There are several notions of necessity that could be driving our intuitions here. Truths of logic and mathematics are necessary in every sense. Likewise conceptual truths like “Nothing is bigger than itself.” Other truths are necessary in a sense, but not the sense we’re interested in.

“The volcano will erupt” – necessary in the sense of guaranteed by initial conditions and laws of nature, but not in our sense, because initial conditions could have been different. (Inevitability)

“Salt dissolves in water” – necessary in the stronger sense of guaranteed just by the laws, but not necessary in our sense, because (it seems) we could have had different natural laws. (Nomic necessity)

“I have hands” – necessary in the sense that I am as sure as can be, but not necessary in our sense; the hands could be an illusion. (Certainty)

“I exist” – necessary in the sense that I can know it just by the light of reason, with no appeal to experience, but not necessary in our sense; God may exist necessarily but not me. (A priori)

Even if not all a priori truths are necessary, someone might think that necessary truths have to be a priori knowable. (Why?) Clearly not all necessary truths are known a priori. Take an arithmetical hypothesis that we haven't yet proved or refuted. Either H is necessary or ~H is. Either way there's a necessary truth we don't know, and so don't a priori know. But perhaps all necessary truths are knowable a priori? One possible counterexample is undecidable arithmetical truths. (Maybe still knowable since a superior being could go through the integers one by one.) Kripke rejects this direction too. Examples of necessary aposteriori truths?

So far been talking about de dicto necessity, or necessity as it applies to truths; now we'll look at de re necessity, or necessity as it relates to things and their properties. “Necessarily, all cats are animals” states a de dicto fact about the embedded dictum (all cats are animals). But now here is a de re fact about my cat: it is necessarily a cat, it is impossible for it to lack that property.

I have highlighted the distinction syntactically, by putting “necessarily” first in the de dicto sentence and between the subject and predicate in the de re sentence. But this is not required: either could have been written either way. The key is in how you read the sentence. So, “All presidents are necessarily officeholders” has the "necessarily" in the middle but to the extent we find it true, we're reading it de dicto. “Necessarily, my favorite furball is a cat” has the "necessarily" at the beginning but to the extent we find it true, we're giving it a de re reading. Often the very same sentence will bear either reading.

“Necessarily, the inventor of bifocals invented bifocals,” read de dicto this expresses the truth that there is no way that someone who uniquely invented bifocals to at the same time not invent them. Read de re, it expresses the falsehood that Benjamin Franklin had to invent bifocals.

A lot of people are happy with de dicto modality but have trouble with de re. Why? De re modality leads naturally to essentialism: the theory that of a thing's properties, some are essential – it couldn't have existed without them – and others are accidental – it could have existed without them. Some find this implausible or even unintelligible. Suppose we say “Socrates is necessarily human.” This is OK because it's naturally read de dicto as attributing necessity to the claim that Socrates is human. If we read it de re, however it implies that for some x, x is necessarily human. It's not Socrates so-conceived has to be human. There’s a thing x that however conceived has this property necessarily. That may seem
problematic. Suppose as some people think that \( x = \text{Socrates} \) is a certain body, or as a certain bunch of particles. Conceived as \( \text{Socrates} \), \( x \) seems essentially human; but conceived as the bunch of particles he doesn't.

Here are three specific objections, variations on a theme. First is from "reductionism." Common to hold that things of one type reduce to or "boil down to" things of another type. So, tables are hunks of matter, people are a kind of animal, and so on. Take numbers are just certain sets, for instance, \( 2 = \{\{\}\} \). The problem is this. Numbers are thought to have their arithmetical properties, e.g., primeness, essentially. But the same is not true of sets; it makes no sense to say a set is essentially prime. Yet by Leibniz's Law, identicals have the same properties. BUT: it's not really essentialism that leads to these problems, because it makes equally little sense to say that \( x \) is set \( x \) is prime full stop. One response is to say: OK, so numbers are not sets. Another is to say: well "prime" as applied to sets goes via the numbers they are identical to. If a given set is the number 7, then some sets \( \text{are} \) prime.

Second objection, due to Kneale and Quine, has to do with description-relativity. According to them, there is no such thing as \( x \) being essentially \( P \) all by itself, there is only \( x \) being \( P \) relative to description \( D \). Here's the trouble you get in if you deny this.

(a) 12 = the number of apostles
(b) 12 is necessarily composite.
(c) The number of apostles is necessarily composite.

It's supposed to be obvious that (c) is false. If it's obvious that's because you're reading (c) de dicto rather than de re. The de re reading is: if \( x = \text{the number of apostles} \), \( x \) is necessarily composite. And that seems right.

Third objection has a similar spirit. Best to read the whole quote from Quine:

perhaps I can evoke the appropriate sense of bewilderment as follows. Mathematicians may conceivably be said to be necessarily rational and not necessarily two-legged. And cyclists necessarily two-legged and not necessarily rational. But what of an individual who counts among his eccentricities both mathematics and cycling? Is this concrete individual necessary rational and contingently two-legged or vice versa? Just insofar as we are talking referentially of the object, with no special bias towards a background grouping of mathematicians as against cyclists, or vice versa, there is no semblance of sense in rating some of his attributes as necessary and others as contingent. Some of his attributes count as important and others as unimportant, yes, some as enduring and some as fleeting; but none as necessary or contingent."

Formally

(a) mathematicians are necessarily rational but not necessarily bipedal
(b) cyclists are the reverse
(c) Paul Zwier is both a cyclist and a mathematician.
(d) PZ is both necessarily rational and not necessarily rational.

But does (d) really follow? Again there's a scope confusion. Argument works only by getting us confused between "necessarily all As are Bs" and "each A is necessarily B." (Compare: "If you know something it's got to be true; so knowledge is only of necessary truths – contingent truths are unknowable.")