Imaging of near-surface heterogeneities by scattered elastic waves

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ABSTRACT

We have developed an elastic reverse time migration (RTM) approach for imaging near-surface heterogeneities, such as karst features, using scattered waves (e.g., body to P-, S-, and surface waves). Knowledge of location and strength of the scatterers helps in seismic imaging, survey planning, and geotechnical site characterization. To model seismic wave propagation for RTM, we use an elastic staggered-grid finite-difference scheme. The scattered body-to-surface waves provide optimal illumination and wavenumber coverage of the near surface as they travel horizontally along the free surface. We tested the elastic RTM approach on synthetic data simulated using a finite-difference solver and found it to be robust.

INTRODUCTION

Near-surface objects, such as karst features, act as strong scatterers and affect the quality of seismic data used for subsurface imaging. Determining, locating, and imaging of such features are important for deep and shallow reflection imaging for resource exploration and geotechnical site characterization, respectively. In this paper, we describe a method of imaging the scattering objects using scattered body and surface waves.

In general, depth migration algorithms are categorized as ray-based (e.g., high-frequency asymptotic methods such as Kirchhoff and beam migration) and wave-equation-based (e.g., one-way and two-way wave-equation-based migrations). The concept based on two-way wave equation migration is known as reverse time migration (RTM) (Baysal et al., 1983; Loewenthal and Mufti, 1983; McMechan, 1983; Whitmore, 1983). Although RTM is computationally expensive, it is more attractive than other imaging algorithms because it can handle multiple arrivals and overturned waves, and it has no restrictions with respect to the complexity of the velocity model or the dip of the structure.

RTM schemes based on the acoustic-wave equation have been more widely used than elastic RTM for imaging complex geologic structures due to the lower computational cost. The earth, however, is elastic, and the data recorded in the field contain all wave types, including P-wave, S-wave, converted, etc. In recent years, there has been more interest in exploiting all the information carried by mode-converted seismic data by using elastic RTM. Sun et al. (2006) introduce a modified RTM approach of transmitted P-to-S converted (PS) waves for salt-flank imaging. Their approach separates the wavefield into pure mode PP- and PS-waves, and the extrapolation is performed using the scalar wave equation with the corresponding P- and S-wave velocities. A similar strategy is proposed by Xiao and Leaney (2010) for salt-flank imaging with vertical seismic profiling, local elastic RTM, and using the vector wave equation to extrapolate the separated PP- and PS-waves. Shang et al. (2012) use teleseismic transmitted P- and S-waves recorded on the surface to perform passive-source RTM to reconstruct steeply dipping interfaces, an approach superior to traditional receiver-function analysis in complex geologic environments.

To address the problem of imaging near-surface scatterers, several studies have formulated solutions of the inverse scattering problems. Blonk et al. (1995), Blonk and Herman (1996), Ernst et al. (2002), and Kasliilar (2007) use a perturbation method based on the Born approximation (single scattering). These methods have difficulties when dealing with large and high-contrast heterogeneities that violate the Born approximation. Campman et al. (2005, 2006) use an inverse scattering approach based on an integral-equation formulation to image the near-surface heterogeneities, but they assume that scattering takes place immediately under the receivers. Other methods, based on solving integral equations using the method of moments, can handle strong contrast and large heterogeneities and can take into account multiple scattering (Riyanti and Herman, 2005; Campman and Riyanti, 2007). However, these
we test the proposed elastic RTM approach on data simulated with the shot-reciever wavefield (recorded multicomponent data) as boundary condition. This equation can be separated into scalar and vector potentials by the Helmholtz decomposition, which applies to the vector field $\mathbf{u}$:

$$\mathbf{u} = \nabla \phi + \nabla \times \mathbf{\varphi},$$

(2)

where $\phi$ and $\mathbf{\varphi}$ are the scalar and vector potentials of the wavefield $\mathbf{u}$, respectively. Substituting equation 2 into equation 1 and applying vector identities gives the equations for the P-wave potential:

$$\nabla^2 \phi - \frac{1}{\alpha^2} \frac{\partial^2}{\partial t^2} \phi = 0$$

(3)

and the S-wave potential:

$$\nabla^2 \mathbf{\varphi} - \frac{1}{\beta^2} \frac{\partial^2}{\partial t^2} \mathbf{\varphi} = 0,$$

(4)

where $\mathbf{\varphi}$ is one component of the vector potential in equation 2, and $\alpha$ and $\beta$ are the P- and S-wave velocities, respectively.

For reverse time continuation, the data (i.e., the separated scattered wavefield) are reversed at the corresponding receiver locations and injected as sources (e.g., $v_x$- and $v_z$-components) into the computational domain using an elastic-wave equation solver. The P-wave mode is separated at each wavefield-extrapolation step by taking the divergence of the wavefield $\nabla \cdot \mathbf{u}$, and an imaging condition

$$\alpha^2 \frac{\partial^2}{\partial t^2} \phi - \frac{1}{\beta^2} \frac{\partial^2}{\partial t^2} \mathbf{\varphi} = 0,$$

(5)

as boundary conditions.

In this paper, we present a prestack elastic RTM for locating and imaging near-surface scatterers. The main idea is to separate the near-surface scattered waves from the total recorded wavefield and to use the scattered waves for receiver wavefield extrapolation.

METHODOLOGY

The main idea underlying elastic RTM for imaging near-surface scatterers is to (1) forward propagating the reference wavefield (modeled data with the estimated source wavelet), (2) back propagating the scattered wavefield (recorded multicomponent data) as boundary conditions, and (3) separating the reference and scattered wavefields into P-wave components (i.e., divergence of the wavefield) before the imaging condition is applied at each image location. The reference and scattered wavefields are also referred to as the source and receiver wavefields, respectively.

The multicomponent source and receiver wavefields are extrapolated in time by solving the seismic elastic-wave equation in isotropic elastic media as

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} - (\lambda + 2\mu) \nabla (\nabla \cdot \mathbf{u}) + \mu \nabla \times (\nabla \times \mathbf{u}) = \mathbf{f},$$

(1)

where $\mathbf{u}$ is the displacement vector, and $\mathbf{f}$ is a force term (e.g., source wavelet or receiver wavefield injected as a boundary condition). This equation can be separated into scalar and vector potentials by the Helmholtz decomposition, which applies to the vector field $\mathbf{u}$:

$$\mathbf{u} = \nabla \phi + \nabla \times \mathbf{\varphi},$$

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where $\phi$ and $\mathbf{\varphi}$ are the scalar and vector potentials of the wavefield $\mathbf{u}$, respectively. Substituting equation 2 into equation 1 and applying vector identities gives the equations for the P-wave potential:

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Figure 3. Finite-difference simulations showing the (a-c) $v_z$-component, (d-f) divergence, and (g-i) curl; panels (a, d, g) show the reference wavefield simulated using the model without scatterers, panels (b, e, h) show the total wavefield simulated using the model with scatterers, and panels (c, f, i) show the scattered wavefield. The color scales indicate amplitudes. The source is located at $(x, z) = (150, 0 \text{ m})$. 

Imaging by scattered elastic waves
is applied to form an image of the scatterers. The zero-lag cross-correlation imaging condition is defined as follows:

\[ m(x) = \sum_{\text{shots}} \int_{0}^{t_{\text{max}}} S(x, t) R(x, t) \, dt, \quad (5) \]

where \( m \) is the value of the migration image at a spatial location \( x \) and \( S \) and \( R \) are the P-wave modes of the forward- and time-reversed wavefields. The imaging condition computes the scalar product of the two wavefields at each time step and summation over all time levels and shot locations.

For simplifying the imaging condition, the S-waves are not included by taking the divergence of the wavefield. However, the P-wave component of the Rayleigh waves and upcoming mode-converted reflections (e.g., SP, SSP, SPP, and PSP) are incorporated in the imaging condition. The reflections of not only P- but also S-waves from the deeper dipping interfaces provide superior angle coverage, and the use of Rayleigh waves plays a key role in broadening the illumination angle to 180°.

### NUMERICAL TESTS

In this section, we demonstrate the application of elastic RTM to synthetic data calculated using elastic finite-difference modeling. We consider a 2D earth model with multiple dipping layers and five scatterers embedded in the uppermost layer (Figure 2). The scatterers are located at 15-m depth below the free surface, and each has a 10-m diameter and an impedance contrast corresponding to 0.36. The material properties are given in Table 1. A vertical point source is used with a Ricker wavelet and 30-Hz center frequency (approximately 75-Hz maximum frequency). The simulations are carried for 19 sources located at the surface with 50-m space intervals. The receivers are at the surface and placed at intervals of 1 m. To image the near-surface scatterers, we assume the reflected body-wave arrivals as the reference wavefield (i.e., the source wavefield).
field), as the direct body and surface waves are modeled and removed during the wavefield extrapolation. This implies that the interfaces (i.e., reflectors) act as seismic sources (Figure 1) for the incident waves on the scatterers. The receiver wavefield (i.e., reflected body waves scattered to body and surface waves) is obtained by first modeling and then subtracting the reference wavefield from the total wavefield (Figure 3).

In Figure 4, snapshots of the $v_z$-component, divergence (P-waves), and curl (S-waves) of the incident and scattered waves are shown. The snapshots clearly show the wavefield decomposition into P- and S-waves by applying the divergence and curl operators. However, it is difficult to identify the separated phases in the shot-gather domain in the case of surface receivers. This is due to mode conversion occurring right at the free surface where the receivers are placed. For example, we can observe weak S-waves in the shot-gather domain (the curl of the wavefield as shown in Figure 3i) recorded with similar slopes to the P-wave arrivals. These wave phases are scattered P-waves converted to S-waves right at the free-surface boundary. The same is also true in the case of the scattered S-wave arrivals converted to P-waves at the free surface and recorded with similar slopes to the S-wave phases (as shown by the divergence of the wavefield in Figure 3f).

The near-surface scatterers’ image is formed by applying the imaging condition (equation 5) to the P-wave components of the extrapolated forward reference and backward-scattered wavefields stored at each time step (Figure 5a). The wavefield extrapolation is carried out using a velocity model with no a priori information of the scatterers, and the image is constructed when the near-surface scattered waves are in-phase with the incident waves at the scatterer locations. All the scatterers are imaged and located accurately. However, the image can be distorted due to limited illumination aperture (Figure 5b) in case the recorded scattered wavefield is simulated with an absorbing boundary at the surface, in which free-surface multiples and scattered body-to-surface waves are not included in the imaging process.

Because the reflected signal is used as the source wavefield, the subsurface interfaces as expected, are not accurately imaged by RTM. The only exceptions, however, are very weak scattered body-to-body waves reflected from the deep interfaces and recorded on the surface. These recorded phases can slightly contribute to imaging the deep reflectors. In general, artifacts in the RTM image can be due to many factors, including the imaging condition, injection of the receiver wavefield as a boundary condition for backward extrapolation, and the one-sided coverage of the receivers.

Previously (Almuhaidib and Toksöz, 2014), we have studied the near-surface scattering of body-to-surface waves and demonstrate the effects of source characteristics and near-surface perturbations (e.g., volume and interface heterogeneities). Body-to-body scattered waves, in general, require illumination from all angles (i.e., receivers surrounding the computational domain) to achieve true relative amplitude migration. In contrast, scattered body-to-surface waves travel horizontally along the free surface and attenuate less with distance than body waves. Therefore, they are recorded by all receivers on the surface and provide optimal illumination of the near-surface layers. However, because the amplitude of surface waves decays exponentially with depth, only near-surface heterogeneities that are close to the free surface (e.g., shallower than one wavelength) can be illuminated and imaged by the scattered body-to-surface waves. As a result, the intensity of the imaged scatterers with scattered surface waves decreases with depth, and therefore, body-to-body-wave scattering contributes more to the image.

**DISCUSSION**

The results presented in this paper demonstrate the feasibility of imaging and locating near-surface heterogeneities using waves scat-

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**Figure 5.** Elastic RTM of near-surface scattered waves with receivers placed on the surface; panel (a) shows imaging the near-surface scattered body to P-, S-, and surface waves, panel (b) shows imaging without free-surface multiples and only scattered body-to-body waves are included (e.g., the recorded scattered wavefield is modeled with an absorbing boundary at the surface), panel (c) shows the same as panel (a) but with almost 10% error in the upper layer’s velocity ($\alpha = 2000$ m/s and $\beta = 1100$ m/s instead of $\alpha = 1800$ m/s and $\beta = 1000$ m/s), panel (d) shows imaging with the back scattered Rayleigh-to-Rayleigh waves included. The color scales indicate normalized amplitudes.
tered at the near surface. There are, however, some remaining questions that may cause difficulties in field data application and need to be answered. In this section, we will address two important questions.

The first question is how to obtain the velocity model for waveform extrapolation. Indeed, a precise depth image depends on the ability to accurately estimate the subsurface velocity model especially for field data. As shown in Figure 5c, small errors in the velocity model can significantly defocus the image results. In such cases, an extension of our approach to update the velocity model based on least-squares migration or waveform inversion would be essential.

The second question is how to perform the wavefield separation in case the velocity model is not accurately estimated. The answer to this question is that scattered body-to-surface waves exhibit different slopes than body-wave reflections, which make them easier to separate (e.g., using a velocity filter) for subsurface imaging. In a previous study, Almuhaidib and Toksöz (2015), we develop a data-based approach (e.g., using a velocity filter) for subsurface imaging. In a pre

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