Essays on Debt Markets
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Abstract

This thesis consists of three chapters on debt markets.

In chapter 1, I consider the interaction between domestic banking and growth in a
DSGE model of sovereign default in order to address (i) the joint existence of sovereign
debt and international reserves, and (ii) the occurrence of twin (domestic banking
and sovereign default) crises.

In chapter 2, joint with Hui Chen and Jun Yang, we build a structural model to explain
corporate debt maturity dynamics over the business cycle and their implications for
the term structure of credit spreads.

In chapter 3, joint with Juan Passadore, we study debt policy of emerging economies
accounting for credit and liquidity risk.

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I dedicate this thesis to my parents, Hong-Kun Xu and Xuan Cai, whose constant love and support have made all this possible.
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Growth, Liquidity Provision, International Reserves, and Sovereign Debt Capacity

Doctoral Dissertation Essay 1

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Growth, Liquidity Provision, International Reserves, and Sovereign Debt Capacity

Abstract

I consider the interaction between domestic banking and growth in a DSGE model of sovereign default in order to address (i) the joint existence of sovereign debt and international reserves, and (ii) the occurrence of twin (domestic banking and sovereign default) crises. In my model, growth requires intermediation which is subject to coordination failure and can result in growth destroying domestic bank runs. This forms the basis for endogenous costs of sovereign default which provide incentives for sovereign debt repayment. Importantly, these costs are robust to the presence of international reserves: bank run concerns give rise to a liquidity constraint which hinders the government's ability to use international reserves to undertake the type of sovereign-debt-capacity-destroying savings strategies in Bulow and Rogoff (1989). In fact, rather than destroy debt capacity, international reserves can help boost debt capacity in my model by preventing domestic bank runs which are times when sovereign default incentives are high. More generally, my model is also able to capture twin crises dynamics by endogenously linking the probability of a domestic banking crises to the government's external balance sheet conditions. Finally, my model makes use of Global Games techniques for bank run analysis and can quantitatively generate reasonable amounts of sovereign debt and international reserves in equilibrium.

Keywords: sovereign debt, international reserves, twin crises, Bulow and Rogoff (1989) puzzle, growth, bank runs
1 Introduction

As the recent Eurozone crises highlight, joint domestic banking and sovereign debt crises, or “twin crises” (Kaminsky and Reinhart (1999)) for short, have serious macroeconomic consequences for aggregate quantities such as output and investment.¹ Moreover, twin crises are usually preceded by surges in sovereign debt (Reinhart and Rogoff (2009, 2011)). Therefore, a prerequisite for understanding twin crises is to first understand why countries are able to borrow so much sovereign debt in the first place.

However, the existence of large amounts of sovereign debt is itself a puzzle. Sovereign debt is fundamentally different from private debt due to a general lack of international legal enforcement for the repayment of sovereign debt, and countries otherwise lack the ability to commit to honoring their obligations.² In the absence of legal enforcement, countries can always strategically default and so there must be other forms of non-legal costs of sovereign default in order to provide countries with repayment incentives. As first formalized by Eaton and Gersovitz (1981), an old understanding of such repayment motives is that countries repay out of fear of losing credit access and, along with that, the ability to borrow and smooth consumption in the future.

This explanation of sovereign debt capacity becomes tentative in the presence of international reserves. A predominant view is that countries accumulate international reserve for precautionary motives—to smooth the domestic economy against adverse shocks such as sudden stops and twin crises.³ While international reserves can stabilize

¹These recent episodes are just the newest additions to an already long list of crisis episodes. For the period 1970-2011, the crises database of Laeven and Valencia (2008, 2012) counts a total of 147 banking crises and 66 sovereign defaults episodes; of these, they count 19 episodes of twin (domestic) banking and sovereign debt crises. Reinhart and Rogoff (2009, 2011) also document the prevalence of twin crises in their extended historical sample.

²It is difficult for countries to credibly pledge international reserves as collateral due to sovereign immunity laws. For example, in the US, central bank assets, which include international reserves, are typically immune from attachment proceedings in the event of a sovereign default under the Foreign Sovereign Immunities Act of 1976. Although it is possible for a country to voluntarily waive its sovereign immunity, countries are typically hesitant to do so. For example, during the 1990s Argentina included bond clauses declaring that its central bank reserves were unattachable. For more information on the legal issues surrounding sovereign defaults, see Sturzenegger and Zettelmeyer (2006) and Panizza et al. (2009).

³In addition, there is also a mercantile view of reserves. My paper focuses on the precautionary view of international reserves (see Aizenman and Lee (2007) for a review of the two perspectives).
the domestic economy and increase a country’s ability to repay its sovereign debt, they can paradoxically reduce debt capacity by decreasing a country’s willingness to repay. This is because a country can now self insure by saving with international reserves and so the ability to borrow in the future diminishes in value. Furthermore, Bulow and Rogoff (1989) show that if exclusion from future borrowing is the only cost of default, then the presence of international reserves can kill sovereign debt capacity altogether by allowing countries to achieve a higher consumption stream by defaulting and subsequently saving in a manner which perfectly replicates the original lending arrangement, but with additional income. Thus, the theoretical challenge is to generate sovereign debt and international reserves in equilibrium. This theoretical conundrum is in sharp contrast to the data in which sovereign debt and international reserves coexist—see Figure 1.1 for a plot sovereign debt and international reserve levels for a cross-section of emerging markets in 2003.

In this paper, I present a model of twin crises and sovereign default costs in the presence of international reserve accumulation. I do so by introducing growth and domestic banking fragility (following Diamond and Dybvig (1983)) into the canonical Eaton and Gersovitz (1981) model of sovereign default. In the model, a necessary component for economic growth is intermediation and maturity transformation by the domestic banking sector. As in Diamond and Dybvig (1983), the domestic banking sector is subject to the possibility of coordination failure and bank runs. Thus, the maturity transformation process inherent in the process of growth generates liquidity risk and implies an domestic liquidity constraint for the economy. Violating this liquidity constraint is very costly: coordination failure will occur in the domestic banking sector resulting in growth destroying bank runs. This gives a natural role for the government to provide public liquidity risk management. Importantly, the government’s ability to do so will be intimately linked to the health of its external

4 The original Bulow and Rogoff (1989) result assumes the ability to save in complete asset markets in which case any lending arrangement can be replicated. When debt contracts are instead non-state contingent (which is the case in most quantitative models following the Eaton and Gersovitz (1981) setting) a non-state contingent savings vehicle suffices for this replication argument to go through. See Appendix 1 for an illustration of this in the canonical Eaton and Gersovitz (1981) model.

5 I provide a more formal summary of the challenges in Appendix 1.
balance sheet which is composed of savings in the form of international reserves and borrowing in the form of (external) sovereign debt. For example, the government’s ability to act as a lender of last resort crucially depends on the amount of liquidity that it can access on short notice, and this provides a motive for international reserves accumulation. Similarly, the amount of sovereign debt on the government’s external balance sheet affects its ability to credibly provide guarantees to the domestic banking sector.

Since this liquidity constraint is the crucial innovation of the model relative to the canonical Eaton and Gersovitz (1981) setting, I take extra steps to microfound it through a bank run analysis. Since it is well known that equilibrium indeterminacy can result in models involving coordination failure and bank runs (Diamond and Dybvig (1983)), I employ equilibrium selection techniques from the Global Games

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6The view of international reserves in my model closely reflects the Thornton (1802) view—international reserves protect and stabilize domestic credit markets against runs. See Obstfeld et al. (2010) for empirical support for this view.
literature (see Morris and Shin (2003) for a review) in order to obtain a unique bank run equilibrium. While this does involve some additional complexity for the model, the advantage of doing so is that I can then characterize the endogenous relationship between bank runs, sovereign debt capacity, international reserve accumulation, growth, and macroeconomic fundamentals. In turn, this rich interplay gives rise to sovereign debt capacity as well as twin crises dynamics.

With liquidity concerns in place, the key mechanism in the model is that these liquidity concerns become more pronounced after a sovereign default. In order to obtain this, my model assumes that domestic banks will have a higher intermediation burden after a sovereign default. This key assumption is motivated by empirical findings of decreased inflows of external private credit to the domestic private sector after a sovereign default (see Arteta and Hale (2008) and Das et al. (2010)).

Thus, conditional on trying to maintain the same level of investment and growth, domestic intermediaries will have to provide credit in place of lost foreign private credit after a sovereign default. In turn, this would lead to a more leveraged domestic banking sector and result in increased chances of a domestic banking crises. Mathematically, this translates into a tightening of the liquidity constraint after a sovereign default.

**Costs of sovereign default.** With the dynamics of domestic liquidity concerns in mind, my model generates endogenous costs of default as follows. First, the above reasoning shows that a sovereign default would lead to higher domestic banking fragility and increased chances of domestic bank runs, which is costly. However, this is not the end of the story as I have ignored the possibility of endogenous policy responses by the government. In particular, the government can try to manage this increased fragility by holding more international reserves. However, this is not without cost as budget constraint considerations imply forgone investments and lost growth opportunities, which is again costly. Therefore, the full cost of a sovereign default in the model is a

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7 This is consistent with a sovereign default having some form of reputation or information spill over effects for the private sector (Cole and Kehoe (1998), Sandleris (2008)). This can also be due to institutional constraints. For example, sovereign ceilings imposed by ratings agencies might mean that domestic firms might be “unjustly” downgraded when a country defaults and as a result lose access to foreign private credit (Almeida et al. (2014)).
Figure 1.2: Output, investment and international reserves around default.
The solid lines in each panel of this figure plot averages responses in (log) output, investment-to-output, and international reserves-to-output around a sovereign default event at time zero. All quantities are relative to their values one year prior to defaulting. The dashed (dotted) lines in each panel plots the one (two) standard error bars. Standard errors are clustered by country and year. Further details are available in Appendix 2.

combination of increased banking fragility and lost growth opportunities.

In order to lend support for this mechanism, I conduct an event study\(^8\) to see how output, investment and international reserves behave around sovereign default episodes in the data. The results are plotted in Figure 1.2. We see that following a sovereign default, there is an output drop (panel A) and an increase in international reserve-to-output ratios (panel C). Note that regardless of whether or not the output drop is permanent or transitory, the usual consumption smoothing logic would imply that countries should run down their international reserve levels. Instead, we see the opposite which is indicative of increased precautionary savings motives. In addition, comparing panels B and C, the increase in international reserves is offset by a decrease in investment rates. This is consistent with the mechanism of my model.

Sovereign debt and international reserves in equilibrium. Still, there are additional factors to take into account in order to obtain both sovereign debt and international reserves in equilibrium.

The first concern is that we must prevent Bulow and Rogoff (1989) type of post-default

\(^8\)Details for this event study are available in Appendix 2.
savings strategies from killing sovereign debt capacity by undoing the above costs of default. Crucially, bank run concerns prevent such savings strategies in my setting. Intuitively, when bank runs are possible, the default and save strategy of Bulow and Rogoff (1989) will generate additional banking fragility so that it will not always be worthwhile to do so. Mathematically, the savings strategies in Bulow and Rogoff (1989) are subject only to a budget constraint and so such savings strategies will be hindered in my setting because of the additional liquidity constraint. Therefore, costs of default in my setting will be robust to international reserve accumulation.

The second concern is that a mechanism which overcomes the Bulow and Rogoff (1989) replication argument and generates sovereign debt capacity may still be insufficient when it comes to generating meaningful levels of international reserves in equilibrium. For example, even when costs of default are exogenously assumed, quantitative models following the Eaton and Gersovitz (1981) setting often generate non-existent or very low levels of international reserves in equilibrium (see Alfaro and Kanczuk (2009), Salomao (2013), Bianchi et al. (2014)). There is still a tension between the precautionary view of international reserve accumulation and strategic sovereign default incentives: international reserve accumulation in the canonical Eaton and Gersovitz (1981) framework allow for better self insurance and increase strategic sovereign default incentives, and so governments optimally choose very low levels of international reserves so as not to run up its costs of borrowing.

Fortunately, bank run concerns in my model provide a countervailing force to this logic. The intuition is as follows: domestic bank runs are times in which sovereign default incentives are very high—by defaulting on its foreign lenders, a government will have more resources with which to prevent a domestic bank run. Hence, holding international reserves allows a country to decrease sovereign default incentives by preventing domestic banking problems from materializing in the first place. This “war chest” property of international reserves is what allows my model to quantitatively generate higher levels of international reserves in equilibrium.
Counter-cyclical sovereign default and twin crises In the model, sovereign default is determined after the government compares the costs of managing a more fragile banking system after defaulting against the costs of debt repayment. These tradeoffs will also vary across the business cycle and co-move with the investment opportunities set. Costs of default are pro-cyclical in my model: defaulting during good times entails a higher opportunity cost in terms of forgone investment opportunities. The opposite is true during bad times—hoarding more international reserves is less costly if investment opportunities are limited to begin with. In this way, sovereign defaults occur in a counter-cyclical fashion in my model.

Finally, my model also generates twin crises. In the model, increased domestic banking fragility increases sovereign default incentives and as a result raise a sovereign's borrowing costs. In turn, this weakens a government's external balance sheet conditions and decreases the government's ability to provide public liquidity support, which means that domestic bank runs are then more likely to take place. This results in a vicious twin crises feedback mechanism which is consistent with the data (see Laeven and Valencia (2008, 2012) and Bulow and Rogoff (2011)).

1.1 Related literature.

This paper is related to a large literature dealing with sovereign debt and default (see Aguiar and Amador (2014) for an excellent review). With a few exceptions, much of this literature assumes exogenous costs of default and does not consider the joint accumulation of sovereign debt and international reserves.

The main reason for this has to do with the Bulow and Rogoff (1989) puzzle. Under fairly general conditions, Bulow and Rogoff (1989) show that if asset markets are sufficiently complete and exclusion from future borrowing is the only form of punishment for defaulting, then a country could always default and subsequently replicate the original lending relationship through state-contingent savings. This action results in a higher consumption path and so in the absence of other costs of default, sovereign debt capacity will not exist. Furthermore, being able to save through complete markets is
not always necessary for this result to hold: Auclert and Rognlie (2014) show that a non-state contingent international reserve asset suffices for the replication argument to go through in the incomplete-markets setting of Eaton and Gersovitz (1981). There have been many proposed solutions to the Bulow and Rogoff (1989) puzzle. These include direct sanctions (Bulow and Rogoff (1989)), inability or unwillingness to save after defaulting (Cole and Kehoe (1995), Kletzer and Wright (2000), Wright (2002), Gul and Pesendorfer (2004), and Amador (2012)), reputation or information spill overs affecting other parts of the domestic economy (Cole and Kehoe (1998) and Sandleris (2008)), bubbles (Hellwig and Lorenzoni (2009)), and non-selective default (Broner et al. (2010) and Gennaioli et al. (2014)). To the best of my knowledge, this paper is the first to show that bank run induced liquidity constraints can help hinder the type of post-default savings strategies of Bulow and Rogoff (1989).

Mendoza and Yue (2012) is the first paper to endogenize sovereign default costs. Default costs in their setting is in terms of decreased trade and output as a result of lost private trade credit following a sovereign default. Gornemann (2013) extends Mendoza and Yue (2012) to further incorporate endogenous growth. While the underlying frictions in Mendoza and Yue (2012) and Gornemann (2013) involve working capital constraints and are thus also liquidity related, they assume away the ability for the economy to meet these working capital constraints through international savings, and therefore avoids dealing with the Bulow and Rogoff (1989) puzzle. This is a key difference from my setting. In addition, costs of default in my model stem from banking fragility, which is distinct from the trade channel considered in Mendoza and Yue (2012) and Gornemann (2013).

To the best of my knowledge, papers which study the joint accumulation of sovereign debt and international reserves international reserves has avoided dealing with the Bulow and Rogoff (1989) puzzle. One strand of this literatures does not deal with strategic sovereign default by assuming that countries can commit to repay its sovereign debt (see Aizenman and Lee (2007), Jeanne and Ranciere (2011), and Hur and Kondo (2013)). Another strand allows for strategic sovereign default but assumes debt capacity
in the form of exogenous costs of default (see Alfaro and Kanczuk (2009), Salomao (2013) and Bianchi et al. (2014)). Quantitatively, equilibrium international reserves levels are often very low in the latter setting as countries will strategically choose low levels of international reserves in order to decrease its default incentives and obtain better borrowing terms. This highlights the tension between the precautionary view of international reserves and the strategic view of sovereign default. In particular, these results indicate that even if a mechanism can overcome the Bulow and Rogoff (1989) puzzle and generate debt capacity, it may not additionally generate meaningful levels of international reserves in equilibrium. In my model, on top of hindering Bulow and Rogoff (1989) replication arguments and generating sovereign debt capacity, liquidity concerns also endows international reserves with a war chest property that allows for higher levels of international reserves in equilibrium.

My paper is also related to crises models in open economy settings (see Lorenzoni (2014) for a review of this literature). Examples include Krugman (1979), Cole and Kehoe (2000), Caballero and Krishnamurthy (2001), Chang and Velasco (2001), and Holmstrom and Tirole (2002). Within this literature, Chang and Velasco (2001) is most similar in spirit to my paper. They also consider how domestic bank runs interact with strategic sovereign incentives, and show its implications for international reserve accumulation as well as domestic investments. The key difference is that their analysis is done in a finite horizon setting in which it is impossible to generate any forms of “reputation” for debt repayment and so they impose exogenous default costs. While their analysis is extremely informative, their theory is still incomplete: as Reinhart and Rogoff (2009, 2011) show, twin crises usually occur after large amounts of sovereign debt have been accumulated, hence a complete theory of twin crises also requires endogenizing sovereign debt capacity. To the best of my knowledge, this paper is the first such “complete” theory which captures twin crises as well as endogenize sovereign debt capacity in a setting with international reserve accumulation.

Many other crises models also stress the link between sovereign default and the domestic banking sector. In particular, a strand of literature explores the implications

\footnote{This is fully anticipated by Chang and Velasco (2001) in their final remarks section.}
of non-selective default. In these models, the government cannot selectively default and so sovereign defaults directly hurt domestic agents (including domestic banks) who hold sovereign bonds. Analysis along these lines include Broner et al. (2010), Brutti (2011), Acharya and Rajan (2013), D’Erasmo and Mendoza (2013), Bocola (2014), Gennaioli et al. (2014), Pei (2014) and Perez (2014). My model instead assumes selective external sovereign default— the goal is to see how far theories of external sovereign debt capacity can get under the original selective default assumptions of Eaton and Gersovitz (1981) and Bulow and Rogoff (1989). Another distinguishing feature is that my model also considers international reserve accumulation which this non-selective default literature is silent about.

Finally, my paper also builds on closed-economy analysis of banking and growth. Papers along these lines include Cooper and Ross (1998) and Ennis and Keister (2003, 2006) who investigate how bank runs (modeled after Diamond and Dybvig (1983)) affect growth through both ex-post liquidations as well as by changing ex-ante incentives to invest. From a methodological perspective, I embed the Global Games analysis of the closed-economy bank run model of Goldstein and Pauzner (2005) into an open economy dynamic stochastic general equilibrium (DSGE) setting. This allows me to microfound the liquidity constraint as well as obtain unique equilibrium predictions without having to impose sun spots.

To summarize, the literature has investigated various combinations of international reserves, endogenizing sovereign debt capacity, and/or twin crises. To the best of my knowledge, this paper is the first attempt at capturing all three phenomena simultaneously.

10Reality lies somewhere in between these two extremes. Selective default can happen in practice (e.g. the government can default and then bail out domestic banks), however obstacles are likely to hinder such behaviour in practice (e.g. Broner et al. (2010) points out one such obstacle). How well a government is able to achieve full selective default in practice is an empirical question which warrants further investigation. Keep in mind that if a sovereign can achieve a sufficiently high level of selective default, then it would be difficult for the government to commit to not selectively defaulting on just its external creditors in the event of an actual sovereign default.
The rest of this paper is structured as follows: Appendix 2 then describes the model. This is done as follows: Section 2.1 provides an overview of the setting, Section 2.2 describes bank runs within the model, Section 2.3 fills in the remaining details for the dynamic setting, and Section 2.4 provides more details on the model’s mechanisms. A quantitative analysis is then conducted in Section 3. Finally, Section 4 concludes. In addition, Appendix 1 may be of interest to the reader as it provides a self-contained and more formal review of the challenges associated with generating international reserves and sovereign debt in equilibrium.

2 Model

2.1 Overview

I first provide an overview of my model. My model consists of a representative agent in a small open economy (SOE). The representative agent is composed of a private and a public sector. The private sector consists of households, firms, and banks. The public sector consists of a government which we assume to be a benevolent planner. The relationship between the private and public sectors is illustrated in Figure 1.3. As a further simplification, I abstract from exchange rate and monetary policy concerns by assuming that the economy is entirely real.11

Production and liquidation. I think of banks and firms as a joint productive entity. Capital $K_t$ is used to produce goods each period, with production being subject to a liquidation friction which I model following Diamond and Dybvig (1983). I follow the Diamond and Dybvig (1983) setting for its convenience, especially when it comes to studying coordination problems and bank runs.12 At the start of each period, each

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11This is a standard simplifying assumption in models of sovereign debt within the Eaton and Gersovitz (1981) setting. Extensions along these lines are beyond the scope of this paper.

12Of course, there are other forms of frictions which can create a similar need for government support and liquidity management; for example, the Holmstrom and Tirole (1998, 2002, 2011) framework readily spring to mind. In addition, the need for government liquidity support can also be driven by solvency concerns following the accumulation of substantial amounts of non-performing loans on bank balance sheets; the recent financial crises of 2008 is a good demonstration of this.
Figure 1.3: The representative agent economy. This figure illustrates the structure of the economy. There is a representative agent consisting of firms, households, banks, and a government. Growth is driven by capital accumulation. Investments are intermediated and partially financed with household deposits. The government accumulates international reserves, issues external sovereign debt, and provides liquidity support for the private sector. NFA denotes net foreign assets.

A unit of capital gives off one unit of seed. It takes a full period for the seed to ripen into fruit which can then be harvested and consumed. All consumption is in terms of "ripened fruit" which also serves as then numeraire. If allowed to mature, each unit of seed produces $e^{z_t}$ units of fruit where $z_t$ is the productivity shock for the period. This total factor productivity (TFP) shock is the only source of uncertainty within our model; it follows an AR(1) process:

$$z_{t+1} = \rho_z z_t + \sigma_z z_{t+1}.$$  \hspace{2cm} (1.1)

Unfortunately, sometimes banks may have funding needs and not be able to wait until fruits fully ripen before harvesting. These funding needs arise in the event of a bank run (I will describe the banking setup shortly). In the event of an early harvest, each unit of seed will instead only be worth

$$L(z_t) < e^{z_t}.$$  \hspace{2cm} (1.2)
Figure 1.4: Within period timing. This figure illustrates the timing of events within each period $t$. The small open economy enters each period with capital $K_t$, international reserves $S_t$, outstanding bank loans $A_t$ and deposits $D_t$, as well as some outstanding stock of sovereign debt $B_t$. In the middle of each period, households observe private signals about productivity $s_{t,t}$. Depositors then choose whether or not to run $a_{t,t}$. Withdrawers are served subject to a “sequential service” constraint. Their total consumption is bounded by the amount of available liquidity, which is given by the sum of international reserves and liquidation proceeds $\ell_t L(z_t)K_t$. Full harvest takes place at the end of each period. Expenditures $X_t$ are then paid. Finally, remaining households consume. Limited liability applies to all households.

units of ripened fruit. The wedge $e^{z_t} - L(z_t) > 0$ is the cost of liquidation, and forms the basis for costs of default. I assume that liquidation losses are larger during good times when productivity is high and the opportunity cost of having idle productive units is also high. That is, the liquidation wedge is increasing in productivity $z_t$. Total output each period is given by

$$Y_t = \left[ \ell_t L(z_t) + (1 - \ell_t)e^{z_t} \right] K_t,$$

(1.3)

where $\ell_t \in [0, 1]$ is the fraction of seeds harvested early (i.e. liquidated).

The timing of production within each period is portrayed in Figure 1.4 which also illustrates the relative timing of other events occurring within the economy.

**Growth.** Growth is entirely driven by capital accumulation by firms in the private sector.\(^{13}\) I think of capital $K_t$ as more than just physical capital, but also include other

\(^{13}\)In the absence of liquidation frictions, the closed economy version of my model reduces to an $AK$ growth model; see Acemoglu (2009) for a textbook treatment of the growth literature. I consider
intangible forms of capital (e.g. human capital) which can be difficult to disinvest. At the aggregate level, the law of motion for capital is given by

\[ K_{t+1} = (1 - \delta) K_t + I_t, \quad I_t \geq 0, \]  

(1.4)

with \( \delta > 0 \) being the depreciation rate. Note that liquidations apply only to seeds and do not directly pertain to capital \( K_t \). However, investment \( I_t \) is still illiquid. This is because an initial capital investment \( I_t \) produces a stream of seeds, all of which are subject to liquidation risk. In addition, investments are irreversible so that \( I_t \geq 0 \). Irreversibility captures technological illiquidity and gives a meaningful role for accumulating international reserves.

Capital accumulation is subject to convex adjustment costs so that the total cost of increasing capital by \( I_t \) units is given by

\[ \phi \left( \frac{I_t}{K_t} \right) K_t, \]  

(1.5)

where the function \( \phi(\cdot) \) captures decreasing returns to scale in capital accumulation. Investment costs form part of expenditures \( X_t \) at the end of each period.

**Intermediation.** I assume that there is a role for banks as intermediaries for investments. There is a representative intermediary which I will refer to as “the bank.” I take the intermediation process as given and explore its implications for sovereign debt capacity, international reserves, and twin crises.\(^{14}\) In particular, a fraction \( \chi \in (0, 1) \) of total investment costs each period, \( \phi \left( \frac{I_t}{K_t} \right) K_t \), is intermediated by banks. Bank loans \( A^{\text{loan}}_t \) evolve according to

\[ A^{\text{loan}}_{t+1} = (1 - m_A) A^{\text{loan}}_t + \chi \phi \left( \frac{I_t}{K_t} \right) K_t. \]  

(1.6)

an \( AK \) setting for its numerical convenience, as capital \( K_t \) can be scaled out.

\(^{14}\)Generating a need for intermediaries from first principals is beyond the scope of this paper. Such first principals have already been extensively explored in the banking literature (see Freixas and Rochet (2008) for a textbook treatment).
Here $\chi \phi \left( \frac{A_t}{K_t} \right) K_t$ is the amount of new investment loans given out by banks in period $t$. The bank's loan book can be thought of as a collection of small loans, with each outstanding loan randomly maturing with probability $m_A \in (0, 1)$ each period. Under this assumption, a fraction $m_A$ of loans comes off the bank's book each period and the average debt maturity of the bank's loan book is $1/m_A$.

Beside financing domestic investments, banks also extend other forms of credit to the domestic economy. Examples of such forms of credit include households loans such as mortgage and auto loans. These loans total

$$A_t^{\text{other}} = a_0 K_t,$$  \hspace{1cm} (1.7)

and are proportional to the size of the economy as measured capital stock $K_t$. The constant of proportionality is given by $a_0$ and will be set so that the size of the banking sector as measured by its total assets

$$A_t = A_t^{\text{loan}} + A_t^{\text{other}}$$  \hspace{1cm} (1.8)

is close to its empirical counterpart.

**Households.** There is a continuum of households with unit mass. The intermediary finances its lending, in part, by taking deposits from households. I assume that the total amount of (demand) deposits is given by

$$D_t = m_L A_t,$$  \hspace{1cm} (1.9)

where I have conveniently assumed a constant capital structure for banks\textsuperscript{15} with $m_L \in (0, 1)$ being the fraction of bank lending financed with demand deposits (or, more generally, short term debt). All depositors have the option of withdrawing

\textsuperscript{15}This allows me to conveniently keep just one of $A_t$ and $D_t$ as the state variable in the numerical implementation. In general, bank capital structure will be time varying. Incorporating time varying intermediary leverage within my model will generate additional amplification effects over the business cycle; see, for example, Brunnermeier and Sannikov (2014) for a model of time varying banking leverage in a closed economy.
and consuming their deposits on short notice. Withdrawers are serviced subject to a *sequential service* constraint. The difference between the maturity rate of bank liabilities, $m_L$, and the maturity rate of bank loans, $m_A$, captures the amount of maturity mismatch on bank balance sheets. Usually, $m_A$ is less than $m_L$ which corresponds to the empirically relevant case where banks finance long term loans with short term deposits (i.e. banks are doing maturity transformation).

Not all households need be depositors—I denote the fraction of depositor households in period $t$ by $n_{\text{max},t}$. The total stock of deposits, $D_t$, is evenly split amongst the $n_{\text{max},t}$ depositors according to

$$D_t = d_t \times n_{\text{max},t},$$

(1.10)

where $d_t$ is the *promised* deposit payout (in terms of fruit). For simplicity, I set the promised payout $d_t$ to be proportional to capital:

$$d_t = d_0 K_t,$$

(1.11)

where $d_0$ is the constant of proportionality.

**Coordination failure and domestic bank runs: implications for international reserves and sovereign debt capacity.** As in Diamond and Dybvig (1983), there is a mismatch between the timing of withdraws and the timing of production. In particular, withdraws occur in the middle of each period when fruits have not yet

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16It is possible to determine $d_t$ endogenously by considering an additional household portfolio problem (e.g. see Gertler and Kiyotaki (2010, 2013) for an implementation of this in a dynamic setting). Usually, this involves introducing household preferences for liquidity. As will be remarked upon shortly, I purposefully do away with household liquidity preferences in order to emphasize coordination failure. The key assumption in fixing $d_0$ is that the contractual terms for banks are sticky and downward rigid so that it is difficult for banks to deleverage on short notice without taking large losses. A possible reason for such downward rigidity is that information asymmetry can become more severe in times of financial distress, which increases the costs of external financing for banks (Myers and Majluf (1984)).
fully ripened. The total payout to withdrawing depositors is given by

\[ n_t C_{\text{run},t}, \]  

where \( n_t \) is the equilibrium number of households who choose to run, with each such household consuming \( C_{\text{run},t} \) on average. The total amount of liquidity available for paying withdrawing households is given by

\[ S_t + \ell_t L(z_t)K_t, \]  

and is the sum of international reserves and liquidation proceeds. When total liquidity is plentiful, each withdrawing household will be paid the full promised amount and we will have \( C_{\text{run},t} = d_t \). Instead, if total liquidity is insufficient, then some withdrawing households may go unserved and end up consuming very little. International reserves act as a buffer against having to liquidate (i.e. being forced to set \( \ell_t > 0 \)) in the event of a bank run. Thus, reserves help stabilize the domestic banking sector.

Total consumption for the remaining \( 1 - n_t \) households (i.e. depositor households that chose not to withdraw, as well as non-depositor households) is given by

\[ (1 - n_t)C_{\text{wait},t}, \]  

where the average consumption per remaining household, \( C_{\text{wait},t} \), is determined by evenly dividing all remaining amounts\(^{17}\) amongst the remaining households at the end of the period.

Each depositor will decide whether or not to run by comparing the expected utilities from running and obtaining consumption \( C_{\text{run},t} \), or waiting and consuming \( C_{\text{wait},t} \). As shown in Diamond and Dybvig (1983), coordination problems can result in equilibrium indeterminacy. To overcome this, I use equilibrium selection techniques from the

\(^{17}\)This residual amount consists of (1) proceeds from full harvest at the end of the period, (2) any unused liquidity from the middle of the period, and (3) expenditures at the end of the period. Expenditures consist of investments, reserve accumulation, and net (external) debt rollover costs. This will be made clear in Section 2.3
Global Games literature and introduce noisy private household signals

\[ s_{i,t} = z_t + \varepsilon_{i,t} \quad (1.15) \]

of the productivity shock \( z_t \), with the noise component \( \varepsilon_{i,t} \) being independent and identically distributed (iid) in the cross-section and over time. The run decision for each depositor will be conditioned on his private signal. I defer details for this Global Games analysis to Section 2.2. For now, I press on with describing the model.

In order to make the results stark, I assume that households do not have intrinsic liquidity needs. That is, households do not have liquidity preferences for "early" consumption, so that all runs within my model are the result of coordination failure. This assumption highlights the following: absent coordination failure, my model reduces to a setting without liquidity concerns (i.e. \( \ell_t = 0 \)). In this case, my model will not have any costs of default other than financial exclusion. Furthermore, my model also allows for savings. It is well known that such a setting will have difficulty generating external debt capacity. In this sense, twin crises and sovereign debt capacity come hand in hand within our model.\(^{18}\)

**Boom-bust cycles.** Implicit in my set up is the assumption that investment booms can lead to banking fragility down the line. Given the assumptions, increased investment \( I_t \) during good times leads to increased amounts of bank lending \( A_t \) and build-up in domestic bank debt \( D_t \). If the level of international reserves \( S_t \) is insufficient when a recession hits, then there may be an inadequate amount of liquidity buffer to prevent bank runs, liquidations and sovereign debt crises from occurring. Empirically, such boom-bust dynamics have been regularly occurring (Reinhart and Rogoff (2009, 2011)). Of course, the government will take into account the possibility of such boom-bust dynamics in formulating the economy's investment, international reserve, and external

\(^{18}\)This result contains some flavor of Allen and Gale (2004). Twin crises are a natural outcome in my model's constrained efficient setting. It may not be optimal for the planner to always avoid crises as this can be overly costly. For example, the planner can choose to hoard large amounts of international reserves to always prevent crises, but this comes at the cost of forgone investment opportunities.
The government. The government is a benevolent planner with Epstein and Zin (1989) and Weil (1989) preferences who maximizes social welfare $V_t$ given (recursively) by

$$V_t = \left\{ (1 - \beta)W_t^{1-\psi^{-1}} + \beta E_t \left[ V_{t+1}^{1-\gamma} \right]^{1-\psi^{-1}} \right\}^{1-\psi^{-1}}$$

(1.16)

where $\psi > 0$ is the elasticity of intertemporal substitution (EIS), $\gamma > 0$ is the planner’s relative risk aversion,\(^{19}\) and $W_t$ reflects the certainty equivalent consumption of the representative household after taking into account the risk of bank runs. Note that in the absence of coordination problems, $W_t$ is just aggregate consumption. Details for the derivation of $W_t$ are deferred to Section 2.2.

The planner oversees domestic investment and external balance sheet adjustments for the small open economy. The planner is, however, constrained by the domestic banking system in implementing its policies. That is, the planner takes the role of the banking sector (as described in equations (1.6) through (1.10)), as well as household coordination problems as given when it comes to choosing its policies. The possibility of coordination failure and bank runs imply a role for the government to manage liquidity risk at the country level; such liquidity risk management is intimately linked to the government’s external balance sheet.

The baseline model studies two instruments for liquidity provision. The first is for the planner to just liquidate projects (i.e. choose $\ell_t > 0$) and incur lost output $e^{z_t} - L(z_t)$, however this is inefficient. The second option is for the planner to accumulate a stock of international reserves $S_t$ which can be thought of as claims on foreign fruit. International reserves are non-state contingent and accumulate at the risk free rate $r$ (e.g. think of an emerging market holding US treasuries). Importantly, international reserves are fully liquid and can be converted into fruit at any time.

\(^{19}\)To the best of my knowledge, the quantitative sovereign debt literature has so far focused on CRRA preferences (i.e. $\psi^{-1} = \gamma$). In my setting where the endogenous costs of default is closely tied to growth, being able to disentangle the EIS $\psi$ and risk aversion $\gamma$ parameters can be of quantitative importance.
However, international reserve accumulation is not without its costs: they lack state contingency and may not be as high yielding as domestic investments $I_t$ (especially during domestic booms).

**Sovereign default and its consequences.** The government can also borrow from international creditors by issuing external sovereign debt. This stock of outstanding debt is denoted by $B_t$. I assume that debt is single period. Debt issuances and repayments are all assumed to take place at the end of the period, and the net proceeds from debt rollover cost form part of the end of period expenditures $X_t$. Under this timing assumption, sovereign debt issues do not *directly* provide liquidity, nor does it *directly* imply liquidity needs. Rather, its effects are more *indirect*. It can influence liquidity supply by financing international reserve accumulation. Similarly, debt rollover costs will affect the amount of resources available for paying non-withdrawing households at the end of the period, which can potentially alter run incentives for depositors.

I follow the Eaton and Gersovitz (1981) framework and assume that the government is unable to commit to repaying its external obligations. In particular, the government’s default decision is strategic: he is always comparing the value of not defaulting, $V_{ND,t}$, to the value of defaulting, $V_{D,t}$, and default occurs at time $t$ if and only if the latter is higher. That is, default occurs on the set $\{V_{D,t} > V_{ND,t}\}$.

Should the planner default, the economy goes into financial autarky and access to foreign credit is lost. However, the planner is still able to save abroad using international reserves. A defaulted country can regain access to international credit markets with probability $\xi > 0$ each period. All debt is written off when the planner reenters international credit markets.20

Furthermore, I assume the domestic banking sector must partake in a larger amount of intermediation after defaulting, so that the fraction of intermediated investments $\chi$

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20 These are standard simplifying assumptions used in the sovereign default literature. Recovery can be incorporated at the cost of additional complexity (for example, see Yue (2010)).
(see equation (1.6)) is larger whilst the small open economy is in default:

\[ XD > XND. \]  

(1.17)

Assumption (1.17) is motivated by empirical findings that sovereign defaults trigger losses in private credit. The percentage decrease in external private credit attributable to sovereign default ranges between 20-40% (see Arteta and Hale (2008) and Das et al. (2010)).\(^{21,22}\) Hence, implicit in assumption (1.17), is the idea that the domestic banking sector will have a heavier intermediation burden after a sovereign default as it will have to substitute credit in place of lost foreign credit to the domestic private sector.

Theoretically, this drop in external private credit is consistent with a sovereign default having some form of reputation or information “spill overs” into other parts of the economy (see Cole and Kehoe (1998) and Sandleris (2008)). Alternatively, sovereign ceilings and institutional constraints may also hinder the ability of institutional investors to invest in the private sector of countries in default (Almeida et al. (2014)). For simplicity, I do not provide further microfoundations for this drop in external private credit. Instead, I take this fact as given and explore its implications for domestic banking fragility.

**Equilibrium and remaining details.** Formally, the equilibrium consists of a game played between households and the government, with international lenders in the background. In order to characterize the Nash equilibrium, I proceed as follows in the remainder of this section: Section 2.2 uses Global Game methods to study equilibrium household run behavior taking as given the planner’s policies; Section 2.3

\(^{21}\)Their results include controls for international competitiveness, investment climate and monetary stability, financial development, macroeconomic fundamentals, political stability, and global capital supply. The average drop in private credit attributable to sovereign default found in Das et al. (2010) is 40%. The magnitude of the drop in private credit found in Arteta and Hale (2008) is smaller. This is due to their expanded definition of a “default event” for their baseline results. They find similar magnitudes once they use a more stringent definition of default.

\(^{22}\)Note that these numbers do not include decreases in other forms of private credit such as trade credit. My channel will only become stronger if I account for alternate sources of increased liquidity needs. For example, trade credit is emphasized in Mendoza and Yue (2012).
then considers the planner's problem taking as given the equilibrium run behavior of households, and characterizes the full equilibrium.

2.2 Equilibrium Bank Runs

In this section, I focus on the within-period decision of depositors' regarding whether or not to run (cf. Figure 1.4). The households' run decision takes as given the actions of firms and the government. As is well known from the original analysis of bank runs in Diamond and Dybvig (1983), bank run models are prone to equilibrium indeterminacy. In order to resolve this problem, I use techniques from the global games literature to obtain unique equilibrium predictions; in particular, I build upon the bank run model of Goldstein and Pauzner (2005).

The structure of the economy as well the timing was described in Section 2.1 (also see Figure 1.4 for a graphical illustration). The analysis here will be static in nature, and so I drop all time subscripts. I index households by \( i \in [0, 1] \). Following Goldstein and Pauzner (2005), the noise component of households' private productivity signals (1.15), is uniformly distributed

\[
\varepsilon_i \sim Unif(-\Delta, \Delta)
\]  \hspace{1cm} (1.18)

with support between \(-\Delta\) and \(\Delta\). Furthermore, the noise components \(\varepsilon_i\) are independent and identically distributed (iid) in the cross section (and over time).

After observing the private signals, depositor households decide whether or not to withdraw their deposit. This decision will ultimately depend on the payoffs, which we...
go through now.

**Payoffs.** Given the realized productivity $z$ and the (still to be determined) equilibrium fraction of households who withdraws early $n \in [0, n_{max}]$, the fraction of seeds subject to early harvest $\ell \in [0, 1]$ is given by

$$
\ell(z, K, d, S, X, n) = \begin{cases} 
0 & \text{if } nd \leq S \\
\frac{nd-S}{L(z)K} & \text{if } nd \in (S, S + L(z)K) \\
1 & \text{if } nd \geq S + L(z)K
\end{cases} .
\quad (1.19)
$$

Here, $d$ is the promise payout to withdrawers (1.11). The total amount of fruit demanded by withdrawing depositors is given by $nd$. When international reserves $S$ are sufficient to cover this amount (i.e. $nd \leq S$), there is no need for early harvest. In the intermediate region where withdraws can only be fulfilled by harvesting some of the unripe fruit (i.e. $S < nd < S + L(z)K$), the bank chooses the minimal required amount of early harvest and sets $nd = S + \ell L(z)K$.

Finally, when withdraws are too large for the total amount of liquidity available (i.e. $nd \geq S + L(z)K$), the bank is forced to harvest early its entire crop of fruit and pay out all the proceeds to withdrawing households.

The realized consumption from running is given by

$$
C_{run}(z, K, d, S, X, n) = \begin{cases} 
d & \text{if } nd \leq S + L(z)K \\
d & \text{with probability } P_{\text{withdraw}} \\
0 & \text{with probability } 1 - P_{\text{withdraw}} \end{cases} .
\quad (1.20)
$$

In the region where the bank has a sufficient supply of liquidity to pay all $n$ withdrawing households (i.e. $nd \leq S + L(z)K$), we see that each withdrawing depositor receives the promised amount $d$. Outside of this region, the bank cannot guarantee payment to all households.

\footnote{This is because early harvest involves a loss relative to late harvest— it is never efficient to harvest early more than is needed and store the proceeds to the end of the period.}
withdrawing households. In this case, households are paid according to the sequential service constraint so that each withdrawing household receives \( d \) with probability

\[
P_{\text{withdraw}} = \min \left\{ \frac{S + L(z)K}{nd}, 1 \right\}
\]  

(1.21)

and zero otherwise.

The corresponding utility obtained by withdrawing households is then given by

\[
U_{\text{run}}(z, K, d, S, X, n) = \begin{cases} 
    u(C_{\text{home}} + d) & \text{if } nd \leq S + L(z)K \\
    p_{\text{withdraw}}u(C_{\text{home}} + d) + (1 - p_{\text{withdraw}})u(C_{\text{home}}) & \text{if } nd > S + L(z)K
\end{cases}
\]

where the probability of a successful withdrawal \( p_{\text{withdraw}} \) is given in (1.21), and the utility function \( u(\cdot) \) is assumed to be in CRRA form

\[
u(c) = \frac{c^{1-\eta}}{1-\eta}
\]

(1.23)

with \( \eta > 1 \) being the depositors' risk aversion. For numerical convenience, I bound utility from below by introducing negligible amounts of home production (in terms of fruit) given by

\[
C_{\text{home}} = c_{lb}K,
\]

(1.24)

with \( 0 < c_{lb} \ll 1 \) set to be very small.\(^{26}\) I specify this amount to be proportional to capital in order to preserve homogeneity.

All non-withdrawing households are residual claimants on the economy at the end of the period. Their consumption can be calculated as follows: after paying off withdrawing depositors, the amount of liquidity carried over to the end of the period is given by \((S + \ell L(z)K - nd)_+\) where the amount of early harvest \( \ell \) is given in (1.19).\(^{27}\)

This amount is then supplemented by proceeds from late harvest (which amounts

\(^{26}\)Think of each household as having a cabbage patch which provides him with a subsistence level of consumption.

\(^{27}\)The notation \((x)_+\) denotes \(\max\{x, 0\}\).
to \((1 - \ell)e^{z}K\) before the bank finally pays end of period expenditures \(X\). The residual amount \((S + \ell L(z)K - nd)_{+} + (1 - \ell)e^{z}K - X\) is then split evenly amongst the remaining \(1 - n\) households who each obtain consumption

\[
C_{\text{wait}}(z, K, d, S, X, n) = \frac{[(S + \ell L(z)K - nd)_{+} + (1 - \ell)e^{z}K - X]}{1 - n},
\]

(1.25)

where \(C_{\text{wait}} \geq 0\) due to limited liability for households.\(^{28}\)

These households obtain utility

\[
U_{\text{wait}}(z, K, d, R, X, n) = u(C_{\text{home}} + C_{\text{wait}}(z, K, d, R, X, n)).
\]

(1.26)

with \(C_{\text{wait}}()\) given by (1.25) and home production given by (1.24).

**Equilibrium.** Based on the noise structure (1.18), the posterior distribution for productivity conditional on household private signal \(s_{i,t}\),

\[
z \mid s_{i} \sim \text{Unif}(s_{i} - \Delta, s_{i} + \Delta),
\]

(1.27)

is uniformly distributed between \(s_{i} \pm \Delta\).

After observing his signal, each depositor then chooses whether or not to run according to

\[
\max_{a_{i} \in \{\text{wait, run}\}} \mathbb{E}[U_{a_{i}}(z, K, d, n_{\text{max}}, S, X, n(a_{-i})) \mid s_{i}].
\]

(1.28)

Conditional on their private signals \(s_{i}\), households decide whether or not to run. Households obtain expected utility \(\mathbb{E}[U_{\text{run}}(\cdot) \mid s_{i}]\) by running or \(\mathbb{E}[U_{\text{wait}}(\cdot) \mid s_{i}]\) by waiting, with the utility from withdrawing (waiting) being given by equation (1.22) (equation (1.26)). Note that in formulation (1.28), each household understands the structure of the coordination problem. In particular, each household \(i\) realizes that equilibrium number of runs \(n = n(\{a_{-i}\})\) depend on the collective actions of other

\(^{28}\)This is only imposed for the determination of bank runs. In the full model of Section 2.3, the presence of the budget constraint means that the limited liability constraint will never bind in equilibrium.
households \( \{a_i\} \), each of whom is facing a similar problem.

I focus on a symmetric equilibrium. The equilibrium run decision is given by a threshold policy where an individual depositor runs if and only if his private signal is below some threshold \( s^*_\Delta (K, d, n_{\text{max}}, S, X) \). That is, in equilibrium the solution to (1.28) is given by

\[
a_i = \begin{cases} 
\text{run} & \text{if } s_i \leq s^*_\Delta (K, d, n_{\text{max}}, S, X) \\
\text{wait} & \text{if } s_i > s^*_\Delta (K, d, n_{\text{max}}, S, X)
\end{cases}
\] (1.29)

It remains to characterize the equilibrium threshold \( s^*_\Delta (K, d, n_{\text{max}}, S, X) \).

Under the conjectured threshold equilibrium (1.29) and the assumed noise structure for households' signals, (1.18), we have the following outcome for the run variable \( n \) given the realization of the productivity shock \( z \):

\[
n(z, s^*_\Delta) = \text{mass} \{ i \in [0, n_{\text{max}}] : s_i \leq s^*_\Delta \}
\] (1.30)

That is, the mass of depositors who end up running is given by the number of depositors with private signals below the run threshold \( s^*_\Delta \).

As in Goldstein and Pauzner (2005), the run threshold \( s^*_\Delta \) can then be characterized by observing that the hypothetical "threshold depositor" who has signal \( s_i = s^*_\Delta \) is indifferent between running and waiting:

**Proposition 1.** (Indifference condition for the run threshold). The run threshold \( s^*_\Delta \) is given by the solution to the following indifference condition:

\[
\int_0^{n_{\text{max}}} U_{\text{run}} (z(n, s^*_\Delta), K, d, S, X, n) \, dn = \int_0^{n_{\text{max}}} U_{\text{wait}} (z(n, s^*_\Delta), K, d, S, X, n) \, dn.
\] (1.31)

where \( z(n, s^*_\Delta) = s^*_\Delta + \Delta - \frac{2\Delta n}{n_{\text{max}}} \).
Proof. Consider the expectations within the maximization problem (1.28) for the “threshold depositor.” Given posterior distribution (1.27), and the conjectured number of runs under the threshold policy (1.30), the threshold depositor will have posterior belief \( n(z, s^*_\Delta) | s_i = s^*_\Delta \sim Unif(0, n_{max}) \). We can then obtain (1.31) through a change of variable given by \( z = z(n, s^*_\Delta) \).

The limiting economy without noise. Finally, I take \( \Delta \downarrow 0 \) and let the noise component vanish\(^{29}\) so that the limiting economy recovers the structure of original economy. The run threshold in the limiting economy is now in terms of the productivity \( z \). The certainty equivalent value of consumption, \( W_t \), appearing in the planner’s objective function (1.16) takes into consideration consumption in the cross-section of households after the equilibrium run outcome. This is summarized below:

**Proposition 2.** (Equilibrium in the limiting economy.) *In the limiting economy as noise goes to zero (i.e. \( \Delta \downarrow 0 \)), the run threshold is given in terms of productivity:*

\[
z^*(K, d, n_{max}, S, X) = \lim_{\Delta \downarrow 0} s^*_\Delta (K, d, n_{max}, S, X).
\]  

(1.32)

Furthermore, this run threshold is implicitly determined as the root to the following equation:

\[
\int_0^{n_{max}} U_{\text{run}}(z^*, K, d, S, X, n) \, dn = \int_0^{n_{max}} U_{\text{wait}}(z^*, K, d, S, X, n) \, dn,
\]

(1.33)

where \( U_{\text{run}}(\cdot) \) and \( U_{\text{wait}}(\cdot) \) are given, respectively, by (1.22) and (1.26).

**Households have symmetric information (they observe realized productivity \( z \)) and run if and only if productivity \( z \) is below the run threshold \( z^* \). The equilibrium number of

---

\(^{29}\)This is standard in the global games literature. Another reason for considering the limiting economy is more subtle. While threshold equilibria are well defined for every value of \( \Delta > 0 \), some of the parameters (e.g. expenditures \( X \)) will be determined by choice variables of the planner in the full model. In the limiting model where signals are fully revealing, there is no additional information content contained in choice variables over and above each household’s signal. This greatly simplifies the characterization of bank run problem (1.28). In particular, it avoids feedbacks of the sort studied in Bond and Goldstein (2014) where government policies contain information over and above agents’ private signals and act as a public coordination device. Considering policy feedbacks of this sort is beyond the scope of this paper.
runs is given by

\[ n_{eq} (z, K, d, n_{max}, S, X) = n_{max} 1_{\{z \leq z^* (K, d, n_{max}, S, X)\}}. \] (1.34)

The associated certainty equivalent household consumption value appearing in the planner’s objective function (1.16) is given by

\[ W (z, K, d, n_{max}, S, X) \equiv u^{-1} \left( n_{eq} U_{run} (z, K, d, S, X, n_{eq}) + (1 - n_{eq}) U_{wait} (z, K, d, S, X, n_{eq}) \right). \] (1.35)

where the equilibrium number of runs \( n_{eq} = n_{eq} (z, K, d, S, X) \) is given by (1.34), and \( u(\cdot) \) is the utility function (1.23).

Proof. The indifference condition (1.33) for the limiting economy can be seen by taking the limit \( \Delta \downarrow 0 \) in (1.31). The rest follow straight from their definitions. \( \square \)

The welfare function \( W(\cdot) \) defined in (1.35) serves as a key input for the dynamic stochastic general equilibrium model of Section 2.3. Note in the special case of CRRA preferences (i.e. \( \eta = \gamma = \psi^{-1} \)), social welfare (1.16) simplifies to

\[ V_t = E_t \left[ \sum_{s \geq t} \beta^{s-t} \left( n_{eq,s} \frac{C_{run,s}^{1-\gamma}}{1-\gamma} + (1 - n_{eq,s}) \frac{C_{wait,s}^{1-\gamma}}{1-\gamma} \right) \right]. \]

To build intuition for the full DSGE model, I first illustrate some of the key properties of the equilibrium run schedule \( n_{eq} \).

**Run thresholds.** I first focus on the endogenous nature of bank runs. In Figure 1.5, I plot the run threshold \( z^* \) (see (1.32)). The run thresholds display intuitive comparative statics.

First, we see that the run threshold decreases as the amount of international reserves on hand increases. A higher level of liquidity buffer prevents the liquidation of projects and ensures final output will be high. In this case, the payoff to waiting becomes higher and households’ incentives to run are decreased.
Figure 1.5: Run thresholds. This figure illustrates run thresholds $z^*$ as a function of international reserves on hand $R$ and end of period expenditures $X$. Panel A (B) plots the run thresholds for low (high) amounts of (total) deposits $D = d \times n_{\text{max}}$. The run threshold is characterized in (1.32) and is given by the solution to equation (1.33).

Second, run thresholds are increasing in (end of period) expenditures $X$. Since waiting households are residual claimants on the bank at the end of the period, higher expenditure levels only serve to decrease the consumption levels associated with waiting and so increase households’ run incentives. For example, if sovereign debt repayments are prohibitively high, then bank runs are likely unless the sovereign defaults.

Third, comparing panels A and B, we see that run thresholds are higher for higher deposit levels. Higher amounts of short term (domestic) debt create larger coordination problems within the domestic banking sector, and make the banking sector more prone to runs.

Fourth, we also observe substantial amount of non-linearity. When domestic leverage is low (panel A), the run threshold falls quickly when reserve levels increase and/or when expenditures decrease. In contrast, when domestic leverage is high (panel B), the run threshold falls more slowly. We see that when the domestic economy is more highly leveraged, more substantial amounts of liquidity on hand and/or larger cut backs in expenditures are necessary in order to decrease run incentives. Instead, when the domestic economy is not so highly leveraged, a small increase in liquidity and/or
a small cut back in expenditures will go a long way to stabilize the domestic banking sector.

**Equilibrium runs.** The corresponding equilibrium run schedule $n_{eq}$ is plotted in Figure 1.6. The equilibrium run schedule inherits the properties of run thresholds according to (1.34): runs are more likely to occur when (i) there is an insufficient amount of international reserves $S$, (ii) expenditures $X$ are too high, (iii) domestic leverage levels $D$ are high, as well as (iv) when productivity $z$ is low. Furthermore, the transition between a no run and a run equilibrium can be very abrupt. When the economy is slightly over the run threshold, a small amount of additional liquidity will be extremely valuable in preventing bank runs. Similarly, a small reduction in expenditures can also go a long way in preventing bank runs. These forces are important in shaping debt capacity in the full model.
2.3 The planner’s problem

Having characterized the equilibrium run behavior and obtained the welfare function \( W(\cdot) \) in Section 2.2, I now provide the remaining details to the full model which was first described in Section 2.1.

Timing. The timing is illustrated in Figure 1.7 which supplements Figure 1.4 with the timing of various choice variables available to the planner. The SOE enters each period with capital stock \( K_t \), international reserves \( S_t \), external debt \( B_t \), banking lending \( A_t \) and bank deposits \( D_t \). The productivity shock \( z_t \) is then realized. At this stage, the planner chooses whether or not to default. Bank runs and liquidations then occur according to (1.34) and (1.19), respectively.

If the planner chooses not to default, debt \( B_t \) is repayed and new debt \( B_{t+1} \) is issued at per-unit price \( Q_t \) which will be determined in equilibrium. In addition, the planner chooses investments \( I_t \) and next period’s reserves \( S_{t+1} \).

In the event of a default, the planner repudiates on outstanding debt \( B_t \) and loses access to international credit markets. Default is also accompanied by a drop in private credit which translates into a higher burden on the domestic banking sector under assumption (1.17). Whilst in default, the ability to borrow is unavailable and the planner can only choose reserve \( S_{t+1} \) and investment \( I_t \) policies. The planner can subsequently regain access to international credit markets in the following period with exogenous probability \( \xi > 0 \). All previous debt is forgiven in the event of a reentry.

Capital accumulates according to (1.4), while bank balance sheets evolves according to equations (1.6) through (1.9). Since I assume a constant capital structure (1.9), I will keep \( D_t \) as the state variable and discard \( A_t \) (only one of the two need to be kept).

Budget constraint. Total end of period expenditures are given by

\[
X_t = \frac{S_{t+1}}{1+r} + \phi \left( \frac{I_t}{K_t} \right) K_t + 1_{\{V_{N.D.t} \geq V_{D.t}\}} \left( B_t - B_{t+1}Q_t \right),
\]

(1.36)
Figure 1.7: Within period timing for the full model. The SOE enters period $t$ with capital $K_t$, international reserves $S_t$, outstanding debt $B_t$, and domestic bank assets $A_t$ and $D_t$, respectively. The productivity shock $z_t$ is then realized. The planner then chooses whether or not to default. Equilibrium runs $n_t$ and liquidations $l_t$ then occur according to the run model of Section 2.2. If default occurs, then there is a loss of private credit resulting in a greater burden for domestic intermediaries ($\chi_D > \chi_{ND}$), the planner then makes investment and reserve accumulation decisions. The SOE can regain access to international credit markets with probability $\xi$ next period. If default does not occur, then the planner repays outstanding debt $B_t$, issues new debt with face value $B_{t+1}$ at per unit price $Q_t$, and makes investment and reserve accumulation decisions.

and consist of reserve accumulation costs for next period, investment costs, and in the case when default does not take place, the net proceeds from debt rollover. The magnitude of the expenditures will in turn influence the equilibrium run outcome $n_t$ given by (1.34).

The planner’s choices are subject to the following budget constraint:

$$\max \{ S_t + \ell_t L(z_t)K_t - n_t d_t, 0 \} + (1 - \ell_t) e^{z_t}K_t - X_t \geq 0. \quad (1.37)$$

In the expression above, the amount of expenditures $X_t$ implied by a particular set of choices must be such that the limited liability constraint for households is never violated in equilibrium.\textsuperscript{30}

\textsuperscript{30}Note that the choice is non-empty: expenditures $X_t$ can always be set to zero by defaulting and then setting investment costs and reserve accumulation costs to zero. Constraint (1.37) will always hold for $X_t = 0$. 

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We can rewrite the budget constraint (1.37) in the more usual form as follows:

\[
\left[ \ell_t L(z_t) + (1 - \ell_t)e^{z_t} \right] K_t = \frac{n_t p_t^{\text{withdraw}} d_t}{C_t} (1 - n_t) C_{\text{wait},t} + \phi \left( \frac{I_t}{K_t} \right) K_t + \frac{S_{t+1}}{1+r} - S_t + 1_{\{V_{ND,t} \geq V_{D,t}\}} (B_t - B_{t+1} Q_t),
\]

where \( C_{\text{wait},t} \geq 0 \), where I have made use of equations (1.20), (1.21), (1.25) and (1.36) to arrive at the above expression. Note that this is just the usual budget constraint in an open economy setting adjusted for runs and liquidations.

**Planner’s problem in default.** The planner’s objective function whilst in default is given by

\[
V_D(z_t, K_t, D_t, S_t) = \max_{I_t, S_{t+1}} \left\{ (1 - \beta) W(z_t, K_t, D_t, S_t, X_{D,t})^{1-\psi^{-1}} + \beta C E_D(z_t, K_{t+1}, D_{t+1}, S_{t+1})^{1-\psi^{-1}} \right\}^{\frac{1}{1-\psi^{-1}}}
\]

where the choice variables are investment \( I_t \) and international reserves \( S_{t+1} \) for the next period, and \( W(\cdot) \) is given by (1.35). End of period expenditures during default are given by

\[
X_{D,t} = \phi \left( \frac{I_t}{K_t} \right) K_t + \frac{S_{t+1}}{1+r},
\]

which is the sum of investment and international reserve accumulation expenditures. The planner gets the autarky continuation value \( C E_D(z_t, K_{t+1}, D_{t+1}, S_{t+1}) \) in which no borrowing is allowed and external private credit decreases so that bank loans accumulate according to \( \chi = \chi_D \) (cf. equation (1.6)).
Planner's problem when not in default. If the country is not in default, then the planner solves

\[ V_{ND}(z_t, K_t, D_t, S_t, B_t) = \max_{I_t, S_{t+1}, B_{t+1}} \left\{ \left(1 - \beta\right) W(z_t, K_t, D_t, S_t, X_{ND,t})^{1-\psi^{-1}} + \beta CE_{ND}(z_{t+1}, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1})^{1-\psi^{-1}} \right\} ^{\frac{1}{1-\psi^{-1}}} \]  

(1.41)

where the choice variables are investment \( I_t \), international reserve policy \( S_{t+1} \), and debt policy \( B_{t+1} \). The net expenditure under the no default regime is given by

\[ X_{ND,t} = \phi \left( \frac{I_t}{K_t} \right) K_t + \frac{S_{t+1}}{1+r} + B_t - B_{t+1} Q_t. \]  

(1.42)

It consists of investment costs, the cost of accumulating international reserves for tomorrow, and the cost of repaying outstanding debt \( B_t \). Note that additional borrowing can be used to offset expenditures for the current period. Similarly, choices are also subject to the budget constraint (1.37).

The SOE gets the credit access continuation value \( CE_{ND}(z_t, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1}) \) under which access to international credit markets is maintained, and external private credit is available to help decrease the burden on domestic banks so that bank loan accumulation (1.6) happens with \( \chi = X_{ND} \).

Note that unlike the planner's problem in default, when international reserves \( S_t \) are too low and/or when debt levels \( B_t \) are very high, no amount of expenditure cutbacks can satisfy the budget constraint (1.37), so that defaulting is the only option.

Bond prices and continuation values. Finally I determine bond prices and continuation values. The planner will always choose the greater of the default, \( V_{D,t} \), and non-default values \( V_{ND,t} \). Default occurs on the set \( \{ V_{D,t} > V_{ND,t} \} \) when defaulting
is more attractive. The credit access continuation value is given by

\[ CE_{ND} (z_t, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1}) \]

\[ = E_{z_{t+1} | z_t} \left[ \max \{ V_{ND} (z_{t+1}, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1}), V_D (z_{t+1}, K_{t+1}, D_{t+1}, S_{t+1}) \} \right]^{1-\gamma} \]  

Whilst in default, the continuation value is given by

\[ CE_D (z_t, K_{t+1}, D_{t+1}, S_{t+1}) = \left( 1 - \xi \right) E_{z_{t+1} | z_t} \left[ V_D (z_{t+1}, K_{t+1}, D_{t+1}, S_{t+1}) \right]^{1-\gamma} + \xi CE_{ND} (z_t, K_{t+1}, D_{t+1}, S_{t+1}, 0)^{1-\gamma} \]  

where \( \xi \) is the probability of reentering credit markets. All outstanding debt is written off when the country reenters credit markets.

Under the small open economy setting, sovereign bonds are priced by international investors who are assumed to be risk neutral.\(^{31}\) Bond prices are given by

\[ Q (z_t, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1}) \]

\[ = \frac{1}{1 + r} E_{z_{t+1} | z_t} \left[ 1 \{ V_{ND} (z_{t+1}, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1}) \geq V_D (z_{t+1}, K_{t+1}, D_{t+1}, S_{t+1}) \} \right] \]

and reflect the probability of repayment next period.

**Equilibrium.** The equilibrium concept is the standard Markov equilibrium:

**Definition (Markov Equilibrium).** A Markov Equilibrium for the economy consists of (i) default and non-default value functions \( V_D \) and \( V_{ND} \), (ii) a bond price schedule \( Q \), and (iii) a run schedule \( n_{eq} \), such that

a. Given the bond price and run schedules \( Q \) and \( n_{eq} \), \( V_D \) and \( V_{ND} \) are respectively characterized by (1.39) and (1.41).

b. The bond prices are consistent with the planner’s default behavior so that the bond price schedule \( Q \) satisfies (1.45).

c. The equilibrium run schedule \( n_{eq} \) captures households’ run incentives and is given by the Nash equilibrium summarized in (1.34).

\(^{31}\)This can be easily relaxed by incorporating a pricing kernel for international lenders. See Borri and Verdelhan (2011) for an example of this.
Numerical implementation. The problem is homogenous of degree one in capital $K_t$. This allows me to scale out capital in the numerical implementation. In addition, the problem as formulated will not converge. This is because the equilibrium run schedule (1.34) is discontinuous at the run threshold. In turn, this makes $W(\cdot)$ and the Bellman equations discontinuous. To achieve numerical convergence, I instead use a smoothed version of the run threshold (1.34). The state space and choice sets are then discretized, and the model is then numerically computed using value function iteration methods. Details for these steps are given in Appendix 3.

2.4 Discussion

The view implicit in the model is that the process of economic growth naturally involves maturity transformation and liquidity risk. This gives rise to an additional liquidity constraint which is embedded in the equilibrium run schedule (1.34). With these liquidity concerns in mind, I now illustrate my model's implications for:

a. Costs of default and sovereign debt capacity

b. Equilibrium levels of international reserves

c. Twin crises dynamics

Sovereign debt capacity. In the model, sovereign defaults place a higher burden on the domestic banking sector when it comes to financing growth, as the domestic banking sector will have to do more amounts of intermediation (see (1.17)). Therefore, maintaining the same investment level after a sovereign default leads to increased bank domestic lending and leverage $D$ (see (1.6)). The properties of the equilibrium run schedule (i.e. $\partial n_{eq}/\partial D \geq 0$; see Section 2.2 for details) implies that, all else equal, higher banking leverage leads to increased chances of bank runs and liquidation losses. Of course, the planner can try to reduce banking fragility by hoarding more international reserves after defaulting (i.e. $\partial n_{eq}/\partial S \leq 0$). However, this is costly as the
planner will have to forgo valuable growth opportunities which will lead to lower future output and consumption. Hence, the full costs of default in my model is a mix of higher domestic banking fragility and lost growth opportunities.

Finally, any discussion of sovereign debt capacity would be incomplete without a discussion of international reserves. In my model, the liquidity constraint is very useful for hindering the replication arguments in Bulow and Rogoff (1989). Mathematically, there is a final point: what if the country could hoard so much reserves so that liquidity concerns no longer becomes a concern? If this is true, then

Equilibrium international reserves.

Twin crises.

3 Quantitative Analysis

3.1 Calibration

The calibration is at a quarterly frequency, and is loosely based on Argentina for the period 1993Q1 to 2001Q4. Much focus has been placed on Argentina by the sovereign default literature (e.g. see Arellano (2008) and Chatterjee and Eyigungor (2012)), and the period 1993-2001 is often chosen because Argentina had a fixed exchange rate vis-a-vis the dollar during this period. The parameters used in the baseline calibration are summarized in Table 1.1.

Productivity. Parameters for the TFP process (1.1) are estimated after linearly detrending and deseasonalizing real Argentine GDP.\textsuperscript{32} The quarterly GDP series is

\footnote{\textsuperscript{32}This is the same procedure used in Chatterjee and Eyigungor (2012).}
Preferences. I set the elasticity of inter-temporal substitution (EIS) parameter \( \psi \) to be 0.5. This is a standard value used in the literature. Both the planner’s and the depositors’ risk aversion parameters, \( \gamma \) and \( \eta \) respectively, are set to be 5. The subjective discount rate \( \beta \) is chosen to be 0.99. Note that the sovereign default literature usually calibrates this number to be much lower in order to increase the frequency of default.\(^{33}\) The literature can afford to do so because they usually assume exogenous output costs to generate debt capacity.\(^{34}\) This is a luxury which my model does not enjoy.

Liquidation. I set liquidation proceeds to be constant \( L(z) = 0.5 \). Since the productivity process (1.1) fluctuates around zero, on average full harvest will yield \( e^z \approx 1 \) units of fruit. This means that on average 50\% of potential output is lost for each unit of fruit harvested early. Note, however, that the liquidation loss at the aggregate level is instead given by \( \ell_t (e^{z_t} - L(z_t)) \) which will be much lower on average since not all fruit will be harvested early each period. In addition, fixing liquidation proceeds to be constant also implies that liquidation is more costly during good times

\(^{33}\) For example, the subjective discount factor is 0.95 in Arellano (2008) and 0.88 in Mendoza and Yue (2012). A more impatient government will care less about the consequences of default and therefore default more often.

\(^{34}\) Mendoza and Yue (2012) is an exception. However, they can still set the subjective discount factor to be quite low as they do not consider reserve accumulation after defaulting.

<table>
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<th>Preferences, (log) TFP, and credit markets</th>
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<td>( \eta ) deposits’ risk aversion</td>
<td>( \phi(i) ) total investment cost, ( i \geq 0 )</td>
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<tr>
<td>( \gamma ) planner’s risk aversion</td>
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<td>( \psi ) EIS</td>
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<td>( \beta ) discount rate</td>
<td>( \chi_{ND} ) bank financing share, not default</td>
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<tr>
<td>( \rho_z ) log TFP autocorrelation</td>
<td>( \chi_{ND} ) banking financing share, default</td>
</tr>
<tr>
<td>( \sigma_z ) log TFP volatility</td>
<td>( m_L ) deposits to loans</td>
</tr>
<tr>
<td>( \tau ) interest rate</td>
<td>( a_0 ) promised deposit payout to capital</td>
</tr>
<tr>
<td>( \xi ) reentry probability</td>
<td>( L(z) ) liquidation proceeds</td>
</tr>
<tr>
<td></td>
<td>( a_0 ) other bank lending to capital</td>
</tr>
</tbody>
</table>

Table 1.1: Baseline parameters.

taken from Neumeyer and Perri (2005). The resulting parameters are \( \rho_z = 0.93 \) and \( \sigma_z = 0.027 \).
when productivity is high.

**Growth.** I set the capital depreciation rate to be $\delta = 0.037$. The total cost of investment takes the form $\phi \left( \frac{L}{K_t} \right) = \phi_0 \left( \frac{L}{K_t} \right)^{\phi_1}$, a common specification in the literature (e.g. Jermann (1998)). I set $\phi_0 = 283000$ and $\phi_1 = 4.5$, which are chosen, in conjunction with the depreciation rate, so that both growth rates as well as investment-to-output ratios are reasonable. Since the production function is $AK$ and lack curvature, larger amounts of curvature in the investment cost function $\phi(\cdot)$ are required in order to achieve these objectives.

**External financing dependence on domestic banks.** The parameters $\chi_D$ and $\chi_{ND}$ are meant to capture domestic firms’ external financing dependence on the domestic banking sector. In general, this is very difficult to measure. First, such data is typically not available, and even if it were, the observed data will reflect equilibrium forces of demand and supply for external financing. In an influential study, Rajan and Zingales (1998) measure external financing dependence for US manufacturing firms as the fraction of investments in excess of a firm’s free cash flows, and find that external financing dependence for investments is, on average, 32%. They then use their US measures of external financing dependence to proxy for firms around the world. I follow their approach in the baseline calibration. However, for my purposes, I still need to know the split between domestic and foreign financing. Gozzi et al. (2013) find that for developing countries, 45% of the proceeds of bond issues are raised abroad on average. Based on these findings, I set the domestic financing dependence to be $\chi_{ND} = 0.32 \times 0.55 = 0.176$ while the country is not in default.

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35Only investments in tangible assets are included in Rajan and Zingales (1998)’s study. In our context where we think of additional forms of intangible investments, there is good reason to believe that external financing dependence will be higher than that assumed in the baseline calibration. In addition, there are substantial amounts of variation in the Rajan and Zingales (1998) measure of external dependence across industries and firm size and age.

36They argue that in a perfect capital market, the actual amount of external funds raised by a firm would equal its desired amount. Since US capital markets are amongst the most advanced in the world, their US measures of external financing dependence can then serve as a good proxy for demand for external financing around the world.
Empirical studies by Arteta and Hale (2008) and Das et al. (2010) show that external private credit decreases in the event of a sovereign default. I assume a drop of 40% based on findings in Das et al. (2010). Based on these findings, I set domestic financing dependence to be $X_D = X_{ND} + 0.32 \times 0.45 \times 0.4 = 0.2336$ while the country is in default. The difference between $X_D$ and $X_{ND}$ reflects that portion of investments no longer financed with external private credit; in the event of foreign private credit outflows, the domestic intermediary will have to step in and provide credit.

I set $m_A = 0.1225$ so that on average 12.25% (49%) of bank loans come due every quarter (year). This is based on average long term debt shares for Argentinean firms considered in Kirch and Terra (2012). This corresponds to bank loans having an average maturity of approximately 8 quarters or 2 years.

**Bank capital structure.** I choose $m_L = 0.71$ based on deposit-to-asset ratios for Argentine banks during 1993 to 2001. The data for this is available from the World Bank’s World Development Indicators (WDI) database. Recall that total bank lending $A_t$ consists of investment loans $A_t^{\text{loan}}$ as well as other forms of credit $A_t^{\text{other}} = a_0 K_t$. I set $a_0 = 0.6$, so that total bank assets to (quarterly) GDP will look reasonable. Based on WDI data for Argentina, bank assets to (quarterly) GDP is 112% during 1993-2001, and 81% for the full sample 1961-2011.

For promised deposit payouts, I set $d_0 = 0.7$. To get a feel for this number, note that in a closed economy, consumption to capital for late withdrawing households will be approximately $c_{\text{wait}} \approx 1 - \phi \left( \frac{L}{K_t} \right)$ on average. The ratio $c_{\text{wait}}/d_0$ can then be thought

---

37 This amount is the portion attributable to a sovereign default after controlling for a whole set of other variables. Such variables include international competitiveness, investment climate and monetary stability, financial development, macroeconomic fundamentals, political stability, as well as global capital supply. The decrease in private credit found in Arteta and Hale (2008) and is between 20-30%. A reason for this difference is that Das et al. (2010) only focuses episodes of sovereign default to private creditors, which is in line with definitions of sovereign default in other empirical studies (e.g. Reinhart et al. (2003), Tomz and Wright (2007), and Panizza et al. (2009)). In contrast, Arteta and Hale (2008) uses an expanded definition of defaulting so that their sample contains “smaller” defaults on average. It is interesting to note that Arteta and Hale (2008) find similar magnitudes for worst case scenario defaults in their sample.

38 Note that this “back of the envelope” adjust only reflects changes in quantities. Given that the private costs of borrowing from external sources also increase after a sovereign default (Agca and Celasun (2012)), $X_D$ is likely to be higher in a price-adjusted calculation.
of as the spread between long term and short term deposits.

**International capital markets.** The quarterly international interest rate is set to be $r = 0.01$ and is based on US short rates. I follow Chatterjee and Eyigungor (2012) and set the probability of reentry after defaulting to be $\xi = 0.0385$. This is based on exclusion periods after Argentine default, and implies an average exclusion period of 26 quarters or 6.5 years.

### 3.2 Results

**Baseline model moments.** Moments from the baseline calibration are shown in Table 1.2. My model is able to generate (external sovereign) debt to GDP of 20%. While this amount falls short of its empirical counterpart, which I take to be 70%, it is still comparable to debt levels generated in previous studies involving single period debt.\(^{40,41}\)

International reserves to (quarterly) GDP averages 76%. Based on the World Bank’s WDI dataset, the mean international reserve to (quarterly) GDP ratio for Argentina over the period 1993 to 2001 is 25%, which is much lower than the model’s counterpart. Figure 1.8 plots average external sovereign debt to GDP and international reserve to GDP ratios for middle income countries. Since the late 80s, there have been a steady decline in debt levels alongside an increase in reserve levels, with the trend in reserves

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\(^{39}\)Actual debt to (quarterly) GDP ratios for Argentina over the period 1993-2001 is 100%. However, because my model does not feature recovery upon default, and since the eventual recovery on debt defaulted in 2001 was around 30%, I follow Chatterjee and Eyigungor (2012) in treating 70 cents out of each dollar as the truly unsecured portion of debt.

\(^{40}\)In an endowment economy setting with exogenous costs of default, Arellano (2008) generates debt to GDP of 6% in a stationary setting, while the non-stationary models investigated in Aguiar and Gopinath (2006) generate debt to GDP ranging between 18% and 27%. Mendoza and Yue (2012) features trade-related endogenous costs of default and generates debt to GDP of 23%. Gornemann (2013) extends the stationary setting of Mendoza and Yue (2012) by further incorporating endogenous growth, and generates debt to GDP of 12%. The studies of Mendoza and Yue (2012) and Gornemann (2013) do not consider reserve accumulation. To the best of my knowledge, there are no other papers featuring both endogenous costs of default and international reserve accumulation with which to compare results.

\(^{41}\)Chatterjee and Eyigungor (2012) was successful in fully matching empirical levels of debt to GDP in an endowment economy setting with long term debt and calibrated exogenous output costs after defaulting.
increasing in a much more pronounced fashion since 2000. Middle income countries have steadily evolved from a high debt-low reserves regime to a high reserves-low debt regime. My model seems to be a better fit for the more recent period. One possibility for this is that liquidity concerns, the central ingredient in my setting, played a more central role more recently after the onset of various liquidity crises (e.g. the 1998 LTCM crises).

The model generates a negligible sovereign spread of 5 basis points when (quarterly) sovereign spreads are instead around 200 basis points in the data. While this is a common pitfall in sovereign default models with single period debt, it is especially pronounced in my baseline calibration. This is due to my stringent requirement of endogenously generating debt capacity in a setting where reserve accumulation is possible. A common strategy in the sovereign debt literature for generating sovereign spreads is to lower the subjective discount factor (a more impatient country will care less about the consequences of defaulting, and hence borrow more and also default more often), and at the same time increase default costs so as to increase debt capacity. Sovereign default models with exogenously specified default costs have a lot of flexibility when it comes the latter, which is a luxury that I cannot afford in my setting. In addition, the requirement that default costs be robust to (post-default) international reserve accumulation is especially stringent.\footnote{For example, in Mendoza and Yue (2012), lost trade credit after a sovereign defaulting directly} Given these constraints,
Figure 1.8: External public debt and international reserves, middle income countries. This figure plots the time-series of external public and publicly guaranteed debt to (quarterly) GDP, and international reserves (inclusive of gold) to (quarterly) GDP ratios for middle income countries for the period 1970-2012. For each period, the averages are GDP-weighted. Middle income countries include both lower middle income and upper middle income country groups as per the World Bank’s definitions. The data is taken from the World Bank’s WDI database.

my framework will first have to generate additional debt capacity before it can generate more realistic levels of sovereign credit spreads.

Average growth rates in the baseline model is 1% per quarter, which is a bit higher than Argentina’s quarterly real growth rate of 0.7%. This growth rate is accompanied by a mean investment-to-GDP ratio of 29% in the model. Average investment, in tangible capital, to GDP is 20% in the data. The difference between our model’s investment rate and the data is attributable to the accumulation of intangible capital which can be difficult to measure. Studies indicate that investments in intangible assets can be substantial. For example, Eisfeldt and Papanikolaou (2013) find that for US firms, the size of the organizational capital stock (i.e. human capital within the firm) is on par with physical capital stock, and that the investment rate in organizational capital translates into losses in trade and output. This is because reserve accumulation is assumed away in their setting so that countries cannot self-finance trade credit when in default. More broadly, this involves tackling the Bulow and Rogoff (1989) puzzle.

43I take a longer sample, 1960-2013, for computing average growth rates and investment rates. The data for this is annual and is from the World Bank’s WDI dataset.
is 11%. This number is lower in less developed countries but can still be substantial. In a study of Brazil, Dutz et al. (2012) find that investment in organizational capital is around 4% of GDP. Furthermore, there are still other forms of investments, such as research and development, and education, which will ultimately affect the capital stock of a country. Finally, the planner will also care about positive spillover effects associated with various types of investments and include these in his accounting of the total capital stock. These considerations imply that total investments (in both tangible and intangible forms of capital) can be substantially higher than 20% of GDP.

The size of the banking sector is, on average, 99% of (quarterly) GDP in terms of bank assets, while deposits to (quarterly) GDP is 70% within the model. These values are below their counterparts in the data of 112% and 81% respectively. While it is possible to further increase debt capacity by increasing the size of the domestic banking sector (e.g. I can increase the $a_0$ parameter), I have chosen not to do so in the baseline calibration as this will lead to too many withdrawals on average. As it stands, we see that withdrawals $n_t$ average 60% in the model. Given $d_0 = 0.7$, an average deposit to GDP of 70% implies that on average we have $n_{max,t} \approx 1$. In other words, on average, all households end up being depositors in the baseline model and a substantial fraction of them (60%) choose to take out their deposits and consume the promised amount $d_t$. Given my timing assumption, investment choice made at the end of the period is very flexible and can be readily adjusted in order to avoid runs. The planner is choosing investments so that there is not too much difference in consumption levels between waiting or running. Note, however, that liquidations are effectively zero on average within the model. This is due to the planner’s high choice of international reserves.

**Output, investment, and international reserves around default.** I now use the baseline model to conduct an event study of key variables around default. Recall the model’s key predictions: the increased burden on the domestic banking sector as

44These are average numbers for Argentina between 1993-2001. There is a lot of variation both in the time series and in the cross-section.
Figure 1.9: Output, investment, and international reserves around default: model. This figure plots (log) output, investment to output, and reserve to output around a default event. Default occurs at time 0. All responses are relative to their respective values right before defaulting. The solid line plots the average response, while the dashed (dotted) lines plots the one (two) standard deviation bounds. This is the model’s counterpart to the empirical responses plotted in Figure 1.2.

A result of decreased foreign private lending after a sovereign default means that the planner will have to accumulate more international reserves after defaulting. As a consequence, this leads to lower investment and lost growth opportunities.

The results for this exercise are shown in Figure 1.9. Qualitatively, the results are consistent with the data (see Figure 1.2)—after defaulting, reserve to output ratios increase, investment to output ratios decrease, and output falls. The differences in the responses are attributable to my model’s simplifying assumptions. Output losses within the model are transitory due to my assumption that liquidations do not destroy capital, whereas output losses contain a permanent component in the data. Also, changes in investment and reserve policies in the model do not feature any transition dynamics, in contrast to the data. This lack of transition dynamics is due to the AK growth setting.45

45This is because of the lack of curvature in a linear production function. See Acemoglu (2009) for details. Transition dynamics can be introduced by incorporating curvature into the production function (e.g. by using a Cobb-Douglas production function). However, this comes at the expense of additional computational complexity.
Figure 1.10: Productivity and domestic leverage around default: model. This figure plots productivity and domestic deposit to GDP around a sovereign default. Default occurs at time 0. All responses are relative to their respective values right before defaulting. The solid line plots the average response, while the dashed (dotted) lines plots the one (two) standard deviation bounds.

**Counter-cyclical sovereign default.** Even though sovereign credit spreads are low in the baseline model, default nevertheless still occur (but rarely). Panel A of Figure 1.10 show productivity $z_t$ around a sovereign default. We see that sovereign default is counter-cyclical and occurs when productivity is low. Default in the baseline model occurs after a sudden large drop in productivity—the average drop in productivity which triggers a default is 0.1 which is a 3.7 standard deviation shock. This is why average sovereign spreads are negligible in the model.

Panel B shows that within the model, domestic leverage increases after a sovereign default as the domestic banking sector raises additional domestic deposits in order finance investments (recall that default triggers a flight in foreign private credit in the model).\textsuperscript{46}

To summarize, the model is able to capture important qualitative aspects of the data. These include banking related costs of default and sovereign debt capacity in

\textsuperscript{46}My model is missing domestic capital flight. In the data, deposits decrease after a sovereign default. Often, these deposits go overseas, perhaps out of fear of hyperinflation (a feature not present within my setting).
References


the presence of international reserve accumulation, as well as the dynamics of key macroeconomic quantities (including output, investments, and international reserves) around a sovereign default.

4 Conclusion

I have introduced growth and domestic banking fragility into the canonical Eaton and Gersovitz (1981) sovereign default model. This allows us to tackle some puzzles in international economics. First, I obtain a theory of sovereign debt capacity that can withstand international reserve accumulation. Second, my model generates twin crises in a dynamic setting. Third, international reserves have “war-chest” like property within the model. Thus my model can account for the high levels of international reserves observed in the data. Finally, my model is able to generate reasonable levels of sovereign debt and international reserves in equilibrium.


Appendix

1 Debt and international reserves: a brief review of the challenges

In this section, I begin with a review of the Eaton and Gersovitz (1981) model of sovereign default. This will then allow me to provide a more formal and self contained discussion of some of the challenges associated with generating sovereign debt and international reserves in equilibrium.

The Eaton and Gersovitz (1981) model. The Eaton and Gersovitz (1981) model has become the canonical framework for thinking about strategic sovereign default. The model consists of a small open economy with an endowment stream (output) of $y_t$ which follows a Markov chain. A benevolent government tries to maximize household welfare given by $V_t = E_t \left[ \sum_{s \geq 1} \beta^{s-t} u(c_s) \right]$. To achieve this, the government tries to smooth household assumption by borrowing from international lenders who, for simplicity, are taken to be risk neutral.

The government borrows by issuing single period sovereign debt $b_t$ according to a bond price schedule $q_{EG}$ which will be determined in equilibrium. Household consumption when the government is not in default is given by

$$c_{ND,t} = y_t - b_t + b_{t+1}q_t,$$

where $b_t$ denotes the repayment of debt due in period $t$ and $b_{t+1}q_t$ is the net proceeds this period from issuing debt, with face value $b_{t+1}$, which will become due next period. When not in default, welfare can be expressed recursively as

$$V^{EG}_{ND}(y,b) = \max_{y' \geq 0} u(y - b + b'q^{EG}(y,b')) + \beta E_{y'|y} \left[ \max \left\{ V^{EG}_{ND}(y',b'), V^{EG}_D(y') \right\} \right].$$

Here the continuation value takes into account the government's option to default next period.

A sovereign default involves two types of costs. First, the government gets excluded from international credit markets and goes into financial autarky for a random period of time. Each period whilst in autarky, the government can regain credit access with probability $\xi$ so that the mean exclusion time is $1/\xi$ periods. For simplicity, it is assumed that all outstanding debt is forgiven upon the government reentering credit markets. In addition, the second costs assumes exogenous output costs of default $\phi(y) \geq 0$. These output costs of default are essential for generating sovereign debt capacity. Taken together, this means that consumption in default is given by

70
\( c_{D,t} = y - \phi(y) \). Therefore, welfare in default is given by

\[
V_D^{EG}(y) = u(y - \phi(y)) + \beta(1 - \xi)\mathbb{E}_{y'|y} \left[ V_D^{EG}(y') \right] \\
+ \beta \xi \mathbb{E}_{y'|y} \left[ \max \{ V_D^{EG}(y', 0), V_D^{EG}(y') \} \right].
\] (1.A.3)

Note that the continuation value here takes into account the possibility of regaining credit access.

Finally, due to a lack of an ability to commit, the government defaults in a strategic fashion. In particular, the government will default whenever \( V_D(y) > V_D(y, b) \). International lenders anticipate this and price this in. In equilibrium, bond prices are given

\[
q^{EG}(y, b) = \frac{1}{1 + r} \mathbb{E}_{y'|y} \left[ 1 \{ V_N^{EG}(y', y') \geq V_D^{EG}(y') \} \right].
\] (1.A.4)

**Challenge I: the Bulow and Rogoff (1989) puzzle.** A challenge comes about when one tries to microfound sovereign debt capacity by endogenizing the output costs of default \( \phi(y) \). This can be especially challenging if the government is also allowed to save by holding international reserves. This concern was raised by Bulow and Rogoff (1989). I will demonstrate their argument in the context of the Eaton and Gersovitz (1981) setting.\(^{47}\) For simplicity, I will suppose that endowment shocks are iid so that the bond price schedule \( q \) will be independent of \( y \). The object of interest is the debt limit

\[
b_{\text{max}} \equiv \max \{ b' : q(b') > 0 \}
\] (1.A.5)

beyond which no further borrowing is feasible. Note that the government will never borrowing more that the debt limit as that would mean it would obtain a zero price. That is, we must have

\[
b_t \leq b_{\text{max}}
\] (1.A.6)

for any rational government.

Bulow and Rogoff (1989)'s idea is as follows. Suppose that there are no exogenous output costs (i.e. \( \phi(y) = 0 \)) so that the only cost of default is financial exclusion. Furthermore, the government is allowed to save in the form of holding non-state contingent international reserves \( s \) which bears interest at rate \( r \). In this case, autarky consumption is given by

\[
c_{D,t} = y_t + s_t - \frac{s_t + 1}{1 + r}.
\] (1.A.7)

Then, should the debt limit ever be reached, say at time \( t = 0 \) so that \( b_0 = b_{\text{max}} \), then the government can default and initiate a sequence of savings given by

\[
s_t = (1 + r)^t b_0 - b_t \geq 0.
\] (1.A.8)

Note that condition (1.A.6) implies the savings sequence is well defined (i.e. \( s_t \geq 0 \)).

\(^{47}\) I am indebted to Juan Passadore for showing me these arguments, which are adapted from Auclert and Rognlie (2014).
Under the savings sequence (1.A.8), we see that defaulting would lead to a higher consumption stream:

\[ c_{ND,t} = y_t - b_t + b_{t+1} g_t \]
\[ \leq y_t - b_t + b_{t+1} \frac{1}{1+r} = c_{D,t}, \]

where the inequality above follows from the fact that the best possible bond price \( q \) corresponds to risk free debt. Furthermore, consumption from defaulting will be strictly higher whenever there are any sovereign default premia involved (i.e. whenever \( q < \frac{1}{1+r} \)). This is the essence of the replication argument shown by Bulow and Rogoff (1989). This argument implies that we must have zero debt capacity in equilibrium. That is \( b_{max} = 0 \).

Therefore, any mechanism which tries to endogenize the costs of default in the presence of international reserves will also have to deal with this replication argument.

### Challenge II: equilibrium international reserves.

An additional challenge for generating both debt and reserves in equilibrium is that a mechanism which generates sovereign debt capacity need not also generate meaningful levels of international reserves in equilibrium. To see this, positive levels of output costs of default \( \phi(y) > 0 \), and allow for reserve accumulation so that consumption is modified as follows:

\[ c_{ND,t} = y_t - b_t + b_{t+1} q_t + s_t - \frac{s_{t+1}}{1+r} \tag{1.A.9} \]
\[ c_{D,t} = y_t - \phi(y_t) + s_t - \frac{s_{t+1}}{1+r}. \tag{1.A.10} \]

This is the setting considered in Alfaro and Kanczuk (2009). The value functions and bond pricing schedules are modified accordingly:

\[ V_{ND}^A(y, b, s) = \max_{b', s' \geq 0} \left( y - b + b' q_{AK}(y, b', s') + s - \frac{s'}{1+r} \right) + \beta \mathbb{E}_{y' | y} \left[ \max \left\{ V_{ND}^A(y', b', s'), V_{D}^A(y', s') \right\} \right] \tag{1.A.11} \]
\[ V_{D}^A(y, s) = \max_{s' \geq 0} \left( y + s - \frac{s'}{1+r} \right) + \beta (1- \xi) \mathbb{E}_{y' | y} \left[ V_{D}^A(y', s') \right] + \beta \xi \mathbb{E}_{y' | y} \left[ \max \left\{ V_{ND}^A(y', 0, s'), V_{D}^A(y', s') \right\} \right] \tag{1.A.12} \]
\[ q_{AK}(y, b', s') = \frac{1}{1+r} \mathbb{E}_{y' | y} \left[ \frac{1}{1 \{ V_{ND}^A(y', b', s') \geq V_{D}^A(y', s') \}} \right]. \tag{1.A.13} \]

Quantitatively, international reserves are low in this setting. The reason for this is that countries intentionally hold low international reserves for strategic purposes. The

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\(^{48}\) Alfaro and Kanczuk (2009) find that international reserves are often zero in equilibrium. It is possible to generate non-zero levels of international reserves in equilibrium in this setting. For example, by considering more flexible specifications for the output costs of default (Salomao (2013)) or debt rollover risk (Bianchi et al. (2014)). However, the underlying conflict between the precautionary
intuition is as follows: holding international reserves allows countries to better self
insure and so going into autarky becomes less painful. As a result, strategic sovereign
default incentives would increase and result in higher costs of borrowing ex-ante. The
government intentionally chooses low reserves to save on borrowing costs. Formally,
we see this as follows. Consumption smoothing motives are more pressing whilst in
autarky, this is because of costs of default which both lowers autarky consumption
(output costs of default) and make it more volatile (the inability to borrow). As a
result, a marginal value of an additional value of international reserves much higher in
autarky. That is, $\partial V^A_K/\partial s$ is likely to be higher than $\partial V_N^K/\partial s$. From the definition
of the bond price schedule (1.A.13), this would mean that there’s a tendency for
additional international reserves to decrease bond prices ($\partial q^A_K/\partial s \leq 0$) and as a
result optimal reserve policy involves holding very low reserve levels whilst not in
default.

**Summary.** Thus the challenge of microfounding the coexistence of sovereign debt
and international reserves involves coming up with an economic mechanism that (1)
generates costs of default in the presence of international reserves, which involves
addressing the Bulow and Rogoff (1989) puzzle, and (2) prevents the government from
strategically choosing low levels of international reserves in equilibrium.

## 2 Details for the Event Study

I investigate how output, investment, and international reserves respond around a
sovereign default. For this, I conduct an event study of the following form:

$$Y_{i,t+k} - Y_{i,t-1} = \alpha_i^k + \gamma_i^k + \beta^k D_{i,t} + e_{i,t}, \quad (1.A.14)$$

where $Y$ is a response variable of interest, the $\alpha$'s and $\gamma$'s are, respectively, country
and time fixed effects, and the $D_{i,t}$'s are indicators for sovereign defaults which take
on a value of one if and only if country $i$ defaults in year $t$. The coefficients of interest
are the $\beta_k$'s which are the average response of the variable of interest, relative to its
value just a year before default, $k$ years since the time of default.

The response variables of interest are motivated by my model and include output in
logs, investment-to-output ratios, as well as international reserves-to-output ratios.
In order to conduct this study, I make use of macroeconomic time series data from
the World Bank’s World Development Indicators (WDI) database.\(^{49}\) In addition, I

\(^{49}\)For output, I use the GNI (current US$) series NY.GNP.MKTP.CD; for international reserves I
use the total reserves (includes gold, current US$) series FI.RES.TOTL.CD; finally, for investments I
use the gross capital formation (current US$) series NE.GDI.TOTL.CD.
Table 1.A.1: Output, investment and international reserves around default.
This table gives estimates for the coefficient $\beta^k$ in the regression (1.A.14). This is done for the following response variables: (log) output, investment to output, and international reserves to output. The table reports estimates for $k$ ranging from -10 to 10 years since a sovereign default. Standard errors are clustered by country and time, and are shown in brackets. The corresponding plot is shown in Figure 1.2.

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<th>res gni</th>
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make use of the crises database of Laeven and Valencia (2008, 2012) to construct the default indicators $D_{it}$. The resulting merged sample is for 155 countries and runs from 1970 to 2011.

The results are tabulated in Table 1.A.1 as well as plotted in Figure 1.2. Following a sovereign default, we see that output drops,\(^{50}\) investment-to-output ratios decrease, while international reserve-to-output ratios increase. These results are consistent with the mechanism of my model.

\(^{50}\)Gornemann (2013) finds that the output drop remains statistically and economically significant when additional controls are included.
3 Numerical Algorithm

3.1 Scaled System

The AK setting allows for numerical tractability as capital $K_t$ can be scaled out to reduce the dimension of the dynamic programming problem. All scaled variables will be denoted with a tilde so that $\tilde{x}_t$ denotes $x_t/K_t$. I first summarize the scaled system of equations.

Per period welfare after scaling is given by

$$\tilde{W}(z_t, \tilde{D}_t, \tilde{S}_t, \tilde{X}_t) = W(z_t, K = 1, \tilde{D}_t, \tilde{S}_t, \tilde{X}_t)$$  (1.15)

with $W(\cdot)$ being given in (1.35).

The scaled value function in default is given by

$$\tilde{V}_D(z_t, \tilde{D}_t, \tilde{S}_t) = \max_{\tilde{I}_t, \tilde{S}_{t+1}} \left\{ (1 - \beta) \tilde{W}(z_t, \tilde{D}_t, \tilde{S}_t, \tilde{X}_{D,t})^{1-\psi^{-1}} + \beta \left[ (1 - \delta + \tilde{I}_t) CE_D(z_t, \tilde{D}_{t+1}, \tilde{S}_{t+1}) \right]^{1-\psi^{-1}} \right\}$$  (1.16)

$$\tilde{X}_{D,t} = \frac{(1 - \delta + \tilde{I}_t) \tilde{S}_{t+1}}{1 + r} + \phi(\tilde{I}_t)$$

$$\tilde{D}_{t+1} = mL \left[ a_0 + \frac{(1 - m_A) \left( \frac{1}{m_L} \tilde{D}_t - a_0 \right) + \chi_{D} \phi(\tilde{I}_t)}{1 - \delta + \tilde{I}_t} \right]$$

where the law of motion for $\tilde{D}_t$ follows from (1.6), (1.7), (1.8), and (1.9).

The scaled value function when not in default is given by

$$\tilde{V}_{ND}(z_t, \tilde{D}_t, \tilde{S}_t, \tilde{B}_t) = \max_{\tilde{I}_t, \tilde{S}_{t+1}, \tilde{B}_{t+1}} \left\{ (1 - \beta) \tilde{W}(z_t, \tilde{D}_t, \tilde{S}_t, \tilde{X}_{ND,t})^{1-\psi^{-1}} + \beta \left[ (1 - \delta + \tilde{I}_t) CE_{ND}(z_t, \tilde{D}_{t+1}, \tilde{S}_{t+1}, \tilde{B}_{t+1}) \right]^{1-\psi^{-1}} \right\}$$  (1.17)

$$\tilde{X}_{ND,t} = \frac{(1 - \delta + \tilde{I}_t) \tilde{S}_{t+1}}{1 + r} + \phi(\tilde{I}_t) + \tilde{B}_t - (1 - \delta + \tilde{I}_t) \tilde{B}_t Q(z_t, \tilde{D}_{t+1}, \tilde{S}_{t+1}, \tilde{B}_{t+1})$$

$$\tilde{D}_{t+1} = mL \left[ a_0 + \frac{(1 - m_A) \left( \frac{1}{m_L} \tilde{D}_t - a_0 \right) + \chi_{ND} \phi(\tilde{I}_t)}{1 - \delta + \tilde{I}_t} \right].$$

Note that only scaled state variables now appear in the bond price schedule $Q(\cdot)$. 

75
Scaled continuation values are given by

$$\tilde{C}_E_{ND} (z_t, \bar{D}_{t+1}, \bar{S}_{t+1}, \bar{B}_{t+1}) = \mathbb{E} \left[ \max \left\{ \tilde{V}_D (z_{t+1}, \bar{D}_{t+1}, \bar{S}_{t+1}), \tilde{V}_N (z_{t+1}, \bar{D}_{t+1}, \bar{S}_{t+1}, \bar{B}_{t+1}) \right\} \Bigg| z_t \right]^{1-\gamma}$$

(1.A.18)

$$\tilde{C}_E_D (z_t, \bar{D}_{t+1}, \bar{S}_{t+1}) = \left( (1 - \xi) \mathbb{E} \left[ \tilde{V}_D (z_{t+1}, \bar{D}_{t+1}, \bar{S}_{t+1})^{1-\gamma} \bigg| z_t \right] \right)^{\frac{1}{1-\gamma}}$$

(1.A.19)

Finally, the bond price schedule in terms of scaled variables is given by

$$Q (z_t, \bar{D}_{t+1}, \bar{S}_{t+1}, \bar{B}_{t+1}) = \frac{1}{1+r} \mathbb{E} \left[ \left\{ \tilde{V}_N (z_{t+1}, \bar{D}_{t+1}, \bar{S}_{t+1}, \bar{B}_{t+1}) \right\} \bigg| z_t \right] .$$

(1.A.20)

### 3.2 Numerical Implementation

**Details for implementing the bank run model.** In order to ensure that the integrals appearing in the threshold condition (1.33) are well behaved, I assume

$$n_{max} \leq N_{max} < 1. \quad (1.A.21)$$

That is, runs are uniformly bounded away from 1. In the numerical implementation, I set $N_{max} = 0.99.51$

Since the population is bounded above, conditions (1.9) and (1.10) may not simultaneously hold when $A_t$ gets large. To account for this, I will instead be using the following definitions for the numerical implementation:

$$d_t = \begin{cases} 
\frac{d_0 K_t}{N_{max}} & \text{if } d_0 K_t N_{max} \leq D_t \\
\frac{d_t}{N_{max}} & \text{otherwise}
\end{cases} \quad (1.A.22)$$

$$n_{max,t} = \begin{cases} 
\frac{D_t}{d_0 K_t} & \text{if } d_0 K_t N_{max} \leq D_t \\
\frac{d_t}{N_{max}} & \text{otherwise}
\end{cases} \quad (1.A.23)$$

That is, definitions (1.9) and (1.10) will normally apply. However, if banks lend large amounts, then households will have to provide more than just $d_0 K_t$ units of deposits. The run threshold (1.32) is found by solving (1.33) numerically.

51 Perhaps some fraction of depositors were away on a fishing trip and therefore inattentive to runs.
**Smoothing the run threshold.** The equilibrium run scheduled \( n(\cdot) \) as defined in (1.34) is discontinuous. In particular, there is a jump at the run threshold \( z^* \). This implies that \( \hat{W}(\cdot) \) as well as the Bellman equations for the value functions will also be continuous. Thus, convergence cannot be achieved in general.

To get around this, I instead use a smoothed version of (1.34):

\[
n_{\text{smooth}}(z, K, D, S, X) = n_{\text{max}} \Phi \left( \frac{e^{z^*(K,D,S,X)} - e^{z}}{h} \right), \tag{1.24}
\]

where \( z^*(\cdot) \) is the run threshold in (1.32), \( \Phi(\cdot) \) is the standard normal cumulative distribution function, and \( h \) is a smoothing parameter. In (1.24), I have used a sigmoid function to approximate the step function; my choice of the "link" function is probit. Note that the approximation converges to the step function as \( h \downarrow 0 \).

**Discrete state space value function iteration.** I first discretize the TFP process (1.1) using the Rouwenhorst (1995) method. This method is known to have good approximation properties, especially for highly persistent processes (Kopecky and Suen (2010)). I then discretize the remaining state variables \( D, S, \) and \( B \). The scaled value functions are then computed over the resulting tensor grid \( \{z_i\} \otimes \{D_j\} \otimes \{S_k\} \otimes \{B_l\} \).

The choice variables \( \bar{I}, \bar{S}, \) and \( \bar{B} \) are discretized in a similar fashion. The discretized choice grids for \( \bar{S} \) and \( \bar{B} \) are the same as their state space counterparts. Note that \( \bar{D}_{t+1} \) may be off grid, so I use linear interpolation when necessary. In order to speed up numerical convergence, I also pre-compute \( \hat{W}(\cdot) \) over a discrete set of points \( \{X_m\} \) using the smoothed run definition (1.24), and subsequently linearly interpolate when necessary. Finally, the scaled system of equations in appendix 3.1 is iterated until convergence.
Systematic Risk, Debt Maturity, and the Term Structure of Credit Spreads

Doctoral Dissertation Essay 2

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This paper is based on joint work with Hui Chen and Jun Yang
Systematic Risk, Debt Maturity, and the Term Structure of Credit Spreads

Abstract

We build a structural model to explain corporate debt maturity dynamics over the business cycle and their implications for the term structure of credit spreads. Longer-term debt helps lower firms’ default risks while shorter-term debt reduces investors’ exposures to liquidity shocks. The joint variations in default risks and liquidity frictions over the business cycle cause debt maturity to lengthen in economic expansions and shorten in recessions. The model predicts that firms with higher systematic risk exposures will choose longer debt maturity, and that this cross-sectional relation between systematic risk and debt maturity will be stronger when risk premium is high. It also shows that the pro-cyclical maturity dynamics induced by liquidity frictions can significantly amplify the impact of aggregate shocks on credit risk, with different effects across the term structure, and that maturity management is especially important in helping high-beta and high-leverage firms reduce the impact of a crisis event that shuts down long-term refinancing. Finally, we provide empirical evidence for the model predictions on both debt maturity and credit spreads.

Keywords: credit risk, term structure, business cycle, maturity dynamics, liquidity
1 Introduction

The aggregate corporate debt maturity has a clear cyclical pattern: the average debt maturity is longer in economic expansions than in recessions. Using data from the Flow of Funds Accounts, we plot in Figure 2.1 the trend and cyclical components of the share of long-term debt for nonfinancial firms from 1952 to 2010. The cyclical component falls in every recession in the sample, with an average drop of 4% from peak to trough.\(^1\) For individual firms, the maturity variation over time can be even stronger. For example, during the financial crisis of 2007-08, 26% of the non-financial public firms in the U.S. saw their long-term debt share falling by 20% or more.

What explains the cyclical variations in corporate debt maturity? How do the maturity dynamics affect the term structure of credit risk? And how effective is maturity management in reducing firms’ credit risk exposures in a financial crisis? To address these questions, we build a dynamic capital structure model that endogenizes firms’ maturity choices over the business cycle, and examine the impact of the interactions between maturity dynamics and macroeconomic conditions on credit risk.

In our model, firms face business cycle fluctuations in growth, economic uncertainty, and risk premia. They choose how much debt to issue based on the tradeoff between the tax benefits of debt and the costs of financial distress. Default occurs when equity holders are no longer willing to service the debt. The need to roll over existing risky debt (by redeeming them at par) leads to the classic debt overhang problem (Myers (1977)), which makes default more likely. A longer debt maturity helps reduce this problem, thus lowering the costs of financial distress. At the same time, investors are subject to idiosyncratic but non-diversifiable liquidity shocks, which endogenously cause longer-term bonds to have larger liquidity discounts and hence to be more costly to issue. The tradeoff between default risk and liquidity determines the optimal maturity choice.

\(^1\)We do not study the long-term trend in debt maturity in this paper. Greenwood, Hanson, and Stein (2010) argue that this trend is consistent with firms acting as macro liquidity providers. Custodio, Ferreira, and Laureano (2012) show that the secular decline in the maturity of public firms was generated by firms with higher information asymmetry and by new public firms in the 1980s and 1990s.
Figure 2.1: Long-term debt share for nonfinancial corporate business. The top panel plots the trend component (via the Hodrick-Prescott filter) of aggregate long-term debt share. The bottom panel plots the cyclical component. The shaded areas denote NBER-dated recessions. Source: Flow of Funds Accounts (Table L.102).

Systematic risk affects maturity choice through two channels. For firms with high systematic risk, default is more likely to occur in aggregate bad times. Since the risk premium associated with the deadweight losses of default raises the expected bankruptcy costs, these firms choose longer debt maturity during normal times to reduce their default risk. As the economy moves into a recession, risk premium rises, and so do the frequency and severity of liquidity shocks. On the one hand, firms with low systematic risk exposures respond to the higher liquidity discounts of long-term bonds by replacing those matured bonds with short-term bonds, which lowers their average debt maturity. On the other hand, firms with high systematic risk become even more concerned about the default risk associated with short maturity. In response, they continue rolling over the matured long-term bonds into new long-term bonds despite the higher liquidity costs, and their maturity structures will be more stable over the business cycle as a result.
Following Duffie, Garleanu, and Pedersen (2005, 2007) and He and Milbradt (2012), we model the illiquidity of corporate bonds via search frictions. When an investor experiences a liquidity shock, she incurs a cost for holding any asset that cannot be liquidated immediately, where the holding costs represent the costs of alternative sources of financing to meet the liquidity needs (instead of using the proceeds from selling the asset). For a corporate bond, this liquidity problem lasts until either the constrained investor finds someone to trade with or until the maturity of the bond, at which point the principal is returned to the investor. For this reason, long-term bonds will have a larger liquidity discount than short-term bonds.

Our calibrated model generates reasonable predictions for leverage, default probabilities, credit spreads, and equity pricing. The model also allows us to analyze a series of questions regarding the impact of debt maturity dynamics on the term structure of credit spreads.

First, like leverage, debt maturity has first order effects on both the level and shape of the term structure of credit spreads. Everything else equal, a shorter maturity raises credit spreads at all horizons and can potentially make the credit curve change from upward-sloping to downward-sloping. For a low-leverage firm (with market leverage of 30%), cutting the average maturity from 8 to 5 years raises the credit spreads by as much as 18 bps in good times and 24 bps in bad times; for a high-leverage firm (with market leverage of 55%), the same change in maturity raises spreads by up to 74 bps in good times and 135 bps in bad times. The maturity effect is stronger at the medium-to-long horizon (8-12 years) for low-leverage firms but at short horizons (2-5 years) for high leverage firms. Moreover, the size of the maturity effect increases nonlinearly as maturity shortens.

Second, pro-cyclical maturity dynamics make a firm’s credit spreads higher and more volatile over the business cycle. Thus, ignoring the maturity dynamics can lead one to underestimate the credit risk. The amplification effect of maturity dynamics is nonlinear in the size of maturity changes over the cycle. For firms with low leverage, the maturity dynamics mainly affect credit spreads at the medium horizon and almost
have no impact on the short end of the credit curve. In contrast, for firms with high leverage, the effect of pro-cyclical maturity on credit risk is not only much stronger, but is highly concentrated at the short end of the credit curve, which reflects the fact that rollover-induced default risk is imminent but temporary.

Third, our model quantifies the effectiveness of maturity management in helping a firm reduce the impact of rollover risk during a financial crisis. The inability to secure long-term refinancing in a crisis means that a firm that enters into the crisis with a large amount of debt coming due can only roll these debt over using short-term debt. The resulting maturity reduction makes the firm more exposed to the crisis than a firm that manages to maintain a long average maturity before the crisis arrives. Thus, firms anticipating a crisis should try to lengthen their debt maturity, especially those with high leverage and high systematic risk exposures. For example, we find that a high leverage firm that enters into a crisis with an average maturity of 1 year experiences an increase in credit spreads of up to 660 bps. Had the same firm chosen an average maturity of 8 years entering into the crisis, the increase in spreads will only be up to 220 bps.

Fourth, our model shows that the endogenous link between systematic risk and debt maturity should be a key consideration for empirical studies of rollover risk. Firms with high systematic risk endogenously choose longer debt maturities and more stable maturity structures. However, their credit spreads (as well as earnings and investment) will likely still be more affected by aggregate shocks because of their fundamental risk exposures. Thus, instead of identifying high-rollover risk firms by comparing the levels or changes in debt maturity, one should also account for the heterogeneity in firms’ systematic risk exposures.

We test the model predictions using firm-level data. Consistent with the model, we find that firms with high systematic risk choose longer debt maturity and maintain a more stable maturity structure over the business cycle. After controlling for total asset volatility and leverage, a one-standard deviation increase in asset market beta raises firm’s long-term debt share (the percentage of total debt that matures in more
than 3 years) by 6.6%. When macroeconomic conditions worsen, for example, during recessions or times of high market volatility, the average debt maturity falls while the sensitivity of debt maturity to systematic risk exposure becomes higher. The long-term debt share is 3.9% lower in recessions than in expansions for a firm with asset market beta at the 10th percentile, but almost unchanged for a firm with asset beta at the 90th percentile. These findings are robust to different measures of systematic risk and different proxies for debt maturity. Furthermore, using data from the recent financial crisis, we find that the effects of rollover risk on credit spreads are significantly stronger for firms with high leverage or high cashflow beta, and they are stronger at shorter horizons, which are again consistent with our model predictions.

The main contribution of our paper is two-fold. First, to our best knowledge, this paper is the first to provide both a dynamic model and empirical evidence for the link between systematic risk and firms' maturity choices over the business cycle. It adds to the growing body of research on how aggregate risk affects corporate financing decisions, which includes Hackbarth, Miao, and Morellec (2006), Almeida and Philippon (2007), Acharya, Almeida, and Campello (2012), Bhamra, Kuehn, and Strebulaev (2010a), Bhamra, Kuehn, and Strebulaev (2010b), Chen (2010), Chen and Manso (2010), and Gomes and Schmid (2010), among others.

On the empirical side, Barclay and Smith (1995) find that firms with higher asset volatility choose shorter debt maturity. They do not separately examine the effects of systematic and idiosyncratic risk on debt maturity. Baker, Greenwood, and Wurgler (2003) argue that firms choose debt maturity by looking at inflation, the short rate, and the term spread to minimize the cost of capital. Two recent empirical studies have documented that firms' debt maturity changes over the business cycle. Erel, Julio, Kim, and Weisbach (2012) show that new debt issuances shift towards shorter maturity and more security during times of poor macroeconomic conditions. Mian and Santos (2011) show that the maturity of syndicated loans is pro-cyclical, especially for credit worthy firms. They also argue that firms actively managed their loan maturity before the financial crisis through early refinancing of outstanding loans. Our measures
of systematic risk exposure are different from their measures of credit quality.

Second, our paper contributes to the studies of the term structure of credit spreads.\footnote{Earlier contributions include structural models by Chen, Collin-Dufresne, and Goldstein (2009), Collin-Dufresne and Goldstein (2001), Leland (1994), Leland and Toft (1996), and reduced-form models by Duffie and Singleton (1999), Jarrow, Lando, and Turnbull (1997), Lando (1998), among others.}

Structural models can endogenously link default risk to firms’ financing decisions, including leverage and maturity structure. This is valuable for credit risk modeling because, while intuitive, it is not obvious theoretically or empirically how to connect debt maturity choice to credit risk at different horizons. For simplicity, earlier models mostly restrict the maturity structure to be time-invariant. Our model allows the maturity structure to change over the business cycle and connects the maturity dynamics to the term structure of credit risk via firms’ endogenous default decisions.

Our model builds on the dynamic capital structure models with optimal choices for leverage, maturity, and default decisions. The disadvantage of short-term debt in our model is that rolling over risky debt gives rise to the debt overhang problem, which increases the risk of default. Importantly, the rollover risk of short-term debt crucially depends on the downward rigidity in leverage, without which short-term debt can actually reduce credit risk. The disadvantage of long-term debt is the illiquidity discount, which is endogenously generated in the model via search frictions (following He and Milbradt (2012)).\footnote{Other possible costs for long-term debt include information asymmetry and adverse selection (Diamond (1991), Flannery (1986)), debt overhang (Myers (1977)), or asset substitution (Leland and Toft (1996)).} We focus on the cost of illiquidity because it can be directly calibrated to the data on liquidity spreads for corporate bonds. Bao, Pan, and Wang (2011), Chen, Lesmond, and Wei (2007), Edwards, Harris, and Piwowar (2007), and Longstaff, Mithal, and Neis (2005) have all documented a positive relation between maturity and various measures of corporate bond illiquidity.
2 Model

In this section, we present a dynamic capital structure model that allows for maturity adjustments over the business cycle. We first introduce the macroeconomic environment and then describe the firm's problem.

2.1 The Economy

The aggregate state of the economy is described by a continuous-time Markov chain with the state at time $t$ denoted by $s_t \in \{G, B\}$. State $G$ represents an expansion state, which is characterized by high expected growth rates, low economic uncertainty, and low risk premium, while the opposite is true in the recession state $B$. The physical transition intensities from state $G$ to $B$ and from $B$ to $G$ are $\tilde{\pi}_G$ and $\tilde{\pi}_B$, respectively. They imply that the probability that the economy switches from state $G$ to $B$ (or from $B$ to $G$) in a small time interval $\Delta$ is approximately $\tilde{\pi}_G \Delta$ (or $\tilde{\pi}_B \Delta$).

Firms generate cash flows that are subject to the large aggregate shocks that change the state of the economy, small systematic shocks, as well as firm-specific diversifiable shocks. Specifically, a firm's cash flow $y_t$ follows the process

$$
\frac{dy_t}{y_t} = \mu(s_t)dt + \sigma^\lambda(s_t)dZ_t^\lambda + \sigma^f(s_t)dZ_t^f.
$$

The two independent standard Brownian motions $Z_t^\lambda$ and $Z_t^f$ are the sources of systematic and firm-specific cash-flow shocks, respectively. The expected growth rate of cash flows is $\mu(s_t)$, while $\sigma^\lambda(s_t)$ and $\sigma^f(s_t)$ denote the systematic and idiosyncratic conditional volatility of cash flows. Although a change in the aggregate state $s_t$ does not lead to any immediate change in the level of cash flows, it changes the dynamics of $y_t$ by altering its conditional growth rate and volatilities.

Investors in this economy are subject to idiosyncratic but uninsurable liquidity shocks. An example of such liquidity shocks is a sudden and large redemption request for banks or hedge funds. In the presence of financing frictions, a liquidity-constrained investor
would prefer to sell her assets to raise funds provided there is a liquid secondary market for the asset. Otherwise, she will have to raise costly funding elsewhere or sell the asset at a discount. These financing costs are a form of shadow costs for investing in illiquid assets. Duffie, Garleanu, and Pedersen (2005, 2007) formalize this argument in a model of the over-the-counter markets with search frictions. He and Milbradt (2012) extend the model to corporate bonds and generate a liquidity spread that is increasing with bond maturity. We follow He and Milbradt (2012) to endogenize the illiquidity of long-term bonds via search frictions.

Specifically, we assume that an unconstrained investor (denoted as type \( \text{U} \)) can become constrained (type \( \text{C} \)) when she receives an idiosyncratic liquidity shock, which occurs with intensity \( \lambda_U(s) \) in state \( s \). Being liquidity-constrained means that the investor will incur a holding cost every period for holding onto an asset (as in Duffie, Garleanu, and Pedersen (2005)). If the asset has a liquid secondary market, the investor will sell it immediately to avoid the holding costs. If it is illiquid, the constrained investor will need to find an unconstrained investor to trade with. The search succeeds with intensity \( \lambda_C(s) \), at which point the investor ceases to incur the holding costs.\(^4\) Alternatively, if the asset has finite maturity, the return of principal at maturity will also resolve the liquidity problem. Thus, the constrained investor incurs holding costs until she finds someone to trade with or until the asset maturity, whichever comes first. The dependence of \( \lambda_i(s) \) (\( i = \text{U}, \text{C} \)) on the aggregate state \( s \) allows both the frequency of liquidity shocks and the search frictions in the over-the-counter markets to differ in good and bad times.

The presence of non-diversifiable liquidity shocks makes markets incomplete, and the equilibrium can only be solved analytically in some special cases (see e.g., Duffie, Garleanu, and Pedersen (2007)). For tractability, we assume that illiquid assets are a very small part of individual investors' portfolios. In the limit, these investors' marginal utilities are unaffected by the liquidity shocks, which means they will not

\(^4\)For simplicity, we abstract away from considering dealers in the over-the-counter market, and we assume the seller has all the bargaining power when trading takes places. Reducing the bargaining power of the seller has similar effects on the model as a higher holding cost.
demand any risk premium for the exposure to liquidity shocks. As for those investors who only hold liquid assets, we assume they effectively face complete markets because the liquidity shocks have no impact on their wealth.

It then follows that there is a unique stochastic discount factor (SDF) \( \Lambda_t \) that is only driven by aggregate shocks. We assume \( \Lambda_t \) follows the process:

\[
\frac{d\Lambda_t}{\Lambda_t} = -r(s_t)\, dt - \eta(s_t)\, dZ_t^A + \delta_G(s_t)\, (e^\kappa - 1)\, dM_t^G - \delta_B(s_t)\, (1 - e^{-\kappa})\, dM_t^B,
\]

with

\[
\delta_G(G) = \delta_B(B) = 1, \quad \delta_G(B) = \delta_B(G) = 0,
\]

where \( r(s_t) \) is the state-dependent risk free rate, and \( \eta(s_t) \) is the market price of risk for the aggregate Brownian shocks \( dZ_t^A \). The compensated Poisson processes \( dM_t^s \equiv dN_t^s - \hat{\pi}_s\, dt \) reflect the changes of the aggregate state (away from state \( s \)), while \( \kappa \) determines the size of the jump in the discount factor when the aggregate state changes. To capture the notion that state \( B \) is a time with high marginal utilities and high risk prices, we set \( \eta(B) > \eta(G) \) and \( \kappa > 0 \) so that \( \Lambda_t \) jumps up going into a recession and down coming out of a recession.

The SDF \( \Lambda_t \) in (2.2) implies a unique risk-neutral probability measure for all the aggregate shocks. Standard risk-neutral pricing techniques apply to the pricing of any liquid asset in the economy. The valuation of an illiquid asset will depend on the type of its investor and the liquidity shocks. Because there is no risk premium associated with the liquidity shocks, their probability distribution remains the same under the risk-neutral measure. This feature significantly simplifies the pricing of illiquid assets.

2.2 The Firm’s Problem

A firm chooses the optimal leverage and debt maturity jointly. The total face value of the firm’s debt is \( P \), with corresponding coupon rate \( b \) chosen such that the debt is

\[\text{See Chen (2010) for a general equilibrium model based on the long-run risk model of Bansal and Yaron (2004) that generates the stochastic discount factor of this form.}\]
priced at par upon issuance at $t = 0$. The optimal leverage is primarily determined by the tradeoff between the tax benefits (interest expenses are tax-deductible) and the costs of financial distress. The effective tax rate on corporate income is $\tau$. In bankruptcy, the absolute priority rule applies, with debt-holders recovering a fraction $\alpha(s)$ of the firm's unlevered assets and equity-holders receiving nothing. For the maturity choice, firms trade off the default risk induced by the need to roll over short-term debt against the illiquidity of long-term debt.

To fully specify a maturity structure, one needs to specify the amount of debt maturing at different horizons as well as the rollover policy for matured debt. For tractability, the existing literature mostly focuses on the time-invariant maturity structure introduced by Leland and Toft (1996) and Leland (1998). For example, Leland (1998) assumes that debt has no stated maturity but is continuously retired at face value at a constant rate $m$, and that all the retired debt is immediately replaced by new debt with identical face value and seniority. This implies that the average maturity of debt outstanding today is $\int_0^\infty t m e^{-mt} dt = 1/m$.

Such a maturity structure has several important implications. First, the maturity structure is time-invariant, which is at odds with the empirical evidence (see e.g., Figure 2.1). Second, it also rules out "lumpiness" in the maturity structure so that the same amount of debt is retired at different horizons. Choi, Hackbarth, and Zechner (2012) find that lumpiness in debt maturity is common in the data, possibly for the purpose of lowering floatation costs, improving liquidity, or market timing. Third, the setting introduces downward rigidity in leverage because firms are always immediately rolling over all the retired debt.

We extend the maturity modeling in Leland (1998) by allowing a firm to roll over its retired debt into new debt of different maturity when the state of the economy changes. While this setting is still restrictive—all firms should in principle be able to adjust their debt maturity at any time, it allows us to capture the business-cycle dynamics of debt maturity, which is the focus of this paper.\(^6\)

\(^6\)We examine the assumption of downward rigidity in leverage extensively in Section 2.3. Chen,
To understand how debt maturity can change in the model, consider the following setting. The maturity structure in state $G$ (good times) is the same as in Leland (1998): debt is retired at a constant rate $m_G$ and is replaced by new debt with the same principal and seniority. When state $B$ (recession) arrives, the firm can choose to replace the retired debt with new debt of a different maturity (the same seniority). This new maturity is determined by the rate $m_B$ at which the new debt is retired. Thus, the firm will have two types of debt outstanding in state $B$. After $t$ years in state $B$, the instantaneous rate of debt retirement is $R_B(t) = m_G e^{-m_G t} + m_B (1 - e^{-m_B t})$.

Finally, when the economy moves from state $B$ back to state $G$, the firm swaps all the type-$m_B$ debt into type-$m_G$ debt.

The rate of debt retirement $R_B(t)$ is time dependent, which complicates this problem. To keep the problem analytically tractable, we approximate the above dynamics by assuming that all the debt will be retired at a constant rate $m_B$ in state $B$, where $m_B$ is the average rate at which debt is retired in state $B$:

$$m_B = \int_0^\infty \pi_B e^{-\pi_B t} \left( \frac{1}{t} \int_0^t R_B(u) du \right) dt. \quad (2.3)$$

Thus, choosing $m_B$ will be similar to choosing $m_B$, provided the value of $m_B$ implied by (2.3) is nonnegative. Since debt will be retired at a constant rate in both states based on this approximation, we define the firm’s average debt maturity conditional on being in state $s$ as $M_s = 1/m_s$ ($s = G, B$).

There is a liquid secondary market for the firm’s equity, but the corporate bonds are traded in the over-the-counter market. As a result, equity prices are not affected by liquidity shocks, while the corporate bond prices will reflect the risk of liquidity shocks and the holding costs that liquidity-constrained investors incur. We assume the holding cost per unit of time is proportional to the face value of the bond and

---

Xu, and Yang (2012) present an extension of this model that captures lumpiness in the maturity structure.
takes the following functional form:

\[ h(m, s) = h_0(s) \left( e^{h_1(s)/m} - 1 \right). \]  

(2.4)

Two key properties of the holding cost are: it is higher in bad times \((h(\cdot, G) < h(\cdot, B))\), and it is increasing with maturity (decreasing in \(m\)) \((h_0(s), h_1(s) > 0)\).\(^7\) The first property is quite intuitive. The second property is not necessary for the qualitative results in the model (a constant holding cost can already make the liquidity spread increase with maturity), but it helps with matching the model-implied term structure of liquidity spreads to the data.

The assumption that \(h(m, s)\) increases with bond maturity is consistent with the notion that the holding cost rises with the amount of time the investor remains constrained. This is implied by dynamic models of financing constraints (e.g., Bolton, Chen, and Wang (2011)) where the marginal value of liquidity rises as the amount of financial slack dwindles over time. To see the intuition, consider the special case where the aggregate state does not change. The actual holding cost for the constrained investor is \(f(\tau)\), with \(\tau\) being the time the investor has spent in the constrained state, where \(f(0) = 0, f'(\tau) > 0\). The average expected holding cost per unit of time for a constrained investor holding a bond with maturity \(1/m\) is:

\[ h(m) = \int_0^\infty (\lambda C + m)e^{-(\lambda C + m)t} \left( \frac{1}{t} \int_0^t f(u)du \right) dt. \]

It follows that \(h(m)\) will be decreasing in \(m\) (increasing in maturity) given that \(f' > 0\), and \(\lim_{m \to \infty} h(m) = 0\), both of which are captured by (2.4).

Another implication of the specification for holding cost in (2.4) is that the holding cost as a fraction of the market value of the bond is increasing as the firm approaches default. This feature is consistent with Longstaff, Mithal, and Neis (2005), Bao, Pan, and Wang (2011), and others that find that bonds with higher default risk are more

\(^7\)Strictly speaking, the holding costs should also be bounded above so that the bond price is never negative. Otherwise the investor can simply abandon the bond. This will never be the case for the parameters and range of maturity considered in our quantitative exercises.
illiquid.

Finally, the firm’s problem is to choose the optimal amount of debt to issue at time 0 (with face value $P$) and the optimal maturity structure for state $G$ and $B$ ($m_G$ and $m_B$) to maximize the equity-holder value at time $t = 0$.

Ex post, the firm also chooses the optimal default policy in the two states. The default policy is characterized by a pair of default boundaries $\{y_D(G), y_D(B)\}$. In a given state, the firm defaults if its cash flow is below the default boundary for that state. In summary, the firm’s policy for capital structure and default is characterized by the 5-tuple $(P, m_G, m_B, y_D(G), y_D(B))$.

In the remainder of this section, we first solve for the value of debt and equity given the firm policy for capital structure and default. Then we characterize the optimal firm policy.

Valuation of Debt and Equity

Due to the presence of uninsurable liquidity shocks, the pricing of illiquid assets such as corporate bonds differs from that of liquid assets such as stocks. We first discuss the pricing of liquid assets, and then present the analytical results for pricing debt and equity.

Risk-neutral pricing for liquid assets

Under the risk-neutral probability measure implied by the SDF in (2.2), the firm’s cash flow process has expected growth rate $\mu(s_t) = \hat{\mu}(s_t) - \sigma_A(s_t)\eta(s_t)$ and total volatility $\sigma(s_t) = \sqrt{\sigma_A^2(s_t) + \sigma_f^2(s_t)}$. In addition, the risk-neutral transition intensities between the aggregate states are given by $\pi_G = e^{e\hat{\pi}_G}$ and $\pi_B = e^{-e\hat{\pi}_B}$. Because $\kappa > 0$, the risk-neutral transition intensity from state $G$ to $B$ is higher than the physical intensity, while the risk-neutral intensity from state $B$ to $G$ is lower than the physical intensity. Jointly, they imply that the bad state is both more likely to occur and tends to last longer under the risk-neutral measure than under the physical measure.

This is based on the assumption that the firm can commit to the maturity policy $(m_G, m_B)$ chosen at time $t = 0$. Letting equity holders choose the debt maturity ex post when the aggregate state changes will generate similar results.
The value of a liquid claim on an unlevered firm, \( V(y, s) \), which pays out a perpetual stream of cash flows \( y \) specified in (2.1) (without adjusting for taxes), satisfies a system of ordinary differential equations (ODE):

\[
 r(s)V(y, s) = y + \mu(s)yV_y(y, s) + \frac{1}{2}\sigma^2(s)y^2V_{yy}(y, s) + \pi_s(V(y, s^c) - V(y, s)) \tag{2.5}
\]

where \( s^c \) denotes the complement state to state \( s \). Its solution is \( V(y, s) = v(s)y \), where the state-dependent price-dividend ratio \( v \equiv (v(G), v(B)) \) is given by

\[
 v = \begin{pmatrix}
 r(G) - \mu(G) + \pi_G & -\pi_G \\
 -\pi_B & r(B) - \mu(B) + \pi_B
\end{pmatrix}^{-1}
\begin{pmatrix}
 1 \\
 1
\end{pmatrix} . \tag{2.6}
\]

This is a generalized Gordon growth formula, which takes into account the state-dependent riskfree rates \( r(s) \) and risk-neutral expected growth rates \( \mu(s) \), as well as possible future transitions between the states. In the special case with no transition between the states (\( \pi_G = \pi_B = 0 \)), equation (2.6) reduces to the standard Gordon growth formula.

**Debt pricing** As in Leland (1998), it is convenient to directly compute the value of all the debt outstanding at time \( t \). Its value to a type-\( i \) investor, \( D(y_t, s_t, i) \) with \( i \in \{U, C\} \), will be independent of \( t \). The total debt value satisfies a system of ODEs:

\[
 r(s)D(y, s, i) = b - h(m_s, s)P1_{\{i=C\}} + \mu(s)yD_y(y, s, i) + \frac{1}{2}\sigma^2(s)y^2D_{yy}(y, s, i) \tag{2.7}
\]

\[
 + m_s(P - D(y, s, i)) + \pi_s(D(y, s^c, i) - D(y, s, i)) \\
 + \lambda_i(s)(D(y, s, i^c) - D(y, s, i)) ,
\]

with boundary conditions at default:

\[
 D(y_D(s), s, i) = \alpha(s)v(s)y_D(s) , \tag{2.8}
\]

where \( v(s) \) is the price-dividend ratio in (2.6), and \( \alpha(s) \) is the asset recovery rate in state \( s \). Proposition 3 in section 1 gives the analytical solution for \( D(y, s, i) \).
By collecting the terms related to liquidity shocks and holding costs, we can rewrite (2.7) as

$$
(r(s) + \ell(y, m_s, s, i)) D(y, s, i) = b + \mu(s) y D_y(y, s, i) + \frac{1}{2} \sigma^2(s) y^2 D_{yy}(y, s, i) + m_s (P - D(y, s, i)) + \pi_s (D(y, s^c, i) - D(y, s, i)) ,
$$

(2.9)

where

$$
\ell(y, m_s, s, i) = \frac{h(m_s, s) P 1_{i=c} + \lambda_i(s) (D(y, s, i) - D(y, s, i^c))}{D(y, s, i)}
$$

(2.10)

can be viewed as the instantaneous liquidity spread that type-\(i\) investor applies to pricing the bond in state \(s\). This liquidity spread is nonnegative for both types of investors (since \(h(m, s) \geq 0\)). It shows that the holding costs for constrained investors lower the market value of debt ex ante, which is a form of financing costs for corporate debt.

A shorter maturity (higher \(m\)) effectively reduces the duration of liquidity shocks: a constrained investor no longer incurs the holding costs once she receives the principal back at the maturity date. As a result, a bond with shorter maturity will have a lower liquidity discount, which is an important factor for firms’ maturity choices.

**Equity pricing** Since equity is traded in a liquid secondary market, its value \(E(y, s)\) will be not be affected by liquidity shocks. The payout for equity holders includes the cash flow net of interest expenses and taxes, as well as any costs associated with issuing new debt to replace retired debt. Whenever the firm issues new debt, unconstrained investors are the natural buyers with the highest valuation. Therefore, the value at which new debt is issued is the value to a type-\(U\) investor, \(D(y, s, u)\). Thus, \(E(y, s)\)
satisfies the following ODEs:

\[ r(s)E(y, s) = (1 - \tau)(y - b) - m_s(P - D(y, s, U)) \]
\[ + \mu(s)yE_y(y, s) + \frac{1}{2}\sigma(s)^2y^2E_{yy}(y, s) + \pi_s(E(y, s^c) - E(y, s)). \tag{2.11} \]

Because equity holders recover nothing at default, the equity value upon default is:

\[ E(y_D(s), s) = 0. \tag{2.12} \]

Proposition 3 in section 1 gives the analytical solution for \( E(y, s) \).

The first term on the right-hand side of equation (2.11), \((1 - \tau)(y - b)\), is the cash flow net of interest expenses and taxes. The second term, \(m_s(P - D(y, s, U))\), is the instantaneous rollover costs to equity holders. If old debt matures and is replaced by new debt issued under par value \((D(y, s, U) < P)\), equity holders will have to incur extra costs for rolling over the debt. These rollover costs are a transfer from equity holders to debt holders, which can lead equity holders to default earlier. This is a classic debt overhang problem as described by Myers (1977). He and Xiong (2012) use this channel to show how debt market liquidity problems affect credit risk.

In our model, the size of rollover costs depends on firm-specific conditions, macroeconomic conditions, and debt maturity. Under poor macroeconomic conditions, low expected growth rates of cash flows, high systematic volatility, and high liquidity spreads will all drive the market value of debt lower, which raises the rollover costs. Moreover, a shorter debt maturity means that debt is retiring at a higher rate \((m is large)\), which will amplify the rollover costs whenever debt is priced below par. Thus, if debt maturity is pro-cyclical as shown in Figure 2.1, the combination of short maturity, high aggregate risk premium, low cash flows, and high volatility can generate particularly high rollover costs and high default risk in bad times.
Optimal default and capital structure decisions

So far we have discussed the pricing of debt and equity for a given set of choices on debt level, maturity, and default boundaries \((P, m_G, m_B, y_D(G), y_D(B))\). We now characterize the optimal firm policies.

First, for a given choice of debt level and maturity, standard results imply that the ex-post optimal default boundaries for equity holders satisfy the smooth-pasting conditions:

\[
E_y(y_D(s), s) = 0, \quad s \in \{G, B\}.
\]  
(2.13)

Next, at time \(t = 0\), the firm chooses its capital structure \((P, m_G, m_B)\) to maximize the initial value of the firm, which is the sum of the value of equity after debt issuance and the proceeds from debt issuance. Thus, the firm’s objective function is:

\[
\max_{P, m_G, m_B} E(y_0, s_0; P, m_G, m_B) + D(y_0, s_0, U; P, m_G, m_B).
\]  
(2.14)

Our solution strategy is as follows. For any given capital structure and default policy summarized by \((P, m_G, m_B, y_D(G), y_D(B))\), we obtain closed-form solutions for the value of debt and equity. We then solve for the optimal default boundaries \(\{y_D(G), y_D(B)\}\) for given \((P, m_G, m_B)\) via a system of non-linear equations implied by (2.13). Finally, we solve for the optimal capital structure via (2.14).

The analysis of debt and equity pricing in Section 2.2 provides the key intuition for the maturity tradeoff. Shorter debt maturity leads to more frequent rollover and higher default risk, whereas longer debt maturity leads to higher liquidity discounts. This tradeoff is influenced by firms’ systematic risk exposures and macroeconomic conditions. All else equal, firms with low exposures to systematic risk are less concerned about debt rollover raising default risk, because default is less costly for them. They will choose shorter maturity debt to reduce the financing costs. The opposite is true for firms with high systematic risk exposures. Thus, the model predicts that in the cross section, debt maturity increases with a firm’s systematic risk exposure.
The maturity tradeoff also varies over the business cycle. On the one hand, debt rollover has a bigger impact on firm value in recessions due to higher default probabilities, higher costs of bankruptcy, and higher risk premium. These factors tend to cause all firms to lengthen their debt maturities in recessions. On the other hand, liquidity risk can rise in recessions as well (due to more frequent liquidity shocks and higher holding costs), which causes firms to shorten debt maturity. The net result on whether maturity will become longer or shorter in recessions is ambiguous. Moreover, since firms with high systematic risk exposures are affected more by higher systematic risk and risk premium, the cross-sectional relation between debt maturity and systematic risk exposure will become stronger in bad times. In Section 3, we analyze the quantitative predictions of the calibrated model.

2.3 Downward rigidity in leverage

Our model of debt maturity dynamics is an extension of Leland and Toft (1996) and Leland (1998), where firms are assumed to always roll over all the retired debt immediately. The direct implication is that the face value of debt outstanding is constant over time. More importantly, this assumption introduces downward rigidity in leverage, a key feature that enables structural credit risk models to generate significant default risk.\textsuperscript{9}

The intuition is as follows. In the absence of other frictions, a firm that can freely adjust its debt level will reduce debt following negative shocks to cash flows. Then, as long as cash flows do not drop too quickly, the firm will be able to lower its leverage sufficiently to avoid default. By doing so, the firm can lower the costs of financial distress and still enjoy a large tax shield in good times. As Dangl and Zechner (2007) show, issuing short-term debt enables equity holders to commit to this type of downward debt adjustment. This is why it is difficult to generate significant default risk in models with one-period debt, which is well documented in models of corporate and sovereign default. Besides low default risk, models that allow downward

\textsuperscript{9}It is straightforward to allow firms to restructure their debt upward in this model.
adjustment in leverage also predict that firms with higher costs of financial distress will choose shorter debt maturity, which is opposite to what our model predicts.

The drastically different implications from the models with and without downward rigidity in leverage highlight the importance in demonstrating the validity of this assumption. Empirically, several studies have shown the difficulty for firms to adjust leverage downward. Asquith, Gertner, and Scharfstein (1994) find that factors including debt overhang, asymmetric information, and free-rider problems present strong impediments to out-of-court restructuring to reduce leverage. Gilson (1997) also shows that leverage of financially distressed firms remains high before Chapter 11. Welch (2004) finds that firms respond to poor performance with more debt issuing activity and to good performance with more equity issuing activity.

There is also evidence of downward rigidity in leverage that is directly related to debt maturity. Mian and Santos (2011) show that instead of rolling over long-term debt in bad times, credit-worthy firms draw down their credit line commitment. Effectively, these firms replace matured long-term debt with short-term debt instead of equity. We also show (see the Internet Appendix) that the speed of leverage adjustment is slow for both firms with long and short maturity. Moreover, the negative correlation between changes in cash flows and changes in book leverage (as in Welch (2004)) is even stronger for firms with shorter maturity, suggesting that firms with shorter maturity do not reduce leverage in bad times.

Theoretically, we can provide micro-foundation for the downward rigidity in leverage by introducing frictions that make it difficult for firms to reduce leverage following poor performances. In Appendix 2, we present a model that builds on Dangl and Zechner (2007) by adding equity issuance costs.\(^{10}\) In the model, firms are not required to roll over the retired debt immediately, yet the downward rigidity in leverage arises endogenously. The intuition is as follows. If a firm does not roll over the retired debt, it has to pay back the existing debt holders using either internal funds or external equity. Thus, the firm will need to raise large amounts of equity precisely when the

\(^{10}\)We thank an anonymous referee for suggesting this model.
cash flows are low. A shorter debt maturity means that the firm needs to issue equity more frequently and at a faster rate. As a result, a convex equity issuance cost not only discourages firms from reducing leverage following poor performances, but also discourages the issuance of short-term debt.

As the results in Appendix 2 show, without equity issuance costs, this model generates very low credit spreads and a negative relation between systematic risk and debt maturity. After adding a convex equity issuance cost, the model is able to generate more realistic credit spreads and a positive relation between systematic risk and debt maturity.

Adding equity issuance costs is the first step towards explaining the downward rigidity in leverage. It is also important to understand what makes equity issuance difficult following poor performance. One possible reason is that uncertainty rises following poor performance, which makes the information asymmetry more severe and raises the costs of issuing equity (Myers and Majluf (1984)). We leave this question to future research.

3 Quantitative Analysis

In this section, we examine the quantitative implications of the model. We first describe the calibration procedure. Then we examine the optimal maturity choice in the cross section and over the business cycle. Finally, we examine the impact of maturity dynamics on the term structure of credit spreads.

3.1 Calibration

We set the transition intensities for the aggregate states of the economy to \( \hat{\pi}_G = 0.1 \) and \( \hat{\pi}_B = 0.5 \), which imply that an expansion is expected to last for 10 years, while a recession is expected to last for 2 years. To calibrate the stochastic discount factor, we choose the riskfree rate \( r(s) \), the market prices of risk for Brownian shocks \( \eta(s) \),
and the market price of risk for state transition $\kappa$ to match the first two moments of their counterparts in the SDF in Chen (2010).

Similarly, we calibrate the expected growth rate $\tilde{\mu}(s)$ and systematic volatility $\sigma_A(s)$ for the benchmark firm based on Chen (2010), which in turn are calibrated to the corporate profit data from the National Income and Product Accounts. The annualized idiosyncratic cash flow volatility of the benchmark firm is fixed at $\sigma_f = 23\%$. The bankruptcy recovery rates in the two states are $\alpha(G) = 0.72$ and $\alpha(B) = 0.59$. The cyclical variation in the recovery rate has important effects on the ex ante bankruptcy costs. The effective tax rate is set to $\tau = 0.2$. To define model-implied market betas and use them as a measure of firms’ systematic risk exposures, we specify the dividend process for the market portfolio to be a levered-up version of the cash-flow process (2.1) absent idiosyncratic shocks. We choose the leverage factor so that the unlevered market beta for the benchmark firm is 0.8, the medium asset beta for U.S. public firms.

In our model, transactions in the secondary bond market occur when a liquidity-constrained investor meets with an unconstrained investor. Conditional on the aggregate state $s$, a fraction $\lambda_U(s)/(\lambda_U(s) + \lambda_C(s))$ of the investors are constrained on average, so that $\lambda_C(s)\lambda_U(s)/(\lambda_U(s) + \lambda_C(s))$ is the model-implied bond turnover rate. For calibration, we pick $\lambda_U(s)$ so that the idiosyncratic liquidity shock arrives 1.5 times per year during expansions and 3 times per year in recessions, and choose the intermediation intensity $\lambda_C(s)$ such that the bond turnover rate is 10% (5%) per month during expansions (recessions) based on the findings from Bao, Pan, and Wang (2011).\textsuperscript{11}

Finally, we calibrate the four holding cost parameters $h_0(s), h_1(s)$ in equation (2.4) by matching the term structure of model-implied bond liquidity spreads with the data. We follow the procedure of Longstaff, Mithal, and Neis (2005) to estimate the

\textsuperscript{11}It is possible that a large part of the bond trading in good times is not due to liquidity shocks, in which case our calibration procedure would overstate $\lambda_C(G)$. However, since we calibrate the holding costs for constrained investors to match the observed liquidity spreads for corporate bonds, the impact of less frequency liquidity shocks will be largely offset by higher holding costs. See Appendix 3 for details.
non-default components in corporate bond spreads at different maturities, which are shown to be largely related to liquidity. We use bond price data from the Mergent Fixed Income Securities Database and CDS data from Markit for the period from 2004 to 2010. To address the possible selection bias that firms facing higher long-term non-default spreads might only issue short-term debt, we restrict the sample to firms that issue both short-term (less than 3 years) and long-term (longer than 7 years) straight corporate bonds. Details of the procedure are in section 3.

The short sample period (due to the availability of CDS data) makes it difficult to identify different holding cost parameters for state $G$ and $B$. There is only one NBER recession in our sample (from December 2007 to June 2009), which is also the period of a major financial crisis. The average liquidity spreads for bonds with maturities of 1, 5, and 10 years are 0, 4, and 12 bps during normal times and rise to 1, 65, and 145 bps respectively during the crisis. Since state $B$ in our model represents a typical recession rather than a financial crisis, we calibrate the baseline holding cost parameters to match the full liquidity spreads in normal times and one third of the liquidity spreads in the financial crisis. Since this choice of calibration target for recessions is admittedly ad hoc, we have also conducted sensitivity analysis to show the robustness of our results to the holding cost parameters (see the Internet Appendix).

A firm's choice of leverage and maturity structure implies a particular term structure of credit spreads. To compute the term structure of credit spreads, we take the firm's optimal default policy (default boundaries) as given and price fictitious bonds with a range of different maturities. These bonds are assumed to default at the same time as the firm, and their recovery rate is set to 44% in state $G$ and 20% in state $B$, which is consistent with the business-cycle variation in bond recovery rates in the data (see Chen (2010)).

Panel A of Table 2.1 summarizes the parameter values for our baseline model.
3.2 Maturity Choice

The main results for the capital structure and default risk of the benchmark firm are summarized in Panel B of Table 2.1. We assume that the firm makes its optimal capital structure decision in state $G$ with initial cash flow normalized to 1. The initial market leverage is 28.5% in state $G$. Fixing the level of cash flow, the same amount of debt will imply a market leverage of 31.6% in state $B$ due to the fact that equity value drops more than debt value in recessions. The initial interest coverage ($y_0/b$) is 2.68. The optimal maturity is 5.5 years for state $G$ and 5.0 years for state $B$. Based on our interpretation of maturity adjustment in equation (2.3), $m_B = 1/5$ corresponds to $\bar{m}_B = 0.31$. That means the firm will be replacing its 5.5-year debt that retires in state $B$ with new 3.3-year debt.

The model-implied 10-year default probability is 4.2% in state $G$ conditional on the initial cash flow and leverage choice. Fixing the cash flow and leverage but changing the aggregate state from $G$ to $B$ raises the 10-year default probability to 5.6%. The 10-year total credit spread is 115.2 bps in state $G$ and 166.2 bps in state $B$ (again based on initial cash flow and leverage). The default components of the 10-year spreads, which are computed by pricing the bonds using the same default boundaries and removing the liquidity shocks and holding costs, are 97.7 bps in state $G$ and 135.2 bps in state $B$. These values are consistent with the historical average default rate and credit spread for Baa-rated firms. Finally, the conditional equity Sharpe ratio is 0.12 in state $G$ and 0.22 in state $B$.

Next, we study how systematic risk affects firms' maturity structures. As discussed at the end of Section 2.2, the tradeoff for debt maturity is as follows. On the one hand, shorter debt maturity generates higher default risk and hence higher expected costs of default. On the other hand, long-term debt has higher liquidity spreads, which raises the cost of debt financing. Since an analytical characterization of the optimal maturity choice is not feasible in this model, we provide the intuition using a numerical example from the calibrated model.

Consider two firms with identical leverage but different levels of systematic volatility
Figure 2.2: Debt maturity tradeoff. Panels A and B plot the model implied liquidity spreads in state $G$ and $B$. For each state $s$, the liquidity spreads are defined as $\lambda_U(s) [D(y_0, s, U) - D(y_0, s, C)] / D(y_0, s, U)$ and computed at the initial cash flow level. Panels C and D plots the annual default rate in the next 10 years conditional on the initial aggregate state $G$ and $B$. The low beta firm is the benchmark firm with asset beta of 0.8. The high beta firm has an asset beta of 1.08 by rescaling the systematic cash flow volatilities of the benchmark firm.

of cash flows. In Panels A and B of Figure 2.2, we plot the annualized liquidity spreads associated with different debt maturity in the two aggregate states. In Panels C and D, we plot the annualized default rates over a 10-year horizon. Within each aggregate state, the liquidity spread increases with debt maturity while the default rate decreases with maturity. Holding the maturity fixed and increasing the systematic risk exposure has negligible effect on the liquidity spreads (see Panels A and B), but it significantly raises the default risk, especially for short-term debt and especially in state $B$ (see Panels C and D). These results imply that as a firm’s systematic risk exposure rises, concern about default risk will cause it to choose longer debt maturity. Moreover, debt maturity will be more sensitive to systematic risk exposure in state
A. Optimal leverage

\[ M_G (\text{optimal } P) \]

\[ M_B (\text{optimal } P) \]

B. Fixed leverage

\[ M_G (\text{fix } P) \]

\[ M_B (\text{fix } P) \]

**Figure 2.3: Optimal debt maturity.** In Panel A, we hold fixed the idiosyncratic volatility of cash flow while letting the systematic volatility vary and then plot the resulting choices of the optimal average maturity in the two states under optimal leverage. In Panel B, we repeat the exercise but hold leverage fixed at the level of the benchmark firm. The benchmark firm has an average systematic volatility of 0.139 and asset beta of 0.8.

Figure 2.3 shows the results. Indeed, as Panel A shows, controlling for the idiosyncratic cash-flow volatility, the optimal debt maturity increases for firms with higher systematic volatility, and the relationship is stronger in state \( B \) than in state \( G \). As the average systematic volatility rises from 0.07 to 0.21, the optimal maturity in state \( G \) rises from 5.1 to 6.1 years, whereas the maturity in state \( B \) rises from 4.1 to 6.0 years.

The graph also shows that the optimal debt maturity drops from state \( G \) to state \( B \) for the same firm. This result does depend on the differences of liquidity frictions.
in the two states. Because firms are more concerned with rollover risk in bad times, they will only reduce debt maturity if the liquidity frictions become sufficiently more severe in state $B$. However, the result of pro-cyclical maturity choice appears robust quantitatively. Even though we have chosen a relatively conservative target for the liquidity spread in state $B$ (one third of the liquidity spreads in the financial crisis), it is enough to make maturity drop in recessions. The combined effect of (1) pro-cyclical maturity choice and (2) higher sensitivity of debt maturity to systematic volatility in recessions is that the debt maturity for firms with high systematic risk will be relatively stable over the business cycle, while the maturity for firms with low systematic risk will be more volatile.

Next, instead of allowing firms with different systematic risk to choose leverage optimally, we fix the leverage for all firms at the same level as the benchmark firm and re-examine the maturity choice. The results, shown in Panel B of Figure 2.3, are qualitatively similar. However, debt maturity in this case increases faster with systematic volatility in both state $G$ and $B$. For firms with sufficiently high systematic risk exposures, the debt maturity in state $B$ can become even higher than the maturity in state $G$, indicating that these firms roll their maturing debt into longer maturity in recessions.

Why does the optimal debt maturity become more sensitive to systematic risk after controlling for leverage? Because of higher expected costs of financial distress, firms with high systematic risk exposures will optimally choose lower leverage. By fixing their leverage at the level of the benchmark firm, firms with high systematic volatility end up with higher leverage than the optimal amount. As a result, it becomes more important for these firms to use long-term debt to reduce rollover risk, especially in bad times.

### 3.3 Maturity Dynamics and Credit Risk

So far we have analyzed how systematic risk and liquidity frictions affect the maturity dynamics over the business cycle. Existing structural credit risk models mostly consider
the setting of time-invariant maturity structures (many of them only consider perpetual
debt), yet it is quite intuitive that these maturity dynamics can have significant impact
on credit risk at different horizons and over the business cycle. The way maturity
dynamics affect default risk hinges on the endogenous responses of the firms, which
are difficult to capture using reduced-form models. Our structural model provides a
tractable framework to analyze these effects. We focus the analysis on the following
questions.

(i) How sensitive is the term structure of credit spreads to the level of
debt maturity? We fix the maturity to be the same in the two aggregate states
\(M_G = M_B\) so that we can separate the effect of maturity dynamics from that of
maturity level. We then compute the term structure of credit spreads in state \(G\) and
\(B\) for a range of maturity choices. We are also interested in how maturity effect differs
for firms with different leverage.\(^2\) Thus, we first set the firm's interest coverage \((y/b)\)
at the optimal leverage of the benchmark firm (low leverage firm), and then repeat
the exercise for a firm with half the interest coverage (high leverage firm), which can
result from the original low leverage firm experiencing a sequence of negative cash
flow shocks.

The results are shown in Figure 2.4. Panels A and B show the term structure of credit
spreads for the low leverage firm in state \(G\) and \(B\). The credit curve is mostly upward
sloping. Shortening the maturity increases the level of credit spreads at all horizons,
but the effect is rather small at the 1 to 2 year horizon\(^3\) and bigger at medium-to-long
horizons (8 to 12 years). Moreover, the incremental effect of shorter maturity on credit
spreads is nonlinear. By cutting maturity from 8 to 5 years, credit spreads rise by up
to 18 bps in state \(G\) and 24 bps in state \(B\); from 5 to 2 years, the increases in spreads

\(^2\)Our model can be used to study the credit risk of firms under a range of different capital structure
choices, not just the optimal capital structure implied by the tradeoff we consider. Firms in practice
have significant heterogeneity. Their capital structures can vary substantially due to transaction
costs and other frictions.

\(^3\)Part of the reason is that the diffusion assumption for cash flows mechanically implies very little
default risk at the shortest horizons (see Duffie and Lando (2001)). The fact that our model has
additional shocks to the aggregate state alleviates this problem to some extent, especially for firms
with high leverage.
Figure 2.4: Maturity choice and the term structure of credit spreads. This figure plots the term structure of credit spreads as debt maturity choice varies. Debt maturity choice is fixed across states so that $M_G = M_B = M$. In Panels A and B, the firm’s initial interest coverage is fixed at 2.68. In Panels C and D, the firm’s initial interest coverage is fixed at 1.34.

are up to 41 bps and 55 bps; from 2 to 1 years, the increase in spreads are up to 31 bps and 46 bps.

For the firm with higher leverage, its credit curve will still be largely upward-sloping (except at the long horizons) if its average maturity is 8 years. The downward-sloping feature becomes more prominent as the maturity shortens. Unlike the low-leverage firm where shortening the maturity mostly affects credit spreads at the medium to long horizons, here the effect is the largest at short horizons (2 to 5 years), especially when the macroeconomic conditions are poor (in state $B$). Moreover, the size of the impact of debt maturity on credit spreads is larger for the high-leverage firm. Cutting the maturity from 5 to 2 years raises the credit spreads by up to 195 bps in state $G$ and up to 400 bps in state $B$.  

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It is well known that structural models can generate an upward-sloping term structure of credit spreads for low-leverage firms and downward-sloping term structure for high-leverage firms. The new finding in our model is that maturity choice also has first order effect on the shape of the credit curve. Moreover, the maturity effect is nonlinear and is magnified by poor macroeconomic conditions and high leverage.

The previous analysis shows that credit spreads are counter-cyclical (i.e., higher in recessions) with constant maturity across states $G$ and $B$. As equation (2.11) shows, if debt is already priced below par in recessions ($D(y, B, U) < P$), the fact that maturity is shorter at such times ($m_B$ is larger) will make the rollover costs higher for equity holders, which makes default more likely and further increases the credit spreads in state $B$.

(ii) How much can the pro-cyclical variation in debt maturity amplify the fluctuations in credit spreads over the business cycle? To answer this question, we conduct the following difference-in-difference analysis. Let $CS^i(\tau, s)$ be the credit spread in state $s$ at horizon $\tau$ for firm $i$. Now consider two firms: firm $X$ has constant maturity $M_G = M_B = 5.5$; firm $Y$ has the same maturity as $X$ in state $G$, but shorter maturity in state $B$. As we lower $M_B$ for firm $Y$, not only will its credit spreads rise in state $B$, they will also rise in state $G$ due to the anticipation effect. Thus, the pro-cyclical maturity variation amplifies the fluctuations in credit spreads over the business cycle if the differences in credit spreads between the two firms are larger in state $B$ than in state $G$,

\[
CS^Y(\tau, G) - CS^X(\tau, G) < CS^Y(\tau, B) - CS^X(\tau, B). \tag{2.15}
\]

Figure 2.5 shows the results of this analysis. In Panels A and B, we plot the differences in credit spreads between the two firms $X, Y$ in the two aggregate states (i.e., the left and right-hand side of inequality (2.15)), where $M_B$ for firm $Y$ ranges from 5.5 years to 1 year. In Panels C and D, we do the same calculations for the two firms with higher leverage.
Figure 2.5: The amplification effect of pro-cyclical maturity on credit spreads. This figure plots the differences in the term structure of credit spreads between firm X with constant debt maturity $M_G = M_B = 5.5$ years and firm Y with $M_G = 5.5$ years but $M_B < M_G$. In Panels A and B, the initial interest coverage is 2.68. In Panels C and D, the initial interest coverage is 1.34.

For any $M_B < 5.5$, firm Y has higher credit spreads than firm X at all horizons. Comparing Panel A vs. B, we do see larger differences in credit spreads between the two firms in state B, suggesting that pro-cyclical maturity dynamics indeed amplify the variation in credit spreads over the business cycle. For example, with $M_B = 2$, the credit spread of firm Y is as much as 39 bps higher than firm X in state G, and 52 bps higher in state B. With higher leverage, the amplification effect can become much stronger (see Panels C and D). When firm Y’s maturity drops from 5.5 years to 5.0 years, the credit spread can rise by up to 12 bps in state G and 25 bps in state B. If firm Y’s average maturity in state B drops to 1 year, credit spreads rise by up to 283 bps in state G, and by up to 782 bps in state B.

Given that the magnitude of the amplification effect is sensitive to the size of the drop
in maturity, it is important to understand the mechanisms that lead to large changes in maturity from state $G$ to $B$. Revisiting the mechanics for how debt maturity is adjusted in Section 2.2, we see that the average maturity will become shorter in state $B$ if the firm rolls the retired debt into new debt with shorter maturity, and if the bad state is more persistent. For example, consider the cases where the average debt maturity falls from 5.5 years to 3 years or 1 year in state $B$. Based on the interpretation of maturity adjustment in equation (2.3) and our calibration of the transition intensities, $m_B = 1/3$ corresponds to $\bar{m}_B = 1.2$, meaning the retired bonds are rolled into new bonds with maturity of 10 months, while $m_B = 1$ corresponds to $\bar{m}_B = 5.7$ or approximately a maturity of 2 months. Alternatively, lumpy maturity structures can also lead to big maturity adjustments over the business cycle (see Choi, Hackbarth, and Zechner (2012) and Chen, Xu, and Yang (2012)).

Another interesting observation is that while the amplification effect of pro-cyclical maturity dynamics is the largest at the medium horizon (5-7 years) for the low leverage firm, it becomes the largest at the short end of the credit curve (1-3 years) for the high leverage firm. The intuition is as follows. With low leverage, the firm faces low default risk. In this case, especially in the near future, newly issued debt will be priced close to par value. Thus, there is no debt overhang problem, and more frequent rollover will not raise the burden for equity holders. As a result, the increase in credit spreads due to shorter maturity is negligible at the short end of the credit curve. In contrast, the impact of shorter maturity on default risk immediately shows up in the case of high leverage, because the newly issued bonds are priced under par already.

(iii) How much can maturity management help firms reduce the impact of a crisis episode on credit risk? Almeida, Campello, Laranjeira, and Weisbenner (2011) find that those firms with more long-term debt coming due in the 2008 financial crisis suffered deeper cuts in investment during the crisis because of the difficulty in rolling over their debt. Hu (2010) uses the same empirical strategy to identify firms facing higher rollover risk and finds that these firms experienced larger increases in credit spreads.
Our model can capture such maturity dynamics in a “crisis” episode. Suppose a financial crisis completely shuts down the demand for long-term debt and firms can only roll over matured debt into one year debt (i.e., $\bar{m}_B = 1$). Then, a firm’s average debt maturity in state $B$ will be fully determined by the average maturity in state $G$ and the average duration of the crisis (see equation (2.3)). In particular, if a firm chooses a longer average maturity (smaller $m_G$) before entering the crisis, it will have a smaller fraction of total debt maturing during the crisis, which implies a smaller reduction in the average maturity.

To quantify this effect, we conduct another difference-in-difference analysis. Again consider two firms $X$ and $Y$. Suppose firm $X$ has a longer average maturity in state $G$ than firm $Y$, $M^X_G > M^Y_G$. The impact of the “crisis” on credit spreads can be measured as the change in credit spreads from state $G$ to state $B$, everything else equal. A longer maturity before the crisis reduces the impact of the rollover risk on credit risk if

$$CS^Y(\tau, B) - CS^Y(\tau, G) > CS^X(\tau, B) - CS^X(\tau, G). \quad (2.16)$$

We present the results of this analysis in Figure 2.6. Panels A and B consider the cases of low leverage and high leverage, respectively. In each panel, we consider 4 firms with different average maturity in state $G$, with $M_G = 1, 2, 5, 8$ years and plot the changes in credit spreads for each of them when the aggregate state changes from $G$ to $B$. Indeed, maturity management in state $G$ matters for firms’ credit risk exposure to the crisis. In the case of a low leverage firm, having an average maturity of 8 years before the crisis helps cap the impact of the crisis on credit spreads at 38 bps, while an otherwise identical firm with an average maturity of 1 year will experience an increase in the credit spreads that is almost twice as large (up to 67 bps).

In the case of the high leverage firm, maturity management becomes even more important. With an average maturity of 8 years before the crisis, the firm’s credit spreads rise by as much as 220 bps entering the crisis state. An otherwise identical firm with an average maturity of 1 year will experience three times as large an increase
Figure 2.6: Maturity and rollover risk. This figure plots the changes in credit spreads when the aggregate state switches from $G$ to $B$ for an initial debt maturity ranging between $M_G = 1$ and $M_G = 8$ in state $G$. For each choice of $M_G$, the effective average maturity in state $B$ is calculated using expression (2.3) with newly issued debt in state $B$ maturing at rate $\bar{m}_B = 1$. The initial interest coverage is 2.68 for the low leverage firms, 1.34 for the high leverage firms.

in its credit spreads (up to 660 bps). Moreover, maturity management is particularly effective in reducing the credit risk at short horizons for a high leverage firm. Besides for firms with high leverage, we also find a stronger effect of maturity management for firms with high cash flow betas.

How can a firm avoid being caught with short average maturity entering into a crisis? The answer is not only to issue longer-term debt, but also to maintain a long average maturity over time. The latter requires the firm to evenly spread out the timing of maturity of its debt rather than having a lumpy maturity structure.

(iv) How does the endogenous maturity choice affect the cross-sectional relation between debt maturity and rollover risk? The results from the previous exercise are consistent with the standard intuition that shorter maturity makes the impact of aggregate shocks on credit spreads stronger. However, this is under the condition that the firms have identical systematic risk exposures. In reality, the impact of aggregate shocks on credit risk will also depend on firms’ systematic risk
Figure 2.7: Credit spread changes under exogenous vs. endogenous maturity choice. This figure plots the increase in credit spreads at various horizons when the aggregate state switches from $G$ to $B$ for different firms. In Panel A, the two firms have the same systematic risk exposures but are given different debt maturity choice exogenously. In Panel B, the two firms endogenously choose different maturity structure due to differences in systematic risk.

exposures, which as we have shown in Section 3.2, endogenously influence the firms’ maturity choices in the first place.

We illustrate this point using a simple example in Figure 2.7. In Panel A, we take two firms with identical asset beta (the same as the benchmark firm), but fix their debt maturity exogenously at 6.8 years and 3.2 years, respectively. In Panel B, we identify two firms with different systematic volatility (but the same average total volatility), which leads them to choose different debt maturities endogenously. One firm has an average systematic volatility of 18.9% (asset beta of 1.08) and sets its debt maturity in state $B$ optimally at 6.8 years. The other has an average systematic volatility of 8.9% (asset beta of 0.52) and sets its maturity in state $B$ at 3.2 years. The leverage for all the firms are fixed at the same level as the benchmark firm. The figure plots the change in credit spreads from state $G$ to $B$, which measures the response of the credit spreads to the aggregate shock. Panel A shows that, with exogenous maturity, the credit spread rises more for the firm with shorter maturity, which is consistent with the standard intuition of rollover risk. In Panel B, however, the firm with longer maturity actually has a bigger increase in credit spreads than the one with shorter
maturity because of its larger exposure to systematic risk.

4 Empirical Evidence

In this section, we test the following predictions that the model generates about the relations between firms' systematic risk, debt maturity, and credit risk:

a. Firms with higher systematic risk exposures will choose longer debt maturity.

b. The sensitivity of debt maturity to systematic risk exposure becomes stronger after controlling for leverage.

c. The sensitivity of debt maturity to systematic risk exposure rises in times of higher risk premium.

d. A longer average maturity before entering into a crisis helps reduce the impact of the crisis on credit spreads. This effect of maturity management is stronger for firms with higher leverage or high systematic risk.

4.1 Data

We merge the data from COMPUSTAT annual industrial files and the CRSP files for the period 1974 to 2010. We exclude financial firms (SIC codes 6000-6999), utilities (SIC codes 4900-4999), and quasi-public firms (SIC codes greater than 8999), whose capital structure decisions can be subject to regulation. In addition, we require firms in our sample to have total debt that represents at least 5% of their assets. All the variables are winsorized at the 1% and 99% level. Finally, we remove firm-year observations with extreme year-to-year changes in the capital structure, defined as having changes in book leverage or long-term debt share in the lowest or highest 1%, which are likely due to major corporate events such as mergers, acquisitions, and spin-offs.

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14 COMPUSTAT first begins to report balance sheet information used to construct our proxies for debt maturity in 1974.

15 Lowering the threshold to 3% generates very similar results.
For each firm, COMPUSTAT provides information on the amount of debt in 6 maturity categories: debt due in less than 1 year (dlc), in years two to five (dd2, dd3, dd4, and dd5), and in more than 5 years. Following existing studies (see e.g., Barclay and Smith (1995), Guedes and Opler (1996), and Stohs and Mauer (1996)), we construct the benchmark measure of debt maturity using the long-term debt share, which is the percentage of total debt that are due in more than 3 years (ldebt3y). For robustness, we also construct several alternative measures of debt maturity, including the percentage of total debt due in more than $n$ years (ldebt$\text{n}y$), with $n = 1, 2, 4, 5$, and a book-value weighted numerical estimate of debt maturity ($\text{debtmat}$), based on the assumption that the average maturities of the 6 COMPUSTAT maturity categories are 0.5 year, 1.5 years, 2.5 years, 3.5 years, 4.5 years, and 10 years.

Our primary measure of firms’ exposure to systematic risk is the asset market beta. Since firm asset values are not directly observable, we follow Bharath and Shumway (2008) and back out asset betas from equity betas based on the Merton (1974) model. Equity betas are computed using past 36 months of equity returns and value-weighted market returns.$^{16}$ In this process, we also obtain the systematic and idiosyncratic asset volatilities ($\text{sys assetvol}$ and $\text{id assetvol}$). Following Acharya, Almeida, and Campello (2012), we also compute the “asset bank beta,” which measures a firm’s exposure to a banking portfolio, and the “asset tail beta,” which captures a firm’s exposure to large negative shocks to the market portfolio.

The various asset betas constructed above could be mechanically related to firms’ leverage, which might affect firms’ maturity choices. We address this concern by using two additional measures of systematic risk exposure. First, we compute firm-level cash flow betas using rolling 20-year windows. The cash flow beta is defined as the covariance between firm-level and aggregate cash flow changes (normalized by total assets from the previous year) divided by the variance of aggregate cash flow changes. Second, Gomes, Kogan, and Yogo (2009) show that demand for durable goods is more cyclical than for nondurable goods and services. Thus, durable-good producers are exposed to higher systematic risk than non-durables and service producers. They

$^{16}$Computing equity betas with past 12 or 24 months of equity returns generates similar results.
classify industries into three groups according to the durability of a firm’s output. We use their classification as another measure of systematic risk exposure.

Previous empirical studies find that debt maturity decisions are related to several firm characteristics, including firm size (log market assets, or \( mkat \)), abnormal earnings (\( abnearn \)),\(^{17}\) book leverage (\( bklev \)), market-to-book ratio (\( mk2bk \)), asset maturity (\( assetmat \)), and profit volatility (\( profitvol \)). We control for these firm characteristics in our main regressions.

Table 2.2 provides the summary statistics for the variables used in our paper. The detailed descriptions of these variables are in the Internet Appendix. The median firm has 85% of the debt due in more than 1 year, 58% due in more than 3 years, and 32% due in more than 5 years. There is also considerable cross-sectional variation in debt maturity. The standard deviation of the long-term debt share \( ldebt3y \) (the percentage of debt due in more than 3 years) is 32%, and the interquartile range of \( ldebt3y \) is from 27% to 79%. Based on our numerical measure of debt maturity, the median debt maturity is 4.7 years, with an interquartile range from 2.5 years to 6.8 years. The median book leverage in our sample is 27%. The median asset market beta is 0.80, whereas the median equity beta is 1.07. The median systematic and idiosyncratic asset volatilities are 12% and 30%, respectively. The correlations among the different risk measures are reported in Panel B of Table 2.2.

4.2 Debt Maturity

Debt maturity in the cross section

To test the model’s prediction on a positive relation between debt maturity and systematic risk exposures across firms, we run Fama-MacBeth regressions with the following general specification:

\[
 ldebt3y_{i,t} = \alpha + \beta_1 risk_{i,t} + \beta_2 X_{i,t-1} + \varepsilon_{i,t}, \tag{2.17}
\]

\(^{17}\)Following Barclay and Smith (1995), we define “abnormal earnings” as the change in earnings from year \( t \) to \( t + 1 \) normalized by market equity at the end of year \( t \).
where \( ldebt3y \) is the long-term debt share; \( risk_{i,t} \) represents various measures of firms' systematic risk exposures; \( X_{i,t} \) represents firm-specific controls, including total asset volatility (assetvol), market assets (mkat), abnormal earnings (abnearn), book leverage (bklev), market-to-book ratio (mk2bk), asset maturity (assetmat), and profit volatility (profitvol).

The results are presented in Table 2.3. We compute robust t-statistics using Newey-West standard errors with 2 lags, except in the case of cash flow beta, where we use 20 lags. The coefficient of the asset market beta in column (1) is positive but insignificant in the univariate regression. After controlling for asset volatility, asset market beta becomes significantly positively correlated with debt maturity (column (2)). The coefficient estimate of 0.084 implies that a one-standard deviation increase in asset beta, keeping total asset volatility constant, is associated with a 5.4% increase in the long-term debt share. Consistent with our model prediction, the effect of asset beta on debt maturity further strengthens to 0.104 after controlling for book leverage (column (3)), implying that a one-standard deviation increase in asset beta raises the long-term debt share by 6.6%. The coefficient estimate on asset volatility is negative and statistically significant, which is consistent with Barclay and Smith (1995), Guedes and Opler (1996), and Stohs and Mauer (1996).

In the cross section, holding asset beta fixed while changing total asset volatility is equivalent to holding systematic volatility fixed while changing idiosyncratic volatility. Our results show that the negative effect of asset volatility on debt maturity as documented by the earlier studies is driven by the negative relation between idiosyncratic volatility and maturity (see column (4)). This result is consistent with the theory of debt maturity based on information asymmetries (see Diamond (1991), Flannery (1986)). Asymmetric information is more naturally associated with firm-specific uncertainty than aggregate uncertainty (managers are unlikely to know more about the market than outside investors), and firms with higher idiosyncratic risk choose shorter debt maturity to signal their quality. It is also intuitive that controlling for asset volatility is key to finding a significant effect for asset beta. Firms with high asset beta
will tend to have higher idiosyncratic volatility, which offsets the effect of systematic volatility on debt maturity.

In column (5), we introduce other firm controls into the regression. The coefficient estimate of the asset market beta is 0.052, smaller than the previous specifications but still highly significant. The smaller coefficient could be due to the fact that firm characteristics such as size and book-to-market ratio are also related to systematic risk. The coefficient on asset volatility becomes much smaller than before, which is because firm controls such as size and profit volatility are highly correlated with idiosyncratic asset volatility.

Columns (6) - (8) report regression results when we replace asset market beta with asset bank beta, asset tail beta, and cash flow beta, respectively. The coefficient estimates on these alternative systematic risk measures are all positive and statistically significant. They imply that a one-standard deviation increase in a firm's corresponding beta measure lengthens its long-term debt share by 2.4%, 2.7%, and 1.0%. Column (9) reports the results when we use the industry classification for producers of durable goods, nondurable goods, and services as proxy for systematic risk exposure. Since this classification is fixed over time, we run a single cross-sectional regression in this case. The results show that the long-term debt share of durable good producers, which have more cyclical cash flows, is 2.3% larger than that of non-durable good producers, and 3.2% larger than that of service producers.

Table 2.4 reports the results for pooled regressions, where we add year dummies to absorb time-specific effects, and industry dummies (3-digit SIC code) to control for industry fixed-effects. We compute the standard errors by clustering the observations at the industry level.18 The results for the pooled regressions are quantitatively similar to the Fama-MacBeth regressions and are in support of the model prediction that debt maturity is increasing in firms' systematic risk exposures.

Besides systematic risk measures, the effects of various other firm characteristics on maturity are consistent with earlier studies. Everything else equal, firms with low

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18We obtain very similar results adjusting standard errors by clustering the observations in the same industry and in the same year.
Figure 2.8: Time series of Fama-MacBeth coefficients for systematic and idiosyncratic volatility. This graph plots time series of coefficient estimates in a cross-sectional regression of long-term debt shares on systematic and idiosyncratic asset volatility. The confidence intervals are at 95% level. The shaded areas denote NBER-dated recessions.

Total asset volatility, large size, high leverage, low market-to-book ratio, long asset maturity, and low profit volatility are more likely to have longer debt maturity.

To further investigate how the cross-sectional relation between debt maturity and systematic risk changes over time, we plot in Figure 2.8 the time series of the coefficients on the systematic and idiosyncratic volatilities from specification (4) in Table 2.3. The 95% confidence intervals are computed using heteroscedasticity consistent standard errors. Panel B shows that the coefficient on the idiosyncratic asset volatility is significantly negative throughout the sample. Panel A shows that the coefficient for the systematic asset volatility is significantly positive for the majority of the sample years. The coefficient tends to rise in recessions, and it becomes insignificant in 1976-77, 1995, and 2005-06, all of which are in an economic expansion. This result is consistent with our model’s prediction that the cross-sectional relation between systematic risk and debt maturity is stronger in bad times, which we test formally in
the following section.

**Impact of macroeconomic conditions**

As a proxy for macroeconomic conditions, we obtain recession/expansion dates from the National Bureau of Economic Research (NBER). Since firms have different fiscal year-ends, we construct for each firm a yearly recession dummy which equals one if the fiscal year-end month for the firm is in a recession, and zero otherwise. Then, we examine the impact of business cycles on debt maturity by adding a recession dummy and the interaction term between the recession dummy and the systematic risk measure to equation (2.17).

To measure the changes in debt maturity over the business cycle, we need to remove the time variation in maturity due to secular trends. As shown in Figure 2.1, the trend component of aggregate debt maturity is U-shaped over the sample period. We use either a quadratic time trend or the trend component from the Hodrick-Prescott filter to control for this effect, where we assume that the loadings on the time trend are the same for all firms. The results are presented in Table 2.5.

In column (1), the long-term debt share is regressed on the recession dummy, asset market beta, and their interaction term after controlling for asset volatility, book leverage, and other firm characteristics. The coefficient estimate of the recession dummy is -0.043, while the coefficient of the interaction term between asset market beta and the recession dummy is 0.025. Taken together, these values imply that (a) the long-term debt share of an average firm drops by about 2.1% from expansions to recessions (based on the average market beta of 0.879); (b) for a firm with an asset market beta at the 10th percentile, its long-term debt share is 3.9% lower from expansions to recessions, whereas the long-term debt share of a firm with asset market beta at the 90th percentile is essentially unchanged from expansions to recessions. In column (2), replacing the quadratic time trend with the trend component from the Hodrick-Prescott filter generates almost identical results.

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The results are quantitatively similar if we categorize a fiscal year as in recession when at least three months of the fiscal year are in recession.
The regression results using asset bank betas and tail betas are in columns (3) - (4). The coefficient estimates of the interaction term between the asset bank beta and the recession dummy and the interaction term between the asset tail beta and the recession dummy are both positive and statistically significant. The economic significance of the coefficient estimates are comparable to those obtained for the asset market beta.

More broadly, our model predicts that the relation between systematic risk and debt maturity is stronger in times of high aggregate risk premium. Besides recessions, another commonly used proxy of the aggregate risk premium is the market volatility index. We use VXO, the implied volatility of the S&P 100 options (a close cousin to the better-known VIX index for S&P 500 options, but VXO has a longer sample), as an alternative measure of macroeconomic conditions. Columns (5) - (7) of Table 2.5 show that debt maturity indeed becomes more sensitive to systematic risk during times of high risk premium.

In the analysis presented so far, we allow the impact of business cycles on debt maturity to depend only on firms' exposure to systematic risk. However, changes in macroeconomic conditions could also affect the relation between debt maturity and other firm characteristics. We find that, in addition to low beta firms, firms with large size and low default probability reduce their debt maturity more from expansions to recessions. This is consistent with the finding of Mian and Santos (2011) that credit worthy firms are more likely not to rollover their long-term debt in bad times. These characteristics are different from our systematic risk measures. We also find that firms with high book leverage reduce maturity more in recessions. This is consistent with Diamond (1991) in that firms with very low credit quality might only be able to issue short-term debt in bad times.

**Robustness checks**

We have conducted a series of robustness checks for the empirical tests presented above, which we summarize below.

---

20 The regression results are presented in the Internet Appendix.
First, in our model, firms are not allowed to hold cash. In practice, firms with high systematic risk exposures not only can choose longer debt maturity, but also maintain a larger cash reserve to reduce the rollover risks. For example, Harford, Klasa, and Maxwell (2013) show that firms increase their cash holdings and save more cash to mitigate the refinancing risk caused by shorter debt maturity. Thus, we expect the impact of firms’ systematic risk exposure on debt maturity (cash holdings) to become stronger after controlling for cash holdings (debt maturity). To test this hypothesis, we estimate a system of simultaneous equations in which both debt maturity and cash holdings are endogenously determined. In the debt maturity equation, we follow Acharya, Davydenko, and Strebulaev (2012) and use the 2SLS method with the following two instrumental variables for cash holdings, the ratio of intangible to book assets and the median ratio of R&D expenditures to book assets in the firm’s three-digit SIC industry.

Table 2.6 reports the estimation results for the structural equation on debt maturity.\textsuperscript{21} Panel A shows that controlling for cash holdings indeed strengthens the effect of firms betas on debt maturity. This is especially true in the case of cash flow beta, where the coefficient nearly doubles (from 0.004 in Table 2.4 to 0.007) after controlling for cash holdings. Panel B shows that the result that the relation between systematic risk and debt maturity is stronger in bad times continues to hold after controlling for cash holdings.

Next, we have computed our asset beta measures by unlevering the equity betas according to the Merton model. This procedure ignores the heterogeneity in debt maturity across firms, which could mechanically generate a relation between the unlevered asset beta and debt maturity even if the true asset beta is unrelated to debt maturity. Our model shows in which direction this bias goes. For the same asset beta, a longer debt maturity lowers the firm’s equity beta. This is especially true for high leverage firms. That means if we do not take into account the heterogeneity in debt maturity when unlevering the equity beta, we would understate the asset beta for those firms with long maturity, which biases us against finding a positive relationship.

\textsuperscript{21}The estimation results of the cash holding equation are in the Internet Appendix.
between asset beta and debt maturity. Moreover, our results hold in the sub-sample of firms with below-median leverage, where the effects of debt maturity on the unlevering procedure is negligible.

Do high-beta firms end up with longer debt maturity by issuing longer-maturity debt in the first place? The answer is yes. Using the FISD issuance data of public bonds, we find that firms with high systematic risk are indeed more likely to issue long-term bonds in normal times. The relation between bond issuance and systematic risk is not significant in economic downturns, which could be due to firms switching from public bond issuance to bank loans and lines of credit in recessions.

Other robustness checks we have performed include the following. We show that the results on the positive relation between beta and debt maturity hold for alternative measures of debt maturity, and they hold in the sample period excluding the recent financial crisis. We also show that the results are not affected by the callability of debt: high-beta firms do not tend to issue more callable debt than low-beta firms, which could have lowered their effective debt maturity. The details of these analyses are in the Internet Appendix.

4.3 Term Structure of Credit Spreads

In this section, we test our model's predictions of the effect of debt maturity dynamics on the term structure of credit spreads. Specifically, we treat the 2008 financial crisis/recession as a significant change in macroeconomic conditions and examine the resulting changes in credit spreads for firms with different maturity structures, leverage, and systematic risk exposures.

We obtain firm-level credit default swap (CDS) spreads with maturity 1 year, 5 years, and 10 years from Markit, and match the data with the COMPUSTAT information. Changes in the CDS spreads during the crisis are measured as the differences in the averages of daily CDS spreads between fiscal year 2007 and 2008. We use the fiscal year 2007 balance sheet information to compute the fraction of long-term debt that matures in 2008 \( ldebt08 = dd1/(dd1 + dltt) \). The larger this measure, the more the
debt maturity could be reduced in the crisis, the higher the rollover risk. Following Almeida, Campello, Laranjeira, and Weisbenner (2011), we treat August 2007 as the onset of the financial crisis, and focus on firms that have the 2007 fiscal year-end month in between September 2007 and January 2008. About 87% of the 375 firms with CDS data in fiscal year 2007 and 2008 meet this criterion.

We then examine the cross-sectional relation between the changes in the CDS spreads from 2007 to 2008 (based on 1 year, 5 year, and 10 year CDS spreads) and firms’ maturity structures in 2007. The financial crisis could exacerbate default risk through other firm characteristics besides maturity. Consequently, we also control for firm characteristics including market leverage, asset volatility, firm size, market-to-book ratio, profitability, tangibility, equity return (past 12 months), credit rating (from the Standard & Poor’s, converted to a numerical scale), and industry dummies (1-digit SIC code) in the regression.

Our model predicts that a bigger drop in debt maturity leads to larger increases in credit spreads in the crisis. Moreover, the maturity effect on credit spreads should be more pronounced for firms with high leverage or high systematic risk. To test these predictions, we split the sample into two halves based on the pre-crisis book leverage, market leverage, and cash flow beta, respectively. We then run cross-sectional regressions of changes in the CDS spreads on the fraction of long-term debt that matures in 2008 ($l_{debt08}$) for firms in each group separately. The regression results are presented in Table 2.7. We also use the regression coefficients to compute the implied impact of a one-standard deviation increase in $l_{debt08}$ on changes in the CDS spreads and present the results in Figure 2.9.

The results show that a firm that has a larger portion of its long-term debt maturing in 2008 would experience a more significant increase in CDS spreads. A one-standard deviation increase in $l_{debt08}$ corresponds to a 50 bps, 40 bps and 32 bps increase in the CDS spreads with maturity of 1 year, 5 years, and 10 years respectively in 2008. These results are consistent with the empirical findings in Hu (2010).

We also find evidence that the maturity drop amplifies the impact of aggregate shocks
Figure 2.9: Impact of long-term debt structure on credit spreads. This graph shows the impact of a one-standard deviation increase in the proportion of long-term debt that matures in 2008 on the changes in the CDS spreads between 2007 and 2008. Panels A and B display the result for firms sorted on book leverage and cash-flow beta, respectively.

on the credit spreads more for firms with higher leverage and higher beta before the crisis. Panel A of Figure 2.9 shows that a one-standard deviation increase in \( l_{\text{debt}08} \) raises the 1-year, 5-year, and 10-year CDS spreads by 84 bps, 77 bps, and 55 bps respectively for firms with above-median book leverage, while the corresponding increase in the CDS spreads is 31 bps, 22 bps, and 21 bps respectively for firms with below-median book leverage. The results are similar if we sort firms based on market leverage. Panel B shows that a one-standard deviation increase in \( l_{\text{debt}08} \) raises 1-year, 5-year, and 10-year the CDS spreads by 70 bps, 63 bps, and 46 bps respectively for firms with above-median cash flow beta, while the corresponding increase in the CDS spreads is negligible for firms with below-median cash flow beta.

One might be concerned that the long-term debt structure in 2007 is endogenous. Mian and Santos (2011) show that firms with good credit quality did actively manage the maturity of syndicated loans before the financial crisis through early refinancing of
outstanding loans. This would imply that those firms with high rollover risk according to \( \text{ldebt08} \) could be the firms with low quality (which are not captured by the controls), which would explain the larger increase in their credit spreads during the crisis. To address this concern, we compute \( \text{ldebt08} \) based on balance sheet information from fiscal years 2004 (\( \text{dd4} / (\text{dd1} + \text{dltt}) \)), 2005 (\( \text{dd3} / (\text{dd1} + \text{dltt}) \)), and 2006 (\( \text{dd2} / (\text{dd1} + \text{dltt}) \)). We then run cross-sectional regression of changes in the CDS spreads from 2007 to 2008 on \( \text{ldebt08} \), industry dummies, and other firm controls measured at the end of the fiscal years 2004-2006, respectively. We obtain almost identical results based on the fiscal year 2006 information (see the Internet Appendix). The results are slightly weaker for 2005 and no longer significant for 2004.

5 Concluding Remarks

Firms' maturity choices are closely linked to their systematic risk exposures and macroeconomic conditions. We build a model to explain the maturity dynamics over the business cycle, as well as their implications for the term structure of credit risk. We also provide empirical evidence for the model predictions.

While we have focused on debt maturity in this paper, there are many dimensions in which firms can manage their exposures to macroeconomic risks, including other financing decisions such as cash holding, lines of credit, payout, and real decisions such as investments and mergers and acquisitions (see e.g., Acharya, Almeida, and Campello (2012), Hugonnier, Malamud, and Morellec (2011), Bolton, Chen, and Wang (2012)). To understand the data, ultimately we need to jointly study the firms’ decisions in all these dimensions. Moreover, as a starting point, our modeling of the maturity structure dynamics is still quite stylized, especially for the purpose of pricing credit-risky securities. It would be useful to extend the model to capture more realistic maturity structures.
Table 2.1: Baseline model parameters and results. Panel A contains the parameters used in the baseline model and the results on capital structure and credit spreads. Parameters that do not vary across the states include: \( \kappa = \ln 2.5, \sigma_f = 0.23, \tau = 0.2 \). All parameters are annualized if applicable. The asset beta for the unlevered firm is 0.8 and is calculated based on a market dividend process with \( \phi = 1.25 \) relative to the cashflows of the baseline firm. The initial capital structure choices are determined in state \( G \). Panel B summarizes results from the baseline calibration.

<table>
<thead>
<tr>
<th>A. Baseline parameters</th>
<th>state ( G )</th>
<th>state ( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate state transition intensities: ( \hat{\pi}_t )</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>Riskfree rate: ( r(s) )</td>
<td>0.0558</td>
<td>0.0261</td>
</tr>
<tr>
<td>Market price of Brownian risk: ( \eta(s) )</td>
<td>0.156</td>
<td>0.24</td>
</tr>
<tr>
<td>Cash flow expected growth rate: ( \bar{\mu}(s) )</td>
<td>0.0617</td>
<td>0.0162</td>
</tr>
<tr>
<td>Cash flow systematic volatility: ( \sigma_A(s) )</td>
<td>0.1361</td>
<td>0.1555</td>
</tr>
<tr>
<td>Recovery rate: ( \alpha(s) )</td>
<td>0.72</td>
<td>0.59</td>
</tr>
<tr>
<td>Liquidity shock intensity: ( \lambda_U(s) )</td>
<td>1.5</td>
<td>3</td>
</tr>
<tr>
<td>Intermediation intensity: ( \lambda_C(s) )</td>
<td>6</td>
<td>0.75</td>
</tr>
<tr>
<td>Holding cost parameter: ( h_0(s) )</td>
<td>( 0.9884 \times 10^{-4} )</td>
<td>0.1774</td>
</tr>
<tr>
<td>Holding cost parameter: ( h_1(s) )</td>
<td>0.4179</td>
<td>0.0043</td>
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</table>

<table>
<thead>
<tr>
<th>B. Baseline model results</th>
<th>state ( G )</th>
<th>state ( B )</th>
</tr>
</thead>
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<tr>
<td>Initial market leverage: ( D/V )</td>
<td>28.5%</td>
<td>31.6%</td>
</tr>
<tr>
<td>Initial interest coverage: ( y_0/b )</td>
<td>2.68</td>
<td>2.68</td>
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<tr>
<td>Debt maturity: ( 1/m_s )</td>
<td>5.5</td>
<td>5.0</td>
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<tr>
<td>5 year default rate</td>
<td>0.6%</td>
<td>1.1%</td>
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<tr>
<td>10 year default rate</td>
<td>4.2%</td>
<td>5.6%</td>
</tr>
<tr>
<td>5 year credit spread (default component)</td>
<td>29.2 bps</td>
<td>55.1 bps</td>
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<tr>
<td>5 year credit spread (total)</td>
<td>33.6 bps</td>
<td>67.5 bps</td>
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<tr>
<td>10 year credit spread (default component)</td>
<td>97.7 bps</td>
<td>135.2 bps</td>
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<tr>
<td>10 year credit spread (total)</td>
<td>115.2 bps</td>
<td>166.2 bps</td>
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<tr>
<td>Conditional equity Sharpe ratio</td>
<td>0.12</td>
<td>0.22</td>
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</table>
Table 2.2: Summary Statistics and Correlations. This table presents descriptive statistics (Panel A) of firm-level variables (definitions see Internet Appendix) and Pearson correlations (Panel B) among risk measures.

<table>
<thead>
<tr>
<th>Variable</th>
<th>mean</th>
<th>std</th>
<th>median</th>
<th>25%</th>
<th>75%</th>
<th>obs</th>
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<td>ldebt1y</td>
<td>0.737</td>
<td>0.287</td>
<td>0.852</td>
<td>0.612</td>
<td>0.954</td>
<td>94,204</td>
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<td>ldebt2y</td>
<td>0.631</td>
<td>0.306</td>
<td>0.723</td>
<td>0.436</td>
<td>0.878</td>
<td>71,406</td>
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<tr>
<td>ldebt3y</td>
<td>0.525</td>
<td>0.315</td>
<td>0.580</td>
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<td>71,406</td>
</tr>
<tr>
<td>ldebt4y</td>
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<td>0.310</td>
<td>0.450</td>
<td>0.138</td>
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<tr>
<td>ldebt5y</td>
<td>0.349</td>
<td>0.295</td>
<td>0.324</td>
<td>0.046</td>
<td>0.582</td>
<td>71,406</td>
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<td>4.733</td>
<td>2.548</td>
<td>6.836</td>
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<td>0.271</td>
<td>0.165</td>
<td>0.398</td>
<td>94,204</td>
</tr>
<tr>
<td>mk2bk</td>
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<td>1.217</td>
<td>0.953</td>
<td>1.734</td>
<td>92,619</td>
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<tr>
<td>assetmat</td>
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<td>6.649</td>
<td>3.907</td>
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<td>0.063</td>
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<td>0.454</td>
<td>1.201</td>
<td>64,496</td>
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<td>asset bank beta</td>
<td>0.507</td>
<td>0.450</td>
<td>0.472</td>
<td>0.230</td>
<td>0.746</td>
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<td>asset tail beta</td>
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<td>0.607</td>
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<td>0.270</td>
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<td>equity market beta</td>
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<td>cash flow beta</td>
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<tr>
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<td>0.121</td>
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B. Correlations

<table>
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<tr>
<th></th>
<th>market beta</th>
<th>bank beta</th>
<th>tail beta</th>
<th>equity beta</th>
<th>cf beta</th>
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<tbody>
<tr>
<td>bank beta</td>
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<tr>
<td>tail beta</td>
<td>0.468</td>
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<td>equity beta</td>
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<td>0.639</td>
<td>0.371</td>
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<td>cf beta</td>
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<td>0.187</td>
<td>0.134</td>
<td>0.327</td>
<td>0.045</td>
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128
Table 2.3: Fama-MacBeth Regressions of Long-Term Debt Share. This table presents regressions of the fraction of debt that matures in more than 3 years on firm-specific variables: asset beta, asset volatility, firm size, abnormal earning, book leverage, market-to-book ratio, asset maturity, and profit volatility. In the Fama-MacBeth regressions (column (1) - (8)), we compute robust t-statistics using Newey-West standard errors with 2 lags, except in column (8) we use 20 lags. In the cross-sectional regression (column (9)), we compute White standard errors. Robust t-statistics are presented in parentheses below parameter estimates. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively.

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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
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<tbody>
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<td>market beta</td>
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<td>0.084***</td>
<td>0.104***</td>
<td>0.052***</td>
<td>0.054***</td>
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<td></td>
<td>-0.023*</td>
<td>-0.032**</td>
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<td>(12.04)</td>
<td>(10.55)</td>
<td>(6.43)</td>
<td>(4.14)</td>
<td></td>
<td></td>
<td>(-1.87)</td>
<td>(-1.98)</td>
</tr>
<tr>
<td>tail beta</td>
<td></td>
<td>0.045***</td>
<td>0.004***</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cf beta</td>
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<td>-0.510***</td>
<td>-0.058**</td>
<td>-0.020</td>
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<td></td>
<td>(-23.37)</td>
<td>(-21.81)</td>
<td>(-9.14)</td>
<td>(-6.77)</td>
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<td>0.048***</td>
<td>0.045***</td>
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<td>(12.01)</td>
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<td>-0.014*</td>
<td>-0.014*</td>
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<td></td>
<td>(-1.84)</td>
<td>(-1.90)</td>
<td>(-1.91)</td>
<td>(-2.98)</td>
<td>(0.09)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>bklev</td>
<td>0.275***</td>
<td>0.275***</td>
<td>0.263***</td>
<td>0.256***</td>
<td>0.253***</td>
<td>0.239***</td>
<td>0.188***</td>
<td></td>
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<td>0.007***</td>
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<td>0.009***</td>
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Observations 51,832 51,832 47,808 47,808 42,570 42,570 42,437 15,235 1,652

$R^2$ 0.004 0.070 0.091 0.090 0.206 0.205 0.205 0.161 0.328
Table 2.4: Panel Regressions of Debt Maturity. This table presents panel regressions of the fraction of debt that matures in more than 3 years on firm-specific variables: asset beta, asset volatility, firm size, abnormal earning, book leverage, market-to-book ratio, asset maturity, and profit volatility. Industry-fixed effects and year-fixed effects are also included in the regression. We adjust standard errors by clustering the observations at the industry level. Robust t-statistics are presented in parentheses below parameter estimates. Significance at the 10%, 5%, and 1% levels is indicated by *, **, and *** respectively.

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<td>0.049***</td>
<td>0.047***</td>
<td>0.043***</td>
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<td>(9.32)</td>
<td>(9.11)</td>
<td>(9.23)</td>
<td>(7.36)</td>
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<td>42,570</td>
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<td>0.159</td>
<td>0.161</td>
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Table 2.5: Debt Maturity: Impact of Macroeconomic Conditions. This table presents regression results of the fraction of debt that matures in more than 3 years on a macroeconomic variable, asset beta, an interaction between the macroeconomic variable and asset beta, firm controls (total asset volatility, firm size, abnormal earning, book leverage, market-to-book ratio, asset maturity, and profit volatility) and industry dummies. We measure macroeconomic conditions using either a recession dummy dated by NBER or the S&P 100 volatility index (VXO). We also include either a quadratic time trend or an aggregate trend generated by the H-P filter on the aggregate long-term debt share. We adjust standard errors by clustering the observations at the industry level. Robust t-statistics are presented in parentheses below parameter estimates. Significance at the 10%, 5%, and 1% levels is indicated by *, **, *** respectively.

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<td>-0.001*** (-2.67)</td>
</tr>
<tr>
<td>market beta</td>
<td>0.047*** (10.13)</td>
<td>0.024*** (2.61)</td>
</tr>
<tr>
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<td>0.025*** (2.85)</td>
<td>0.001* (1.91)</td>
</tr>
<tr>
<td>bank beta</td>
<td>0.041*** (6.47)</td>
<td>-0.000 (-0.02)</td>
</tr>
<tr>
<td>bank beta x macro</td>
<td>0.037*** (2.98)</td>
<td>0.001** (2.51)</td>
</tr>
<tr>
<td>tail beta</td>
<td>0.041*** (11.10)</td>
<td>0.025** (2.59)</td>
</tr>
<tr>
<td>tail beta x macro</td>
<td>0.020*** (2.80)</td>
<td>0.001* (1.85)</td>
</tr>
<tr>
<td>assetvol</td>
<td>-0.090*** (-5.61)</td>
<td>-0.069*** (-3.69)</td>
</tr>
<tr>
<td>m Kat</td>
<td>0.046*** (17.14)</td>
<td>0.054*** (17.12)</td>
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<td>-0.022*** (-3.39)</td>
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<td>bkev</td>
<td>0.246*** (8.90)</td>
<td>0.335*** (10.76)</td>
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<td>-0.015*** (-3.59)</td>
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<td>0.005*** (8.67)</td>
<td>0.005*** (5.98)</td>
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<td>profitvol</td>
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<td>-0.301*** (-4.15)</td>
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<th>HP Trend</th>
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<td>No</td>
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131
Table 2.6: The Effect of Cash Holdings on Debt Maturity. This table presents the second-stage regression results for the structural equation that explain debt maturity using the 2SLS methodology. The second-stage structural equation that explains debt maturity has the fraction of total debt maturing in more than 3 years as the dependent variable and the independent variables are the predicted value of the natural logarithm of cash holdings, asset beta, and firm controls (total asset volatility, firm size, abnormal earning, book leverage, market-to-book ratio, asset maturity, and profit volatility) and industry dummies. In Panel A, we include year dummies to control for year-fixed effects. In Panel B, we include a recession dummy dated by NBER, an interaction term of beta and the recession dummy, and either a quadratic time trend or an aggregate trend generated by the H-P filter on the aggregate long-term debt share. We adjust standard errors by clustering the observations at the industry level. Robust z-statistics are presented in parentheses below parameter estimates. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively.

### A. Year-Fixed Effects

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<td>-0.027</td>
<td>-0.031</td>
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<tr>
<td>bank beta</td>
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<tr>
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<td>0.040***</td>
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<tr>
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<td></td>
<td>0.040***</td>
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### B. Macroeconomic Condition

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<td>-0.042***</td>
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<td>bank beta</td>
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<td>37,304</td>
<td>37,304</td>
<td>37,184</td>
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Table 2.7: Long-Term Debt Structure and Credit Spreads. This table presents cross-sectional regression results of yearly changes in CDS spreads from fiscal year 2007 to 2008 on the proportion of long-term debt maturing in 2008, firm controls (market leverage, total asset volatility, firm size, market-to-book ratio, profitability, tangible, equity return, and credit rating), and industry dummies based on the 1-digit SIC code. The regressions are estimated for the entire sample and separately for sub-samples of firms formed on the basis of firm characteristics at the end of fiscal year 2007. For three firm characteristics, the sub-samples comprise firms with market leverage, book leverage, and cash flow beta above and below the sample median, respectively. Robust t-statistics are presented in parentheses below parameter estimates. Significance at the 10%, 5%, and 1% levels is indicated by *, **, *** respectively.

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<tr>
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Appendix

1 Model Solution

We first state a general result that will be the basis for analytically pricing both equity and debt.

**Proposition 3.** Let $f : \mathbb{R} \to \mathbb{R}^n$ be a vector-valued function satisfying the following system of ordinary differential equations:

$$ Af(x) = a_0 + \sum_{i=1}^{\ell} g_i (a_i e^{hx}) + Bf'(x) + Cf''(x) \tag{2.A.1} $$

where $A, B \in \mathbb{R}^{n \times n}$ are real-valued matrices, $C \in \mathbb{R}^{n \times n}$ is a real and non-singular matrix, $\ell \in \mathbb{N}$, $a_0 \in \mathbb{R}^n$, $a_i \in \mathbb{C}^n$, $b_i \in \mathbb{C}$, and $g_i$ is either the operator $Re(\cdot)$ (the real part of a complex number) or $Im(\cdot)$ (the imaginary part). Then $f(x)$ takes the following form:

$$ f(x) = f_0 + \sum_{i=1}^{\ell} g_i (f_i e^{hx}) + \sum_{j=1}^{n_c} \omega_j^R \Ree^\lambda_j^R e^\lambda_j^R x + \sum_{k=1}^{n_c/2} \left( \omega_k^C \Ree^\lambda_k^C \Ime^{\lambda_k^C x} + \omega_k^C \Ime^\lambda_k^C \Ree^{\lambda_k^C x} \right) \tag{2.A.2} $$

with $f_i (i = 0, ..., \ell)$ defined by

$$ Af_0 = a_0 \tag{2.A.3} $$

$$ (A - b_i B - b_i^2 C) f_i = a_i, \quad i = 1, ..., \ell \tag{2.A.4} $$

and $\omega_j^R, \omega_k^C, \Ree^\lambda_k^C, \Ime^\lambda_k^C \in \mathbb{R}$ are real-valued coefficients.

The pairs $(\lambda_j^R, \nu_j^R)$ are the real solutions to the following Quadratic Eigenvalue Problem (QEP):

$$ Av = \lambda Bv + \lambda^2 Cv. \tag{2.A.5} $$

while the pairs $(\lambda_k^C, \nu_k^C)$ are the complex solutions to the QEP with $\Im(\lambda_k^C) > 0$. In total, there are $2n = n_R + n_C$ pairs of such solutions (unique up to scaling) with $n_C$ being even. Furthermore, in the special case where $A = (a_{ij})$ satisfies

$$ a_{ij} \geq 0, \quad (i \neq j) \tag{2.A.6} $$

$$ \sum_k a_{ik} < 0, \quad \forall i \tag{2.A.7} $$

exactly half of the eigenvalues for the QEP will be located in the left (complex) half plane with the other half in the right half plane.

**Proof.** The conditions (2.A.3), (2.A.4) and (2.A.5) are obtained after substituting (2.A.2) into (2.A.1). Next, the assumptions for $A$, $B$, and $C$ imply that there are $2n$ finite eigenvalues to the QEP (2.A.5), with eigenvalue-eigenvector pairs either being real or in conjugate pairs...
(see Tisseur and Meerbergen (2001)). Since the weights $\omega_k^{C,Re}$ and $\omega_k^{C,Im}$ can be freely chosen, for each conjugate pair of solutions to (2.A.5), we may keep the solution with $\text{Im} \left( \Lambda_k^C \right) > 0$ w.l.o.g. Finally, the result for the location of the eigenvalues when conditions (2.A.6)-(2.A.7) are satisfied is shown in Barlow, Rogers, and Williams (1980).

Next, we derive the analytical solutions to debt and equity. The solution forms are similar to those obtained in Jobert and Rogers (2006) and Chen (2010), but our characterization is more general in that it allows for complex eigenvalue-eigenvector solutions to the QEP.

### 1.1 Debt Valuation

We now characterize the solution for debt taking the default boundaries $\{y_D(G), y_D(B)\}$ as given. We will work in terms of log cash flows, which we denote by $x = \log(y)$. Given a pair of (log) default boundaries $x_D(G)$ and $x_D(B)$, we first reorder the macroeconomic states so that the default boundaries are increasing:

$$x_D(\underline{s}) \leq x_D(\bar{s})$$

with $\underline{s}$ ($\bar{s}$) being the macroeconomic state with the lower (higher) default boundary.

The default boundaries define three regions:

$$\mathcal{R}_1 = (-\infty, x_D(\underline{s}))$$
$$\mathcal{R}_2 = [x_D(\underline{s}), x_D(\bar{s})]$$
$$\mathcal{R}_3 = (x_D(\bar{s}), \infty)$$

We will successively characterize debt values $D_{[i]}$ for each region $\mathcal{R}_i$.

Equity holders will always default when cash flows fall within region $\mathcal{R}_1$. Therefore debt value will be given by the bankruptcy recovery value in equation (2.8):

$$D_{[1]}(x, s, i) = \alpha(s)v(s)e^x$$

In region $\mathcal{R}_2$, equity holders will default whenever the state is $\bar{s}$. This gives

$$D_{[2]}(x, \bar{s}, i) = \alpha(\bar{s})v(\bar{s})e^x, \quad i = U, C.$$  

When the state is $\underline{s}$, debt value in region $\mathcal{R}_2$, $D_{[2]}(x, \underline{s}, i) = (D(x, \underline{s}, U), D(x, \underline{s}, C))^t$, is characterized by the following system:

$${W}_D D_{[2]}(x, \underline{s}) = {d}_{0[2]} + {d}_{1[2]}e^x + {U}_{2[2]} D_{[2]}(x, \underline{s}) + {V}_{2[2]} D''_{[2]}(x, \underline{s}).$$ (2.A.14)

The coefficients in (2.A.14) are obtained in a straightforward manner from stacking (2.7) in vector form, and so we do not state them explicitly. The solution is characterized by Proposition 3 and takes the following form:

$$D_{[2]}(x, \underline{s}) = D_{0[2]} + D_{1[2]}e^x + \sum_j \omega_{D[2],j}^{R} v_{D[2],j}^{R} e^{\lambda_{D[2],j}^{R}x} + \sum_k \left( \omega_{D[2],k}^{C,Re} \text{Re} \left( v_{D[2],k}^{C,Re} e^{\lambda_{D[2],k}^{C,Re}x} \right) + \omega_{D[2],k}^{C,Im} \text{Im} \left( v_{D[2],k}^{C,Im} e^{\lambda_{D[2],k}^{C,Im}x} \right) \right).$$ (2.A.15)
Debt value in region $R_3$, $D_3(x) = (D(x, \underline{s}, U), D(x, \underline{s}, C), D(x, \bar{s}, U), D(x, \bar{s}, C))'$, is characterized by the system

$$W_{D_3}D_3(x) = d_{03} + U_{[3]}D_3'(x) + V_{[3]}D_3''(x). \quad (2.A.16)$$

The solution here is similar to (2.A.15) and takes the following form:

$$D(x) = D_{03} + \sum_j \omega_{D_3}^R \nu_{D_3}^R x^j + e^{\lambda_{D_3}^R x} \sum_k \left( \omega_{C,R}^k R \Re \left( \nu_{C_3}^k e^{\lambda_{C_3}^k x} \right) + \omega_{C,Im}^k \Im \left( \nu_{C_3}^k e^{\lambda_{C_3}^k x} \right) \right). \quad (2.A.17)$$

Debt value needs to be bounded as $x$ goes to infinity (where debt becomes risk-free). This condition means that those coefficients $\omega_{D_3}$ in (2.A.17) corresponding to the eigenvalues that lie in the right half plane will all be equal to zero. According to Proposition 3, there are four such eigenvalues.

Finally, we are left with eight weights, four from region $R_2$ and four from region $R_3$:

$$\omega_D = \bigcup_{i=2}^3 \left\{ \omega_{j^R}^i \right\} \cup \left\{ \omega_{C,R}^i \right\} \cup \left\{ \omega_{C,Im}^i \right\}. \quad (2.A.18)$$

These coefficients are exactly identified from eight value matching and smoothness conditions at $x_D(\underline{s})$ and $x_D(\bar{s})$:

$$D(x_D(s), s, i) = \alpha(s)v(s)e^{\varepsilon_D(s)}, \text{ for } s \in \{\underline{s}, \bar{s}\} \text{ and } i \in \{U, C\}. \quad (2.A.19)$$

$$D_{[2]}(x_D(\bar{s}), \bar{s}, i) = D_{[3]}(x_D(\bar{s}), \bar{s}, i), \text{ for } i \in \{U, C\}. \quad (2.A.20)$$

$$D_{[3]}'(x_D(\bar{s}), \bar{s}, i) = D_{[3]}'(x_D(\bar{s}), \bar{s}, i), \text{ for } i \in \{U, C\}. \quad (2.A.21)$$

These boundary conditions give a system of linear equations for the weights $\omega_D$ that can be solved analytically.

### 1.2 Equity Valuation

Similar to debt, we can analytically characterize the equity value taking the default boundaries as given. In region $R_1$, equity holders will always default, and so equity will always have zero value:

$$E(x, s) = 0 \quad \text{for } x \in R_1 \quad (2.A.22)$$

Similarly, equity holders default on region $R_2$ whenever the state is $\bar{s}$ so that

$$E(x, \bar{s}) = 0 \quad \text{for } x \in R_2. \quad (2.A.23)$$

In order to solve for the remaining equity values, notice that when we plug in our solution for debt into (2.11), the resulting ODE for equity satisfies the assumptions of Proposition 3.
Thus, equity value takes the form

$$E_{[i]}(x) = E_{0[i]} + E_{1[i]} e^x$$

(2.A.24)

$$= E_{0[i]} + E_{1[i]} e^x + \sum_j \omega_{D[i],j} E_{D[i],j} e^{\lambda_{D[i],j} x}$$

$$+ \sum_j \omega_{C[1],j} E_{C[1],j} e^{\lambda_{C[1],j} x} + \sum_j \omega_{D[i],j} \text{Im} \left( E_{D[i],j} e^{\lambda_{D[i],j} x} \right)$$

$$+ \sum_k \omega_{E[i],k} \text{Re} \left( E_{E[i],k} e^{\lambda_{E[i],k} x} \right) + \sum_k \omega_{E[i],k} \text{Im} \left( E_{E[i],k} e^{\lambda_{E[i],k} x} \right).$$

The second line in (2.A.24) is deduced by matching terms involving the debt coefficients from (2.11) with the weights and exponents coming from the debt solution given in the previous section. Similar to the case of debt, asymptotic growth conditions will rule out half the eigenvalue-eigenvector pairs from region $\mathcal{R}_3$.

There remains four weights for equity, two from region $\mathcal{R}_2$ and two from $\mathcal{R}_3$, which are determined by the following value matching and smoothness conditions:

$$E(x_D(s), s) = 0, \text{ for } s \in \{ \bar{s}, \bar{s} \}.$$  

(2.A.25)

$$E_{[2]}(x_D(\bar{s}), \bar{s}) = E_{[3]}(x_D(\bar{s}), \bar{s}).$$  

(2.A.26)

$$E_{[2]}(x_D(\bar{s}), \bar{s}) = E_{[3]}(x_D(\bar{s}), \bar{s}).$$  

(2.A.27)

These conditions give a set of four linear equations for the equity weights which can be solved analytically.

Finally, the optimal default boundaries $x_D(G)$ and $x_D(B)$ are solutions to the two smooth-pasting conditions (2.13). This allows us to characterize the default boundaries as the solution to the following system of non-linear equations:

$$E_{[2]}'(x_D(s), s) = 0$$  

(2.A.28)

$$E_{[3]}'(x_D(\bar{s}), \bar{s}) = 0$$  

(2.A.29)

where the derivatives are again sums of exponential functions following from (2.A.24).

## 2 Endogenizing downward rigidity in leverage

In this section, we analyze a model with endogenous rollover decisions. The model builds on Dangl and Zechner (2007), but adds equity issuance costs and liquidity spreads for corporate bonds. The key feature of the model is that the firm does not have to roll over the matured debt immediately. Instead, it can choose when to reissue the debt and how much to issue. However, the presence of convex equity issuance costs helps the model endogenously generate downward rigidity in leverage.

For simplicity, we consider a special case of the benchmark model in our paper with a single macro state. In that case, the aggregate stochastic discount factor simplifies to

$$\frac{d\Lambda_t}{\Lambda_t} = -\tau dt - \eta dZ_t^\Lambda,$$  

(2.A.30)

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Figure 2.A.1: Credit spreads in models with endogenous vs. exogenous rollover. Equity issuance costs are set to be zero. We fix $P_0 = 7.7$, implying an initial coverage ratio $y_0/b_0 = 2.6$.

and the firm's cash flows $y_t$ follow the process

$$\frac{dy_t}{y_t} = \bar{\mu} dt + \sigma dZ_t,$$  \hspace{1cm} (2.A.31)

where $\text{cov}(dZ^A_t, dZ_t) = \rho dt$. Then, the systematic volatility of the firm's cash flows is $\rho \sigma$, which is our measure of a firm's systematic risk exposure in this model. The growth rate of cash flow under the risk-neutral measure is $\mu = \bar{\mu} - \rho \sigma \eta$. The tax rate on corporate income is still $\tau$.

The initial face value of debt is $P_0$, with coupon $b_0$. Debt is retired at a constant rate $m$, so that the remaining face value and coupon rate at time $t$ are $P_t = P_0 e^{-mt}$ and $b_t = b_0 e^{-mt}$, respectively. The firm can lever up at any time by buying back all the existing debt at face value and then issuing new debt with a new maturity. Debt issuance has a proportional cost $q$. Upon default, the asset recovery rate is $\alpha$. Finally, the initial debt maturity rate $m$ and face value $P_0$ are optimally chosen at $t = 0$.

Rather than endogenizing the liquidity discount via search frictions, here we take a reduced form approach and directly specify a liquidity spread that investors demand for holding a corporate bond with maturity rate $m$:

$$\ell(m) = \ell_0/m. \hspace{1cm} (2.A.32)$$

Next, we assume the proportional cost for issuing $x$ dollars of equity is $\xi(x)$, where

$$\xi(x) = \frac{\theta x}{y_0 e^{-mt}} \times 1_{\{x > 0\}}. \hspace{1cm} (2.A.33)$$

This specification of equity issuance costs follows Henessy, Levy, and Whited (2007). It
implies that the total dollar equity issuance cost is quadratic in the amount of issuance. We normalize the cost with $y_0$, the cash flow from the previous point of upward restructuring, to ensure that the marginal equity issuance cost is stationarity as the firm grows. The additional scaling factor $e^{-mt}$ is added for analytical tractability.²² Because the amount of equity issuance in this model is decreasing in the firm’s cash flows, equation (2.A.33) implies that the costs of equity injection will rise in a convex fashion as the firm moves closer to default.

The details of the solution are in the Internet Appendix. In short, after applying the scaling property as in Goldstein, Jr., and Leland (2001) and a change of variable as in Dangl and Zechner (2007), the problem becomes time-independent, and we are able to characterize the solution to debt, equity, and the default and restructuring boundaries analytically.

Before examining the results from the full model, we first consider the special case without equity issuance costs, i.e., $\xi(x) = 0$. We compare this model to a model where 100% of the retired debt are rolled over immediately, which highlights the importance of the downward rigidity to default risk and maturity choice.

In Figure 2.A.1, we plot the 5-year credit spreads for a firm with different average debt maturity (ranging from 1 to 10 years). In the model with endogenous rollover, credit spreads are very low. They also increase with maturity, contrary to the notion of rollover risk. The spreads are only about 15 bps when the average maturity is 10 years, and fall to essentially zero when the average maturity is under 3 years. This is because leverage declines quickly with a short debt maturity, and the firm will only reissue debt if its cash flows are sufficiently high. In contrast, in the model with exogenous rollover, credit spreads are significantly higher, and they are decreasing with debt maturity.

The fact that shorter maturity helps decrease default risk in the endogenous rollover model without equity issuance costs directly affects the maturity choice. Even though issuing

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²²It allows for a convenient change of variable to remove the time dependence of the solution.
Figure 2.A.3: Maturity choice in the model with endogenous rollover and costly equity injection. Initial interest rate coverage is fixed at 6 for all firms; this corresponds to the optimal leverage choice for the baseline firm which has a systematic volatility of 0.14. Equity issuances costs are specified with $\theta = 2/3$.

short-term debt increases the debt floatation costs, it is outweighed by the benefit of reducing bankruptcy costs. As a result, in the cross section, firms with higher systematic risk exposures choose shorter maturity in the model with endogenous rollover (Panel A of Figure 2.A.2), opposite to the results in the model with exogenous rollover (Panel B).

The contrast between the results from the two models of rollover in Figure 2.A.1 and Figure 2.A.2 shows that (1) downward rigidity in leverage is crucial for a model to generate meaningful default risk, and (2) the channel through which default risk arises is tightly connected to the tradeoff for debt maturity. The difficulty to reduce leverage following negative cashflow shocks is why firms choose to default ex post. It is also why short term debt will increase default risk, as in the model of exogenous rollover, rather than decrease it, as in the model of endogenous rollover.

Finally, we study the case with equity issuance costs. In this case, after choosing short debt maturity, reducing leverage when cash flows are low will be prohibitively expensive because it requires a large amount of equity injection in a short period of time. Figure 2.A.3 shows the result of optimal maturity choice in the endogenous rollover model with equity issuance costs. By making equity issuance more costly for firms with low cash flows, credit spreads are significantly higher, especially for firms with shorter debt maturity, and firms with higher systematic risk exposures will optimally choose longer maturity, as in our benchmark model.

3 Calibrating the liquidity friction parameters

We compute the empirical targets for the liquidity spreads at different maturities as follows. First, we compute the bond-CDS spread as the difference between the bond spread and the CDS spread for the same company at the same maturity. Bond transaction data and characteristic information such as coupon rates, issue dates, maturity dates, and issue
amounts are obtained from the Mergent Fixed Income Securities Database for the period between 2004 and 2010. To compute the bond-CDS spread, we focus on senior-unsecured fixed-rate straight corporate bonds with semi-annual coupon payments. We keep bonds with investment grade ratings as Mergent’s coverage of transactions on speculative grade bonds is small. We delete bonds with embedded options such as callable, puttable, and convertible. We also delete bonds with credit enhancement and less than one year to maturity. The corporate spread is computed as a parallel shift of the riskless zero curve, constructed from the libor-swap rates with maturity of 3 months to 10 years, such that the present value of future cash flows equals to the current bond price under the assumption of no default. The corresponding CDS spread with the same maturity is computed by interpolating CDS spreads with maturity of 6 months, 1 year, 2years, 3 years, 4 years, 5 years, 7 years and 10 years.

Before running regressions to investigate the relation between the bond-CDS spread and bond maturity, we need to address a possible sample selection bias: firms facing higher long-term liquidity spreads will likely choose to issue short-term bonds. Following Helwege and Turner (1999), we restrict the data to firms issuing both short-term bonds (maturity less than 3 years) and long-term bonds (maturity longer than 7 years) during the sample period. We then run a regression of the bond-CDS spread on bond maturity, bond characteristics (bond age, issuing amount, and coupon rate), and firm characteristics (systematic beta, size, book leverage, market-to-book ratio, and profit volatility). To identify the effects of the business cycles, we separately run the regression using the pre-crisis sample (January 2004 to June 2007) and the crisis sample (August 2007 - June 2009). The regression results are presented in Table 2.A.1. Based on the coefficient estimates, we compute average bond-cds spreads for maturity of 1 year, 5 years and 10 years respectively in both the pre-crisis period and crisis period.

We have calibrated the intensity of liquidity shocks $\lambda_U(s)$ and the matching intensity $\lambda_C(s)$ to fit the bond turnover rate in the data. It is possible that a large part of the bond trading in good times is not due to liquidity shocks, in which case our calibration procedure would overstate $\lambda_C(G)$. However, a change in $\lambda_U(G)$ would also affect the holding cost parameters, because we calibrate the holding costs for constrained investors to match the observed liquidity spreads in the data while taking $\lambda_U(s)$ and $\lambda_C(s)$ as given. If we lower the intensity of liquidity shocks, the holding cost will have to become higher in order to keep the bond liquidity spreads unchanged. This adjustment in holding cost will largely offset the impact of lower frequency of liquidity shocks on bond pricing and maturity choices.

As a robustness check, we set $\lambda_U(G)$ to 0.15, which is one-tenth of the baseline value, while keeping $\lambda_C(G)$ unchanged. Then, we examine the following two cases: (A) leaving the holding cost parameters unchanged from the baseline case; (B) recalibrating the holding cost parameters to match the bond liquidity spreads.

Figure 2.A.4 shows the optimal maturity choices for firms with different systematic risk exposures under these alternative calibrations. In Panel A, we see that by reducing the frequency of liquidity shocks and keeping the rest of the parameters fixed, long-term debt becomes more attractive. As a result, all firms raise their debt maturity in state $G$, while the maturity choice in state $B$ remains approximately the same as before. Notice that it is still

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23However, restricting the sample to firms that have issued both short-term and long-term bonds introduces another selection bias in that it rules out firms with really high liquidity spreads for long-term bonds. Our estimate can be biased downward.
true that the sensitivity of debt maturity to systematic risk is stronger in state $B$ than in state $G$. Next, if we recalibrate the holding costs so that the model-implied liquidity spreads still match the data, then the optimal maturity choices across firms become essentially identical to those in the benchmark case (Figure 2.3, Panel A).
Table 2.A.1: Liquidity Spread and Bond Maturity. This table presents regression results of the bond-CDS spread on bond maturity, asset beta, bond characteristics (rating, bond age, issuing amount, and coupon rate), and firm controls (firm size, book leverage, market-to-book ratio, and profit volatility) in two sample periods: before the crisis (January 2004 - July 2007) and during the crisis (August 2007 - June 2009). We restrict the sample to firms that have issued both short-term bonds (maturity less than 3 years) and long-term bonds (maturity longer than 7 years) straight corporate bonds in the period of 2004 - 2010. We adjust standard errors by clustering the observations at the bond issue level. Robust t-statistics are presented in parentheses below parameter estimates. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively.

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References


Illiquidity in Sovereign Debt Markets

Doctoral Dissertation Essay 3

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This paper is based on joint work with Juan Passadore
Illiquidity in Sovereign Debt Markets

Abstract

We study debt policy of emerging economies accounting for credit and liquidity risk. To account for credit risk we study an incomplete markets model with limited commitment and exogenous costs of default following the quantitative literature of sovereign debt. To account for liquidity risk, we introduce search frictions in the market for sovereign bonds. In our model, default and liquidity will be jointly determined. This permits us to structurally decompose spreads into a credit and liquidity component. To evaluate the quantitative performance of the model we perform a calibration exercise using data for Argentina. We find that introducing liquidity risk does not harm the overall performance of the model in matching key moments of the data (mean debt to GDP, mean sovereign spread and volatility of sovereign spread). At the same time, the model endogenously generates bid ask spreads, that can match those for Argentinean bonds in the period of analysis. Regarding the structural decomposition, we find that the liquidity component can explain up to 50 percent of the sovereign spread during bad times; when the sovereign is not close to default, the liquidity component is negligible. Finally, regarding business cycle properties, the model matches key moments in the data.

Keywords: sovereign debt, default default, secondary market liquidity
1 Introduction

The quantitative literature of sovereign debt studies business cycles in economies with endogenous spreads due to the risk of default. This literature has mainly focused on credit risk as the factor explaining spreads and debt capacity in sovereign nations.\(^1\) However, the recent financial crisis in the US and the sovereign crisis in Europe have highlighted that there is substantial liquidity risk associated with sovereign lending.\(^2\) Sovereign bonds are mostly traded in over-the-counter markets, where an investor who wants to sell a bond must search for a trading counterparty. While searching for a counterparty, this seller might incur in losses, and for this reason, investors need to be compensated to hold less liquid assets; this implies a higher risk premium.

In this paper we study debt policy of emerging economies taking into account both credit and liquidity risk. To account for credit risk, we will study an incomplete markets model with limited commitment and exogenous costs of default, as in Aguiar and Gopinath (2006), Arellano (2008), and Chatterjee and Eyigungor (2012); default arises endogenously because of the relative costs and benefits of default. At the same time, we introduce search frictions in the market for Sovereign bonds, as in Duffie, Garleanu, and Pedersen (2005). At any given point in time an investor can receive a liquidity shock; in our model this means that the investor now has a higher discount rate on payoffs until he sells the asset. Due to search friction, it takes time for the investor to find a counterparty. The time until he sells depends on the probability of finding a trading counterparty. The fact that some investors are liquidity constrained introduces a wedge between the valuations of the liquidity constrained and unconstrained investors. Intermediaries will exploit these wedges and bid ask spreads will emerge endogenously.

In the model credit and liquidity risk are jointly determined. Because of the liquidity risk, the sovereign pays higher spreads today which affects the default decision. Therefore, liquidity affects default risk. At the same time, higher default risk feeds back into worse liquidity conditions because investors anticipate that the liquidity conditions will be worse during default. Therefore, default affects credit risk. As a consequence of this joint determination, the model enables us to quantify the relative contribution of credit and liquidity risk in sovereign spreads.

To illustrate quantitatively the ability of the model to match key moments in the data, structurally decompose credit spreads, and resemble business cycles, we calibrate our model using data for Argentina. We find that introducing liquidity concerns does not harm the overall performance of the model in matching key moments of the data (mean debt to GDP, mean sovereign spread and volatility of sovereign spread). At

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\(^2\)Liquidity risk in sovereign debt markets has been recently documented by Pelizzon, Subrahmanyam, Tomio, and Uno (2013) and Bai, Juilliard, and Yuan (2012).
the same time, the model endogenously generates liquidity spreads, that can match the ones for Argentinean bonds in the period of analysis. Regarding the structural decomposition, we find that the liquidity component can explain up to 50 percent of the sovereign spread during bad times; when the sovereign is not close to default, the liquidity component of spreads is negligible. Finally, regarding business cycle properties, the model matches key moments in the data.

**Literature Review.** We build on the setting of the quantitative models of sovereign debt as in Aguiar and Gopinath (2006) and Arellano (2008); these two papers, extend the Eaton and Gersovitz (1981) framework of endogenous default to study business cycles in economies with risk of default. These early quantitative implementations study economies with short-term debt and no recovery on default. In our setting both long-term debt and recovery are crucial for the joint determination of credit and liquidity risk. Long-term debt was introduced by Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012). Chatterjee and Eyigungor (2012) introduce randomization to guarantee convergence of the numerical algorithm and show the existence of an equilibrium pricing function. We follow this approach to modeling long-term debt. Endogenous recovery of defaulted debt was introduced by Yue (2010) by explicitly modeling the bargaining process between the sovereign and investors in the debt restructuring process. In our model recovery is exogenous.3

We build on the setting of over-the-counter markets first studied by Duffie, Garleanu, and Pedersen (2005).4 This framework was extended by Lagos and Rocheteau (2009) to allow for arbitrary asset holdings for investors. Lagos and Rocheteau (2007) studies the entry of dealers into the market.5 Our paper structures the debt market as in Duffie, Garleanu, and Pedersen (2005) but to keep the model numerically tractable we follow He and Milbradt (2013) and we do not keep track of the asset holdings of high and low valuation investors.

Our paper is closely related to He and Milbradt (2013), which extends the models of corporate default as in Leland and Toft (1996) by introducing an over-the-counter market as in Duffie, Garleanu, and Pedersen (2005). This, uncovers a joint determination of liquidity and credit risk. We are also closely related to Chen, Cui, He, and Milbradt (2013). Our paper extends the model of sovereign debt with long-term debt instruments as in Chatterjee and Eyigungor (2012) to account for liquidity frictions.

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3 We abstract from this bargaining process because it is not crucial for our model. However, exogenous recovery implies the sovereign is willing to issue debt at low prices because he is certain that he will only repay a fraction; this behavior implies high mean and volatility of spreads. To rule out this behavior we introduce a reduced form cost of defaulting that depends on the level of debt. In a setting as in Yue (2010) this cost arises endogenously.


5 There has been an extensive literature following Duffie, Garleanu, and Pedersen (2005). Some examples are Lagos, Rocheteau, and Weill (2011) which studies crises in over-the-counter markets; Afonso and Lagos (2012) which studies high frequency trading in the market for federal funds; Atkenson, Eisfeldt, and Weill (2013) which studies the decisions of financial intermediaries to enter and exit an over-the-couter market.
as in Duffie, Garleanu, and Pedersen (2005). Despite the similarities, there is one crucial qualitative difference between the sovereign and corporate settings. The value of default in our model is endogenously determined whereas in the corporate setting this value is fixed (does not depend future liquidity conditions) and is zero in most cases.

Recent studies show that liquidity is a factor explaining sovereign spreads. Pelizzon, Subrahmanyam, Tomio, and Uno (2013) study market micro-structure using tick by tick data and document the strong non-linear relationship between changes in Italian sovereign risk and liquidity in the secondary bond market. Bai, Julliard, and Yuan (2012) find that most of the spread variations before the European sovereign debt crisis were due to liquidity and that most of the spreads were explained by credit risk in the onset of the crisis. Beber, Brandt, and Kavajecz (2009), on the contrary, show that for the Euro area, the majority of the spread is explained by credit risk. 6

The evidence showing that liquidity is a factor explaining the spread of corporate bonds is more established. Longstaff, Mithal, and Neis (2005) use data of credit default swaps to measure the size of the default and non default component of credit spreads. They find that most of the spread is due to default risk and that the non default component is explained mostly by measures of bond illiquidity. Bao, Pan, and Wang (2011) show that there is a strong link between illiquidity and bond prices. Edwards, Harris, and Piwowar (2007) study transaction costs in OTC markets and find that transaction costs decrease significantly with transparency, trade size, and bond rating, and increase with maturity. Friewald, Jankowitsch, and Subrahmanyam (2012) liquidity effects account for approximately 14 per cent of the explained market-wide corporate yield spread changes. Chen, Lesmond, and Wei (2007) also find that liquidity is priced into corporate debt for a wide range of liquidity measures after controlling for common bond-specific, firm-specific, and macroeconomic variables.

Layout. The paper is structured as follows. Section 2 describes the model environment and defines the equilibrium. Section 4 describes the calibration for Argentina and the numerical results. Section 5 concludes.

2 Model

2.1 Small Open Economy

Time is discrete and denoted by \( t \in \{0, 1, 2, \ldots \} \). The small open economy receives a stochastic stream of income denoted by \( y_t \). Income follows a first order Markov process \( \mathbb{P}(y_{t+1} = y' \mid y_t = y) = F(y', y) > 0 \). The government is benevolent and wants to maximize the utility of the households. To do this it trades bonds in the

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international bond market smoothing the households consumption. The household evaluates consumption streams according to

\[ E \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right] \]

The sovereign issues long-term debt\(^7\). To simplify the maturity structure of debt, we follow Chatterjee and Eyigungor (2012)\(^8\). Each unit of outstanding debt will mature with probability \(m\). If the unit does not mature, it pays a coupon \(z\). The advantage of this formulation of debt is that it is memory-less; whether debt was issued 1 or \(n\) periods before, the probability that this debt will mature next period will be \(m\). Therefore, the relevant state variable to measure the obligations of the government due in next period is the face value of debt.

There is limited enforcement of debt. Therefore, the government will repay debts only if it is more convenient to do so. There are two consequences of default. First, the government loses access to the international credit market so it is effectively in autarky. It regains access next period with probability \(\theta(b)\)^9. Once the government regains access the face value of debt will be \(fb\). Second, during default output is lower and given by \(y - \phi(y)\).

There are two markets for debt. In the primary market, the government can sell bonds at a price \(q_t\). The price of debt will depend on next periods bond position and current income. Our convention is that \(b_{t+1} > 0\) denotes debt and \(b_{t+1} < 0\) denotes savings. In the case of borrowing, after paying debt that matured this period \(mb_t\), and the coupon on outstanding debt \((1 - m)zb_t\), the country increases its debt position to \(b_{t+1}\). The capital inflow that the country receives today is given by \(q_t[b_{t+1} - (1 - m)b_t]\). The budget constraint for the economy is then

\[ c_t = y_t - [m + (1 - m)z]b_t + q_t[b_{t+1} - (1 - m)b_t] \]

In the secondary market, government debt can be reselled.

### 2.2 Investors

There are two types of investors (high valuation and low valuation) and two markets (primary and secondary). High valuation investors are risk neutral and discount payoffs at the rate \(r_U\). They are the only type of investors in the primary market buying debt from the government. An investor with high valuation receives an idiosyncratic

\(^7\)We assume that there is a single type of bond in this economy.

\(^8\)Long-term debt was introduced by Hatchondo and Martinez (2009) and Arellano and Ramnarayan (2012); these two papers model long-term debt as consols. The approach is analogous.

\(^9\)The probability of re-entering the market will be an increasing function of the amount of debt the government has at the moment it defaults. This assumption is consistent with models of endogenous renegotiation as Yue (2010) and is important only for the quantitative performance of the model. We discuss this in detail in the calibration section.
liquidity shock that is un-insurable; with probability $\zeta$ the investor will become liquidity constrained and his discount factor will now be $r_C$, with $r_C > r_U$. Once a high valuation investor receives a liquidity shock he becomes a liquidity constrained (low valuation) investor and is a natural seller of the asset; he values the asset less than the high valuation investors.

The liquidity constrained investor will sell the bond in the secondary market. As in Duffie, Garleanu, and Pedersen (2005), there is a search friction: a low valuation investor will meet a counter-party with probability $\lambda$. Once a low valuation investor meets an intermediary and sells, she exits the market. We will denote the valuations of the international investors by $q^H_D, q^L_D$ for debt before default and $q^H_D, q^L_D$ for debt in default for the high and low valuation investors respectively.

### 2.3 Intermediaries

This section follows He and Milbradt (2013). There is a continuum of intermediaries (broker dealers) in perfect Bertrand competition holding no stock as in Duffie, Garleanu, and Pedersen (2007). The intermediary buys from high valuation investors (H) and resells immediately to low valuation investors (L). The intermediaries contact low valuation investors with probability $\lambda$. We will assume that there is a big mass of high valuation investors ready to buy in the primary market or in the secondary market. There is Nash bargaining between the intermediary and the investors. We assume that the bargaining power of the high and low valuation investors zero and $\alpha$ respectively.\(^{10}\)

**Ask Price.** The surplus for an intermediary that is trading with the high valuation investors is given by

$$S_H = A - M$$

where $A$ is the asking price at which they are buying from high valuation investors and $M$ is the price at which the intermediary buys in the inter-dealer market. This surplus is zero because of Bertrand competition, the assumption that there is a high mass of high valuation investors, and the zero inventory restriction. Therefore, $S_H = 0$ and this implies $A = M$. The surplus of the high valuation investors is $(q^H_i - A) - q^H_{i,0}$, where $q^H_{i,0}$ denotes the valuation of the high valuation investor that has no bonds (where $i \in \{D, ND\}$). Because they have no bargaining power, they have a surplus of zero. Also, $q^H_{i,0} = 0$, since the value of not having the asset is the claim on any future surplus; because this surplus is zero, the price is zero. Then

$$A = M = q^H_i \quad (3.1)$$

\(^{10}\)This is natural because we assume that there are more high valuation investors than low valuation investors. These investors are ready to jump in and buy the bonds. The assumption that the number of type $H$ investors is much higher than the number of type $L$ investors is for tractability since it allows us to avoid keeping track of the distribution of asset holdings.
**Bid Price.** Trading between the intermediary and the low valuation investor determines the selling price. The surplus for an intermediary trading with the low valuation investors is given by

\[ S_L = M - B = q^H_i - B \]

The surplus of the low valuation investors is given by \( (B - q^L_i) - q^L_i \). Because the low valuation investors exit the market once they sell, \( q^L_i = 0 \). The total surplus (investors plus intermediary) is then \( q^H_i - q^L_i \). The bid price is such that the intermediary gets \((1 - \alpha)\) (from Nash bargaining) of the total surplus and is given by

\[ B = q^L_i + \alpha(q^H_i - q^L_i) \]  

(3.2)

**Bid-Ask Spread.** From (3.1) and (3.2) the bid ask spread will be

\[ A - B = (1 - \alpha)(q^H_i - q^L_i) \]

**2.4 Timing**

In this subsection we spell out the timing of the model.

**Before Default.** In period \( t \), if the government is not in default, it starts the period with \( b_t \) bonds outstanding. For these bonds the government will have to pay a coupon and pay principal as they mature. The total amount due in period \( t \) is \([m + (1 - m)z] b_t\). Then, income \( y_t \) is realized. After income is realized, the government decides whether to default or not \( d_t \in \{0, 1\} \). If the government does not default, it issues \([b_{t+1} - (1 - m)b_t] \) debt in the primary market to the high valuation investors at a price \( q^H_{ND}(y_t, b_{t+1}) \). If the government decides to default, consumption this period is \( c_t = y_t - \phi (y_t) \). The investors who started the period as low valuation investors will find an intermediary with probability \( \lambda \) and will sell at a price \( q^L_{ND}(y_t, b_{t+1}) \). Then, with probability \( \zeta \) the high valuation investors will receive a liquidity shock, so their effective discount rate will increase to \( r_C \) from \( r_U \). If the government decides to default in period \( t \), it will re-access the debt market in period \( t + 1 \) with debt \( f \times b_t \).

**During Default.** In period \( t \), if the government is in default, it starts the period with current defaulted debt \( b_t \). Income \( y_t \) is realized and consumption \( c_t \) is given by \( y_t - \phi (y_t) \). Investors who started the period as low valuation investors will find an intermediary with probability \( \lambda \) and will sell at a price \( q^L_{ND}(y_t, b_{t+1}) \). With probability \( \zeta \) the high valuation investors receive a liquidity shock, so their effective discount rate will be \( r_C \). With probability \( \theta \) the government will re-access the international debt market in \( t + 1 \) with outstanding debt \( f \times b_t \). Figure appendix 2.4 summarizes the timing.
2.5 Decision Problem of the Government

We represent the infinite horizon decision problem of the government as a recursive dynamic programming problem. The model has one endogenous state variable \( b \) and one exogenous state variable \( y \). We focus on a Markov equilibrium with state variables \((b, y)\).

**Value of the Option.** The value of a government that has the option to default \( V^{ND} \) is the maximum between the values of defaulting on its debt and repayment. At a particular state \((b, y)\) this value is given by

\[
V^{ND}(b, y) = \max_{\{D, C\}} \{V^D(b, y), V^C(b, y)\}
\]

where \( V^D(b, y) \) and \( V^C(b, y) \) are the values of defaulting and repaying respectively.
Value of a Government in Default. The value of a government that defaults on its debt is

\[ V^D(b, y) = u(y - \phi(y)) + \beta \mathbb{E}_{y'} \left[ \theta(b) V^{ND}(f \times b, y') + (1 - \theta(b)) V^D(b, y) \right] \]

The first term measures the flow utility: because the government defaults, the household consumes \( y - \phi(y) \) instead of \( y \). In the next period, with probability \( \theta(b) \) the government will regain access to the international debt market with an outstanding debt of \( b \). With probability \( (1 - \theta(b)) \) it will remain in default.

Value of Repayment. The value of a government that chooses to repay its debt is given by

\[ V^C(b, y) = \max_{b'} \left\{ (1 - \beta)u(c) + \beta \mathbb{E}_{y'} [V^{ND}(b', y')] \right\} \]

where consumption is given by the budget constraint

\[ c = y - [m + (1 - m)z] b + q^H_{ND}(y, b') [b' - (1 - m)b] \]

Default Sets. The default policy can be characterized by default and repayment sets. Let \( D(b) \) be the income levels such that the government prefers to default on its debt

\[ D(b) = \{ y \in Y : V^C(b, y) < V^D(b, y) \} \]

When the borrower repays its debt, the policy function for debt issue is given by

\[ b' = b'(b, y) \]

2.6 Valuations of Debt: Before Default

In this section we define the valuations of the high and low valuation investors before default. Suppose that the government has not decided to default in the state \((b, y)\).

High Valuation. The value of debt for the high valuation investors if the government wants to issue \( b' - (1 - m)b \) so that total debt increases to \( b' \) is \( q^H_{ND}(b', y) \) solves the following functional equation

\[
q^H_{ND}(b', y) = \mathbb{E}_{y'} \left\{ \left( 1 - d(b', y') \right) \frac{m + (1 - m) \left[ z + \zeta q^H_{ND}(b'', y') + (1 - \zeta) q^H_{ND}(b'', y') \right]}{1 + r_U} \right\}
\]

\[ + d(b', y') \frac{q^H_D(b', y') + (1 - \zeta) q^H_D(b', y')}{1 + r_U} \] (3.3)

The payoffs for the investor are as follows. If the government does not default on its debt in the next period, \( d(b', y') = 0 \), the investors will receive the fraction \( m \) of the
debt that is maturing and the coupon on the remaining fraction \((1 - m)\) given by \(z(1 - m)\). With probability \(\zeta\) in the next period they will receive a liquidity shock, so their remaining debt \((1 - m)\) will have a value \(q^L_{ND}(b'', y')\) for them. With probability \((1 - \zeta)\) they will receive no liquidity shock and will value debt at \(q^H_{ND}(b'', y')\). Note that \(b''\) is the optimal policy for the government in the next period in the event they do not default. Should default occur, the government cannot borrow but keeps the defaulted debt \(b'\).

If the government does default on its debt in the next period, \(d(b', y') = 1\), the investors will receive neither principal nor coupon payment; the debt will be valued \(q^L_D(b', y')\) and \(q^H_D(b', y')\) if they receive a liquidity shock and if they do not, respectively. Note that, since these investors are not currently liquidity constrained, they discount at the rate \(r_U\).

**Liquidity Constrained.** The price of debt for a liquidity constrained investor solves the following functional equation:

\[
q^L_{ND}(b', y) = \mathbb{E}^y_{y'} \left\{ (1 - d(b', y')) \frac{m + (1 - m) \left[ z + (1 - \lambda)q^L_{ND}(b'', y') + \lambda q^{Sale}_{ND}(b'', y') \right]}{1 + r_L} + d(b', y') \frac{(1 - \lambda_D)q^L_D(b', y') + \lambda_D q^{Sale}_{ND}(b', y')}{1 + r_L} \right\}
\]

(3.4)

If the government does not default on its debt in the next period, \(d(y', b') = 0\), investors will receive the fraction of debt \(m\) that is maturing and the coupon on the remaining fraction of debt \((1 - m)\) given by \(z(1 - m)\). With probability \(\lambda\) in the next period they will find an intermediary to trade their debt and will sell it at a price \(q^{Sale}_{ND}(y', b'')\). Otherwise, the investor will keep the debt and his valuation for it will be given by \(q^L_{ND}(y', b')\). Again \(b''\) is the optimal policy for the government in the next period. The sale price is the outcome from the bargaining with the intermediary and is given by

\[
q^{Sale}_{ND}(b', y) = (1 - \alpha)q^L_{ND}(b', y) + \alpha q^H_{ND}(b', y)
\]

If the government does default on its debt next period, \(d(y', b') = 1\), the investors will receive neither debt nor coupon payment; the debt will be valued \(q^L_D(y', b'')\) and \(q^H_D(y', b'')\) if they receive a liquidity shock and if they do not, respectively. The sale price in this case is

\[
q^{Sale}_D(b', y) = (1 - \alpha)q^L_D(b', y) + \alpha q^H_D(b', y)
\]

Note that we assume that the probability of finding a counterparty to trade is lower when the investor is liquidity constrained.
Figure 3.2: The figure details the bond market if the sovereign is not in default and does not default in period t. It starts by issuing debt $b_{t+1}$. This debt is bought by the high valuation investors in the primary market. After that, with probability $\lambda$ the low valuation investors will meet an intermediary. They will sell their bonds at the price $q_{t,ND}^L = \alpha q_{t,ND}^H + (1 - \alpha) q_{t,ND}^L$. After selling their bonds they exit the market.

The low valuation investors that do not meet an intermediary will try to sell their bonds next period. Then, with probability $\zeta$, the high valuation investors will receive a liquidity shock. They will have the opportunity to sell the bond next period in the secondary market. Both the high and low valuation investors will receive the debt service $m \times b_t$ and the coupon $z \times b_t$.

2.7 Valuations of Debt: After Default

Suppose that the government decides to default or enters the period without market access, with current outstanding debt $b$, and income realization the income realization is $y$.

**High Valuation.** The value of debt for the high valuation investors when the government is in default solves the following functional equation

$$q_{t,ND}^H(b, y) = \frac{1 - \theta(b)}{1 + r_U} \mathbb{E}_y \left[ \zeta q_{t,ND}^H(b', y') + (1 - \zeta) q_{t,ND}^L(b', y') \right] + \theta(b) f q_{ND}^H(y, f \times b)$$

(3.5)

With probability $(1 - \theta(b))$ the default does not get resolved. Therefore, the value of the debt next period will be $q_{t}^H(y', b)$ and $q_{t}^L(y', b)$ if they receive or not the liquidity shock, respectively. With probability $\theta(b)$ default gets resolved and the investors
receive a fraction $f$ for every dollar of debt they have. They value this debt at $q^H_{ND}(y, f \times b)$ given by (3.3).

**Liquidity Constrained.** The value of debt for the low valuation investors when the government is in default solves the following functional equation

$$q^L_D(y, b) = \frac{1 - \theta(b)}{1 + r_C} E_{y'} [\lambda_D q^\text{sale}_D(b, y') + (1 - \lambda_D) q^L_D(b, y')] + \theta(b) f q^L_{ND}(y, f \times b) \tag{3.6}$$

With probability $(1 - \theta(b))$ the default does not get resolved. With probability $\lambda_D$ the liquidity constrained investors find an intermediary and they will sell the defaulted bond at a price $q^\text{sale}_D(y', b)$ given by

$$q^\text{sale}_D(b, y) = (1 - \alpha_D) q^L_D(b, y) + \alpha_D q^H_D(b, y)$$

With probability $(1 - \lambda_D)$ they do not find an intermediary so they keep the unit of debt which they value it at $q^L_D(b, y')$. With probability $\theta(b)$ the default gets resolved, they collect $f$ for every unit of debt they had. Their valuation for this debt is $q^L_{ND}(f \times b, y)$ given by equation (3.4).

### 2.8 Equilibrium

We focus in a Markov equilibrium with state variables $(b, y)$.

**Definition (Markov equilibrium).** An equilibrium is a set of policy functions for consumption $c(b, y)$, default $d(b, y)$, and debt issue $b'(b, y)$ such that: taking as given the bond valuation $q^H_{ND}$, the policy function for consumption $c(b, y)$, debt issue $b'(b, y)$ and the default set $D(b)$, solve the borrowers optimization problem; the bond valuation functions

$$q^H_{ND}(b, y), q^L_{ND}(b, y), q^L_D(b, y), q^H_D(b, y)$$

satisfy (3.3) (3.4), (3.5) and (3.6) when default $d(b', y')$ is consistent with $D(b')$.

### 2.9 Numerical Algorithm

We follow a discrete state space method to solve for the equilibrium. As is discussed in Chatterjee and Eyigungor (2012) grid based methods have poor convergence properties when there is long-term debt. To overcome this problem, they propose a randomization procedure. We follow the prescription in Chatterjee and Eyigungor (2012) and compute a “slightly” perturbed version of the model described in this section. The details are given in the Numerical Appendix.

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11An alternative would be to solve the model using Chebyshev polynomials as in Hatchondo and Martinez (2009). Hatchondo, Martinez, and Sapriza (2010) report the performance of the discrete state space techniques.
Figure 3.3: The figure details the bond market if the government is in default or defaults in period $t$. There is no debt issue or debt service. The sovereign has an outstanding balance of debt $b_t$. The low valuation investors will meet an intermediary with probability $\lambda$. They will sell their bonds at the price $q_{t,D}^{Sale} = \alpha q_{t,D}^{H} + (1 - \alpha) q_{t,D}^{L}$. After they sell the bond they exit the market. The low valuation investors that do not meet an intermediary will try to sell next period. Then, with probability $\zeta$, the high valuation investors will receive a liquidity shock. They will have the opportunity to sell next period in the secondary market. Finally, with probability $\theta$, the government resolves the default and re-accesses the next period with face value of debt $b_t \times f$.

3 Calibration

We calibrate the model developed in the previous section to the case of Argentina. We choose to work with Argentina for two reasons. First, it makes the comparison with previous studies that focused on this case easy.\(^{12}\) Second, they had a recent episode of default with secondary market trading of debt. We will focus on the period of 1993:I and 2001:IV when Argentina had a fixed exchange rate with the dollar and was borrowing in international debt markets with the bonds traded in the secondary market.\(^{13}\)

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\(^{12}\)Examples are Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009) and Arellano (2008).

\(^{13}\)This is also the period analyzed in Arellano (2008), Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009).
Preferences, Output. The utility function is CRRA \( u(c) = \frac{c^{1-\gamma}}{1-\gamma} \). The endowment process follows
\[
\ln y_t = \rho \ln y_{t-1} + u_t
\]
with \( \rho \in (0, 1) \) and \( u_t \sim N(0, \sigma_e^2) \).\(^\text{14}\)

Default Costs Following Chatterjee and Eyigungor (2012) the loss in terms of output during default is given by\(^\text{15}\)
\[
\phi(y) = \max\{0, d_0 y + d_1 y^2\}
\]
The convexity of output costs is crucial to obtain spreads with, simultaneously, a high mean and a low volatility.\(^\text{16}\) We also introduce a functional form for the probability of reentering the international market after default. The functional form is
\[
\theta(b) = \begin{cases} 
\frac{1}{8} & \text{if } b \leq 0 \\
\frac{1}{40} & \text{if } 1.4 \leq b \\
\theta_1 b & \text{if } 0 < b < 1.4
\end{cases}
\]
(3.7)
So, the expected time in autarky after a default is a linear function of defaulted debt, with a minimum of 8 quarters and a maximum of 40 quarters. We introduce a state dependent probability of reentry to associate default costs with the amount of debt that is defaulted. One of the differences in our paper from Chatterjee and Eyigungor (2012), Arellano (2008), Hatchondo and Martinez (2009), and Arellano and Ramanarayanan (2012) is that we introduce recovery in the case of default. Recovery is crucial for our mechanism. At the same time, it affects the incentives to borrow before default. In particular, the government finds it appealing to borrow at high interest rates prior to default, because it effectively knows that will repay only a fraction of the face value. Therefore, in settings such as Chatterjee and Eyigungor (2012), Arellano (2008), Hatchondo and Martinez (2009), and Arellano and Ramanarayanan (2012) introducing recovery increases the volatility of spreads because the government is borrowing at high interest rates prior to default. To disincentivize this behavior, we

\(^{14}\)In the Appendix we provide the details of the iid random shock to income \( \epsilon_t \sim U[0, \epsilon_{max}] \) that is introduced to guarantee convergence of the numerical procedure.

\(^{15}\)As is explained in Chatterjee and Eyigungor (2012) this function nests several cases in the literature. In particular, when \( d_0 < 0 \) and \( d_1 > 0 \) the cost is zero when \( 0 \leq y \leq -\frac{d_0}{d_1} \) and rises more than proportional with output \( y > -\frac{d_0}{d_1} \). Alternatively, when \( d_0 > 0 \) and \( d_1 = 0 \) the cost is a linear function of output. The case studied in Arellano (2008) features consumption in default that is given by mean output if output is over the mean and equal to output if output is less than the mean. This implies a cost function \( \phi'(y) = \max\{y - E(y), 0\} \), which closely resembles the case of \( d_0 > 0 \) and \( d_1 = 0 \).

\(^{16}\)The intuition is that during good times the probability of default is low because the costs of default are high and therefore spreads are high. Borrower's impatience implies that debt is built up during good times. However, during bad times, spreads increase quickly because the default costs decrease and the option of defaulting is more attractive.
introduce a reduced form cost of default that depends on debt.\footnote{This reduced form cost of default is qualitatively similar to what we would obtain with explicit micro-foundations, as in Yue (2010) and Bai and Zhang (2012). These two papers explicitly micro-found recovery after default by modeling the bargaining process between the government and the international lenders. Once the government defaults, it bargains with the international investors over a surplus generated by repayment. If the government re-accesses the debt market and agrees to repay a fraction of debt it is better off because they can smooth consumption; on the other hand, the international investors recover some of the principal they lent. The actual fraction that the government repays depends on the relative bargaining powers and the outside option. Once they agree, the government starts repaying debt and reaccesses the market when it payed all of the debt.} We discuss the role of the debt dependent costs of default in the numerical results section.

**Parameters.** With these functional forms, the model has 9 parameters that are standard in the literature of long-term debt: $\beta, \gamma$ are preference parameters; $\rho_y, \sigma_y$ are the parameters for the process of output; $\varepsilon_{\text{max}}$ is the width of the support of the randomization variable; $m, z$ rate at which debt matures and coupon rate; $d_0, d_1$ output costs parameters. Our paper introduces an over-the-counter market and endogenous time in autarky after default. So, we introduce additional parameters: $r_U, r_C$ are the discount factor of the unconstrained and the constrained investor; $\zeta$ is the probability of receiving a liquidity shock; $\lambda_{ND}, \lambda_D$ are the probabilities of meeting a dealer in the case of default and not default; $\alpha_{ND}, \alpha_D$ are the bargaining powers of the intermediaries; $f$ is the recovery rate the time in autarky function; $\theta_1$ is the parameters in the probability of re-access.

**Preferences.** Risk aversion $\gamma$ is set to 2 and this is a standard value in the RBC literature and sovereign debt literature.

**Endowment.** The parameters for output are estimated from linearly detrended data adjusted for seasonality of the real GDP of Argentina. The data is quarterly and the period is 1980:I and 2001:IV; the source is Neumeyer and Perri (2005). The estimated values are $\rho_y = 0.929$ and $\sigma_y = 0.027$. The width for the randomization parameter is set to be $\varepsilon_{\text{max}} = 0.01$ in the baseline model. In the computations, we approximate the AR(1) process with Rouwenhorst (1995) in 15 states for output.

**Discount rate.** The discount rate of unconstrained investors is 1 percent, to match the risk free real quarterly return of the 3 month treasury bill in the period of study.

**Maturity.** Regarding the parameters of debt maturity we match the average maturity and coupon information in Broner, Lorenzoni, and Schmukler (2013) as used in Chatterjee and Eyigungor (2012). The maturity $m = \frac{1}{20}$ is chosen to match the median maturity of Argentina’s bonds that is equal to 20 quarters reported in Chatterjee and Eyigungor (2012). The coupon rate is set to $z = 0.03$ implying a coupon rate of 12 percent close to the 11 percent value weighted coupon rate for Argentina reported in Chatterjee and Eyigungor (2012).
Recovery. We fix recovery \( f \) in 30 percent of face value following the target in Yue (2010).

Market Reentry. We set (arbitrary) \( b_{\text{max}} - b_{\text{min}} \) at 1.4. This implies that if the country defaults with 140 percent of debt to GDP it will reenter with the lowest probability. In terms of the probabilities of reentering the financial market, there is a wide range of values used in the literature\(^{18}\); we obtain \( \theta_{\text{max}}, \theta_{\text{min}} \) from them. Beim and Calomiris (2001), report that for the 1982 default episode, Argentina spent until 1993 in a default state. For the 2001 default episode, Benjamin and Wright (2009) report that Argentina was in default starting in 2001 until 2005 when it settled with most of its bondholders. Chatterjee and Eyigungor (2012) fix \( \theta = 0.0385 \) and this implies an average exclusion period of 26 quarters or 6.5 years. We choose \( \theta_{\text{max}} - \theta_{\text{min}} \)

\[
\theta(b) = \frac{\theta_{\text{max}} - \theta_{\text{min}}}{b_{\text{max}} - b_{\text{min}}}
\]

such that the expected time in autarky ranges from 2 to 10 years.

Matching Moments. So far, the parameters that remain to be calibrated are

\[
\Theta = [\beta, d_0, d_1, \lambda_{ND}, \lambda_D, \alpha_{ND}, \alpha_D, r_C, \zeta]
\]

where \( \Theta^{sd} = (\beta, d_0, d_1) \) and \( \Theta^{oic} = (\lambda_{ND}, \lambda_D, \alpha_{ND}, \alpha_D, r_C, \zeta) \). In the sovereign setting, as opposed to the corporate setting, calibrating \( \Theta^{oic} \) is challenging because of data availability. This is particularly the case for Argentina. He and Milbradt (2013) use turnover data to calibrate \( \lambda_D, \alpha_{ND} \) and data from intermediaries profits\(^{19}\) to calibrate \( \alpha_{ND}, \alpha_D \). Then, with data from bid ask spreads, \( r_C, \zeta \) could be calibrated. So, \( \Theta^{oic} \) is identified.

For Argentinean bonds, there is no data on turnover and intermediaries profits; but, there is data on bid ask prices. So, we will rely on this for the calibration. We set the bargaining power \( \alpha_{ND}, \alpha_D \) of the investors to zero; all of the gains from trade go to the intermediary. Second, the cost of a liquidity shock, given that an investor receives a liquidity shock, is pinned down by \( r_C \) (discount factor of the constrained investor) and \( \lambda_{ND}, \lambda_D \) (probabilities of meeting an intermediary). It can be shown that increasing \( r_C \) is analogous to decreasing \( \lambda_{ND}, \lambda_D \). We fix \( r_C = 0.2 \) arbitrarily and will use \( \lambda_{ND}, \lambda_D \) to match moments. The probability of receiving a liquidity shock will be fixed in 0.25. So, the set of parameters that we will use to match moments are

\(^{18}\)As is discussed in Chatterjee and Eyigungor (2012), the definition of market access matters for these computations. Dias, Richmond, and Wang (2012) define a country as having normal market access whenever the country receives net resource transfers of 1 percent of the GDP. Using this measure half of defaulting countries do not regain access until 7 years after a default. Gelos, Sahay, and Sandleris (2011) measure the period without market access as the period up until the country issues public and publicly guaranteed bonds or syndicated loans. Using this measure, the exclusion after the default in 1982 lasted only 4 years.

\(^{19}\)The data comes from Feldhutter (2011).
\[
\Theta' = [\beta, d_0, d_1, \lambda_{ND}, \lambda_D]
\]

These parameters will be chosen to match 5 moments: average debt to GDP ratio, mean and volatility of spreads, mean and volatility of bid-ask spreads

\[
\left[ \mathbb{E}\left[\frac{b_t}{y_t}\right], \mathbb{E}\left[spr{d}_t\right], \sigma\left(spr{d}_t\right), \mathbb{E}\left[bi{d} - as{k}_t\right], \sigma\left(bi{d} - as{k}_t\right) \right]
\]

**Target Yield and Bid Ask Spreads** For the target mean and volatility of spreads we use the series in Neumeyer and Perri (2005). Over the period 1993:I and 2001:IV the mean and standard deviation of spreads was 0.0815 and 0.0443, respectively. The internal rate of return of bonds issued in the primary market is computed as \(q^H(y, b') = \left[m + (1 - m)z\right] / \left[m + r^H(y, b')\right]\). The spread is then computed as \((1 + r^H(y, b'))^d - 1" minus \((1 + r^L) - 1\). We will match this with the analogs in the data. We will use 100 basis points as target bid ask spread.\(^{20}\) The model counter-parties are computed according to \((q^H - q^L) / \frac{1}{2} (q^H + q^L)\).

**Target Debt Capacity** For the target debt capacity we use the same average debt level as in Chatterjee and Eyigungor (2012) equal to 70 percent of the GDP. As is explained in Chatterjee and Eyigungor (2012), the database of World Bank development finance does not take into account coupon payments as debt because they only measure obligations at the face value. Therefore, the model analog of debt as reported in this database is just the face value of current obligations \(b\).

**Summary of Parameter Calibration** The final parameter values can be found in Table 3.1.\(^{21}\)

**Model Moments.** The results from our baseline calibration are summarized in Table 3.2. The last column lists the baseline results from Chatterjee and Eyigungor (2012) for comparison. Our baseline model generates mean debt to gdp of 54% which is below the empirical target of 100%. Despite having recovery, which intuitively

---

\(^{20}\)We are working in a more detailed analysis of the bid-ask spreads for Argentinean bonds. The three biggest issues (in terms of face value) defaulted in 2001 were issued in 2001, so there is only one year of data. For these biggest issues, the average bid ask spread was 95 basis points. This is comparable to the bid ask spreads of Corporate and Sovereign bonds of similar credit quality. Pelizzon, Subrahmanyan, Tomio, and Uno (2013) find that the bid ask spreads for European bonds have a median of 43 basis points and can rise up to 125 basis points (period June 2011 to November 2012). Chen, Cui, He, and Milbradt (2013) report bid ask spreads of 50 basis points during normal times for junk bonds and 218 during bad times.

\(^{21}\)In order to facilitate comparisons we have purposefully restricted most of our parameters to be equal to the ones in Chatterjee and Eyigungor (2012). In particular, the default costs are the same as in their paper. The only differences in the calibration are in (a) additional liquidity parameters, and (b) our specification of reentry probabilities which depend on the level of defaulted debt (see equation (3.7)).
Table 3.1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Sovereign's discount rate</td>
<td>0.954</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Sovereign's risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>( \rho_y )</td>
<td>Persistence of output</td>
<td>0.9485</td>
</tr>
<tr>
<td>( \sigma_y )</td>
<td>Volatility of output</td>
<td>0.0271</td>
</tr>
<tr>
<td>( \varepsilon_{\text{max}} )</td>
<td>Width of randomization parameter</td>
<td>0.01</td>
</tr>
<tr>
<td>( m )</td>
<td>Rate at which debt matures</td>
<td>0.05</td>
</tr>
<tr>
<td>( z )</td>
<td>Coupon rate</td>
<td>0.03</td>
</tr>
<tr>
<td>( 1 - \theta )</td>
<td>Probability of reentry</td>
<td></td>
</tr>
<tr>
<td>( d_0, d_1 )</td>
<td>Output costs for default is ( {0, d_0y + d_1y^2} )</td>
<td>( d_0 = -0.18819, d_1 = 0.24558 )</td>
</tr>
<tr>
<td>( r_U )</td>
<td>Discount rate for unconstrained investors</td>
<td>0.01</td>
</tr>
<tr>
<td>( r_C )</td>
<td>Discount rate for constrained investors</td>
<td>0.02</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Probability of getting a liquidity shock</td>
<td>0.25</td>
</tr>
<tr>
<td>( \lambda, \lambda_D )</td>
<td>Probability of meeting a market maker</td>
<td>( \lambda = 0.8, \lambda_D = 0 )</td>
</tr>
<tr>
<td>( \alpha, \alpha_D )</td>
<td>Bargaining power</td>
<td>1</td>
</tr>
<tr>
<td>( f )</td>
<td>Recovery rate for sovereign bonds</td>
<td>0.3</td>
</tr>
</tbody>
</table>

\[ b_{\text{max}} - b_{\text{min}} = 1.4 \]

Table 3.2: Model moments.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
<th>CE (2012), Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean debt to gdp</td>
<td>1.0</td>
<td>0.54</td>
<td>0.7</td>
</tr>
<tr>
<td>mean sovereign spread</td>
<td>0.0815</td>
<td>0.0975</td>
<td>0.0815</td>
</tr>
<tr>
<td>vol. sovereign spread</td>
<td>0.0443</td>
<td>0.0519</td>
<td>0.0443</td>
</tr>
<tr>
<td>mean bid-ask spread</td>
<td>0.0100</td>
<td>0.0136</td>
<td>-</td>
</tr>
<tr>
<td>vol. bid-ask spread</td>
<td>0.002</td>
<td>0.0014</td>
<td>-</td>
</tr>
</tbody>
</table>

should lessen default incentives and lead to better borrowing terms ex-ante, debt levels are lower in our setting. Part of this is due to liquidity effects, which raise the cost of borrowing: our model’s sovereign spreads have a mean of 0.0975 with a volatility of 0.0519 both of which are higher than Chatterjee and Eyigungor (2012). Our model’s mean bid-ask spread\( ^2 \) is 0.0136 in line with the data for Argentinean bonds. Note that the Bid-Ask spread is a quantitatively important component of borrowing costs.

4 Numerical Results

For the parameter values calibrated in Section 3, we structurally decompose credit spreads, we study business cycles, and we discuss our modeling assumptions.
Figure 3.4 plots model implied bond prices and bid-ask spreads as a function of output $y$ and debt choice $b'$. First, panels A and C plot bond prices in the primary market $q^H (b', y)$ during the credit access and autarky regimes respectively. Note that standard comparative statics apply for bond prices; they are increasing in output and decreasing in debt. Also, they are always positive (event in autarky); this follows because the model features positive recovery upon default. Note that prices are much lower during autarky since recovery is set at 30% in the baseline calibration. Second, panel B plots bid-ask spreads during the credit access regime. Note that bid-ask spreads are small and flat when output is sufficiently high and default is not a concern, and rise.

\[^{22}\text{Bid-ask spreads within the model are computed as } \frac{q^H - q^L}{\frac{1}{2} (q^H + q^L)}. \text{ According to our market structure assumptions, market makers buy at price } q^L \text{ and sell at price } q^H. \text{ The mid-quote is then } \frac{1}{2} (q^L + q^H).\]
as output falls and default becomes more likely. This is because prices are forward looking and take into account the possibility of worsening liquidity conditions for defaulted bonds. Finally, panel D plots bid-ask spreads for defaulted bonds. Because during default there are no bond issues, the state variable is the amount of debt in default $b$ (and not debt choice $b'$). Note that bid-ask spreads are an increasing function of the level of debt in default. This is due to our assumption that the probability of reentry decreases for higher levels of defaulted debt and is a standard feature in sovereign debt models with renegotiation (see for example, Yue (2010), Bi (2008), and Benjamin and Wright (2009)). As in the credit access regime, bid-ask spreads are also higher when output is lower; this is due to default concerns after re-accessing credit markets.

Liquidity Feedback. In Figure 3.4 we can observe that the liquidity-credit feedback loop highlighted in He and Milbradt (2013) for corporate bonds is also present in the sovereign setting. For example, from panel B, is clear that bid-ask spreads increase as the country nears default. On one hand, wider liquidity spreads traduce in higher ex-ante borrowing costs for the country. This in turn leads to increased debt rollover costs and increases default incentives. On the other hand, higher default risk implies that worse liquidity conditions are forecasted in the event of a default, because bid-ask spreads are higher during default. These effects are nonlinear, in particular the feedback mechanism is stronger when output is low and/or when debt levels are high.

Sovereign Spread Decomposition. One of the advantages of a model where liquidity and credit risk are jointly determined is that we can decompose spreads in credit and liquidity components. To investigate this in more detail, we follow He and Milbradt (2013) and decompose the total sovereign spread into default and liquidity
components

\[ CS = CS_D + CS_L \]  

(3.8)

where \( CS \) is the sovereign spread at the time of issuance\(^{23} \), \( CS_D \) is the default component of the spread, and \( CS_L \) is the liquidity component of the spread.

The default component of the sovereign spread \( CS_D \) is computed as follows. Take an individual investor without liquidity concerns operating in a marketplace that otherwise has liquidity concerns as a whole. That is, the bond price associated with \( CS_D \) is still computed using equilibrium default and debt policies that take into account liquidity spreads, but discounting is done by an investor who faces no liquidity problems. The interpretation is that while there are liquidity concerns for the overall market (and the planner takes this into account in choosing debt and default policies), individual investors are heterogeneous and in particular there may be some investors without liquidity concerns who discount at the risk free rate. The liquidity component is just the residual \( CS_L = CS - CS_D \).

The above decomposition is plotted in Figure 3.5. Panel A plots the total sovereign spread \( CS \) as a function of current output \( y \) and debt choice \( b' \). Panels B and C plot respectively, the default component \( CS_D \) and the liquidity component \( CS_L \) as a fraction of the total spread \( CS \). Panel A highlights standard comparative statistics results for sovereign bonds: sovereign spreads increase during bad times (when output \( y \) is low) and when debt levels are high. Panels B and C show that when default risk is low (i.e. when output is high and/or debt levels are low) default risk is the predominant component, while the liquidity component becomes first order as overall default risk increases. For example, we see in Panel C that the fraction of the total sovereign spreads attributable to liquidity rises from around 0 to 50% as we move from right to left (i.e. from the low default risk region to the in default region). Even though the bonds are in default in the latter region, they nevertheless influence sovereign bond prices far away from default due to forward looking investors. These results are consistent with the feedback mechanism highlighted in He and Milbradt (2013).

**Discussion.** In the corporate setting (as in He and Milbradt (2013)) spreads can be decomposed in four terms

\[ CS = CS_{D,D} + CS_{D,L} + CS_{L,D} + CS_{L,L} \]  

(3.9)

where \( CS_{D,D} \) is a pure default component (default policies of a world without liquidity frictions), \( CS_{D,L} \) is a liquidity induced credit component (how liquidity is changing default policies), \( CS_{L,D} \) is a default induced liquidity component (calculated as a residual) and \( CS_{L,L} \) is a pure liquidity component (abstracting from default risk, and

\[^{23}\text{More precisely, the annualized credit spread depends on current output } y \text{ and the choice of debt } b' \text{ so that } CS = CS(b', y). \text{ For bonds at issue it is computed as } CS(b', y) = 4 \left[ m + (1 - m) \left( \frac{1}{q_{D}(b', y)} + (1 - q_{D}(b', y)) \right) \right]. \]
only taking into account liquidity risk). All of these components are positive. Within the sovereign default setting there are additional complications that are not present within the corporate default setting: debt policy and the autarky continuation value are endogenous. A fixed debt policy is standard in a corporate setting; the rational is that the bond issue might have a covenant that restricts further issues, and this covenant is enforceable in a court. This assumption is usually for simplicity. Furthermore, in the corporate setting, the “autarky continuation value” for equity-holders corresponds to bankruptcy the value of which is usually exogenously fixed at zero (they optimally liquidate the firm when it has no value for them). On the contrary, in the sovereign debt setting debt policy responds to changes in liquidity conditions (because of changes in spreads) and the autarky continuation is endogenously determined. So, an increase in the liquidity friction might imply a decrease in credit risk, due to a different default policy; there is not guarantee that all the terms in (3.9) are positive.\footnote{In fact, in our simulations some of these terms are negative}

### 4.2 Business Cycle Properties

The model’s business cycle properties are summarized in Table 3.3. The second column lists the empirical moments in the data, while the last column lists the results from Chatterjee and Eyigungor (2012), for comparison. Qualitatively, the model performs well. As in the data, consumption is as volatile as output and nearly perfectly correlated with output. The volatility of the current account relative to output volatility is 0.26 in the model which is close to its empirical counterpart of 0.17. Sovereign spreads have a correlation of -0.46 with output. Also, qualitatively the model does a good job of capturing counter-cyclical sovereign credit risk although quantitatively the correlation still falls short of the empirical moment of -0.79. The model generates debt service (as a fraction of output) of 4.4% and a default frequency of 2.3%. While these two numbers are qualitatively correct, they are too low quantitatively. The reason being that the debt level is too low in the baseline model. As previously mentioned,
if we allow for additional flexibility in choosing default costs so as to increase debt
levels, then the quantitative performance of the model along these dimensions will also
improve. Finally, note that the baseline model generates a positive correlation of 0.1
between the current account and output. This is an undesirable quality of the baseline
model. However, the reason for this positive correlation is the functional form for
exclusion times $\theta(b)$ that for the current calibration is steep.\textsuperscript{25} Since defaulting with
a high level of debt entails longer exclusion times, the optimal debt policy involves
paying down debt during good times so that debt levels will on average be lower
when defaults occur. For this reason, the current account and output are positively
correlated in the model. Quantitatively, improvements can be made by decreasing
the slope of $\theta(b)$ so that the incentives for implementing the above mentioned debt
reduction policies are not as strong.

### 4.3 State dependent time in autarky?

To correctly capture the volatility of sovereign spreads, in our setting with recovery, it
is crucial to impose additional costs of default that depend on the amount of debt
defaulted. In particular, as we discussed before, we choose expected exclusion times
that are increasing in the amount of defaulted debt (that is $\theta'(b) < 0$). We now
provide an intuition on why this helps in keeping volatility of spreads low.

First consider a setting in which the reentry probability is constant and independent
of the amount of debt in default. In this setting, Figure 3.6 compares default and debt
policies for a model without recovery (i.e. $f = 0$) and one with 10 percent recovery
(i.e. $f = 0.1$). Panels A and C respectively plot default policies for a model without
recovery and one with recovery. Default occurs in the northwest region. The default
threshold is “fuzzy” due to the randomization component $\varepsilon$ (see the appendix for
details). Panels B and D respectively plot debt policy in the continuation region for
the model without and with recovery; the plot fixed the randomization component at
$\varepsilon = 0$. In Panel B, we see that debt policy in the model without recovery is continuous.
In contrast, we see in Panel D that debt policy in a model without recovery contains
a jump\textsuperscript{26} in the region where both output $y$ and debt $b$ is low. This jump involves
the sovereign issuing debt at very low prices.

The intuition for this jump in debt policy is as follows. In a setting with recovery,
bond prices are always positive (see Figure 3.4), and as a result, the sovereign always
has the option to issue additional debt to smooth consumption even if this means
having to issue at extremely low prices. The discontinuity region is one where the
sovereign is precisely doing that. In fact, the level of debt is high enough such that
in simulations default almost always occurs in the next period. This turns out to be
optimal for the sovereign since upon reentering credit markets the country will only

\textsuperscript{25}As we previously hinted, having this feature is nevertheless very important.

\textsuperscript{26}Note that this jump does violate the theorem of the maximum. This is because the numerical
algorithm is discretized so that the choice set is not continuous. See app:numerical appendix for
details for the numerical algorithm.
**Figure 3.6: Default and debt policy with constant reentry probabilities.** This figure compares default and debt policies for a setting without recovery ($f = 0$) and one with a recovery rate of $f = 10\%$. Reentry probabilities are constant and do not depend on the amount of defaulted debt. Panels A and C respectively plot default policies for a model without recovery and one with recovery. Default occurs in the northwest region. The default threshold is "fuzzy" due to the randomization component $\varepsilon$ (see the appendix for details). Panels B and D respectively plot debt policy in the continuation region for the model without and with recovery. The plot fixed the randomization component at $\varepsilon = 0$.

be responsible for the recovered amount of debt which is very small in comparison; in conjunction with delayed repayment, this high debt issuance strategy becomes attractive.

In terms of the model, note that the benefit of issuing $\Delta b > 0$ units of debt is an increase consumption of $\Delta c = \Delta b \times q_{ND}^H (b + \Delta b, y)$. When bond prices are extremely low, a large amount of debt $\Delta b$ must be issued in order to increase consumption by a small amount. This implies that $\Delta b \gg \Delta c$. Should default subsequently occur the country goes into autarky with additional debt $\Delta b$, the cost of which is having to repay an additional $f \Delta b$ units of debt with certainty after regaining credit access (where $f < 1$ is the recovery rate). For low recovery rates (which is the case empirically), the marginal cost of default within the region where debt policy jumps is too low to discourage such behavior in the first place. The effect of discounting only further decreases marginal default costs within this reason.
Figure 3.7: Default and debt policy with $\theta(b)$. This figure plots default and debt policies for the baseline model which contains positive recovery rates and whose reentry probabilities are decreasing in the amount of defaulted debt (i.e. $\theta'(b) < 0$). Panel A plots the default policy. Default occurs in the northwest region. The default threshold is “fuzzy” due to the randomization component $\varepsilon$ (see the appendix for details). Panel B plots debt policy in the continuation region (more precisely, the plot fixed the randomization component at $\varepsilon = 0$).

So, the sovereign has incentives to issue debt right before default at very high rates making spreads volatile. In simulations, the volatility of sovereign spreads are often many times larger than that of the mean of sovereign spreads. Furthermore, this is entirely attributable to the “jump” region for debt policy; in fact, if we truncate the period immediately prior to default in simulations (and hence ignore the “jump” part in debt policy), then the volatility of sovereign spreads will once again look reasonable.

Second, Panel B plots debt policies when there is no recovery. There are no such jumps present when there is no recovery. Such policies of issuing a lot of debt at extremely high yields is obviously not feasible when bond prices are zero (and yields are infinite). As a result, the model without recovery can generate reasonable volatilities for sovereign spreads even when reentry probabilities are constant and independent of the amount of defaulted debt.

Since having positive recovery is crucial for generating a feedback loop between liquidity risk and sovereign credit risk (as well as an important feature empirically), additional costs are required to rule out the above mentioned jumps in debt policy. When reentry probabilities depend on the amount of defaulted debt, issuing a lot debt at extremely high yields no longer becomes attractive (even when recovery rates are positive and bond prices are always positive). Such a debt policy implies high levels of debt upon default which is costly due to extended exclusion times. To see this,
note that defaulting after issuing $\Delta b$ units of debt will decrease chance of regaining credit access by $\theta (b) - \theta (b + \Delta b) \approx -\theta' (b) \Delta b$ or equivalently the average time spent in autarky increases by $\frac{1}{\theta'(b)\Delta b}$ periods approximately. This additional cost is able to rule out jumps in debt policy. This is illustrated in Figure 3.7 which plots default and debt policies for the baseline model in which reentry probabilities decrease with the amount of debt in default. Notice that there are no longer any jumps regions in debt policy. By implication, this allows the model implied volatility of sovereign spreads to become reasonable.

5 Conclusion

We studied debt policy of emerging economies taking into account credit and liquidity risk. To account for credit risk, we followed the quantitative literature of sovereign debt in studying an incomplete markets model with limited commitment and exogenous costs of default. To account for liquidity risk, we introduced search frictions in the market for sovereign bonds. By introducing liquidity risk in an otherwise standard model of sovereign debt, default and liquidity risk are now jointly determined.

To illustrate quantitatively the ability of the model to match key moments in the data, structurally decompose credit spreads and resemble business cycles, we calibrated our model using data for Argentina. We found that introducing liquidity concerns does not harm the overall performance of the model in matching key moments of the data: mean debt to GDP, mean sovereign spread and volatility of sovereign spread. At the same time, the model generated endogenously liquidity spreads, which can match the ones for Argentinean bonds in the period of analysis. We also found that the liquidity component can explain up to 50 percent of the sovereign spread during bad times and the model matched key business cycle fluctuations data.

\footnote{Another way of addressing this issue is to have upward adjustment costs for debt levels in order to rule out sudden large increases in debt issuance.}
6 Numerical Appendix

It is well known that numerical convergence is often a problem in discrete time sovereign debt models with long-term debt. To get around this problem, we adopt the randomization methods introduced in Chatterjee and Eyigungor (2012). Should the government choose to repay its debt, total output is given by $y_t + \varepsilon_t$ where $\varepsilon_t \sim \text{Unif} (0, \varepsilon_{\text{max}})$ is a small noise component that is iid across time. As shown in Chatterjee and Eyigungor (2012), this noise component guarantees the existence of a solution of the pricing function equation. Qualitatively, it does not otherwise alter the model. A government that chooses to repay its debt will obtain

$$V^C (b, y, \varepsilon) = \max_{b'} \{(1 - \beta) u (c) + \beta \mathbb{E}_{y'|y} [V^{ND} (b', y')]\} \tag{3.A.1}$$

where the budget constraint is now given by

$$c = y + \varepsilon - b \left[ m + (1 - m) z \right] + q_H^{ND} (y, b') \left[ b' - (1 - m) b \right] \tag{3.A.2}$$

which contains the randomization component $\varepsilon$. Debt choice is denoted as $b' (b, y, \varepsilon)$. We impose that $\varepsilon_t = 0$ during the autarky regime. The value to defaulting remains the same and is given by

$$V^D (b, y) = (1 - \beta) u (y - \phi (y)) + \beta \mathbb{E}_{y'} \left[ \theta V^{ND} (f \times b, y') + (1 - \theta) V^D (b(y)) \right] \tag{3.A.3}$$

Note that the lack of choice variables in autarky means that randomization is not necessary for overall numerical convergence. The default decision is given by

$$d (b, y, \varepsilon) = 1_{\{V^C (b, y, \varepsilon) \geq V^D (b, y)\}} \tag{3.A.4}$$

and contains the randomization component. The continuation values are now adjusted as follows

$$V^{ND} (b, y) = \mathbb{E}_{\varepsilon} \left[ \max \{V^D (b, y), V^C (b, y, \varepsilon)\} \right] \tag{3.A.5}$$
in order to take into account the randomization component. Finally, bond prices are adjusted accordingly so as to take into account the additional randomization variable:

\[ q_{ND}^H (b', y) = \mathbb{E}_{y', e'} [\frac{1-d(y', e')}{1+r_U} z + \zeta q_{ND}^H (b' (b', y', e'), y') + (1-\zeta) q_{ND}^H (b', y', e'), y') ] \]

\[ q_{ND}^L (b', y) = \mathbb{E}_{y', e'} [\frac{1-d(y', e')}{1+r_C} z + \zeta q_{ND}^L (b' (b', y', e'), y') + (1-\zeta) q_{ND}^L (b', y', e'), y') ] \]

The rest of the numerical scheme is standard and follows the routine outlined in Chatterjee and Eyigungor (2012). We summarize the scheme in 4 steps:

a. Start by discretizing the state space. This involves choosing grids \{y_i\}_{i=1}^{N_y} and \{b_j\}_{j=1}^{N_b} for output and debt. The grid points and transition probabilities for output is chosen in accordance with the Rouwenhorst (1995) method. In the baseline model the number of states for output is chosen to be \( N_y = 15 \). The grid points for debt values are uniformly distributed over the range \([0, b_{max}]\) with the upper limit \( b_{max} \) chosen large enough so as never to be binding in simulations. The baseline calibration has \( b_{max} = 1.4 \) and \( N_b = 41 \). In addition, the width for the randomization parameter is set to be \( \varepsilon_{max} = 0.01 \) in the baseline calibration.

b. Next perform value function iteration. Given bond prices, update value functions \( V_C \) and \( V_D \). The debt and default policies \( b' (\cdot) \) and \( d (\cdot) \) are constructed using the algorithm outlined in Chatterjee and Eyigungor (2012). Where necessary, linear interpolation is used to obtain terms involving \( f \times b \).

c. Given debt and default policies, bond prices are then updated.

d. The above steps are iterated until both value functions and bond prices converge.
References


