-pass
Figure 2.6: Optimization of different dimensions as a function of camera format \( x_c \) is shown with realistic parameter values. The dimensions include a) heights \( x_b \) and \( x_a \), b) lengths \( L_b \) and \( L_c \), c) total area, d) total volume, and e) view angle. The parameters used for optimization are shown in the bottom right corner.

would require a more powerful actuator to achieve a bandwidth similar to the single-pass design (that only needs to move a single mirror). A more powerful actuator would
need to be larger (if optimal material choices have already been made) or would need larger, higher power electronics. The electro-optic modulation design has no moving parts therefore requiring no actuator. However, the design requires additional complex high voltage electronics for modulation leading to even larger spectrometer volumes. Based on this analysis, the single-pass Michelson interferometer was chosen due to its preferable scaling qualities at high spectral resolutions.

2.4 Interferometer Ray Tracing

There are several methods for estimating the optical properties of an interferometer design. A simple first order estimation can be done by using the ray transfer matrix technique [147]. This method, however, is difficult to use for accurate estimation of interactions with complex curved surfaces including achromatic and aspheric lenses. For more precise optical estimation, a ray tracing method should be used. Programs such as Zemax and WinLens3D allow users to solve and optimize complex optical systems more accurately. For this work a simple ray tracer was designed with limited capabilities for handling complex curved surfaces and for assessing interference effects.

Each module of the ray tracing program works by taking in three types of inputs, the ray information, the intersection point data, and the dimensions and properties of the optical component. Each module is broken up into different surface interactions and propagation through a variety of media. For a simple lens, there are two interaction surfaces and propagation through the lens as well as the free space before entering the lens. A simple mirror has only one interaction surface and one free space propagation. An achromat has three surfaces and three propagation spaces. For each surface, the program calculates the intersection point of an input ray with the surfaces and the angle at which the ray exist. The array of intersection points are collected and can be graphically displayed for verification.
With this architecture, two-dimensional and three-dimensional simulations can be constructed. In addition, components can be rotated or tilted by multiplying the mirror and lens objects with simple rotation matrices. The rays themselves can also travel in any direction and apertures can be constructed by simply terminating the incoming rays.

For mirrors or lenses with spherical surfaces, only the radii, thicknesses, and material types (to determine the refractive index) need to be provided. The intersection points between a spherical surface and a line are easy to calculate. In order to account for rotation and translation of the lens in the design, all intersection points are calculated in the frame of reference of the optical component. For a two-dimensional simulation, the incoming ray information (position \( x_{r0} \) and \( y_{r0} \) and propagation angle \( \alpha_{r0} \)) must first be converted to the lens frame of reference and rotated by the angle \(-\theta\), resulting in the values for \( x_r \), \( y_r \), and \( \alpha_r \). Next, the intersection points and exit ray angle are calculated using,

\[
A_L = (1 - \tan^2(\alpha_r)), \quad (2.20)
\]
\[
B_L = 2D_L\tan(\alpha_r) - 2w_L, \quad (2.21)
\]
\[
C_L = D_L^2 + w_L^2 - r^2, \quad (2.22)
\]
\[
D_L = y_r - x_r\tan(\alpha_r), \quad (2.23)
\]
\[
x_L = \frac{-B_L + \text{sgn}(r)\sqrt{B_L^2 - 4A_L C_L}}{2A_L}, \quad (2.24)
\]
\[
y_L = y_r - x_r\tan(\alpha_r) + x_L\tan(\alpha_r), \quad (2.25)
\]
\[
\phi_L = \text{sgn}(r)\tan^{-1}\left(\frac{y_L}{\text{sgn}(r)(w_L - x_L)}\right), \quad (2.26)
\]
\[
\alpha_L = -\phi_L + \sin^{-1}\left(\frac{n_L \sin(\phi_L + \alpha_r)}{n_{l2}}\right), \quad (2.27)
\]

where \( r \) is the radius of the lens at that particular interface, \( t_L \) is the thickness of the lens at the center and \( w_L = r \) for the first interface and \( w = r + t_L \) for the second
interface. After determining the intersection points in the coordinate system of the lens, they can be translated back to the main coordinate system by rotating by $\theta$ and translating the $x$ and $y$ points.

For aspheric lenses, the equations are not as straightforward and cannot be solved generally. Therefore, a Levenberg-Marquardt nonlinear minimization technique [125] is used to solve for the intersection points $x_L$ and $y_L$ from the constants provided by the lens manufacturer (or from existing Zemax files) $k_L$ and $c_i$.

$$y_r - x_r \tan(\alpha_r) - x_L \tan(\alpha_r) - y_L = 0, \quad (2.28)$$

$$\frac{y_L^2}{r(1+\sqrt{1-(1+k_L)y^2/r^2})} + \sum_{i=1}^{8} c_i y^{2i} - x_L = 0. \quad (2.29)$$

Once the intersection points are calculated and plotted, the total path length can be evaluated by simply summing the vector magnitudes. The path length through the fixed arm versus the moving arm of the interferometer can be compared to determine the OPD variation as a function of different parameters and components.

These equations were used to simulate the scenario shown in Figure 2.7a. The simulation includes an input lens, a beam splitter, a fixed mirror, a moving mirror, an output achromat, and an imaging plane. The simulation varies the height and angle of the input light and serves as a more accurate estimate of the OPD as a function of input angle than the estimate provided by Equation 2.12. The point spread function simulation for the test case in Figure 2.7c was computed using WinLens3D. The simulations correspond to the high speed version of the IFTS design described in Section 4.4 in the special case where there are no application specific optics. Additional conditions include a 2 mm entrance aperture, a 6 mm imaging lens aperture, and a 4 degree limit on entrance ray angle.

The shape of the OPD curves as a function of angle at different input heights is very similar to the curves that can be calculated with Equation 2.12. As the angle
Figure 2.7: a) The custom ray tracing simulation created in Matlab can be used to evaluate different optical arrangements and components. In this case, the lenses are 30 mm FL achromats and the light wavelength simulated is 580 nm. b) For the arrangement shown above, simulations were conducted to test the variation in OPD as a function of the entrance angle and entrance height for a full stroke of 3 mm (6 mm OPD). c) The point spread function or spot diagram for the 30 mm FL imaging achromat (Edmund 47-694) is shown for the simulated condition of a path length of 28 mm between the entrance aperture and the lens. The simulations are at 0, -100 or -200 μm focal point offset and 0 or 4 degrees entrance ray angle. The simulated input ray height is 1 mm. The black circle has a radius of 10 μm.

increases, the optical path difference changes by a small percentage near 0.25 % for a 30 mm FL lens. However, if more than a single input height is overlapped, this may cause some averaging of different fringes across the image for any given input angle that is not close to zero. The variation that appears in Figure 2.12 at 4 ° is about 230 nm for this lens and increases as the entrance angle increases. However, this variation may not always be small. The delay variation as a function of entrance height may vary by as much as several micrometers or more depending on the choice
of lens and wavelength.

The most accurate high spectral resolution images can be obtained when the entrance aperture (not the imaging lens aperture) is small. This is generally not a problem for images that have low spectral resolution, such as simple evaluations of LED color or fluorescence spectra. Using a small entrance aperture can affect the integration times of the images for higher resolution spectra like those for Raman spectroscopy.

One approach to solving this problem is to specifically design or choose a lens configuration which minimizes this effect for a variety of wavelengths. It is possible to design a specific optical delay component to perfectly compensate for the OPD variation inside the single pass Michelson interferometer as well as the OPD variation inside the imaging lens. If custom optical components are not used, either accurate position measurement correction (using non-uniform sampling Fourier transform solvers) or rough wavelength correction [131] can be easily implemented to provide more accurate results.
Chapter 3

Design and Manufacturing

A miniature imaging Fourier transform spectrometer (IFTS) can be used for many different applications. By following the scaling laws for miniaturization, it is possible to design a generalized small, handheld spectrometer with the layout indicated in Figure 3.1. In this figure, there are two main sections, a general imaging Fourier transform spectrometer component and separate application specific optics section. The application specific optics section includes an input light block for applying white, Raman or fluorescence light inputs utilizing lasers, light emitting diodes (LEDs), or thermal sources.

The generalized IFTS section is composed of several important subsystems, the first being the positioning subsystem indicated by the purple light path based on a narrow line-width vertical-cavity surface-emitting laser (VCSEL) laser and a series of photodiodes that can be used to measure the relative tip, tilt and translation of the two mirrors for real-time control. The instrument has a Michelson interferometer for input image autocorrelation. This incorporates a moving mirror which requires a relatively high stroke actuator which can produce high resolution spectra (down to 1.6 cm\(^{-1}\) for an optical path difference of 6.25 mm). The other mirror which can have a tip, tilt or translation actuator is another subsystem. As the mirrors are
displaced relative to one another, the light from the image interferes constructively and destructively to create dark and light patterns. The last subsystem is the camera and associated aperture and lens. The camera captures images from different mirror positions and reconstructs them into high-resolution spectra for each pixel.

This chapter discusses the manufacturing techniques used for IFTS including the construction of mechanical and optical components. Then the design of actuators is discussed along with the layout of the electronics and software. The different implementations of the design are discussed in the next chapter.

### 3.1 Manufacturing Techniques

Several manufacturing techniques were explored for the design of the miniature IFTS. The most important aspect of the design, outside of actuators, are designs for holding the optics in place while blocking undesirable scattered light. One possible solution would have been to glue or epoxy components to a substrate while completing an alignment process. This however, requires active testing during every step in assembly.
Instead, a different approach is taken. Once the optical components are placed in the system, the manufacturing technique should allow for the creation of features which can adjust the relative position of optical components to take up small errors in alignment during assembly.

Several manufacturing techniques were considered including micro machining, injection molding, casting, metal extrusion molding, laser machining, high aspect ratio MEMS techniques such as LIGA, and additive manufacturing methods such as stereolithography and metal sintering. Many different materials from plastics to composites were also considered. Based on the scale and feature sizes, electric discharge machining (EDM) was chosen as the main manufacturing technique for the mechanical components. For custom thin optical components, masking and sputtering techniques were chosen.

3.1.1 Electric Discharge Machining

Electric discharge machining is a method where conductive materials can be cut by using microscopic electric sparks at kilohertz to megahertz rates between the material and an electrode. In wire EDM, a thin wire (30 to 250 μm) serves as the electrode and can be used to create high aspect ratio features that are the size of the diameter of the wire plus spark gap (which is a function of the material type and cutting parameters). Several different components used in the IFTS which were manufactured using wire EDM are shown in Figure 3.2 using a 250 μm wire and a Charmille Robofil 1020SI machine. Parts designed in SolidWorks 2013 were converted to G-code using FeatureCAM for early versions and CAMWorks 2013 for later versions. A complete list of EDM parts for each implementation is listed in Appendix A.1.

This figure shows the many materials, assembly methods, post processing techniques, and geometries that were used in the design of the IFTS. The wire EDM
Figure 3.2: Components made by electric discharge machining (EDM) manufacturing are shown including a) a three process aluminum flexure, b) a copper heat sink, c) a small preload spring and layers of aluminum and stainless steel assembled in a stacked fashion, and d) a low-carbon steel yoke (a.k.a. iron yoke) with an aluminum bobbin and flexure holder assembled in an orthogonal fashion.

process can cut through thick metal parts like steel and aluminum. The more conductive materials like copper take longer to process. Wire EDM lends itself well to a layered manufacturing technique as shown in Figure 3.2c where several layers with different features are stacked together. However, it is also capable of making skewed cuts. Multiple cuts on the same part can also be done as shown in Figure 3.2a, where three different cuts along three different axes have been made for the same part. These layered components can also be assembled orthogonally as shown in Figure 3.2d.

Through holes can be cut on the EDM by adding slot features (300 μm across) as shown in Figure 3.2c. These features can be tapped for threads as small as M1. The parts can be post processed with additional milled features or through hole features. The EDM can be used to mark the location of drill holes (similar to a center punch) to help with post processing steps. Precise alignment of stacked structures can be achieved by using hole and slot features for placing M0.8 or M1 dowel pins. Notches can also be designed into the part to allow orthogonal components to snap fit into the structure as shown by the aluminum flexure holder and aluminum bobbin in Figure 3.2d. The parts can be held together using M1 screws with threaded features in layers thicker than 1 mm.
In order to adjust the relative position of optical components, flexures were designed and manufactured using wire EDM. These built-in tip and tilt flexures allow the elements to be adjusted to passively align (or factory align) the optics to take up errors in manufacturing. When properly designed, the adjustment should only be necessary once after assembly. Rotational flexures [80, 82, 155], that can be used to adjust the angle of mirrors, can be constructed by producing thin areas in the material. Flexures as thin as 150 \( \mu m \) have been produced. Spring structures that can be used for preloading can also be manufactured as shown in Figure 3.2c.

In traditional flexure designs, it is important to design the system with a preload and use a screw or actuator to work against the preload to adjust the position. In the design of the IFTS, a pair of screws is used instead of a preload for flexure manipulation as show in Figure 3.2a. When one screw is tightened, the two faces of the flexure are brought closer together. When the other screw is tightened the two faces are pushed apart. The angle between the two faces can be adjusted by manipulating the two screws and when both of them are tightened, the stiffness of the joint increases and provides a more permanent alignment of the components. If the positions of the two screws are aligned along the axis of the flexure, it is possible to achieve tip and tilt motions with a single flexure by adjusting the relative load of the two screws. Since linearity, flexure stiffness and plastic deformation are not important to the design of this type of double-loaded flexure system, there is more flexibility in design geometry.

For adjusting flexures, M1 screws are used because the thread pitch is relatively tight at 0.25 \( mm \). For every full rotation of the M1 screw, the travel of the flexure changes by 250 \( \mu m \) allowing for fine adjustment. For rotational flexures, using longer lever arms gives more angular positioning resolution.

The spectrometer layered components are typically made from non-magnetic met-
als as to not interfere with the magnetic structure of the linear actuator. The aluminum alloy 7075 was chosen due to his high strength to Young’s modulus ratio and suitability as a non-magnetic flexure material for thickness from 0.6 mm up to 12 mm. For material thicknesses less than 0.6 mm, precision 300 series stainless steel shim stock is used in both the structure of the IFTS and for the high stroke flexure systems. This material is slightly magnetic on the surfaces from the cold working process. Actuator material selection and design is discussed in Section 3.2. The thermal coefficient of expansion of the material is also important but the active degrees of freedom in the interferometer can be used to account for dimensional changes as a function of temperature. Invar, for example, is magnetic and has a relatively low yield stress to Young’s modulus ratio compared to the types of aluminum and steel that are used in this design, making it not suitable for the design requirements.

### 3.1.2 Aluminum Anodization

By using a layered manufacturing process, it is possible to completely encase the optical components to prevent external light from reaching the spectrometer. However, this does not prevent light from scattering off of the internal surfaces of the spectrometer. Therefore the internal surfaces of the aluminum can be colored using an anodization process. For small custom components, a simple anodization process was utilized involving several different steps:

- Surface preparation: The anodization process can add anywhere from 5 to 40 μm to the size of the parts and will tend to matte the finish. The anodized layer will also be more brittle and less conductive. Any parts that need to remain soft or conductive should be masked using anodization masking tape. Threads may need to be re-tapped after the anodization process. Dust and EDM residue are removed using sonication and oils are scrubbed off with soap.
Chemical cleaning: Aluminum alloys will often have metals that must be removed from the surface before anodization. All chemical processing is completed in a hood. The parts are first etched using a mixture of lye in water (0.4 % up to 4%) for a few minutes. This process can also be used to remove anodization. Alloys like 7075 aluminum will turn black during this process. Immediately afterwards, the parts should be submerged in distilled water. Next, the part needs to be desmutted using a concentration of 1.5 % nitric acid in water for a few minutes. This cleaning process may change the dimensions of the part if left in solution too long. The part should be submerged in distilled water when finished. A properly prepared part will be hydrophilic (no droplets form on the aluminum).

Anodization: An anodization bath for small parts is created and is shown in Figure 3.3a. The cathode is made from 6061 aluminum sheet and high purity aluminum wire is used to connect the part to the large anode sitting outside the bath. The anodization bath is a solution of 15 % sulfuric acid in distilled water. The system needs to be worked-in by running a short anodization test for the
best results. The parts to be anodized are attached firmly to the aluminum wire by looping the wire through holes in the part and the conductivity is tested. Only aluminum components should be in this bath. Any location of the part that is in firm contact with the aluminum wire will not become anodized. The surface area of the parts is calculated and the typical current density used is between 130 and 250 $A/m^2$. The parts are left to anodize in the bath for about 30 minutes. A constant current is set on the power supply and the voltage of the bath will slowly increase over time as the anodization deposits on the surface of the part. The part will start to produce bubbles during this process as shown in Figure 3.3b. The gas produced should not be inhaled. When the part is finished, it can be submerged in distilled water.

- Dyeing and sealing: The anodized layer of aluminum is porous and can be dyed using 1% Castwell black anodization dye in water. The solution is heated to 60 °C and the part is allowed to soak for 15 to 30 minutes. The part can then be sealed using 0.08% Castwell anodization sealant in water heated to 98 °C for another 15 to 30 minutes. The final result is shown in Figure 3.3c.

The anodization, especially processes with thick anodized layers, produces a very dark surface which reflects very little light due to its matte finish, dark dye and porous nature. Anodization is also not as conductive which is good for preventing shorts when attached to electrical components. This, however, can make it difficult to case-ground the device. For case grounding, the parts can be partially masked or can be drilled or sanded after the anodization process to provide a electrical connection.
3.1.3 Thin Optical Components

Small, thin optical components are important for a small imaging spectrometer. The quality of components is most important for the optics that are in the interferometric path, including the two main mirrors and the beam splitter. Small cube beam splitters with no ghosting effects are purchased. The mirrors can also be purchased as larger 1 \text{mm} thick mirrors that are 20 \text{mm} across (thinner mirrors tend to be less flat). The quality of these large mirrors can be lower than the final specification, for example, a 4 \lambda first surface mirror that is 20 \text{mm} in size becomes a 1 \lambda mirror when cut down to the desired 5 \text{mm} size. Mirrors are cut by a scoring and breaking process using a carbide tip that is normally used for fiber optic scoring. The edges are then ground on a diamond grinder to prevent any cracks from propagating.

It is normally difficult to find mirrors that are thinner than 1 \text{mm}. For optics that are not directly in the interferometric path, it is possible to make thin, lower quality optics by sputtering metal on thin sheets of glass like microscope cover slips. These cover slips are on the order of 130 to 250 \mu m thick with good surface finish. Gold or silver at different thicknesses is sputtered directly onto the glass using a Denton Vacuum Desk V sputter coater as shown in Figure 3.4a. Different thickness can be used to produce mirrors or beam splitters of different split ratios. Thin layers of

![Figure 3.4: a) Sputter coating of thin mirrors with gold or silver is shown. For a sputtering current of 40 mA the split ratio of the transmission to reflection can be controlled as a function of sputtering time for a) gold and b) silver tested at 780 nm.](image-url)
Figure 3.5: The sputter coating of thin mirrors a) for scratch resistance is shown in using b) a three layer process. c) Miniature beam splitters and mirrors can be made using this process. d) The ratio of transmission and reflection and e) overall absorption for different stages in the manufacturing process are shown for sputtering silver.

Sputtering can potentially cause some scattering and undesirable absorption of light leading to some loss at the beam splitter.

Sputtering gold or silver directly on glass is not a long term solution for making mirrors as these coatings are very delicate and difficult to assemble without damage. The silver beam splitters are especially prone to tarnish. Therefore, layers of titanium are first sputtered onto the glass to promote adhesion and an additional layer of titanium is sputtered on top of the silver to prevent scratches and damage from finger oils. The sputtering conditions are first titanium at 100 mA for 10 minutes, followed by silver at 80 mA for either 15 s for beam splitters or 40 s for mirrors, followed finally by titanium at 100 mA for another 10 minutes. The results of this three layer process are shown in Figure 3.5a and a diagram of the multi-layer sputtering is shown in Figure 3.5b. Based on this method, small beam splitters and mirrors can be made for the positioning stage optics. The glass is first cut using a scoring and breaking process, and then sputtered using the three layer process as shown in Figure 3.5c.

82
Figure 3.6: The sputter coating of mirror levels is shown. a) The desired levels are achieved via masking and sputter coating. b) The SEM can be used to show the structure of the mirror and c) the relative mirror level heights can be measured in a detuned interferometer.

The effect of different layers on the beam splitter is shown in Figure 3.5d and e. When only silver is sputtered onto the glass, the absorption is fairly low. When titanium is added on top or below, the absorption increases and is highest with two layers of titanium. The final product tends to have a golden color which may be due to titanium nitrides forming at the surface from excess nitrogen in the sputtering chamber or the deposition of other impurities from inside the sputtering chamber.

Sputtering opens up other possibilities for manufacturing custom optical components. For the positioning system, it is desirable to have different optical delays. Additional details are outlined in Section 5.1. Different levels can be sputtered onto a mirror by masking between sputtering processes. For the layered structure shown in 3.6a, the areas 1, 2 and 3 are left exposed while the rest of the mirror is masked. After the first sputtering run, the mirror is masked all except for the area 1 and another sputtering iteration is completed. Additional layers of titanium can be added to promote adhesion and prevent tarnish.
An SEM of the mirror structure is shown in Figure 3.6b. In order to measure the thickness of the sputter coating, the multi-level mirror is used as one of the mirrors inside a larger interferometer. A green 532 nm laser is used to illuminate the mirrors and the output of the interferometer can be used to determine the thicknesses of the layers by comparing the displacement of the detuned interferometer pattern as shown in Figure 3.6c. The fully aligned interferometer image is shown in the inset. By comparing the difference of the edges, it was determined that the height of each of the levels is 180.9 ± 34.1 nm for a sputtering time of 276 seconds at 40 mA for the silver layers. The edges of the full mirror tend to show smoothing from the sputtering method but the level edges near the center are more discrete.

3.2 Lorentz Force Voice Coil Motor Design

The most important component of the design is an actuator for the moving mirror. This actuator must be able to travel several millimeters, have a high positioning resolution on the order of micrometers, and have a high bandwidth in order to move quickly. Traditionally, FTIR instruments and other interferometers sweep through the different positions and only control the speed at which the actuator moves, and not the position. These systems use fast sampling on fast photodiodes in a continuous fashion. Imaging interferometers based on a camera sensor for imaging must be operated differently. Because snapshots are taken rather than continuous data, it is important that the moving mirror actuator be able to hold the same position for the entire imaging time or else the interferometric pattern will smear in time across several frames degrading the signal quality. This is discussed in more detail in Chapter 5.

Based on these criteria, several actuators were considered including piezoelectric actuators, stepper motors, MEMS pop-up mirrors [55, 170], and rotary to linear conversion motors. The design that minimized the motor size for the desired stroke and
Figure 3.7: a) The Lorentz force voice coil motor and flexure design is shown along with b) preliminary linear translation results as a function of input voltage to the motor for different blade flexure thicknesses.

reduced the effect of friction (which may cause unpredictable parasitic rotation on the mirrors) was a linear Lorentz force voice coil motor with a flexure based return spring. This system, theoretically, has infinite resolution and is only limited by the digital to analog converters (DAC) and amplifiers used to drive it. The flexures also suspend the motor so that there is no friction between the bobbin and the housing. A simple diagram of the motor design is shown in Figure 3.7a and preliminary translation results were obtained for flexures of different thicknesses in Figure 3.7b.

Several different design iterations were done on the motor, some of which are shown in Figure 4.1. The overall design of the motor uses a cube-like geometry. This geometry is easy to manufacture using wire EDM, easy to assemble and can easily ac-
commodate high rotational-stiffness blade flexures. By attaching the mirror at some point closer to the coil, the overall IFTS assembly can also be made smaller. This is easier to accomplish with a cube-like geometry than with a cylindrical geometry. Magnets are also more readily available for this type of design because it does not require axial magnets or circular segment magnets that may be required by a cylindrical geometry. The packing density of a cube and ease of assembly with other components is also preferable. Using a cube-like structure also has downsides including corners which are sources of loss and leakage in the magnet structure.

The actuator yoke is made from low-carbon 1018 steel, which has a high magnetic permeability. The blade flexures are made from thin 300 series stainless steel shim stock, which can be made with a wire EDM if clamped to thicker pieces of metal on the top and bottom. Although this material is slightly magnetic, the high yield strength to Young’s modulus ratio makes it an ideal material for blade flexures.

In this design, there are two facing magnets (gold colored cube that is 3.175 mm on each side, K&J Magnetics B222G-N52) which connect at an iron or stainless steel plate. The field lines then go from the steel plate through free space and then through the iron or stainless steel yoke and back though the magnets. The part of the field that passes through the coil contributes to the force produced by the motor when a current is applied. The coil is typically between 5 to 8 Ω and wound with 32, 34, or 36 gauge (0.127 to 0.202 mm OD) copper magnet wire depending on the amount of space available for the bobbin. The coil is attached to the blade flexure system through a bobbin. When a force is produced in the coil, the force acts against the flexure stiffness.
3.2.1 Magnet Structure

Design optimizations for the magnet structure were done in Comsol 4.3b. The magnets chosen for this design are N52 neodymium-iron-boron (NdFeB) block magnets with high field densities and a high magnetic energy density of 52 MGOe (414 kJ/m³). The 3D simulation of the original two magnet design is shown in Figure 3.8a. It is clear from the figure that there are many areas in the yoke that have a relatively low magnetic field density and many areas that may be saturated (dark blue). After several iterations, a six magnet design was developed as shown in Figure 3.8b. The original two magnets are present (highlighted in pink) plus two extra magnets placed on the original yoke and two more magnets situated in the new yoke. These features are highlighted in green in the figure. The flux density diagram shows many more areas that have a higher magnetic flux. The areas that were originally saturated have also been improved.

The most important parameters can then be optimized under space and stroke constraints to produce higher output forces and smaller force deviations across the

![Figure 3.8: Simulations of the magnetic field distribution of two different motor designs, a) a two magnet design and b) a six magnet design.](image)
Figure 3.9: The motor magnet designs with simulation specifications are shown along with different implementations. Three distinct designs are shown with a) two magnets, b) four magnets, c) and six magnets.

Figure 3.10: a) Orientation and b) simulation of the flexure translation is shown.

full stroke. The effect of thicknesses of different components including the coil, the yoke dimensions and the center iron segment are simulated and optimized. Three different designs with performance predictions are shown in Figure 3.9 with images of different implementations on the multi-axis version of the IFTS.

3.2.2 Flexure Design

An offset double-compound flexure system is used in these designs, which provides low stiffness in the actuation direction and high linear and torsional stiffness in other
directions while also saving space. The flexure design also provides pure translation without much rotation or vertical shift. The analytical approximations [80, 155] for a double compound blade translation flexure (both planar and offset) using the coordinate system defined in Figure 3.10a are,

\[ K_z = 2Eb_f \left( \frac{d_f}{L_f} \right)^3, \]  
(3.1)

and

\[ q_{max} = \frac{2\sigma_{max}L_f^2}{3Ed_f}. \]  
(3.2)

The first equation defines the stiffness of the design in the direction of travel \( K_z \) and the second equation defines the maximum repeatable linear translation before the flexures enter a plastic or failure regime \( q_{max} \). The Young’s modulus is \( E \), the width of a flexure is \( b_f \), the thickness of the flexure is \( d_f \), the length of the flexure is \( L_f \), and the maximum yield stress of the material is \( \sigma_{max} \). The second equation is used to define the desired travel to determine the necessary length and thickness of the flexure. Then the first equation can be used in conjunction with the known force output of the motor design \( K_z = \frac{F_{max}}{q_{max}} \) to define the width of the flexure blade. Additional analytical equations exist for stiffness in other directions as well as rotational stiffnesses for the planar double compound motor case. In other cases, such as for the offset double compound design, these values need to be simulated. A simulation of the flexure during translation is shown in Figure 3.10b.

The resonant frequency of the motor system can be approximated from a second order fit for the stiffness of the flexure \( k_t \) and the total moving mass \( m_t \) of the system \( \omega_n = \sqrt{\frac{k_t}{m_t}} \). It is however, clear from the design that the system is not a simple second-order system. Note that the designed stiffness \( K_z \) is similar to but not exactly the same as the total stiffness from a simplified second order fit \( k_t \). There are two major masses, the motor mass and the mass of the crossbar at the top of
Figure 3.11: The simulated motor a) stiffness specifications, displacement difference across a 5 mm mirror as a function of displacement and b) dynamics are shown for a 100 μm flexure blades. Simulations were completed in Comsol and Matlab. The displacement of the motor coil is designated as $z_2$ and the displacement of the crossbar mass of the flexure system is designated $z_1$.

the flexure. The system can be modeled as a fourth order system with two masses and two springs in Section 5.3 and it can be compared to simulations from Comsol as shown in Figure 3.11b. It is clear that the two models match well and that there are two resonant peaks for the position to force (motor current control) transfer function. Although this system is essentially fourth order, the high frequency response slope
Figure 3.12: Two offset double-compound flexures are used in the a) design for linear motor and b) the stroke of the design is simulated.

for $z_2/F$ is essentially second order. If voltage control is used, the resonant peaks are less pronounced and can be treated as second order up to 500 Hz.

Simulations of the different stiffness of the design are illustrated in Figure 3.11a. The lowest stiffness is clearly in the translation direction and is relatively high for other directions. The lowest rotational stiffness is rotation about the y-axis $K_\beta$. This parasitic rotation can cause undesirable rotation and misalignment of the image if subjected to external disturbance forces or if the flexure is improperly loaded. The graph shows that the largest misalignment as a function of displacement comes from rotation about the x-axis. For a 5 mm mirror displaced by 2 mm from the center (4 mm OPD) the difference between the two sides of the mirror would be a displacement of 100 nm which can cause image misalignment. The main cause for this is the lack of symmetry from the actuator and offset double-compound flexure system. If a planar double-compound system is used and the force from the motor acts on the center of the flexure system this effect is minimized.

Figure 3.12a shows the design for a flexure system that uses two sets of planar double-compound flexures offset from each other. This design is scaled up from a 6 magnet design and has much larger flexures. It produces larger forces and is used
for moving more massive mirrors for the high speed version of the IFTS. In this case, the parasitic rotational motions are minimized through the use of symmetry. The addition symmetry came at the cost of much more complex assembly. The displacement simulation is shown in Figure 3.12b.

### 3.2.3 Flexure Damping

The motor and flexure arrangement can produce linear motion with resonant frequencies around 100 to 200 $Hz$. These resonances, as indicated by Figure 3.11b are under-damped and require high gains to achieve precise control quickly. A damping scheme was developed for the flexure system and the system with and without the damping material are compared in Figure 3.13a and b. Several different materials were tested for use as the damping substance at the flexure joints. A material that was thin, matched the stiffness of the flexures the best while not over-damping the flexures was used. The polyolephin-based damping material (two layers of adhesive backed material with a thickness of 280 $\mu m$) absorbs energy and changes the damping of the actuator. The open loop step responses can be compared in Figure 3.13c and d. Although the closed loop responses of the two systems are similar, the open loop response of the flexure that has damping material is much more highly damped.

Eddy currents can also be used as a form of damping. The bobbins of the initial designs were made through a stereolithography process, which does not have any eddy current damping. The bobbins of later designs were made through the wire EDM process from aluminum which can provide some eddy current damping, have better high temperature characteristics and help improve heat transfer. Both flexure damping and eddy current damping helped to improve the dynamic response of the motor.
Figure 3.13: a) A motor with no damping material on the flexures is compared with b) a similar motor with damping material on the flexures. The open and closed loop responses of the c) motor with not damping material and d) motor with damping material are shown.

### 3.3 Piezoelectric Actuator Design

For any particular linear actuator implementation, angular misalignment as a function of actuator travel will always exist. It is therefore important to design a system that can compensate for the angular misalignment in real time.

The actuator speed depends in part on the desired hyperspectral image acquisition rate: it should be fast enough to move the mirrors such that the spectrometer samples the spectrum at the desired rate. In other words, the actuator bandwidth should be greater than the maximum spectral sampling rate (detector frame rate), which in turn should be greater than the hyperspectral image frame rate. For example, if the desired frame rate (spectral sampling rate) is 2000 fps, then the actuator bandwidth should be about 2 kHz to about 20 kHz or more. If the desired frame rate is 30 fps, then the actuator bandwidth should be about 30 Hz or more.

Based on the design of the linear actuator, the maximum bandwidth or stepping
speed of this linear actuator is on the order of 100 to 200 Hz depending on the controller implementation. Increasing the actuator bandwidth by a factor of ten increases the flexure stiffness and energy required by a factor of one hundred. It is therefore difficult to use the long stroke actuator for higher frame rates. In this case, it would be desirable to use a actuator with faster dynamics for high speed positioning. It is generally difficult to get both high speed and high stroke in the same actuator especially when there are space and power constraints. Therefore, it would be useful to come up with a hybrid system, one part of system producing high stroke and one part producing fast dynamics. This hybrid actuation can be combined with feed-forward and feedback control to cancel misalignment due to external vibrations, temperature changes, and other environmental perturbations.

There are several actuators that could be used for angular misalignment compensation and for high speed hybrid control. This could be accomplished with both Lorentz force voice coils or with piezoelectric actuators. The current motor can be modified by wrapping additional coil around the linear actuator bobbin in different orientations to produce small tip and tilt forces. This, however, will have bandwidths similar to the voice coil actuator. Due to the design of the linear flexures, it would also require higher forces and more power to create torsional motions.

It is generally easier to implement a short stroke piezoelectric actuator (typical displacements are less than 1 % of their length) with high stiffness (producing higher forces for less power and higher bandwidths for faster positioning speeds) than it is to implement a small linear motor because the piezoelectric actuator can be monolithic and require fewer custom components. Piezoelectric actuators can, however, be hysteretic and would require closed loop feedback for accurate positioning.

Different topologies including piezoelectric tubes and piezoelectric bend sensors were considered. In terms of overall size and the displacement to input voltage ra-
tio, piezoelectric stacks were chosen. Small $3 \times 3 \times 5$ mm piezoelectric actuators from Steiner and Martins Inc. (SMPAK15553D4) were chosen as the main building block for the compensation stage. They are capable of up to 4 $\mu m$ displacement and 330 N at 150 V, but are used up to 60 V in the design of the IFTS. The capacitance of the actuators is $0.15 \mu F$. The force density of the piezoelectric actuator is approximately $323 \, kN/kg$.

The circuit design of the piezoelectric actuator driver is integrally tied to the properties of the piezoelectric actuator, especially the capacitance of the actuator and the maximum voltage. Although piezoelectric actuators are charge-based, the voltage roughly defines the displacement and little current is drawn when the actuator is at a constant displacement. However, the drive circuitry for the actuator must still be able to provide a large amount of current at the instant when the actuator is going from one displacement to another. If the drive circuitry is unable to provide this current, the bandwidth of the actuator and the ability to control it at high speed suffers. The consequences for this are discussed in Section 5.3.

These actuators can be arranged in series, where each actuator is responsible for tip, tilt or translation motions. This arrangement is easier to control since the axes are independent. They can also be arranged in parallel where the combined motion of the actuators produces tip, tilt or translation. This arrangement is stiffer and therefore has a higher bandwidth.

### 3.3.1 Series Actuation

An early design implementation, used for the multi-axis design, for the piezoelectric actuator stacked in series is shown in Figure 3.14a. A close-up of the series stack for tip, tilt and translation is shown in Figure 3.14b. Each actuator is in charge of one degree of freedom. In reality, the tip actuator in Figure 3.14c will produce some
amount of tip at the same time as some amount of translation when it expands. The
same is true of the tilt actuator in Figure 3.14d. Two screws are used in this case to
apply a small preload to the actuator and to help change zero point of the alignment.
To prevent the screw from damaging the ceramic material, an additional stainless
steel plate is placed at the end of the actuator to help distribute point loads.

The translation actuator works slightly differently. When it expands, it forces the
flexures to stretch and causes a contraction in the orthogonal direction. The screw
is being used to apply a preload to the piezoelectric actuator. This arrangement can
be used to amplify translation and is a function of the angle between the flexure and
the piezoelectric actuator $\theta_{T3}$.

\[ \frac{\Delta y}{\Delta x} = -\cot(\theta_{T3}). \quad (3.3) \]

As the piezoelectric actuator expands in the x-direction (vertically in the picture),
the contraction in the y-direction is a function of the angle. As the angle becomes
shallower, the displacement becomes larger. At an angle of 30 degrees the predicted
contraction is 1.7 times the expansion of the piezoelectric actuator. In practice, the
deformation of other mechanical components may prevent the design from reaching
the predicted value. Table 3.1 lists the simple command directions on each of the
three piezoelectric actuators that can be used to tip, tilt and translate the fixed
mirror using a series configuration. A preload force or preload voltage is required for
bidirectional tip, tilt and translation.

The simulation of the actuation of each axis is shown in Figure 3.15. These figures
show how each actuator is able to produce angular rotation as well as translation
with relatively low stresses on the metal flexure components. The performance of
this design is outlined in Section 5.3. There are several undesirable resonances in the
response of the actuator, but because it is a compact design, the resonant frequency
Figure 3.14: A series piezoelectric tip-tilt-translate actuator design is presented. a) The implementation is shown for b) the tip, tilt and translate axis actuators. c) The tip actuator motion shown along with the d) tilt motion and the translate motion.

Table 3.1: Command table for the series piezoelectric actuator

<table>
<thead>
<tr>
<th></th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip +</td>
<td>+</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Tilt +</td>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>Translate +</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

of the translate actuator is between 2.3 and 2.7 kHz which is capable of positioning the actuator much faster than having a linear actuator alone. The flexure components were designed to be thin in order to reduce mass and save space. In general, there are many parasitic deformations that occur in this type of flexure design. In designs where more space is available, parts that should not deform are made thicker.

Figure 3.16 shows a similar implementation for the high speed design which is much larger. In this design, longer flexures are used outside the piezoelectric actuator flexure systems to help increase the rotational resolution of adjustment. Ideally, the piezoelectric actuator should be close to the center of rotation in order to increase the range of the actuator. The opposite is true for hand adjusting the zero position. For hand adjustments, it is desirable to have more adjustment resolution so longer
Figure 3.15: The series piezoelectric actuator design is simulated in the a) tip, b) tilt, and c) translation directions. The simulation conditions specify a maximum of 4 \( \mu m \) of displacement at the actuator without exceeding the maximum force output of the actuator.

Figure 3.16: The series piezoelectric tip-tilt-translate actuator simulation for a) the high speed implementation are shown. The b) tip, c) tilt, d) and translate axis actuator are simulated. The simulation conditions specify a maximum of 4 \( \mu m \) of displacement at the actuator without exceeding the maximum force output of the actuator.

This design utilizes a preload spring shown in Figure 3.2c instead of two screws to load the piezoelectric actuator. This design also moves the translation actuator much closer to the mirror in order to reduce the amount of mass that would be translated. However, the design is not very compact leading to longer beams that
have low frequency bending resonances on around 1.2 \textit{kHz} which cannot be controlled as detailed in Section 5.3. Because the mirror is larger and heavier in this design, it is also not possible to use the previous design to achieve the same resonant frequencies. The resonant modes can be improved by going to a stiffer design that does away with long beams and by going to a parallel actuation system.

### 3.3.2 Parallel Actuation

Unlike series actuation, parallel actuation does not allow each actuator to control a different axis of tip, tilt or translation. Instead, each actuator contributes to each of these directions as shown in Figure 3.17. The consequences are that the control is slightly more complex, but less mass needs to be moved (do not need to move the additional mass of the other actuators). The system can be stiffer leading to higher resonances as well as lower overall stroke. In the design implemented in Figure 3.17c, tip-tilt-translation motion can be accomplished with three piezoelectric actuators but four actuators are used to provide symmetry in mounting thereby reducing undesirable dynamics. The preload on each of the actuators can be adjusted by small screws. Table 3.2 lists the command directions on each of the four piezoelectric actuators that can be used to tip, tilt and translate the fixed mirror for a parallel architecture piezoelectric actuator design. Note that this design allows for both positive and negative tip and tilt and does not require a preload force or preload voltage for those motions.

Simulations of the different actuation directions are shown in Figure 3.18. In the tip mode, the difference between the top and bottom of the mirror is relatively small due the arrangement of the piezoelectric actuators with respect to the pivot points. On the other hand, the tilt motion shows a relatively large difference between the two sides of the mirror. In translation, the mirror moves almost uniformly a distance of about 3.5 to 4 \textit{mm} with some amount of asymmetry due to the design mounting.
Figure 3.17: The piezoelectric actuator designs for the high speed implementation showing a) tip-tilt-translation motion. b) A series actuator design (tip of 1 \( \mu m \), tilt of 0.5 \( \mu m \), translation of 0.3 \( \mu m \), and lowest resonance at 1.2 \( kHz \)) and b) a parallel actuator design are shown (tip of 1.1 \( \mu m \), tilt of 2.15 \( \mu m \), translation of 1.1 \( \mu m \), and lowest resonance 2.5 \( kHz \)).

<table>
<thead>
<tr>
<th>Command Table for the Parallel Piezoelectric Actuator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip +</td>
</tr>
<tr>
<td>Tip -</td>
</tr>
<tr>
<td>Tilt +</td>
</tr>
<tr>
<td>Tilt -</td>
</tr>
<tr>
<td>Translate +</td>
</tr>
</tbody>
</table>

It is important to note that this design does not have one pivot point but several, which causes some parasitic translation during rotation or rotation during translation at the sensor points on the mirror.

The parallel design clearly has thicker metal components that improve the stiffness and increase the frequency of resonant modes. The four lowest modes are shown in Figure 3.19. Since the accuracy of the mode resonances are dependent on the accuracy of the boundary conditions and the preload conditions, these values are used only as a relative comparison between different designs. Here the tip resonant mode is the lowest parasitic mode followed by a tilt mode. All of the resonant modes are larger.
Figure 3.18: a) The parallel actuator design is implemented. Parallel piezoelectric tip-tilt-translate actuator simulations are conducted for the for high speed implementation in b) tip, c) tilt, and d) translation. The simulation conditions specify a maximum of 4 \( \mu m \) of displacement at the actuator without exceeding the maximum force output of the actuator.

Figure 3.19: Resonant modes are analyzed for the high speed implementation and the first four modes are shown with the resonant mode frequencies listed.

than the desired actuation bandwidth of 2 \( kHz \). In practice, this design is capable of a bandwidth of 2.5 \( kHz \) up to 7 \( kHz \) in translation.

3.4 Software and Electronics

The software and electronics architecture for the IFTS requires combining several sensors, actuators and control systems. For a research device, where different control and data processing algorithms would be developed and tested, the electronics architecture revolves around a desktop computer for data storage with a user interface and
a permanent power source. For a handheld product where algorithms are fully developed, it would be advantageous to revolve the architecture around a microprocessor system, an on-board data storage and display system, and a battery power source. The electronics architecture for the IFTS is laid out in Figure 3.20 showing the sensor, actuator and video pipelines. The power system, light system and laser system are not shown in this diagram since they are not a part of any data pipeline. This particular design is the most general and the prototypes that have been developed are some variation of this design. The detailed circuit schematics are available in the Appendix A.2.

In this figure, the video pipeline at the top centers on the camera, which can be either a conventional 30 fps camera (TCM8230MD) or a high speed camera (LUPA 300) that is commonly used for scientific imaging [106, 139]. The registers on this camera are controlled via the serial peripheral interface (SPI) or inter-integrated...
circuit (I2C) communications protocols from a microcontroller and the frame trigger information can be sent directly from the camera to the microcontroller. The camera data are clocked out as either 8-bit or 10-bit data with two extra bits for line valid and frame valid information. To reduce noise for long communication cables, a low voltage differential signal (LVDS) serializer and deserializer are used. Since the data being clocked off the LVDS deserializer and high speed universal serial bus (USB) 2.0 device are at different frequencies and both devices act as master devices (control the data clock), an intermediate first-in-first-out (FIFO) buffer is used to store, buffer, and control requests from the two devices. The high speed USB device then sends the video data via the USB port to the computer. A discussion of the choice of high speed USB 2.0 over other protocols is located in Section A.2. The maximum uncompressed data rate of this pipeline is 280 Mbits/s.

In this pipeline, there are several dotted lines leading to and from the field programmable gate array (FPGA). There are several cases where the FPGA is necessary for controlling and encoding the data and the LVDS portion of the pipeline can be bypassed. First, the high speed USB pipeline can only handle 8-bit camera data. In the case where 10-bit camera data are desired, an FPGA is required for encoding 10-bit data to 8-bits. Second, depending on the camera, there are circumstances where the start of a frame or start of a line is unclear or inconsistent from just the 8-bit stream. The line valid and frame valid information can also be used in further encoding to add start bits or end bits to the data stream to indicate when a picture starts and ends. The design of the FPGA code is discussed in Section 3.4.3.

Below the video pipeline is the sensor and actuator control system orchestrated by an ARM Cortex M4F microcontroller (STM32F405RG). The microcontroller receives the photodiode readings through transimpedance amplifiers and analog to digital converters (ADC) and sends commands to different digital to analog converters (DAC)
which control motor drivers and piezoelectric actuator drivers. It also controls settings to the camera and FPGA. Up to four of the six photodiodes are used for the tip-tilt-translation positioning system and up to two of the photodiodes are used for finding the center of the interferometer or zero path difference (ZPD) point. The two motor drivers are linear operational amplifiers that are used to control the Lorentz force voice coil actuator. These amplifiers are capable of up to $5 \, \text{V}$ at $2 \, \text{A}$. Technically, the minimum coil resistance should be $2.5 \, \Omega$ but this operation is inefficient due to the losses inside the amplifier, therefore the typical coil resistance is at least $5 \, \Omega$.

The piezoelectric drivers (either transconductance amplifiers or linear operational amplifiers) are used to control up to four different piezoelectric actuator stacks.

Operational amplifiers were chosen over h-bridge style amplifiers to minimize noise in the sensor system and to reduce the overall complexity. Operational amplifiers are relatively inefficient and can be replaced with h-bridge style amplifiers and charge pump designs for portable designs.

In a typical control cycle, the camera trigger indicates the start of a frame to the microcontroller, which then gathers information from the photodiodes, calculates the position and determines the necessary control commands to send the motors and piezoelectric actuators to move the appropriate mirrors. The mirrors are then moved using feed-forward and closed loop feedback signals until the mirror achieves the desired position and orientation while rejecting external vibrations. The controller holds this position until the camera triggers again on the next frame of an image and this process repeats. The signals and positioning information is sent via a universal asynchronous receiver/transmitter (USART) signal to a USB converter which sends the information to the computer. The hardware and software for all of these systems were custom designed and manufactured for the implementations of the IFTS.
3.4.1 Electronics Design and Manufacturing

The electronic are developed for the IFTS implementations using a free tool chain starting with TinyCad 2.8 for the schematics, FreePCB 1.359 for the layout, and ViewMate 11.2.18 for validation. The layouts for the power and programming boards that are common to most of the implementations are shown in Figure 3.21 along with pictures of the final circuit boards. The detailed schematics for these boards are in Section A.2. The boards are designed to be modular to allow for different boards to be used in different implementations, debugged and upgraded separately. For example, the 3.3 V and 5 V power board and programming board are common across most of the IFTS implementations and the piezoelectric amplifier boards can be upgraded without altering any of the other boards. The layouts for the other components are implementation specific and are shown in their respective sections.

The layouts are typically two or four layers and these boards are manufactured by Sunstone Circuits. When the boards are received, they are first cut into separate

![Layout](image1)

![Front](image2)

Figure 3.21: The power and programming circuit layouts and implementations are shown. These boards are common across most implementations of the IFTS.
boards on a diamond band saw using water to reduce fiberglass particulates in the air. Each component is then placed on the boards and soldered in place either by hand or by using a printed circuit board (PCB) oven. When size is a major constraint, several methods are used to maximize the usage of space including using four-layer boards, using undersized footprints, and placing components on both sides of the board. This may make it more difficult to assemble using traditional methods and most parts must be hand soldered under a microscope. Some parts, such as the conventional camera which has plastic lenses, cannot be placed in the PCB oven.

The boards that will come in contact with the metal surfaces of the IFTS are then covered in non-conductive Kapton (polyimide) film to prevent shorting. Wiring between boards may then be added using coated magnet wire that is twisted into signal pairs. The power wires are 32 gauge (0.202 mm OD) with polyimide coatings and the signal wires are 34 gauge (0.160 mm OD) using a bondable wire that has a coating that can be heat stripped during soldering.

### 3.4.2 Microcontroller Architecture

Most of the electronics is managed directly from the microcontroller and the microcontroller state diagram design is shown in Figure 3.22. The microcontroller is programmed in C using the IAR Embedded Workbench IDE 6.5. A complete map of the registers and commands is listed in Table B.1 and explained in Section B.1. When the system starts up, it goes into a booting algorithm that sends the settings to the camera and initializes all the variables. After the boot state, it enters a standby state where it polls for a command to complete a task. This design implements registers on the microcontroller which hold information on different variable constants such as position calibration constants, motor constants, piezoelectric actuator constants, observer constants and camera parameters. The default parameters on the system
allow all the other programs to run but send only zeros to the actuators to prevent possible damage. In order to run the system, a set of non-zero parameters must be sent from the user interface. This system is flexible because it can use the same set of code for several implementations of the IFTS by simply uploading different register constants.

These parameters can be obtained by running simple calibration functions by sending test commands to the microcontroller. These commands include a ramp response which can be used to calibrate the photodiode constants, a step response with controller which can be used to calibrate controller constants for the motor and piezoelectric actuator and stochastic system identification algorithms that can be used to determine feed-forward constants.

After the calibrations functions are run and the register constants are calculated and sent to the microcontroller, the system can be run in one of several functions for.
spectra acquisition. The first functions are unsynchronized with the camera trigger and are driven entirely by the user interface commands. They are typically slower (around 3 fps) because they are limited by the hand-shake time between the microcontroller and the user interface running Windows. The modes include a simple mode using two photodiodes for positioning, a normal mode with 4 photodiodes for positioning and a slow mode which uses the camera's non-destructive readout (NDR) function to integrate more photons onto the sensor. After each of these modes is completed, the microcontroller is sent to a hold position state where the controller stays on until an end command is received. This allows the mirror to reject disturbances while the microcontroller downloads data to the computer and the computer processes the data and decides the next step.

The next functions are synchronized with the camera trigger. These functions typically run much faster (15 fps to greater than 2000 fps). At these speeds, it is not desirable to send data back and forth, so these modes stream video and positioning data directly to the computer. There are three main modes including a simple mode using two photodiodes for positioning, a normal mode using 4 photodiodes for positioning and an extended mode which uses 4 photodiodes and controls the motor and up to 4 piezoelectric actuators. There is an additional slow mode, which uses the NDR function for camera synchronized motion. In each of these modes, the microcontroller first turns on an interrupt for the camera trigger and then goes into a hold state where it holds the current position using feedback control until the next edge of the camera trigger. On the next trigger edge, the algorithm increments the position using feed-forward and feedback control until the number of frames captured is equal to the number of frames desired. The algorithm then goes back to holding the desired position using a feedback controller until the trigger is turned off before entering the general hold-position state to wait for data to be downloaded. The user-interface
can then trigger several another acquisition of N frames at different desired mirror positions once the data have been received.

The slowest step in this design is the hand-shake time between the computer and the microcontroller. For the camera-synchronized functions, hand-shaking is avoided and data are either downloaded directly from the microcontroller and video pipelines or commands are sent directly from the user interface. Therefore, the time where the microcontroller goes from a general hold position state through standby back to a new iteration of a camera synchronized function is designed to be extremely short and positioning is not lost between iterations. This provides better positioning accuracy and disturbance rejection.

### 3.4.3 FPGA Design Architecture

The FPGA used for this design is a Xilinx Spartan 6 (XC6SLX9) implemented on either the Mojo board by Embedded Micro or on a custom FPGA board running on a 50 MHz oscillator. The device is simulated and synthesized in Verilog using the ISE Design Suite 14.7 and code is boot loaded onto the chip. The Xilinx CORE generator is used to generate internal phase locked loop (PLL) clocks for clocking the camera at different frequencies.

Figure 3.23 shows a colored block diagram of how the FPGA interacts with the video stream data. When the FPGA is used in the video pipeline, the FPGA provides the clock signal (light blue) to the high speed camera. The camera provides a line valid (LV), a frame valid (FV) and 10-bit pixel intensity data. The rising edge of frame valid indicates the start of an image and the rising edge of line valid indicates the beginning of a line. When line valid is low, the data on the 10-bit line are not part of the image. The code on the FPGA implements several different clock frequencies using a PLL and two different clocking algorithms for clocking out image as 8-bit
The field programmable gate array (FPGA) imaging sensor data handling block diagram is shown. In these test bench simulations, the video data can be clocked out through the FPGA as either b) 8-bit data or as c) 10-bit data. Each of these requires a different clocking scheme to encode data, frame start bytes and line start or line end bytes.

In the 8-bit scheme, five bytes of 0xFF are sent to indicate the frame start (rising edge of frame valid) and five bytes of 0xFE are sent at the end of every line (falling edge of line valid). When both line valid and frame valid are high, the most significant 8-bits of camera data are clocked out through the FPGA data output at a delay of one cycle on the FPGA clock output, which is the same as the clock provided to the camera.

In the 10-bit scheme, it is necessary to group the 10-bit data into four groups and clock the information as five groups of 8-bits. This condition requires that two different clock frequencies be used, a slower clock for the camera and a faster clock output to the FIFO in order for the input data to be clocked out with minimum data storage. Based on this condition, the FPGA clock output occurs in groups of five. As
with the previous scheme, the frame start bytes trigger on the rising edge of frame valid. This scheme, however, has line start bytes that trigger on the rising edge of line valid because the LUPA 300 camera does not have enough room between the falling edge of line valid and the rising edge of the next line valid for both the line end byte and the last group of data. The data are received by the user interface on the computer and are decoded into image information depending on the bit-depth scheme selected.

3.4.4 User Interface Design

The user interface was created in C# and uses Math.Net Numerics libraries for complex computations. The USB communications is achieved using the FTDI D2XX drivers that can utilize the full data bandwidth of the high speed USB port. The main user interface window shown in Figure 3.24 allows the operator to grab data from the data port and the video port and display video in real time. It can graph the positioning data, auto-calibrate constants using different algorithms, as well as save and load different data sets. It can save video as compressed audio video interleave files (AVI) or as raw binary (BIN) files. Several different calibration functions are available and each calibration constant can be manually altered. The system can also track the interferogram of any pixel in the image and use different solvers to compute the Fourier transform. The main screen also has the ability to direct different algorithms including adaptive sampling and recursive sampling algorithms. Additional windows allow the user to check the relative photodiode phase, adjust different registers on the camera and FPGA, run stochastic system identification tests, as well as simulate and adjust parameters for adaptive and recursive sampling algorithms. A full explanation of the different functions of the main window and the special function windows of the user interface is located in Section B.2.
Figure 3.24: The user interface main window is shown. This window allows the user to connect to and control the microcontroller registers and functions. It graphs the data provided by the microcontroller and can run different calibration functions. It can save and load data and video files. It displays the camera image in real-time and can control the acquisition of the spectral data as well as process the Fourier transform spectra using different algorithms.
The most speed intensive operation on the user interface is converting the image data to a video in real-time. For an 8-bit image at video graphics array (VGA at 640 × 480 pixels) resolution, 307.2 kB of information (384 kB for 10-bit images) need to be processed at 30 to 78 frames per second (9.22 MB/s to 30.0 MB/s). There are two implementations: the real-time implementation (used for display and aligning the camera) and the polling implementation (used for gathering spectral data). The real-time implementation is created for frame rates up to 30 fps by using C#'s locked bits and marshalling functions which are fast implementations appropriate for real-time video. It becomes more difficult to update the screen at higher frame rates and in that case, an entire frame is grabbed and displayed and then another frame is grabbed, dropping intermediate images. The refresh rate at the screen can be up to 30 fps even if the true frame rate is 2000 fps. While the real-time implementation can drop images, the polling implementation is not capable of dropping images and is used directly for spectrum analysis. It grabs a specified number of images at the same time that the camera synchronized spectral acquisition programs are running on the microcontroller. The image data can then be dumped directly in a file to decrease computation time. As a slower option, the images can be shown on the screen during computation if desired.
Chapter 4

Implementations

Several implementations of the imaging Fourier transform spectrometer were attempted, starting with designs that focused on the development of a small, long-stroke linear actuator. Several actuator designs were prototyped and are shown in Figure 4.1a. After a magnet and flexure structure type was chosen, several different manufacturing methods and geometric layouts were explored including stereolithography and laser machining as shown in Figure 4.1b. Eventually, a more precise stacked layout structure was chosen and prototypes with different layouts were tested using laser machining techniques shown in Figure 4.1c and d. The first full prototype design incorporating features for holding the optical components in place was developed and converted for manufacturing with wire EDM. The wire EDM process had several benefits including the ability to create built in flexures, the ability to tap the material and the ability to create relatively high precision, high aspect ratio components. From these first prototypes, several other prototype layouts with different applications and specifications were developed including the first full prototype design, the multi-axis actuation design, the robotic endoscope design, and a high speed camera design. The decisions and layout details for each of these designs is described in the following sections.
Figure 4.1: Development of initial prototype actuators, layouts, and manufacturing techniques are shown. a) Several different actuators were first developed before b) different manufacturing techniques were attempted. c) After deciding on a layered manufacturing technique, d) several different layouts were tested.

4.1 Prototype Design

The first full prototype design used a single circuit board that incorporated all the positioning optics and camera components in order to simplify wiring and assembly. Additional space on the circuit board was used for test points of the electronics for debugging. The mechanical components were then designed around the circuit board and mounted via four through holes on the circuit board. The assembled view and the exploded view are shown in Figure 4.2a and b.

The structure of the design centers around the optics layer that aligns directly over the circuit board sensors. The optics layer (including the spectrometer and
sensor components) contains the components that hold the mirrors, beam splitter, and flexure components. These separate components are mounted to an attachment layer which contains all the tapped holes. The motor and flexure systems are also mounted to this layer. Additional spacing layers and cover layers are used to provide spacing between moving layers and to block external light.

The arrangement of the optical components and the light paths in the assembled design are shown in Figure 4.3a. The location of the sample or the incoming light is on the left side of the spectrometer. The light enters the spectrometer and interfaces with a hot mirror or short pass filter which allows light that has a wavelength less than 700 nm to pass through and reflects light with longer wavelengths. The light then passes through the interferometer system with adjustment flexures on both the stationary and moving mirror. The light then reaches the camera (which has its own lens and aperture system) at the bottom. The reference laser light leaves the 850 nm VCSEL at the laser location and passes through a small 2 mm focal length (FL) lens
Figure 4.3: a) The layout and optical path of the prototype design are shown along the b) front, bottom, and back of the design.

that collimates the laser light. This beam bounces off a mirror and passes through a beam splitter (half the beam also goes through the beam splitter and interfaces with the triangle-shaped cutout in the metal which reduces reflections to the photodiodes), reflects off the short pass filter and passes through the interferometer. The reference laser light also goes toward the camera but is blocked by the camera aperture to prevent the reference laser light from saturating the hyperspectral image. The light also returns through the short pass filter and beam splitter before entering a -3 mm FL lens and is projected on two photodiodes.

The front, bottom and back views of the design are also shown in Figure 4.3b. The motor is the tallest component in the design with flexures that are constrained at the attachment layer. The magnet structure of this design can be positioned independently to adjust the center of force relative to the flexure. The overall dimensions of this design without the electronics test points is $21 \times 23 \times 38$ mm. The thickness of the stacked structure is only 17 mm without the motor flexures.

The designs of the custom circuit boards for the full prototype are shown in Figure 4.4. The base board contains the microcontroller (72 MHz ARM Cortex M3)
Figure 4.4: a) The circuit board layout and the front and back of the implementation of prototype design are shown.

and video pipeline. The maximum closed loop control for this design was limited at 20 kHz due to the processor, analog encoder conversion, floating point multiplications and SPI command speeds. The sensor board contains the camera, motor drivers, laser and photodiode amplifiers for a two-photodiode positioning system, as well as a large number of test points. Some of those test points are connected directly to the output of the transimpedance amplifier and are then connected to an oscilloscope for real-time photodiode alignment. The magnitude and relative phase of the two interferometric signals can be checked from these test points to determine the proper alignment for the photodiode sensors. The connection between the two boards is located at the top of the sensor board where thin, twisted pair magnet wire connections are protected with Kapton tape. To save board space, components are located on both the top and bottom of each board.

The final assembled design of the full prototype IFTS is shown in Figure 4.5a. The test electronics which include a 3.3 V and 5 V power board and a programming board are shown in Figure 4.5b. The typical load of the entire system for low strokes is 200 to 300 mW on an 8 V supply (1.6 to 2.4 W). Most of the power is used by the video pipeline system.
Because the design of this system is relatively simple, there are several issues associated with some of the design choices that were improved in later designs. First, because the position of the laser with respect to the photodiodes is fixed, it was difficult to align the positioning system. The laser was re-soldered several times and the photodiodes were ground down to allow them to be placed closer together. The positioning sensor alignment also involved moving the entire metal assembly with respect to the circuit board, which was very imprecise. The use of the short pass filter, beam splitter and multiple passes through the interferometer also made the positioning signal weaker. Because the motor was built directly into the spectrometer, it was more difficult to assemble or replace. The default position of the moving mirror with respect to the stationary mirror is also fixed in this design. Lastly, by using the built-in optics of the small camera, the usable pixels of the camera were limited to $100 \times 100$ pixels.
4.2 Multi-Axis Design

Due to the asymmetric nature of the small motor design, a parasitic tip and tilt exist in the mirror alignment as a function of mirror travel. In order to compensate for this misalignment as a function of travel and other small tip and tilt errors, a multi-axis design was developed incorporating piezoelectric actuators for tip and tilt compensation. This system also has the additional benefit of a fast translation piezoelectric actuator that can be used to increase the speed of positioning and spectral acquisition. The tip-tilt-translate actuators are placed on the stationary mirror rather than the moving mirror in order to decrease the moving mass thereby increasing the bandwidth of the actuators. In order to sense tip and tilt, four photodiodes were used and a tip-tilt observer was implemented.

This design also fixes some of the issues of the previous design. It separates the laser and photodiode alignment from the camera alignment. The camera optics are also improved by removing the existing optics (by sanding down the plastic lenses) on

Figure 4.6: a) The unexploded and b) exploded views of multi-axis (V0.60) design are shown.
the camera sensor and replacing them with a custom aperture and lens to extend the usage of the camera pixels. The motor was designed as a separate module which can be easily replaced or shimmed to change the default position of the moving mirror with respect to the stationary mirror. The motor module mass is on the order of 6 grams with a motor and flexure assembly force density of approximately 170 $N/kg$. Test points and extra dead space was also removed to shrink down the size of the spectrometer while extending the functionality. The diagrams showing the assembled and exploded views of this design are in Figure 4.6a and b.

This figure shows several different modules including the optics layer containing all the optical components (both for the spectrometer axis slice and sensor axis slice), the motor module, the tip-tilt-translation piezoelectric actuator module, as well as top and bottom covers. The electronic boards are also separated by function. In this design, the top and bottom cover are mounted directly to the optics layer, which contains all the tapped holes. The top and bottom covers contain several layers in order to account for differences in component thicknesses.

Figure 4.7 shows the path of the light passing through the imaging spectrometer in 4.7b and the light from the reference laser in 4.7c, which are on different orthogonal planes of the multi-axis IFTS design. The spectrometer axis slice shows the motor module and the moving mirror as well as the fixed mirror and one of the associated piezoelectric actuators. The spectrometer axis slice also shows how the light enters the spectrometer at the sample location and passes through the beam splitter and the two mirrors. The light then passes through an aperture and the imaging lens, which is a 6.325 $mm$ outer diameter (OD), 13 $mm$ FL plastic aspheric lens (Edmund Optics 83-684). The light then hits a mirror and is directed into the page. In Figure 4.7c, the light bouncing into the mirror is directed downward and focused onto the camera.

The positioning sensor optics are also shown on this plane. The reference laser
Figure 4.7: a) The bottom of the spectrometer design is used to show the different optics planes in the multi-axis design. b) The layout and optical path of the spectrometer optics are shown along with c) the positioning sensor optics in an orthogonal axis.

Figure 4.8: The circuit board layout and implementation of the multi-axis design are shown with the camera boards, photodiode board, motor board, 35 V piezoelectric actuator board and base board. Not all faces of the boards are shown.

light leaves the VCSEL and is focused by the lens. It then hits a thin beam splitter manufactured via the sputtering process. Half the light travels to the right into the
Figure 4.9: a) The internal layout and build process of the multi-axis design are shown. b) The initial assembly of the components is shown before anodization and c) after anodization. d) The final design assembly with circuit boards and board connector wires is also shown. The images were taken of the back of the spectrometer.

triangle-shaped structures to prevent reflections. The remaining light travels towards a thin mirror before entering the interferometer. The light returns through the same thin mirror and thin beam splitter and enters the negative focal length lens. The light hits another thin mirror and projects onto four photodiodes used for tip, tilt and translation sensing. The implementation of the layout of the positioning layer is shown in Figure 4.9a.

The custom electronics for this design are shown in Figure 4.8. There are two camera boards to accommodate the camera, LVDS, power regulation and clocking chips. This design included the option to use a variable clock to allow frame rates to be changed in real time. However, the variable clock contained too much jitter to be used by the LVDS and was not utilized. These boards are stacked on top of each other and attached to the main spectrometer structure via two clips that can me moved vertically to change the focus of the image via a screw and slot system.
The photodiode board contains four small photodiodes along with transimpedance amplifiers and ADCs. The motor board is capable of driving the Lorentz force voice coil with a 16-bit DAC. A separate 24-bit DAC board has also been developed for this design but is not necessary for achieving the desired positioning resolution. The final small board is a 35 V piezoelectric amplifier board that is capable of driving the three piezoelectric actuators. A separate variant of the piezoelectric amplifier board has also been developed to run at 60 V and is combined with the motor board. The performance difference between the two variants is discussed in Section 5.3.2. The updated variant is also mounted in a different location on the spectrometer and has better heat transfer characteristics due to the larger board area. A base board was also developed for this design incorporating a more powerful processor with single cycle floating point multiplication and a faster clock speed (168 MHz ARM Cortex M4F). This allowed the system to run faster positioning control loops up to 100 kHz.

The build process for the multi-axis design is shown in Figure 4.9. The initial assembly is shown in Figure 4.9b and the anodized assembly is shown in Figure 4.9c. The final instrument with electronics as well as power and communications wires is shown in Figure 4.9d. The final design and test electronics are shown in Figure 4.10. The electronics take up a large portion of the volume of the design. The final dimensions of the design are $18 \times 23 \times 25$ mm, which is 56 % of the volume of the previous prototype design.

Despite the many improvements over the full prototype, this design still had a few issues that needed to be addressed. Because the components are much thinner than in previous designs, there were issues with undesirable parasitic deformations in the flexures. The flexures were also thin, and would sometimes break during the nitric acid cleaning and anodization processes. The 35 V piezoelectric drive circuitry did not have the full bandwidth needed to take advantage of the bandwidth of the
The use of high voltage switching circuits and piezoelectric actuators also increased the noise of the system, which could be mitigated with case grounding and the addition of extra ground planes to mitigate radiated noise. Due to the small size of the motor board, it has difficulty dissipating heat for long strokes. A separate 60 V piezoelectric amplifier system with motor controllers and a larger board area were developed for this design to help mitigate these issues.

Even though the camera system was separated from the positioning system, the positioning system continued to be difficult to align. Ideally, the laser and the photodiodes would be placed on two independent boards, as it would be in later designs.

4.3 Endoscope Hyper-Spectral Imaging Design

In a departure from spectrometer development, a separate IFTS was developed for the application of hyperspectral imaging for robotic endoscopes similar to the one shown in Figure 4.11a. Robotic endoscopes and other conventional endoscopes are 15 mm OD or smaller devices which can be used to look for cancerous lesions inside the body. Some endoscopes are also therapeutic, allowing for tools to be passed through them.
Figure 4.11: a) The mechanical design of a robotic endoscope without the outer sheath is shown. b) The circuit board design for the camera module of the robotic endoscope without hyperspectral imaging with and without the plastic cover is also shown.

The robotic endoscope system shown in the figure has motorized bending joints which allow the endoscope to bend at several points along the body [36, 37, 39]. There are also many alternative methods that can be used to move the long body of the endoscope [28, 35, 40, 41]. A miniature motor module using two electric motors for bending each unit is also shown. The angle of each of these modules is controlled by taking measurements with a gyroscope or a polymer bend sensor [38].
the endoscope to navigate through the colon or other body cavities by either utilizing the walls of the body cavity or by avoiding them. For this robotic system, a camera module was created with a small camera, LVDS communications system, and lighting system. These boards and connectors have a size of 15 mm OD and are 23 mm long. For the development of this advanced endoscope system, a new hyperspectral imaging board was created to fit the form factor and imaging needs of endoscope application. An imaging spectrometer used at the tip of an endoscope would circumvent many of the issues associated with fiber-based systems. The direct incorporation of the imaging system with the spectrometer system allows a physician to obtain both a raw color image as well as a hyperspectral image from the same field of view. When the hyperspectral imaging system is not in use, the camera module can be used simply as an endoscope camera.

The robotic endoscope implementation of the IFTS is shown in Figure 4.12. The system consists of boards in the front of the spectrometer, which house a white lighting
Figure 4.13: a) The layout of the sensor layer and b) spectrometer layer of endoscope hyperspectral imaging design are shown. c) The small motor module design is also shown.

system as well as photodiode boards. This is followed by the main spectrometer which has several layers, a spectrometer layer, a photodiode positioning layer, and several spacing layers. A miniature motor module is also shown. The back of the module contains the camera board, the motor board, and the back cover. The design of this implementation is simplified and only contains a motor for moving one mirror.

The layout of this design is also a departure from previous designs. There are two separate, distinct layers which contain optical components. The sensor layer, shown in Figure 4.13a, indicates how reference laser light goes through a focusing lens and the travels through a thin beam splitter before hitting the moving mirror and a thin fixed mirror which is attached to the same flexure as a thicker mirror used in the spectrometer layer. The light then expanded through a negative focal length lens, reflects off another thin mirror before being projected onto two photodiodes for positioning.

The spectrometer layer of this design uses a larger 5 mm beam splitter and projects the hyperspectral light onto two larger mirrors. The light exits the system and hits a mirror before reaching an aperture. The imaging lens in this system is a plastic aspheric lens that has ground down edges allowing it to fit inside the allotted space. The design also uses an input aperture to limit the incoming light. Separating the two optical systems has the benefit that there is no light leakage from the positioning
Figure 4.14: The circuit board layout and implementation of the endoscope hyperspectral imaging design is shown with the laser, light and photodiode board, the motor board, the camera board and the base board.

laser into the hyperspectral imaging system. However, there are not enough degrees of freedom for adjusting the alignment of the optical components making the system difficult to align for both the image and the positioning system. In order to align the design, the hyperspectral image is first aligned, then the thin beam splitter on the sensor layer is manipulated, shimmed and epoxied in place. The invisible laser beam can be viewed through an image intensifier screen for infrared frequencies.

The motor module for this design is smaller than previous designs with shorter, stiffer flexures. The corners of the flexure were clipped to allow it to fit inside the defined space. The stroke of this actuator is also limited to 714 µm (1428 µm OPD).

Figure 4.14 shows the layout and implementation of the different custom optics for this design. This includes the laser, lighting and photodiode board, the motor power board, and the camera board. The boards were octagonal but were cut down on a diamond band saw and diamond grinder to have a more circular shape. These boards have a few mounting holes for M1 screws so that they can be mounted to
Figure 4.15: a) The endoscope hyperspectral imaging design implementation and b) test electronics are shown.

The fully assembled spectrometer without the outer casing is shown in Figure 4.15a. The design with the other casing has an outer diameter of 15 mm and a length of 35 mm. The test electronics are also shown with the assembled spectrometer with its outer casing. The lights are also powered on. The robotic endoscope application requires a long cable to provide power and communications from the spectrometer to the base board, power board and programming board as shown in the figure.

### 4.4 High Speed Design

In order to achieve video rate spectra, a high speed camera is necessary. Several small high speed cameras exist both in charge-coupled device (CCD) and complementary metal-oxide-semiconductor (CMOS) format. A 1/2” format camera, the LUPA 300, was chosen for this application. When clocked at 24 MHz, it is capable of more than 2000 fps at 100 x 100 pixel format. Because this camera is larger, the scaling
equations show that the size of the entire system must increase to take advantage of all the pixels in the array. When the size of the optics increases, the actuators must also increase in stiffness or overall force in order to meet bandwidth specifications.

Because the focus of this design is speed and functionality and not size, the layout was optimized to improve stiffness and heat transfer by placing the motor and piezoelectric amplifier boards separate from the other spectrometer components and adding heat sinks. The size of the heat sinks is over sized and is generally only necessary when running a large series of stochastic system identification tests one after the other. The design is also more modular with larger circuit boards that provide more debugging and design-change options.

The layout for the high speed design is shown in Figure 4.16. In this design, the optimal beam splitter size is rounded up to the nearest available size of 10 mm. This gives additional space for a sensor layer that shares the same beam splitter. This design has a larger motor and a larger motor mounting system as well as a piezoelectric actuator and flexure system. The high speed camera board, photodiode
Figure 4.17: a) The internal structure of the implementation of the larger motor for the high speed implementation is shown along with b) the full motor incorporating the six magnet motor design (which uses 8 total magnets), the two double-compound flexure system, the mirror flexures, and the motor flexures.

...
Figure 4.18: a) The layout and optical path of the sensor layer and b) spectrometer layer for the high speed design are shown. c) The outer anodized case is also shown indicating the location of the two layers.

holder contact the bobbin at the top and bottom. This requires two tilt flexures and one long tip flexure. The mirror can be aligned with these flexures while the full motor can be aligned with the actuator tip and tilt flexures. These flexures account for assembly and manufacturing errors and allow the motor to translate in a straight line with respect to the fixed mirror. The actuator translate flexure allows fine adjustment of the default ZPD of the moving mirror with respect to the fixed mirror.

The layout of the optical light paths is shown in Figure 4.18. The sensor layer contains the VCSEL reference laser which is directed at a collimating lens that couples light straight into the interferometer. The output light enters another lens that expands the light and projects onto 4 photodiodes for tip, tilt and translation sensing. There is also a separate white light LED source which also passes through a collimating lens and the interferometer. The low coherence light is directed onto a pair of photodiodes and can be used to find the interferometer center or ZPD. Other potential light sources that could accomplish this include superluminescent LEDs and thermal emitters.

The spectrometer layer layout shows how the incoming light enters the interferometer. It then passes through an aperture and an imaging lens, a 6.25 mm OD,
Figure 4.19: The circuit board layout and implementation of the high speed design are shown with the photodiode board, high speed camera board, laser and light board, actuator amplifier board and base board. The three piezoelectric actuator amplifier variant is shown in this figure.

30 mm FL achromat (Edmund Optics 47-694). The light then hits two mirrors before being projected onto the camera. The total mass of the spectrometer with motor is approximately 32 grams.

The custom electronics for this design are shown in Figure 4.19. The photodiode board contains six photodiodes and associated transimpedance amplifiers and ADCs. The high speed camera board includes the clocking electronics an LVDS and the high speed CMOS camera. The laser and light boards utilize a VCSEL laser for positioning and a white LED for finding the center of the interferometer. The actuator amplifier board has both the motor amplification system (two operational amplifiers) and piezoelectric actuator amplification system. There were two variants on the piezoelectric actuator amplifier design for series actuation (three operational amplifiers for three actuators) or parallel actuation (four operational amplifiers for
The final assembly of the high speed IFTS is shown in Figure 4.20. The overall size of the system is $25 \times 40 \times 42 \text{ mm}$. The test electronics for this design are also far more complex including the off-board actuator amplifier board with copper heat sinks, the $20\, V$ and $60\, V$ power board to drive the piezoelectric amplifiers, and an external FPGA board.

This design fixes several issues present in previous designs. The light and laser source boards are separate from each other and from the photodiode board making alignment more straightforward. The additional motor flexures also make alignment faster and more accurate for longer actuator strokes.

For example, in order to find and align the center of the interferometer, the camera board can be removed and a white light as well as long coherence length laser light can be projected into the IFTS. The output light from the camera board location
can be projected on a small screen. Then the motor voltage is ramped slowly on the user interface while adjusting the alignment (using the mirror and motor alignment flexures) of the interferometer based on the long coherence reference laser. When the center of the interferometer is found based on the short coherence white light interference pattern, final alignment adjustments are made and the motor voltage and corresponding position relative to the default location is noted. The camera board can then be replaced and the spectrometer can be used. The ZPD point does not change appreciably over time once aligned and can be found easily later using the photodiodes that sense the short coherence white light.

The interferometer can be moved without changing the alignment but needs to be realigned if jolted. This is because the motor alignment flexure design shown in Figure 4.17 contains a weak connection point which causes some change in alignment if knocked. Later design and manufacturing modifications address and correct this issue.

The motor and piezoelectric actuator boards are separate from the spectrometer leading to better heat transfer and longer operation for full stroke tests. The flexures are thicker allowing for less parasitic deformation. When the sensor layer and spectrometer layers share a beam splitter and mirrors, however, light will tend to scatter onto the camera, thereby limiting its sensitivity to external signals, especially for fluorescence and Raman applications. For these higher sensitivity applications, additional improvements are necessary. Possible solutions include moving the reference light paths or changing the reference laser to wavelengths that cannot be measured with the camera, such as using a 1550 nm VCSEL.
4.5 Handheld Prototype Design

A handheld prototype can be constructed from any of the designs. The most capable system, the high speed version, was chosen to create the first handheld prototype. The majority of the design of the spectrometer optics and mechanics was maintained. The handheld prototype design process focuses on miniaturizing the electronics and adding hardware that would be useful in portable applications. Figure 4.21 shows solid models of the handheld device. Figure 4.22 shows all six sides of the fully assembled handheld prototype.

The spectrometer, input aperture, and circuit boards are attached to the outer case through three custom mounting plates and custom standoffs made on the wire EDM. The outer case of the design is constructed from aluminum corner posts made using the wire EDM process. These posts hold clear acrylic sides that allow the internal structure to be viewed. The dimensions of the device are $35 \times 53 \times 98$ mm. The overall dimensions were constrained by the size of the screen and buttons as well as the thickness of the spectrometer.

As shown in Figure 4.23, the device is smaller than an iPhone 4S in footprint.

Figure 4.21: The solid model of the handheld prototype using the high speed spectrometer design is shown.
but much thicker. The device has a mass of 194 grams. The input aperture of the device is approximately 6.5 mm in diameter with no application specific optics. For some applications, additional adjustable lenses could be added to the input aperture including tunable electrostatic or bending membrane lenses from Varioptic or Optotune.

This new device includes a 240 × 400 pixel 18-bit RGB display screen (Crystalfontz CFAF240400D) and several buttons for changing scan settings. This system has a custom FPGA board with a Spartan 6 chip and microcontroller. When not connected to a computer, the device is designed to feed the camera video to the screen using the FPGA. Alternatively, the device can be used in the original configuration with two USB ports connected to a computer with the user interface. There are additional connectors for programming the microcontrollers and the FPGA. There is also a power connector for charging the battery and a switch for turning the system on and off. Several indicator LEDs are also used to show the charge status of the battery, the condition of the power lines, and the state of the microcontroller and FPGA, and the information transfer on the USART lines.

A new custom battery charger board with a 3.5 to 4.35 V buck-boost converter
Figure 4.23: a) A close-up view of the handheld prototype is shown along with b) a comparison of the prototype with an iPhone 4S.

with efficiencies in the range of 89 % to 93 % was designed for this device. There are also separate 5 V boost and 3.3 V buck converters on this board capable of efficiencies in the range of 94 %. The layouts for these boards are listed in Appendix A.2. Custom layouts were created for the base microcontroller board, the 20 V and 60 V power board, and the actuator amplifier board. Designs were updated for the remaining laser & light board, photodiode board, and high speed camera board. The LVDS 8-bit video pipeline was removed in this version. The electrical layout and final assembled boards are shown in Figure 4.24. Much of the board area is used by connectors between different boards so future versions may consolidate the number of boards to reduce the connection points.

This particular design includes an 850 mAh lithium polymer batter at 3.7 V. The power budget for this device includes 0.45 W for the FPGA board, 0.28 W for the screen, 3.3 W for the 20 V and 60 V boost converters, and 1.45 W for the microcontroller and sensor circuits. If the motor is at maximum stroke, it draws up to 2.4 W. In full scanning mode, the motor only draws about 0.8 W. When taking boost
Figure 4.24: The custom electronics for the handheld prototype are shown along with the new custom battery charger and FPGA boards.
converter efficiency into account, low resolution scanning with the screen off allows the device to operate for up to 85 minutes. With the screen on and the device running in full scanning mode, the operating time is approximately 54 minutes. If high speed scanning (using piezoelectric amplifiers) at maximum motor stroke is completed with the screen on, the device should operate using external power supplies (plugged into the wall) or two separate batteries and battery charger boards as to not exceed current limits on a single battery charger.

4.6 Implementation Comparison

In each implementation, the design process was improved. Table 4.1 shows the specifications of the different implementations of the IFTS. The first full prototype has a low field of view which was improved in the multi-axis and endoscope designs. The endoscope design is 34% of the size of the prototype design but has a lower overall stroke. The multi-axis implementation improved the settling time and full stroke of the system and is only 56% of the size of the prototype. The high speed design is much larger but has a faster settling time, a longer stroke, and a higher frame rate. Each of these designs can then be characterized, controlled and used to make different hyperspectral imaging measurements.
Table 4.1: Summary of the specifications of the different implementations

<table>
<thead>
<tr>
<th>Specification</th>
<th>Prototype (V 0.53)</th>
<th>Multi-Axis (V 0.60)</th>
<th>Endoscope (V0.70)</th>
<th>High Speed (V0.80)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camera</td>
<td>TCM 8230 1/6&quot; format (1.8 x 2.4 mm) 3.75 μm pixel</td>
<td>TCM 8230 1/6&quot; format (1.8 x 2.4 mm) 3.75 μm pixel</td>
<td>TCM 8230 1/6&quot; format (1.8 x 2.4 mm) 3.75 μm pixel</td>
<td>LUPA 100 1/2&quot; format (4.8 x 6.4 mm) 9.9 μm pixel</td>
</tr>
<tr>
<td>Dimensions (with circuit boards)</td>
<td>21 x 23 x 38 mm (16 % of prototype)</td>
<td>18 x 23 x 25 mm (16 % of prototype)</td>
<td>15 x 35 mm (34 % of prototype)</td>
<td>25 x 40 x 42 mm</td>
</tr>
<tr>
<td>Field of View</td>
<td>100 x 105 pixels</td>
<td>640 x 480 pixels</td>
<td>640 x 480 pixels</td>
<td>640 x 480 pixels</td>
</tr>
<tr>
<td>Controlled axes</td>
<td>1 voice coil (translation)</td>
<td>1 voice coil (translation)</td>
<td>1 voice coil (translation)</td>
<td>1 voice coil (translation)</td>
</tr>
<tr>
<td>Setting Time for a 1 μm step</td>
<td>10 ms (up to 100 fps)</td>
<td>1.2 ms (up to 720 fps)</td>
<td>10 ms (up to 100 fps)</td>
<td>0.44 ms (up to 2300 fps)</td>
</tr>
<tr>
<td>Maximum OPD</td>
<td>3174 μm</td>
<td>6010 μm</td>
<td>1428 μm</td>
<td>6600 μm</td>
</tr>
<tr>
<td>Imaging Speed</td>
<td>Up to 30 fps</td>
<td>Up to 30 fps</td>
<td>Up to 30 fps</td>
<td>Up to 2300 fps</td>
</tr>
</tbody>
</table>
Chapter 5

Measurement and Control

Fourier transform spectrometers are typically instruments that measure the instantaneous intensity of the output light of the interferometer. This typically involves moving one of the mirrors of the interferometer at a constant velocity (using velocity control) and sampling the output intensity of the interferometer when a reference laser crosses certain points along its interferometric pattern. In this traditional method, the mirror only moves in one direction at a time so it is not necessary to know the direction of travel nor is it important control the position of the mirror accurately. A simple, low resolution position sensor, a single reference laser, and a simple actuator can be used in this case.

This, however, is not necessarily true for certain applications of Fourier transform spectroscopy, including for the measurement of low light levels requiring long integrations of the incoming photons (Raman spectroscopy), for the case of imaging spectrometers, or for the case where irregular, non-uniform sampling techniques are used. This is illustrated in Figure 5.1. The first column shows the traditional system utilizing velocity control and instantaneous sampling. The middle column illustrates what happens if this topology is used on systems that integrate the incoming light intensity over time. The movement of the interferometer will tend to blur during the
Figure 5.1: Different mirror displacement topologies are compared with different data sampling techniques. a) In the standard Fourier transform spectrometer topology, the mirror is displaced at a constant speed and the sampling is instantaneous. b) If constant velocity control is used on a system that does photon integration over time, sampling error can appear. c) If step position control is used in conjunction with photon integration, the measurement error is reduced opening up possibilities for irregular non-uniform sampling, long integration sampling, and other applications.

integration and the average value of the light will be represented instead of the instantaneous value. In this case, the accuracy of the measurement can drop significantly especially as the desired velocity, step size or the integration time increases.

The solution presented in this work uses position control instead of velocity control to accurately hold the moving mirror in place over the entire photon integration period of the imaging sensor. This is shown in the last column of Figure 5.1. For each sample, the mirror is quickly moved to the desired position and held in place. The resulting intensity of the signal output of the interferometer will also be constant over this period thereby providing an accurate signal regardless of the step size or the integration time. This can be used effectively for long integration applications, imaging spectroscopy and irregular non-uniform sampling techniques.

In order to do this, several new components are necessary. First, high precision a method for measuring the position of the mirror in real time is needed. This method must be able to compensate for the mirror moving forward or backward over
distances on the order of tens of nanometers. This sensing method must also be compact. Second, a control system needs to be developed to accurately control the mirror position in real time. Lastly, there needs to be a system that coordinates the positioning system with the image capture system so that the two are synchronized. This chapter describes each of these components and the consequences of different design decisions. The system dynamics are modeled, characterized and translated into different types of feedback and feed-forward controllers for implementing the coordinated positioning and imaging system. Finally, the resulting blur reduction results are discussed.

5.1 Position Measurement

Several different high resolution position measurement techniques exist including the use of confocal sensors [21], angled mirror and photodiode arrays [156], and micro-strain sensors [86, 143]. The most compact methods for position measurement would utilize the existing interferometer components. Traditionally, a single position reference helium neon (HeNe) laser tube would be used because of the fine linewidth of the laser. This, however, would be prohibitively large for a miniature IFTS. Instead, a small VCSEL laser is used. VCSEL lasers have some of the finest linewidths of any semiconductor laser and are commonly used for optical fiber applications. A surface mount 0603 package VCSEL at 850 nm with a bandwidth of 0.057 to 0.1 nm FWHM is used for this application. The maximum spectral resolution that can be accomplished with this reference laser is 1.38 to 0.788 cm⁻¹ without the use of any spectral line correction algorithms.

There are also several different interferometric techniques for utilizing a reference laser to measure both position and direction. Some of these homodyne techniques include using polarized beam splitters [68, 169] or optical delay plates [122] to create
a phase difference between measurements at two different photodiodes. Heterodyne methods utilize two lasers with different wavelengths [31]. The interferometric method chosen for this design uses mirror tilt (or a step in the mirror) as the method for producing the delay, requires only two to four photodiodes, and the speed is only limited by the electronic amplification of the photodiode signals. Other more complex designs use cameras and the speed is limited by the camera frame rate [87]. The method used in the IFTS design does not require additional components in the interferometric path and is sensitive to mirror tip and tilt, which also makes it a great method for aligning two mirrors for tip and tilt control. The drawback of this method is that the phase difference between the two measurements are not always in perfect quadrature and requires additional calculations, calibrations and observers to determine the position, tip and tilt accurately.

5.1.1 Two-Signal Analog Encoder

The basic concept for the tilted mirror (or stepped mirror) interferometric measurement system is shown in Figure 5.2. When one mirror in the interferometer is tilted with respect to a second mirror, the paths of the light reflected from those two mirrors (pink and blue) will not follow the same path thereby projecting an interference pattern on the probe plane in Figure 5.2b. This interferometric pattern is sinusoidal and is a function of the relative position of the two mirrors as well as the relative tilt. When the moving mirror is translated, the dark and bright portions of the light will translate left or right. By placing two photodiodes in different locations on this pattern, it is possible to measure two parts of this curve with a fixed phase delay. With two signals, it is possible to calculate the position and direction of travel by using an analog encoder scheme where the phase delay resulting from a tilt angle \( \theta \) is found through calibration.
Figure 5.2: a) The setup for a tilted mirror interferometric position measurement system is shown along with b) the projected pattern at the probe plane and c) the intensity as a function of position.

The solution concept for calculating the position from two analog signals is modified from electronic analog encoder systems operating under non-interferometric principles [157]. The signal at the two photodiodes can be used to calculate the relative position using the following scheme where the two signals at positions 1 and 2 are $a(z)$ and $b(z)$ respectively,

\begin{align}
    a(z) &= \cos \left( \frac{2\pi z}{\lambda} \right), \\
    b(z) &= \sin \left( \frac{2\pi z}{\lambda} - \phi \right) = \sin \left( \frac{2\pi z}{\lambda} \right) \cos(\phi) - \cos \left( \frac{2\pi z}{\lambda} \right) \sin(\phi).
\end{align}

The phase difference between the two signals is a function of tilt angle $\theta$ and the distance between the two sensors $d$,

\begin{align}
    \phi &= -\frac{\pi}{2} + \frac{2\pi}{\lambda} \Delta, \\
    \Delta &= d \tan(\theta).
\end{align}

In order to solve for the position $z$, the two measure signals can be divided and the
Figure 5.3: a) The raw photodiode signals, b) Lissajou plot and c) calculated position are shown for the step response of the mirror. d) In this case, the mirror is on a thin cantilever causing tilting motion as a function of mirror velocity.

position can be found by taking an inverse tangent,

$$\frac{b(z)}{a(z)} = \tan \left( \frac{2\pi z}{\lambda} \right) \cos (\phi) - \sin (\phi),$$  \hspace{1cm} (5.5)

resulting in the final equation for determining the relative position of the two mirrors,

$$z = \frac{\lambda}{2\pi} \tan^{-1} \left( \frac{b(z) + a(z)\sin(\phi)}{a(z)\cos(\phi)} \right).$$  \hspace{1cm} (5.6)

In order to solve this equation, the input data must first be scaled such that the data is between -1 and 1. Then the position can be calculated by using the arctangent function with two arguments, being careful to properly unwrap the phase of the output data. The relative phase delay can be calibrated by running a fast ramp on the actuator. Then a Lissajou plot can be made from the two signals and
a quick fit can be made to determine the phase delay. A typical Lissajou plot for a calibration looks like the red oval curve in Figure B.2. This calibration step takes about 2 seconds to complete with the user interface. Because the IFTS and motor flexure system are relatively stable, the calibration remains valid for a long time.

A typical set of data acquired from this position sensor is shown in Figure 5.3. This figure shows a typical step response with the two raw photodiode scaled signals, the Lissajou plot, and the resulting calculated position. This particular set of data is interesting because it uses a weak cantilever to hold the mirror, which causes the mirror to tilt back and forth as a function of the mirror velocity due to drag [52]. Since the trace of the Lissajou plot does not go over itself, this shows that the phase changes over time due to mirror tilt. By calculating the instantaneous phase difference and the velocity of the mirror, it is clear that the two signals match up well. This effect disappears when the component holding the mirror is strengthened, thereby changing the Lissajou plot into a perfect oval with a constant phase delay.

This type of analog encoder, theoretically, has infinite positioning resolution. It is however, limited by the resolution of the ADC used to measure its magnitude and the inherent noise in the circuit system and on the photodiode (e.g. dark current). The resolution limitation for a single photodiode is roughly \( \Delta z = \lambda / (2 \frac{\Delta V}{V_{\text{max}}^B} B_{\text{ADC}}) \), which is a function of the reference wavelength \( \lambda \) and the number of bits of the ADC between the signal maximum and signal minimum \( \frac{\Delta V}{V_{\text{max}}} B_{\text{ADC}} \). The maximum positioning resolution for this wavelength is approximately 0.1 nm for a 12-bit ADC. In practice, the resolution of the system can also be limited by the resolution of the look-up table used to calculate the sine, cosine or inverse tangent.
5.1.2 Four-Signal Analog Encoder

There are several possible methods for measuring two axis mirror tip and tilt using the analog encoder topology with either three or four signals. With three signals, a fitting procedure or observer algorithm is required for determining position, tip and tilt. A four signal design is shown in Figure 5.4. With a two dimensional probe plane, the interferometric pattern can be tilted and be projected differently on the four photodiodes with a relative phase difference in the tip direction and in the tilt direction. The signals on the four sensors are,

\[
a(z) = \cos\left(\frac{2\pi z}{\lambda}\right),
\]
\[
b(z) = \sin\left(\frac{2\pi z}{\lambda} - \phi_{ab}\right),
\]
\[
c(z) = \sin\left(\frac{2\pi z}{\lambda} - \phi_{ac}\right),
\]
\[
d(z) = \sin\left(\frac{2\pi z}{\lambda} - \phi_{ad}\right).
\]

Figure 5.4: a) The setup for a tip and tilt measurement system is shown along with b) the projected pattern at the probe plane and c) the intensity as a function of position. The sensor numbering orientation is a mirror image of the tip-tilt mirror numbering orientation.
The positions calculated by comparing the signals between each of these points is,

\[
\begin{align*}
    z_{ab} &= \frac{\lambda}{2\pi} \tan^{-1} \left( \frac{b(z) + a(z) \sin(\phi_{ab})}{a(z) \cos(\phi_{ab})} \right), \\
    z_{ac} &= \frac{\lambda}{2\pi} \tan^{-1} \left( \frac{c(z) + a(z) \sin(\phi_{ac})}{a(z) \cos(\phi_{ac})} \right), \\
    z_{bd} &= \frac{\lambda}{2\pi} \tan^{-1} \left( \frac{d(z) + b(z) \sin(\phi_{bd})}{b(z) \cos(\phi_{bd})} \right), \\
    z_{cd} &= \frac{\lambda}{2\pi} \tan^{-1} \left( \frac{d(z) + c(z) \sin(\phi_{cd})}{c(z) \cos(\phi_{cd})} \right).
\end{align*}
\]

The relative phase angles between the signals is then,

\[
\begin{align*}
    \phi_{ab} &= -\frac{\pi}{2} + \frac{2\pi}{\lambda} \Delta_x, \\
    \phi_{ac} &= -\frac{\pi}{2} + \frac{2\pi}{\lambda} \Delta_y, \\
    \phi_{bd} &= -\frac{\pi}{2} + \frac{2\pi}{\lambda} \Delta_y, \\
    \phi_{cd} &= -\frac{\pi}{2} + \frac{2\pi}{\lambda} \Delta_x, \\
    \phi_{ad} &= -\frac{\pi}{2} + \frac{2\pi}{\lambda} (\Delta_x + \Delta_y).
\end{align*}
\]

Fundamentally, the phase differences are based on the \( x \) and \( y \) distance between the four sensors as well as the relative tip \( \theta_x \) and tilt \( \theta_y \) such that,

\[
\begin{align*}
    \Delta_x &= z_{bd} - z_{ac} = d_x \tan(\theta_x), \\
    \Delta_y &= z_{bd} - z_{ac} = d_y \tan(\theta_y).
\end{align*}
\]

The coordinate system is labeled in Figure 5.5a. It is clear that there are several geometric factors that can affect the measurement of the angular resolution as well as
Figure 5.5: The effect of a) laser light projection placement and c) laser light projection area on angular resolution and sensitivity are shown. There are two methods for obtaining phase delay between the four signals, b) the mirror can be tilted and/or d) a raised level mirror can be constructed.

the angular range of the measurement. When the sensor area or laser light projection area is large, the interferometric pattern of the tilted mirror will be averaged over the sensor and will therefore lead to lower angular resolution. This could be corrected if the input laser is split into several small points or if the light incident on each sensor is intercepted by an aperture. For the same tip and tilt angle, more fringes will appear on the sensors if they are placed further apart than when they are placed closer together. For the tip and tilt calculation schemes that cannot handle $n\pi$ transitions, this can lead to a lower sensor range with higher angle sensitivity. If the tip and tilt calculation scheme can handle $n\pi$ transitions, distributed sensing points increase the angle sensitivity of the design.

Mirror tilt is a good way of describing the phenomenon that is being measured. In practice, too much mirror tilt would distort the interferometer signal but some phase difference is required for the signal to be calculated. A different way of producing the same result would be to use a step in the mirror. The manufacturing process for
producing this mirror step is described in Section 3.1.3. By using this method, it is possible to then measure additional mirror tip and tilt in order to do active control on mirror misalignment while being able to maintain a zero path difference across the interferometer image.

Figure 5.5c and d show how a raised level mirror can be used to create a phase difference across the sensors without tilting the mirror. For a perfect quadrature on an 850 nm laser, the height of the levels should be near 212.5 nm.

5.1.3 Phase Calculation

There are several different ways that the relative phase, or tip and tilt, can be calculated from four different signals. The first, most obvious method, is to calculate each of the relative positions and take the difference between the measured positions to determine tip and tilt. This direct method utilizes Equations 5.20 and 5.21. It is interesting to note that if the two rotational axes are stationary, the resulting signal can give a quick and accurate phase difference. If only one of the axes is moving, then phase noise will appear in the signal of the other axis as shown in the simulations in Figure 5.6c. This effect is undesirable and can be removed using an iterative algorithm.

In the iterative version, the $\Delta_x$ and $\Delta_y$ from Equations 5.20 and 5.21 can be plugged back into Equations 5.11 through 5.14 iteratively and the position can be solved at for every data point. The resulting simulated signals produce very good results with very little error. The low noise error of the iterative method is as low as 1.3 nm in the position estimate and as low as 3.5 nm in the phase estimate across a 4.5 mm mirror. The method is at least twice as computationally intensive as the direct method.

Although these methods work well in simulation and are tolerant to noise, the iter-
Figure 5.6: The simulation results of the direct phase calculation method (a and c) and the iterative calculation method (b and d) are shown indicating the position and tip and tilt of the system when the position is changed sinusoidally and the tip angle is altered linearly.

The iterative method is not very tolerant to estimation errors. Small errors in the estimation of the relative position of the sensors can cause the algorithm to produce inaccurate results. Other, more error tolerant methods are more appropriate for implementation in a real system.

5.1.4 Phase Observer

There are several non-ideal elements in the measurement of signals while the system undergoes tip and tilt. All of these non-ideal characteristics must be captured by the algorithm used to determine the tip and tilt angle of the system. One of the signals obtained when the mirror is tipped or tilted is shown in Figure 5.7a. This Lissajou plot shows how the relative phase changes when the tip or tilt piezoelectric actuator is ramped up and ramped down. Notice how the line retraces itself along the same path but does not end up in the same position. Also, notice how the range (maximum and minimum) of the signals change as the phase changes.

A phase observer can be used to capture the tip and tilt of the system by using the algorithm outlined in Figure 5.7b. The first step in the algorithm is to calculate
a) A typical Lissajou plot shows the variation of the signals when the mirror is tipped or tilted. The data were obtained from the multi-axis version. b) The phase observer algorithm is used to account for the artifacts that appear in measured signals by using phase fitting.

all four relative positions for a small chunk of the data, then the phase is fit over this full range for all four signals. These absolute phase values are then converted to relative phase values between the different signals. These relative phases are then used in future calculations of the position. Sensor data is then grabbed in small groups, used to calculate the position, and then utilized to fit the phase keeping in mind that the variation in phase should be relatively small so only values that are close to the current estimated values are checked. This prevents \( n\pi \) wrapping issues. Then the values are converted to relative phase and the magnitudes (maximum and minimum on each signal) are recalculated after traveling more than a distance \( \lambda \) to rescale the range of the data. This is repeated for all the position data.

This algorithm works fairly well and can be used on measured signals as shown in Figure 5.8. In this experiment, a ramp is put on the motor and the tilt piezoelectric actuator is ramped up and down. It is clear that the signal magnitudes change in the raw sensor signals, and that the phase observer algorithm was able to tease out the difference between the large translation and the small tip and tilt angle on the piezoelectric actuator. In practice, the tip and tilt angle results are better when a
Figure 5.8: a) The measured raw signals from four photodiodes is used to determine the b) the translation and c) the tip and tilt of a mirror by using the phase observer algorithm. The data were obtained from the multi-axis version.

The phase observer algorithm is compared to the direct calculation algorithm in Figure 5.9. In this case, the motor is not ramped so the phase estimates are made only from the signal changes due to the tip, tilt and translate piezoelectric actuators alone. The results therefore require more averaging. Although the time resolution is lower for the phase observer, it is clear that it does a better job of determining the value of the tip, tilt and translation. It is clear that the tip and tilt actuators have some parasitic translation which comes from their flexure designs.

The tip and tilt actuators are not perfectly aligned with the sensor array. Therefore coordinate rotation of the measured signals is required. Figure 5.10 first actuates the tip actuator and then the tilt actuator by ramping the voltages up and down. After
Figure 5.9: The direct calculation method (a, b and c) is compared to the phase observer method (d, e and f) using experimental data of the translate, tip and tilt piezoelectric actuators. The data were obtained from the multi-axis version.

Figure 5.10: Coordinate rotation is necessary for aligning the sensor signals to the actuator axes. a) The signal before coordinate rotation can be improved to b) the signal after coordinate rotation when the tip and tilt actuators are ramped separately. The data were obtained from the multi-axis version.

coordinate rotation, the parasitic components of the tip and tilt are removed from the orthogonal signal. This helps facilitate accurate and stable angle control.
5.2 Positioning System Characterization

The characterization of the positioning system is important for developing algorithms and optimizing the design. The two most important static characteristics are the stroke of the actuator, which determines the spectral resolution, and the temperature profile as a function of stroke, which determines the maximum steady state stroke that can be used.

5.2.1 Stroke

In an ideal system, the interferometric signal on the positioning system would remain at a constant level for the full stroke. In a real system, however, with external vibrations and misalignment as a function of angle, the signals are not ideal and require additional compensation algorithms.

Figure 5.11a shows how two photodiode signals can vary as a function of stroke. By using the phase observer algorithm, it is possible to track the changes in signal magnitude and the relative signal phase to determine the true relative position of the mirrors. Figure 5.11b shows the high resolution spectra taken from a stroke of 1600 $\mu m$ on two lasers, one is the reference VCSEL at 850 nm and the other is a different laser at 843 nm.

Figure 5.12 shows one such test from the prototype version tested at $\pm 5$ V. The measured travel for this test was approximately 2700 $\mu m$ and the measured phase tilt was approximately $\pm 20$ degrees.

Similarly, a four-sensor system (multi-axis version) can be used to identify tip and tilt for a long stroke as shown in Figure 5.13. The overall stroke of this test was about 2640 $\mu m$ with a phase tip of $\pm 90$ degrees and a phase tilt of $\pm 400$ degrees for this
Figure 5.11: a) Raw encoder signals from two photodiodes in a two sensor system for a long stroke is shown. b) The spectra from the long stroke experiment are exhibited including the spectrum from the reference laser and from a separate external laser. The data were obtained from the spectrometer camera on the prototype version.

Figure 5.12: A long stroke experiment was conducted on the prototype version of the IFTS showing the stroke for a ±5 V test for a travel of 2700 μm OPD and a phase tilt of approximately ± 20 degrees.

particular alignment. This actuator has better heat transfer characteristics, outputs more force, and has lower flexure stiffness allowing it to achieve the same stroke for a lower input voltage as the previous design. Unlike the previous designs, the full stroke of the high speed version can be easily imaged and is displayed in Figure 5.14. This figure shows the negative translation, center and positive translation of the long-stroke linear actuator.

One interesting point to notice is curvature of the position to voltage function near the extremes of the stroke in Figure 5.12. This is not an indication of the nonlinearity
Figure 5.13: A long stroke experiment was conducted on the multi-axis version of the IFTS showing the stroke for a ± 2 V test for a travel of 2640 μm OPD, phase tip of ± 90 degrees and phase tilt of ± 400 degrees.

![Graph showing position and voltage](image)

Figure 5.14: The long stroke on the high speed version of the IFTS mirror is visualized.

of the stroke but an indication of the actuator heating up and is purely a factor of the speed at which the test was conducted. When the actuator is powered at 5 V, it tends to heat up. When the copper in the actuator coil heats up, the resistivity of the copper increases thereby decreasing the current passing through the actuator and decreasing the overall force at a given input voltage. If this test was conducted at a faster speed where the temperature of the coil was not allowed to increase, the overall stroke of the actuator would be longer (on the order of 3500 μm). The longest stroke actually measured for this design was about 3174 μm.

By comparing the measured position with the predicted position of the actuator,
it is possible to approximate the average temperature of the copper portion of the actuator from the prototype version (assuming that there is no friction or obstruction blocking the actuator and that the actuator is perfectly linear) using,

\[ z - z_0 = K_m V \left( \frac{1}{R} - \frac{1}{R_0} \right), \quad (5.22) \]

\[ R = (1 + \alpha \Delta T) R_0, \quad (5.23) \]

where \( K_m \) is the current to position constant, \( R \) is the resistance of the coil at any temperature, \( \alpha \) is the temperature coefficient of the actuator, and \( R_0 \) is the resistance of the actuator at room temperature (approximately 24 °C). The measured position is \( z \) and the predicted position when there is no temperature change is \( z_0 \). For this system, \( \alpha = 0.003862 \, K^{-1} \), \( R_0 = 7.2 \, \Omega \) and \( K_m = 2520 \, \mu m\Omega/V \). By combining these equations and solving for the change in temperature \( \Delta T \), the following equation is obtained,

\[ \Delta T = -\frac{(z - z_0)R_0}{\alpha(z - z_0)R_0 + \alpha K_m V}. \quad (5.24) \]

Using this equation, the predicted temperature increase at the extremes of the stroke of the actuator in Figure 5.12 is between 80 and 100 degrees based purely on the curvature of the positioning curve. As long as the positioning controller is calibrated and turned on, this method can potentially be used to monitor the temperature of the coil in real time without the use of an additional sensor. This is a significant increase in temperature and shows how the design of the heat transfer mechanisms for the actuator is very important to the overall design of the IFTS. Rather than being limited by the force output of the actuator, the designs are limited by the heat transfer characteristics.
5.2.2 Temperature

The heat transfer characteristics are an important factor in the design of Lorentz force voice coil actuators for several reasons. First, the steady state temperature needs to remain low to prevent certain components from melting, short circuiting or expanding. The temperature must also remain low in order to not degrade the strength of the NdFeB magnets. The two center magnets that appear in all the designs are N52 grade. The permanence coefficient for these magnets is 3.07, therefore the maximum operating temperature (before demagnetization occurs) obtained from the BH curve is just above 110 °C.

Rather than using an indirect method for calculating temperature as outlined previously, a direct method is used to check the temperature of the coil as well as the amplifiers for the motor using a Fluke Ti125 thermal imaging camera. The temperature as a function of position is recorded in steady state for the prototype version and multi-axis version in Figure 5.15 and for the endoscope and high speed versions in Figure 5.16. In these figures, the system is shut down immediately if any of the components reaches more than 110 °C.

The original prototype has a large circuit board and a small, enclosed actuator with a relatively stiff flexure system. This caused the system to require more power to obtain the same stroke. The board temperature tended to level out while the temperature of the coil grew rapidly. The multi-axis version, on the other hand, had a small circuit board and more exposed area for the actuator. Therefore, the temperature of the coil grew slowly while the temperature of the board reached much higher values due to the smaller heat transfer area.

When the smaller endoscope version was created, the flexure system was designed to be stiffer and the stroke was designed to be shorter. The smaller board and smaller and stiffer actuator cause the design to have higher temperatures. On the
Figure 5.15: The temperature of the coil and the circuit board near the motor amplifiers as a function of mirror travel is shown for the prototype version (a and b) and the multi-axis version (c and d).

Figure 5.16: The temperature of the coil and the circuit board near the motor amplifiers as a function of mirror travel is shown for the endoscope version (a and b) and the high speed version (c and d).

On the other hand, the high speed design had a much larger actuator and more board area thereby allowing much lower steady state temperatures for both components.

By plotting the steady state power against the temperature, it is possible to
Figure 5.17: The temperature profiles as a function of the steady state input profiles are shown for a) the prototype version, b) the multi-axis version, c) the endoscope version and d) the high speed version.

determine the thermal resistance between the coil and the ambient. This is exhibited in Figure 5.17. In this figure, it is clear that the coil temperature as a function of steady state input power is linear. These values were used to calculate the thermal resistance of the actuator design. The board temperature, on the other hand, does not have a linear relationship to power or to position. The thermal resistance of each of the designs is listed in Table 5.1.

In normal operation, the actuator is not held at some position for long periods of time. Instead, the actuator would be swept back and forth quickly from the negative stroke to the maximum positive stroke. Therefore, the average power used by the actuator during this sweeping process is the value necessary for calculating the steady state temperature of the actuator during operation. The average power on the actuator is calculated by integrating the power used along the stroke assuming that equal time is being spent at each position between the minimum voltage and the
maximum voltage. By calculating the average integral of $P = V^2/R$ over all voltage values, the solution is,

$$P_{\text{avg}} = \frac{1}{RV_{\text{max}}} \int_0^{V_{\text{max}}} V^2 dV = \frac{1}{RV_{\text{max}}} \frac{V_{\text{max}}^3}{3} = \frac{V_{\text{max}}^2}{3R}. \quad (5.25)$$

This equation indicates that the average power $P_{\text{avg}}$ is a function of the maximum voltage $V_{\text{max}}$ and the resistance of the coil $R$. The average power can be converted to an average temperature for operation up to full stroke by using the equation $T_{\text{avg}} = R_T P_{\text{avg}}$ where $R_T$ is the thermal resistance. These values are tabulated in Table 5.1. If the ambient temperature is assumed to be 24 °C, then the temperature of all the designs is less than 82 °C operating at full stroke (5 V).

Since the average temperature is only a function of the maximum voltage, coil resistance, and the thermal resistance, other factors like the positioning gain and the maximum stroke do not play a part in the thermal design of the actuator. The stroke length can then be independently controlled by changing the positioning gain of the actuator through altering the flexure blade dimensions. This, in turn, affects the dynamic performance.

### Table 5.1: Summary of the temperature and power characteristics of different implementations

<table>
<thead>
<tr>
<th>Design</th>
<th>Coil Resistance (Ω)</th>
<th>Thermal Resistance (k/W)</th>
<th>Positioning Gain $k_v$ (V/µm)</th>
<th>Average Power while Operating up to Full Stroke (W)</th>
<th>Average Temperature while Operating up to Full Stroke (°C) (Ambient at 24 °C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prototype</td>
<td>7.2</td>
<td>45.9</td>
<td>0.0028</td>
<td>1.16</td>
<td>77</td>
</tr>
<tr>
<td>Multi-Axis</td>
<td>8.5</td>
<td>50.1</td>
<td>0.0015</td>
<td>0.98</td>
<td>67</td>
</tr>
<tr>
<td>Endoscope</td>
<td>5.15</td>
<td>35.9</td>
<td>0.007</td>
<td>1.62</td>
<td>82</td>
</tr>
<tr>
<td>High Speed</td>
<td>5.3</td>
<td>31.2</td>
<td>0.0015</td>
<td>1.57</td>
<td>73</td>
</tr>
</tbody>
</table>
5.3 Dynamic Modeling and System Identification

Although the static characteristics of the system are important to the design, the dynamics of the system are important for control. In order to identify the dynamics of the system quickly and repeatably, a stochastic system identification method is used on all the translation actuators. This method is shown in Figure 5.18. This method uses a random input profile that contains a large number of frequencies to interrogate the actuator system. The output is measured and the response to all these different frequencies can be teased apart by using different system identification algorithms in the time domain and in the frequency domain. Frequency domain techniques are useful for determining model order, phase delay, and control strategies while time domain techniques are useful for parametric fitting of model-based feedforward control systems.

For time domain stochastic system identification, a Toeplitz matrix inversion technique utilizing cross-correlation is used. This can only be applied to systems that approximate well as causal FIR models. The component to be identified is the impulse response $\hat{h}$ and this can be done by calculating the input autocorrelation $\varphi_{xx}$ and the input output cross-correlation $\varphi_{xy}$,

$$
\varphi_{xx}(n) = \frac{1}{N} \sum_{i=1}^{N} x(i)x(i + n - 1),
$$

(5.26)

![Figure 5.18: The actuator system can be identified using a stochastic system identification scheme utilizing a random input profile and by measuring the corresponding output profile.](image)
\[ \varphi_{xy}(n) = \frac{1}{N} \sum_{i=1}^{N} x(i)y(i+n-1). \quad (5.27) \]

The input autocorrelation is then used to create the Toeplitz matrix up to the desired size \( M < N \),

\[
T = \begin{bmatrix}
\varphi_{xx}(1) & \varphi_{xx}(2) & \varphi_{xx}(3) & \cdots & \varphi_{xx}(M) \\
\varphi_{xx}(2) & \varphi_{xx}(1) & \varphi_{xx}(2) & \cdots & \varphi_{xx}(M-1) \\
\varphi_{xx}(3) & \varphi_{xx}(2) & \varphi_{xx}(1) & \cdots & \varphi_{xx}(M-2) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\varphi_{xx}(M) & \varphi_{xx}(M-1) & \varphi_{xx}(M-2) & \cdots & \varphi_{xx}(1)
\end{bmatrix}. \quad (5.28)
\]

Using a smaller vector size \( M \) will improve the calculation speed. These equations can be utilized to solve for the impulse response up to size \( M \) by using,

\[
\hat{h}(m) = F_s \left[ T^{-1} \varphi_{xy}(m) \right], \quad (5.29)
\]

where \( F_s \) is the sampling frequency. If the input autocorrelation is perfectly uncorrelated, the input is considered white and the Toeplitz matrix is an identity matrix. In this case, the cross-correlation is directly proportional to the impulse response of the system.

### 5.3.1 Lorentz Force Voice Coil Characterization

The Lorentz force voice coil motor and flexure system can be modeled by treating the system as a series of springs and masses. This is illustrated in Figure 5.19. This linear translation system has two main masses and several flexure springs in parallel, which can be modeled as a single spring. Linear movement in other axes can be ignored for this analysis. The largest mass \( m_2 \) includes the mass of the mirror and mirror adjustment flexures. The force generated by the motor is applied at \( m_2 \).
Figure 5.19: a) A simple dynamic model of the motor and flexure system is presented with four energy storage elements. b) This is broken down into a series of springs, dampers and masses.

This system can be broken down into a state space model as a function of displacements and velocities,

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
z_1 \\
z_2
\end{bmatrix} =
\begin{bmatrix}
\frac{-b_2}{m_2} & \frac{-b_2}{m_2} & \frac{-b_2}{m_2} & \frac{k_2}{m_2} \\
1 & 0 & 0 & 0 \\
\frac{b_2}{m_1} & \frac{b_2}{m_1} & -\frac{(b_1+b_2)}{m_1} & -\frac{(k_1+k_2)}{m_1} \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
z_1 \\
z_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
\frac{F}{m_2}
\end{bmatrix}.
\tag{5.30}
\]

Two transfer functions can be written for the displacement of the main mass \( m_2 \) and the crossbar mass \( m_1 \) as a function of the force generated by the motor \( F \),

\[
z_2 = \frac{z_2}{F} = \frac{m_1 s^2 + (b_1 + b_2)s + (k_1 + k_2)}{(m_2 s^2 + b_2 s + k_2)(m_1 s^2 + (b_1 + b_2)s + (k_1 + k_2)) - (b_2 s + k_2)^2},
\tag{5.31}
\]

\[
z_1 = \frac{z_1}{F} = \frac{b_2 s + k_2}{(m_2 s^2 + b_2 s + k_2)(m_1 s^2 + (b_1 + b_2)s + (k_1 + k_2)) - (b_2 s + k_2)^2}.
\tag{5.32}
\]

The Bode plots of both these equations are shown in Figure 3.11. Equation 5.31 is the main transfer function of the system. It is fourth order with four poles and two zeros. This can be converted into a high order model for voltage to position...
Figure 5.20: The step responses of the four different IFTS designs are shown. Each has a different resonance and damping factor.

relationship by using,

\[ F = \frac{V - K_e z^2 s}{Ls + R} K_f. \] (5.33)

This equation models the actuator as an electrical system with a resistance \( R \), an inductance \( L \) and motor constants \( K_e \) and \( K_f \). The coil resistances are listed in Table 5.1 and the coil inductance is on the order of 0.2 mH depending on the design implementation. Using the voltage to position transfer function results in a more heavily damped system response.

This model can be compared with measured results. The step responses of the different prototypes are shown in Figure 5.20. Each prototype has its own natural frequency and damping depending on the design of the flexure dimensions and flexure damping materials. The endoscope version has the highest natural frequency and the multi-axis version has the slowest natural frequency.

Stochastic system identification techniques are applied to the motor system and the impulse response obtained using the time domain technique is shown in Figure 5.21a. Frequency domain techniques utilizing windowing and direct Fourier trans-
Figure 5.21: The system identification of the Lorentz force voice coil actuator is shown. 
a) The impulse response of the system, b) the mean squared coherence, c) the Bode plot 
magnitude and d) the Bode plot phase are exhibited. Multiple tests were conducted to 
obtain the bode plot and each test is presented in a different color. The second order model 
is presented as a thick black line.

form methods were used to obtain the mean squared coherence and the Bode plots 
for the $\frac{z_2}{V}$ transfer function. The system appears to be approximately second order 
with natural frequencies between 100 and 200 Hz depending on the version. By using 
the second order approximation for this design, the majority of the lower frequency 
characteristics can be captured.

Due to the limitation on the size of the vectors that can be saved on the micro- 
controller, it is not possible to obtain very long records on the input and output data. 
Several tests are conducted and the Fourier transform is taken on each. The data are 
then averaged. This method reduces the variance in the same way as the Bartlett 
method or the Welch method with no overlapping segments.

The mean squared coherence of this system is generally greater than 0.6 especially 
near its resonance. High coherence near one indicates that the system can be modeled 
as a purely linear system. Several factors can contribute to a decrease in coherence
including noise, sensor reading nonlinearities, and short data lengths. The low coherence at lower frequencies is most likely due to noise from external disturbances like table vibrations or air currents. The lower coherence at higher frequencies is most likely due to lower signal magnitudes leading to low signal to noise ratios.

5.3.2 Piezoelectric Actuator Characterization

Piezoelectric actuators can similarly be characterized using frequency and time domain techniques for controller design. Piezoelectric actuators are well-known to be hysteretic if voltage is used to control the displacement but linear if charge is used to control the displacement. A simple model with hysteresis can be derived following Goldfarb [67] in the Laplace domain,

\[ q = T_{pm}z + C_{pm}v_t, \]  
\[ F = T_{pm}v_t, \]  
\[ (m_p s^2 + b_p s + k_p)z = F, \]

where \( q \) is the charge, \( T_{pm} \) is the transformer ratio, \( C_{pm} \) is the piezoelectric actuator capacitance, \( v_t \) is the back electromotive force, \( m_p \) is the total system mass, \( b_p \) is the total system damping, and \( k_p \) is the total system stiffness. The hysteretic component can be modeled as,

\[ v_t = V + \sum_{i=1}^{N} V_i, \]

\[ V_i = \begin{cases} \frac{q - q_{hi}}{C_i} & \text{if } \left| \frac{q - q_{hi}}{C_i} \right| < v_i, \\ v_isgn(i) \text{ and } q_{hi} = q - C_i v_isgn(i) & \text{else.} \end{cases} \]

The total voltage applied \( V \) and the hysteretic voltage terms \( V_i \) both contribute
to the system dynamics. The hysteretic terms are a sum of an ensemble of Maxwell slip units with block charge $q_i$, break away voltage $v_i$, and unit capacitance $C_i$. If the system is charge controlled, the transfer function is linear,

$$z = \frac{T_{pm}/C_{pm}}{m_p s^2 + b_p s + k_p + T_{pm}^2/C_{pm}}.$$  \hspace{1cm} (5.39)

The approach taken here is to look at the linear dynamics which contribute to the high speed positioning aspects of the actuator and allow the controller to capture as much of the hysteretic response as possible. It is also possible to use feedback linearization techniques or other nonlinear methods for controlling a voltage input system. The dynamics in the case when hysteretic terms are ignored ($V_i = 0$) can be approximated as,

$$\frac{z}{V} = \frac{T_{pm}}{m_p s^2 + b_p s + k_p}.$$  \hspace{1cm} (5.40)

Similar to the motor, the piezoelectric stack actuators have an inherent mass, damping and stiffness that act against flexure elements. In the simplified linear form in Equation 5.40, the actuator force output is roughly proportional to the voltage across the actuator multiplied by the transformer ratio [67]. This is a much simpler relationship than with the Lorentz force voice coil actuators. In the most ideal case where the stiffness of the translation flexure is much lower than any other mechanical stiffnesses (including the stiffness of the actuator), the system can be modeled as second order with the stiffness of the flexure and the mass of the flexure and moving mirror as energy storage elements. In other scenarios, other mechanical resonances caused by off-axis bending or low stiffnesses in other parts of the system may cause uncontrollable resonances in the system.

The dynamics are the most important for the translation actuator as it is designed to move at higher frequencies. The step responses from the series topology multi-axis
Figure 5.22: The step response of the tip, tilt and translate actuators in the piezoelectric actuator from the multi-axis version are shown.

version piezoelectric actuators are shown in Figure 5.22. The translation actuator resonances are on the order of 2273 to 2778 Hz. The system is also clearly not second order. The tip actuator, on the other hand, does appear to have a roughly second order step response. This may be due to the fact that it is the actuator that is directly connected to the mirror and thus there are fewer spring elements between it and the point of measurement.

An impulse response can be obtained from this translation actuator system and is shown in Figure 5.23a along with a close-up of the step response in Figure 5.23b. The corresponding frequency domain responses are shown in Figure 5.24. These figures
show the actuator with two different amplifiers. The first amplifier version uses the 35 V system while the second amplifier version uses the 60 V supply with higher output current. The difference between the two is fairly clear. The 35 V system has a lower current output and therefore exhibits a slight time delay for high frequency inputs. This can be seen in Figure 5.23a where the V1 amplifier impulse response lags the output of the V2 amplifier impulse response slightly. In Figure 5.23b, the larger current limit of the V2 amplifier allows a faster response with more amplitude swing. The frequency response clearly shows how the V1 amplifier has a larger phase lag compared to the V2 amplifier (red ovals).

Overall, however, the flexure design for this system is not ideal. There are a pair of poles very close together causing the system to be fourth order up to 4 kHz. This makes the system slightly harder to control. Another example of a system that is more difficult to control is shown in Figure 5.25. These graphs correspond to the series piezoelectric actuator from the high speed version of the IFTS. The translate...
Figure 5.25: a) The step responses of the tip, tilt and translate actuators are shown for the series piezoelectric actuators from the high speed version. The corresponding Bode plots (b and c) are shown indicating a large number of resonances that are difficult to model.

actuator has several high frequency components in its response including resonances at 1.2 kHz and 5.3 kHz. These resonances are all relatively close together as indicated by the Bode plot of the system. These resonances are difficult to model past 3 kHz and would be challenging to control past 1 kHz.

By changing the design to a parallel piezoelectric actuator stack, the system can be modeled more simply as a second order system. Figure 5.26 shows the step response of this system at different preloads. The over-tightened system shows a 7 kHz resonance and very little translation. When the system is properly preloaded, the resulting translation resonance is at 2.5 kHz. Each of the four actuators in the parallel design has the same resonance but each has a slightly different phase relationship depending on their location relative to the bending points and their location relative to the optical sensors. The T3 actuator is the closest to the optical sensing point on the mirror and has a fairly clear second order transfer function.

The time domain and frequency domain response of the T3 actuator (\(z/V\) transfer function) is shown for the parallel configuration of the high speed version of the IFTS
Figure 5.26: Step responses for the parallel configuration of the high speed version are displayed. a) The 7 kHz resonances of the four piezoelectric actuators are shown when the actuator stack is over-tightened. b) When properly preloaded, the 2.5 kHz resonances on the four actuators can be seen. Each of these actuators has a different step response with the T3 actuator showing a true second order response.

Figure 5.27: The system identification of the translate motion of the parallel configuration piezoelectric actuators is shown for the high speed version. a) The impulse response of the system, b) the mean squared coherence, c) the Bode plot magnitude and d) the Bode plot phase are exhibited.

in Figure 5.27. The impulse response is very close to a second order model as is the Bode plot up to 5 kHz, where additional resonances begin to appear. The system also has a pure delay which comes from the way that the sensor input and actuator command output are clocked on the microcontroller. This system would be relatively
Figure 5.28: The hysteresis curves of the actuators are shown for the parallel configuration of the piezoelectric actuators in the high speed version. a) The hysteresis for each of the actuators and b) the translation from all four actuators acting together are shown. The c) translation and d) angle change on the tilt motions are indicated along with the e) translation and f) angle change on the tip motions.

straight forward to control up to 2 kHz making it ideal for obtaining spectra at up to 2000 fps.

The mean squared coherence for the piezoelectric actuator is fairly low, averaging around 0.7, for the entire frequency range. In addition to external disturbances contributing at lower frequencies, actuator hysteresis contributes across the entire frequency range thereby making it difficult to model the system using only linear elements. These hysteretic responses can be seen in Figure 5.28 in different measurement configurations including individual translation, group translation, translation from combined tip and tilt actuation, and rotation angle from combined tip and tilt actuation. Each of these curves is measured by simply ramping the voltage on the actuator and measuring the corresponding displacement. The noise or wobble on each curve comes from the phase observer algorithm. Because a motor ramp is not used at the same time as the ramp on the piezoelectric actuators, the results will tend to include more estimation error. By identifying the statics and dynamics of each
actuator, it is possible to design appropriate control systems and ideal feed-forward elements from these models.

### 5.4 Feedback Controller Architecture

A feedback controller can be used to accurately position the mirror and increase the speed of response of the actuator. The controller architecture for a simple feedback system is shown in Figure 5.29. In this system, there is a feedback controller element \( C(s) \) which combines with the system or plant transfer function \( G(s) \). An additional feed-forward component can be used if something is already known about the system from system identification or from calibrations. In this section, the feed-forward component is relatively simple. More complex model-based feed-forward designs will be considered in the next section. The control system will also need to reject disturbances \( D(s) \) which enter the control loop after the system block. Other nonlinear blocks can be included such as input shaping blocks to improve the system control.

The canonical transfer functions for the true displacement as a function of the desired displacement and disturbance rejection are,

\[
\frac{z(s)}{z_{\text{des}}(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)} + \frac{F(s)G(s)}{1 + C(s)G(s)}, \tag{5.41}
\]

Figure 5.29: An overview of the feedback and feed-forward controller transfer function architecture is exhibited.
Figure 5.30: A full block diagram of the different components in the motor translation, tip-tilt control, and fast hybrid translation control schemes is laid out. This arrangement is valid for the series piezoelectric actuator topology and can be modified for the parallel piezoelectric actuator topology.

\[
\frac{z(s)}{D(s)} = \frac{1}{1 + C(s)G(s)}.
\]  

(5.42)

These simple relationships are then used to control different functions in the IFTS. The three main control functions are motor translation control, tip-tilt control, and fast hybrid translation control. The components involved with each of these systems are shown in Figure 5.30.

5.4.1 Translation Control

The motor control system is the simplest, involving a nonlinear input shaping block, a simple controller and motor drivers for the Lorentz force voice coil. Because the motion is mostly linear, only two of the four photodiodes are necessary and a fast position decoding algorithm located on the microcontroller can be used. The translation component of the full phase observer can also be used in positioning the mirror. Figure 5.31 shows the step control of the system with and without the nonlinear in-
Figure 5.31: a) The closed loop step response of a system with no input shaping is shown. b) The closed loop step response of a similar system with input shaping is shown allowing for less overshoot and higher proportional gains leading to better disturbance rejection.

put shaping block. In Figure 5.31a, the input shaping block is removed and the step response is shown with different controller parameters. A simple proportional and derivative (PD) controller is sufficient for obtaining good position control with minimal overshoot but does a poor job with steady-state errors. When integral control is used on the system (PID) to reduce steady state errors, the error at the beginning of the step accumulates in additional overshoot throughout the step response. This effect is undesirable and difficult to remove with traditional linear control.

There is a fundamental speed limit on the motor actuator because the ramping time on the actuator is limited by its dynamics. The rise time of under-damped second order systems from 0 to 100 % is,

$$ T_r = \frac{1}{\omega_n \sqrt{1 - \zeta^2}} \left( \pi - \tan^{-1} \left( \frac{\sqrt{1 - \zeta^2}}{\zeta} \right) \right), \quad (5.43) $$

where $\omega_n = \sqrt{k_t/m_t}$ is the natural frequency and $\zeta = b_t/(2\sqrt{m_tk_t})$ is the damping ratio. Although it is theoretically possible to push the motor actuator up to 400 or 500 fps based on the rise time, the actuator phase is very low at those frequencies as seen in Figure 5.21d. A single lead-lag compensator could be added at the cost of additional computational resources to boost the response of the system. Generally,
the best ways to improve the speed of an actuator are through design rather than through control. The use of hybrid control, discussed later in this section, will extend the frame rate much further while not sacrificing stability.

Taking advantage of the rise time of the actuator, a nonlinear input shaping block can be added to incorporate this speed limit with other filtering to generate a desired input that maximizes the speed while accounting for the limitations of feedback control. The results of incorporating the nonlinear input shaping block are shown in Figure 5.31b. The input shaping block can also be used to eliminate high frequencies in the input with additional filtering, which results in the rounded corners in the input. The input shaping block also uses a slightly less aggressive slope than that allowed by the rise time of the actuator, thereby reducing ringing and increasing controller stability. By using this type of input, the integral term does not accumulate too much error since the difference between the desired position and the measured position in closed loop are very small at all times. Using input shaping also allows the user to increase the proportional gain, thereby increasing the disturbance rejection. For the motors used in this design, the maximum frame rates that can be obtained are on the order of 100 to 200 fps with this controller.

5.4.2 Multi-Axis Tip and Tilt Control

The multi-axis tip and tilt control system was implemented to account for parasitic tip and tilt from misalignment or off-axis forces from the actuator. This effect is fundamentally small. Therefore, a very slow control loop can be used for tip and tilt compensation. The tip-tilt controller uses all the actuators in the system including the phase observer and the coordinate rotation system. The translation piezoelectric actuator is included in this system because the tip and tilt actuators have some parasitic translation associated with their motions that need to be compensated.
Figure 5.32: Low frequency tip and tilt control during a long translation is shown. a) The translation control is completed at 20 kHz while the tip and tilt control is completed at between 1 and 2 Hz by the user interface. The open loop response, the feed-forward only response and the closed loop response of tip and tilt control are shown.

Figure 5.32 shows tip-tilt control implemented with translation control. When the moving mirror displaces by 180 μm at 0.3 μm steps, the tip-tilt controller runs in the background and compensates for up to 30 degrees of phase angle in tip and 40 degrees of phase angle in tilt. While the translation controller runs at a 20 kHz control loop, the tip-tilt controller runs at a much slower 1.0 to 2.0 Hz on the user interface. By first running a calibration, it is possible to determine the amount of tip or tilt in the system. Then, a simple feed-forward gain can be used to account for the misalignment. On the other hand, closed loop tip-tilt controller can also account for this misalignment as well as reject any unexpected disturbances.

5.4.3 Fast Positioning Hybrid Control

The translation piezoelectric actuator can be used for small displacements at high speeds and is therefore useful for extending the dynamic response of the IFTS. The closed loop step responses of the translation piezoelectric actuators on the high speed version are shown in Figure 5.33. The largest contribution of the controller is the removal of the low frequency noise from air currents and table vibrations through the integral term. By using a shaped input, the actuator ringing can be reduced.
Figure 5.33: The closed loop step responses of translation in the parallel configuration piezoelectric actuator from the high speed version are shown.

Because the stroke of the piezoelectric actuators are very low, hybrid control is proposed for combining the dynamics of the piezoelectric actuator with the stroke of the Lorentz force voice coil. Because only the relative motion of the two mirrors can be observed, only a single sensor can be used for both actuators which means that input shaping becomes very important. The total desired trajectory of a single high speed step is shown in black in Figure 5.34a. This is created from the addition of the piezoelectric actuator motion and motor motion. Since the fast motion comes from the piezoelectric actuator, the initial motion must come from a fast ramp on the piezoelectric actuator. However, the piezoelectric actuator must return to zero so that it can repeat its motion for the next step. As the motion on the piezoelectric actuator decreases, the motion on the linear actuator must increase.

The curves in Figure 5.34a are created by generating the total desired trajectory first. This total trajectory takes into account the rise time of the piezoelectric actuator. Then the input shape is generated for the motor. By subtracting the two curves, the piezoelectric actuator’s shaped input is found. This design is implemented and the resulting trajectories are shown in Figure 5.34b.

With only motor input shaping, the position ramps up slowly. However, when
Figure 5.34: a) Input shaping and b) final trajectories for the high speed hybrid translation control scheme are shown combining the motions of the fast piezoelectric actuator with the long-stroke motor using only a single relative position sensor for both actuators.

In this design, the motor input shape is essentially a ramp and the piezoelectric actuator input is essentially a saw tooth. The measure position matches very well with the total desired trajectory. At lower stepping frequencies, in this case corresponding to 450 fps, the response matches the desired trajectory well. At higher frame rates, the two will begin to deviate more due to the motor ramp dynamics. The part of the curve in Figure 5.35b that matches the least is at the beginning near the first step and the third step in the sequence. This is due to the oscillations from the motor.
Figure 5.35: The multi-step input shaping algorithm for the high speed hybrid position control algorithm is shown with a) the shaped inputs and b) the resulting final trajectory. These results were obtained from the multi-axis version and correspond to image acquisition at 450 fps.

ramp response. This effect can be removed and the stepping speed increased with a model-based feed-forward system that takes the system dynamics into account when generating shaped inputs.

5.4.4 Disturbance Rejection

The disturbance rejection of the controllers can be simulated and experimentally tested. The disturbance rejection of the motor only control system, piezoelectric actuator only control system and the motor and piezoelectric actuator hybrid control system are shown in Figure 5.36. These experimental results were obtained by vibrating an external piezoelectric actuator at different frequencies. At each frequency, the amplitude of the response was recorded with the controller on and with the controller off. The two values are divided and the results are shown. The motor only response has good disturbance rejection up to about 200 Hz.

When the piezoelectric actuator is incorporated, the response ratio decreases significantly. With hybrid control the motor resonance still plays a part in the dynamics of the controller and has a similar resonance peak as the motor only controller. For the controller with only the piezoelectric actuator, the disturbance rejection is extended
Figure 5.36: a) Disturbance rejection simulation and b) disturbance rejection experiments were conducted with a large external piezoelectric actuator. Different control schemes were used to reject the external disturbance including the motor only controller, the hybrid control system with the motor and piezoelectric actuator, and the piezoelectric actuator alone.

In general, external disturbances come from 1/f noise and are generally fairly low frequency, well below 100 Hz. Sources of disturbances include temperature drift, external vibrations, air currents, and motions associated with a human holding or bumping the device. Human motion vibrations are in the range of 1 to 20 Hz which can be mostly rejected by these controllers. Additional passive rejection elements, such as sandwiched composites and vibration-absorbing thin foam layers can be used to further reject external disturbances for better handheld operation.

5.5 Full Inversion Feed-Forward

Model-based feed-forward controller design is a method of generating the optimal input for any desired output based on the known dynamics of the system. There are many different ways to accomplish this [23, 85]. Here, full inversion feed-forward (FIFF) is chosen and it is characterized by the block diagram in Figure 5.37. This implementation improves the step response dynamics of the actuator can produce the desired step or ramp response quickly. This can be used in a variety of cases to
improve position control of the motor and improve hybrid control.

More advanced control methods that are related to feed-forward control can also be considered here. Adaptive control is a method of determining the system model at the same time as providing feed-forward and feedback control, instead of doing these as two separate steps. It is useful if the system transfer function changes over time due to un-modeled effects like temperature changes. One such method is to combine stochastic system identification methods with feed-forward and feedback control. In this method, a stochastic signal in the form of a dither, is added to the desired position. As the system moves to different desired positions, the input and output data can be used to compute the dynamic transfer function of the system and this information can be used to update the system model in real time. The dither must activate the frequency modes in the actuator so that there is enough information to create an accurate model. The dither, however, may also cause the actuator to vibrate slightly, adding to the smearing in the image data. The real-time computational load of an adaptive control algorithm must also be considered. Since feed-forward control system can be computed off-line, it is an ideal addition for better control.

Figure 5.37: The full inversion feed-forward block diagram is shown indicating the feed-forward block, the low pass filter block and the rate filter block.
5.5.1 Position Control

In the FIFF design for position control of the motor, the input shaping block is replaced by a simple nonlinear rate filter. The feed-forward compensator block contains the full inverse of the identified model with an additional low-pass filter term to ensure causality. For example, identified system can be simplified to second order such that,

\[ G(s) = \frac{1}{m_t s^2 + b_t s + k_t}. \]  

(5.44)

The FIFF compensator block is therefore the inverse of the system with an additional low-pass filter with constant \( a_L \) representing the filter frequency,

\[ F(s) = \frac{m_t s^2 + b_t s + k_t}{(s + a_L)^4}. \]  

(5.45)

In order to make sure that the desired position matches the feed-forward input, a low pass filter with the same dynamics as the low-pass filter used in the feed-forward block is used,

\[ L(s) = \frac{1}{(s + a_L)^4}. \]  

(5.46)

In this paradigm, all of the desired trajectory information is captured by the feed-forward compensator, and the controller is only responsible for rejecting disturbances. This allows larger gains with lower oscillations and therefore better disturbance rejection. Bode plots of this design are shown in Figure 5.38. The original high order model is simplified to a second order model and a feed-forward compensator is designed for it. The feed-forward compensator has a notch that perfectly compensates for the resonance of the actuator and a low pass filter at high frequencies to help produce a causal impulse response and reduce the effects of high frequency noise. The total response, which is equivalent to the desired output, is simply the fourth order low-pass filtered result.
In order to calculate the feed forward compensation \( V(t) \) and low pass filtered result \( L(t) \) for a desired input, it is necessary to use the time domain versions of these blocks. There are several ways that it can be implemented including the use of IIR digital filters. An FIR digital filter was chosen instead because it is easy to convert from a time domain design. Because it can be pre-calculated, it is not necessary to use real-time computational resources for computing any of the FIFF profiles. Hence, there is no great benefit to using an IIR design.

The profile \( V(t) = K_0/G_F (N(t) * \tilde{F}(t)) \) can be calculated from the convolution of the output of the nonlinear rate filter \( N(t) \) with the impulse response for the feed-forward compensator. For the design obtained for the system \( F(s) \), the impulse response \( \tilde{F}(t) \) can be obtained from common Laplace transform tables [64],

\[
\tilde{F}(t) = k_t \left( \frac{1}{6} e^{-aLt} t^3 \right) + b_t \left( -\frac{1}{6} e^{-aLt} t^2 (aLt - 3) \right) + m_t \left( \frac{1}{3} e^{-aLt} t \left( \frac{1}{2} aLt^2 - 3aLt + 3 \right) \right),
\]

(5.47)

where \( G_F \) is the area under the curve \( \tilde{F}(t) \) and \( K_0 \) is the scaling constant in units of \( V/\mu m \). Similarly, the low pass filtered result can be generated from \( L(t) = 1/G_L (N(t) * \tilde{L}(t)) \),

\[
\tilde{L}(t) = \frac{1}{6} e^{-aLt} t^3,
\]

(5.48)

where \( G_L \) is the area under the curve \( \tilde{L}(t) \). By choosing this solution method, it is
clear that only causal impulse responses can be used. Additional benefits can possibly be gained from using the non-causal components of the impulse response. Although it is mathematically possible to obtain infinite fast responses with such methods, it is not practical to implement faster feed-forward responses. The fundamental reason is that the speed of the system response is limited by the frequency at which the system model begins to break down. Fast responses require good models up to the frequency of response. As seen in the system identification section, the model for the motor begins to break down before 400 Hz and the model for the piezoelectric actuator begins to break down near 5 kHz. Better modeling would lead to more complex dynamics and increase the difficulty of the computation. Therefore, the low-pass filter parameter \( a \) is carefully chosen to be less than the frequency at which the models break down, thereby ensuring performance and accuracy up to that frequency.

The resulting impulse responses are shown in Figure 5.39a. The simulated responses are also shown for a given nonlinear rate filter response. The feed-forward voltage in red is convoluted but the resulting feed-forward only response matches perfectly with the desired position, completely eliminating the ringing that was initially evident in the open loop position.
Figure 5.40: A comparison of input shaping with full inversion feed-forward techniques with open and closed loop responses on motor control is shown. The a) open loop with input shaping results show a lot of ringing while the b) closed loop with input shaping reduces much of this ringing. c) The open loop full inversion feed-forward is able to achieve good results with no ringing by predicting the perfect input profile. d) By adding closed loop to full inversion feed-forward, the system is able to reject external disturbances.

These techniques can be implemented on the motor position control and the results are shown in Figure 5.40. On the left side, the system uses only the traditional input shaping techniques in open loop and closed loop. By implementing full-inversion feed forward techniques, the open loop result almost perfectly matches the desired position. The scaled feed-forward profile, which matches the voltage sent directly to the actuator, is exactly the profile necessary for obtaining the desired position without feedback.

In fact, the feed-forward profile is exactly the same as the input to the motor if a perfect feed-back controller was implemented. This reflects the fact that for any desired output, there is only one input profile that can produce that particular output.
Figure 5.41: A comparison of the input shaping and full inversion feed forward techniques with open and closed loop responses on high speed hybrid control is shown. The input shaping results (a and b) show significant ringing from the long-stroke motor while the full inversion feed-forward implemented on the motor (c and d) is able to reduce ringing from the motor. The remaining error comes from the piezoelectric actuator and from external disturbances.

Therefore, the choice of controller type is not important if it is able to produce the desired output and reject disturbances. Figure 5.40d shows the system under FIFF with closed loop control. This produces additional ringing but is better at rejecting external disturbances. Overall, by using the FIFF algorithm, the ramp speed of the actuator was increased by 30 to 50% without changing the closed loop controller parameters.

5.5.2 Fast Positioning Hybrid Control

The FIFF technique can be applied further to hybrid control by correcting the motions of the linear actuator. The results for a single step are shown in Figure 5.41. The left column shows the traditional technique of using input shaping to generate the
desired profiles in open loop and closed loop. Note that the ramp on the motor is faster in this case than in Figure 5.34. When full inversion feed-forward is used, the input profile for the motor changes dramatically. Note that the piezoelectric actuator feed-forward profile is still equivalent to the difference between the desired position and the motor input shaping (and not the motor feed-forward profile).

The open loop profile of this technique has very little ringing and is in many ways better than the open or closed loop responses of the non-FIFF responses on the left. When closed loop control is added, the response has low ringing and a fast response time. The remaining high frequency ringing comes from the un-modeled frequencies in the piezoelectric actuator.
This technique can be further applied for multi-step hybrid control. For hybrid control at up to 2030 fps, simple input shaping profiles and resulting trajectories are shown in Figure 5.42a and b. At these speeds, the measured trajectory does not match the total desired trajectory well because of the ramp response on the motor. When the motor dynamics are taken into account, the FIFF inputs are shown in Figure 5.42c. The feed-forward input of the motor has an additional bump to account for the ramp response. The piezoelectric actuator feed-forward profile is raised to account for the motor lag during a ramp input. By using these profiles, the measured trajectory matches with the total desired trajectory very well showing little to no ringing from the motor dynamics. The profile can be further improved by incorporating the piezoelectric actuator dynamics.

5.6 Controller and Imaging Coordination

Coordinating the controller with the imaging sensor trigger is essential to obtaining accurate spectral data. Three different modes are considered including simple timing for a video rate 30 fps camera, timing for a high speed global shutter camera, and timing for a long integration camera with non-destructive readout. The design of the coordination algorithms allow the user to request blocks of images from $N_{frames} = 1$ image to up to $N_{frames} = 750$ images at VGA resolution (limited by the amount of RAM allocated to storing the incoming data). The positioning holding algorithms allow multiple blocks of images to be requested for a single test.

5.6.1 Video Rate Coordination

Coordinating video rate data with mirror positioning is relatively straight forward. The process is indicated in Figure 5.43. The algorithm starts by taking the camera reset low and clearing all the buffers on the FIFO and USB. Then the program starts
Figure 5.43: Coordination of camera timing and mirror positioning algorithms for a video rate 30 fps camera is shown. This figure shows how $N_{frames} = 3$ frames could be acquired from $N_{frames}+1$ clocking cycles.

by taking the camera reset high. The frame valid line is used as an interrupt trigger which starts the controller algorithm on the rising edge. On each subsequent rising edge, the controller will reposition the mirror. If the desired number of frames is $N_{frames} = 3$, the mirror is positioned 3 times and held in position until the next command.

Although the camera image is acquired at the same time as the positioning period, it is not clocked out until the next frame. Therefore, one extra frame $N_{frames}+1$ must always be acquired in order to obtain $N_{frames}$ of information. This algorithm is utilized for the TCM8230MD camera. This camera has a rolling shutter, making it difficult to be certain how the pixels in each frame coordinate with the frame valid rising edge or the moving portion of the control algorithm. Ideally, the moving portion of the control would not occur when the camera is integrating photons. This type of camera is not ideal for use with large step sizes or control algorithms with longer ramp portions, but would work well for other cases.

5.6.2 High Speed Camera Coordination

The coordination algorithms for high speed cameras with global shutter control can be much more complex. Figure 5.44 shows the controller and camera timing coordi-
Figure 5.44: Coordination of camera timing and mirror positioning algorithms for high speed cameras with global shutters is shown. This figure shows how $N_{frames} = 3$ frames could be acquired from $N_{frames}+2$ clocking cycles. For the lower speed applications $< 170$ fps, the mirror positioning can be accomplished by using the motor only. For higher speed applications, the hybrid control with the piezoelectric actuator can be used. This splits the mirror positioning between two actuators. This is highlighted in green.

In high speed modes, there is generally less time to do positioning, so it is important that the correct trigger is used for coordinating movement. In this case, the camera integration falling edge is used to coordinate the trigger because it occurs earlier than the rising edge of the frame valid trigger. While the camera integration is low, the ramp portion of the mirror positioning occurs so that when the camera actually starts integrating photons, the position of the mirror is held in place. For slower speed applications (frame rates $< 170$ fps), the linear motor is capable of providing fast enough steps. For faster frame rates, hybrid control is necessary and the positioning is split between the piezoelectric actuator and the motor.

As with the previous scheme, at least $N_{frames}+1$ images need to be clocked out in order to receive information for $N_{frames}$ images. In this situation, the first frame is dropped as well leading to $N_{frames}+2$ frames being acquired. There are two reasons
for this. First, the LUPA 300 high speed camera may continue to integrate photons even with the camera reset low causing the first frame to be noisy in some cases. Second, the high speed hybrid model requires that the piezoelectric actuator voltage to be slowly ramped (reset) when switching between the high speed controller and the slower hold position algorithm. When transitioning from the hybrid controller to the slower controller, this reset algorithm allows the steady state positioning errors to be taken up by the high stroke motor rather than being held by the short stroke piezoelectric actuator and potentially causing it to hit its positioning limits.

5.6.3 Long Integration Coordination

For low-light applications, the LUPA 300 has a non-destructive readout (NDR) function that can read out pixels of an image without clearing the charge on each pixel. This allows the pixels to continue to integrate photons over time leading to much longer integration times for fluorescence and Raman applications. The intermediate
intensities can be clocked out and used to extend the dynamic range of an image. For example, if one part of the image is bright and another part of the image is dark, the NDR function can be used to get grab the bright part of an image without it saturating and continue integrating pixels for darker parts of the image.

In order to use this function, the number of frames to integrate over must first be chosen, in the case shown in Figure 5.45, \( N_{int} = 3 \). The algorithm starts with one dropped frame followed by the NDR function being turned on via changing the camera settings via the SPI port. Then a series of images are clocked out before the NDR is reset for the next set of images. The images that can be used include every \( N_{int} \)-th frame. If dynamic range extension is desired, other intermediate images can be used. The final number of frames that needs to be acquired for this algorithm is \( N_{frames}N_{int}+2 \).

5.7 Blur Considerations

Proper integration of the controller with the imaging is important for obtaining accurate results. Several different situations were tested including step size, integration location, controller type and frame rate to determine the effect of each on the measurement accuracy. Preliminary tests were also conducted on the effect of human motion on readings taken with the spectrometer.

5.7.1 Timing and Step Size Effect

The accuracy of measurements depends heavily on step size and the quality of the controller. To illustrate this, the control system was tuned such that the system allowed up to 20% overshoot. Several different step sizes and integration locations were tested. The first type of integration location is early/fast integration where the integration period happens directly after the ramp component of the step. The
Early Camera Integration (Fast Acquisition)

Late Camera Integration (Slow Acquisition)

b 250 Fast/Early Acquisition

--.

1 pm steps --

0.2

n steps

200

0.5 pm steps

-+1.0

AM steps

150 --. 5

m steps

-2.0 IpWAm

steps

100

4.7

- 0O-1

pm -

-.2 -0.2 JAM step

42

0.5 1" step

50

3.7 - 1.0

p~m

sy

3.2

Ps

2.7

2 4 6 8 10Position (srn)

2.2350

Slow/Late Acquisition

1.7-----35

1.2

300

0.7

250-- --

0.2 4 200

0.01

0.03 0.05 0.07 0.09 0.11 0.13

Pm

1150 -0.2

Pm steps

0.5

Time (s) 110

0.s

pm steps

.. 0

1.0

pm steps

so -e1.

5

pmn

steps-0-.0IAm

steps

0

2 4 6 8 10

Position (sIm)

Figure 5.46: The effect of step size and integration location on the accuracy of signal measurements is tested for a 850 nm signal. a) The step sizes are varied, each with 20 % over shoot, and the relative location integration times is shown. The resulting magnitude of the measurements is indicated for b) fast or early acquisition and c) slow or late acquisition.

second type of integration location is slow/late integration, where the integration period happens later. The results of these tests are shown in Figure 5.46.

For fast or early acquisition, the larger steps that are about 2 μm will tend to have larger overshoot up to 0.2 μm. This causes the interferogram to smear. In Figure 5.46b, it is clear that the larger steps will have lower measured intensity variations as an effect of this blurring process. For slow or late acquisitions, the error between the measured and desired positions is much lower leading to less blurring in the measurement. This means that the magnitude of the signal is recovered more accurately.

Figure 5.47a shows the how the peak range drops significantly for fast acquisition as the step size increases but remains relatively steady for slow acquisition. For step sizes around 0.2 μm, this effect is minimal and integration location can be ignored. As the step size increases, the standard deviation of the difference between the desired
Figure 5.47: a) The peak range accuracy as a function of step size is shown for the two acquisition methods. b) The positioning error (blue) and measurement error (red) are plotted as a function of step size. c) The spectra for the slow and fast acquisition methods are shown on normalized plot for 2 \( \mu m \) steps. d) The images taken during spectral acquisition of a laser spot are shown with the fast method producing blurring of the dark/bright transitions in the interferogram.

position and the measured position stays about the same but the error of measured signal increases quickly. Figure 5.47c shows how the spectral measurement can also be affected by integration location with 2 \( \mu m \) steps. The intensities are scaled to show that, although the wavelength is accurate, the signal to noise ratio is better for the slow/late acquisition and there is less peak broadening.

This blurring effect can also be seen in the images taken with the spectrometer. In Figure 5.47d, two representative images from the spectral acquisition are shown. The spectrometer mirrors are tilted slightly so that the laser light source shows several different interferometric bands. Ideally, the bands would show high contrast as indicated by the slow acquisition image. However, when fast acquisition is used, the image is smeared and the dark to bright transitions do not show up clearly.
5.7.2 Controller and Solver Effect

For high speed acquisitions, the step size, type of controller used and solution method also have an effect on the data accuracy. Figure 5.48 shows the effect of these parameters for sampling at 2010 fps. For small steps on the order of 0.2 μm, the type of controller and the solution method do not heavily affect the measured data. At step sizes of 1.0 μm, the motor only control method produces extra peaks in the data that are not present when hybrid control (motor and piezoelectric actuator) is used.

The extra peaks in the data can also be eliminated by calculating the Fourier transforms using the measured positions rather than using the equally spaced desired positions. Since the motor only controller cannot accurately position the mirror at 2010 fps, the measured position and desired positions are very different. By using a

![Graphs showing the effect of step size, controller time and solution type on spectrum accuracy.](image)

Figure 5.48: The effect of step size, controller time and solution type on spectrum accuracy is shown for 850 nm laser light data acquired at 2010 fps. The left column (a and b) represent small step sizes of 0.2 μm while the right column (c and d) represent large step sizes of 1 μm. The top graphs (a and c) utilize only motor control while the bottom graphs (b and d) utilize the hybrid controller that use both the motor and the piezoelectric actuator.

203
least squares solver or any other solver that is capable of incorporating the sampling location, the error due to positioning inaccuracies can be removed.

5.7.3 Human Disturbances

One of the important aspects of a small imaging spectrometer would be for handheld applications, where either the spectrometer or the source being tested may be held by a human. Humans are not good at maintaining accurate positions and are a source of vibrations. Therefore, it is important to consider the effect of human disturbances on the accuracy of measurements. For preliminary tests of this effect a large laser light source that has a projected size of approximately 100 pixels in diameter and contains 3 to 5 wavelengths of tilt at 850 nm is used. The projected image is similar to Figure 5.47d. Reference data are obtained at both 78 fps and at 2010 fps. Then tests are conducted with a human subject holding the spectrometer and then the human subject holding the light sources. The results are shown in Figure 5.49.

The measured human vibrations are on the order of 10 to 15 Hz, which is easily rejected by the position controller but cause significant error in the readings taken at 78 fps due to the movement of the image across the sensor. This also leads to less accurate results when the spectra are calculated. It is clear in Figure 5.49b that the handheld results are much less accurate than the reference data and exhibit much broader peaks. When sampled at 2010 fps, however, the human vibrations can be seen directly as oscillations in the interferograms. The resulting spectra are also more similar to the reference. When data are taken at faster frame rates, the effect of human motion is smaller and the resulting data are more accurate.

Figure 5.50 shows the average deviation from the ideal spectrum for the different cases sampled at different frame rates. Five tests were conducted for each handheld case. It is clear that the error for data sampled at 78 fps is higher than the data
Figure 5.49: The effect of frame rate on the accuracy of spectral data is explored. The images on left (a and b) are from readings taken at 78 fps. The images on the right (c and d) are from reading taken at 2010 fps. The top graphs show the raw interferograms while the bottom graphs show the resulting spectra.

Figure 5.50: The average deviation from an ideal spectrum is calculated for each handheld case at 78 fps and 2010 fps. Five tests were conducted for each handheld case.
sampled at 2010 fps. This information, however, is only applicable to the case were a large, coherent source is used. In the most extreme case, the light source may only occupy one pixel in the image and it would then be impossible to hold its position relative to the spectrometer by hand. Ideally, in that case, image tracking and machine vision methods would be used in conjunction with high speed acquisition to reduce the effect of human motion on the accuracy of spectral data. Note that there is an inherent difficulty with using traditional edge tracking algorithms as the edges of the interferograms will tend to move from image to image for some experiments. For situations where there is no tilt, the image brightness will instead change from image to image. These difficulties will need to be overcome with any image tracking methods used for position tracking in imaging Fourier transform spectroscopy.
Chapter 6

Imaging Fourier Transform

Applications

There are several real world applications for the IFTS. The simplest uses would be to map the color of an array of different light sources like lasers or LEDs. Measurements of light reflection, absorption and transmission can be made without the use of pinholes or fiber optics. With the appropriate input light, the IFTS can be used as a white light interferometer or a blood oxygen imaging sensor. With a microscope objective, the IFTS can be used to measure fluorescence for sample characterization. The imaging capabilities of the IFTS allow measurements to be made from the entire image at the same time and allow many custom filtering options to be done in software. The IFTS can also be used for Raman imaging measurements provided that the input light intensity is sufficiently high.

Because the system is inherently an interferometer, there are many other possible applications. These include using the system as a femtosecond or picosecond pulse beam imaging device. Different imaging sensors can also be used to make infrared or ultraviolet imaging measurements.
6.1 Intensity Measurements

Measurements of known sources are important for assessing the characteristics of the instrument. In particular, the location of different peaks should be checked against known sources. In addition, the transmission characteristics of the camera are important. By using known incandescent sources, the response of the sensor can be corrected.

6.1.1 Input Light and Filters

Several different types of results can be obtained from this imaging spectrometer. The spectrometer takes a single image at each spectral position and each of the pixels on this image can produce an interferogram when a series of images are taken as shown in Figure 6.1a.

This figure shows the intensity of three different pixels in a series of 100 × 100 pixel images as the position is changed. One of these images is shown in the inset of Figure 6.1a. The three pixels chosen are from an incoherent green LED (green dot), an 850 nm laser (purple dot), and the ambient light (white background). The resulting

![Figure 6.1: a) The interferometric measurement and b) the spectrum of an 850 nm laser, a green LED, and background fluorescent light showing the two major peaks. The data were acquired with the TCM8230MD camera.](image-url)
spectra from the Fourier transform of the information are shown in Figure 6.1b. The expected broad peak from the green LED at 570 nm is shown along with the sharp peak for the 850 nm laser. The ambient light comes from fluorescent lighting and shows the two main peaks of fluorescent lights at 544 to 546 nm (terbium and mercury peaks) and 611 nm (europium peak) [145].

The types of spectra that can be obtained with the imaging spectrometer can include light sources with low coherence such as white LEDs or incandescent lamps as shown in Figure 6.2a. The white LED clearly shows the blue LED peak and the broader, yellow phosphorescent peak. Depending on the pixel chosen, the relative intensity of those two peaks may change as some parts of the image will include more of the blue LED and some parts of the image will include more phosphorescence. The incandescent light also shown in this image does not look like the traditional curve that is obtained for thermal sources. This is because the TCM8230MD camera used in this version of the IFTS has a built-in hot mirror with a cutoff before 700 nm. Strong lasers at longer wavelengths can penetrate this filter and the sensor has been used to measure wavelengths up to 1064 nm.

In addition to measuring pure light sources, fluorescence samples can also be
Figure 6.3: a) A series of long pass filters and band pass filters are measured using a white LED source. b) A series of colored glass filters are measured using a white LED source. The data were acquired with the TCM8230MD camera.

measured. Figure 6.2b shows the input light at 532 nm and the light emitted by the fluorescence of a sample dyed with propidium iodide with a peak at 615 nm. The true tail of this fluorophore extends well beyond 700 nm but cannot be picked up effectively with this particular sensor.

Figure 6.3 shows the response curves of several different filters when a white LED is used as the source. The measured responses are very similar to what is expected especially for the long pass filters. The 475 nm band pass filter has a better response near 450 nm due to the lack of input light near 475 nm. The same is true of the blue colored glass filter, which appears to be split between a peak at 450 nm and a broader peak near 550 nm. The noise levels on some of the signals is relatively high (near 20 %) as some of this data were taken with the prototype version of the IFTS before many controller modifications were developed.

Despite the limitations of the TCM8230MD camera, it can be used effectively for measuring many different phenomena in the visible wavelengths, especially for lower cost applications. The camera has a built-in Bayer mask system that can capture the color of different sources making it easier to align in some situations. For extended wavelength sensitivity and a higher number of bits per pixel, the LUPA 300 camera is a more ideal sensor.
6.1.2 Compensation Curves

Although the LUPA 300 camera does not have a built-in infrared filter, it does have a non-uniform response as a function of input wavelength. This is shown in Figure 6.4b. Because of this oscillatory response, a compensation curve must be used on all the measurements made with this sensor. The calibration curve is created by first measuring the interferogram of a known broadband light source, such as a Quartz Tungsten Halogen bulb. Here 700 points are taken with a sampling interval of 200 nm. The temperature of the bulb can be calculated and estimated in order to create a true curve from Plank’s law. This procedure is shown in Figure 6.4a.

Next, the ideal curve is divided by the measured lamp data curve to create a derived response for the LUPA 300 camera as shown for the red curve in Figure 6.4b. It is clear the overall response is very similar to the curve provided by the manufacturer [121] although the peak locations are slightly offset. The noise level of measurements made with this sensor is also lower. Thermal lamp calibrations are typically sufficient for wavelengths above 500 nm. For the region below 500 nm, different sources are needed for proper calibration. Typically, ultraviolet deuterium arc lamps are used for calibrations in the region between 200 nm to 500 nm. For most of the experiments,

![Figure 6.4: a) The intensity of the LUPA 300 high speed camera can be compensated for by using a halogen thermal lamp. b) The calculated compensation curve can be compared to the compensation curve provided by the manufacturer [121].](image-url)
including fluorescence and Raman experiments, only the thermal compensation curve is needed.

6.2 White Light Interferometry

Full image analysis can also be used to determine the characteristics of the interferometer. White light interferometry is a method which uses a low coherence white light to measure the delay characteristics of components within different arms of interferometer. This can be used to measure mirror tip, tilt, surface shape, and flatness. This method can also be used to measure the phase delays imparted by reflective objects that are placed in the arms of the interferometer.

In Figure 6.5, a white light source is placed over the entire imaging area with a diffuser and 400 points of the resulting spectra are measured. The data from three pixels at different positions in the image are shown in Figure 6.5a. In this figure, the spectra do not line up perfectly on the position axis because the mirror is slightly tilted and each point on the image has different center point on the interferometer. By cross-correlating the spectra of different pixels in the image, it is possible to show how far the offset of each pixel is with respect to the others. For the full image, the

![Image]

Figure 6.5: The IFTS can be used as a white light interferometer. Here the a) interferogram, b) white light image, and c) calculated offset are shown. The purple spot in the white light image and the blue spot on the calculated offset are from a specular reflection from the 850 nm positioning laser. The data were acquired with the TCM8230MD camera.
overall offset from corner to corner of Figure 6.5c is about 1 \( \mu m \). A specular reflection from the positioning laser can be seen on the camera as a purple dot resulting in an area where the mirror offset is not well predicted.

### 6.3 Color Measurement

Interferometric data are inherently black and white but can be processed to produce color images by creating filters that integrate intensity in different areas of the spectrum. This can be used to create false color images, where colors are assigned arbitrarily, or to create true color images, where an attempt is made to recreate the real color of an image as seen by a human. The underlying spectrum, however, contains much more information. For example, a purple LED may be a single output at a wavelength near 400 \( nm \), or it may be a combined output of two different, longer wavelengths, blue and red.

This section covers the results of these experiments when the final full spectrum is used for color filtering. This means that a series of several hundred black and white images are used to create one high spectral resolution color image. Section 7.5 covers a different digital filtering technique where several high spectral resolution color images can be obtained from the same number of black and white images.

#### 6.3.1 Narrowband Laser Light

Three different laser wavelengths can be false-colored into the RGB color space by assigning a different color vector to each wavelength. In this case, a 532 \( nm \) laser is assigned to blue, an 808 \( nm \) laser spot is assigned to green and the 850 \( nm \) laser is assigned to red. Once the spectra are calculated and the filters are applied to each pixel, a final image can be built and is shown in Figure 6.6b. The raw black and white image has three distinct circles each with lines across the spot indicating
Figure 6.6: The light from three different lasers can be processed as different colors from a) 78 fps black and white data. b) The false color image shows an orange spot for 850 nm laser, a green spot for the 808 nm laser and a blue spot for the 532 nm laser. c) The spectra for the input lasers are shown. The data were acquired with the black and white LUPA 300 camera.

The interference from a source that is tilted relative to the interferometer and an interferometer that is not at its center alignment. These black and white transitions will move as the mirror is positioned and the frequencies at which those lines move relative to the mirror movement determine the wavelength of the light for that laser spot.

This particular set of data comes from a series of 500 images taken at 78 fps. The noise level for this data is especially low. Due to the variation in peak magnitudes, the image brightness and contrast need to be adjusted using tone mapping techniques such as gamma compression. Although there are clearly three different laser color spots, the spot on the right is orange and the green spot is slightly yellow. This is because the two spectra overlap slightly so that the red spot has some amount of green in it and the green spot has some amount of red. If the two lasers were at wavelengths that were further apart, the colors would be more distinct. The direct spectral integration method does allow for color overlap which makes it ill-suited for classification purposes but does a good job of representing the true overlap of different wavelengths in the image for determining concentrations and ratios.

The data set in Figure 6.7 was obtained from 500 images sampled at 2010 fps.
Figure 6.7: The light from three different lasers can be processed as different colors from a) 2010 fps black and white data. b) The false color image shows an orange spot for 850 nm laser, a green spot for the 808 nm laser and a blue spot for the 532 nm laser. c) The spectra for the input lasers are shown. The data were acquired with the black and white LUPA 300 camera.

Due to the high speed positioning, there are some more artifacts and slightly higher noise that appear in the spectral data, especially for the 532 nm spot. The image format is also smaller for this frame rate and the laser spots therefore appear larger.

6.3.2 Broadband LED Light

Broadband light can also be measured using the IFTS. While the mirror alignment for measuring narrowband laser light is more forgiving, the mirror alignment for broadband light is much less forgiving, since interference for broadband light only appears near the center of the interferometer. Therefore, almost no dark and bright bands will appear across the image to indicate the interference pattern. Instead, the entire image will go from dark to bright.

Figure 6.8 shows an array of color LEDs that are measured using the spectrometer. This series of LEDs spans the visible range from purple though blue on the first column on the left, followed by a green to yellow column, a yellow to red column and finally a white LED column with different temperatures with an infrared LED at the top right corner. Table 6.1 shows the manufacturer supplied wavelength specification and white LED temperature specification for each LED. A series of 200 color images are
Figure 6.8: The spectra from different colored LEDs can be measured with the IFTS. The a) test array, b) the image captured by the IFTS, and c) the spectra calculated using the IFTS are shown for four of the LEDs. The IFTS data were acquired with a TCM8230MD camera with a Bayer mask.

Table 6.1: LED calibration plate arrangement and manufacturer provided wavelengths and specifications

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Peak Wavelength</th>
<th>Column 2</th>
<th>Peak Wavelength</th>
<th>Column 3</th>
<th>Peak Wavelength</th>
<th>Column 4</th>
<th>Peak Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Violet</td>
<td>460 nm</td>
<td>Blue</td>
<td>472 nm</td>
<td>Red</td>
<td>660 nm</td>
<td>Infrared</td>
<td>830 nm</td>
</tr>
<tr>
<td>Pink</td>
<td>440 nm</td>
<td>Aqua</td>
<td>505 nm</td>
<td>Red</td>
<td>630 nm</td>
<td>White</td>
<td>3100 K</td>
</tr>
<tr>
<td>UV1</td>
<td>400 nm</td>
<td>Green</td>
<td>525 nm</td>
<td>Orange</td>
<td>605 nm</td>
<td>White</td>
<td>4800 K</td>
</tr>
<tr>
<td>UV2</td>
<td>380 nm</td>
<td>Yellow</td>
<td>588 nm</td>
<td>Yellow</td>
<td>595 nm</td>
<td>White</td>
<td>13850 K</td>
</tr>
</tbody>
</table>

taken by the spectrometer, converted to black and white intensity levels and lastly Fourier transformed for each pixel. In positions 1 through 4, which show red, orange, green and blue LEDs, respectively, the spectra can be obtained and are shown in Figure 6.8c. These four LEDs in particular have spectra that are clearly a single broadband wavelength and match well with the values specified in the table.

The Fourier transform information preserves the color information and can be reconstructed as a color image by mapping the spectra of each pixel into red, green and blue color bins shown in Figures 6.9a, b and c. These color bins can be remapped into a RGB image and the intensity and saturation levels can be manipulated to create the reconstructed color image shown on the right of Figure 6.9e. For clarity, pixels with light intensities below a threshold are colored black. This figure matches well
Figure 6.9: The color image can be reconstructed from the black and white intensity data by integrating light in the a) blue, b) green, and c) red areas in the interferogram. d) The information can be combined together for a color image. e) The saturation and intensity can be adjusted until the final processed color image is created. This can be compared to the original color image obtained on the TCM8230MD camera on the IFTS.

Figure 6.10: a) An image can be created by plotting the peak wavelength for each pixel in the image. b) This is compared to the color image obtained by the TCM8230MD camera on the IFTS.

with one of the original Bayer masked color image obtained from the spectrometer in Figure 6.9f. Because of the filter inside the camera, the light from the infrared LED at the top is not visible.

The spectral data can also be organized in other ways. Figure 6.10 shows the
image produced when the color of the pixel is organized by the location of the peak spectral wavelength with a total of 55 different color bands available. From this second image, it is clear that the white LEDs have a peak wavelength in the orange. The LEDs that are pink or violet have peak wavelengths that are located in the orange or red. This may be because purple and pink LEDs are typically created by combining several LEDs, using phosphors, or simply utilizing colored plastic. The ultraviolet LEDs also have peaks in the orange range. Even from the visible image, it is clear that the ultraviolet LEDs show up more blue or white. One of the reasons for the color response may be due to the filtering of the Bayer mask on the camera causing more of the orange components of these LEDs to be measured rather than more of the blue components.

6.4 Blood Oxygen Levels

Aside from measuring the properties of different light sources, different input light designs can be created to measure properties of materials in transmission. In this example, the IFTS is used to measure the blood oxygen content of the finger by comparing the oxygenation levels of the blood at two different wavelengths (e.g., 650 nm and 980 nm). Figure 6.11a shows the input design for such a measurement. The two wavelengths enter the finger and each wavelength will have a different transmission depending on the concentration of oxyhemoglobin versus deoxyhemoglobin.

The relative blood oxygen saturation can be calculated by comparing the intensity of light transmitted at both wavelengths \( I_1 \) and \( I_2 \) with respect to the transmission when the finger is not in the setup \( I_{10} \) and \( I_{20} \). The ratio is defined as [154],

\[
R_I = \frac{\log_{10} (I_1/I_{10})}{\log_{10} (I_2/I_{20})}. \tag{6.1}
\]
Figure 6.11: a) The design of the blood oxygen measurement input optics are shown along with b) the molar extinction coefficient for oxyhemoglobin and deoxyhemoglobin [154].

This ratio can then be used to calculate the blood oxygen saturation if the absorption coefficients of deoxyhemoglobin $\alpha_{r1}$ and $\alpha_{r2}$ are known with respect to the absorption coefficients of oxyhemoglobin $\alpha_{o1}$ and $\alpha_{o2}$ using

$$SaO_2 = \frac{\alpha_{r2} R_I - \alpha_{r1}}{(\alpha_{r2} - \alpha_{o2}) R_I - (\alpha_{r1} - \alpha_{o1})}.$$  \hfill (6.2)

This method is commonly used to measure blood oxygen content through pulse oximetry. A pulse oximeter uses two LEDs that are placed close to one another to measure not only the relative blood oxygen but also the pulse rate of the individual. The blood oxygen content will tend to change as a function of the heart rate because each beat will bring in fresh blood leading to more light absorption and fresh oxygen. Compensating for the pulse rate can lead to more accurate readings. A pulse oximeter can be placed anywhere on the body where there is a sufficient blood supply and where the tissue is thin, such as the finger or the ear lobe.

There were several ways that this experiment could be set up, including using LEDs or broadband near IR sources. Lasers at the two different wavelengths were chosen for the initial experiment because it allows for higher transmission intensities.

For the preliminary study, the finger was tested with free blood flow and with blocked blood flow for less than 30 seconds. For the blocked blood flow case, a
Figure 6.12: The oxygen saturation ratio is shown for a finger with a) free blood flow and b) with blocked blood flow for less than 30 seconds. One of the raw black and white images taken from the IFTS is shown in the inset. c) The measurements taken from several trials are compared for free and blocked blood flow. The data were obtained using the multi-axis version.

thick string was tied around the finger at the joint to temporarily restrict blood flow. Results measured with the IFTS were calibrated against a commercial pulse oximeter (ChoiceMMed MD300C1). The experimental procedure was approved by the Committee on the Use of Humans as Experimental Subjects (COUHES) at MIT.

The results of the experiment are illustrated in Figure 6.12.

The gray inset of this figure shows one of the raw images acquired from the interferometer. The image shows the tip of the finger and part of the nail. A total of 300 images were taken for each experiment with a step size of 300 nm. Some post filtering is used to set areas with low intensities to black and to smooth out the image. Ring patterns on the image are caused by the shape of the interference pattern due to the diffraction of the laser light. More collimated input beams and better imaging lenses would help eliminate these patterns.

Typical images of the calculated blood oxygen saturation ratios are shown in Figures 6.12a and b. These images also show the finger and part of the nail. It is clear that the finger with free blood flow has a higher average oxygen saturation ratio than the finger with blocked blood flow. The results of several different tests are shown in Figure 6.12c. The error bars indicate the standard deviation of the signal.
across the entire image. It is clear that the blocked blood flow is distinct from the free blood flow.

There are several ways in which the error bars could be reduced. First, the image readings can be synchronized to the pulse of the individual. Second, the calibration constants were assumed to be constant across the entire image. Because the light beams are not uniform, more accurate results could be obtained from calibrating each point in the image separately.

Imaging spectrometers make several interesting experiments possible. With this simple setup, it was possible to look at the blood oxygen content of the human finger and see how it may be distributed. In this case, the blood oxygen is fairly evenly distributed. With higher input intensities, it may be possible to look at the blood oxygen concentration of veins versus arteries in the finger.

6.5 Fluorescence Applications

Fluorescence today is an important tool in biological imaging that can be used to distinguish different parts of a cell or different cell processes through staining. Fluorescence is a process where an input light (laser or broad band light) provides energy to the fluorophore. If the energy level is in the absorption range of the fluorophore, the material will fluoresce, allowing some of the input energy to fall down different energy levels before reaching an energy level which gives off photons of a different, longer wavelength. The energy levels which produce visible photons tend to be broad and unique. These can be used to distinguish between different fluorophores that are either added as a dye or are already inside the tissue as autofluorescence [22, 107, 111]. The lifetime of the fluorophore and the energy transfer between fluorophores can be used to provide more information about the system [42, 126, 141].

Depth resolved fluorescent imaging is typically executed with confocal scanning...
Figure 6.13: a) The input design for a fluorescence input is shown. b) The measured interferogram may be stable or may change over time due to input laser instabilities and fluorescence bleaching.

Techniques that would be incompatible with an imaging spectrometer with this topology. Structured light methods [45] which include methods like HiLo imaging [12, 173], however, are compatible with these imaging spectrometers and can be used for depth resolved imaging.

In order to make fluorescent measurements, a simple input design was created with a single wavelength laser, a notch filter and a microscope objective as shown in Figure 6.13a. This setup is in an epi-illuminated arrangement and uses the input light from the 532 nm laser and reflects the light off of an OD6 notch filter (532 nm center wavelength, 17 nm FWHM, Edmund Optics 86-125) onto the sample. The returning fluorescence is able to pass through the notch filter where the input light is again reflected. The IFTS can then be used to image the remaining light.

Fluorescence results depend heavily on the input light intensity and can oscillate if the input light is not perfectly stable. The intensity of the fluorescent material will also tend to drop over time as it quenches. Most commercial fluorophores are more stable over time as long as the input light intensity is not too high. Some of these effects can be seen in the interferogram of Figure 6.13 with Dragon Green used as the fluorophore.
Because the oscillations in the interferogram are typically lower frequency, the FFT of the data can separate out the lower frequency oscillations from the more important fluorescent data. It is important to note that if the readings oscillate or the light intensities fall over time, the choice of the solution type for completing the Fourier transform becomes more important. A straightforward FFT will provide the proper result. However, an under-sampled spectrum using the LS algorithm or an adaptive algorithm that does not include the low frequency components cannot be used directly in this case. Including the low frequency components can still make these algorithms viable. Alternatively, the interferogram data could first be corrected with some form of moving average filter before more advanced algorithms can be used. Further discussion of these algorithms is in Section 7.2.

Figure 6.14 shows the fluorescence spectrum from three pixels in an image of the fluorophore Alexa Fluor 555 from Life Technologies. The interferogram can be corrected and the raw spectrum is shown in Figure 6.14b. This raw spectrum can be intensity corrected using the correction curves for the LUPA 300 camera and the final compensated and filtered spectrum is shown in Figure 6.14c. The readings from the three different pixels match very well with the reference spectrum (black) provided by the manufacturer [104].

Once a clear method is established for measuring the spectrum, different images of calibration polystyrene beads can be taken. Beads are placed in solution, mixed with mounting solution (DakoCytomation faramount aqueous mounting medium) and pipetted onto glass slides with cover slips. Some of these are shown in Figure 6.15. In this figure, there are two types of beads, one type is a larger 41.5 µm bead that is dyed with Dragon Green provided from Bangs Laboratory, Inc. while the second is a smaller 10 µm clear polystyrene bead that is set inside free Alexa Fluor 555 dye forming the background.
One of the 700 raw black and white images is shown on the left. The data for the images are processed to the produce spectra and simple spectral integration ranges are used to produce color images. It is clear from these images that the fluorescent beads look very different from the arrangement with a fluorescent background. Not only do the colors appear different for the fluorophores even though the same filter designs were used, but the image structures are different. The fluorescent beads have a dark background while the fluorescent background has a much brighter colored background. The lighting for the fluorescent bead is internal to the bead while the lighting for the fluorescent background appears backlit, showing dark areas inside the beads.

The spectra for the two different dyes are shown in the figures on the right. The dragon green fluorescence is much broader starting from 500 nm up to about 600 nm,
Figure 6.15: Fluorescent beads are assessed in the IFTS. The 41.5 μm dragon green fluorescent beads are measured showing the a) raw black and white image data, b) the processed color image, and c) the resulting spectrum. Here, the notch filter is visible. The 10 μm beads in a fluorescent background with Alexa Fluor 555 are shown with d) raw black and white image data, e) the processed color image, and f) the resulting spectrum.

having a light green and slightly bluish color. There is also a notch in the center of the graph which indicates the area of the spectrum that was filtered out by the notch filter. The Alexa Fluor 555 spectrum ranges from around 540 nm to 630 nm and appears much more yellow or orange. The integration areas in this case make it appear more red.

The IFTS system inherently allows the design of custom color filters to be applied on the measured data. This gives the user more choices for the types of fluorophores that can be used. Instead of designing experiments with fluorophores that are non-overlapping, it is possible to use overlapping fluorophores and post processing techniques to further separate the different fluorophores. This gives a distinct advantage in that the full image with multiple colors can be acquired with a single input laser (rather than multiple different lasers). In addition, the user does not need to purchase special filters for each fluorophore range. This makes the imaging interferometer
system much more flexible for many possible applications.

In order to test this method, two fluorophores were selected with similar, overlapping fluorescence profiles. Tetramethylrhodamine (TMR) and Envy Green are two fluorophores from Bangs Laboratory, Inc. used in this experiment. The former is used for calibration in cell counting setups and the latter is used for calibration in fluorescence microscopes. Both beads are 10 μm in size. The two fluorophores are combined by centrifuging the samples and removing the liquid from the top. Equal amounts of each fluorescent bead are combined into a single solution and mounting solution is added. Lastly, a drop of this mixture is placed on a microscope slide with a cover slip.

The image of the material taken with a conventional camera is shown in Figure 6.16a. The beads are very similar in color, one is slightly green and the other is slightly more yellow. The measured spectrum from the IFTS for each bead is shown in Figure 6.16b. The TMR fluorescence is slightly broader lending to a more yellow color. The IFTS can also be used to distinguish between the two different color beads. One of the 700 raw images is shown in Figure 6.16c and the true color processed image is shown in Figure 6.16d. It is clear that the processed color image from the IFTS matches the color of the conventional camera very well.

Since the two fluorophores are so similar in color, it is sometimes difficult to distinguish between the two depending on lighting conditions and reflections. Therefore, a separate algorithm is developed. In this algorithm, the spectrum between 590 to 630 nm is scaled by the spectrum at 575 nm, and this value is used to determine how much of the pixel is assigned a color of green versus a color of red. The resulting image is shown in Figure 6.16e. This image makes a much clearer distinction between the beads that are TMR versus Envy Green. Figure 6.17 shows the distribution of the pixels as a function of the distinction ratio of the two fluorophores.
Figure 6.16: The fluorescence of two similar fluorophores, Tetramethylrhodamine (TMR) and Envy Green, can be separated by using custom post-processing optical filtering algorithms. a) The image taken with a conventional color camera is shown. b) The spectra for the two fluorophores measured by the IFTS are compared. c) The raw black and white image data are processed into d) the true color image and e) can be further processed with custom post-processing color filters to obtain the false colored image with more contrast between the two types of beads.

at several different intensity limits. For high intensity magnitudes, the distribution has two clear peaks indicating the two different types of beads. It is also possible to get a concentration of the two fluorophores from this method. Some of the pixels in the image are slightly green transitioning into red. This does not indicate that the bead is two different fluorophores. Rather, it indicates that there are reflections on a green bead of some of the red fluorescence.

More advanced algorithms [126] are necessary for measuring relative concentrations of different fluorophores that overlap. One of these methods is called linear unmixing [150]. It utilizes a fitting method where the spectrum is acquired for at least
Figure 6.17: a) The intensity magnitude of the spectroscopic image for Envy Green and TMR beads is shown. b) The distribution of the distinction ratio is shown indicating two peaks, one for the Envy Green bead and one for the TMR beads.

N+1 points if there are N different fluorophores to be fit. Ideally, all the fluorophore responses are not linear combinations of any of the other fluorophores, allowing the fit to be unique.

There is one important limitation to this technique related to the relative intensity of different fluorophores. Some fluorophores may be present in higher concentrations than other fluorophores in a particular experiment and will either wash out the other fluorophores or will saturate the camera while the other fluorophores are not bright enough to see. When this occurs, a straightforward application of the imaging spectrometer cannot be used. Instead, the long integration method with NDR can be used. The method is described in detail in Section 5.6.3. In this method, the NDR will provide images with different amounts of photon integration. The darker images can be used for the brighter fluorophores and the brighter images can be used for the fainter fluorophores. By combining the dark image data with the bright image data, a full spectral image with higher dynamic range can be obtained for a larger range of fluorescence intensities.
6.6 Raman Applications

Raman scattering spectroscopy is a method for distinguishing between materials by analyzing the inelastic scattering properties of light. Figure 6.18a shows the different scattering processes that may occur in a sample. In order to analyze some of the vibrational or rotational energy states inside a material, it is possible to use infrared absorption methods. It is also possible to use different Raman scattering techniques. Normally, a photon will hit a material causing it to enter a virtual state. Then the material will generally scatter the light at the same wavelength. This is the elastic scattering process or Rayleigh scattering.

In some situations, perhaps one in one billion photons, the emitted light will be at a different, longer wavelength because the atom will store the remaining energy in a vibrational or rotational state. This is called a Stokes shift. The Raman scattering intensity scales with the wavelength of the driving laser as $\lambda^{-4}$. Therefore, the shorter the wavelength of the pump laser, the stronger the Raman scattering becomes.

A Stokes shift can also happen in fluorescence. The key difference between fluorescence and Raman is that there are many real energy levels in fluorescence causing broad peaks and the photons are excited to real energy levels rather than virtual energy states. For Raman phenomenon, the scattering is instantaneous while flu-

![Diagram of elastic and inelastic scattering and diamond fluorescence](image)

Figure 6.18: a) The concepts for elastic and inelastic scattering are described. b) The fluorescence of diamond from an input laser at 532 nm is shown. The reference diamond fluorescence is from [9].
orescence phenomenon is a resonant process which may have some associated time lag. Because Raman scattering and fluorescence phenomenon both occur at longer wavelengths, there is a tendency for readings from both to overlap.

Figure 6.18b shows the fluorescence spectrum of diamond when excited by a 532 nm laser. This fluorescence spectrum overlaps with some of the Raman scattering measurement and will sometimes obscure the results. A reference fluorescence line is also provided [9].

A shorter wavelength can also be emitted if the initial state of the atom is not at its ground state. This is called an anti-Stokes shift. The anti-Stokes shift does not overlap with fluorescence but are more rare than the Stokes phenomenon making it harder to measure. Resonant techniques like Coherent Anti-Stokes Raman Scattering spectroscopy (CARS) have been developed to take advantage of this phenomenon [50, 120]. As these methods typically use pulsed lasers at fixed wavelengths, they are not generally compatible with the IFTS measurement method.

There are several ways in which the Raman input can be designed for use with the IFTS. Lasers with fine linewidths are necessary for obtaining high quality spectra. The most readily available and scalable fine linewidth laser is the diode pumped solid state (DPSS) green laser. A neodymium-doped yttrium aluminum vanadate (Nd:YAG) crystal is typically pumped with 808 nm diode, producing a 1064 nm wavelength and then this frequency doubled inside a potassium titanyl phosphate (KTP) crystal. The resulting 532 nm wavelength has a fairly narrow linewidth but can be plagued by unstable light intensity or mode jumping depending on the design.

Figure 6.19 shows two different designs for making Raman scattering measurements. The first arrangement uses a beam splitter. This topology is simpler to miniaturize but loses half the input light and half the output light in the beam splitter. The maximum input light incident on the sample is approximately 100 mW for
Figure 6.19: Two different arrangements for Raman input light are shown. a) The first uses a beam splitter while the second b) uses the notch filter. The corresponding layouts are shown in c) and d).

this laser. This arrangement is much easier to align and calibrate because the beam splitter can be used for inserting calibration sources.

The second arrangement uses a notch filter for reflecting the light directly toward the source. This arrangement is more difficult to align and not as scalable. It is also difficult to insert calibration sources into this design. It is, however, more efficient than the previous design producing more Raman scattering photons than the previous design. The amount of light incident on the sample is about 200 mW for this layout.

This arrangement is similar to the fluorescence setup except for a few lenses and the long pass filter. Because the Raman scattering intensity is so much less than the elastic scattering intensity, the notch filter is not sufficient for filtering out the input 532 nm light. An additional long pass filter is needed to produce reliable Raman scattering data. An additional aperture is also present in these designs. The aperture helps limit the light coming from the sample. Because both the notch filter and long pass filter have angle-dependent filtering properties, the aperture helps remove
additional light that scatters toward the spectrometer at 532 nm at odd angles. It can also be used to change the light intensity from the sample by rejecting rays that are scattered at the more extreme angles.

### 6.6.1 Dispersive Measurements

In order to compare the speed and spectral results from the IFTS, a simple dispersive spectrometer was created using the same input design. The dispersive setup is shown in Figure 6.20a showing a pinhole (30 μm), two lenses, a blazed grating, and a SLR camera (Cannon 7D). The SLR camera was chosen because of its long integration times and its built-in gain via ISO settings. A smaller pinhole allows less light but also less angular dispersion in the incoming light leading to higher wavelength resolution.

The spectral lines for the interferometer were calibrated by using different lasers at 532 nm and 650 nm as well as white light LED sources. The edges of the notch filter were also used as calibration sources. A quadratic fit was used to convert the pixel position to the desired wavelength.

Several different materials can be measured in a Raman spectrometer. The most common materials that are studied include crystalline materials like diamond and

![Figure 6.20:](image)

Figure 6.20: a) The layout for a dispersive Raman interferometer are shown with a pinhole, grating and a camera. b) The dispersive Raman measurements of several different fluorescent diamonds are shown. The inset shows the diamond array fluorescing under ultraviolet light and under white light.
graphite [62] or materials with aromatic rings like benzene or acetamidophenol (active ingredient in Tylenol).

Figure 6.20b shows how a small array of diamonds can be measured. Pure, non-fluorescent diamond crystals have a single strong Raman peak at 1332 cm$^{-1}$ due to the pure crystalline structure. The inset of this figure shows how each diamond has different fluorescence and Raman peaks due to small differences in their chemical makeup due to nitrogen vacancies. Three different diamonds in the array were measured and their spectra are plotted. The first diamond has a slightly higher fluorescence while the second diamond has a nitrogen vacancy peak $NV^0$ close to the main peak. The third diamond has a small second order Raman peak at 2D [4].

Common plastics can also be measured using a Raman spectrometer. In Figure 6.21, two different plastics, polyethylene and polystyrene are compared. The more pure samples of each plastic, in pellet form, are used for this measurement. Other impure forms will have more absorption and more fluorescence. The measured peaks for both polyethylene and polystyrene match well with their respective reference data. The intensity of the Raman peaks are much less than those for diamond. The integration time on the plastic samples are two to three times longer than for diamond.
6.6.2 Diamond Characterization

The IFTS can be used to image the Raman spectrum of for several points. Figure 6.22 shows the raw interferogram for the Raman spectrum of diamond. This can be translated into the raw spectrum and then can be compensated using the curve for the LUPA 300. Next the spectrum can be smoothed by comparing the Raman spectrum with the fluorescence spectrum (measured at a lower intensity), smoothing only the fluorescent component. The final spectrum is shown in Figure 6.22d.

Figure 6.23 compares the measurement from the dispersive spectrometer and the IFTS measurement. By combining the integration time and internal gain on the dispersive spectrometer camera, the total integration time is 256 seconds. The total integration time for the IFTS measurement is 93 seconds. For this particular spec-

![Figure 6.22](image)

**Figure 6.22:** The compensation procedure for Raman data is outlined starting with the a) interference pattern and b) the raw measured spectrum. c) The intensity compensated curves can be derived and d) the fluorescent component can be separated from the Raman component, filtered, and recombined to produce the final curve.
Figure 6.23: a) The raw image of the diamond array is shown along with b) the 40× magnification on one corner of a diamond. c) The processed color image from IFTS data shows the deep red fluorescence of the diamond at the focal laser spot. The Raman spectra of fluorescent diamond from d) dispersive measurements and e) IFTS measurements can be compared. The reference Raman spectrum for fluorescent diamond can be found in [4, 73].

trum which includes a broad fluorescence, under-sampling techniques described in Chapter 7 cannot be used effectively because the spectrum is not especially sparse. The IFTS measurements have a shorter integration time due to the throughput advantage because there is no pinhole in the IFTS design. The dispersive measurement uses a camera with a 700 nm cutoff with an 8-bit intensity depth while the IFTS system has a 1000 nm cutoff with a 10-bit depth per image. For both curves, the resolution at the Raman peak is approximately 5 nm.

The signal to noise ratio for the dispersive measurement is about 15 dB while the signal to noise ratio for the IFTS measurement is between 11 and 18 dB. The two measurements are fairly similar. The near IR filter cutoff for the dispersive spectrometer removes much of the infrared fluorescence so it does not appear as
strongly in the dispersive measurement, hence the wavenumber axis does not reach as high a value.

The black and white image shown in the Figure 6.23b is the image taken by the camera. The white dot indicates the high intensity at the focal point of the microscope objective and the location at which the pixels are selected for the measurement. The edges of the diamond can also be seen. For the IFTS measurement, intensity of the light is decreased so that the camera does not saturate. The red dot in Figure 6.23c is the true color image created from the Raman spectral measurement. Because most of the measurement is dominated by the fluorescence, the image appears red. If only the Raman peak appeared, then the image would be yellow.

Because only 200 $mW$ are available from the input laser in this small setup, there is only enough light to illuminate 20 to 40 pixels for a Raman scattering experiment when taking measurements that are between one and two minutes long. Longer integrations would provide better imaging of the Raman spectra over more pixels in the image.
Chapter 7

Irregular Sampling Theory

The Fourier transform can be used to convert interference patterns into spectra. This transform however has many limitations. First the data have to be sampled at evenly spaced intervals at the Nyquist frequency. This means that a large number of samples are needed. For example, if a 5 nm resolution at $\lambda_0=850$ nm is desired, over 700 samples would be needed. If the camera is 30 frames per second, gathering a full hyperspectral image may take up to 23 seconds. Even for a high-speed camera at 2000 frames per second, a video rate spectrum could not be achieved. This is a clear limitation of Fourier transform methods that utilize traditional uniform sampling at the Nyquist frequency. Improving sampling is an important part of making imaging Fourier transform spectrometers competitive with other topologies in terms of speed.

The choice of sampling technique may depend on the amount of previous information known about the spectra being acquired (sample being imaged) as shown in Table 7. If there is no prior knowledge of the spectra, a uniform or regular sampling algorithm may be used in conjunction with a fast Fourier transform solution technique or some form of $L_2$ solver. If the band limits are known [53], such as in the case were the sensor bandwidth or the fluorescence emission band are known, then aliased uniform sampling or under-sampling can be used.
If the spectrum is known to be sparse, as in the case of Raman spectra or laser light, it is possible to use random sampling in conjunction with compressed sensing techniques that utilize $L_1$ or other types of solvers [59, 84, 101, 161]. Because of the sparsity of the data in the spectrum, it is possible to under-sample heavily. In order to do so without knowing the peak locations (which is not possible with simple under-sampling), the sampling interval should be uncorrelated to spread out the aliasing noise. This leads to random sampling schemes for sparse systems. This has been applied effectively in many optical imaging, holography and tomography systems [152, 153]. Fast orthogonal search (FOS) solution techniques can also be used in this case [91, 93]. If the discontinuous spectral locations are known, a different sampling algorithm can also be used [63, 65, 158, 159]. Other non-uniform sampling methods also exist [60, 92, 129]. All these algorithms require some prior knowledge of the spectrum.

In order to reduce the sampling rate while maintaining the desired resolution, the IFTS designs may use one of a variety of irregular sampling methods in order to
provide full spectral images quickly and accurately. Examples of irregular sampling
techniques that were developed for the IFTS include under-sampling, non-uniform
sampling, optimal sampling, adaptive sampling, and recursive update techniques.
These methods can be used in conjunction with advanced solution methods other
than the fast Fourier transform (FFT) or the discrete Fourier transform (DFT). The
advanced solution methods include techniques like the $L_1$ or $L_2$ (least squares) tech-
niques or fast digital filtering methods for real-time custom color filtering.

7.1 Generalized Solution Method

The solution method presented here is derived from fast orthogonalization techniques
used for sampling Fourier transform data and for solving Volterra kernels [91, 94].
The general solution for any sampling method can be obtained from setting up the
problem in an equation of the form,

$$y(n) = \sum_{m=1}^{M} A_m P_m(n) + e(n), \quad (7.1)$$

with matrix form $Y = PA + E$, where

$$P_1(n) = 1 \quad (7.2)$$

$$P_{2i}(n) = \cos(\omega_i z(n)/c) \quad (7.3)$$

$$P_{2i+1}(n) = \sin(\omega_i z(n)/c). \quad (7.4)$$

Here $y(n)$ is the interferogram intensity, $e(n)$ is the error and $A_m$ contains the spectral
information. The matrix $P_m(n)$ is constructed from real trigonometric polynomials
and contains the guesses for different frequencies $\omega_i$ and the locations where mea-
surements are taken $z$. To convert frequencies to wavelengths, the equation $\lambda_i = \frac{2\pi v}{\omega_i}$
where \( v = c/n_t \) can be used. For air, the refractive index is \( n_t = 1 \) and \( c \) is the speed of light. Note that this is a Hartley transform and not a Fourier transform as all constants are real. Also note that there is no restriction on sampling interval thereby allowing non-uniform sampling. There are many possible ways this can be solved including using an \( L_2 \) norm solution, also known as least squares (LS) algorithms,

\[
\hat{A} = (P^T P)^{-1} P^T Y,
\]

or using \( L_1 \) norm solutions using basis pursuit [56], linear programming, positivity solutions [176] or other compressive sensing solution algorithms,

\[
\min_{A} ||A||_1 \quad \text{s.t.} \quad Y = PA.
\]

Once the solution \( A \) is obtained, the results can be mapped back to a magnitude and phase,

\[
J_i = \sqrt{A_{2i}^2 + A_{2i+1}^2},
\]

\[
\phi_i = \tan^{-1}(A_{2i+1}/A_{2i}).
\]

Note that there is no restriction on sampling interval thereby allowing non-uniform sampling and under-sampling. The flexible form of the matrix \( P \) allows for any choice of desired frequencies, even no-continuous groups of frequencies indicating many different sets of spectral limits, thereby enabling adaptive sampling. This matrix form can also be used for recursive update algorithms.

Because the true relative phase content of the input light is not preserved in an interferometric measurement, it is therefore not necessary to preserve the phase content in the transform. The phase information can therefore be ignored. The
magnitude information maps directly to the spectrum of the measurement. If the amount of data acquired is more than twice the number of wavelengths present in the data (and wavelengths queried), then the $L_2$ algorithm can be used. This is generally the case. If the data are sparse, and the amount of data acquired is less than twice the number of wavelengths queried, then an $L_1$ algorithm or an adaptive or iterative $L_2$ algorithm may be necessary.

If the same data were used to calculate the FFT as the LS algorithm where the same frequencies were used in both the FFT solution and the LS solution, then the solutions of the FFT and the LS algorithm should be exactly the same. The LS algorithm allows the user to drop frequencies of interest and therefore requires less data to be acquired if some prior information is known about the spectrum (like band limits). However, if the sampling is uniform, the most optimal spacing between the desired frequencies $\omega_i$ in the LS algorithm is the same as the spacing for the FFT. Therefore, it is generally not advisable to make arbitrary choices of frequencies for solving the LS algorithm, since there is a direct relationship between the sampling spacing and the optimal spacing of the frequencies.

One clearly useful application of this algorithm is for correcting the spectrum when the sampling locations are not uniformly spaced. This is used in Figure 5.48 in Section 5.7.2. By using the measured positions rather than the desired positions, the spectrum can be effectively corrected and many positioning artifacts in the spectrum can be eliminated.

The following sections will cover a few different applications of this solution method in under-sampling, adaptive sampling and recursive update methods. The discrete filtering method described at the end of this chapter does not utilize this solution method.
7.2 Non-Baseband Sampling Methods

In traditional Fourier transform sampling methods, the sampling spacing must be half the spacing of the lowest wavelength of interest in order to avoid aliasing (Nyquist sampling $\lambda_{min} = 2\Delta z$), the maximum wavelength is equal to the sampling range $\lambda_{max} = N\Delta z$, and the number of samples obtained at this spacing determines the resolution of the spectrum. The wavenumber (generally presented in units of cm$^{-1}$), which is the inverse of the wavelength $\nu = 1/\lambda$, can be used to determine the resolution of the spectrum,

$$\Delta\nu = \frac{1}{z_{max} - z_{min}} \approx \frac{\Delta\lambda}{\lambda^2}. \quad (7.9)$$

If the sampling is equally spaced, then $\Delta\nu = \frac{1}{N\Delta z}$ is also true. The wavelength resolution is approximately $\Delta\lambda \approx \frac{\lambda^2}{N\Delta z}$ at any given wavelength $\lambda$. This wavelength range with this resolution is defined here as baseband sampling.

By looking at these equations, it is clear that resolution is driven only by the range of sampling. Therefore, it may be possible to ignore the Nyquist sampling limit to achieve a desired wavelength resolution by under-sampling the spectrum if the incoming light is limited only to a certain band of wavelengths. For example, if silicon-based photodiodes are used, the wavelengths are limited to between about 300 and 1100 nm when the devices are not cooled.

If the band limits $\lambda_{max}$ and $\lambda_{min}$ are known, then the Shannon non-baseband sampling rate [136] is,

$$\Delta z = \frac{1}{2} \lambda_{min} \left[ \frac{\lambda_{max}}{\lambda_{max} - \lambda_{min}} \right]. \quad (7.10)$$

The brackets in this equation indicate taking the integer floor value of the ratio inside the brackets. By utilizing the known band limits of the spectrum, the number of samples necessary for reconstructing the spectrum can be reduced by three to five
times thereby increasing the sampling speed. This gain in speed does not come for free, however, as fewer samples reduce the ability of the transform to reject noise.

### 7.2.1 Under-Sampling

Under-sampling or non-baseband sampling is a technique where the sampling interval is larger than the Nyquist sampling interval. Traditionally, this type of sampling is discouraged because it tends to cause aliased data to fold into un-aliased data. However, if the band limits of the system are known, it is possible to prevent folding and use the aliased data directly. This dramatically cuts down on the number of data points needed and can dramatically increase the spectra update rate.

Figure 7.1 shows a simulation of how the sampling can be reduced without any loss to the resolution. An FFT is taken of the original data and this requires taking a total of 2579 samples. By decreasing the range of sampling by a factor of two, the number of samples required is decreased, but the resolution is poor as can be seen in the black line in the inset. If the data are instead downsampled by a factor of two and solved with a traditional FFT, the data are aliased. Instead, if that same data are solved using the least squares solver, it is possible to recover the same original

---

**Figure 7.1:** Normally sampled data are compared with data that have been decreased in range, data that are downsampled and solved with an FFT, and data that are downsampled and solved with the LS algorithm.
Figure 7.2: a) Normally sampled high spectral resolution data requires many sample points, in this case 710 samples for a resolution of 5 nm. b) Under-sampled data solved with the LS algorithm can achieve higher resolutions such as 3.6 nm with only 100 sample points by reducing the wavelength range.

Table 7.2: Sampling parameters for normal and under-sampling measurements featured in Figure 7.2

<table>
<thead>
<tr>
<th>Step Interval (µm)</th>
<th>Number of Samples</th>
<th>Total Travel (µm)</th>
<th>Resolution at 850 nm</th>
<th>Restricted Wavelength Range</th>
<th>Solution Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>710</td>
<td>140</td>
<td>5 nm</td>
<td>Baseband</td>
<td>FFT</td>
</tr>
<tr>
<td>0.2</td>
<td>100</td>
<td>20</td>
<td>36 nm</td>
<td>Baseband</td>
<td>FFT</td>
</tr>
<tr>
<td>0.5</td>
<td>100</td>
<td>50</td>
<td>14.5 nm</td>
<td>&gt;500 to &lt;1000 nm</td>
<td>L₂</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>100</td>
<td>7.25 nm</td>
<td>&gt;667 to &lt;1000 nm</td>
<td>L₂</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>200</td>
<td>3.6 nm</td>
<td>&gt;800 to &lt;1000 nm</td>
<td>L₂</td>
</tr>
</tbody>
</table>

spectrum while maintaining the same resolution. By knowing the band limits of the original data, it is possible to determine the appropriate downsampling rate and recover the spectrum without losing detail.

This effect can also be tested on experimental data. Figure 7.2a shows a regularly sampled spectra with a 200 nm sampling interval, taking about 710 samples total. This results in a 5 nm resolution at 850 nm. However, under-sampling around 850 nm, as shown in Figure 7.2b, yields 7.25 nm resolution for only 100 samples sampled at 1 µm steps and 3.6 nm resolution for 100 samples sampled at 2 µm steps. This can drastically reduce the sampling time while maintaining high resolutions. The sampling parameters for this graph are presented in Table 7.2.1. The non-baseband
sampling techniques have a limited wavelength range and require the use of the $L_2$ least squares solver.

As the step interval increases, more noise becomes present in the spectrum. This is because baseband sampling spreads out measured noise across a large number of wavelengths and also reduces noise by averaging, which is inherently done through the least squares process. When under sampling is implemented, the reduction in the number of samples also reduces the averaging effect, thereby producing more noisy spectra. When the sampling interval increases, the noise is distributed over fewer wavelengths thereby giving the appearance that the spectrum sampled at 2 $\mu m$ is more noisy.

### 7.2.2 Optimal Under-Sampling

Because of the inherent trade-off in terms of speed and noise, a cost function can be created to determine the optimal under-sampling interval for a given weight of importance of noise relative to sampling and computation time. In this test case, the system is restricted to a constant spectral resolution with a constant set of desired spectral points. An optimal under-sampling interval can be determined using a cost function where the noise ratio $f(\Delta z)$, data acquisition time $g(\Delta z)$ and solution time $h(\Delta z)$ are considered with relative scaling constants $K_1$, $K_2$ and $K_3$. The sampling interval region is restricted by the Nyquist sampling rate, the maximum possible sampling step size, the maximum allowable sampling time, and the maximum allowable signal to noise ratio,

$$
\arg \min_{\Delta z} K_1 f(\Delta z) + K_2 g(\Delta z) + K_3 h(\Delta z).
$$

subject to, $\Delta z \in [\max(\Delta z_{\text{Nyquist}}, T_{\text{max}}), \min(\Delta z_{\text{Shannon}}, \text{SNR}_{\text{max}})]$.

Based on these criteria, simulations and experiments were performed to deter-
Figure 7.3: a) The noise as a function of sampling interval and b) the computational time as a function of sampling interval are shown. c) The simulated and measured inputs for this experiment is a laser at 850 nm. d) The optimal sampling interval can be obtained from a cost function, here the relative scaling between the cost of computation time and noise ratio is 5 to 1.

mine the optimal sampling region where the cost function is at a minimum. In Figure 7.3a and b, the sampling and computation times drop drastically with the decrease in the number of sampled points while the noise level increases slowly as the data are downsampled more and more. This can be seen in Figure 7.3c.

Figure 7.3d shows the optimal sampling region when the relative scaling of importance between time to noise is 5 to 1. Each simulation point is computed from 100 trials on data with white noise. This is then compared to measurements made on the IFTS. The measurements and the simulation data match well for the computation time. The noise ratio from the measurements is much more flat than the simulated noise level. For this set of parameters, the optimal sampling interval that uses a small number of samples but has a relatively low noise level is around 1 µm, which is much greater than the Nyquist sampling rate for a laser at 850 nm. This can be applied to other configurations to determine the optimal under-sampling intervals.
7.3 Adaptive Under-Sampling

Previously, it was discussed how the band limits of a system can be used to decrease the number of samples taken. In many cases, nothing is known about the spectra before the data are obtained. However, it is possible to slowly learn more and more about the spectrum as the data are acquired and adapt the sampling intervals as more information is obtained. Adaptive under-sampling takes advantage of the fact that the act of obtaining more data points in the spectra automatically yields more spectral data. Since there is more information about the underlying spectrum, it is then possible to tailor the input to converge on the optimal, wider sampling interval over a single sampling sweep thereby producing the desired spectral resolution faster. For example, a controller implementing adaptive sampling may slowly increases the sampling interval as it learns more about the underlying spectra.

There is a tangentially related body of work on adaptive sampling for different dispersive or diffractive optical configurations [19, 66], for non-spectral imaging purposes [56], and for radio spectra [77, 146] or certain frequency bands [130], which involves non-stationary spectra (the spectra changes in time and you cannot go back and forth in the spectra to acquire more data). There are also several “adaptive” basis pursuit algorithms used in compressed sensing for finding a solution to sparsely sampled systems [56]. These algorithms, however, are purely mathematical solution techniques and do not provide direction on how the data should be adaptively sampled.

The spectral data obtained from imaging Fourier transform spectra presents a unique case where the true underlying spectrum is fixed for a given sample and can be sampled adaptively at any desired position at any time. In addition, IFTS spectra have incoherent or broadband data concentrated at the center of the interferometer alignment which must be treated differently than the narrowband data. The IFTS designs presented in this work have significant computational resources and
can compute complicated spectra at the same time as obtaining the data leading to the ability adapt while the data are being acquired. These factors lead to a unique adaptive under-sampling algorithm that can be effectively implemented for imaging Fourier transform spectrometers.

It is important to point out that compressive sensing is a closely related field that can be used to solve sparsely sampled optical spectra [53, 84]. The solution methods used in this field (such as basis pursuit) tend to perform best if the underlying system is very sparse and will often be unable to solve dense systems [56]. The adaptive algorithm presented here does not have this limitation and can be used to solve sparse, dense and intermediate systems. A comparison of how these algorithms tend to scale is also discussed in this section.

### 7.3.1 Algorithm

The adaptive sampling algorithm is presented in Figure 7.4a. In the first step, the user determines the desired resolution and initial sampling rate which is related to the known band limits $\lambda_{\text{max}}$ and $\lambda_{\text{min}}$,

$$\Delta z_0 = \frac{1}{2} \lambda_{\text{min}} \left| \frac{\lambda_{\text{max}}}{\lambda_{\text{max}} - \lambda_{\text{min}}} \right|, \quad (7.12)$$

where $z_i = z_{i-1} + \Delta z_0$. Then a small number of initial samples $N_0$ are collected near the interferogram center. Once this is completed the low-resolution spectra is calculated using the generalized solver. Using the small number of initial samples also forces the system to sample the center of the interferogram more heavily thereby capturing more of the broadband spectral data.

When this result is obtained, the limits (locations in the spectra where information exists and the value is above $\delta_L$) can be determined thereby allowing us to calculate the occupancy (ratio of occupied to total spectral data points). Determining the
occupancy allows us to then select the next sampling step with some relaxation term $R \geq 1$ and an occupancy $\Gamma_i = \frac{N_{\text{occupied}}}{N_{\text{total}}}$ where,

$$
\Delta z_i \leq \frac{1}{2} \lambda_{\text{min}} \frac{R}{\Gamma_i} \left[ \frac{\lambda_{\text{max}}}{\lambda_{\text{max}} - \lambda_{\text{min}}} \right],
$$

and $z_i = z_{i-1} + \Delta z_i$. Once some samples are taken, the process recalculates the spectra using the algorithm outlined above and repeats the procedures until the desired resolution is achieved. Note here that an $L_2$ norm algorithm requires that the limits become smaller (only occupied parts of the spectra are set up in the calculation of $\omega_i$) because there would otherwise not be enough data to solve the problem. An $L_1$ norm algorithm would not have such a restriction but would run much slower. The shrinking limits in the algorithm make the $L_2$ norm solution feasible.

This process is repeated until the limits stop changing or at some predefined number of iterations. The algorithm then switches to constant sampling at the maximum interval until the final desired resolution is reached.

One important consideration in the algorithm solution is how to convert the sam-
pling step limitation $\Delta z$ to a value $\Delta z_i$. There are several possible ways to do this including using multicoset algorithms, packing algorithms, and direct constrained solutions. It is also possible to set $\Delta z = \Delta z_i$ which may lead to a risk of aliasing.

A random sampling algorithm can also be used. This equates the expectation of the distribution with the sampling step size limitation $E(\Delta z_i) = \bar{\Delta z}$ and samples randomly from the distribution, such as a uniform distribution $\Delta z_i \sim U(0, 2\Delta z)$. There are a few consequences to choosing a randomized step size. First, rather than aliasing causing algorithm failure, the aliasing noise will be randomly distributed throughout the spectrum in addition to the other types of noise present in the measurement.

### 7.3.2 Convergence of Simplified Algorithm

Convergence, accuracy and stability are important aspects of this algorithm. When there is no noise and $\delta_L$ is small, the convergence rate for a single narrowband wavelength with a true occupancy of $\Gamma_{true}$ can be simplified and approximated as,

$$N_{totali} = \left\lfloor \frac{\nu_{max} - \nu_{min}}{\Delta \nu_{i-1}} \right\rfloor,$$

(7.14)

$$\Gamma_i = \frac{\Gamma_{true} N_{totali} + N_m}{N_{totali}},$$

(7.15)

$$\Delta \nu_i = \frac{1}{\sum_{j=1}^{i} \Delta z_i} = \frac{1}{\sum_{j=1}^{i} \frac{1}{2} \lambda_{min} \frac{R}{\Gamma_j} \left\lfloor \frac{\lambda_{max}}{\lambda_{max} - \lambda_{min}} \right\rfloor}.$$

(7.16)

The total number of bins in each iteration of the algorithm is $N_{totali}$. The value $N_m$ is the extra tolerance or buffer used at the edges of the wavelength limits. This is used in the algorithm to ensure that $\Gamma_{true} < \Gamma_i$ and will be a larger number if there are separate groups of wavelength limits. In practice, this helps ensure that the limits estimated by the algorithm are no affected too greatly by noise. Notice that this simplified convergence rate does not go through any Fourier transform process,
The algorithm is simulated by varying different parameters including the initial number of samples $N_0$, the extra tolerance $N_m$ and the true occupancy $\Gamma_{true}$. The final number of samples required for converging on a desired spectral resolution for a variety of occupancy ratios is shown with a variety of values for $N_0 = 5, 100, 200, 300, and 400$. Both the guessed occupancy $\Gamma_{guess}$ and true occupancy $\Gamma_{true}$ are plotted. Full occupancy at the desired spectral resolution is defined at 919 samples.

This set of equations can be simulated in Figure 7.5 and a few conclusions can be drawn. First, with the simplifications used in this convergence rate, the algorithm will always converge toward the true occupancy. Although, due to the discrete nature of the algorithm, some instability may occur. Second, depending on the value of $N_m$, the estimated occupancy will never be equal to true occupancy for any desired resolution $\Delta \nu_{desired}$ that is not infinitely small. There will always be some offset between the true occupancy and the estimated occupancy that is controlled by the number of wavelength limit groups and the true occupancy. This can be clearly seen in Figure 7.5b where $\Gamma_{true}$ and $\Gamma_{guess}$ are never equivalent except for full occupancy.

There are also diminishing returns on incorporating a large number of samples in the algorithm. For these simulation cases, after about 250 to 300 samples, there is not
much gain in the occupancy ratio and therefore the step sizes do not increase greatly. Instead of recomputing the step size each time, it would be beneficial to sample at the rate that occurs at 250 samples for the remaining iterations of the algorithm.

The choice of \( N_0 \) has an important effect on the convergence as can be seen in Figure 7.5b. The smaller the value of \( N_0 \), the fewer sampling cycles necessary for converging on the known occupancy. Some nominal value of \( N_0 \) must however be used to help stabilize the adaptive algorithm. Choosing a value of \( N_0 < 100 \) does not greatly affect the number of cycles necessary for obtaining the desired spectral resolution. Choosing a large number for \( N_0 \), such as \( N_0 > 200 \) does not help the algorithm converge to the desired spectral resolution faster but can help to slightly reduce the number of cycles over which the adaptive algorithm is run to achieve a desired spectral resolution. In general, it is beneficial to recompute the sampling interval every 10 to 20 samples rather than after every sample.

### 7.3.3 Full Simulation Performance

The performance of the full algorithm can be evaluated on simulated spectral data, taking into account aliasing effects, noise, and signals of different bandwidths. For these simulations, a random sampling spacing \( \Delta z_i \sim U(0, 2\Delta z) \) is used for the adaptive algorithm. Figure 7.6 shows the results for two different spectra, one which is occupied at around 43 % and another that is very sparsely occupied at 6.8 % (occupation measured with respect to the non-baseband limits). As the algorithm evolves over time the band limits (light green) get closer and closer to the occupied parts of the spectrum. As these limits shrink, the sampling interval increases allowing the algorithm to converge faster.

These are compared with Nyquist sampled and non-baseband sampled systems. The densely populated spectra, when sampled adaptively, required only 612 samples...
Figure 7.6: a) The adaptation to a broadband signal (43% occupancy) as the number samples taken increases is shown. b) The adaptation to a sparser, narrowband signal (6.8% occupancy) as the number of samples acquired increases is also shown. Both are compared to Nyquist sampled and non-baseband sampled measurements.

which is only 22% of the total number of samples that were needed for Nyquist sampling. The sparse spectra required only 10.5% the total number of samples. Without any prior knowledge, the algorithm was able to terminate intelligently taking a very small number of samples for each spectrum.

In the next figure, the performance is compared to completely randomly sampled spectra. In Figure 7.7, for densely populated broadband spectra, the adaptive algorithm performs much better than the pure random algorithm. This is because of the denser sampling near the center of the interferogram and because a true random sampling algorithm only has some probability of success for a given number of samples. For the sparse spectra, the purely randomly sampled algorithm does a little better because the adaptive algorithm has a fixed convergence rate as described earlier.

253
Figure 7.7: For a) broadband and b) narrowband input signals, the performance of the adaptively sampled algorithm is consistent. The randomly sampled measurements have some probability of success for a given number of samples acquired and are much noisier (less successful) in the broadband case than in the narrowband case when compared to the adaptive algorithm. The interferograms are also shown (c and d) indicating the distribution of sampling for each algorithm.

Figure 7.8a shows how the occupancy ratio drops as more samples are taken for two different types of spectra when the full algorithm is simulated. Notice the similarities of the convergence rates between this figure and Figure 7.5. One of the spectra in this graph is a relatively sparse spectrum (narrowband) with 6.8 % occupancy and the other is a spectrum with broadband spectral information with 43 % occupancy. The spectra with less occupancy drops more quickly but both convergence algorithms are stopped at around 240 samples due to diminishing returns. Both types of spectra converge after crossing the Landau bound (red circles), which specifies that two data points must be obtained for each spectral point in order to cover both the sine and cosine phases of the solution algorithm. In addition, both lines do not converge down to their true occupancies due to the number of points used to buffer the band limits.
as indicated by the arrows.

Random sampling can actually work faster than adaptive sampling for sparse cases as exhibited by the magenta line. This line represents the number of samples necessary to solve for a given occupancy at a given probability of success \cite{27, 57, 58, 132}. This line scales as,

\[
\Gamma_{\text{rand}} = \frac{iC_r}{N_{\text{total}} \log_{10}(N_{\text{total}})},
\]

where \( C_r \) is related to the probability of success, \( i \) is the sample number and \( N_{\text{total}} \) is the total number of bins at the desired resolution. The constant is chosen at \( C_r = 1 \) to have a high probability of success similar to those presented in \cite{27, 56}.

The magenta line crosses the 6.8 \% line at the magenta circle indicating that at that number of samples, it is able to reconstruct the sparsely occupied spectrum with some high probability. This shows why random sampling is able to reconstruct narrowband data in Figure 7.7 accurately. Notice, however, that the magenta line does not cross the 43 \% line which indicates that it is not able to reproduce more dense spectra as effectively with a certain probability of success.

Figure 7.8b shows the Landau bound and the random sampling bounds plotted with the simplified adaptive sampling bounds. These bounds were simulated with Equations 7.14 to 7.16 for a series of occupancies up to full occupancy at \( \Gamma_{\text{true}} = 1 \). The number of samples where convergence occurs is plotted against the true occupancy \( \Gamma_{\text{true}} \) and the measured occupancy \( \Gamma_{\text{guess}} \) when the desired resolution is reached for \( N_m = 20 \). The termination iteration of the three simplified equations match well with full adaptive algorithm simulation with LS matrix inversion for all the occupancies tested (blue dots). This shows that the simplified equations can do a good job of predicting convergence performance. This graph also shows that for true occupancies that area greater than about 8 to 10 \% for this set of parameters (including the probability of success for random sampling), the adaptive algorithm would perform
Figure 7.8: a) Convergence and performance comparison for the full adaptive algorithm with respect to the Landau bound and a random sampling bound for a given probability of success. b) Simulations of the simplified adaptive sampling algorithm convergence equations show areas where the adaptive algorithm performs better than random sampling.

better than random sampling. It is interesting to note that the adaptive sampling bound curves up towards the end near an occupancy ratio of one to perfectly match the Landau curve at full occupancy. This clearly shows that it can do as good as but never do better than the ideal Landau sampling bound, which matches well with intuition.

Because random sampling only has some probability of success, it is difficult to rely on it to produce good spectra repeatably. The random sampling process cannot be used directly to determine limits, which means that random sampling alone cannot be used with an $L_2$ least squares solver and the solution cannot be obtained adaptively. Instead, the spectrum must be sampled randomly and then solved using the slower $L_1$ or FOS solution methods. If the sampling is insufficient, the $L_1$ algorithms will not be able to produce any useful information and more samples must be obtained and the $L_1$ solution must be attempted again. Instead of getting iteratively better data, the random sampling method only provides information once there are enough data.

Overall, adaptive sampling is much more flexible in that it can handle broadband and narrowband spectra. Adaptive sampling also produces spectra as it learns more
about the underlying system therefore allowing the spectrometer to confirm the shape of the spectrum over time. Without any prior knowledge, the spectrometer was able to terminate intelligently taking a very small number of samples for each spectrum. Adaptive sampling is ideal for obtaining the first pass on any set of data. Once the spectral locations are known, adaptive sampling can be used for broadband spectrum (to help cover the center of the interferogram better, where the information for broadband data is concentrated) while random sampling can be used for narrowband data using the limits defined by the first pass using the adaptive algorithm.

Adaptive sampling can be robust to noise by using the relaxation parameters $R$, $N_m$, and $\delta_L$, causing the algorithm to take a few extra data points. Figure 7.9 shows the same broadband and narrowband signals when subjected to a large amount of noise, in this case 7-bit quantization noise and an overall signal to noise ratio in the interferogram of 24 $dB$. By taking more data points, 819 for the broadband spectrum and 341 for the narrowband spectrum, the spectral information was recovered adaptively. This shows that the algorithm can be used effectively even when there is noise and other forms of uncertainty.

Other types of spectra, such as a sparse Raman spectrum can also be simulated as shown in Figure 7.10. Here the Raman spectrum of Benzene is simulated. Baseband sampling produces this spectrum with 1229 samples while non-baseband sampling can reduce the number of samples to 246 samples and adaptive sampling is able to
reduce it further to 154 samples. Once the limits were obtained, random sampling was used to reproduce the same spectrum at this resolution with only 96 samples.

7.3.4 Experimental Results

Figure 7.11 shows the results of implementing adaptive sampling with the IFTS system. This implementation solves the adaptive spectrum and limits once every 20 samples before updating the occupancy. This requires fewer intermediate solutions while still adapting quickly. This implementation in the software also uses equal spacing \( \Delta z = \Delta z_i \) rather than random spacing for simplicity. Three laser sources were used for this experiment at 532 nm, 850 nm, and 1064 nm. The photodiodes in the positioning system are used to detect the reference beam at 850 nm and three other pixels in the spectrometer camera sensed light at 532 nm and 1064 nm. The inset of Figure 7.11a shows part of the full image from the imaging interferometer (from which the pixels are chosen).

The data plotted in Figure 7.11a where Nyquist sampled at an interval of 200 nm and took over 75 seconds to complete at 30 frames per second. The data shown in 7.11b were adaptively sampled and took less than 25 seconds to complete at 30
Figure 7.11: a) Nyquist sampled data can be compared to b) adaptively sampled data showing similar peaks. c) The occupancy shows how the algorithm converges quickly and d) the sampling speed shows how the adaptive algorithm achieves the desired resolution almost four times faster than the traditional Nyquist sampled algorithm.

frames per second. The resulting spectrum has the same resolution as the Nyquist sampled spectrum. Due to the adaptive sampling settings, only occupied spectral bands are solved for and plotted. The data peaks are slightly different due to experimental variation. Figure 7.11c shows how the occupancy ratio decreases quickly and steadily to the occupancy bound, which is greater than the true occupancy of the spectrum. Notice how the convergence rate for this curve is very similar to the convergence rates from the simple simulations in Figure 7.5. Figure 7.11d shows the spectral acquisition time. Both Nyquist sampling and adaptive sampling stop at around the same position, which leads to both having the same spectral resolution. As the experiment shows, the adaptive sampling works well in practice.
7.4 Recursive Spectral Update

For many applications, it is desirable to do continuous spectral sampling. When the spectra of the image changes slightly during the acquisition process, it would not be desirable to completely resample the spectra. Instead old data points from the interferogram can be forgotten and replaced with information from new data points. This allows a gradual adaptation or recursive updating of the calculated spectra and gives the user intermediate information about the system.

While adaptive sampling reduces the number of images necessary to produce a full spectral image, recursive sampling can be used to update the spectral information between full spectral images. For example, if 500 images taken at 2000 frames per second are required to create a single full spectral image, the full spectral image rate would be 4 full spectra per second. If a recursive sampling algorithm were used, it would be possible to make small updates the spectral image after each image; this allows the recursively sampled full spectral image rate to be as high as 40 (update group size of 50) up to 2000 (update group size of 1) frames per second. This can be useful for pushing the spectral update to the user at faster rates. Possible applications could be video monitoring of changes in biological fluorescence to events during chemical mixing causing changes in the Raman spectra. Alterations in blood oxygen content of an entire image as a function of heart rate could also be monitored.

7.4.1 Algorithm

In order to implement recursive sampling, the underlying model should not change (the size of the spectra of interest \( m = 1 \ldots M \) and the locations of \( \omega_i \)) so that the spectral resolution and sampling range are fixed. This is different from the adaptive algorithm where these values are changing each time the occupancy is updated. The initial underlying model can be set up from data obtained through adaptive sampling.
Recursive update tends to have a fast calculation time and can be used to decrease noise by obtaining more samples.

For recursive sampling, the model relation is, \( Y = PA + E \). Defining two variables, \( \Phi(n) = P^T P \) and \( \Psi(n) = P^T Y \), makes it possible to derive update equations for a single data point update using the familiar recursive least squares algorithm [64, 74],

\[
K(n) = \frac{\Lambda^{-1}\Phi^{-1}(n-1)P_m^T(n)}{1 + \Lambda^{-1}P_m(n)\Phi^{-1}(n-1)P_m^T(n)},
\]

\[
\Phi^{-1}(n) = \Lambda^{-1}\Phi^{-1}(n-1) - \Lambda^{-1}K(n)P_m(n)\Phi^{-1}(n-1),
\]

\[
\hat{A}(n) = \hat{A}(n-1) + \Phi^{-1}(n)P_m^T(n)\left(Y(n) - P_m(n)\hat{A}(n-1)\right).
\]

At the current iteration \( n \), the \( \hat{A}(n) \) vector is used to obtain the desired magnitude and phase. The parameter \( \Lambda \) is a forgetting factor where \( \Lambda = 1 \) means that no past information is forgotten. Due to the nature of recursive sampling, a full matrix inversion is not required to update the full spectrum and the spectrum can be updated quickly. To initialize recursive sampling, a guess can be made of the spectrum or a full set of spectral images can be obtained first.

The update equations above are suitable for updates after each image \( g = 1 \). But it may be desirable to update the spectrum after acquiring a few data images \( g > 1 \). This is beneficial since the spectra updates change slowly and updates for \( g = 1 \) may be in the noise. For \( g \geq 1 \), it is possible to define,

\[
F = \begin{bmatrix} P_m(n-g+1), & P_m(n-g+2), & \ldots, & P_m(n) \end{bmatrix},
\]

\[
G = \begin{bmatrix} \Lambda^{g-1}P_m(n-g+1), & \Lambda^{g-2}P_m(n-g+2), & \ldots, & P_m(n) \end{bmatrix},
\]

\[
H = \begin{bmatrix} \Lambda^{g-1}P_m(n-g+1), & \Lambda^{g-2}P_m(n-g+2), & \ldots, & P_m(n) \end{bmatrix},
\]
\[ Y_g = \begin{bmatrix} Y(n - g + 1), & Y(n - g + 2), & \ldots, & Y(n) \end{bmatrix}. \]  

(7.24)

The update equations are then derived for this application using the Woodbury matrix identity (matrix inversion lemma),

\[ K(n) = \Lambda^{-g}\Phi^{-1}(n - g)G^T \left[ I + \Lambda^{-g}G\Phi^{-1}(n - g)G^T \right], \]

(7.25)

\[ \Phi^{-1}(n) = \Lambda^{-g}\Phi^{-1}(n - g) - \Lambda^{-g}K(n)G\Phi^{-1}(n - g), \]

(7.26)

\[ \hat{A}(n) = \hat{A}(n - g) + \Phi^{-1}(n)H^T \left( Y_g - F\hat{A}(n - g) \right), \]

(7.27)

with a zero order hold on the estimates of \( \hat{A}(n) \) for all intermediate values. With the group update equations, it is possible to grab a group of spectral images and update the spectral information at the same time as grabbing the next group of data.

One limitation of recursive sampling is that the intermediate spectral updates may not track fast changes and may have a time constant on the order of the number of samples acquired for a full spectra. Therefore, the true update speed may be limited by the spectral resolution and range. Recursive sampling works well for both narrowband (coherent) and broadband (incoherent) signals.

### 7.4.2 Noise and Forgetting-Factor Considerations

Figures 7.12a and c show simulations of recursive sampling starting with an undersampled spectrum where 200 samples are taken to cover the spectral range between 750 nm and 950 nm. In Figure 7.12 (for \( \Lambda = 1, g = 1 \)), the original spectral information (black) is shown and is used to initialize the algorithm (magenta). At the first iteration, the spectra changes instantaneously to the new spectrum (blue) and the result at update iteration 200 is shown (red). The recursive estimation is slowly updating and by iteration 4000, has fully converged to the new spectrum.
**Figure 7.12:** At iteration zero, the wavelength of the input laser is switched. This simulation shows the response of the recursive algorithm to different forgetting factors ($\Lambda = 1.0$ or $\Lambda = 0.98$) at iteration 200 (a and c) and at iteration 4000 (b and d). Lower forgetting factors adapt more quickly but tend to be noisier.

Two different forgetting factors of $\Lambda = 1.0$ and $\Lambda = 0.98$ are shown side by side. It is clear that $\Lambda = 1.0$ converges much slower at iteration 200 but has a lower noise level by the last iteration. These effects can be seen for tests at many different values of $\Lambda$.

Figure 7.13a shows how the peak at 900 nm changes as a function of iteration for different forgetting factors. The lower the forgetting factor, the faster the adaptation and the larger the noise. For a forgetting factor of 1.0, the adaptation is slow but the RMS noise as a function of iteration drops to lower levels as shown in Figure 7.13b. Lower forgetting factors can speed up adaptation but reduces the ability to reject noise. From this data, it is clear that a spectrometer executing recursive sampling can clearly sense changes in the spectra at sub-Shannon sampling intervals.
Figure 7.13: At time zero, the wavelength of the input laser is switched and the peak magnitude and noise levels are monitored. These simulations show the effect of forgetting factor on a) adaptation speed and b) on noise rejection over time.

### 7.4.3 Experimental Results

The recursive update algorithm was implemented on the IFTS in the group update form. The spectra from three pixels in the image are tracked as a function of time. One experiment was conducted where a 532 nm laser is turned off quickly followed by a 850 nm laser that is turned on. This is illustrated in Figure 7.14 where the top row indicates an experiment conducted with $\Lambda = 1.0$ and the bottom row indicates an experiment conducted with $\Lambda = 0.998$. The time steps where each of these snapshots are taken are indicated in Figure 7.15a.

Initially, it is clear that the 532 nm laser is on, then there is a transition period where the laser is turned off and the next laser is turned on. In reality, there was no point where one laser light was fading and one laser magnitude was growing as turning the lasers on and off is instantaneous. The transition period is an artifact of the algorithm. As the algorithm converges, it is clear that the lower forgetting factor has more oscillation in the peak value while the higher forgetting factor has a more steady peak value.

Figure 7.15a shows the experimental results of several experiments where $g = 10$ and the number of images used for a full spectra is 100 samples. For $\Lambda = 1$, there is less oscillation but the data updates more slowly, increasing the settling time.
Figure 7.14: Two different forgetting factors were used on experimental data, a) $\Lambda = 1.0$ and b) $\Lambda = 0.998$. A laser at 532 nm is turned off at the frame labeled 1 and a 850 nm laser is immediately turned on. The evolution of the spectra is tracked over time. The frames correspond to those indicated in Figure 7.15a.

Figure 7.15: a) The response of the two peak wavelengths, 532 nm and 850 nm are tracked for different forgetting factors. b) In a separate experiment, noise levels are tracked over time to determine how forgetting factor affects noise over time.

Figure 7.15b shows how the noise evolves as a function of time for different forgetting factors. For the largest forgetting factor, the noise level drops quickly and stays low. For the lower forgetting factors, however, the noise levels remain high and tend to oscillate back and forth. This slow oscillation only appears in experimental data and may be due to positioning and imaging artifacts.

If this spectral data had been more traditionally updated, e.g., for the whole
period of 1000 iterations, only 10 full spectral updates would have been completed since the number of images per spectra is 100. However, by using recursive sampling, the spectrometer can acquire at least 90 to 900 spectra over the same period. This greatly increases the availability of the spectral update to the user.

### 7.5 Digital Filtering

Unlike previous methods that attempt to improve sampling speed or update rates, the digital filtering methods presented in this section attempt to create color images from the inherent “black and white” data (that are obtained from the spectrometer) in a fast and effective manner that does not require taking a full FFT or solving the LS equations. The idea is to do so with a minimal number of simple calculations per pixel and to be able to track motions at video rates while reproducing color images from the black and white data. Examples of applications include tracking different fluorescent peaks, tracking different Raman peaks while collecting a minimal amount of data or simply converting data from a black and white camera to color without the use of Bayer masks or rotating filter disks.

These color images can be created from any set of wavelength bands and as many colors can be assigned to as many wavelength bands as desired. In general, broadband colors, similar to the Bayer filters in traditional color cameras, are relatively quick and easy to reproduce from interferometric data because those filters are so broad that only a few terms are needed to create a filter for those wavelengths. The more challenging case is when there are narrowband colors, such as two laser wavelengths that are very close together and a third that is further away. This is the case shown in Figure 7.16b. This case does not allow optimal use of non-baseband sampling and requires more complex digital filter designs. For this reason, this more complex arrangement was used as the test system for digital filtering.
Figure 7.16a shows the experimental system used to test the digital filtering algorithms. Three laser wavelengths are used. One laser produces 532 nm light and 808 nm light. A second laser source produces 850 nm light and it passes through a chopper. This chopper is used to modulate the light value to simulate fast “motions” to determine how quickly and effectively the algorithm can track changes. For imaging purposes, these different wavelengths were mapped to three different colors, the 532 nm laser to blue, the 808 nm laser to green and the 850 nm laser to red. Once

Figure 7.16: a) A schematic for a system that incorporates three lasers, one of which is chopped at different frequencies, is shown. This is used to test the frame rates of the digital filter. b) The wavelengths being tested are 532 nm, 808 nm and 850 nm. The last two wavelengths are very close together.

Figure 7.17: The process for converting black and white data to color data is outlined. Each pixel in a series of black and white interferometric images (video) is sent to a band pass filter to separate out the different wavelengths. Then a moving average is applied and the resulting red, green, and blue intensities are reconstructed into color video.
the data are obtained, the digital filtering scheme shown in Figure 7.17 can be used. Each black and white image that is grabbed from the high speed camera at different positions along the interferogram is sent to a filter which splits the image into different pixels and each pixel is passed through three different bandpass filters that cover distinct wavelength ranges. The output of the filter is then sent to a moving average filter and the resulting data can be reconstructed into a color image. A series of black and white images with interferometric data therefore become a series of color images through this process. Different filter designs are explored including filters that require uniform sampling algorithms and filters that can handle non-uniform sampling algorithms.

7.5.1 Uniform Sampling Experimental Results

There are two major classes of digital filters, infinite impulse response filters (IIR) and finite impulse response filters (FIR). An IIR filter has the form,

$$y(n) = \frac{1}{a_0} \sum_{i=0}^{Q_b} b_i x(n - i) - \sum_{j=1}^{Q_a} a_j y(n - j),$$

(7.28)

with both feed-forward filter coefficients $b_i$ and feedback filter coefficients $a_j$. Generally, fewer terms are necessary in the IIR domain than in the FIR domain for similar filter cutoffs due to the feedback terms. These IIR filters are generally more difficult to convert from any time-domain design requiring a bilinear transform or Tustin transform [64] and are much more straightforward to design directly in the digital domain. IIR filters were designed for the digital filter application and are shown in Figure 7.18a.

A Butterworth filter was designed for the wavelength at 532 nm (indicated by the blue filter) because it is sufficiently far away from the other two wavelengths. Elliptical filters were designed for the 808 and 850 nm filters. Elliptical filters have
sharper cutoffs for the same filter order but have more ripple. The wider the filter in frequency space, the less delay there is in time. For this reason, the edges of the red and green filters are used to separate the 808 nm from the 850 nm while allowing the filters to be fairly wide overall.

The filter delay as a function of frequency is shown in Figure 7.18b. The Butterworth filter has low and relatively consistent group delay while the group delay for the elliptical filters is much more varied. The corresponding estimated real time update rate or frame rate tracking for 2010 fps is plotted along the right side. In general, these filters should be able to track motions up to about 40 fps when video is taken at 2010 fps.

FIR filters generally require more computations for the similar filter cutoff characteristics but they have a constant delay equal to half their filter order. FIR filters are also easier to generate from continuous domain filter designs. An FIR filter has the form,

$$y(n) = \sum_{i=0}^{Q_b} b_i x(n - i).$$  \hspace{1cm} (7.29)

The FIR filters are designed in Figure 7.19a. The red and green filters are very narrow and have an order of 100 (delay of 50). The blue filter is much broader and has an order of 30 (delay of 15). Because these filters do not have a flat region, it is not

Figure 7.18: a) Three IIR filters (two elliptical filters and one Butterworth filter, all 8th order) are designed and b) the resulting filter delay at the frequencies of interest are shown.

269
Figure 7.19: a) Three FIR filters are designed with constant filter delays. The red and green filters have an order of 100 and the blue filter has an order of 30. b) This graph shows a comparison of the FFT of the full data length with data that are gathered through the IIR filters of these data. These data indicate that a 50 sample set (40 fps) is unable to distinguish between 808 and 850 nm while the digital filters are able to do so at that frame rate.

possible to use just the edges of the filter to separate the two wavelengths as with the IIR filter. For this same reason, the rejection ratio of the green filter for information at 850 nm and the red filter for information at 808 nm is potentially more problematic for the FIR designs.

Figure 7.19b shows how the performance of these filters compares with the performance of taking the FFT of different sizes. The red, green and blue filters are able to separate out the information for their respective wavelengths fairly well. By using a 50 sample FFT, it is clear that the peaks at 808 nm and 850 nm cannot be separated and therefore it would not be possible to use the FFT method for doing video up to 40 fps. When 100 samples are used for the FFT, it becomes easier to separate out the two wavelengths but the frame rate is reduced to 20 fps. As discussed earlier, the IIR filters used to separate the different colors can track data up to 40 fps or faster easily. The IIR filters are also less computationally intensive than a 50 or 100 point FFT or the LS solution and therefore the digital filtering technique is ideal for this application.

Figure 7.20 shows how the raw black and white data are processed into the colored
Figure 7.20: Raw data from three different pixels in the image with the red pixel chopped at a) 15.7 Hz and b) 60.9 Hz are shown. The corresponding processed data are shown with the chopping frequencies evident in the red pixel data (c and d).

The filters pick up only the frequencies associated with each color and the signals are passed through a moving average filter. The processed data are shown in Figures 7.20c and d. The filters are able to pick out the chopper oscillations with some delay. The slower 15.7 Hz chopper does a better job of distinguishing the on and off points than the 60.9 Hz chopper, although the chopping frequency is still clearly visible.
Figure 7.21: The black and white images and processed color images are shown at different times at two different chopper frequencies. The data from the 15.7 Hz chopped images (a and b) come from Figure 7.20a and the data from the 60.9 Hz chopped images (c and d) come from Figure 7.20b.

The images associated with the different time points 1 through 4 and 5 through 8 are shown in Figure 7.21. These figures show the black and white raw data of the three laser spots as well as the final processed color data. There is a clear green spot, smaller blue spot and a red spot that is chopped so it appears and disappears periodically. The contrast for the red laser spot for the 15.7 Hz chopper images is much better than the contrast on the 60.9 Hz chopper images, showing that this method has a clear frequency cut off.
7.5.2 Non-Uniform IIR Digital Filtering

The initial startup period in Figures 7.20c and d of up to approximately 30 $\mu m$ is associated with the filter initialization and is partially due to positioning errors which cause the sampling to be non-uniform in this area. Filters with the ability to handle non-uniform spacing can be used to correct for those small positioning errors and can be further used for other applications with non-uniform sampling, such as adaptive sampling or random sampling schemes.

The first filter type explored is the IIR filter. Although IIR filters are more computationally efficient for uniform spacing, they are not necessarily more efficient for the non-uniform sampling. In order to create a non-uniform sampling IIR filter, a consistent master design must be created in the continuous domain and converted to the digital domain. The more accurate conversion processes requires a Tustin transform and this involves a matrix inversion, although there are some other approximations that are slightly more efficient [61].

A continuous domain filter design in state space form can be converted to the digital domain. Here, the input vector is $x(z)$, the output vector is $y(z)$, the state variable is $w(z)$ and they are all a function of position $z$,

\[
\frac{dw(z)}{dz} = A_d w(z) + B_d x(z), \tag{7.30}
\]

\[
y(z) = C_d w(z) + D_d x(z). \tag{7.31}
\]

There are several possible approximations that can be made. The first is an Euler approximation,

\[
\frac{w_n - w_{n-1}}{\Delta z_n} = A_d w_{n-1} + B_d x_{n-1}, \tag{7.32}
\]
that has a solution of the form,

\[ w_n = (I + \Delta z_n A_d) w_{n-1} + B_d \Delta z_n x_{n-1}, \quad (7.33) \]

\[ y_n = C_d w_n + D_d x_n. \quad (7.34) \]

The second type of approximation is the bilinear approximation or Tustin transform,

\[ \frac{w_n - w_{n-1}}{\Delta z_n} = A_d \frac{w_n + w_{n-1}}{2} + B_d \frac{x_n + x_{n-1}}{2}, \quad (7.35) \]

which has a solution of the form,

\[ w_n = (I - \frac{\Delta z_n}{2} A_d)^{-1} \left( I + \frac{\Delta z_n}{2} A_d \right) w_{n-1} \]

\[ + \frac{1}{2} \Delta z_n \left( I - \frac{\Delta z_n}{2} A_d \right)^{-1} B_d (x_n + x_{n-1}), \quad (7.36) \]

\[ y_n = C_d w_n + D_d x_n. \quad (7.37) \]

If the non-uniform sampling intervals are not predictable or are different for every sample, a matrix inversion would be required for every frame obtained. This would therefore make it less computationally efficient and makes the non-uniform sampling FIR digital filters seem more tractable.

### 7.5.3 Non-Uniform FIR Digital Filtering

The benefit of using FIR filter designs for non-uniform sampling filter designs is that FIR models translate easily from continuous domain designs [13]. After creating a continuous domain filter, it is possible to simply resample the filter at different points corresponding to the non-uniform sampling points. From a time-domain or frequency domain master filter (with all windowing removed), two vectors are generated. The
first being the master filter and the second being the associated uniformly sampling space vector. In general, the master filter should be over-sampled slightly to improve resampling accuracy later. The master filter $b^m$ is shown as a blue line in Figure 7.22. For uniform spacing, the filter vector is,

$$b_i = b^m_i \Delta z_i.$$  \hspace{1cm} (7.38)

The non-uniform filter is slightly more complex as shown in Figure 7.22b. Because the sampling is non-uniform, the filter must also have the same non-uniform spacing as the sampling. If the sampling position vector is known then,

$$b_i = b^m_i \Delta z_i \text{ or } b_i = \frac{b^m_i + b^m_{i+1}}{2} \Delta z_i,$$  \hspace{1cm} (7.39)

where the first equation uses the instantaneous value and the second equation uses the trapezoid rule. The master filter must be resampled every time the filter is updated (once for each new frame) which can take longer to compute but is relatively simple to execute.

Figure 7.23 shows simulations were the spectra are sampled non-uniformly. Three different lasers are simulated on a single pixel and these lasers are turned on and off. Figure 7.23a shows the sample step sizes which vary as a function of sample

![Figure 7.22](image_url)

Figure 7.22: a) The filtering scheme for a uniform FIR digital filter is shown along with b) the non-uniform FIR digital filtering scheme.
Figure 7.23: a) The sampling for this set of simulated data is sinusoidal and non-uniform. b) The input magnitudes for two of the colors are chopped. c) The output obtained with a uniform digital filter shows artifacts from the non-uniform sampling. d) The output obtained with the non-uniform FIR digital filter is much cleaner, is better at rejecting errors from the sampling scheme, and is similar to the original intensity modulation.

in a sinusoidal manner. The input is also modulated by the intensities shown in Figure 7.23b. If a traditional uniformly sampled FIR filter is used, the sampling errors show up directly in the calculation and it is difficult to determine when the laser is on or off. However, if the non-uniform FIR filter is used, it becomes clear that the three different wavelengths can be picked out from the same signal based on when they are being turned on and off.

7.5.4 Non-Uniform Sampling Experimental Results

The non-uniform FIR filter design can be easily implemented on the IFTS measurements since the position at which each frame is taken is recorded. The real step size as a function of sample is shown in Figure 7.24. The sampling intervals are clearly non-uniform and oscillate heavily for the first 150 samples.

The non-uniform FIR digital filters were applied to video taken with these data and are shown in Figure 7.25. The top figures are from the uniform spacing filters while the bottom figures are from the non-uniform spacing filters with the exact same
Figure 7.24: The sampling intervals from an experiment are shown to be non-uniform, especially for the first 150 samples.

master filter designs. While the uniform filter is only able to come to steady state at around a position of 40 $\mu m$ (corresponding to sample 200), the non-uniform filter is able to come to steady state much faster, setting around 10 to 20 $\mu m$ (corresponding to samples 50 and 100 respectively). Most of the ramping time is associated with the large FIR filter order and the moving average filter.

This ramp-up time has consequences for the perceived color of the images as shown in Figures 7.25c and d. The first two pictures for the uniform spacing filters are very red and the blue laser spot is completely the wrong color. In contrast, the first two pictures for the non-uniform spacing filter have colors that are slightly more orange but are mostly the correct color. The positioning errors clearly create errors that can be perceived when converting the data to color images.

The filter width also has consequences for color separation and frame rate. Figure 7.26 shows the same data processed through the non-uniform spacing filters of different filter lengths. The first filter is narrow, with an order of 100. It is clear that the red and green color separation is good but the processed signal shows that it is not able to capture the chopping frequency very well, the chopped data for the red pixel are almost uniform. On the other hand, the broader filter with a filter order of 40
Figure 7.25: The a) uniform and b) non-uniform digital filter results are compared. c) The resulting images show that the uniform filter is darker and have more color distortion in the initial portion of the data while d) the non-uniform filtered data have much less color distortion.

Figure 7.26: a) A narrow filter (filter order = 100) is compared to d) a broad filter (filter order = 40). The filter designs are shown (b and e) as are the processed signal data (c and f). The narrow filter has better color separation but poorer frequency response.

has some trouble separating the red and green colors. In fact, they are both slightly orange. The filter, however does a better job of capturing the chopping frequencies.
Figure 7.27: a) The rejection ratio of the red filter and the green filter with respect to each other's wavelengths are shown along with the estimated frame rate as a function of FIR filter order. b) The peak to peak intensity output of the chopped data is measured as a function of the chopper frequency, indicating the true frame rates of different filters.

and it is clear that the red pixels are being turned on and off.

This information is best captured in Figure 7.27a. The FIR filter order is changed and the red and green filter rejection ratio is measured along with the estimated frame rate (rate at which the chopper frequency can still be distinguished). It is clear that a lower filter order leads to less rejection but higher frame rates.

The frame rates can also be measured from real data by measuring the peak to peak intensity ratio on the chopper data. Figure 7.27b shows the intensity ratio as a function of the chopper frequency for the IIR filter with an order of 8 and the FIR filter with an order of 40. Both of these filters have a peak to peak ratio of 50% at around 45 frames per second and they can continue to track changes up to 60 fps.

Previously, the problem of inaccurate positioning was addressed using the LS algorithm with the measured positions rather than the desired positions. It can also be addressed by using FIFF control. In this section, this problem was addressed by using the modified non-uniform sampling forms of the digital filtering equations. By developing non-uniform sampling forms for the digital filtering process, it is possible to extend the applications of these digital filters for adaptive, random, or any other type of non-uniform sampling process. As this work has shown, there are several
synergies with different sampling techniques and solution techniques that can be used to increase the speed of imaging interferometers to video rates and produce color images from inherently “black and white” data quickly and efficiently.
Chapter 8

Conclusions and Recommendations

The unique goals of this work have driven the creation of several novel designs and mathematical techniques. In Chapter 2, this work has reviewed several different designs for imaging spectroscopy and determined the design with the minimum volume for high resolution spectroscopy through simple scaling laws. Based on the scaling law theory, several different implementations of the imaging Fourier transform spectrometer were designed and manufactured using electric discharge machining and other techniques and layouts listed in Chapters 3 and 4. Mirror positioning and tip-tilt control were implemented along with a hybrid high speed control algorithm to quickly position the interferometer and reject external disturbances in Chapter 5. By using these control algorithms, several measurements were made on LED light, transmission, fluorescence and Raman phenomenon in Chapter 6. Finally in Chapter 7, this work discussed several irregular sampling paradigms including under-sampling, adaptive sampling, recursive update methods, and uniform and non-uniform digital filtering.

This body of work has led to several practical contributions:

- The volumetric scaling laws developed for some common imaging spectrometer designs serves as a method for understanding the how imaging spectrometers
should scale and how to choose the best design for the desired specifications in a quantitative manner. The ray tracing program developed in conjunction with this work is then used to evaluate the consequences of specific design choices for the optics and serves to point out areas where correction algorithms would be necessary.

- Several small Lorentz force linear actuators with compact flexures was developed, modeled and characterized for long stroke (> 3 mm) and high resolution positioning.
- Several parallel and series piezoelectric actuators designs for tip, tilt and translation were developed and characterized for high speed, high resolution positioning.
- An unique and compact method for measuring the relative tip, tilt and translation of the interferometer mirrors was designed and a fast real-time implementation on the microcontroller and user interface was also developed. A raised-level mirror manufacturing process was also created for zero-tilt translation. An observer was also developed for determining the tip and tilt of the system independently from the translation.
- Several unique implementations of the IFTS were manufactured to show how the design could be miniaturized and packaged for different applications. These include implementations for robotic endoscopy and for high speed imaging.
- Feedback control and input shaping techniques were developed for fast and accurate mirror translation and lower speed tip-tilt control.
- Control algorithms were developed for hybrid high speed positioning with the linear motor and piezoelectric actuators utilizing input shaping and a single position sensor.
- Algorithms were developed which use the parameters from stochastic system
identification methods for model-based full inversion feed-forward control.

- Several different modes were tested using the IFTS designs including creating color images from black and white information. Blood oxygen levels, fluorescence, and Raman applications were also explored.

- A novel adaptive sampling technique was developed to handle both narrow-band and broadband (sparse and dense) spectra. Convergence metrics for this algorithm are shown and compared to compressive sensing techniques. For denser spectra, the adaptive sampling technique clearly outperformed compressive sensing and traditional sampling methods.

- Recursive update methods were developed for a single sample as well as for a group of samples. This method reduces the computational load for updating the spectrum in real time.

- Uniform and Non-uniform sampling digital filtering techniques were developed to create color images and color video from black and white data in real time. For data sampled at 2010 fps, the color image update can be up to 40 to 60 fps.

8.1 Project Goals

By analyzing scaling laws, creating a novel device, utilizing modeling and controls methods, and deriving special sampling techniques, this work has accomplished the original project goals:

- **Create an imaging spectrometer**: The IFTS design is inherently a full imaging spectrometer with imaging formats ranging from 100 × 100 pixels to VGA resolutions. It has been used to image different calibration light sources, blood oxygen content in the finger and the fluorescence of different beads in a mixture.

- **Create a small device**: Size optimization for several design topologies were
conducted and the design that scales the best as a function of OPD was chosen. The different iterations of IFTS are all compact and can fit in the palm of the hand as shown in Figure 8.1. One version is designed for robotic endoscopy allowing it to fit the form factor of a 15 mm outer diameter with the bulk of the electronics sitting outside the endoscope body. Other implementations with full test electronics are light and compact at around a total of 100 grams to 200 grams and can be used for many portable applications. A low cost video-rate camera was utilized for the prototype design to show how the IFTS device can potentially be economical.

- **Create a device that is optical-table-free:** The controller for mirror positioning was designed to utilize high disturbance rejection allowing accurate positioning in noisy environments up to 100 Hz and higher. The high speed
camera can also track human motions that typically go up to 15 Hz. By utilizing image tracking methods, the device is fully optical-table-free and can be used in the field.

- **Create a device with high spectral resolution:** The full stroke of the IFTS determines the maximum spectral resolution of the device. The full stroke is approximately 3 mm (or 6 mm OPD) which leads to a maximum spectral resolution of 1.67 cm⁻¹. This is equivalent to 0.12 nm resolution at 850 nm. Most applications that require high resolutions can use less than 500 μm OPD of the stroke, equivalent to 1.2 nm resolution at 850 nm.

- **Create a device that is capable of getting spectra quickly:** The high speed implementation of the device is capable of obtaining data at up to 2000 fps, producing high resolution spectra in less than a second. In order to make this a reality, a hybrid piezoelectric actuator and Lorentz force linear motor system was designed and controlled with the use of full inversion feed-forward and feedback control systems.

- **Develop mathematical techniques for getting spectra quickly:** Several advanced sampling and solution techniques were developed for this work. These include the adaptive sampling technique, which can take as few as 10% of the necessary samples to achieve a high resolution spectrum for sparse systems. This reduces the sampling time by up to a factor of 10. The recursive update algorithm and the digital filtering techniques can both be used to increase the speed so that the user sees the spectrum update in real time. Uniform and non-uniform sampling digital filtering techniques are computationally simple and can provide false-colored images for the user at video rates.

Solving these problems has led to several unique contributions that can be applied to miniature systems as well as larger optical systems. The sampling techniques, in
particular, can be applied to many different systems including advanced radio frequency sampling or even imaging applications. The adaptive sampling technique can be applied in compressive sensing applications for systems that are not sparse. The unique actuators designed for the IFTS can also be applied elsewhere in medical, automation and robotics applications. The tip-tilt-translate encoder positioning system is very compact and can potentially be used as a small, accurate positioning sensor for a variety of applications.

8.2 Future Work

There are many additional exciting directions for this body of work. Many design improvements can be implemented for making the device more reliable and easy to use. Additional imaging applications can also be explored including different methods for advanced sampling methods. Some of the possible future challenges include:

- **Design improvements**: There are several possible areas to improve in the design of the high speed implementation, especially going from a research device to a product. Improvements can be made to miniaturize the electronics and port more of the user-interface code onto a custom DSP, FPGA or microcontroller for portable operation. The positioning light path and the imaging light path can also be separated to prevent light leakage for longer image integration for more sensitive measurements. Alternatively, a different reference wavelength can be used for the position sensing that does not interfere with the camera measurements.

- **Miniaturization of the input light module**: Application specific optics can also be miniaturized for applications such as fluorescence imaging or Raman spectroscopy. This would make the instrument more useful in the field especially as a handheld device.
• **Depth resolved imaging application:** By using structured light, the IFTS can be used for depth resolved imaging to create fluorescence and Raman images similar to those obtained with confocal imaging.

• **Optical coherence tomography application:** A modified version of the IFTS could possibly be used for white light interferometry of different surfaces as well as optical coherence tomography via structured light inputs.

• **Implementation for infrared imaging:** By changing the camera and modifying the optical components for infrared wavelengths, it would be possible to use the same topology for infrared imaging. This would make the device useful for several new applications including infrared absorption spectroscopy and thermal imaging. Specific applications include using the spectrometer to study absorption due to different gases in the atmosphere [165].

• **Adaptive intensity integration:** By measuring different pixel intensities, it would be possible to adapt the camera to adjust its integration time to extend the dynamic range of the sensor over time. For pixels that are darker, the integration time (via the NDR function) can be longer and for pixels that are bright, the integration time could be shorter (as to not saturate the pixel). This can be used to improve the signal to noise ratio of the data similar to the technique used by Kinast and Gehm for dispersive measurements [88, 66].

• **Implementation for optical pulse imaging:** Although a Fourier transform spectrometer is not compatible with pulsed optical systems such as CARS, it is possible to use the IFTS to measure the pulse width of a laser. The imaging sensor can be used to effectively image the difference in pulse width across the entire pulse shape.

• **Synergies between imaging and spectral acquisition:** Tilting the mirror slightly yields extra position offsets for every image captured (each pixel is at a
different position offset). If the image covers areas that have the same spectral information, the data collected from these areas can be combined thereby decreasing the noise of the spectra and further reducing the number of images or data positions that have to be sampled. Image tracking methods can be combined with the adaptive sampling algorithm so that if the image moves slightly during an acquisition, the full spectra does not need to be re-acquired but can be mapped with the original data obtained before the disturbance. This may improve the acquisition process and make it more useful in practice.

- **Compressive imaging with compressive spectral sampling:** Section 7.3 covers compressive and adaptive sampling for the spectral domain. There is, however an additional dimension where compressive sensing can be applied. It is possible to compressively (or adaptively) sample the image as well. For an imaging spectrometer which samples and sends the entire image, this may not seem necessary. However, when sending real-time image data, the data rates are on the order of $9.22 \text{ MB/s}$ to $30.0 \text{ MB/s}$. A compressive or adaptive algorithm can be implemented inside the FPGA to deliver or store less data therefore allowing the system to be smaller and more efficient without losing too much important spectral or spatial information. By reducing the load on the video pipeline, it would be possible to increase the image resolution and the frame rate of the camera for real-time high speed operation. The current camera is clocked at $25 \text{ MHz}$ due to pipeline limitations. The full speed of the LUPA 300 is $80 \text{ MHz}$, which is 3.2 times faster. The LUPA 3000 is an even faster camera and is clocked at $206 \text{ MHz}$ (8.24 times faster) with a higher resolution. Using compressive or adaptive sampling for spatial information can greatly increase the real time speed of the camera system without having to use any post-sampling full image compression technique.
In addition to these possible areas for future work, there could be several other interesting improvements and applications that can make these instruments useful for everyday personal diagnostics, process monitoring, or material identification. As a product, the IFTS can help take diagnostics and identification from the lab into the field.
Bibliography


Appendix A

Hardware Design

A.1 Mechanical Parts Lists

One of the consequences of choosing electric discharge machining as the manufacturing technique is that the part counts are large. One single component, such as a cover or a flexure, may require several different layers to be cut on the EDM. Molding or micro machining processes may be used in the future to reduce the component count and increase the ease of assembly. Due to the large number of parts, the drawings for each part are not provided in this thesis. Instead, a list of parts detailing file names, part thicknesses and post processing techniques is shown. The post processing techniques include drilling holes on the sides of the parts on the axis that is not made via the EDM process. These holes are then threaded for M1 screws. Some surfaces are premilled or post-milled to remove material for assembly in tight spaces, to recess screw heads, or to prevent part interferences. The lists only detail the components that are manufactured via EDM and parts made via other processes are not listed.

The prototype version parts list is available in Table A.1. The parts are grouped by assembly. There are several standard materials and standard thicknesses used for these designs. The bobbin in this version was manufactured via stereolithography,
and the blade flexures are cut via a separate process. Thus, these parts do not appear in the list. A few parts are relatively small, have the same thicknesses and can be cut on a single EDM file and are therefore listed together as a group. For this version, a total 17 parts are manufactured using the EDM and anodization processes. The grayed out rows indicate that there are later versions of this design that were implemented via the iterative design process.

The multi-axis version parts list is shown in Table A.2. There are three components on this list that require several different profiles to be cut on the EDM to create a single part. The multi-axis version has several different implementations with different motor designs. The first (2 magnet) and second versions (4 magnets) have 36 parts each while the third version (6 magnets) has 38 parts. The increase in the component part count is mainly due to the introduction of the piezoelectric actuator assembly and additional components for compact mounting of electronics. The use of a metal bobbin material to improve the temperature characteristics of the design and EDM cut flexure blades also increases the part count.

<table>
<thead>
<tr>
<th>File Number</th>
<th>Part Name</th>
<th>Qty</th>
<th>Thickness</th>
<th>Material Type</th>
<th>Notes</th>
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<td>1018 Steel</td>
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<td>1.6 mm</td>
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<tr>
<td>V0533-EC009</td>
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<td>7075 Aluminum Post Drill, Tap</td>
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</tr>
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<td>7075 Aluminum Post Drill, Tap</td>
<td></td>
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Version 0.53.3 Total Part Count 17
## Table A.2: Multi-axis version EDM parts list

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<th>Material Type</th>
<th>Notes</th>
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<td>Post Drill, Tap</td>
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Version 0.60.2 Total Parts Count: 36
Version 0.60.3 Total Parts Count: 36
Version 0.60.4 Total Parts Count: 38

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A3
The endoscope version of the IFTS goes back to a smaller size and a simplified architecture as shown in Table A.3. In this design, there are 25 parts including a part that needs to be pre-milled.

The high speed version has the most complex design due to its larger symmetric motor topology as shown in Table A.4. The motor uses 8 magnets and has an intricate assembly procedure which requires many components to be split into two separate parts that are later screwed together. This design also has two implementations with different piezoelectric actuator flexure designs. The series design has 51 parts while the parallel design has 58 parts. This implementation also includes heat sink designs and additional flexures for repositioning the motor alignment to improve the long stroke characteristics. The additional metal mounting components (not including clear side-wall components) necessary for conversion to a handheld device are listed.
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<tr>
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<th>Thickness</th>
<th>Material Type</th>
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Version 0.80.6: Total Part Count: 51
Version 0.80.3: Total Part Count: 58
Version 0.80.3: Handheld Mounting Part Count: 15
A.2 Electronic Schematics

The schematics for the layouts of circuit board designs are designed in TinyCad 2.8. As it would be prohibitively long to show schematics from all the different implementation, most of the schematics shown are from the high speed (V0.80) implementation of the IFTS and designs from other versions are basically simplified variations of these schematics. The schematics begin with the power and programming boards implemented in Figure 3.20, followed by the base, sensor, and actuator boards implemented in Figure 4.19. Section A.2.5 and Section A.2.6 describe the schematics for the handheld variant of the high speed IFTS design including the portable power board and the FPGA board.

A.2.1 Power and Programming

The power circuits were designed to be able to source more current than needed in order to avoid issues during testing. Therefore, they are much larger than necessary for the operation of the IFTS. The basic power board for the system is shown in Figure A.1. This power board uses a buck converter to provide 5 V power at up to 5 A for the operation of the FPGA, video pipeline, sensors and the linear Lorentz force voice coil motor. A linear regulator is used for the 3.3 V supply and is capable of providing up to 1 A for the microcontroller and video pipeline circuits.

For the piezoelectric actuator system, a high voltage system that is capable of providing sufficient current is necessary to optimize actuator performance and reduce delay. In order to use 60 V operational amplifiers to drive the piezoelectric actuators, a double-boost topology was developed to provide 60 V power to the system at 250 mA. The first boost stage shown in Figure A.2 takes in 5 to 8 V power and boosts the voltage to 20 V at 1 A. The second boost stage shown in Figure A.3 takes in 20 V and converts it to 60 V. The handheld variation includes additional connections for

A6
enabling and disabling the power supplies to reduce power consumption when not in use.

A programming adapter board in Figure A.4 was also developed to adapt J-Link programmers to smaller connectors for programming the microcontrollers. Single wire debug (SWD) was chosen because it frees up more ports on the microcontroller than the joint test action group (JTAG) protocol. The programming adapter also houses the microcontroller reset button to free up space on the base board.

**A.2.2 Base Board**

The base board contains the microcontroller and video pipeline for the IFTS. The first base board schematic shown in Figure A.5 shows the Arm Cortex M4F microcontroller (STM32F405RG) capable of single cycle floating point multiplication and...
Figure A.2: The schematic of the 20 V boost power board is shown.

Figure A.3: The schematic of the 60 V boost power board is shown.
Figure A.4: The schematic of the programmer adapter board is shown.

clock speeds of up to 168 MHz. Older versions of the system design from the prototype IFTS used an Arm Cortex M3 microcontroller (STM32F101T8) which ran at 72 MHz. The handheld variation on the base boards includes additional enable pins for different power converters and chips (to help conserve power), communication pins for buttons as well as connections to monitor the portable power board. Due to the large number of floating point calculations needed for calculating the position and implementing the controllers, the microcontroller was upgraded in order to meet timing specifications. The board also contains an FTDI USART to USB converter chip for basic data communications with an external computer. The baud rate setting for the communications is set to 921600. This speed is fast but may occasionally drop bits (≪ 1 % of the time), leading to the implementation of data confirmation algorithms and error alert algorithms.

The second schematic for the base board shows the video pipeline schematic. At the time this thesis was written, USB 3.0 implementations were rare on desktop computers and laptops. Other formats like PCI (peripheral component interconnect) Express and SCSI (small computer system interface) were becoming increasingly uncommon. Most computers also had only one Ethernet connection and only some
computers had Firewire connections. Due to the ubiquitous nature of USB 2.0, the availability and ease of use of USB 2.0 chips and drivers, and relatively high data rates, a video pipeline was developed for High Speed USB 2.0.

The video information is received at the LVDS deserializer (SN65LV1124B) with 10 bits at 24 MHz. The LVDS allows for transmission of data over longer distances (long cables) while being more immune to noise. This data is then sent to the first-in-first-out (FIFO) buffer (IDT72245LB15). The FIFO buffer stores the data until the FTDI bit-bang to USB converter (FT2232H) is ready for data. The addition of the FIFO chip has several advantages. First, the FIFO chip allows both the LVDS and the USB converter to request the reading or writing of data at any time (the FIFO serves as the slave while the LVDS and USB converter are both masters). The FIFO chip also allows for the LVDS and USB converter to be clocked at different
Figure A.6: The schematic of the base board video pipeline components is shown with an LVDS deserializer, a FIFO buffer and the high speed USB converter.

frequencies and provides an extra data buffer in the event that one component is slow in either requesting data or providing data.

The fastest mode for USB 2.0 is high speed USB (up to 280 Mbit/s effective throughput). This can be implemented using the FTDI bit-bang to USB converter chip in synchronous FIFO mode. This mode clocks out data in bursts at 60 MHz. This mode must be set using the EEPROM (93LC46B) and FT Prog software on the computer after the board is assembled.

Note that because the LVDS system is 10 bits wide (8 bits for data, 2 bits for line valid and frame valid), it cannot be used when transmitting all the data from the high speed camera, which is 12 bits wide (10 bits for data, 2 bits for line valid and frame valid). It is possible to use only the 8 most significant bits of the camera data.
Figure A.7: The schematic of the base board connections is shown.

For the high speed camera implementation with 10 bits of data, the pipeline can be implemented with the FPGA encoding process. An additional switch is added to turn off the LVDS chip, therefore allowing the FIFO buffer to be clock directly from the FPGA at any rate desired. Without the LVDS, more noise appears on the camera lines and more noise overall is radiated to other analog sensors across the system.

The last schematic for the base board in Figure A.7 shows off-board connections, switches and debugging ports. For the high speed IFTS implementation, additional connections are used to communicate with the FPGA and additional switches are used to switch between the LVDS implementation of the pipeline and the FPGA implementation of the pipeline. Several connections are also provided as test points for debugging. Each off-board connection powers different sensors and actuators and capacitors are used to reduce power fluctuations on the base board.
A.2.3 Sensor and Illumination Boards

The first sensor board is the laser and light source board. It provides regulated power to the surface mount VCSEL laser (VCS-F85S20-S from Lasermate) for the analog quadrature system and a broadband white light source for finding the center of the interferometer. Due to the sensitivity of the relative alignment of the laser and light relative to the optics and photodiodes, it is preferable to mount this board separately from other components.

Figure A.9 shows the photodiode board containing six separate photodiodes, amplification systems, and analog to digital converters. In other versions of this board, only two or four photodiodes are used. In the high speed implementation, four photodiodes are being used for the analog encoder system and two photodiodes are being used for finding the center of the interferometer. Each photodiode is connected to a transimpedance amplifier (OPA2380) which amplifies the signal. This signal is read by the 12-bit simultaneous sampling analog to digital converter (ADC122S625) which can be read on the SPI bus. Under the most optimal conditions (full scale signal,
Figure A.9: The schematic of the photodiode board is shown here with 6 separate photodiode measurements.

minimal noise, perfect quadrature phase), this ADC is capable of producing position resolution on the order of 0.1 nm for a reference wavelength of 850 nm.

The next sensor is the camera board shown in Figure A.10 and this is the schematic used for the multi-axis IFTS implementation (V0.60). This is a 1/6" format camera (TCM8230MD) used typically for webcam applications. It outputs data in 16-bit red green blue (RGB) 565 format or luma and chrominance colorspace (YUV) format and has on-board color correction, white balance, and gain selection. In order to use this camera for spectrometer applications, the auto-white balance, auto-gain and other settings must be turned off using the I2C interface clocked at 400 kHz. It is a rolling-shutter camera which means that it is not possible to completely control when the integration of the light incident on the pixels occurs. This camera is typically clocked at 24 MHz for 16-bit (each pixel is composed two groups of 8-bits) color
Figure A.10: The schematic of the camera board is shown with the LVDS serializer.

Images at about 30 fps. It can be clocked faster for slightly higher frame rates or slower for lower frame rates. The number of pixels per image range from 128 x 96 in quarter common intermediate format (subQCIF) up to 640 x 480 in VGA format.

The TCM8230MD camera comes with a plastic lens which reduces the field of view for the spectrometer. This lens can be ground off and replaced with custom optics. This camera also comes with a hot mirror (short pass filter) which reduces spectral response of the camera to about 650 to 700 nm for dim sources. Wavelengths of up to 1064 nm have been measured with this camera when higher intensity lasers have been used. This camera board also contains an LVDS serializer (SN65LV1124A) for transferring the data across longer distances (on the order of meters) using a twisted pair.

Several different high speed cameras were considered for this application including
The Kodak (currently TrueSense Imaging) KAI-0340 and the LUPA 3000. The KAI-0340 is a small 1/3" CCD camera that requires high voltage clocking and off-chip ADCs ($50 to $90 depending on grade). It is capable of up to 210 fps at VGA resolutions (40 MHz clock) in dual read-out mode. The LUPA 3000 is a larger than 1" format with greater than VGA resolution. It is capable of VGA outputs at up to 2653 fps (206 MHz maximum clock speed). It is, however, much more expensive ($2200) and much more difficult to interface with, requiring BGA (ball grid array) mounting and 32 channels of LVDS.

The last sensor board is the high speed CMOS camera board shown in Figure A.11. The high speed camera chosen for this implementation is the LUPA 300, a 1/2" format camera typically used for machine vision applications ($150). It is typically clocked at 80 MHz and can output VGA data at 250 fps. However, due to pipeline limitations,
is clocked at up to $25 \text{ MHz}$. The camera produces 10-bit black and white pixels up to $640 \times 480$ per image at 75 fps. The number of pixels output by the camera can be adjusted to any size smaller than the maximum (such as $100 \times 100$ which results in images in excess of 2000 fps). Hence, the camera frame rate can be increased by simply reducing the number of pixels per image without running into the video pipeline limit. The camera also has an adjustable global shutter, adjustable gain, and a non-destructive read-out system that can be used to control the integration time and brightness of all the pixels in the image. The specifications are comparable to CCD cameras with a sensitivity of $3200 \text{ Vm}^2/\text{Ws}$, a dark current of $300 \text{ mV/s}$, a saturation charge of $35,000 \text{ e}^-$, a conversion gain of $34 \text{ uV/e}^-$, a parasitic sensitivity of $1/5000$, and a peak efficiency of $45\%$. The settings are controlled using an SPI interface clocked at a rate of $10.5 \text{ MHz}$.

This black and white camera has a non-uniform spectral response due most likely to the layered manufacturing process of the sensors, which can be corrected in software. There are no hot mirrors and the spectral response of the camera has been characterized at up to $1000 \text{ nm}$. This camera also has a fairly high fixed-pattern noise due to the four internal analog to digital converters which need to be corrected in software as indicated in Figure B.4.

This particular board allows for the most significant 8 bits of the camera data to be clocked out using the LVDS system. The board can also be switched into an off-board clocking mode where the camera is clocked using the FPGA and all 10 bits of data per pixel can be captured and encoded for the video pipeline. When the camera is clocked using the FPGA, it is possible to reduce the clock rate in order to increase the integration time of photons and hence the brightness of dim images, such as measurements of fluorescence or Raman.
A.2.4 Actuator Boards

High precision positioning is important to the operation of the IFTS. The Lorentz force linear voice coil actuator board shown in Figure A.12 is used to position the moving mirror in the spectrometer system. The board contains two operational amplifiers (OPA567) which allow the motor to be driven forward or backward depending on relative voltage of each. When driven at 5V, the bandwidth of the amplifier is about $60 \, kHz$. The total gain-bandwidth is $1.2 \, MHz$ and the slew rate is $1.2 \, V/\mu s$. The commands are sent via SPI and the 16-bit digital to analog converter (DAC8552). For a maximum single direction of travel of 3 mm OPD, the positioning resolution of the DAC under the most optimal conditions is on the order of 45 nm.

There are several ways of achieving higher positioning resolution. The first method is to decrease the total travel by changing the actuator and flexure characteristics. Higher positioning accuracy can also be achieved with higher resolution DACs. A separate 24-bit DAC was developed for the motor board using an I2S communications protocol. Finally, resolution can also be improved using a separate piezoelectric actuator (with shorter overall travel) in conjunction with the Lorentz force linear voice coil actuator.

The first design of the piezoelectric amplifier board is shown in Figure A.13. This design comes from the multi-axis version (V0.60). In this design, each of the piezoelectric actuators have their own driver (LT3469), which is composed of a 35 V boost converter and a transconductance amplifier. Each driver is small but has a limited slew rate and a limited bandwidth for piezoelectric actuators on the order of $150 \, nF$. The digital to analog converter (AD5664) is a 16-bit quad output converter. For an actuator and flexure assembly output of 2 $\mu m$ the positioning resolution under optimal conditions is on the order of 30 $pm$.

A separate scheme was then developed for the piezoelectric amplifier board that
Figure A.12: The schematic of the motor power board is shown.

extends the slew rate, bandwidth, and stroke of the actuators. Figure A.14 shows the 60 V piezoelectric amplifier board. Here, each actuator has its own operational amplifier (OPA552) capable of 60 V operation at up to 200 mA. The amplifier gain-bandwidth product is 12 MHz with a slew rate of 24 V/μs. For a 200 mA load, the estimated bandwidth of the amplifier is approximately 10 kHz. The power for each amplifier comes from the off-board 60 V double boost power supply. Since the power capacity of the amplifier board is much higher, it may trigger a thermal shutdown on the chip when the junction temperature reaches 160 °C. For long term operation in high power modes, such as during stochastic system identification, this board is heat-sinked. For normal lower power modes, no heat-sinking is necessary.
Figure A.13: The schematic of the 35 V piezoelectric amplifier board is shown.

Figure A.14: The schematic of the 60 V piezoelectric amplifier board is shown.
A.2.5 Portable Power

For the handheld version of the device, the power boards were changed to a lithium polymer battery charger circuit and less powerful buck and boost converters. The battery charging board (Texas Instruments BQ24192) in Figure A.15 is capable of charging the battery as well as providing continuous 3.5 to 4.35 V at up to 1.3 mA at 89 % to 93 % conversion efficiency from the power plug or from the battery. The charging characteristics for this chip are controlled using an I2C protocol and the status is indicated through several LEDs and status registers on the device. The new buck converter at 3.3 V and 1 A (TPS62086) as well as the boost converter at 5 V and 2.1 A (TPS61230DRC) are capable of conversion efficiencies near 94 % as shown in Figure A.16. This design is much more compact than previous power converter designs.

Figure A.15: The schematic of the portable power board battery charger is shown.
Figure A.16: The schematic of the portable power board 3.3 V buck and 5 V boost converters is shown.

### A.2.6 Field Programmable Gate Array

The handheld version also includes an FPGA board based on the Spartan 6 system with a bootloader topology based on the open source Mojo V3. The first schematic in Figure A.17 shows the microcontroller (ATMEGA32U4) and flash memory used for boot loading the FPGA. In this version of the design, the boot loader is installed using an in-system programmer through SPI. Boot loading the FPGA code is then completed using the USB port. The second schematic in Figure A.18 shows the Spartan 6 FPGA and the third schematic in Figure A.19 shows all the board connections to the camera, microcontroller, video pipeline and screen. The new button board for the handheld device is shown in Figure A.20.
Figure A.17: The schematic of the FPGA board microcontroller is shown.

Figure A.18: The schematic of the Spartan 6 FPGA is shown.
Figure A.19: The schematic of the FPGA board connections is shown.

Figure A.20: The schematic of the button board is shown.
Appendix B

Software Design

B.1 Microcontroller Code Structure

The microcontroller state diagram is shown in Figure 3.22 and the detailed implementation of the microcontroller architecture is outlined in this section. The microcontroller interface is driven by the user interface commands. There are several different settings and commands outlined by the table in Table B.1. The first column indicates the start-byte or command mode. ASCII characters are used for start-bytes because this allows the functions to be accessed directly from a command line. The second column indicates the function of the start-byte and the third column shows the communications format. The next column lists the sensors and actuators that are controlled by this function and the last column provides a brief description of the function.

The first set of commands is used for setting and checking the registers to determine the motor constants, piezoelectric actuator constants, photodiode constants, controller constants, observer constants, safety or emergency limits and camera registers. The “confirm” output type is when the microcontroller sends a start-byte followed by a set of parameters and then waits for the microcontroller to return all
Table B.1: The microcontroller register and command map

<table>
<thead>
<tr>
<th>Start Byte</th>
<th>Function</th>
<th>Output Type</th>
<th>Actuators &amp; Sensors</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>register</td>
<td>confirm</td>
<td>Motor offsets</td>
<td>Motor offsets</td>
</tr>
<tr>
<td>r</td>
<td>register</td>
<td>confirm</td>
<td>Motor increments</td>
<td>Motor increments</td>
</tr>
<tr>
<td>n</td>
<td>register</td>
<td>confirm</td>
<td>Position calculation constants</td>
<td>Position calculation constants</td>
</tr>
<tr>
<td>p</td>
<td>register</td>
<td>confirm</td>
<td>Motor controller constants</td>
<td>Motor controller constants</td>
</tr>
<tr>
<td>f</td>
<td>register</td>
<td>confirm</td>
<td>Spectra constants</td>
<td>Spectra constants</td>
</tr>
<tr>
<td>b</td>
<td>register</td>
<td>value</td>
<td>Get current sampling rate</td>
<td>Get current sampling rate</td>
</tr>
<tr>
<td>d</td>
<td>register</td>
<td>confirm</td>
<td>Emergency limits</td>
<td>Emergency limits</td>
</tr>
<tr>
<td>v</td>
<td>register</td>
<td>confirm</td>
<td>Observer constants</td>
<td>Observer constants</td>
</tr>
<tr>
<td>M</td>
<td>register</td>
<td>confirm</td>
<td>Set piezo voltages</td>
<td>Set piezo voltages</td>
</tr>
<tr>
<td>P</td>
<td>register</td>
<td>confirm</td>
<td>Fast controller constants</td>
<td>Fast controller constants</td>
</tr>
<tr>
<td>D</td>
<td>register</td>
<td>confirm</td>
<td>Camera timing Registers</td>
<td>Camera timing Registers</td>
</tr>
<tr>
<td>B</td>
<td>register &amp; run</td>
<td>confirm</td>
<td>Camera, FPGA</td>
<td>Update registers on high speed camera and FPGA</td>
</tr>
</tbody>
</table>

Calibration & Test Functions

<table>
<thead>
<tr>
<th>Run Type</th>
<th>Function</th>
<th>Actuators &amp; Sensors</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>run</td>
<td>none</td>
<td>Wait, do nothing</td>
</tr>
<tr>
<td>t</td>
<td>run</td>
<td>none</td>
<td>4 Photodiodes Startup warmup</td>
</tr>
<tr>
<td>h</td>
<td>run</td>
<td>none</td>
<td>Camera Sleep camera</td>
</tr>
<tr>
<td>e</td>
<td>run</td>
<td>none</td>
<td>2 Motors Stop and reset motors</td>
</tr>
<tr>
<td>u</td>
<td>run</td>
<td>none</td>
<td>2 Motors Update motor voltage with initial offsets</td>
</tr>
<tr>
<td>g</td>
<td>run</td>
<td>short ('s')</td>
<td>2 Motors, 2 Photodiodes Step response</td>
</tr>
<tr>
<td>k</td>
<td>run</td>
<td>short ('s')</td>
<td>2 Motors, 2 Photodiodes Ramp response with 2 photodiodes</td>
</tr>
<tr>
<td>q</td>
<td>run</td>
<td>short ('s')</td>
<td>2 Motors, 2 Photodiodes Step position control with 2 photodiodes</td>
</tr>
<tr>
<td>G</td>
<td>run</td>
<td>none</td>
<td>4 Piezos Update piezo voltage with initial offsets</td>
</tr>
<tr>
<td>K</td>
<td>run</td>
<td>long ('S')</td>
<td>2 Motors, 4 Photodiodes Ramp response with 4 photodiodes</td>
</tr>
<tr>
<td>Q</td>
<td>run</td>
<td>long ('S')</td>
<td>2 Motors, 4 Photodiodes Step position control with 4 photodiodes</td>
</tr>
<tr>
<td>L</td>
<td>run</td>
<td>long ('S')</td>
<td>2 Motors, 4 Piezo, 4 Photodiodes Ramp and step response for combinations of motor and piezos</td>
</tr>
<tr>
<td>U</td>
<td>run</td>
<td>long ('S')</td>
<td>2 Motors, 4 Piezo, 4 Photodiodes Fast step position control</td>
</tr>
<tr>
<td>R</td>
<td>run</td>
<td>long ('S')</td>
<td>4 Piezo, 4 Photodiodes Piezo only position control</td>
</tr>
<tr>
<td>Y</td>
<td>run</td>
<td>long ('S')</td>
<td>2 Motors, 4 Piezo, 4 Photodiodes Run stochastic system ID on motor or piezos</td>
</tr>
<tr>
<td>V</td>
<td>run</td>
<td>none</td>
<td>Runs feed-forward and LPF calculations</td>
</tr>
<tr>
<td>N</td>
<td>run</td>
<td>none</td>
<td>Report input shaping results</td>
</tr>
</tbody>
</table>

Unsynchronized Functions

<table>
<thead>
<tr>
<th>Run Type</th>
<th>Function</th>
<th>Actuators &amp; Sensors</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>z</td>
<td>run</td>
<td>short ('s')</td>
<td>2 Motors, 2 Photodiodes Initialize all controller constants</td>
</tr>
<tr>
<td>s</td>
<td>run</td>
<td>short ('s')</td>
<td>2 Motors, 2 Photodiodes Run spectra by increment</td>
</tr>
<tr>
<td>i</td>
<td>run</td>
<td>max-min short ('s')</td>
<td>2 Motors, 2 Photodiodes Go to initial startpoint</td>
</tr>
<tr>
<td>t</td>
<td>run</td>
<td>none</td>
<td>2 Motors, 2 Photodiodes Keep controller on to hold position</td>
</tr>
<tr>
<td>T</td>
<td>run</td>
<td>long ('S')</td>
<td>2 Motors, 2 Photodiodes Initialize all controller constants</td>
</tr>
<tr>
<td>s</td>
<td>run</td>
<td>long ('S')</td>
<td>2 Motors, 4 Photodiodes Run spectra by increment</td>
</tr>
<tr>
<td>I</td>
<td>run</td>
<td>max-min long ('S')</td>
<td>2 Motors, 4 Photodiodes Go to initial startpoint</td>
</tr>
<tr>
<td>A</td>
<td>run</td>
<td>none</td>
<td>2 Motors, 4 Photodiodes Keep controller on to hold position</td>
</tr>
<tr>
<td>V</td>
<td>run</td>
<td>none</td>
<td>Camera Slow camera integration turn on, change setting</td>
</tr>
<tr>
<td>W</td>
<td>run</td>
<td>none</td>
<td>Camera Slow camera integration turn off</td>
</tr>
</tbody>
</table>

Camera Synchronized Functions

<table>
<thead>
<tr>
<th>Run Type</th>
<th>Function</th>
<th>Actuators &amp; Sensors</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>start interrupt function</td>
<td>short ('s')</td>
<td>Camera, Interrupt, 2 Motors, 2 Photodiodes Camera and continuous report on</td>
</tr>
<tr>
<td>C</td>
<td>start interrupt function</td>
<td>long ('S')</td>
<td>Camera, Interrupt, 2 Motors, 4 Photodiodes Camera and continuous report on</td>
</tr>
<tr>
<td>@</td>
<td>start interrupt function</td>
<td>micro (4 bit)</td>
<td>Camera, Interrupt, 2 Motors, 4 Photodiodes Camera and continuous report on</td>
</tr>
<tr>
<td>o</td>
<td>run</td>
<td>none</td>
<td>Interrupt Continuous report off</td>
</tr>
</tbody>
</table>

The settings. The user interface then compares the data that is sent and received and sends a warning if the two are not the same. This ensures that the settings are sent properly. The "value" type sends a value directly from the microcontroller to the user interface in a special messaging mode.
The next set of functions is for calibration and testing. They include functions that turn the camera on and off, run ramp responses, run step responses with different controllers, and run stochastic system identification algorithms and feed-forward calculations. Each of these functions starts by zeroing the control constants, computing the input command, checking the input command against safety limits, converting those commands to 16-bit variables and then sending those variables to the respective amplifiers via SPI. At the same time that data is being clocked out on the master output line, photodiode data is being clocked in on the master input line from the ADCs clocked at 21 MHz. The sampling rate can be adjusted by changing different register settings and can vary anywhere from 7.5 kHz (used for long integrations) up to 100 kHz (used for high speed control).

Once the photodiode data is received, it is directed to one of several different position calculation algorithms using function pointers. These functions use the photodiode calibration settings to compensate for the maximum and minimum values of the sensor as well as the relative phase between any two signals. The arctangent function with two arguments is then computed by first determining the appropriate quadrant of the solution. The two values are then divided, scaled and a 9-bit lookup table is used to return the desired value (10-bits if both positive and negative values are considered). This final value is then used in a simple phase unwrapping algorithm to produce the final position. The look-up table method is used in order to speed up calculations of the arctangent and is especially useful for processors without single cycle float point multiplication. The look-up table is, however, less accurate and has less resolution than the brute force calculation.

There are two types of output types listed here. Each function is typically 1000 samples long and can be run at a variety of sampling rates. The “short (‘s’)” type outputs a start-bit which starts with an ‘s’ followed by 16 bytes of data which
include information about two photodiodes, two motor amplifiers, desired position, measured position, and error codes. The “long (‘S’)” type has a start-byte of ‘S’ followed by 34 bytes of data which include information on four photodiodes, two motor amplifiers, 4 piezoelectric amplifiers, desired position, measured position, desired tip-tilt, measured tip-tilt and error codes.

The next set of functions control unsynchronized (not synchronized with the camera trigger) spectral acquisition functions that initialize the controller constants, go to the initial start point, increment the position on the controller and hold a position using the controller until interrupted. There are also slow camera integration algorithms that utilize the non-destructive readout function of the high speed camera. The running lengths of these functions are variable and can be short or as long as desired. The functions that go the initial start position have special calibration functions which determine the maximum and minimum of the photodiode signals on the microcontroller to auto-calibrate the position sensors and determine positioning errors. The “max-min short” and “max-min long” output modes for these algorithms are a special messaging mode that sends the maximum and minimum photodiode calibration values.

The last set of functions are for synchronized (synchronized with the camera trigger) spectral acquisition. These functions turn on the camera trigger interrupt on the microcontroller and run hold position and step position functions depending on the setting chosen. These functions are timed and will automatically exit after the desired number of cycles or frames acquired. The internal camera trigger interrupt can be turned off using an additional function. For synchronized positioning algorithms, the amount of time is relatively short so it is not possible to send all the values from each cycle back to the user interface. The information in “short (‘s’)” and “long (‘S’)” format is stored on a direct memory access (DMA) function for 9 samples out of a
single positioning cycle (one camera image), and clocked out of the DMA in a burst at the end of one positioning cycle. This provides good control and a low-resolution monitoring signal to the user interface.

For high speed positioning applications, both of the formats previously discussed were too long and could not be implemented at 2000 fps. Therefore, a new output type called “micro (4-bit)” was used. This format has a 4-bit start signal and contains information from one photodiode, two motor controllers, one grouped piezoelectric actuator, the desired position, the measured position, and a shortened error code. This data is only sent three times per positioning cycle for monitoring. Because the high speed positioning system is so fast, a single DMA cannot be used as there is a high incidence of one DMA being written to at the same time that it is being clocked out in burst mode from the previous cycle. Therefore, two DMA buffers are used in this mode. One DMA bugger is being written to while the other DMA buffer is being clocked out and the DMA buffers are swapped at the end of each sampling cycle.

These commands can be used effectively by the user interface to coordinate different sampling algorithms. Some of the coordinated motion algorithms are listed in Table B.2. This table shows the user interface coordinated unsynchronized motions in simple, normal and slow mode. It also shows the camera driven synchronized motions in simple, normal, extended and slow modes. Lastly, it shows the control sequences used for adaptive and recursive sampling algorithms.

The command list is organized as setup commands (used to set the initial constants and zero values), initialization commands (used to send the motor to the initial desired position), commands used for data collection (which are repeated until the desired number of samples is acquired, each set starts with a space to exit the previous algorithm), and finalization commands (used to stop the motors, turn the cameras back on and set the microcontroller back to standby mode). There are a few unique
Table B.2: Spectral acquisition function commands

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Setup Commands</th>
<th>Initialization Commands</th>
<th>Data Collection Commands (Repeated)</th>
<th>Finalization Commands</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ul Driven – Simple Mode</td>
<td>u z</td>
<td>i a</td>
<td>_ s a</td>
<td>_ e E w</td>
<td>2 photodiode control</td>
</tr>
<tr>
<td>Ul Driven – Normal Mode</td>
<td>u Z</td>
<td>i A</td>
<td>_ s A</td>
<td>_ e E w</td>
<td>4 photodiode control</td>
</tr>
<tr>
<td>Ul Driven – Slow Mode</td>
<td>u Z</td>
<td>i A</td>
<td>_ V S A W</td>
<td>_ e E w</td>
<td>4 photodiode control</td>
</tr>
<tr>
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commands in this table. For example, the slow mode uses two special functions ‘V’ and ‘W’ for setting and clearing the NDR mode. The extended mode uses an ‘M’ function which sets the registers for the initial offsets of the piezoelectric actuator and the ‘G’ function which initializes the piezoelectric actuator voltages.

The adaptive and recursive sampling algorithms both use the same set of functions. For the adaptive algorithm, the sampling spacing vector is sent after the command ‘f’ with the spacing determined by the calculated spectral occupancy. The recursive algorithm also uses this vector but will use constant spacing and will only change the sign of the value to scan the same region of the spectrum back and forth. Other combinations of the algorithms on the microcontroller can be used through a command prompt or through the UI for other advanced functions.

B.2 User Interface Design

In order to develop and debug the experimental system, a user interface was designed to help run experiments and gather data quickly. The user interface system was developed in C# and uses Math.Net Numerics for complex mathematical operations.
The C# programming environment was chosen due to its availability (free to use) and ability to handle both pointers and objects. Because the .NET framework is a Windows based software platform, the code developed here cannot be applied to other operating systems, but the overall architecture can be reworked for other platforms. Because a real-time operating system is not being used for the user interface, additional considerations regarding hand-shake times must be taken into account.

B.2.1 Main Window

Figure 3.24 shows the main user interface screen. The top left set of buttons allows the user to connect to the USB device data streams via the FTDI device drivers (the FTD2XX.DLL D2XX Direct Drivers for C#). The main program where data is grabbed from the data and video streams (check boxes) can be started or stopped. The data displayed in the graphs and status boxes can also be cleared.

Below this area is a box showing the data statistics and box controlling the saving and loading of files such as BIN files for raw image data (“Save BIN Files”), compressed AVI video (“Record”), and raw graph data in XML format (“Save Data” and “Load Data”). The last large box on the bottom left shows the connection status and program debugging outputs of algorithms running in the user interface software.

The “Microcontroller Command Status” shows a list of commands that were sent to the microcontroller and a status message indicating if the data was properly received. To the right of this is a section which allows the user to send raw commands as ASCII characters for fast debugging. Below this is a series of tabs that provide settings for a variety of different registers on the microcontroller. The information for these registers is indicated in Figure B.1. Constants for the photodiodes, motor, piezo, motor controller, tip-tilt controller, fast positioning hybrid controller, phase observer and safety limits and timing parameters are controlled from this bank of
Figure B.1: The user interface for the main window options is shown. This includes a) the photodiode calibration constants, b) the motor offsets and increments, c) the piezo offsets and ramping parameters, d) the controller constants for linear motor controller, e) the constants for the tip-tilt controller and coordinate rotation system, f) the constants for the fast position hybrid controller, g) the constants for the phase observer, and h) the safety limits and timing parameters.

tabs. The user interface saves this information in a user parameters file when they are changed.

The most common calibration functions are shown in the column of buttons in Figure 3.24 where all the microcontroller registers can be updated (“Send Constants”), a ramp response can be sent for calibrating the analog encoder system (“Ramp Voltage”), and the closed-loop controller constants can be checked for an input step (“Send Position”). The “Calculate Position” function with the “Update” check box checked
allows the user to automatically update the system variables based on a calibration ramp. The “Run Observer” function updates additional parameters associated with the phase observer. Lastly, the “Emergency Stop” button ends algorithms controlled by the user interface to end spectral acquisitions.

The block of graphs in the middle of the interface show the data received from the hardware. The top left graph shows the raw signal from the photodiodes used for the analog encoder. The graph directly beneath shows the voltage commands to the motor and piezoelectric actuator amplifiers. The following graph shows the desired and measured position of the mirror. Lastly, the bottom graph shows the calculation of the relative change in phase of the signals to determine mirror tip and tilt using the phase observer algorithm.

The graph on the top right shows the intensity of signals as a function of the position at which they are acquired. The graph beneath it shows the Fourier transform (transform calculated by any algorithm) of the data acquired from images. By left clicking on any part of this graph, it is possible to determine the intensity and wavelength of that location. By right clicking, it is possible to snap to the nearest data point and display its value. The series of options below this graph allow the user to change the axis maximum and minimum for any of the graphs and allow the user to show or hide the legends for each graph.

The top right area contains options for the acquisition of spectral data. The “Snapshot Number” box sets the number of images to acquire for a particular test and the progress bar indicates the status of the acquisition. Three tracking points can be chosen from the image by first clicking on any desired pixel in the image and then choosing “Track Point 1” or any of the other two buttons.

The buttons directly beneath this area run the different spectral acquisition algorithms. The “Run Spectral Series” algorithm is completely user interface driven in
that the user interface asks for each picture to be taken and waits for the data to be acquired and process the data before asking for a new picture. This algorithm is slow, dominated by the handshake time between the computer and the microcontroller, and operates at around 3 fps. The next button “Fast Spectra” is a microcontroller driven algorithm that positions the mirror and acquires images on the camera trigger at the speed of the camera frame capture (up to 2000 fps or more) and downloads the data to the user interface. The “Slow Spectra” button is also a microcontroller driven algorithm that operates using the non-destructive readout functions of the high speed camera. Three special spectral acquisition functions also exist including the “Adaptive Spectra”, “Recursive Spectra” and “Unequal Spacing” algorithms.

The box on the top right of the main user interface shows the options for computing the spectra from the interferogram. Options for intensity scaling, normalization, LS solver algorithm, wavelength maximum and minimum, data clipping, and use of measured versus desired positions are available. The two buttons beneath allow the user to compute a traditional discrete Fourier transform (“Calculate DFT”) or a brute force least squares Hartley transform with a single limit group from the maximum to minimum wavelength (“LS Solution”).

The real time video from the camera is shown directly below and the options for image zoom, clearing the image, and showing the image while doing spectral acquisition computations (un-checking this option increases the calculation speed) are also included.

### B.2.2 Observer Window

In addition to this main window, there are several other windows that can be accessed from the tool bar under “File” on the top left. One of these additional windows is the observer calculation window shown in Figure B.2. This window graphs the relative
Figure B.2: The user interface for the observer calculations window is shown.

phase diagrams of the four different photodiode signals (left), the predicted signal relative the measured signal (middle) and the predicted position, tip and tilt (right). Three plotting algorithms are listed. The “Plot Lissajou” algorithm is the most simple and is used for checking the relative phase of all the signals. The “Plot Observer” and “Plot Spectra” algorithms plot all the parameters from the phase observer algorithm and the final position, tip and tilt.

B.2.3 Video and Image Grabber Windows

The “Video Options” window shown in Figure B.3a allows the user to open videos, rotate the videos, compress the videos or convert the videos into a series of images. As the original saved AVI videos have some amount of compression, it is preferable to use the uncompressed BIN files for any external processing in Matlab. The “Image Grabber” window houses the options for the LUPA 300 high speed camera. The first column of options labeled “LUPA 300 Registers” on the left are the register values for controlling the image dimensions, the integration time, the image gains, and other parameters for the on-board ADCs. There is also an option box that allows the user to select and 8-bit or 10-bit image mode from the FPGA. The second column of options labeled “Microcontroller Options” are settings for controlling the timing of
the microcontroller driven spectra acquisition algorithm. The “Camera Parameters” column contains settings that indicate the image size, clock speed, integration cycles, and overhead bits of the incoming data. The parameters can be used to calculate the frame rate, bit rate, rise time, and integration time in the next two columns of data by clicking “Calculate.” The parameters are converted to hexadecimal parameters for the camera and other parameters for the microcontroller using the “Change Registers” button.

A sequence of images can also be grabbed from the camera to test different functions by clicking “NDR Grab”, “Grab Images” and “Process Images.” These images are displayed in the picture box below and each can be checked using the scrollbar at the bottom. The images can be saved as an image series, a set of BIN files, or as a video by using the buttons on the top right. A reference image can be recorded or loaded from file to eliminate the fixed pattern noise (FPN) from the camera data.

An example of FPN removal is shown in Figure B.4. The raw image from the camera contains noise from the on-board ADCs. This fixed pattern noise can be
removed by first blocking the input to the interferometer and capturing a black image. This information can then be subtracted from any raw image to form the final image where the noise has been eliminated and the image is clearly visible. A reference FPN image should be acquired for any group of camera settings.

B.2.4 System ID and Feed-Forward Generator Window

The “System ID and Feed-forward Generator” window in Figure B.5 is used for generating and calculating the feed-forward constants. The “Get Current Profiles” button sends a request to the microcontroller to download the current feed-forward profiles for the motor, piezoelectric actuator or hybrid controller depending on the options selected from the drop down menu to the right. The “Run System ID” button clears the plot windows and implements a stochastic system identification algorithm on the microcontroller and plots the input and output data in the main window (two graphs on the left). By clicking the “Load Test Data” button, the information is brought into the current widow and saved. Multiple tests can be conducted and loaded into this window for averaging. The algorithm also computes
Figure B.5: The user interface for the system identification window is shown.

the input autocorrelation, input-output cross correlation (graph in the top middle), and system impulse response (graph in the bottom middle). The "Run Parametric Fit" function uses the settings to the left to compute either the motor model or the piezoelectric actuator model parameters and plots the calculated impulse responses for the model, the feed-forward compensator and associated low-pass filter (graph in the top right). The simulated step feed-forward voltage input and response are shown in the graph on the bottom right. This data can be saved or loaded as XML files.

There are two separate feed-forward modes that can be run from this program. The first mode is a simple feed-forward mode where only rate and acceleration limiting is conducted. The second mode is the full inversion feed-forward mode which can be selected in the check box in the top left labeled “Run In Full Inverted FF
Figure B.6: The user interface for the adaptive sampling window is shown.

Mode.” In this mode, all the model parameters are used in a calculation of the feedforward profiles directly on the microcontroller. This method was selected to reduce the number of parameters sent between the user interface and microcontroller. The algorithm used on both the user interface and the microcontroller for this calculation first generates impulses responses that are then convolved with the rate limited position profile.

**B.2.5 Adaptive and Recursive Sampling Windows**

Adaptive sampling for spectrum acquisition is a complex algorithm that requires several matrix inversions. Therefore the matrix inversion component of the algorithm is handled by the user interface and the parameters for the algorithm are set in the “Adaptive Plots” window shown in Figure B.6. This window sets the initial baseband
wavelengths, the initialization samples, the maximum number of samples, the safety factors, the rejection ratios and the grouping size of the algorithm. This window also runs a simulation of the parameters using the “Run Simulation” button. Once the parameters are set, the “Adaptive Spectra” button on the main window can be pressed. This first acquires a group of data, uses the current occupancy and limits to calculate the spectrum (using the least squares Hartley transform with multiple limit groups) and predicts the new occupancy and therefore the next step size to use. The next set of step sizes is sent to the microcontroller and the algorithm repeats until the maximum number of steps or desired resolution is reached. By clicking “Plot Adaptive Data,” the user can plot the occupancy ratio, evolution of the step size, and the calculation time from the real experiment on the four graphs on the right. This data can be saved or loaded from an XML file.

The last window is used to control the parameters of the recursive spectral acquisition algorithm. The “Recursive Plots” window shown in Figure B.7 contains several settings for the wavelength limits, sampling range, maximum samples, group size, forgetting factor, and wavelengths to track. Simulations can be run with these parameters by clicking the “Run Simulation” button. Once the settings are satisfactory, the experiment can be run using the “Recursive Spectra” button on the main window. This algorithm acquires a group of data, uses the data to update the recursive algorithm, and then grabs another set of data. The recursive algorithm information can be brought into the window by pressing the “Plot Recursive Data” button and the intensity evolution and noise evolution can be plotted on the two graphs on the right. The algorithm records the spectrum evolution after every group of data is acquired in a set of arrays. This information can be played as a video by clicking “Replay Recursive Data.” The final data can be saved or loaded as an XML file and the replay video can be saved as a video or as the original series of arrays.
Figure B.7: The user interface for the recursive sampling window is shown.