POST CRITICAL HEAT FLUX HEAT TRANSFER

by

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ABSTRACT

An experimental and theoretical analysis of the post critical heat flux heat transfer has been performed. The transient experimental technique included a long tubular preheater section for creating a dispersed flow and a short tubular transient test section for collecting the heat transfer data (heat flux vs. wall superheat data). Liquid nitrogen was used as a test fluid. The mass velocities varied from 30,000 to 220,000 lb/ft²hr.

Dispersed flow (a flow pattern associated with post critical heat flux heat transfer) consists of liquid drops dispersed in a continuous vapor phase. The theoretical study included: the analysis of the structure of a dispersed flow (the analysis of a drop size and drop size distribution); the analysis of the deposition motion of liquid drops (the migration of drops toward the wall); the analysis of the possible successive states of drops-wall interaction, and heat transfer to a drop deposited on the heated wall.

Based on the above analyses the expression for the heat flux from the wall to dispersed flow has been developed. It includes the heat flux to liquid drops deposited on the wall, the heat flux to vapor component of the flow, and radiative heat flux between the wall and dispersed flow. The developed expression for the heat flux has given good agreement with the experimental data.

The work, presented in this study, was undertaken as the post critical heat transfer has come to importance in recent years due to advancements in various technologies, such as in cryogenics, materials, rocketry, and especially in design and safety analysis of nuclear reactors.

Thesis Supervisor: Warren M. Rohsenow
Title: Professor of Mechanical Engineering
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F

G

h

H

k

K

N

Nu

P(a)

Pr

Q

q/A

Re

S

S

S

t

T

U

U

V

V

We

x

reaction force [Eq. (4.38)], lb
mass flux, lbm/ft\textsuperscript{2}hr
heat transfer coefficient, Btu/ft\textsuperscript{2}hr-\textsuperscirc{°}F
latent heat of evaporation, Btu/lbm
conductivity, Btu/ft-hr-\textsuperscirc{°}F
constant [Eq. (4.15)]
drop deposition flux, \# drops/ft\textsuperscript{2}hr
Nusselt number
drop size distribution [Eq. (3.7)], 1/ft
Prandtl number
heat transferred to single drop, Btu
heat flux, Btu/ft\textsuperscript{2}hr
Reynolds number
slip ratio [Eq. (4.42)]
stability number [Eq. (3.2)]
time, s
temperature, °R, °F
gas or vapor velocity in the flow direction, ft/s
friction velocity [Eq. (4.44)], ft/s
gas or vapor velocity (vector), ft/s
drop velocity, ft/s
vapor velocity [Eqs. (4.26)-(4.30)], ft/s
Weber number [Eq. (3.1)]
axial distance, ft
x    vapor quality
y    radial distance, ft
f    cumulative (deposition) factor [Eq. (4.46)]
v_o  drop deposition velocity, ft/s [Eq. (4.44)], v_o = 0.15 \frac{G_x \sqrt{f}}{\alpha p g}
U_o  mean gas (vapor) velocity, ft/s (Note: U_o = V_g)
u_o  initial drop velocity in the flow direction at the edge of the laminar sublayer, ft/s [Eq. (4.41a)]

GREEK LETTERS

\alpha  void fraction
\gamma  weight density, lb/ft^3
\delta  boundary or laminar sublayer thickness, ft
\epsilon  thermal emissivity
\kappa  constant
\mu  viscosity, lbm/ft-hr
\mu m  micron
\rho  density, lbm/ft^3
\omega  angular velocity, rad/s
Stefan-Boltzmann constant, Btu/ft^2hr-°R^4
surface tension, lb/ft
SUBSCRIPTS

a  drop
b  drag
c  drop
e,E  equilibrium
g  gas or vapor
g  gravity
i  inertia
l  liquid
L  lift
m  maximum
mp  most probable
r  radiation
s  surface
s  saturation
sv  volume-surface
sat  saturation
v  vapor
v  volume
w  wall or surface
wl  wall-liquid
wv  wall-vapor
CHAPTER I: INTRODUCTION

1.1 Post Critical Heat Flux Heat Transfer

The formation of a two-phase mixture by vapor generation in a vertical heated tube is shown in Figure 1.1. The presence of a heat flux through the tube wall alters the flow pattern from that which would have occurred in a longer unheated tube at the same local flow conditions [6]. At some position up the uniformly heated tube (Figure 1.1) the wall temperature exceeds the saturation temperature and vapor bubbles grow at the wall surface and by detaching form a bubbly flow. The bubble population increases with length and coalescence takes place forming slug flow. An annular flow then forms along the tube where now vapor generation is a result of evaporation at the liquid film-vapor core interface. Increasing velocities in the vapor core will cause entrainment of liquid in the form of drops. The depletion of the liquid from the film by this entrainment and by evaporation finally causes the film to dry out completely [6]. Drops continue to exist and are slowly evaporated until only single-phase vapor is present, Figure 1.1. The region between the dryout point and the transition to dry vapor has been termed the post critical heat flux heat transfer region (the post

†Numbers in parentheses refer to references, found on p. 122
critical heat transfer region, the liquid deficient region, the dispersed flow heat transfer region; the terms are used interchangeably), and is associated with the dispersed flow pattern. Figure 1.1 shows, in diagrammatic form, the various flow patterns in addition to dispersed flow patterns encountered over the length of the tube, together with the heat transfer regions and wall and fluid temperatures variation and heat transfer coefficient variation. When dryout point is reached the wall temperature undergoes a sudden increase (Figure 1.1) due to a fundamental transition in the mechanism of heat transfer (the heated wall is no longer wetted by a thin liquid film through which heat is very efficiently carried away from the wall by forced convection in the film to the liquid film-vapor core interface, where evaporation occurs).

Dispersed flow is a particular but very important configuration of two-phase flow. In this configuration the liquid phase is the discontinuous phase, in form of drops, dispersed in the continuous vapor phase. The drops dimensions are very small, up to a few hundred microns, and therefore the surface tension acts in such a way that the drops behave as rigid spheres [10]. They are impelled by the vapor into a high turbulent motion; their velocity has transversal components, and the drops migrate toward the heated wall. Actually, only in the case of a heated wall does dispersed flow exist since in the case of an adiabatic wall, the drops deposited on the wall will form a thin liquid layer, changing the dispersed flow pattern into the annular flow pattern.

There is clear evidence [65,32] of a significant contribution of the
liquid drops to the heat transfer in dispersed flow but less clear are the mechanisms of this transfer. The mechanism of heat transfer to dispersed flow at wall temperatures below the Leidenfrost temperature is transition boiling, known as a mechanism where nucleate and film boiling both exist side by side. For the case where wall temperature is above Leidenfrost temperature the film boiling is the mechanism of heat transfer associated with dispersed flow. In this region the radiation heat transfer can be also significant, especially if the wall temperature is very high.

Post critical heat transfer has come to be important in recent years due to advancements in various technologies, such as in cryogenics, materials, rocketry, steam generators, and nuclear reactor design and safety. Much of the current interest in post critical heat transfer is due to its role in analysing and predicting heat transfer during a postulated loss of coolant accident in nuclear reactors.
Figure 1.1 Regions of Heat Transfer in Convective Boiling (Uniformly Heated Tube, Low Heat Flux)[6,68]
1.2 Literature Survey

Post critical heat transfer has been studied experimentally. Experimental data for steam-water mixtures in the pressure range 2000-3500 psi were published by Schmidt (1959), Swenson et al (1961), and Herkenrath et al (1967). Their data indicated the characteristic sharp increase in wall temperature at the dryout point followed by decreasing wall temperature as the vapor velocity increases with increasing vapor quality (Figure 7.11, ref. [6]). Wall temperatures increase once again in the superheat region as a result of the increase in bulk fluid temperature [6].

Similar studies to the above for cryogenic fluids flowing under forced convection in circular ducts are reported by Walters (1961), von Glahn and Lewis (1960), and Lewis et al (1962).

A number of empirical equations have been presented by various investigators for the estimation of heat transfer in the post critical region. Almost all of these correlations are modifications of the well-known Dittus-Boelter type relationship [6] for single-phase flow where various definitions of the "two-phase velocity" and physical properties are used. These correlations are listed in Table 1 of reference [24]. They generally predict a heat transfer coefficient based on the temperature difference between wall and saturation. As mentioned by Groeneveld (1975) they are simple to use but have a limited range of validity and should not be extrapolated outside the recommended range. Groeneveld's correlation, developed from post critical water data for tubes and annuli, is analyzed in reference [50]
where the region of applicability is mentioned. Recently published correlation by Tong and Young (1974) covers the transition and film boiling heat transfer in a steady water flow and provides good prediction of the experimental data from references [3, 14].

Several MIT reports deal with a theoretical model for the prediction of the post dryout wall temperature as reported by Laverty (1964) and later by Forslund (1966) and Hynek (1969). In this model all parameters (drop size, vapor velocity, liquid velocity, void fraction, slip, etc.) are initially evaluated at the dryout location. It is assumed that heat transfer takes place in steps:

(a) from the heated wall to the vapor and then from the vapor to the drops;
(b) from the heated wall to liquid drops.

The heated channel is subdivided axially and the axial gradients in drop diameter, quality, vapor velocity, and liquid velocity are calculated at each node. The vapor superheat is evaluated from a heat balance at each node. The conditions at the downstream nodes are found by step-by-step integration along the heated channel. The wall temperature is then calculated at each node using superheated vapor heat transfer correlations. A similar model was independently developed by Bennett et al (1967).

Hynek (1969), in his thesis, clearly described two types of dryout which were experimentally observed. For the first type of dryout (Figure 6, reference [31]), low void fraction dryout, the inverted annual flow was observed downstream of dryout and a dispersed flow was created at certain
distance downstream after the liquid core was broken into liquid drops. The second type of dryout (Figure 1.1, or Figure 7 of ref. [31]) had all regimes of two phase flow existing upstream of dryout and dispersed flow starting from the dryout point.

Plummer et al (1974) and Iloeje et al (1974) extended work by Hynek by introducing the transient experiment for the experimental analysis of heat transfer in dispersed flow. They obtained experimental data in the film boiling region. In addition, Plummer presented a prediction scheme for correlating data in the film boiling region. Iloeje analysed the rewet phenomena in dispersed flow and proposed the additional mode of heat transfer in dispersed flow; heat transfer from the wall to liquid drops not touching the wall [32]. He also presented phenomenological prediction scheme of post critical heat transfer and compared it with the experimental data in the film boiling region.


Further literature review was omitted in this section. It will be presented in the chapters III, IV, and V of this study where a closer look will be taken at some specific theoretical and experimental results, available in the literature and relevant to this study.
1.3 Objectives

The objectives of the present investigation include the following experimental and theoretical work:

1.3.1 Experimental work

The experimental work includes the analysis of the post critical heat flux heat transfer using transient experimental technique. Obtained experimental data will be presented in the boiling cure form (i.e., heat flux vs. wall superheat for a constant mass flux and vapor quality) covering both transition and film boiling regions.

1.3.2 Theoretical Work

The theoretical work includes the following analysis and derivations:

(1) Analysis of the structure of the fully-developed dispersed flow. Utilization of the relations for calculation of the maximum drop size, average drop size, and drop size distribution, in terms of mass flux and vapor quality.

(2) Analysis of the drop deposition in dispersed flow and development of drop deposition model capable of predicting a drop deposition flux (number of drops deposited on the heated wall per unit surface and unit time).

(3) Analysis of the experimental data in the literature concerning
the heat transfer to a single drop deposited on a wall. Characterization of the possible successive states of the drop-wall interaction and selection of an empirical relation for the heat transfer to a single drop.

(4) Derivation of the relation for the heat flux to dispersed flow which will cover both the transition boiling region and the film boiling region, applying results from (1), (2), and (3).

(5) Comparison of the experimental data with the relation which will be developed in Item (4).
CHAPTER II: EXPERIMENTAL PROGRAM

2.1 Apparatus

The loop diagram for the experimental apparatus is given in Figure 2.1. It is a once through system employing liquid nitrogen as the test fluid. It was constructed to allow for both a steady-state and transient test run, but the results analyzed here were obtained in the transient section.

The steady-state test section, a uniformly electrically-heated eight-foot long Inconel 600 tube 0.50 in. O.D. by 0.40 in. I.D., operates as a preheater for the transient test section. In this manner a two-phase flow mixture with a particular vapor quality and mass flux can be supplied to the transient section. The transient test section, Figure 2.2, consists of a one inch tube 0.40 in. I.D. by 1.00 in. O.D. supported and encased to be independently heated with steam supplied at a temperature of 220-250°F. The specimen is insulated from the supporting structure by micarta insulators (thermal conductivity of 0.2 Btu/ft²·hr). Three thermocouple holes 0.042 inches in diameter were drilled radially into the test pieces to a depth 1/32 of an inch from the inside radius [50]. The holes were spaced at three axial positions along each test piece with each hole circumferentially spaced 120 degrees apart. The thermocouples were coated with a conductivity gel and their leads were exited from the steam jacket through conex glands, to the recording devices. Further discussion of the apparatus and the instrumentation for monitoring of the test loop operations and data
acquisitions from the transient test section can be found in references [50] and [32]. In all cases copper-constantan thermocouples were utilized as the temperature sensing device.

2.2 Experimental Method

The following sequence of operations was carried out for obtaining dispersed flow heat transfer data. The steam supply to the transient section was turned on which allowed the specimen to reach an initial temperature of 220-250°F. Liquid nitrogen at 100 psia was allowed to flow through two heat exchangers that cooled it down sufficiently so that it remained liquid as it passed through the flow control valve. The subcooling was achieved by bleeding part of the main flow into a vacuum line, which forms the outer part of the two concentric-tube heat exchangers. The nitrogen subcooled 3-5°F was initiated into the preheater at about 20 psia. When the preheater thermocouples registered a temperature near the saturation temperature of the liquid, power was applied to the preheater. The flow rate and power were adjusted to give the desired value of mass flux and exit quality to the transient section for the particular run. When the steady state was achieved in the preheater the transient was initiated by closing off the steam flow to the transient section. The temperature transient was recorded following the data flow diagram in Figure 7, ref. [50]. Because of the nature of the test, all regimes of boiling existed on the test piece
during the transient. A run was terminated when nucleate boiling was re-established. Since the test piece was short, it was possible to assume that the quality variation with distance and time was negligible.

The nitrogen, before passing through the voltmeters (Figure 2.1), goes through one or two concentric-tube heat exchangers which serve to heat up the nitrogen to roughly room temperature. The steam or water that flows through the outer part of these heat exchangers also flows through another heat exchanger that heats up the vacuum line so that the vacuum pump does not freeze up.

2.3 Data Deduction

The three thermocouple outputs, the temperature-time history

\[ T = T(t) \quad (2.1) \]

of the transient test section, did not differ significantly with the published thermocouple tables, within the published range. Figure 2.3 shows a typical temperature-time history curve with all regimes of boiling existing.

Considering the transient test section as a lumped heat source (no internal temperature gradients), the heat flux to the fluid during quenching is given as

\[ \frac{q}{A} = \frac{V}{A} \rho C_p \frac{dT}{dt} \quad (2.2) \]
where

\[
\begin{align*}
V &= \text{volume of the transient test section}, \\
\rho &= \text{density of the test section material}, \\
C_p &= \text{specific heat of the test section material}, \\
A &= \text{heat transfer area of the test section}, \\
T &= \text{temperature}.
\end{align*}
\]

The Biot number was calculated to be 0.01, 0.018, and 0.24 for the copper, aluminum, and inconel transient test section, respectively [32]. The characteristic dimension \( L \) on which Biot number was based was obtained by dividing the volume of the transient test section by its heat transfer area.

The heat flux to the two-phase mixture, for a particular quality and mass flux, was obtained by introducing the temperature-time data [Eq. (2.1)] into Eq. (2.2).

Figures 5.3 - 5.12 show the heat flux data obtained by the author. Appendix II-1 gives the estimate of the heat losses from the transient test section.
Figure 2.1 Nitrogen Test Loop
Figure 2.2 Transient Test Section
Fluid: Nitrogen
Test Section Material:
Copper
\[ G = 175,000 \text{ lbm/ft}^2\text{ hr} \]
\[ x = 0.25 \]

- B Minimum Film Boiling
- A-B Film Boiling
- B-C Transition Boiling
- C-D Nucleate Boiling and then Forced Convection

Figure 2.3 Temperature-Time History
CHAPTER III: STRUCTURE OF DISPERSED FLOW

The major concerns of this chapter are characterization of drop size and drop size distribution of a dispersed flow in terms of a mass flux and vapor quality. The distribution of mass of the liquid phase in a dispersed flow is important to the heat transfer and pressure drop characteristics of the flow.

Small liquid drops dispersed in a gas stream (dispersed flow) usually attain spherical shape due to surface tension. They are uniformly (statistically) mixed independently of their dimensions and the local gas velocity [10].

3.1 Maximum Drop Diameter

The maximum drop diameter is an important parameter because it evidently shows the maximum dimension that a drop can have in the particular conditions of dispersed flow. It constitutes in a certain sense a measure of the external forces which act on the drop, opposing surface-tension forces.

The most important dimensionless group for determining the stability of a single drop and its maximum size is the Weber number based on the relative velocity and the gas density [68],

\[
We = \frac{\rho_g (V_g - V_l)^2}{\sigma} \quad \text{(3.1)}
\]
where

\[ \sigma = \text{surface tension}, \]
\[ a = \text{drop diameter}, \]
\[ \rho_g = \text{gas density}, \]
\[ V_g = \text{gas velocity}, \]
\[ V_L = \text{liquid drop velocity}. \]

A calculation of \( V_g \) and \( V_L \) for a given mass flux \( G \) and quality \( x \) is presented in Appendix III-2.

Critical Weber numbers have been measured experimentally. Isshiki (1959) found that \( We_c = 6.5 \) agreed with his measured water drop diameters which were breaking up in an accelerating stream. This value agrees approximately with Forslund's (1966) value. He found \( We_c = 7.5 \) for liquid nitrogen drop in its vapor. Walles (1969) suggested that for nonviscous fluids the critical value of the Weber number above which drops will break up is about 12.

When a liquid drop is introduced into a gas stream at high value of \( We \), several generations of drops will be produced by successive break up. In the eventual process of break up the drop is punched into a bag-like shape by the dynamic pressure of the gas acting at the stagnation point [68].

Liquid viscosity has a stabilizing effect on the drop shape which is scaled by the stability number \( S_n \) defined as [68]

\[ S_n = \frac{\mu_L^2}{\rho_g a \sigma} \]  \hspace{1cm} (3.2)

where \( \mu_L \) is the liquid viscosity.
If $S_n < 0.01$ a drop break up is not likely to occur unless the drop is suddenly accelerated [27,23].

In our dispersed flow study it was decided to use Forslund's value for the critical Weber number, so

$$a_m = \frac{7.5 \sigma}{\rho_g (V_g - V_L)^2}$$  \hspace{1cm} (3.3)

where $a_m$ is the maximum drop diameter.

### 3.2 Mean Drop Diameter

A wide spectrum of drop diameters is present in dispersed flow. The mean drop diameter can be approximately predicted using Nukiyawa-Tanazawa equation [12,68,57],

$$\bar{a} = \frac{1.83}{(V_g - V_L)} \left( \frac{\sigma}{\gamma_L} \right)^{\frac{1}{2}}$$  \hspace{1cm} (3.4)

where

- $\bar{a}$ = mean drop diameter,
- $\gamma_L$ = liquid weight density.

Equation (3.4) has been widely used for predicting the mean drop diameter for atomization (shattering a continuous jet or sheet of liquid) with air. This equation is not dimensionless and care must be taken concerning the units which are $a$: m; $V_g$ and $V_L$: m/sec; $\sigma$: kg/m; and $\gamma_L$: kg/m$^3$. For a low mass flux $G$ and quality $x$, $\bar{a}$ calculated from Eq. (3.4) is larger
than the experimentally observed mean drop diameter [58]. This is particularly true for a dispersed flow with heat addition, where a drop size is influenced not only by aerodynamics and surface forces, but also by evaporation at the drop interface (Appendix III-1). The correction factor $C$ has been introduced into Eq. (3.4) so that a mean drop diameter in dispersed flow with heat addition is calculated using the relation

$$\bar{a} = \frac{1.83 \, C}{(V_g - V_l)} \left( \frac{\sigma}{\gamma} \right)^{1/3}$$  \hspace{1cm} (3.5)

The values for $C$, deduced from the experimental data shown in Figures 5.3 - 5.12, are presented in Figure 5.13, which shows that $C$ is a function of quality and mass flux.

3.3 Most Probable Drop Diameter

The most probable drop diameter $a_{mp}$ satisfies the relation that $P(a)$ has a maximum value for $a = a_{mp}$ in the interval $0 < a < a_m$, where $P(a)$ is a drop size distribution. MacVean (1967) found that a great deal of data satisfied the relation

$$a_{mp} = \frac{\bar{a}}{2}$$  \hspace{1cm} (3.6)

This will be explained in detail in Chapter V.
i.e., the mean drop diameter is twice as large as the most probable drop diameter.

3.4 Drop Size Distribution

The actual histogram of the size distribution of drops in a dispersed flow may, for purposes of this analysis, be replaced by a continuous curve, \( P(a) \). The meaning of \( P(a) \) is the probability of a drop having diameter between \( a \) and \( a + da \). A large number of drop size distributions have been devised by experimenters in order to correlate data. Many authors [68,59] have used the normal and log-normal distributions which are also in common use for describing crushed particles.

In our study the following drop size distribution has been utilized:

\[
P(a) = 4 \frac{a}{\bar{a}^2} e^{-2(a/\bar{a})^2} \tag{3.7}
\]

where \( \bar{a} \) is the mean drop diameter given by Eq. (3.5).

This type of distribution has been widely used for analyzing and solving mass transfer and evaporation problems of various dispersed flow systems [56,29,49]. It is important to mention here that the maximum drop diameter \( a_m \) calculated from Eq. (3.3) satisfies the relation

\[
P(a_m) = 0 \tag{3.8}
\]

for \( 25000 < G < 220,000 \text{ lb/ft}^2\text{hr} \) and \( 0.1 < x < 0.9 \).
From Eqs. (3.7) and (3.8),

\[ \int_{0}^{a_m} P(a) \, da = 1.0 \]  

(3.9)

i.e., \( P(a) \) has properties of a probability density function for \( 0 < a < a_m \).

The effects of the mass flux and quality on the drop size distribution [Eq. (3.7)] are presented in Figures 3.2 and 3.1, respectively. Figures 3.1 and 3.2 show that for higher mass fluxes and qualities, drops are smaller.

The above equations (3.3), (3.5), and (3.7) are sufficient to characterize the constitution of a dispersed flow, at least for the experimental conditions considered.

Experimental research on highly dispersed flow has been performed by Cumo et al (1973) employing Freon 12 at different pressures. The drops size distribution, and drops spatial concentration have been studied by means of a visualization technique for different thermohydraulic conditions (mass flux, quality and pressure). The drop size distribution expressed by the relationship

\[ P(a) = \frac{a}{a_{mp}^2} e^{-a/a_{mp}} \]  

(3.9a)

has been deduced from their experimental data, where \( a_{mp} \) is the most probable drop diameter. Eq. (3.9a) is not very different numerically from Eq. (3.7).

Snyder (1959) also performed experimental study of drops size employing water at atmospheric pressure. The drops were trapped on a glass which was
coated with magnesium oxide and were measured by a traversing microscope. His three sets of the drop size distribution data, shown in Figure 1 of his thesis, follow basically the distribution law given by Eq. (3.7).

In the case when the drop size distribution is known (usually experimentally determined) the average (mean) drop diameters are defined according to [59],

\[
\text{Mean diameter (length)} = \overline{a}_l = \frac{\int a \, P(a) \, da}{\int P(a) \, da} \quad (3.10)
\]

\[
\text{Mean volume diameter} = \overline{a}_v = \left[ \frac{\int a^3 \, P(a) \, da}{\int P(a) \, da} \right]^{1/3} \quad (3.11)
\]

\[
\text{Mean surface diameter} = \overline{a}_s = \left[ \frac{\int a^2 \, P(a) \, da}{\int P(a) \, da} \right]^{1/2} \quad (3.12)
\]

\[
\text{Mean volume-surface diameter} = \overline{a}_{sv} = \frac{\int a^3 \, P(a) \, da}{\int a^2 \, P(a) \, da} \quad (3.13)
\]

Depending on the transport phenomena involved, \( \overline{a} \) is given by one of these relations, Eqs. (3.10)-(3.13). For instance, when dealing with momentum transport of small drops at low relative velocity to fluid (Chapter IV), the mean diameter based on length is applicable. The mean surface diameter is related to heat and mass transfer and reaction at the fluid-particle interface.
The mean volume-surface diameter is related to rate processes affected by volume-to-surface ratio [59]. The mean volume diameter is related to determination of the void fraction (the total volume of drops per unit volume of space).

In the next two chapters we will be using drop size distribution $P(a)$ given by Eq. (3.7). The change of the mean drop diameter $\bar{a}$ [Eq. (3.5)] vs. vapor quality is demonstrated in Figure 3.3.

It is important to mention that even if there is no change in drop size because of the evaporation, the flow oscillations may produce the change of spectral distribution of sizes. For example, if the cloud of drops is accelerated, different sizes of drops will accelerate at different rates, and the differences in drop speeds will alter the relative concentrations per unit volume of the several drop sizes. These types of problems were not considered since no flow oscillations (except, of course, the small fluctuations that were masked by the throttling orifices) were present in any of the experimental data reported in this thesis.

In virtually all theoretical investigations of a dispersed flow the actual size distribution of drops was replaced by a simple model ("equivalent" size model) comprising a constant number of drops of uniform size. In some of these investigations the change in size of drops was allowed [3,23,17]. All these "equivalent" modes can give misleading results concerning drops-wall interactions (drops deposition). This will be discussed in more detail in the next chapter.
Figure 3.1  The Effect of Quality on the Drop Size

Fluid: Nitrogen
\( G = 200,000 \, \text{lbm/ft}^2\,\text{hr} \)

Drop Size Distribution, ft\(^{-1}\)

Drop Diameter \( \times 10^3 \), ft
Figure 3.2 The Effect of Mass Flux on the Drop Size
Figure 3.3 The Effect of Quality on the Mean Drop Diameter (Equation (3.5), $C=0.26$)
CHAPTER IV: MECHANISM OF LIQUID DROP DEPOSITION IN DISPERSED FLOW

The behavior of liquid drops suspended in a fluid stream (drop or dispersed flow) is of interest in a wide range of areas of technical importance. A knowledge of the trajectories of drops is important in the design of a number of industrial applications such as evaporators, nuclear reactors, spray coolers, and combustion devices involving sprays of liquid fuels. The theoretical approach in analyzing drop motion is generally similar to that of analyzing bubble motion. Equations of the motion of bubbles (bubbly flow) are analogous to drop motion. In spite of that, there are qualitative differences between the behavior of drops and bubbles. Those differences are most noticeable when the density difference between the components is high, as in gas-liquid systems at low pressure. In bubbly flow most of the inertia is in the continuous phase and as a result the drag forces on bubbles are large compared with their momentum. Bubbles therefore follow the motion of the surrounding fluid very closely in forced convection flow. Drops, however, take far longer to adjust to the motions of the surrounding gas. Also, for the dispersed flow heat transfer, some additional specific forces are associated with the drop motion.

The most important phenomenon in the dispersed flow is the deposition motion of drops. By deposition motion, we mean the migration of drops toward the wall. Up to now, a theoretical analysis of drop deposition from the gas stream on the hot wall has not been reported. The explanation of the deposition phenomena in the two-phase flow is associated with the theory of the
single drop motion inside the boundary layer.

4.1 Previous Work

The motion of the liquid drop inside the boundary layer is a well-known problem of particle migration in a shear field. In recent years there have been several investigations related to this problem. When a particle enters a shear flow it moves across fluid streamlines as a consequence of fluid dynamic forces. This kind of a particle motion can also be a consequence of the temperature gradient in the fluid stream, as we will show in this study. The forces on a particle in the fluid stream analyzed in most of the studies (Brenner, 1966) are inertia, drag, gravity, and buoyancy forces. In some of those studies, the lift force on a particle also has been considered but conclusions about the magnitude and importance of this force are not unique.

Deposition of water droplets in adiabatic turbulent downward air flow has been studied by Kondic (1970). The micron-size water droplets were generated at atmospheric pressure by the volumetric expansion of slightly superheated steam, mixed with the filtered atmospheric air. New techniques are introduced for the measurement of flow rates, drops deposition rate, and state variables of both components. No drop coagulation occurred. The balance of forces on the representative particle included the Magnus force (Swanson, 1961). A discussion of the magnitude and extension of validity of this force, for the conditions under which the experiment was performed, is presented. A reasonable agreement has been achieved between the computed and experimental results.
The theoretical analysis of the rigid particle motion within the laminar sublayer has been done by Rouhiainen and Stachiewicz (1970). The expression for the lift force developed by Saffman (1965) has been included in the analysis. The authors concluded that the lift force has a major effect on the trajectories of the particles in the certain range of the particle sizes. The numerical value of $K$ [see Eq. (4.15) of this article] in Saffman's expression for the lift force was 81.2 instead of 6.46. This numerical error, in the original Saffman paper of 1965 where a factor of $(4\pi)^{-1}$ is missing from the second line of the Eq. (4.11), was discovered by Harper and Chang (1968). Most of the numerical results in the Rouhiainen and Stachiewicz analysis are affected by the value of $K$.

The characteristics of the motion of a drop in a two-phase boundary layer on a flat plate have been reported by Deich and Ignat'evskaya (1970). In their study they included the lift force derived by Rubinow and Keller (1961). They obtained the experimental relationship of the change of liquid film thickness formed by drop deposition as a function of the initial parameters of a two-phase flow. The effect of the lift force on their experimental results has not been presented.

Denson et al (1966) analyzed the motion of a single rigid sphere entrained in a glycerine-water solution flowing downward through a cylindrical tube. A range of particle Reynolds numbers was from 6 to 120, tube Reynolds numbers of 208 to 890, and particle-to-tube diameter ratios of 0.12 to 0.19. Trajectories of the sphere, calculated for various particle Reynolds numbers by using the Rubinow-Keller expression for the lift force, were found to agree
4.2 Analysis

Dispersed flow heat transfer is high void fraction flow. Because of this, the drop concentration is low, the interference of neighboring drops may be neglected, and the single drop may be treated as if it were alone in the medium.

Also, since we are focusing our analysis on the drop motion through the boundary layer (laminar sublayer), we are assuming that the drop diameter is small compared to the thickness of this layer. This is the kind of continuum satisfactorily with experimentally determined trajectories when the particle Reynolds number was below 40.

A critical review of the work published prior to 1966 on investigations of the radial motion of particles and liquid drops in fluid stream has been presented by Brenner (1966).

The comparison of the theoretical and experimental values for the deposition velocities of liquid drops and solid particles in dispersed flow systems has been summarized by Liu and Ilori (1973). The experimental data obtained by Farmer (1970) are included in their analysis.

In a recent paper by Gauvin et al (1975) the work on spray dryers has been summarized and extended.

In spite of a large number of published studies, the theory of the drops deposition in dispersed flow with heat addition (dispersed flow heat transfer) has remained largely unexplored.
criterion to be satisfied. This basically means that the forces on the drop are dependent on the local vapor velocity and temperature of the stream.

In order to obtain some useful and reliable results the additional following simplifying assumptions for the motion inside the laminar sub-layer are made:

(a) Drop has a spherical shape.
(b) Flow in the two-component, two-phase stream is laminar, two-dimensional, steady, and incompressible.
(c) The change of the drop diameter due to evaporation during deposition motion is not significant.

The list of forces during accelerated motion of the drop includes:

1. Inertia force
2. Drag force
3. Gravity and buoyancy forces
4. Lift force
5. Reaction force
6. Other forces.

4.2.1 Inertia Force

Inertia force acts on the drop as the resultant force

\[ F_i = \frac{\pi a^3}{6} \rho g \frac{dV_a}{dt} \]  \hspace{1cm} (4.1)
where
\[ a = \text{drop diameter}, \]
\[ \rho_l = \text{density of the drop (particle)}, \]
and \( V_a = \text{drop velocity}. \)

4.2.2 Drag Force

Since the drop Reynolds number for our conditions is close to one, the drag of the medium due to viscosity \( \mu_d \) is, according to Stokes Law,

\[ F_d = 6\pi \frac{\mu_g a}{2} (V - V_a) \quad (4.2) \]

where \( V \) is the velocity of the fluid (vapor or gas).

The motion of the drop inside the boundary layer is unsteady, but it is customary to assume that the Stokes law may be used (Hidy, 1970). On the other hand, the drag problem becomes more complicated because of the evaporation of the drop and the possibility for circulation of liquid inside the drop. Evaporation can reduce the drag coefficient due to mass flux from the surface, for which Bailey et al (1970) suggests the following correlation:

\[ C_{D_0} = \frac{C_D}{1 + B} \quad (4.3) \]

where \( B \) is the transfer coefficient (Spalding number), given by

\[ B = \frac{C_g \Delta T}{H_g} \quad (4.4) \]
where

\[ C_D = \text{drag coefficient corresponding to no evaporation}, \]
\[ C_g = \text{specific heat of the diffusing vapor}, \]
\[ T_v = \text{vapor temperature}, \]
\[ \Delta T = (T_v - T_s) = \text{temperature difference}, \]
and \[ H_{lg} = \text{latent heat of evaporation}. \]

The Spalding number is generally small compared to unity for an evaporating drop. This will be numerically proven below.

The circulation of the liquid inside the moving drop has been studied by Rybczynski and Hadamard and summarized by Hidy and Brock (1970). They found that the Stokes drag was modified by a factor involving the ratio of viscosities of two fluids, or

\[ C_{D_0} = C_D f_\mu \]

where

\[ f_\mu = \frac{1 + 0.75 \left( \frac{\mu_g}{\mu_l} \right)}{1 + \left( \frac{\mu_g}{\mu_l} \right)} \quad (4.5) \]

For example, for a water drop in air, the correction \( f_\mu \) is 0.994 at 20°C; for a liquid nitrogen drop \( f_\mu = 0.99 \) at atmospheric pressure. Therefore, for most practical considerations, one may disregard the correction due to the circulation inside the drop.

The Stokes drag for the sphere is derived for the case of uniform flow passing a sphere. On the other hand, the motion of a drop inside the boundary
layer is a shear flow problem as the velocity gradient is very high. This has been the subject of the experimental study by Kohlman and Mollo-Christensen (1965). They demonstrated that shear flow does not affect the drag for low Reynolds number. The reader is referred to their study for more details.

4.2.3 Gravity and Buoyancy Forces

The resultant force of these two forces is

\[ F_g = \frac{\pi a^3}{6} (\rho_f - \rho_g) g \]  \hspace{1cm} (4.6)

where \( g \) is the gravity vector and \( \rho_f \) fluid density.

4.2.4 Lift Force

Rotation of the drop occurs in the presence of a velocity gradient in the boundary layer (laminar sublayer). This is a well-known phenomenon that the velocity gradient can cause a solid particle to rotate. At low Reynolds numbers rotation causes fluid entrainment, increasing the velocity on one side of the body and lowering the velocity on the other side. This is known as the Magnus effect and tends to move the particle toward the region of higher velocity (Soo, 1967). The radial migration of solid particles and liquid drops in shear flow has been experimentally studied as we mentioned before. Most of the experimental data of this kind have been interpreted on the basis of Rubinow-Keller's Eq. (4.7) for the lift force on a spinning translating sphere in an unbounded fluid at rest (or in uniform flow) at
infinity.

By using Stokes' and Oseen's expansions and solving the Navier-Stokes equation, inertia term retained, Rubinow and Keller (1961) found that for the flow about a spinning sphere moving in a viscous fluid, the sphere experiences a force $F_L$ (lift force) orthogonal to its direction of motion, in addition to the drag force determined by Stokes. The force is given by

$$F_L = \frac{\pi a^3}{8} \rho g \omega \times (V - V_a) \left[ 1 + O(Re_a) \right]$$  \hspace{1cm} (4.7)

Here $a$ is the diameter of the sphere, $\omega$ is its angular velocity, $(V - V_a)$ is the linear velocity of the sphere relative to the fluid, $\rho g$ is the fluid density, and $Re_a$ is the sphere Reynolds number,

$$Re_a = \frac{\rho g (V - V_a) a}{\mu g}$$  \hspace{1cm} (4.8)

For the steady motion the angular velocity of the sphere is equal to the angular velocity of the flow. This is experimentally verified by Kohlman and Mollo-Christensen (1965) in the constant shear flow.

For the two-dimensional motion, the angular velocity of the flow is determined by the expression

$$\omega_g = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$  \hspace{1cm} (4.9)

In the boundary layer,

$$\omega_g \approx -\frac{1}{2} \frac{du}{dy}$$  \hspace{1cm} (4.10)
so the spin of the drop is now given by Eq. (4.10). Rather remarkably, the Rubinow-Keller lift force is independent of viscosity for small values of $\text{Re}_a$. This force is comparable to the well-known Magnus force (Goldstein, 1938), arising at very high Reynolds numbers, usually invoked to explain such phenomena as the curving of a pitched baseball. Also, this force is very similar to the lift formula for two-dimensional potential flow about an airfoil.

Saffman (1965) derived the net force acting on a small translating sphere which is simultaneously rotating in an unbounded, uniform, simple shear very viscous flow, the translation velocity being parallel to the stream lines. In his analysis he applied the inner and outer solution technique to the equation of motion. Three independent Reynolds numbers based on sphere diameter arise in the analysis:

\begin{align}
\text{Slip:} \quad \text{Re}_a &= \frac{\rho g(V - V_a)a}{\mu g} \\
\text{Shear:} \quad \text{Re}_k &= \frac{\rho g a^2}{\mu g} \frac{dV}{dy} \\
\text{Rotation:} \quad \text{Re}_\omega &= \frac{\rho g a^2}{\mu g} \omega
\end{align}  

(4.11) \quad (4.12) \quad (4.13)

The analysis is valid when

\[ \text{Re}_a, \text{Re}_k, \text{Re}_\omega \ll 1.0 \quad \text{and} \quad \text{Re}_k, \text{Re}_\omega \gg \text{Re}_a^2 \]  

(4.14)
and the transverse force is given by

\[ F_L = K \frac{a^2}{4} \left( \rho_d \mu_d \right)^{\frac{1}{2}} \left( \frac{dV}{dy} \right)^{\frac{1}{2}} (V - V_a) \] (4.15)

plus a smaller term\(^+\) perpendicular to the flow direction which acts to deflect the particle towards the streamlines. A numerical constant \( K \) has a value of \( K = 6.46 \) obtained by a numerical integration.

When the conditions given by Eq. (4.14) are met, the slip-shear Saffman lift force [Eq. (4.15)] can be an order of magnitude larger than the slip-spin Rubinow-Keller lift force [Eq. (4.7)]. On the other hand, these conditions [Eq. (4.14)] are not satisfied in many kinds of dispersed flow systems, specifically those with high flow Reynolds number, which makes the range of the possible applicability of Saffman's lift force limited.

The directions of the lift force on the small sphere (drop) inside the boundary layer are shown in Figure 4.1 for the cases:

(a) when the velocity component of the drop is less than the velocity of the fluid (slip flow);

(b) when the velocity component of the drop is greater than the velocity of the fluid.

\[ ^+ \text{The smaller term which is due to rotation is given by} \]

\[ \pi \frac{a^3}{6} \rho_d \omega (V - V_a) \] (4.15a)
4.2.5 Reaction Force

As we mentioned before, this force is due to asymmetrical drop evaporation inside the boundary layer. There is a high temperature gradient in the fluid stream close to the wall because the wall temperature is much higher than the fluid temperature. Since the fluid temperature is larger than the drop temperature (saturation temperature assumed), the evaporation of the drop occurs. The difference in these two temperatures is larger for the drop side facing the wall than for the other side of the drop because of the temperature gradient in the fluid stream. This causes the vapor velocity generated on the drop side facing the wall (bottom side) to be higher than the vapor velocity generated on the other side. This produces a reaction force which tends to prevent a deposition of the drop on the wall.

The derivation of the reaction force is based on the following assumptions:

(a) The temperature profile inside the boundary layer is a linear one:

\[ T = T_s + (T_w - T_s) \left(1 - \frac{y}{\delta}\right) \]  \hspace{1cm} (4.16)

where \( y \) = the distance from the wall,
\( \delta \) = laminar sublayer thickness,
\( T_w \) = wall temperature,
and \( T_s \) = saturation temperature.

The temperature gradient is now

\[ \frac{dT}{dy} = - \frac{(T_w - T_s)}{\delta} \]  \hspace{1cm} (4.17)
(b) The Nusselt number related to the drop heat transfer is given by the relation (Bird et al, 1960)

\[
Nu_a = 2.0 + 0.6 \text{Re}_a^{0.5} \text{Pr}^{0.33}
\]  
(4.18)

and the circumferential variation of this number is negligible. Since the drop Reynolds number for our condition is close to one, then

\[
Nu_a = 2.6 \text{ as } \text{Pr} \approx 1
\]  
(4.19)

Equation (4.18) is good for small heat transfer rate. In the case of high evaporation rate (drop is very close to the hot wall), the correction to the Nusselt number is needed, similar to that suggested by Bailey (1970).

(c) The boundary layer thickness around the spherical drop moving in shear flow is approximately given by the potential flow theory. This thickness is then of the correct order of magnitude of drop diameter when the drop Reynolds number is close to one (Schlichting, 1970).

As we mentioned before, the drop diameter is much smaller than the thickness of the laminar sublayer. The above leads us to the conclusion that the drop stay time (deposition time or time measured from the moment a drop enters the boundary layer until it touches the wall) is much larger than the time for a heat diffusion through the drop boundary layer. As long as this conclusion is valid, the drop motion inside the boundary layer is influenced
by the temperature distribution in it.

The evaporation of the drops occurs due to convective heat transfer from the fluid stream (vapor) to the drop and radiative heat transfer from the hot wall to the drop. The vapor participation in radiant energy exchange is assumed to be small. We will first analyze the evaporation due to convective heat transfer.

Let the drop be at distance \( y \) from the wall (Figure 4.2). The corresponding vapor temperature at that distance is \( T_i(y) \). As we assumed the linear temperature distribution, the average vapor temperatures for the upper and lower portions of the drop are then, respectively,

\[
T_{i+1}(y) = T_i(y) + \frac{dT_i}{dy} \Delta y
\]

\[
T_{i-1}(y) = T_i(y) - \frac{dT_i}{dy} \Delta y
\]

where \( \Delta y = \frac{a}{\kappa} \) and \( \kappa = 4 \). The value of \( \kappa \) was obtained by calculating the average fluid (gas) temperature for the upper or lower surface portion of the sphere.

Introducing Eq. (4.17) in (4.20) and (4.21) gives

\[
T_{i+1}(y) = T_i(y) - \frac{(T_w - T_s)}{\delta} \frac{a}{4}
\]

\[
T_{i-1}(y) = T_i(y) + \frac{(T_w - T_s)}{\delta} \frac{a}{4}
\]
The average heat transfer rates for the upper and lower portion of the drop are, respectively,

\[
\frac{q}{A}_1 = h \left[ T_{i+1}(y) - T_s \right] \tag{4.24}
\]

\[
\frac{q}{A}_2 = h \left[ T_{i-1}(y) - T_s \right] \tag{4.25}
\]

where \( h \) is the heat transfer coefficient defined by Eq. (4.19). The average vapor velocities leaving the upper and lower portion of the drop are, respectively,

\[
\upsilon_1 = \frac{(q/A)_1}{H_{lg} \rho_g} \tag{4.26}
\]

\[
\upsilon_2 = \frac{(q/A)_2}{H_{lg} \rho_g} \tag{4.27}
\]

Here, \( H_{lg} \) is the latent heat of evaporation, and \( \rho_g \) is the vapor density.

Introducing equations (4.22)-(4.25) into (4.26) and (4.27) gives

\[
\upsilon_1 = \frac{h}{H_{lg} \rho_g} \left[ (T_w - T_s) \left( 1 - \frac{V}{d} \right) - \frac{(T_w - T_s)}{d} \frac{a}{4} \right] \tag{4.28}
\]

\[
\upsilon_2 = \frac{h}{H_{lg} \rho_g} \left[ (T_w - T_s) \left( 1 - \frac{V}{d} \right) + \frac{(T_w - T_s)}{d} \frac{a}{4} \right] \tag{4.29}
\]

Since \( \upsilon_2 > \upsilon_1 \) the upward force on the spherical drop is produced. We choose the volume of the spherical drop as a control volume. The upward
force (reaction force) is then approximately given as the net rate of efflux of momentum through the control surface in the \( y \) direction,

\[
F_{cy} = \oint_{c.s.} \rho g \vec{v} \cdot d\vec{A} \tag{4.30}
\]

where

\[
\vec{v} = \text{vapor velocity (vector)},
\]

\[
\vec{v}_y = \text{vapor velocity in } y \text{ direction (scalar)},
\]

and \( A = \text{surface of the sphere}. \)

Introducing Eqs. (4.28) and (4.29) into (4.30) gives (Appendix IV-2),

\[
F_{cy} = \frac{\pi a^2 h^2}{4 H_{lg}^2 \rho_g} \left( T_w - T_s \right)^2 \frac{a}{\delta} \left( 1 - \frac{y}{\delta} \right) \tag{4.31}
\]

It is seen from Eq. (4.31) that the reaction force, \( F_{cy} \), is a function of the drop distance \( y \) from the wall for the given wall temperature and drop diameter.

If the distance of the drop from the wall is less than the drop boundary layer thickness, the expression (4.31) is not valid any more. In that case, the thermal conduction through the vapor film existing between the wall and the bottom of the drop is the most important mechanism of heat transfer. If we assume that conduction is in the \( y \) direction only, then the velocity of the vapor leaving the lower portion of the drop due to evaporation is

\[
\vec{v}_2 = \frac{k_g}{H_{lg} \rho_g} \left( \frac{T_w - T_s}{y} \right)
\]
where $\bar{y}$ is the current mean film thickness through which heat is conducted and $k_g$ is the thermal conductivity of vapor.

In this case the pressure increase in the vapor layer does influence saturation temperature $T_s$ and the latent heat of evaporation $H_{lg}$. Also, the effect of the wall roughness may be important. Further details of this analysis include application of Eq. (4.30) and iterative evaluation of the saturation temperature (Appendix (IV-4)). They are omitted here because the most critical effects of the resistance forces on the drop occur near the edge of the laminar sublayer, or sufficiently far away from the wall. More clearly, whenever a drop approaches the distance that is equal to or less than the drop boundary layer thickness, deposition of the drop will occur. This will be shown below.

The evaporation of the drop due to radiative heat transfer becomes more important as the wall temperature increases. Assuming that the wall and the lower half of the drop are separated by a non-absorbing, non-emitting medium, the rate of the radiative heat transfer between these two bodies is

$$q_2 = A_2 F_2 c (T_w - T_s)$$

(4.32)

where $F_2$ is the overall interchange factor for radiation between the hot wall and the lower half of the liquid drop. $F_2$ is given for a system of two-zone, source-sink surfaces as (McAdams, 1954),

$$\frac{1}{F_2} = \left( \frac{1}{e_L} - 1 \right) + \frac{A_2}{A_w} \left( \frac{1}{e_w} - 1 \right) + \frac{1}{F_2}$$

(4.33)
where

\[ A_w = \text{area of hot wall}, \]
\[ \varepsilon_L = \text{thermal emissivity of the liquid}, \]
\[ \varepsilon_w = \text{thermal emissivity of the wall}, \]
\[ \sigma = \text{Stefan-Boltzmann constant}, \]

and \( A_2 \) = surface area of the lower half of the drop.

Since \( A_2 \ll A_w \), Eq. (4.33) can be further simplified to

\[
\frac{1}{F_2} = \left( \frac{1}{\varepsilon_L} - 1 \right) + \frac{1}{F_2} \quad (4.34)
\]

Similarly, the overall interchange factor for radiation between the wall and the upper half of the liquid drop is

\[
\frac{1}{F_1} = \left( \frac{1}{\varepsilon_L} - 1 \right) + \frac{1}{F_1} \quad (4.35)
\]

\( F_1 \) and \( F_2 \) are the average configuration factors for radiation between the lower and upper halves of the spherical drop and wall, respectively. The value for \( F_1 \) and \( F_2 \) are 0.318 and 0.682, respectively (Gottfried, 1966).

Substituting the value for \( F_2 \) into Eq. (4.34), and then into Eq. (4.32), the radiation heat transfer between the wall and the lower half of the drop is

\[
q_2 = \frac{A_2 \varepsilon_w (T_w^4 - T_s^4)}{[1/(\varepsilon_L) - 1] + 1/0.682} \quad (4.36)
\]

Similarly, the radiative heat transfer between the wall and the upper half of
the drop is

\[ q_1 = \frac{A_1 \cdot (T_w^4 - T_s^4)}{[(1/\varepsilon_L) - 1] + 1/0.318} \]  

(4.37)

where \( A_1 \) is the surface area of the upper half of the drop.

Introducing Eqs. (4.36) and (4.37) into (4.26) and (4.27) and then into Eq. (4.30) gives approximately the reaction force on the spherical drop due to radiation heat transfer,

\[ F_{\text{ry}} = \frac{\pi a^2 \sigma^2}{4H^2 \xi g_0 g} (T_w^4 - T_s^4)^2 \left\{ \left[ \frac{1}{\varepsilon_L} - 1 \right] + \frac{1}{0.682} \right\} \left[ \left[ \frac{1}{\varepsilon_L} - 1 \right] + \frac{1}{0.318} \right] \]  

(4.38)

The total reaction force on the drop due to non-uniform evaporation is then the sum of the forces given by Eqs. (4.31) and (4.38). It is not likely that the shape of the spherical drop will be changed due to evaporation. As we mentioned before, a drop rotates inside the boundary layer due to velocity gradient. Because of the rotation the whole surface of the drop is equally exposed to the hot wall during deposition.

For the case of the single drop motion inside the tube the component of the reaction force due to radiation heat transfer, \( F_{\text{ry}} \), is zero as the average configuration factors \( \bar{F}_1 \) and \( \bar{F}_2 \) have the same numerical values.

4.2.6 Other Forces

A full list of forces known to influence the drops motion in addition to the forces we analyzed so far includes:
(6a) Force on the drop due to the pressure gradient in the fluid stream. For a drop of radius $a$ in a pressure gradient $\frac{dP}{dx_i}$, this force is given by (McCormack, 1973)

$$F_p = -\frac{\pi a^3}{6} \frac{dP}{dx_i}$$

where $x_i$ is the spatial coordinate in the direction of $F_p$ and is in the opposite direction to the pressure gradient. Unless there is a very large pressure gradient this force will be very small.

The local increase in pressure when an evaporating drop is approaching the hot wall can produce this type of force. However, this is already included in the derivation of the expression for the reaction force due to non-uniform drop evaporation.

(6b) Basset Acceleration force due to the deviation of the flow pattern from the steady state (McCormack, 1973). It is an instantaneous flow resistance and accounts for the acceleration history of the particle. The expression for this force is

$$F_b = \frac{3}{2} a^2 (\pi \rho g u_g)^{1/2} \int_{t_p}^{t} \frac{(d/d\tau)(V - V_a)}{(t_p - \tau)} \, d\tau$$

where $\tau$ is the time constant for the effect. This force becomes significant only when the drop (particle) is accelerated at high rate, when the observed drag force becomes many times the steady drag coefficient (Hughes, 1952). So only in cases of extreme
particle accelerations should this force be considered.

(6c) Forces arising from proximity to the wall and the effects of irregularities in the wall surface on the local flow.

(6d) Forces produced by possible fluctuations in pressure.

(6e) Intermolecular forces. They are symmetrical in action and their influence on the drop motion is unimportant.

(6f) Forces arising because of thermophoresis and photophoresis phenomena (Waldmann, 1966).

(6g) Forces arising because of diffusiophoresis phenomenon (Waldmann, 1966).

All these forces are important only for the motion of the submicron drop size and they are omitted in our analysis.

4.3 Drop Deposition Model

Denoting the coordinates of the center of a spherical drop by \(x\) and \(y\) and applying Eqs. (4.1), (4.2), (4.6), (4.7), and (4.31), we write the equations of motion of a drop moving in the laminar sublayer when the main flow in the channel is directed vertically upward:

\[
\rho L \frac{\pi}{6} \frac{a^3}{dt^2} = \frac{\pi}{16} a^3 \rho g \frac{dU}{dy} \left( \frac{dy}{dt} \right) - 6\pi \mu g \frac{a}{2} \left( \frac{dx}{dt} - U \right) - \rho_L - \rho_g) g \frac{\pi}{6} a^3
\]  

(4.39a)

\[
\rho L \frac{\pi}{6} \frac{a^3}{dt^2} = -\frac{\pi}{16} a^3 \rho g \frac{dU}{dx} \left( \frac{dx}{dt} - U \right) - 6\pi \mu g \frac{a}{2} \left( \frac{dy}{dt} \right) + \frac{\pi a^2 h^2}{4H \rho_L \rho g} (T_w - T_s) \frac{a}{\delta} \left( 1 - \frac{\nu}{\delta} \right)
\]  

(4.39b)
Equation (4.39a) is obtained from the equilibrium of forces acting on the drop in the direction of flow (x-direction), x being taken as positive upward. Forces acting upwards in the x direction are taken as positive. Equation (4.39b) is obtained from the equilibrium of forces acting on the drop in the y-direction, y being taken as zero at the wall, increasing positively toward the center line of the channel (tube). Subscripts \( l \) and \( g \) refer to liquid drop and stream (gas or vapor), respectively. The first terms on the right side of Eqs. (4.39a) and (4.39b) represent the lift forces (Rubinow-Keller expression used) of the stream flow acting on the periphery of the drop with variation in velocity around the drop, i.e., in the gradient of the stream flow.

The second terms on the right side of Eqs. (4.39a) and (4.39b) represent the drag forces on the drop. The third term on the right side of Eq. (4.39a) represents the gravity and buoyancy force. The third term on the right side of Eq. (4.39b) represents the reaction force on the drop due to non-uniform evaporation of the drop inside the laminar sublayer. The terms on the left side of Eqs. (4.39a) and (4.39b) represent the inertial forces acting on a drop of mass \( m = \frac{\pi a^3}{6} \rho_l \).

In order to solve the equations (4.39a) and (4.39b) the linear velocity profile of the gas stream across the laminar sublayer was assumed:

\[
U = U_0 \left( \frac{y}{\delta} \right) \quad (4.40)
\]

where \( U_0 \) is the mean gas velocity. Also, the following initial boundary
conditions were used:

\[ t = 0 \quad , \quad x = 0 \quad , \quad \frac{dx}{dt} = u_0 \quad \text{(4.41a)} \]

\[ t = 0 \quad , \quad y = \delta \quad , \quad \frac{dy}{dt} = v_0 \quad \text{(4.41b)} \]

where \( \frac{dx}{dt} \) and \( \frac{dy}{dt} \) are the velocity components of the drop in \( x \) and \( y \) directions, respectively. The slip ratio \( S \) at \( y = \delta \), the ratio of \( U_0 \) and \( u_0 \), is defined as

\[ S = \frac{U_0}{u_0} \quad \text{(4.42)} \]

The initial velocity in the radial direction \( v_0 \) is usually called the deposition velocity of the drop. Equations (4.39a) and (4.39b) were solved analytically (Appendix IV-1).

The main results of calculating the trajectories of the drops in the region of the laminar sublayer are presented on Figures 4.3 - 4.6. The effects of the drop diameter, wall temperature, initial drop velocities in the \( x \) and \( y \) directions, and the flow Reynolds number on the drop trajectories are separately analyzed. Also, the magnitude of all forces on the drop are calculated. The drop Reynolds number during the drop motion inside the laminar sublayer is calculated. Nitrogen data were used in the calculation.
A. Effect of Initial Drop Velocity on the Flow Direction (Slip Effect)

The trajectories of the drop of 20 μm diameter for two different values of the initial drop velocity in the flow direction at the edge of laminar sublayer are presented on Figure 4.3. The other parameters of drop motion, the wall temperature $T_w$, drop deposition velocity $v_0$, and the thickness of a laminar sublayer $\delta$, are specified (Figure 4.3). For the first case where $S = 1.18$ [$S$ is the slip, from Eq. (4.42)], the deposition of the drop was not achieved. From the shape of the corresponding trajectory one can see that the drop starts moving toward the wall and after a few milliseconds reverses direction and starts moving away from the wall. Since the initial velocity of the drop in the flow direction is less than the local stream velocity at the edge of the laminar sublayer ($u_0 < U_0$, i.e., $S > 0$), the lift force has been directed toward the center line. The drop then has been decelerated because of the additive effect of lift, Stokes, and reaction forces. After the drop reverses direction of motion it starts accelerating toward the center line. It is important to mention here that the intensity of the lift force is proportional to the intensity of $(V - V_a)$ as is evident from Eq. (4.7).

For the second case where $S = 1.11$ (Figure 4.3), the drop was deposited on the wall. In this case the slip is less compared to the previous case. Because of this, the drop penetrated sufficiently close to the wall, so that it entered the region in which its velocity in the direction of flow $\frac{dx}{dt}$ becomes greater than the local fluid velocity. The lift force is now being
directed towards the wall and becomes larger than the sum of the Stokes and reaction forces, causing the drop to travel to the wall.

B. Effect of Drop Diameter

The effect of drop diameter on the drop trajectory is presented on Figure 4.4. Other parameters of drop motion for this case are specified (Figure 4.4). Since drop deposition velocity is the same for \( a = 20 \, \mu m \) and \( a = 50 \, \mu m \), the larger drop enters the boundary layer with the higher momentum. Because of this the drop of \( a = 50 \, \mu m \) reaches a region where \( (U - \frac{dx}{dt}) \) becomes negative and the direction of the lift force is reversed, i.e., the lift force now is directed toward the wall, assisting deposition. The drop of \( a = 20 \, \mu m \) returns to the main stream.

C. Effect of Wall Temperature

The reaction force on the drop due to non-uniform drop evaporation inside the sublayer is proportional to the square of the wall superheat \( (T_w - T_s) \), Eq. (4.31). The effect of the wall temperature on the drop motion inside the sublayer is presented in Figure 4.5. Three trajectories are plotted in this figure for the three different wall temperatures, 400°R, 450°R, and 500°R. The parameters \( v_0, u_0, U_0 \) and \( \delta \) are kept the same for the three different trajectories. As can be seen in Figure 4.5, the drop of \( a = 20 \, \mu m \) penetrates the sublayer to some depth and then is forced back into the main stream if the wall temperature is equal to or greater than 450°R. For the wall temperature of 400°R the same drop is deposited.
D. Effect of Deposition Velocity

The effect of the deposition velocity on the drop trajectory is presented in Figure 6. For the higher value of the deposition velocity, \( v_0 = 371 \) ft/hr, the drop is deposited as it enters the sublayer with higher momentum, overcoming the resistance of the Stokes and reaction forces and the initial resistance of the lift force until \( (U - \frac{dx}{dt}) \) is positive. As we mentioned before, the lift force is directed toward the main stream (center line) when \( (U - \frac{dx}{dt}) \) is positive.

E. Effect of Reynolds Number

As the Reynolds number (flow Reynolds number) increases, the boundary layer becomes very thin, the velocity gradient \( \frac{du}{dy} \) becomes steeper. Now the distance that the drop must travel into the sublayer for the lift force to reverse directions is very small. As \( \frac{du}{dy} \) is now very large, the lift force becomes very large in both regions: in the region where \( (U - \frac{dx}{dt}) \) is positive, especially at the beginning of this region, and the lift force is directed toward the center line; and in the region where \( (U - \frac{dx}{dt}) \) is negative and the lift force is directed toward the wall. Also, as the Reynolds number increases, the drop deposition velocity \( v_0 \), which is imparted to the drop by the transverse velocity fluctuations of the gas stream which carries it, increases.

On the figures 4.3-4.6 the radial position of the drop was plotted
against time, instead of the axial position. This was done to make evident the regions of the drop acceleration and deceleration.

In order to summarize the studies on the drop motion inside the sublayer the history of the drop that penetrated the sublayer to a certain depth and was forced back to the main stream is presented in Figure 4.7 and Table 1. On the Figures 4.7b-c the forces on the drop in the $y$ and $x$ directions during drop motion are plotted against time. In Table 1 the radial position of the drop, the axial and radial velocities of the drop, and the difference between the axial velocities of the stream and drop are tabulated as functions of time. The drop Reynolds number is calculated and plotted in Figure 4.7d.

The same analysis for the drop that has been deposited on the wall is presented in Figure 4.8 and Table 2.

From Figures 4.7a-b one can see that the direction of the lift force has been changed at $y = 0.8865 \times 10^{-3}$ft as $(U - \frac{dx}{dt})$ becomes negative (see Table 1), but the lift force has been overcome by the reaction and Stokes forces and the drop returned to the main stream.

So if $(U - \frac{dx}{dt})$ is negative the lift force on the drop is directed toward the wall as we mentioned before, but this is not sufficient condition for the deposition of the drop. From Figure 4.8 and Table 2 one can see that the drop penetrates sufficiently far into the sublayer where, at $y = 0.88 \times 10^{-3}$ ft, the lift force overcomes the reaction and Stokes forces, and the drop begins to accelerate toward the wall.

When the drop is approaching a wall the drop Reynolds number is becoming
larger than 1.0 due to drop acceleration (Figure 4.8d). The same is true for the drop leaving the sublayer (Figure 4.7d). In this case the Stokes expression for the drag of the drop is not applicable, but since the lift force dominates in these regions, the correction to the drag coefficient does not play a significant part in the drop trajectory.

By analyzing the values for the lift force and the drop Reynolds number along the drop trajectory we can conclude that the lift force increases with increased drop Reynolds number (Figure 4.7 and 4.8).

The analysis of the drop trajectories on Figures 4.3, 4.4, 4.5, 4.6, and 4.7, and the numerical values for the drop deposition in Table 2, lead to the conclusion that the drops are being deposited on the wall with a definite impact. This is in qualitative agreement with the results of Cousin and Hewitt (1968). Their results of the photographic studies in two-phase up-flow of water drops suspended in air indicate that drops reach the surface with high velocities without being slowed as they enter the region of low gas velocity adjacent to the surface.

The significant conclusion concerning the drop trajectories in Figures 4.3-4.8 is that since the most critical effects of the lift, reaction, and drag forces on the drop deposition take place sufficiently far away from the wall, the effect of the wall may be neglected, as was assumed in Eqs. (4.39a,b). Also, the effect of the evaporation on the drag coefficient and drop Nusselt number is negligible as the values of the transfer coefficient $B$ for $y > y_{cr}$ are small ($y_{cr}$ is the critical distance, i.e., the distance where
the critical effects of the forces occur). For example, the values of $B$ for the trajectories in Figures 4.3, 4.6, 4.7a at $y = y_{cr}$ are 0.007, 0.098, and 0.106, respectively.

Since the vapor superheat increases when a drop is approaching a wall, then $B$, which is a function of the local vapor superheat, increases and effects of the evaporation on the drag coefficient and drop Nusselt number may be significant. On the other hand, the lift force dominates when the drop is approaching a wall, as we mentioned before, and any corrections for the drag coefficient and drop Nusselt number do not play a significant part in the drop trajectory.

The equations (4.39a,b) were solved assuming that decrease of the drop diameter due to evaporation during deposition motion inside the sublayer was negligible. This assumption holds very well for larger drops ($d > 10 \text{ \mu m}$) since their stay time inside the sublayer is very short (Appendix IV-5). For a small drop, a diameter of the order of magnitude of 1 \text{ \mu m}, the effect of the curvature on the surface tension should be included when analyzing the rate of decrease of drop mass due to evaporation. This was omitted in our study because small drops do not usually penetrate the sublayer (Figure 4.9).

Little error is introduced in the analysis by assuming that the angular velocity of the drop is the same as that of the fluid stream. The actual angular velocity of the drop is given by the following relation (Rubinow-Keller, 1960);
\[ \omega = - \frac{1}{2} \frac{dU}{dy} \left[ 1 - \exp \left( - \frac{60u_g}{a^2 \rho_f} t \right) \right] \] (4.43)

For the drop of \( a = 20 \ \mu m \) (Figure 4.8, Table 2), the angular velocity of the drop and stream are becoming practically the same within two milliseconds. The time of two milliseconds is much smaller than the drop stay time (Table 2) and the fact that the drop undergoes unsteady motion can be neglected. The same drop made 23 revolutions before being deposited. This was calculated using Eq. (4.43) and the drop deposition history from Table 2.

In our study of the drop deposition through the sublayer the lift force on the drop derived by Rubinow and Keller is applied instead of Saffman's lift force. The fact is that we are dealing with the dispersed flow gas (vapor) drops where flow Reynolds number is very high. For such types of flow (drops of size from 1 \( \mu m \) to several hundred microns) the conditions for the validity of Saffman's expression for the lift force as specified by Eq. (4.41) are satisfied only for small drops with a diameter of the order of magnitude of 1 \( \mu m \). The same was concluded and discussed with more details by Rouhiainen and Stachiewicz (1970). The reader is referred to Figure 5 of their article. It is also important to mention here that the cumulative mass of drops with a diameter of 1 \( \mu m \) and less is negligible compared to the total cumulative mass of the drops in the dispersed flow under consideration.

The Rubinow-Keller lift force was not derived for a shear flow but on
the other hand, the experimental and theoretical studies of the drop trajectories by Denson et al (1966), Goldsmith (1962), Repetti (1964), and Kondic (1970) indicated that it yields a force of the correct order of magnitude for such a flow.

It is interesting to mention that Saffman's and Rubinow-Keller's lift forces become numerically the same for the larger drops \((a > 10 \, \mu m)\) for the dispersed flow considered in this study (Appendix IV-3).

4.4 Application of Drop Deposition Model

So far we have been analyzing the motion of the single drop through the sublayer. In order to determine the drops deposition rate on the wall (i.e., the number of drops deposited per unit area of the wall and until time), a drops size spectrum \(P(a)\) was introduced into the analysis.

Predictions of drops trajectories for \(a_0 < a < a_m\) (where \(a_0\) and \(a_m\) are minimum and maximum drop diameter, respectively, in dispersed flow under consideration) was done by increasing initial drop diameter \(a_0\) step by step and solving the equations (4.39a,b) for each value of \(a\).

The initial boundary condition given by Eq. (4.41b) was expressed as a function of the friction velocity,

\[
v_0 = k U^* = k U \sqrt{\frac{g}{2}}
\]

(4.44)
Relation (4.44) was obtained from the summarized study of deposition of solid particles and liquid drops in two-phase mixtures by Liu and Ilori (1973). On the Figure 2 of their article, particle deposition velocity was plotted vs. particle relaxation time. The particle dimensionless relaxation time \( t^* \) was defined in terms of particle diameter, particle density, and viscosity of the main stream, i.e., \( t^* = \frac{a^2 \rho \sqrt{\gamma}}{18 \mu g} \). By careful examination of Figure 2 of their article one can see that the particle (drop) deposition velocity is independent of the particle diameters, ranging from several microns to several hundred microns. The value of \( k \) [Eq. (4.44)], related to the deposition velocity of drops at the edge of the sublayer, is always less than one but is actually a parameter. In this analysis we used \( k = 0.15 \) as suggested by Iloeje et al (1974).

The initial drop velocity in the flow direction, \( u_0 \), at the edge of the sublayer, can be approximately determined by equating the drag force on the drop and its weight. Cumo et al (1972) performed visual observations of the dispersed flow employing high speed cinematography. Analyzing more than 10,000 drops, they found that the drop velocity in the direction of flow is independent of drop diameter and almost equal for all drops. They also found that the drop transversal velocity profile in the flow channel is very close to the vapor velocity profile with local slip ratio close to
one (see for instance Figure 9 in their article). Since their observation was basically outside of the laminar sublayer one can assume that the drop velocity in the flow direction at the edge of the sublayer is

\[ u_0 = \frac{U}{S} \]  

(4.45)

where \( S \) is the slip ratio at the edge of the laminar sublayer. The methods of the evaluation of the slip ratio \( S \) are referenced in the Appendix III-2.

For a given mass flux \( G \) and vapor quality \( x \) the initial boundary conditions, Eqs. (4.44) and (4.45), were evaluated. Equations (4.39a,b) were then solved and trajectories obtained for each value of \( a, a_0 < a < a_m \), for one particular value of the wall temperature. The summarized results of calculation for \( G = 215,000 \) lb/ft\(^2\)hr and \( x = 0.49 \) are presented in Figure 4.9.

For example, if the wall temperature was equal to the saturation temperature (no heat addition) then drops with a diameter \( a > 10 \) \( \mu \)m were deposited on the wall and drops with a diameter \( a < 10 \) \( \mu \)m were returned to the main stream (Figure 4.9). The diameter \( a = 10 \) \( \mu \)m for this case is so called the deposition diameter \( a_c \), i.e., \( a_c = 10 \) \( \mu \)m.

For the same conditions and the wall temperature of 700°C, \( a_c = 260 \) \( \mu \)m, etc. (Figure 4.9). The curve in Figure 4.9, deposition diameter vs. wall temperature, was obtained for the step increase in drop diameter of \( \Delta a = 5 \) \( \mu \)m, for one particular value of the wall temperature. The step increase in wall temperature was \( \Delta T_w = 20^\circ C \).
The important point to be noted here is that small drops, \( a < 10 \, \mu m \), were not deposited. Their trajectories were obtained assuming that deposition velocity at the edge of the sublayer was given by Eq. (4.44) but the actual deposition velocity of such small drops is less as it can be seen from Figure 2 of Liu and Ilori's article. This leads to the conclusion that small drops, of a diameter of the order of magnitude of 1 \( \mu m \), do not penetrate the sublayer under normal conditions, i.e., when their deposition velocities are imparted to them by the transverse velocity fluctuations of the main stream. Because of this, the detailed analysis of the evaporation and its effect on the drop trajectory for such small drops was omitted in our study.

The deposition model, Eqs. (4.39a,b), related to the drop motion inside the sublayer, is applicable in the buffer layer too, where quasi-laminar motion can be assumed, i.e., velocity fluctuations of the main fluid can be neglected. The constant \( k \) [Eq. (4.44)] will have a greater value at the edge of the buffer zone than that at the edge of the sublayer. As soon as we move out in the turbulent core, velocity fluctuations of the main fluid (gas) cannot be neglected and the application of the model is questionable. On the other hand, the drop deposition occurred along trajectories mainly developed in the laminar sublayer.

The effect of the wall temperature on the drops deposition is once again demonstrated in Figure 4.9.

For the known value of the deposition diameter \( a_c \), and drop size distribution law \( P(a) \), Eq. (3.7), we define the cumulative factor \( f(a_c) \)
as
\[
f(a_c) = \frac{\int_{a_m}^{a_c} a^n P(a) \, da}{\int_{a_m}^{a_c} a^n P(a) \, da}
\]
(4.46)

For \( n = 3,2,1 \), \( f(a_c) \) represents mass, surface, and momentum cumulative factor, respectively.

From Eq. (4.46) one can see that \( 0 \leq f(a_c) \leq 1.0 \). If the wall temperature increases \( a_c \) increases (Figure 4.9) and \( f(a_c) \) as defined by Eq. (4.46) decreases since the number of drops deposited on the wall decreases. For any value of \( a_c \), \( f(a_c) \) is calculated using Eq. (3.7) for \( P(a) \). The value of \( f(a_c) \) is used in calculating drop deposition flux (number of drops deposited per unit area of the wall and unit time) as explained later in section 5.2.

The accuracy of the deposition model developed in this study in determining the deposition diameter \( a_c \) is based on the validity of several assumptions mentioned before, related to the magnitude of the forces of the drop.

The region of the model inaccuracy, as indicated in Figure 4.10, may produce a small consistent overestimate or underestimate in calculating the cumulative factor \( f \). On the other hand, the drops deposition phenomenon is in general well described by the model.
Figure 4.1 Direction of the Lift Force Acting on the Drop in a Boundary Layer

a. Velocity Component of the Drop is Less Than the Velocity of the Fluid (gas)

b. Velocity of the Drop is Greater Than the Velocity of the Fluid

Figure 4.2 Direction of the Reaction Force Acting on the Evaporating Drop in a Thermal Boundary Layer
Figure 4.3 The Effect of the Slip Ratio on the Drop Trajectory

Figure 4.4 The Effect of Drop Diameter on the Drop Trajectory

Figure 4.5 The Effect of the Wall Temperature on the Drop Trajectory

Figure 4.6 The Effect of Drop Deposition Velocity on the Drop Trajectory
Figure 4.7a Trajectory of Drop

\[ \delta = 300 \mu \]
\[ a = 20 \mu \]
\[ T_w = 450 ^\circ R \]
\[ v_o = 271 \text{ ft/hr} \]
\[ U_o = 49342 \text{ ft/hr} \]
\[ u_o = 0.885 U_o \]

Figure 4.7b Forces on the Drop in the \( y \)-Direction

Figure 4.7c Forces on the Drop in the \( x \)-Direction

Figure 4.7d Drop Reynolds Number
<table>
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<th>t (ms)</th>
<th>y (ft)</th>
<th>$\frac{dy}{dt}$ (ft/hr)</th>
<th>$\frac{dx}{dt}$ (ft/hr)</th>
<th>U (ft/hr)</th>
<th>$U - \frac{dx}{dt}$ (ft/hr)</th>
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$t$ = time
$x$ = axial position of drop
$y$ = radial position of drop

$\frac{dx}{dt}$ = drop velocity in $x$ direction
$\frac{dy}{dt}$ = drop velocity in $y$ direction

$U$ = vapor velocity

**TABLE 1. History of Drop Motion** ($a = 20 \mu; T_w = 450^\circ R$)
Figure 4.8a Trajectory of Drop

\[ \delta = 300 \mu \]
\[ \rho = 20 \mu \]
\[ T_w = 400 \, ^\circ R \]
\[ v_0 = 271 \, \text{ft/hr} \]
\[ U_c = 49345 \, \text{ft/hr} \]
\[ u_o = 0.885 \, U_o \]

Figure 4.8b Forces on the Drop in the \( y \)-Direction

Figure 4.8c Forces on the Drop in the \( x \)-Direction

Figure 4.8d Drop Reynolds Number
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$t =$ time  
$x =$ axial position of drop  
$y =$ radial velocity of drop  
$\frac{dx}{dt} =$ drop velocity in x direction  
$\frac{dy}{dt} =$ drop velocity in y direction  
$U =$ vapor velocity

**TABLE 2. History of Drop Motion**  
($a = 20 \mu$, $T_w = 400^\circ R$)
Figure 4.9 Effect of Wall Temperature on the Drop Deposition Diameter

Fluid: Nitrogen
\( G = 215,000 \text{ lb}_m/\text{ft}^2 \text{ hr} \)
\( x = 0.49 \)
Figure 4.10 Drop Size Distribution

Region of Model Inaccuracy
CHAPTER V: DISPERSED FLOW HEAT TRANSFER

As was mentioned in Chapter I, heat transfer from the wall to the dispersed flow is the sum of

(a) heat transfer to liquid drops deposited on the wall, \( (q/A)_d \);
(b) heat transfer to the bulk vapor component of the flow, \( (q/A)_v \);
(c) radiation heat transfer between the wall and dispersed flow, \( (q/A)_r \);

or

\[
q/A = (q/A)_d + (q/A)_v + (q/A)_r \tag{5.1}
\]

In deriving an expression for \( (q/A)_d \) we will apply our results concerning the structure of dispersed flow (Chapter III) and dynamics of drops deposition on the hot wall (Chapter IV). First, however, we will analyze the heat transfer to the single drop deposited on the wall.

5.1 Heat Transfer to Single Drop

In dispersed flow with heat addition liquid drop penetrating the boundary layer collides with the heated wall and cools it by its evaporation. The physical variables which characterize the drop-wall collision and drop evaporation are very numerous, including all thermal and dynamic properties of the gas and liquid phase of the fluid, the thermal properties and surface conditions of the heated wall. A shape of the liquid drop in contact with solid wall varies with its dimensions. When the drop is very
small it will keep its spherical shape because of the surface tension effect. In the case of a large drop (diameter of the order of several hundred microns or more) it will form disc-shaped liquid film. When liquid drop touches the wall a contact boundary temperature is immediately established which depends [32] on the initial liquid and wall temperature and on the nature of the liquid and wall. Approximately [45],

\[
\frac{T_c - T_L}{T_w - T_c} = \left[ \frac{(k \rho C_p)_D}{(k \rho C_p)_L} \right]^{\frac{1}{2}}
\]

(5.2)

where

- \( T_c \) = contact boundary temperature,
- \( T_w \) = initial wall temperature,
- \( T_L \) = initial liquid temperature,
- \( k \) = conductivity,
- \( \rho \) = density,
- \( C_p \) = specific heat,

and subscripts \( L \) and \( W \) refer to liquid and wall, respectively.

In spite of the above information, very little is known exactly, in technical literature, about thermal behavior of liquid drop deposited on the wall, over a wide range of the wall temperature and the drop impact velocity. In reference [10], where the experimental study of the dispersed flow drop behavior has been performed using high speed photography, it has been concluded that:
(a) As the wall temperature increases, the pictures show fewer and fewer drops touching the wall. The deposition model presented in Chapter IV of this study predicts this phenomenon.

(b) Drop-wall contact time is decreasing with wall temperature increase.

Several other investigators have studied wall-drop interaction [23].

Parker and Grogh (1961) studied steam-water dispersed flow heat transfer at qualities above 89% at 30 psia. In their study they concluded that if the wall temperature was below the minimum film boiling temperature (Leidenfrost\(^*\) temperature, the temperature where the wall is so hot that the momentum of the rapidly generated vapor between the liquid drop and the hot surface forms a steam cushion to support the drop and prevent the liquid from wetting the surface), the heat transfer coefficients were three to six times the dry steam value. For wall temperature above the minimum film boiling temperature the heat transfer coefficients were almost identical to those for dry steam [47,23].

The drop splattering process, defined as the disintegration of drops

\(^*\)Leidenfrost temperature, as described in technical literature, is usually related to the phenomenon of the evaporation of the stationary liquid drops deposited on the wall. The drops are of the order of several thousand microns or more [2]. In our case the drops are of the order of tens of microns and drop-wall contact times are very short (few milliseconds or less) [10,2]. Although there is a difference in the experimental conditions the existence of the Leidenfrost phenomenon appears also in the dispersed flow under consideration.
striking a hot wall without significantly wetting the wall was studied by McGinnis (1969). Splattering was found to occur at the higher drop velocities. The heat transferred per drop was expressed as a function of fluid properties and reached a maximum at surface temperature of 329°F above saturation for water, acetone, and alcohol drops. This maximum, for the impact angle of 27°, was given by the following correlation:

\[
\frac{Q_{\text{max}}}{\rho_l a^3 H_{\text{lg}}} = 8.44 \times 10^{-4} \left| \frac{\rho_l^2 v_0^2 a}{\rho_{\text{lg}} \sigma} \right|^{0.341}
\]

(5.3)

where

- \( \rho_{\text{lg}} \) = density of the vapor at the film temperature,
- \( H_{\text{lg}}' = H_{\text{lg}} + C_p(T_w - T_{\text{sat}})/2 \),
- \( \rho_l \) = liquid density,
- \( a \) = drop diameter,
- \( \sigma \) = surface tension,

and \( H_{\text{lg}} \) = latent heat of evaporation.

Equation (5.3) has a limited applicability to dispersed flow heat transfer since it is based on very large drops \((a > 2.5 \text{ mm})\).

Dispersed flow heat transfer has been studied by Forslund (1966) and later by Hynek (1969). Forslund's work is one of the first studies where drops-wall heat transfer has been analytically treated applying Baumeister's (1966) equation for the heat transfer coefficient for a stationary drop on the hot surface.
Wachters and Westerling (1966) also studied heat transfer from individual drops. They obtained data for drops 2300μ in diameter, approach(impact) velocities varying from 3.54 to 4.82 ft/s, and surface temperatures varying from 410°F to 662°F. Their data indicated a very low effectiveness [effectiveness is defined by Eq. (5.4) below] ranging from a maximum of 1.5 percent at 410°F to a minimum of 0.15 percent at 660°F.

Corman (1966) and Gauglev (1966) also studied drop-hot wall heat transfer. Their data will be shown below.

Cumo and Farello (1967) made a significant contribution to the understanding of disperse flow heat transfer phenomena. From a photographic study they concluded that between heated surface temperatures of 320°F and 482°F there is a sudden reduction in surface wettability [10,23]. Above 482°F his photographic results do not permit distinction between drops touching the surface or being insulated from it by a thin vapor blanket [10,23]. The reader is referred to their study for more details.

Pederson (1967) obtained heat transfer data for individual water drops impinging upon a heated surface. The drop diameters ranged from 200μ to 400μ, and the approach velocities from 8 to 33 ft/s. The effect of surface temperature variation from the saturation temperature to 1800°F was studied. Photographs of the impingement process are presented which show that even the small drops studied break up upon impingement at moderate approach velocities. These results, effectiveness of evaporation ε, are shown on Figure 5.1 together with results of other investigators. The effectiveness of evaporation ε is defined as:
Figure 5.1 Effectiveness vs. Surface Temperature
\[ \varepsilon = \frac{Q}{m \left[ H_{lq} + C_{pl}(T_{sat} - T_{l}) \right]} \] (5.4)

where

- \( Q \) = heat transferred per drop,
- \( m \) = mass of drop \( (m = \frac{\pi}{6} a^3 \rho_L) \),
- \( H_{lq} \) = latent heat of evaporation,
- \( C_{pl} \) = liquid specific heat,
- \( T_{l} \) = liquid (drop) temperature,
- \( T_{sat} \) = saturation temperature.

The effectiveness of evaporation measured by Pedersen was generally higher than the corresponding values measured by the other investigators (Figure 5.1). This may be due to the dry air atmosphere used by Pedersen rather than a saturated steam atmosphere as suggested by Groeneveld (1972).

Figure 5.1 clearly demonstrates the effect of the wall (surface) temperature on the evaporation of the deposited drops. It is evident from this figure that the wall cooling due to the direct removal of heat as latent vaporization heat of deposited drops is much less significant at the higher wall temperature.

Toda (1972) studied a heat transfer cooling phenomenon by mist drops on a heated surface at a temperature above the saturation temperature. Drops were sprayed by any method, collided with the heated surface, and cooled it by their evaporation. It was revealed that the heat transfer characteristics of mist cooling can be classified into the three regions, according to the thermal behavior of liquid film formed from mist drops on
the heated surface:

(a) low surface temperature region,
(b) high surface temperature region,
(c) combination of (a) and (b).

Toda developed the semi-theoretical equations for the time-averaged heat flux for three regions listed above which included four experimentally determined constants. His theory is in good agreement with his experimental data.

A comprehensive study of the evaporation of the liquid drop deposited on the wall has been performed by Iloeje (1974). He assumed that after a drop was deposited on the wall, nucleation and bubble growth from liquid drop take place (pool boiling model assumed). At the end of bubble growth period, a part of the liquid drop is ejected into the main stream by escaping vapor bubbles and a part is left on the wall to evaporate (see Figure 44 of his thesis where proposed model is drawn). His analytical procedure which involves several assumptions [32] gives heat transferred to a single drop as

\[ Q = C_0 C_1 \frac{\pi \alpha^2}{4} \left( \frac{\rho_L}{\rho_g} \right) \frac{R_v \sigma \rho_L c_{pl} T_{sat}^2}{0.213 P_L H_{lg}} \]  

(5.4a)

where

- \( R_v \) = gas constant,
- \( \sigma \) = surface tension,
- \( C_{pl} \) = liquid specific heat,
- \( P_L \) = liquid pressure,
- \( \alpha \) = average drop diameter in dispersed flow as defined by Forslund [17],
and \( C_0 \) and \( C_1 \) are constants with values of 0.683 and 0.455, respectively. The value of \( Q \) calculated from Eq. (5.4a) is only a few percent of the maximum value which is given as \( Q_{\text{max}} = \frac{ma^3}{6} \rho g \). This makes Iloeje's expression applicable only in the very high temperature region \( (T_w >> T_{\text{min}}) \), \( T_{\text{min}} = \) minimum film boiling temperature).

The successive states of drop-wall interaction are shown in Figure 5.2. This figure was made after analyzing available experimental results [10, 63, 47, 44, 2, 67, 48, 46] related to the deposition of the drop and its evaporation on the heating wall.

At the high wall temperature and high drop impact velocity, tiny drops rebound on the wall with little cooling action until the wall temperature is reduced to a certain value (the minimum film boiling temperature) at which point the drops wet the wall with a significant cooling action [10] (Figure 5.2a). For the same condition the large drop behavior as shown in Figure 5.2b where a part of drop is ejected into the main stream by escaping vapor bubbles [46, 10].

In the high temperature region and low impact velocity \( (v_0 < 6.5 \text{ ft/s}) \) the liquid film formed from the deposited drop is found to be in a film boiling-line state (Figure 5.2c) in which a vapor layer is formed between the liquid film and the heated surface [46, 63]. The evaporation of the liquid film on the liquid-vapor layer interface is induced by convective heat transfer through the vapor layer and radiation heat transfer from the heated wall.

The low wall temperature and low impact velocity case is shown on
Figure 5.2 Successive States of Liquid Drop - Wall Interaction
Figure 5.2d where the liquid film evaporates from the liquid-vapor interface. At the initial stage of formation of a liquid film on a heated wall, heat is transferred by conduction from the heated wall to the liquid film. When the superheated thermal layer in the liquid film is fully developed, evaporation of the liquid film at the liquid-vapor interface takes place. This liquid-vapor interface moves with time [63,46].

Not all possible states of the drop-wall interaction are covered in Figure 5.2. Only the most frequently observed states are shown.

The list of variables that have a certain influence on the wall-drop heat transfer includes:

- wall temperature,
- nature of the fluid,
- drop impact velocity,
- nature of the heating wall and coupling of fluid and wall,
- drop temperature,
- surface state of heating wall (microunghness, oxidation, etc.).

The "behavior" and the further existence of the liquid film formed from the deposited drop is mostly prescribed by the wall temperature (Figures 5.1 and 5.2). The wall temperature is the most important variable associated with this problem.

The data for water obtained by Pedersen [48] show that drop impact velocity is the dominant variable affecting drop heat transfer in the very high temperature region ($T_w = T_{\text{min}} + 500^\circ\text{F}$). The effectiveness of the drop evaporation as defined by Eq. (5.4) was very low ($\varepsilon = 5\%$) for the above range
of the wall temperature. The increase in $\varepsilon$ (about 100%) due to the increase in the impact velocity does not increase significantly the total heat transfer to the liquid in the case of dispersed flow $[(q/A)_d, \text{Eq. (5.1)}]$ since the drop deposition flux (number of drops deposited per unit area of the wall per unit time) is very low in such a high temperature region. In the region of high drop deposition flux and high $\varepsilon$ (the wall temperature in the vicinity of and below the minimum film boiling temperature) even small increases in $\varepsilon$ due to increases in drop impact velocity will significantly increase total heat transfer to the liquid $[(q/A)_d, \text{Eq. (5.1)}]$.

Oxide films and crud [32] can reduce the effective liquid drop-surface contact angle and liquid contacting the surface will spread over a wider area, increasing the heat transfer directly to liquid in spite of a high surface thermal resistance layer created by the oxide.

The wall microunghness increases the heat transfer to liquid drop. When a liquid drop hits the wall it is possible that it will be dragged along the wall due to its velocity in the flow direction (see Table 2). Therefore, the liquid wets the wall penetrating into the microscopic surface depressions and heats up rapidly, beginning on very fast evaporation [32,10].

The effect of the drop temperature (drop subcooling) on the drop heat transfer is mentioned in reference [48]. The drop subcooling increases the heat transfer since it lowers the contact boundary temperature (liquid drop-wall contact boundary temperature) as shown by Eq. (5.2).

Decreasing the contact angle (the contact angle between the wall and
the liquid drop is measured through the liquid) increases the wettability of the surface, which in turn increases the heat transfer to drop. As shown in Figure 5.2, the contact angle is important for the case (d) (low wall temperature, \( T_w < T_{\text{min}} \)) and less important for cases (a), (b), and (c) (high wall temperature), where there is not full contact of the liquid with wall.

The wall-drop heat transfer under stationary vapor conditions (a sessile drop or drop impinging upon a wall [48,63,47]) and under flowing vapor conditions (a drop deposited from the vapor stream [10,44,16]) are, to a certain extent, different processes. The presence of a vapor and drop velocity parallel to the wall, for example as in Figure 5.2c, will reduce the vapor layer thickness between drop and wall, thus increasing the heat transfer through this layer. The successive states of drop-wall interaction shown in Figure 5.2 have been observed in both cases (with and without vapor flow). Of course this does not eliminate the need for the caution that should be exercised in extrapolating the data for these two cases [32,16].

The experimental data and proposed correlation for \( Q \) (heat transferred to a single drop evaporating on the wall) available in the literature [62,48,10,44,16] were analyzed in deriving \( (q/A)_d \). The relation between \( Q \) and \( (q/A)_d \) will be shown in the next section. When incorporated into our heat transfer prediction scheme none of the correlations for \( Q \) provided satisfactory prediction of our experimental data. All the experimental data for \( Q \) [63,47,44,48,18,67,7,10] show the exponential decay in \( Q \) with increase of wall temperature (Figure 5.1). Using this information, we
introduced the following simple expression for the heat transferred to a single drop on the wall, deposited from the fluid stream:

\[ Q = \frac{\pi \alpha^3}{6} \rho L H_{\text{lg}} e^{1-(T_w/T_{\text{sat}})^2} \]  

Although this expression included only the nature of the fluid and the wall temperature as variables, it provided good prediction of the experimental data obtained by the author. This will be shown below.

5.2 Drop Deposition Flux and \((q/A)_d\)

The deposition of a drop in dispersed flow with heat addition has been analyzed in Chapter IV. The mass flux of liquid drops migrating toward the wall, entering the laminar sublayer, is by definition

\[ M = v_0(1 - \alpha)\rho L \]  

where
\[
\begin{align*}
M &= \text{mass flux (lb/ft}^2\text{hr),} \\
\alpha &= \text{void fraction (by homogeneous model),} \\
\rho L &= \text{liquid density,} \\
\text{and } v_0 &= 0.15 \frac{G_x}{\alpha \rho g} \sqrt{\frac{F}{g}} \text{, drop deposition velocity, Eq. (4.44).}
\end{align*}
\]

The mass flux of liquid drops at the wall [32] is then

\[ M_w = v_0(1 - \alpha)\rho L f \]  

(5.6a)
where $f$ is the mass fraction of the drops entering the laminar sublayer which reach the wall, Eq. (4.46).

The drop deposition flux (the number of the drops deposited per unit area of the wall per unit time) is then

$$N = \frac{M_w}{\pi \bar{a}^3 \rho} \frac{6v_0(1 - \alpha)}{\pi \bar{a}^3} \left[ \frac{\# \text{ drops}}{\text{ft}^2 \cdot \text{hr}} \right]$$  \hspace{1cm} (5.7)

where $\bar{a}$ = average drop diameter for $a_c < a < a_m$, defined by Eq. (3.11).

Applying Eq. (5.5), the heat transfer from the wall to the liquid drops is

$$(q/A)_d = N \cdot Q = v_0(1 - \alpha)\rho L e f e w s a t \left(1 - \frac{T_w}{T_{sat}}\right)^2$$ \hspace{1cm} (5.8)

The equivalent procedure to the above, in deriving $(q/A)_d$, is presented in Appendix V-1.

5.3 Heat Transfer to Vapor, $(q/A)_v$

Heat transfer from the wall to the bulk vapor component of the dispersed flow is given by the McAdams equation, using a vapor flow Reynolds number,

$$(q/A)_v = 0.023 \frac{k_q}{D} \text{Re}^{0.8} \text{Pr}^{0.4} \left(\frac{T_w - T_{sat}}{T_{sat}}\right)$$ \hspace{1cm} (5.9)

where $\text{Re} = \frac{Gx}{\alpha u_g}$.
Fluid properties are calculated at bulk vapor temperature. It was assumed in the above equation that the surface void fraction $\alpha_s$ (percentage of the wall area available for the heat transfer to the vapor flow) was equal to unity (Appendix V-2).

5.4 Radiation Heat Transfer, $(q/A)_r$

In dispersed flow with heat addition the heated wall is also cooled by radiation. It is given as the sum of the radiation heat transfer from the surface to the liquid drops and to the vapor,

$$(q/A)_r = \Phi_{we} \sigma (T_w^4 - T_{sat}^4) + \Phi_{wv} \sigma (T_v^4 - T_{sat}^4) \quad (5.10)$$

assuming that the liquid drops and vapor are at the saturation temperature. $\Phi_{we}$ and $\Phi_{wv}$ are gray body factors between the wall and drops and between the wall and vapor, respectively. $\sigma$ is the Stefan-Boltzmann constant. The evaluation of $\Phi_{we}$ and $\Phi_{wv}$ was presented in ref. [60] where dispersed system was assumed gray and diffuse, the absorption and emission of the mixture was incorporated into the network analysis by treating the system as an enclosure filled by a radiating gas and a cloud of liquid drops. Details of this analysis are omitted here as $(q/A)_r$ was negligible in the nitrogen dispersed flow under consideration (Table 3).
The emissivity of the test section surface is given in reference [41].
5.5 Total Heat Flux, $q/A$

The total heat transfer from the wall to the dispersed flow under consideration can be calculated for the given mass flux, vapor quality, and system pressure, using Eq. (5.8), developed in this study, and Eq. (5.9),

$$q/A = v_0 (1 - \alpha) \rho \frac{H_l q}{f} e^{1 - (T_w/T_{sat})^2} +$$

$$+ 0.023 \frac{k}{h} Re^{0.8} Pr^{0.4} (T_w - T_{sat})$$

(5.11)

The comparison of Eq. (5.11) with the experimental data obtained by the author, for a relatively wide range of mass fluxes and vapor qualities, is presented in Figures 5.3 - 5.12. The minimum value of $q/A$ [Eq. (5.11)] corresponds to the minimum film boiling temperature (the rewet wall superheat). The equation (5.11) represents a well-known boiling curve for specified values of mass flux, vapor quality, and system pressure. It covers a low and high wall superheat dispersed flow heat transfer, i.e., flow transition boiling and flow film boiling, respectively. The effect of the mass flux and vapor quality on $q/A$, given by Eq. (5.11), is as follows:

(a) When the mass flux increases (constant vapor quality assumed), $q/A$ increases and $T_{min}$ increases.

(b) When the vapor quality increases (constant mass flux assumed), $(q/A)_d$ decreases, $(q/A)_v$ increases, and $T_{min}$ decreases. This affects $q/A$ in such a way that $q/A$ increases in the high
temperature region (film boiling region) and decreases in the low
temperature region (transition boiling region).

These conclusions are experimentally supported (Figures 5.3 - 5.12) within
the experimental error of the data (Appendix II-1).

5.6 Value of C, Equation (3.5)

The values of $C$ in Eq. (3.5), selected for the prediction of the experimental data in Figures 5.3 - 5.12, are summarized in Figure 5.13. Figure 5.13 shows that $C$ increases when the mass flux and vapor quality increase. In references [12,57], where Nukiyawa-Tanazawa equation [Eq. (3.4)] was applied for calculation of $\bar{a}$ (mean drop diameter), the vapor velocity was at least ten times higher than in our case. We may assume that $C$ will approach unity in our case but at very high values of $G$ (mass velocity) and $x$ (vapor quality), since at such high values of $G$ and $x$ the vapor velocity will be very high, i.e., we will be approaching the conditions similar to those for which the Nukiyawa-Tanazawa equation was developed.

From Figure 5.13 one can see that the value of $C$ is larger at higher mass velocity. It does not mean that drops are smaller at lower values of $G$; it simply shows that at a lower value of $G$, the conditions concerning vapor velocity are further from the conditions of the Nukiyawa-Tanazawa equation and the numerical correction of the equation (3.4) is larger.

The effect of gravity on the drop size in our experiment and the experiments relevant to ours [58] is probably very important as the vapor velocity
is not very high. Also, the effect of the drop evaporation (heat and mass transfer at the drop interface) on the drop size can be very important. Neither of these two effects, gravity or evaporation, are included or considered in the Nukiyawa-Tanazawa analysis [12, 68].

The value of $C$ affects the value of $\bar{a}$ [mean drop diameter, Eq. (3.5)], while the drop size distribution is affected by $\bar{a}$ [Eq. (3.7)]. The drop cooling rate at the wall is affected by $P(a)$ [Eq. (5.8)], etc. All this emphasizes the importance of knowing the actual value of the mean drop diameter in predicting the heat transfer in a dispersed flow.

The prediction of our experimental data with constant value of $C = 0.26$ is analyzed in Appendix V-3. Since the predictions in Figures V-2, V-4, V-10 are not good, the constant value of $C$ is not recommended. It is important to mention that $C$ is not a dimensionless parameter, as is evident from Eq. (3.5).

5.7 Thermal Non-Equilibrium in Dispersed Flow

A significant degree of thermal non-equilibrium can exist in a dispersed flow, i.e., a vapor can be superheated. The vapor temperature is, in that case, given by energy balance between the evaporating liquid and superheating vapor,

$$x_E H_{lg} = x \left[ H_{lg} + C_p g (T_v - T_{sat}) \right]$$

(5.12)

where

$T_v = \text{vapor temperature,}$

$x = \text{actual quality,}$

and $x_E = \text{equilibrium quality.}$
The actual quality, the ratio of mass flow of vapor to total mass flow, is less than or equal to the value calculated from the equilibrium energy balance (equilibrium quality) for the case of a dispersed flow. In order to find \( T_v \) from Eq. (5.12) we need one more relation between \( x \) and \( x_E \). Forslund (1966) experimentally obtained the relation between \( x \) and \( x_E \) for nitrogen (Figure 19, ref. [17]). Plummer (1974) deduced the relation between \( x \) and \( x_E \) using Forslund's data. Applying Forslund's data or relation suggested by Plummer (Eq. (4.1), ref. [50]), \( T_v \) can be calculated from Eq. (5.12). Groeneveld (1972) proposed the relation between the actual and equilibrium quality which is based on the large number of the experimental data (Eq. (4), ref. [24]). Knowing \( T_v \), the driving force for the convection heat transfer [second term, Eq. (5.11)] is \( (T_w - T_v) \) instead of \( (T_w - T_{sat}) \).

The analysis of the drop deposition presented in Chapter IV is also valid for the case when vapor is superheated, since the assumption of the linear temperature distribution inside the laminar sublayer holds as well as when vapor was not superheated.

In the experimental data reported in this study (Figures 5.3 - 5.12), no thermal non-equilibrium was observed since the equilibrium vapor qualities were relatively low and the preheater section was very long (Figure 2.1, Section 2.1), so that the preheater flux was low.
Figure 5.3 Heat Flux vs. Wall Superheat

Fluid: Nitrogen
Test Section Material: Copper
G = 70,600 lbm/ft² hr
x = 0.63
Data
— Eqn. 5.11
Fluid: Nitrogen
Test Section Material: Copper
$G = 175,400 \text{ lbm/ft}^2 \text{ hr}$
$x = 0.25$
□ Data
— Eqn. 5.11

Figure 5.4 Heat Flux vs. Wall Superheat

Heat Flux, Btu/ft$^2$ hr

$(T_w - T_{sat}), ^\circ R$
Figure 5.5 Heat Flux vs. Wall Superheat
Fluid: Nitrogen
Test Section Material: Copper
G = 178,800 lbm/ft² hr
x = 0.56

Data

Eqn. 5.11

$\frac{Q}{A} = \frac{h_A}{2} = \frac{h_A}{2} \left( \frac{T_w - T_{sat}}{T_w - T_{sat}} \right)$

Figure 5.6 Heat Flux vs. Wall Superheat
Fluid: Nitrogen
Test Section Material: Copper
G = 184,800 lbm/ft² hr
x = 0.69
Data
— Eqn. 5.11

Figure 5.7 Heat Flux vs. Wall Superheat
Figure 5.8 Heat Flux vs. Wall Superheat

Fluid: Nitrogen
Test Section Material: Copper
G = 215,600 lbm/ft² hr
\( x = 0.49 \)
Data
— Eqn. 5.11
Figure 5.9 Heat Flux vs. Wall Superheat

Fluid: Nitrogen
Test Section Material: Inconel 600
G = 126,400 lbm/ft² hr
x = 0.31

- Data
- Eqn. 5.11
Fluid: Nitrogen
Test Section Material: Inconel 600
G = 127,200 lbm/ft² hr
x = 0.52

Figure 5.10 Heat Flux vs. Wall Superheat
Heat Flux vs. Wall Superheat

Fluid: Nitrogen
Test Section Material: Inconel 600
G = 205,400 lbm/ft² hr
x = 0.31

Data - Eqn. 5.11

Figure 5.11 Heat Flux vs. Wall Superheat
Fluid: Nitrogen
Test Section Material: Inconel 600
\( G = 61,300 \text{ lbm/ft}^2 \text{ hr} \)
x = 0.50

Figure 5.12 Heat Flux vs. Wall Superheat
Figure 5.13  Mean Drop Diameter Correction Factor vs. Quality (Eqn. 3.5)
CHAPTER VI: SUMMARY AND CONCLUSIONS

1. The experimental and theoretical analysis of the dispersed flow heat transfer was performed in this study.

2. The obtained experimental data (Figures 5.3 - 5.12) are with small experimental error (Appendix II-1) and cover both the transition boiling and the film boiling regions.

3. The characterization of a drop size and a drop size distribution in a dispersed flow in terms of a mass flux and vapor quality was performed (Chapter III). Equations for the maximum drop diameter [Eq. (3.3)], the average drop diameter [Eq. (3.5)] and the drop size distribution [Eq. (3.7)] were utilized for the description of the structure of the dispersed flow and the analysis of drop-wall interaction.

4. The above equations for the maximum drop diameter, average drop diameter, and drop size distribution indicate that, for the higher mass fluxes and vapor qualities, the drops are smaller (Figures 3.1, 3.2, 3.3). This is in agreement with the experimental data for the drop size and the drop size distribution reported by several investigators [58,10,16].

5. The deposition of liquid drops in a dispersed flow was studied (Chapter IV). The expression for the reaction force on the drop due to non-uniform drop evaporation inside the laminar sublayer was developed [Eq. (4.31)]. This force, as well as other forces acting on the drop inside the laminar sublayer--lift, drag, gravity, buoyancy, and inertia--was analyzed and the trajectories for the drop were calculated.
6. The trajectory of the drop inside the laminar sublayer is prescribed by the magnitude of the forces mentioned above. The effects of the drop diameter, deposition velocity, slip ratio, wall temperature, and flow Reynolds number on the drop trajectory were examined.

7. The drop deposition model developed in this study (Chapter IV) explains the mechanism of drop deposition in dispersed flow including vapor-drop interaction. The model predicts:
   (a) the decrease of drop deposition rate (number of drops deposited per unit area of the wall and unit time) with increase of the wall temperature;
   (b) the increase of drop deposition rate with increase of the mass flux;
   (c) the decrease of drop deposition rate with increase of the vapor quality.

8. In the literature available, the analyzed experimental data for $Q$ (heat transferred to liquid drop evaporating on the wall) indicated that the wall temperature has the most significant effect on $Q$. An expression for $Q$ was introduced in this thesis [Eq. (5.5)]. It provided good prediction of our experimental data as discussed in Section 5.5. The effects of the drop impact velocity, drop temperature, contact angle, and surface state of heating wall (microughness, oxidation) on $Q$ were summarized in Section 5.1.

9. Based on the analysis of the structure of dispersed flow, drop deposition and drop-wall heat transfer, the relation, Eq.(5.8), for the heat
flux to the liquid drops was developed.

10. The total heat transfer to the dispersed flow considered in this study was given as the sum of the heat transfer to liquid drops \((q/A)_d\), and heat transfer to the vapor component of the flow \((q/A)_v\), i.e. [Eq. (5.11)],

\[
q/A = \frac{kq}{D} \left( 1 - \frac{T_{sat}}{T_w} \right)^2 + 0.023 \frac{kq}{D} \text{Re}^{0.8} \text{Pr}^{0.4} (T_{W} - T_{sat})
\]

since the radiation heat transfer was negligible (Table 3). The comparison of the experimental data with the prediction by the above equation was shown in Figures 5.3 - 5.12. Figures 5.3 - 5.12 show that the above equation is capable of predicting the experimental data.

11. The effect of the mass flux and vapor quality on \(q/A\), given by the above equation, was as follows:

(a) When the mass flux increases (constant vapor quality assumed),
\(q/A\) increases and \(T_{min}\) (minimum film boiling temperature) increases.

(b) When the vapor quality increases (constant mass flux assumed),
\((q/A)_d\) decreases, \((q/A)_v\) increases, and \(T_{min}\) decreases. This affects \(q/A\) in such a way that \(q/A\) increases in the high temperature region (film boiling region) and decreases in the low temperature region (transition boiling region).

These conclusions are experimentally supported (Figures 5.3 - 5.12) within the experimental error of the data (Appendix II-1).
12. Our extensive analysis of drop trajectories indicated that the drops that entered laminar sublayer and did not penetrate it, i.e., were returned to the main stream, did not penetrate very far into the laminar sublayer (Figures 4.3-4.7a). Therefore, we concluded that there was no appreciable heat transfer from the wall to these drops. This was confirmed by the fact that the experimental data were predicted without including this component of heat transfer which was proposed in reference [32].

13. By comparing the predictions of the experimental data in Figures 5.3 - 5.12 to the predictions of the same data, but with constant value of C, in Figures V.1 - V.10, one can see that the prediction [Eq. (5.11)] is very sensitive on the value of C [Eq. (3.5)], especially in the vicinity of and below the minimum film boiling temperature. The value of C affects the value of $\bar{a}$ [mean drop diameter, Eq. (3.5)]. The drop size distribution $P(a)$ is affected by $\bar{a}$. The drop cooling rate at the wall is affected by $P(a)$.

From the above it was concluded that the actual knowledge of the mean drop diameter in dispersed flow is of the extreme importance in predicting the heat transfer in a dispersed flow.

14. Additional work on the average drop size in dispersed flow and mechanism of the heat transfer to single drop deposited on the wall from the fluid stream will improve the work presented in this thesis.
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APPENDIX: II-1

II.1 Calculation of the Maximum Experimental Errors in \((q/A)\) due to the Axial Conduction in the Transient Test Section

A detailed drawing of the transient test section design is given on Figure II-2. This design is a modification of an earlier design (Figure 6 of ref. [1]) which was tested and found to have certain deficiencies concerning extraneous heat additions. It has been verified experimentally that the design in Fig. II-2 still has some problems with heat losses or gains. Appendix A, ref. [1], gives a total estimation of the heat losses or gains of the transient test section. The maximum errors in \((q/A)\) due to the axial conduction losses for the data shown in Figures 5.3 - 5.12 are presented in Table 4. These errors are in the vicinity of the minimum film boiling temperature, \(T_{min}\). For example, the error in heat flux for data on Figure 5.3, Table 4, is 18% for \(T = T_{min}\) but for \(T = T_{min} + 30^\circ R\) and \(T = T_{min} - 30^\circ R\) the errors are \(1 \times 10^{-2}\%\) and 5.0%, respectively.

The effect of the axial conduction on the experimentally obtained boiling curve is summarized in Figure II-1. More about the axial conduction effect on the minimum slope point of \(T(t)\) curve, Eq. (2.1), [or \((q/A)\) vs. wall superheat curve] can be found in references [1] and [2].

\(^{+}\)Numbers in parentheses refer to references, found at the end of Appendix.
TABLE 4: Maximum Experimental Errors in 
\((q/A)\) due to the Axial Conduction 

<table>
<thead>
<tr>
<th>Data in Figure #</th>
<th>(\epsilon (%))</th>
</tr>
</thead>
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<tr>
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</tr>
<tr>
<td>5.4</td>
<td>2.70</td>
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<td>5.9</td>
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<td>5.10</td>
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<td>5.11</td>
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</tr>
<tr>
<td>5.12</td>
<td>(1.0 \times 10^{-3})</td>
</tr>
</tbody>
</table>

\(\epsilon\) = maximum estimated error due to the axial conduction during quench interval

\[
\epsilon = \frac{q'}{M C_p \frac{dT}{dt}} \times 100
\]

\(q' = 0.063(T_{PB} - T)\), heat gains in Btu/hr (Eq. (A.15), ref. [1]),

\(M\) = mass of the transient test section,

\(C_p\) = specific heat of the transient test section,

\(T\) = temperature of the transient test section,

\(t\) = time,

\(T_{PB}\) = "preheater brass base" temperature (see Figure II-2).

NOTE: \(T\) and \(T_{PB}\) are recorded during quench period.
II.2 Determination of System Variables and Experimental Errors

The important system variables calculated in the experimental program are mass flux, equilibrium quality to the transient section, and the saturation temperature of liquid nitrogen at the transient section [1]. The uncertainties in these are related to the independent quantities which go into their evaluation. The equation used in the determination of the quantities described above and the errors for each are presented in Appendix B, reference [1].

REFERENCES

Figure II-1  The Effect of the Axial Conductance on the Experimentally Obtained Boiling Curve

1. Boiling Curve Without Heat Conduction

2. Boiling Curve With Heat Conduction
   a. Conduction Heat Transfer from the Transient Test Section to Preheater (Heat Losses)
   b. Conduction Heat Transfer from the Preheater to the Transient Test Section (Heat Gains)
Figure II-2 Test Block Design
APPENDIX III-1:  
SNYDER'S DATA ANALYSIS

For mass velocity $G = 4 \times 10^{-4}$ lb/sec, tube diameter $D = 0.18$ inches and quality $x = 0.7$ the experimental value for the most probable drop diameter was $a_{mp} = 42.2 \mu m$ (see Figure 1, page 9 of Snyder's thesis, ref. [1]). Applying Eq. (3.6) the mean drop diameter $\bar{a}$ is then

$$\bar{a} = 2a_{mp} = 84.4 \mu m$$

A calculated value for $\bar{a}$, from Eq. (3.4) for the above conditions, was $\bar{a} = 336 \mu m$. The correction factor $C$, Eq. (3.5), is then

$$C = \frac{84.4}{336} = 0.3984$$

i.e., the theoretical value for $\bar{a}$ is almost four times larger than the experimental value.

REFERENCES


†Refers to reference at the end of this Appendix.
APPENDIX III-2

Note that for a given \( G \) and \( x \),

\[
V_g = \frac{G \cdot x}{\rho_g \alpha} \quad (\text{III}-2.1)
\]

and

\[
V_g = \frac{G(1-x)}{\rho_x(1-\alpha)} \quad (\text{III}-2.2)
\]

where

\[
\alpha = \frac{x}{x + \frac{\rho_g}{\rho_x} s(1-x)} \quad (\text{III}-2.3)
\]

The slip ratio \( S \) is given in Refs. [1,2] for a specified \( G \) and \( x \).

Fluid properties are evaluated at the saturation temperature.

REFERENCES

APPENDIX: IV-1

SOLUTION OF DIFFERENTIAL EQUATIONS (4.39a,b)

By introducing the following substitutions:

\[
U = U_0 \frac{Y}{\delta} ; \quad A = \frac{3}{8} \frac{\rho g}{\rho \delta} U_0 ; \quad B = \frac{18U_0}{\rho \alpha^2} ; \quad C = B \frac{U_0}{\delta} ;
\]

\[
D = \frac{(\rho_L - \rho_g)g}{\rho_L} ; \quad E = 3 \frac{h^2(T_w - T_s)^2}{H^2 g \rho_L \rho_g \alpha^2} ; \quad F = (AU_0 - E) \frac{1}{\delta} ;
\]

\[
\frac{dx}{dt} = x' ; \quad \frac{d^2x}{dt^2} = x'' ; \quad \frac{dy}{dt} = y' ; \quad \text{and} \quad \frac{d^2y}{dy^2} = y''
\]

the system of equations (4.39a,b) becomes:

\[
x'' = Ay' - Bx' + Cy - D \quad \text{(IV-1.1)}
\]

\[
y'' = -Ax' - By' + Fy + E \quad \text{(IV-1.2)}
\]

As explained in section 4.3 the following initial boundary conditions are used:

\[
x(t=O) = 0 \quad \text{(IV-1.3)}
\]

\[
y(t=O) = \delta \quad \text{(IV-1.4)}
\]

\[
x'(t=O) = u_0 \quad \text{(IV-1.5)}
\]

\[
y'(t=O) = v_0 \quad \text{(IV-1.6)}
\]
From Eq. (IV-1.2) by differentiation:

\[ y''' = -Ax'' - By'' + Fy' \quad (IV-1.7) \]

Also, from Eq. (IV-1.2),

\[ x' = -\frac{1}{A} y'' - \frac{B}{A} y' + \frac{F}{A} y + \frac{E}{A} \quad (IV-1.8) \]

From Eqs. (IV-1.1) and (IV-1.8),

\[ x'' = \frac{B}{A} y'' + \left(\frac{B^2}{A} + A\right) y' + \left(C - \frac{FB}{A}\right) y - \left(\frac{BE}{A} + D\right) \quad (IV-1.9) \]

From Eqs. (IV-1.7) and (IV-1.9),

\[ y''' + 2By'' + (B^2 + A^2 - F)y' + (CA - FB)y - (BE + DA) = 0 \quad (IV-1.10) \]

The solution of the last equation is

\[ y = c_1e^{r_1t} + c_2e^{r_2t} + c_3e^{r_3t} + \frac{(BE + DA)}{(CA - FB)} \quad (IV-1.11) \]

where \( r_1, r_2, r_3 \) are roots of the characteristic polynomial which corresponds to Eq. (IV-1.10) and \( c_1, c_2, c_3 \) are constants.

From Eq. (IV-1.11),

\[ y' = c_1r_1e^{r_1t} + c_2r_2e^{r_2t} + c_3r_3e^{r_3t} \quad (IV-1.12) \]

\[ y'' = c_1r_1^2e^{r_1t} + c_2r_2^2e^{r_2t} + c_3r_3^2e^{r_3t} \quad (IV-1.13) \]
From Eqs. (IV-1.8), (IV-1.12), and (IV-1.13),

\[ x' = \frac{c_1}{A} (F - Br_1 - r_1^2) e^{r_1 t} + \frac{c_2}{A} (F - Br_2 - r_2^2) e^{r_2 t} + \]

\[ + \frac{c_3}{A} (F - Br_3 - r_3^2) e^{r_3 t} + \frac{E}{A} \]  

(IV-1.14)

From Eq. (IV-1.14),

\[ x = \frac{c_1}{r_1 A} (F - Br_1 - r_1^2) e^{r_1 t} + \frac{c_2}{r_2 A} (F - Br_2 - r_2^2) e^{r_2 t} + \]

\[ + \frac{c_3}{r_3 A} (F - Br_3 - r_3^2) e^{r_3 t} + \left[ \frac{F}{A} \left( \frac{BE + DA}{CA - FB} \right) + \frac{E}{A} \right] t + c_4 \]  

(IV-1.15)

Note that

\[ x = \text{axial position of drop [Eq. (IV-1.15)]}; \]
\[ y = \text{radial position of drop [Eq. (IV-1.11)]}; \]
\[ x' = \text{drop velocity in x direction [Eq. (IV-1.14)]}; \]
\[ y' = \text{drop velocity in y direction [Eq. (IV-1.12)]}. \]

Four numerical constants, \( c_1, c_2, c_3, \) and \( c_4 \) are now determined by the method of determinants using initial boundary conditions given by Eqs. (IV-1.3)-(IV-1.6) (Appendix V-4).
APPENDIX: IV-2

As $U_1$ and $U_2$ are given by Eqs. (4.26) and (4.27), then Eq. (4.30) can be written in the following form:

$$F_{cy} = \frac{A}{4} \rho_g \left( U_2^2 - U_1^2 \right)$$  \hspace{1cm} (IV-2.1)

By substituting Eqs. (4.28) and (4.29) into the above equation, we have

$$F_{cy} = \frac{\pi a^2 h^2}{4 H^2 \rho_g} \left( T_w - T_s \right)^2 \frac{a}{\delta} \left( 1 - \frac{y}{\delta} \right)$$
APPENDIX: IV-3

Rubinow-Keller's force is given by Eq. (4.7). Saffman's force is given by the sum of Eqs. (4.15) and (4.15a). The ratio of Saffman to Rubinow-Keller force, RF, is calculated for different drop size at certain values of flow parameters, Tables 5 and 6.

**TABLE 5. Values of RF at G = 60,000 lb/ft²hr, x = 0.50**

<table>
<thead>
<tr>
<th>a [microns]</th>
<th>RF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19.4</td>
</tr>
<tr>
<td>5</td>
<td>4.68</td>
</tr>
<tr>
<td>10</td>
<td>2.84</td>
</tr>
<tr>
<td>100</td>
<td>1.184</td>
</tr>
<tr>
<td>1000</td>
<td>1.0184</td>
</tr>
</tbody>
</table>

**TABLE 6. Values of RF at G = 210,000 lb/ft²hr, x = 0.50**

<table>
<thead>
<tr>
<th>a [microns]</th>
<th>RF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.3</td>
</tr>
<tr>
<td>5</td>
<td>2.46</td>
</tr>
<tr>
<td>10</td>
<td>1.73</td>
</tr>
<tr>
<td>100</td>
<td>1.073</td>
</tr>
<tr>
<td>1000</td>
<td>1.0073</td>
</tr>
</tbody>
</table>

Note that the forces become numerically the same for the larger drops (a > 10μm).
APPENDIX: IV-4

In order to estimate the pressure increase in the vapor layer between the drop and heated wall (see figure VI-1 on the next page) the following approximate relation has been used:

\[ P_A - P_0 = \rho_g \nabla_A^2 \]

where \( P_0 \) is ambient (system) pressure and \( P_A \) local pressure below the drop (point A, Figure IV-1) and \( \nabla_A \), is the vapor velocity generated at the lower portion of the sphere given as

\[ \nabla_A = \frac{k_g}{y} \frac{(T_W - T_S)}{H_{fg} \rho_g} \]

where \( y \) = current distance from the wall (Figure IV-1).

Figure IV-1 shows that the pressure below the drop increases when the drop is approaching the wall. Calculation was terminated at \( y = 5 \mu m \), i.e., the wall roughness of \( 5 \mu m \) was assumed.† The change of the saturation temperature \( T_s \) and the latent heat of evaporation \( H_{fg} \) due to the pressure increase for the performed calculation was negligible.

†In the process of generating drop trajectories inside the boundary layer (Section 4.4 explains this process), if the distance of the drop from the wall was less than or equal to \( 5 \mu m \) we assumed that the drop was deposited on the wall.
Figure IV-1 The Pressure Increase Below a Drop Approaching a Heated Wall
The amount (mass) of vapor generated during deposition motion of the evaporating drop is approximately given by the following relation:

\[ \sum m = \frac{\pi a^2 h (T_v - T_s)}{H \cdot g} \int_0^{t_0} dt \]  

(IV-5.1)

where \( a \) = drop diameter

\( h \) = heat transfer coefficient [Eq. (4.18)]

\( T_v \) = current vapor temperature

and \( t_0 \) = deposition time.

For a nitrogen drop of \( a = 100\mu \) and \( T_w = 640^\circ R \) (note that \( T_v = T_w \) at \( t = t_0 \)) and \( T_s = 140^\circ R \) and \( t_0 = 100 \text{ ms} \) (very conservative value),

\[ \sum m = 2.33 \times 10^{-12} \text{ lb} \]

Since the mass of a drop with \( a = 100\mu \) is \( m = 1.0 \times 10^{-9} \text{ lb} \), then

\[ \frac{\sum m}{m} \cdot 100 = 0.233\% \text{, which is very small.} \]
APPENDIX: V-1

The mass flux of liquid drops migrating toward the wall, entering the laminar sublayer, is by definition

\[ M = v_0(1 - \alpha) \rho_\ell \text{ [lbm/ft}^2\text{hr]} \]  

(V-1.1)

The drop deposition flux related to \( M \) is then

\[ N = \frac{v_0(1 - \alpha) \rho_\ell}{\frac{\pi}{6} \rho_\ell \int_{0}^{a_m} a^3 P(a) \, da} \]  

(V-1.2)

where

\[
[N] = \left[ \frac{\text{# of drops of size } a, \ 0 < a < a_m}{\text{ft}^2 \cdot \text{hr}} \right]
\]

For a certain value of the wall temperature only drops having diameter \( a > a_c \) (\( a_c \) is the deposition diameter, Chapter IV) will be deposited on the wall. The heat flux from the wall to the deposited drop is

\[ (q/A)_d = N \int_{a_c}^{a_m} Q P(a) \, da \]  

(V-1.3)

where \( Q = \) heat transferred per drop of any size, Eq. (5.5),

\( P(a) = \) drop size distribution,

and \( a_m = \) maximum drop diameter, Eq. (3.3).
Substituting the values for $Q$ and $N$ [Eq. (V-1.2)] into Eq. (V-1.3),

\[
(q/A) = v_0(1 - \alpha)\rho_{xH} e^{1-(T/W_{sat})^2} \frac{1}{a_c^{m}} \int^a_{a_c} a^3 P(a) \, da \\
\int^a_{a_m} a^3 P(a) \, da
\]

Since [Eq. (4.46)]

\[
f = \frac{a_m^{m}}{\int^a_{a_m} a^3 P(a) \, da}
\]

Eq. (5.8) and Eq. (V-1.4) are the same. Substituting the value for $P(a)$ [Eq. (3.7)] into Eq. (V-1.5) and performing the integration,

\[
f = \left\{ \left[ \left( \frac{a_m}{a} \right)^3 + \frac{3}{4} \left( \frac{a_m}{a} \right) \right] \exp \left[ -2 \left( \frac{a_c}{a} \right)^2 \right] - \left[ \left( \frac{a_m}{a} \right)^3 + \frac{3}{4} \left( \frac{a_m}{a} \right) \right] \exp \left[ -2 \left( \frac{a_m}{a} \right)^2 \right] + \right\} \]
\[
+ \frac{6}{16} \sqrt{\pi/2} \left[ \text{erf} \left( \sqrt{\frac{a_m}{a}} \right) - \text{erf} \left( \sqrt{\frac{a_c}{a}} \right) \right] \left\{ \frac{6}{16} \sqrt{\pi/2} \text{erf} \left( \sqrt{\frac{a_m}{a}} \right) - \right\}
\]
\[
- \left[ \left( \frac{a_m}{a} \right)^3 + \frac{3}{4} \left( \frac{a_m}{a} \right) \right] \exp \left[ -2 \left( \frac{a_m}{a} \right)^2 \right]^{-1} \right\}
\]

(V-1.6)
The values of $a_m$ and $\bar{a}$ were given by Eqs. (3.3) and (3.5), respectively. The value of $a_c$ was calculated for the particular value of the wall temperature as explained in Chapter IV.
APPENDIX: V-2

Void fraction $\alpha$ can be defined as the ratio of the gas (vapor) flow area to the total flow area. If a homogeneous void distribution is assumed, then $\alpha$ is the ratio of gas phase area to any area inside the flow. In Chapter IV we concluded that not all drops entering the boundary layer penetrate it so the surface void fraction $\alpha_s$ is generally less than $\alpha$ as defined above. Having this in mind we may write the following relation between the homogeneous and surface void fraction,

$$1 - \alpha_s = (1 - \alpha)f$$  \hspace{1cm} (V-2.1)

where $f$ is now the surface cumulative factor given by Eq. (4.46) for $n = 2$. From Eq. (V-2.1),

$$\alpha_s = 1 - f(1 - \alpha)$$  \hspace{1cm} (V-2.2)

In fully-developed dispersed flow the minimum value for $\alpha$ is about 0.8. For the relatively low wall superheat ($T_w = T_{min}$), $f$ is a few tenths of one, so for all practical purposes, $\alpha_s = 1$. 
The value of $C = 0.26$ is used to predict the data in Figure V-1 to V-10. The prediction in Figures V-2, V-4, V-10 is not good.
Fluid: Nitrogen
Test Section Material: Copper
G = 70,600 lbm/ft² hr
x = 0.63

Figure V-1  Heat Flux vs. Wall Superheat
Figure V-2 Heat Flux vs. Wall Superheat

Fluid: Nitrogen
Test Section Material: Copper
G=175,400 lbm/ft² hr
x=0.25
Data
-Eqn. 5.11
Figure V-3  Heat Flux vs. Wall Superheat

Fluid: Nitrogen
Test Section Material: Copper
G=178,100 lbm/ft$^2$ hr
x=0.41
Data
---Eqn. 5.11
Fluid: Nitrogen
Test Section Material: Copper
G=178,800 lbm/ft² hr
x=0.56

Figure V-4 Heat Flux vs. Wall Superheat
Figure V-5  Heat Flux vs. Wall Superheat

Fluid: Nitrogen  
Test Section Material: Copper  
G = 184,800 lbm/ft² hr  
\( x = 0.69 \)  
□ Data  
— Eqn. 5.11
Fluid: Nitrogen
Test Section Material: Copper
G = 215,600 lbm/ft² hr
x = 0.49
Data
Eqn. 5.11

Figure V-6 Heat Flux vs. Wall Superheat
Fluid: Nitrogen
Test Section Material: Inconel 600
G=126,400 lbm/ft² hr
x=0.31
Data
— Eqn. 5.11

Figure V-7 Heat Flux vs. Wall Superheat
Fluid: Nitrogen
Test Section Material: Inconel 600
G=127,200 lbm/ft² hr
x=0.52

Data

Eqn. 5.11

Figure V-8 Heat Flux vs. Wall Superheat
Fluid: Nitrogen
Test Section Material: Inconel 600
\( G = 205,400 \text{ lbm/ft}^2 \text{ hr} \)
\( x = 0.31 \)

- Data
- Eqn. 5.11

Figure V-9 Heat Flux vs. Wall Superheat
Fluid: Nitrogen
Test Section Material: Inconel 600
G = 61,300
x = 0.50

Data: 
- Eqn. 5.11

Figure V-10 Heat Flux vs. Wall Superheat
APPENDIX V-4
COMPUTER PROGRAM LISTING
DISPERSED FLOW HEAT TRANSFER

PROGRAM Calculates HEAT FLUX VS. WALL SUPERHEAT FOR GIVEN MASS FLUX AND VAPOR QUALITY

REAL KGv, MUGv, KGL, M, M2, K1, K2
READ (8, 1) KGv, MUGv, RHOG, RHCL
READ (8, 1) CPG, CPL, HFG, TSAT
READ (8, 1) SIGMA, RV, P, KGL
READ (8, 11) AL, AG, VIS
READ (8, 111) GM, XE, S1, SLIF, D
READ (8, 112) PL, GC, WE, DCPI

1 FORMAT (4F10.6)
11 FORMAT (3F10.6)
111 FORMAT (5F10.6)
112 FORMAT (4F10.6)
4 FORMAT (1X, F15.3, E20.5)

THE VALUES OF SLIP FOR PREDICTIONS IN FIGURES 5.3-5.12 WERE BETWEEN 1.3 TO 1.1. IT WAS OBTAINED FROM MODIFIED RESULTS OF REF (50) FOR FULLY DEVELOPED DISPERSED FLOW

S1 IS A SYMBOL FOR C IN FIGURE 5.13 SO VALUES OF S1 CAN BE OBTAINED FROM FIGURE 5.13

DATA PROPERTY FOR NITROGEN, REF (32) PAGE 207

KGV-VAPOR CONDUCTIVITY (BTU/FT-HR-R)
KGL-LIQUID CONDUCTIVITY (BTU/FT-HR-R)
MUGV-VAPOR D. VISCOITY (LBM/FT-HR)
VIS-VAPOR C. VISCOITity (FT2/Hr)
XSCl(I)-ARRAY CONTAINING SCALE FOR THE AXES
RHOL-LIQUID DENSITY (LBM/FT3)
RHOG-VAPOR DENSITY (LBM/FT3)
CPG-VAPOR SPEC. HEAT (BTU/LBM-R)
CPL-LIQUID SPEC. HEAT (BTU/LBM-R)
C    HFG-LATENT HEAT (BTU/LBM)
C    TSAT-SATURATION TEMPERATURE (R)
C    SIGMA-SURFACE TENSION (LBF/FT)
C    RV-GAS CONSTANT (BTU/LBM-P)
C    P-SYSTEM PRESSURE (LBF/FT2)
C    AL-LIQUID THER. DIFFUSIVITY (FT2/HR)
C    AG-VAPOR THER. DIFFUSIVITY (FT2/HR)
C    D-TUBE DIAMETER (FT)
C    XE-EQUILIBRIUM QUALITY
C    VN-DROPS DEPOSITION VELOCITY (FT/HR)
C    M-LIQUID DEPOSITION FLUX (LBM/HR-FT2)
C    TCHF-INITIAL VALUE OF WALL TEMPERATURE
C    ALFAE-VOID FRACTION BASED ON XE
C    SLIP-SLIP (VG/VL)
C    VG-VAPOR VELOCITY (FT/HR)
C    VL-LIQUID VELOCITY (FT/HR)
C    WE-WEBER NUMBER
C    RE-REYNOLDS NUMBER
C    PR-PRANDTL NUMBER
C    DM-MAXIMUM DROP DIAMETER (FT)
C    DAV-AVERAGE(MEAN) DROP DIAMETER (FT)
C    DMP-MOST PROBABLE DROP DIAMETER (FT)
C    DCR-DEPOSITED DROP DIAMETER GIVEN BY DRCF DEPOSITION MODEL (FT)
C    TWAL-WALL TEMPERATURE
C    DW-AVERAGE DROP DIAMETER FOR DEPOSITED DROPS (FT)
C    DL-AVERAGE DROP DIAMETER FOR NOT DEPOSITED DROPS (FT)
C    F-DEPOSITION FACTOR
C    QD-HEAT TRANSFERED BY SINGLE DEPOSITED DROP (BTU)
C    Q1-HEAT TRANSFERED BY DEPOSITED DROPS (BTU/FT2-HR)
C    DELTA-BOUNDARY LAYER THICKNESS (FT)
C    Q2-HEAT TRANSFERED BY NOT DEPOSITED DROPS (BTU/FT2-HR)
C    TV-VAPOR TEMPERATURE (R)
C    H-VAPOR HEAT TRANSFER COEFFICIENT (BTU/FT2-HR-R)
C    Q3-HEAT TRANSFERED BY VAFCR (BTU/FT2-HR)
C    QT-TOTAL HEAT FLUX TO FLUID (BTU/FT2-HR)
C    PR-VAPOR FRICTION FACTOR

PGM 10037
PGM 10038
PGM 10039
PGM 10040
PGM 10041
PGM 10042
PGM 10043
PGM 10044
PGM 10045
PGM 10046
PGM 10047
PGM 10048
PGM 10049
PGM 10050
PGM 10051
PGM 10052
PGM 10053
PGM 10054
PGM 10055
PGM 10056
PGM 10057
PGM 10058
PGM 10059
PGM 10060
PGM 10061
PGM 10062
PGM 10063
PGM 10064
PGM 10065
PGM 10066
PGM 10067
PGM 10068
PGM 10069
PGM 10070
PGM 10071
PGM 10072
CALL ENG(VG, VL, VN, TWAL, DELTA, DCR, KG, MUG, RHOM, RHOG, HFG, C PG, TSAT, D 1)

IF(DCR.LT.DSAVE) DCR=DSAVE
DCR1=DCR/DAV
DM1=DM/DAV
IF(DCR1.GE.8.66) GO TO 599
IF(DM1.GE.8.66) GO TO 598
F=(((DCR/DAV)**3+0.75*(DCR/DAV))**2)*EXP(-2.*((DCR/DAV)**2)) - 1(((DM/DAV)**3+0.75*(DM/DAV))**2)*EXP(-2.*((DM/DAV)**2)) +
2  \(0.47 \times (\text{ERF}(1.41 \times \text{DM}/\text{DAV}) - \text{ERF}(1.41 \times \text{DCR}/\text{DAV})) / \)
3 \((-((\text{DM}/\text{DAV})^3 + 0.75 \times (\text{DM}/\text{DAV}) \times \exp(-2 \times (\text{DM}/\text{DAV})^2)) + \)
4 \(0.47 \times \text{ERF}(1.41 \times \text{DM}/\text{DAV}) \) 
\]
\[\text{IF}(\text{DCR} \geq \text{DM}) \quad F = 0. \]
\[\text{DW} = \text{DAV}^3 \times ((\text{DCR}/\text{DAV})^3 + 0.75 \times (\text{DCR}/\text{DAV}) \times \exp(-2 \times (\text{DCR}/\text{DAV})^2) - \]
1 \((\text{DM}/\text{DAV})^3 + 0.75 \times (\text{DM}/\text{DAV}) \times \exp(-2 \times (\text{DM}/\text{DAV})^2) + \)
2 \(0.47 \times (\text{ERF}(1.41 \times \text{DM}/\text{DAV}) - \text{ERF}(1.41 \times \text{DCR}/\text{DAV})) / \)
3 \((\text{exp}(-2 \times (\text{DCR}/\text{DAV})^2) - \text{exp}(-2 \times (\text{DM}/\text{DAV})^2)) \)
\[\text{DW} = \text{DW}^3 \times 0.333 \]
\[\text{GO TO 59} \]
598 \text{CONTINUE} 
\[F = ((\text{DCR}^3 + 0.75 \times \text{DCR}) \times \exp(-2 \times \text{DCR}^2) + 0.47 \times (\text{ERF}(1.41 \times \text{DM}) - \]
1 \(\text{ERF}(1.41 \times \text{DCR})) / (0.47 \times \text{ERF}(1.41 \times \text{DM})) \)
\[\text{DW} = \text{DAV}^3 \times ((\text{DCR}^3 + 0.75 \times \text{DCR}) \times \exp(-2 \times \text{DCR}^2) + 0.47 \times (\text{ERF}(1.41 \times \]
1 \(\text{DM}) - \text{ERF}(1.41 \times \text{DCR})) / \exp(-2 \times \text{DCR}^2) \)
\[\text{DW} = \text{DW}^3 \times 0.333 \]
\[\text{GO TO 59} \]
599 \text{CONTINUE} 
\[F = (\text{ERF}(1.41 \times \text{DM}) - \text{ERF}(1.41 \times \text{DCR}) / \text{ERF}(1.41 \times \text{DM}) \]
\[\text{DW} = 0.47 \times \text{DAV}^3 \times (\text{ERF}(1.41 \times \text{DM}) - \text{ERF}(1.41 \times \text{DCR})) \]
\[\text{DW} = \text{DW}^3 \times 0.333 \]
59 \text{CONTINUE} 
\[\text{C} \]
\[\text{QD} = \text{DW}^3 \times \text{RHCL} \times \pi \times \text{HFG} / 6. \]
\[\text{EFEC} = \exp(1 - (\text{TWAL} / \text{TSAT})^2) \]
\[\text{QD} = \text{QD} \times \text{EFEC} \]
\[\text{M} = \text{RHOL} \times \text{VH} \times (1 - \text{ALFAE}) \]
\[\text{IF}(\text{DCR} \geq \text{DM}) \quad F = 0. \]
\[\text{Q1} = \text{M} \times \text{QD} / (\text{RHOL} \times \text{DW}^3 \times 3 \times \pi / 6.) \]
\[\text{PR} = \text{HUGV} \times \text{CPG} / \text{KV} \]
\[\text{H} = 0.023 \times (\text{KV} / \text{D}) \times (\text{RE} \times 0.8) \times (\text{PR} \times 0.4) \]
\[\text{TV} = \text{TSAT} \]
\[\text{Q3} = \text{H} \times (\text{TWAL} - \text{TV}) \times \text{ALFAE} \]
\[\text{Q3} = \text{H} \times (\text{TWAL} - \text{TV}) \]
\[\text{QT} = \text{Q1} + \text{Q3} \]
\[\text{DELT} = \text{TWAL} - \text{TSAT} \]
WRITE(5,3) DELT,QT
4 TWAL=TWAL+10.
END
THE SUBROUTINE ENG CALCULATES DEPOSITION DIAMETER VS. WALL
TEMPERATURE BY GENERATING DROP TRAJECTORIES INSIDE THE
BOUNDARY LAYER. THE DETAILS CONCERNING DROP TRAJECTORY
CAN BE OBTAINED BY SIMPLY CALLING TMS, X, Y, U, V.

TMS-TIME (MSEC)
X-AXIAL POSITION OF DROP (EQUATION 4-1.15)
Y-RADIAL POSITION OF DROP (4-1.11)
V-RADIAL DROP VELOCITY (EQUATION 4-1.12)
AXIAL DROP VELOCITY (EQUATION 4-1.14)

SUBROUTINE ENG (VG, VL, VN, TWAL, DELTA, DCR, KGV, MUGV, RHOL, RHOG, HPG, CPG,
1 TSAT, D)

REAL KGV, MUGV, KSI
DIMENSION XCOF (4), COF (4), ROOTR (3), ROOTI (3), Y (1000)
UV=VG
UO=VL

DATA FOR DCR1 (IN MICRONS), PI, AND G ARE NEEDED

PI=PI
DIAM=DCR1
G=GC
DIAM1=DIAM/(3.28E-6)
DELTA1=DELTA/(3.28E-6)
Y (1) = DELTA
DO 8 J=1, 100
YP=DELTA+0.95*DELTA

NOTE FOLLOWING IDENTITIES FROM APPENDIX 4-1
A=A
B=B
C=C
Q=D
ALFA=E
BETA = F

A = 3. * RHOG * UV / (8. * RHOL * DIAM * 2)
B = 18. * MUGV / (RHOL * DIAM * 2)
C = B * UV / DELTA
Q = (RHOL - RHOG) * G / RHOL
H = 2.6 * KGV / DIAM
ALFA = 3. * H**2 * (TWAL - TSAT)**2 / (HFG**2 * RHOL * RHOG * DELTA)
BETA = (A * UV - ALFA) / DELTA
XCOF (1) = A * C - BETA * B
XCOF (2) = B**2 - BETA + A**2
XCOF (3) = 2. * B
XCOF (4) = 1.
CALL BAIRS (XCOF, COF, 3, ROOTR, ROOTI, IER)
R1R = ROOTR (1)
R1I = ROOTI (1)
R2R = ROOTR (2)
R2I = ROOTI (2)
R3R = ROOTR (3)
R3I = ROOTI (3)
TMS = 1.
DO 1 I = 2, 1000
TS = TMS / 1000.
S = TS / 3600.
CO = C * A - BETA * B
IF (CO < 0.) 98, 97, 98
97 R2 = -A**2 + BETA
IF (R2 < 0.) 94, 95, 96
96 CONTINUE

ROOTS ARE REAL AND DIFFERENT

R01 = -B + SQRT (-A**2 + BETA)
\[ R^2 = -B - \sqrt{-A^2 + \beta} \]
\[ A10 = \Delta + 2 \cdot B \cdot (B \cdot \alpha A + Q \cdot A) / (B^2 + A^2 - \beta)^2 \]
\[ A20 = -(\Delta N + (2 \cdot \beta \cdot B \cdot A \cdot Q \cdot A) / (A \cdot (B^2 + A^2 - \beta)^2) + (B \cdot (B \cdot \alpha A + Q \cdot A) / (A \cdot (B^2 + A^2 - \beta)^2)) - \alpha A / A \]
\[ A13 = 1 \cdot (B^2 + A^2 - \beta) \]
\[ A31 = (B \cdot \alpha - B \cdot R01 - R01^2) / (R01^2 \cdot A) \]
\[ A32 = (B \cdot \alpha - B \cdot R02 - R02^2) / (R02^2 \cdot A) \]
\[ A41 = A31 \cdot R01 \]
\[ A42 = A32 \cdot R02 \]
\[ A43 = A13 \cdot B / A \]
\[ DC = R02 \cdot A43 + R01 \cdot A43 - A13 \cdot R01 \cdot A42 + A13 \cdot A41 \cdot R02 \]
\[ DC1 = -A10 \cdot A43 \cdot R02 + A20 \cdot A43 - A13 \cdot A42 + A13 \cdot A40 \cdot R02 \]
\[ DC2 = -A20 \cdot A43 \cdot A10 \cdot R01 \cdot A43 - A13 \cdot R01 \cdot A40 + A13 \cdot A20 \cdot A41 \]
\[ DC3 = -A40 \cdot R02 + A42 \cdot A20 + A40 \cdot R01 - A20 \cdot A41 + A10 \cdot R01 \cdot A42 + A10 \cdot R02 \cdot A41 \]
\[ DC4 = A20 \cdot A32 + A43 \cdot A20 + A31 \cdot A43 + A13 \cdot R01 \cdot A32 + A40 \cdot A13 \cdot R02 + A31 \cdot A40 \]
\[ A13 \cdot A42 \cdot A13 \cdot A20 + A43 \cdot A13 \cdot A42 + A10 \cdot R01 \cdot A32 + A43 \cdot A19 \cdot R02 \cdot A31 \cdot A43 \]
\[ C1 = \text{DC} \cdot DC \]
\[ C2 = \text{DC} \cdot DC \]
\[ C3 = \text{DC} \cdot DC \]
\[ C4 = \text{DC} \cdot DC \]

\[ Y(I) = \]
\[ 1 \quad C1 \cdot \exp(R01 \cdot S) + C2 \cdot \exp(R02 \cdot S) + ((B \cdot \alpha A + Q \cdot A) / (B^2 + A^2 - \beta)) \cdot S^2 + 1 \quad A13 \cdot C3 - 2 \cdot B \cdot (B \cdot \alpha A + Q \cdot A) / ((B^2 + A^2 - \beta)^2) \]
\[ X = C1 \cdot A31 \cdot \exp(R01 \cdot S) + C2 \cdot A32 \cdot \exp(R02 \cdot S) + (B \cdot \alpha A + Q \cdot A) / ((B^2 + A^2 - \beta)^2) \]
\[ V = C1 \cdot R01 \cdot \exp(R01 \cdot S) + C2 \cdot R02 \cdot \exp(R02 \cdot S) + (B \cdot \alpha A + Q \cdot A) / ((B^2 + A^2 - \beta)^2) \]
\[ U = C1 \cdot A41 \cdot \exp(R01 \cdot S) + C2 \cdot A42 \cdot \exp(R02 \cdot S) + (B \cdot \alpha A + Q \cdot A) / ((B^2 + A^2 - \beta)^2) \]

95 GO TO 44

CASE ROORS ARE REAL AND EQUAL

A10 = \Delta + 2 \cdot B \cdot (B \cdot \alpha A + Q \cdot A) / ((B^2 + A^2 - \beta)^2)
A20 = -(V + (B*ALFA+Q*A) / (B**2+A**2-BETA))
A40 = UO + 2.*BETA*B* (B*ALFA+Q*A) / (A*(B**2+A**2-BETA)**2) + 
1 B*(B*ALFA+Q*A) / (A*(B**2+A**2-BETA)) **2 - ALFA/A
A13 = 1. / (B**2+A**2-BETA)
A32 = -(1./A-BETA/(A*B**2))
A43 = BETA/(A*(B**2+A**2-BETA))
DC = -A43*A13*B**2/A+A13*BETA/A
DC1 = -A10*A43-A13*A20*B/A+A13*A40
DC2 = -A20*A43-A10*B/A43+A13*B**2/A+A13*A20*BETA/A
DC3 = -A10*A20*B/A+A10*B**2/A+A10*BETA/A
DC4 = A20*A32-A13*B**2/A32*A40+A13*A40*BETA/(A*B) - A13*A20*BETA/
1 (A**2) - A13*A20*A32*BETA/A + A10*B A32*A43-A10*A43*BETA/(A*B)
C1 = DC1/DC
C2 = DC2/DC
C3 = DC3/DC
C4 = DC4/DC

Y(I) =
1 (C1+C2*S)*EXP(-B*S) + ((B*ALFA+Q*A) / (B**2+A**2-BETA)) * S+A13*C3-
2.**B*(B*ALFA+Q*A) / ((B**2+A**2-BETA)**2)
X = ((C1*BETA+C2*B) / (A*B)) * EXP(-B*S) - (C2*BETA/(B*A)) * (S+1./B)*
1 EXP(-B*S) + (BETA* (B*ALFA+Q*A) / (2.*A*(B**2+A**2-BETA)))*S+2*
2 (A43*C3-A40+UO) * S+C4
V = ((C2-C1*B-C2*B*S) * EXP(-B*S) + ((B*ALFA+Q*A) / (B**2+A**2-BETA))
U = ((C1*BETA+C2*B) / (A)**B*S + (C2*BETA+S/A) * EXP(-B*S) + (BETA* 
1 (B*ALFA+Q*A) / (A*(B**2+A**2-BETA)))*S+A43*C3-A40+UO
GO TO 44
94 CONTINUE

ROOTS ARE COMPLEX NUMBERS

KSI = SQRT(+A**2-BETA)
A10 = DELTA+2.*B*(B*ALFA+Q*A) / ((B**2+A**2-BETA)**2)
A20 = -(V + (B*ALFA+Q*A) / (B**2+A**2-BETA))
A40 = UO + 2.*BETA*B*(B*ALFA+Q*A) / (A*(B**2+A**2-BETA)**2) + 
1 B*(B*ALFA+Q*A) / (A*(B**2+A**2-BETA)) **2 - ALFA/A
A13 = 1. / (B**2+A**2-BETA)
A31 = -BETA*B/(A*(B**2+KSI**2))
A32 = KSI* (A**2-B**2)/(A*(B**2+KSI**2))
A42 = B*KSI/A
A43 = BETA*A13/A
DC = -KSI*A43+A13*B*A42-A13*KSI*A
DC1 = -A1O*KSI*A43-A13*A20*A42+A13*KSI*A40
DC2 = -A20*A43-A10*B*A43+A13*B*A40-A13*A*A20
DC3 = -KSI*A40+A20*A42+A10*B*A42-A10*KSI*A
DC4 = A20*A32*A43-A13*A10*B*A43+A13*A10*KSI*A
1 A13*A2C*A+A32+A10*B*A32*A43+A10*KSI*A31*A43
C1 = DC1/DC
C2 = DC2/DC
C3 = DC3/DC
C4 = DC4/DC

C1*(COS(KSI*S) + C2*SIN(KSI*S)) * EXP (-B*S) + ((B*ALFA+Q*A)/(B**2+B**2-BETA))
C1*A13*C3-2. * B*(B*ALFA+Q*A)/(B**2+B**2-BETA)**2)
X = ((C2*A+B*KSI*C1/A) * (B*SIN(KSI*S) + KSI*COS(KSI*S)) + (B*KSI*C2/A -
1 C1*A) * (KSI*SIN(KSI*S) + KSI*COS(KSI*S)) * EXP (-B*S)/*(B**2-KSI**2) +
2 (BETA*(B*ALFA+Q*A)/(B**2+B**2-BETA))) * S**2 + (C3*A43-A40 +
3 U0) * S + C4
V = ((C2*KSI-B*C1) * COS (KSI*S) - (C1*KSI+B*C2) * SIN (KSI*S)) * EXP (-B*S) +
1 (B*ALFA+Q*A)/(B**2+B**2-BETA)
U = (- (C2*A+B*KSI*C1/A) * SIN (KSI*S) + (B*KSI*C2/A-C1*A) * COS (KSI*S))
1 * EXP (-B*S) + (BETA*(B*ALFA+Q*A)/(A*(B**2+B**2-BETA))) * S + C3*A43-A40
2 + U0
GO TO 44
98 CONTINUE
IF (R21.NE.0.0 .AND. R31.NE.0) GO TO 61
IF (R1R.EQ.0.0) GO TO 50
IF (R2R.NE.0.0) GO TO 800
R2R=R1R
R1R=0.
GO TO 50
800 IF (R3R.NE.0.0) GO TO 60
R3R=R1R

PGM 20145
PGM 20146
PGM 20147
PGM 20148
PGM 20149
PGM 20150
PGM 20151
PGM 20152
PGM 20153
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PGM 20167
PGM 20168
PGM 20169
PGM 20170
PGM 20171
PGM 20172
PGM 20173
PGM 20174
PGM 20175
PGM 20176
PGM 20177
PGM 20178
PGM 20179
PGM 20180
R1R=0.

50 CONTINUE

CASE R1R=0.

A10 = DELTA - (B*ALFA + Q*A) / (C*A - BETA*B)
A32 = (BETA - B*R2R - R2R**2) / (B2R*A)
A33 = (BETA - B*R3R - R3R**2) / (B3R*A)
A42 = (BETA - B*R2R - R2R**2) / A
A43 = (BETA - B*R3R - R3R**2) / A
A40 = UO - (BETA*((B*ALFA + Q*A)/(C*A - BETA*B) + ALFA))/A
DC = - R2R*A43 + R3R*A32 - R2R*BETA/A + R2R*BETA/A
DC1 = - A10*R2R*A43 + A10*R3R*A42 - VN*A43 - R3R*A40 + VN*A42 + R2R*A40
DC2 = VN*A43 + A40 - R3R - A10*R3R*BETA/A - VN*BETA/A
DC3 = - R2R*A40 - VN*A42 + VN*BETA/A + A10*R2R*BETA/A
DC4 = R2R*A33 + A40 - R3R*A32 - A40 - VN*A32 - A43 + VN*A42 + A33

1 VN*A33*BETA/A + VN*A32*BETA/A - A10*R2R*A33*BETA/A - A10*R3R*A10*A32

2BETA/A
C1 = DC1/DC
C2 = DC2/DC
C3 = DC3/DC
C4 = DC4/DC

Y(I) =
1 C1 + C2*EXP(R2R*S) + C3*EXP(R3R*S) + (B*ALFA + Q*A)/(C*A - BETA*B)
X = C2*A32 + C3*A33 + EXP(R3R*S) + (BETA*(B*ALFA + Q*A))/(C*A - BETA*B)

1 BETA*B + ALFA*BETA*(C1)*S/A + C4
V = C2*B2R*EXP(R2R*S) + C3*R3R*EXP(R3R*S)
U = C1*BETA/A + C2*A42*EXP(R2R*S) + C3*A43*EXP(R3R*S) + (BETA*(B*ALFA + Q*A))

1/(C*A - BETA*B) + ALFA)/A

GO TO 44

60 CONTINUE

NO ZERO ROOTS

A10 = DELTA - (B*ALFA + Q*A)/(C*A - BETA*B)
A40 = UO - (BETA*(((B*ALFA + Q*A)/(C*A - BETA*B)) + ALFA)/A
\[ A31 = \frac{(\beta - b \cdot R1 \cdot R1 \cdot R1^2)}{(R1 \cdot R1 \cdot A)} \]
\[ A32 = \frac{(\beta - b \cdot R2 \cdot R2 \cdot R2^2)}{(R2 \cdot R2 \cdot A)} \]
\[ A33 = \frac{(\beta - b \cdot R3 \cdot R3 \cdot R3^2)}{(R3 \cdot R3 \cdot A)} \]
\[ A41 = \frac{(\beta - b \cdot R1 \cdot R1 \cdot R1^2)}{A} \]
\[ A42 = \frac{(\beta - b \cdot R2 \cdot R2 \cdot R2^2)}{A} \]
\[ A43 = \frac{(\beta - b \cdot R3 \cdot R3 \cdot R3^2)}{A} \]
\[ DC = -R2 \cdot R2 \cdot A43 + R3 \cdot R3 \cdot A42 + R1 \cdot R1 \cdot A41 - R3 \cdot R3 \cdot A40 + R2 \cdot R2 \cdot A41 \]
\[ DC1 = -A10 \cdot R2 \cdot R2 \cdot A43 + A10 \cdot R3 \cdot R3 \cdot A42 + VN \cdot 143 + R3 \cdot R3 \cdot A40 + VN \cdot A42 + R2 \cdot R2 \cdot k40 \]
\[ DC2 = VN \cdot A43 + R3 \cdot R3 \cdot A40 + A10 \cdot R2 \cdot R2 \cdot A43 + VN \cdot H3R \cdot A41 - R1 \cdot R1 \cdot A40 - VN \cdot A41 \]
\[ DC3 = -R2 \cdot R2 \cdot A43 - VN \cdot k42 + R1 \cdot R1 \cdot A40 + VN \cdot A41 - A10 \cdot R1 \cdot R1 \cdot A42 + A10 \cdot R2 \cdot R2 \cdot A41 \]
\[ DC4 = R2 \cdot R2 \cdot A33 \cdot A40 - R3 \cdot R3 \cdot A32 \cdot A40 - VN \cdot A32 \cdot A43 + VN \cdot A42 \cdot A33 - R1 \cdot R1 \cdot A33 \cdot A40 + 4R3 \cdot A31 \cdot A40 + VN \cdot A31 \cdot A43 - VN \cdot A41 \cdot A33 + R1 \cdot R1 \cdot A32 \cdot A40 - R2 \cdot R2 \cdot A31 \cdot A40 + 5VN \cdot A31 \cdot A42 + VN \cdot A41 \cdot A32 + A10 \cdot A32 \cdot A43 + R1 \cdot A10 \cdot R2 \cdot A32 \cdot A43 + R1 \cdot R1 \cdot A33 \cdot R1 \cdot R1 \cdot A41 + 6A10 \cdot R2 \cdot A31 \cdot A43 - A10 \cdot R2 \cdot A41 \cdot A33 - A10 \cdot R3 \cdot R3 \cdot A41 \cdot A32 \]
\[ C1 = DC1 / DC \]
\[ C2 = DC2 / DC \]
\[ C3 = DC3 / DC \]
\[ C4 = DC4 / DC \]
\[ IF ((R1 \cdot R1 \cdot S) \geq 150.) GO TO 100 \]
\[ IF ((R2 \cdot R2 \cdot S) \geq 150.) GO TO 100 \]
\[ IF ((R3 \cdot R3 \cdot S) \geq 150.) GO TO 100 \]
\[ X01 = 0. \]
\[ IF (((R1 \cdot R1 \cdot S) + 120.) \geq 0.) X01 = \exp (R1 \cdot R1 \cdot S) \]
\[ X02 = 0. \]
\[ IF (((R2 \cdot R2 \cdot S) + 120.) \geq 0.) X02 = \exp (R2 \cdot R2 \cdot S) \]
\[ X03 = 0. \]
\[ IF (((R3 \cdot R3 \cdot S) + 120.) \geq 0.) X03 = \exp (R3 \cdot R3 \cdot S) \]
\[ Y(I) = \]
\[ 1 \quad C1 \cdot X01 + C2 \cdot X02 + C3 \cdot X03 + ((B \cdot ALFA + Q \cdot A) / (C \cdot A - BETA \cdot B)) \]
\[ X = C1 \cdot A31 \cdot X01 + C2 \cdot A32 \cdot X02 + C3 \cdot A33 \cdot X03 + (BETA \cdot (B \cdot ALFA + Q \cdot A) / (C \cdot A - BETA \cdot B)) \]
\[ 1 + ALFA) \cdot S / A + C4 \]
\[ V = C1 \cdot R1 \cdot X01 + C2 \cdot R2 \cdot X02 + C3 \cdot R3 \cdot X03 \]
\[ U = C1 \cdot A41 \cdot X01 + C2 \cdot A42 \cdot X02 + C3 \cdot A43 \cdot X03 + (BETA \cdot (B \cdot ALFA + Q \cdot A) / (C \cdot A - BETA \cdot B)) \]
\[ 1 \quad (C \cdot A - BETA \cdot B) + ALFA) / A \]
\[ GO TO 104 \]
\[ 61 CONTINUE \]
ROOTS ARE COMPLEX NUMBERS

\[ R_{2i} = \text{ABS}(R_{2i}) \]
\[ A_{10} = \Delta \text{TA} - (B \cdot \text{ALPHA} + Q \cdot A) / (C \cdot A - \text{BETA} \cdot B) \]
\[ A_{40} = U - (\text{BETA} \cdot (B \cdot \text{ALPHA} + Q \cdot A) / (C \cdot A - BETA \cdot B)) + \text{ALPHA} / A \]
\[ A_{41} = (\text{BETA} - R_{2R} \cdot R_{2R} - R_{2I} \cdot R_{2I}) / A \]
\[ A_{42} = (B \cdot \text{ALFA} + Q \cdot A) / (C \cdot A - BETA \cdot B) / A \]
\[ A_{43} = -(B \cdot \text{R2I} + 2 \cdot B \cdot \text{R2R} \cdot \text{R2I}) / A \]
\[ \text{IF}(R_{1R} \neq 0, \text{GO TO 100}) \]
\[ A_{31} = A_{41} / R_{1R} \]
\[ A_{32} = (B \cdot \text{R2I} - B \cdot \text{R2R} \cdot E - R_{2R} \cdot 2 \cdot E) / A \]
\[ A_{33} = -(B \cdot \text{R2I} \cdot (B \cdot \text{R2R} + B \cdot \text{R2I} \cdot 2 - E) + B \cdot \text{R2R} \cdot E) / A \]
\[ \text{DC} = \text{R2R} \cdot A_{42} + \text{R2I} \cdot A_{42} + \text{R1R} \cdot A_{42} - \text{R2I} \cdot A_{42} \]
\[ \text{DC1} = -A_{10} \cdot B \cdot R_{2R} + A_{10} \cdot A_{2R} + B \cdot V \cdot A_{42} - V \cdot A_{43} - \text{R2R} \cdot A_{40} \]
\[ \text{DC2} = V \cdot A_{42} + A_{40} \cdot V \cdot A_{43} - A_{10} \cdot R_{1R} \cdot A_{42} + A_{10} \cdot B \cdot R_{2R} \cdot A_{41} \]
\[ \text{DC3} = -B \cdot \text{R2R} \cdot A_{42} + A_{10} \cdot B \cdot R_{2R} \cdot A_{42} - B \cdot \text{R2I} \cdot A_{42} + A_{10} \cdot R_{1R} \cdot A_{42} \]
\[ \text{DC4} = B \cdot \text{R2R} \cdot A_{42} - B \cdot \text{R2I} \cdot A_{42} - V \cdot A_{43} + V \cdot A_{42} - A_{33} \cdot A_{40} - \text{R2I} \cdot A_{42} \]
\[ 1 \cdot R_{2R} \cdot A_{31} + A_{40} \cdot V \cdot A_{31} + A_{42} - V \cdot A_{42} - A_{33} \cdot A_{10} \cdot R_{1R} \cdot A_{32} + A_{10} \cdot B \cdot R_{2R} \cdot A_{32} \]
\[ 2 \cdot A_{10} \cdot B \cdot R_{2R} + A_{31} \cdot A_{43} - A_{10} \cdot B \cdot R_{2R} + A_{41} \cdot A_{33} - A_{10} \cdot R_{2I} \cdot (A_{31} \cdot A_{42} - A_{41} \cdot A_{32}) \]
\[ C_{1} = \text{DC1} / \text{DC} \]
\[ C_{2} = \text{DC2} / \text{DC} \]
\[ C_{3} = \text{DC3} / \text{DC} \]
\[ C_{4} = \text{DC4} / \text{DC} \]
\[ E_{1} = B \cdot R_{2R} \cdot A_{43} + B \cdot \text{R2R} \cdot A_{43} - R_{2I} \cdot A_{41} \]
\[ G_{1} = R \cdot B \cdot R_{2R} \cdot C_{2} \]
\[ E = B \cdot R_{2R} \cdot E_{1} + B \cdot R_{2R} \cdot G_{1} \]
\[ G = B \cdot R_{2R} \cdot G_{1} - B \cdot R_{2R} \cdot E_{1} \]
\[ A = (B \cdot \text{BETA} \cdot C_{2} \cdot E - B \cdot E_{1}) / A \]
\[ B = (B \cdot \text{BETA} \cdot C_{3} - G \cdot G - B \cdot G_{1}) / A \]
\[ Y(I) = C_{1} \cdot \text{EXP}(R_{1R} \cdot S) + (C_{2} \cdot \text{COS}(R_{2R} \cdot S) + C_{3} \cdot \text{SIN}(R_{2I} \cdot S) \cdot \text{EXP}(R_{2R} \cdot S) + 1 \cdot (\text{DELTA} - A_{10}) \]
\[ V = C_{1} \cdot R_{1R} \cdot \text{EXP}(R_{1R} \cdot S) + (E_{1} \cdot \text{COS}(R_{2I} \cdot S) + G \cdot G_{1} \cdot \text{SIN}(R_{2I} \cdot S) \cdot \text{EXP}(R_{2R} \cdot S) \]
\[ X = C_{1} \cdot A_{31} \cdot \text{EXP}(R_{1R} \cdot S) + \text{EXP}(R_{2R} \cdot S) \cdot ((A \cdot E_{2R} \cdot R_{2I} \cdot BB) \cdot \text{COS}(R_{2I} \cdot S) + \text{DELTA}) - A_{10} \]
\[ PGM20253 \]
\[ PGM20254 \]
\[ PGM20255 \]
\[ PGM20256 \]
\[ PGM20257 \]
\[ PGM20258 \]
\[ PGM20259 \]
\[ PGM20260 \]
\[ PGM20261 \]
\[ PGM20262 \]
\[ PGM20263 \]
\[ PGM20264 \]
\[ PGM20265 \]
\[ PGM20266 \]
\[ PGM20267 \]
\[ PGM20268 \]
\[ PGM20269 \]
\[ PGM20270 \]
\[ PGM20271 \]
\[ PGM20272 \]
\[ PGM20273 \]
\[ PGM20274 \]
\[ PGM20275 \]
\[ PGM20276 \]
\[ PGM20277 \]
\[ PGM20278 \]
\[ PGM20279 \]
\[ PGM20280 \]
\[ PGM20281 \]
\[ PGM20282 \]
\[ PGM20283 \]
\[ PGM20284 \]
\[ PGM20285 \]
\[ PGM20286 \]
\[ PGM20287 \]
\[ PGM20288 \]
1 \frac{(AA*E2I+BB*E2R)*SIN(R2*I*S))/(R2R**2+R2I**2)+(U0-A40)*S+C4}{1(U0-A40)}

CONTINUE

IF(Y(I)-0.) 43,43,177

GO TO 18

IF(Y(I)-Y(I-1)) 1,8,8

1 TMS=TMS+1.

GO TO 8

100 CONTINUE

8 DIAM=DIAM+10.*3.28E-6

18 DCR=DIAM

RETURN

END
APPENDIX V-5

SUMMARY OF DISPERSED FLOW HEAT TRANSFER
CALCULATION PRESENTED IN THIS STUDY

The total heat flux from the heated wall to dispersed flow is given by Eq. (5.11), i.e.,

\[
\frac{q}{A} = v_0(1 - \alpha)\rho_l h_g f_e \left(1 - \left(\frac{T_w}{T_{sat}}\right)^2\right) + 0.023 \frac{k_l}{D} \frac{g}{\rho_l} \text{Re}^{0.8} \text{Pr}^{0.8} (T_w - T_{sat})
\]

The units for \( v_0, T_w, \) and \( T_{sat} \) in this equation are

\[
\begin{align*}
\text{ft/hr} & \quad \text{for } v_0 \\
^\circ R & \quad \text{for } T_w, T_{sat}
\end{align*}
\]

The units for the fluid properties are listed on page 12.

For a given \( G \) and \( x \), the slip ratio \( S \) (\( S = \frac{V_g}{V_g} \)) was calculated using technique introduced by Plummer [50] (Equation 4.12 on page 98 of Plummer's report was used to evaluate \( S \)) which is based on Hynek's study of dispersed flow film boiling [31].

Knowing \( S \),

\[
\alpha = \frac{x}{x + \frac{\rho_g}{\rho_l} S(1 - x)}
\]

and then
\[ V_g = \frac{Gx}{\alpha \rho g} \]

\[ V_x = \frac{G(1 - x)}{\rho_x (1 - \alpha)} \]

\[ v_0 = 0.15 U^* = 0.15 \frac{Gx}{\alpha \rho g} \sqrt{f/g} \quad \text{where} \quad f_g = 0.0791/Re^{0.25} \]

and

\[ Re = \frac{GxD}{\alpha \mu g} \]

Deposition factor \( f \) [note that \( P(a) \) is given by Eq. (3.7)]:

\[
f = \frac{a_m}{\left( \int a^3 P(a) \, da \right)} \left[ \frac{a_m}{\left( \int a^3 P(a) \, da \right)} \right] \exp \left[ -2 \left( \frac{a_c}{a} \right)^2 \right] - \left[ \frac{a_m}{\left( \int a^3 P(a) \, da \right)} \right] \exp \left[ -2 \left( \frac{a_m}{a} \right)^2 \right] + \]

\[
+ \frac{6}{16} \sqrt{\pi/2} \left[ \text{erf} \left( \sqrt{2} \frac{a_m}{a} \right) - \text{erf} \left( \sqrt{2} \frac{a_c}{a} \right) \right] \left[ \frac{6}{16} \sqrt{\pi/2} \text{erf} \left( \sqrt{2} \frac{a_m}{a} \right) \right] - \left[ \frac{a_m}{\left( \int a^3 P(a) \, da \right)} \right] \exp \left[ -2 \left( \frac{a_m}{a} \right)^2 \right] \right] \]
In order to find \( f \) we first calculate \( a_m, \bar{a}, \) and \( a_c \).

Maximum drop diameter \( a_m \):

\[
a_m = \frac{W_e \sigma}{\rho_g (V_g - V_\infty)^2}, \quad \text{where } W_e = 7.5
\]

Mean drop diameter \( \bar{a} \):

\[
\bar{a} = \frac{1.83 \frac{C}{(V_g - V_\infty)}}{\sqrt{\sigma/\gamma_\infty}}
\]

where for a given value of \( G \) and \( x \), the value of \( C \) can be obtained from figure 5.13.

Deposition diameter \( a_c \) is calculated for the specified value of the wall temperature (note: \( a_c \) varies with \( T_w \)) by generating drop trajectories inside the boundary layer. The values of \( V_g, V_\infty \), and \( V_0 \) which we need to calculate \( a_c \) are already calculated above. Calculation of \( a_c \) is summarized in subroutine ENG on page 165. In the process of generating the drop trajectory, if the distance of the drop from the wall was less than or equal to 5 \( \mu m \), we assumed that the drop is deposited on the wall (i.e., the wall roughness of 5 \( \mu m \) was assumed). Note that we do not need to know the drop size distribution \( P(a) \) to calculate \( a_c \) as it was explained in Section 4.4.
BIOGRAPHICAL NOTE

Ejup N. Ganić was born and raised in Yugoslavia, and is a citizen of Yugoslavia. He attended the University of Belgrade during 1965-1972, receiving his B.S. and M.S. degrees in Chemical Engineering.

During 1971 he did his master's thesis research in the Thermal Physics Laboratory, Boris Kidric Institute of Nuclear Sciences, Belgrade. In June of 1972 he joined the Institute for Chemistry, Technology, and Metallurgy, Belgrade, where he still holds a post. He worked in Warsaw, Poland, during the summer of 1969, and in Birmingham, England, during the summer of 1970.

In September of 1973 he entered graduate school at MIT. From that time to date he has been a Research Assistant in the Heat Transfer Laboratory.

He is the author of several publications in the areas of heat transfer, applied mathematics, and energy conversion.

Mr. Ganić has a membership with the American Institute of Chemical Engineers, American Society of Mechanical Engineers, American Nuclear Society, and Sigma Xi.