Interval Attenuation Estimation

by

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Abstract

The seismic quality factor (Q) is estimated from synthetic and ultrasonic laboratory shotgathers using the spectral ratio method. Synthetic seismograms are used to estimate constant and frequency dependent Q functions of a target layer. The overburden anelasticity causes target Q estimates to increase with offset. Error caused by the attenuation of the overburden is negligible, relative to other sources of error, for highly attenuating targets (Q<30). The accuracy of estimating frequency dependent Q functions degrades as the reflectivity induced amplitude losses increase. As a result, inversion for frequency dependent Q from far offset traces failed to retrieve the target Q function accurately.

In the laboratory, Q values of Lucite, rubber and Berea sandstone slabs are estimated using ultrasonic top and bottom reflections of each slab. Each target was submerged in a water tank and a shotgather is acquired using a source with center frequency of 250 kHz and receiver hydrophones with offsets ranging from 2 cm to 16 cm at 2 cm trace interval. Both the source and the receiver are located at the water surface. The Lucite and Berea sandstone Q estimates are 25% and 15% off the actual value respectively. The rubber Q estimate contains about 40% error which is due to inherent limitation in the spectral ratio method. The width of the error bounds is found to be inversely proportional to the S/N ratio.

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Chapter 1: Introduction

Estimation of seismic wave attenuation is of practical importance due to the relation between attenuation and lithology (Klimentos and McCann 1990) and fluid and transport properties in a reservoir (Akbar et al. 1994; Dvorkin et al. 1994, Akbar et al. 1993). Many authors have attempted to estimate Q from a variety of seismic data types. Stainsby and Worthington (1987) calculated attenuation from VSP data. Jacobson (1987) investigated the relation between attenuation and velocity dispersion using refraction profiles over thick sedimentary sections. Liao and McMechan (1997) estimated Q using cross-well data. Klimentos and McCann (1990) used the reflection method (Winkler and Plona 1982) to measure Q from core samples using ultrasonic measurements to study the effects of porosity and clay content on attenuation. Dasgupta and Clark (1994) used an adaptation of the reflection method to estimate Q from CMP gathers mainly to differentiate between thick sedimentary sections and crystalline basement.

In this study, an approach similar to that used by Dasgupta and Clark (1994) is used to compute Q from synthetic and ultrasonic laboratory shotgathers. The spectral ratio method is used to calculate Q from P-wave reflections generated at the top and bottom of a target layer. Unlike Winkler and Plona (1982) who measured Q from only zero offset traces, and Dasgupta and Clark (1994) who used the Q estimates from different offsets as an aid to calculate the zero offset Q value, in this study each trace is used to compute an independent Q measurement. Also, in this study the target thickness is much smaller than in the previous studies. The target thickness in the numerical test is 1.75 wavelengths, while the thicknesses of the targets varied from 4 to 9 wavelengths in the ultrasonic lab experiments.
The thickness of the targets are reduced relative to the previous two studies in order to test the accuracy of Q estimates for targets whose thicknesses are closer to those of actual reservoirs. Q estimates from different offsets are used to study the behavior of Q estimation with increasing source-receiver offset. All the models used in this study consist of horizontally layered beds. Although the estimation procedure still holds for dipping layers, interpreting Q values from non-horizontal targets for transport properties, however, may be difficult since parameters such as lithology, saturation and differential pressure which affect Q values (Johnston et al. 1979) may change across such reservoirs. When the target is horizontal most of these factors do not vary significantly across the reservoir, and attenuation values may be interpreted in terms of fluid transport properties in the target.

This study starts with presenting the definition of Q and the spectral ratio method in chapter 2. Also, the effects of the anelasticity of the overburden is analytically developed. In chapter 3, synthetic shotgathers are used to estimate constant Q values and to investigate the overburden interference in the target Q value. In addition, frequency dependent Q functions are estimated from synthetics. In chapter 4, Q is estimated from ultrasonic laboratory experiment in a water tank. Slabs of different materials are submerged in water and a shotgather is acquired for each target. Q is estimated using the target top and bottom reflections at each offset. The conclusions are summarized in chapter 5.
Chapter 2: Method of Determining Q

2.1 Definition of Q

Q and its inverse are the most common measures of attenuation. Q is a dimensionless quantity which represents the ratio of the stored energy in a system to the dissipated energy. If the energy loss is small (Q>10), then intrinsic Q may be defined as:

\[ Q = -\frac{\omega E}{dE/dt} = -\frac{2\pi W}{\Delta W} \quad (2.1) \]

where \( E \) is the instantaneous energy of a system, \( dE/dt \) is the rate of energy loss, \( W \) is the stored elastic energy at maximum stress and strain and \( \Delta W \) is the energy loss per cycle (Toksoz and Johnston 1981). Relating Q to the amplitude (A) of an excitation rather than the elastic energy is more useful for data analysis. For linear wave propagation:

\[ W \propto A^2 \]

then:

\[ \Delta W \propto 2A\Delta A \]

and

\[ Q = -\frac{\pi A}{\Delta A} \quad (2.2) \]

where \( \Delta A \) is the amplitude loss per cycle. In the case of 1-D propagation, \( \Delta A \) can be expressed in terms of the wavelength (\( \lambda \)), and the amplitude change per length can be expressed as:

\[ \Delta A = \lambda \frac{dA}{dx} \]

Replacing \( \lambda \) by \( \frac{2\pi c}{\omega} \), where \( c \) is the velocity and \( \omega \) is the angular frequency of the excitation, yields:
\[ \Delta A = \frac{2\pi c}{\omega} \frac{dA}{dx}. \]  

(2.3)

Substituting (2.3) in (2.2), and rearranging, relates the change in wave amplitude to Q and propagation distance (x) as:

\[ \frac{1}{A} \frac{dA}{dx} = -\frac{\omega}{2cQ} dx. \]  

(2.4)

Equation (2.4) can be solved for the wave amplitude as a function of distance and frequency:

\[ A(x, \omega) = A(0, \omega) \cdot \exp\left[-\frac{\omega}{2cQ} x\right] \]

\[ = A(0, \omega) \cdot \exp[-\alpha x] \]  

(2.5)

where \( A(0, \omega) \) is the initial amplitude spectrum of the wave and \( \alpha = \left(\frac{\omega}{2cQ}\right) \) is defined as the attenuation coefficient of the medium (Baranowski 1980). Equation (2.5) indicates that the amplitude of a propagating wave in an attenuating medium exhibits exponential decay behavior, with higher decay at high frequencies (for constant Q), and that \( \exp[-\alpha x] \) is the amplitude of the attenuation impulse response. Also, equation (2.5) is the basis for the spectral ratio method used to estimate Q of different materials.

2.2 Spectral ratio method

The spectral ratio method, which is a widely used to evaluate attenuation from seismic data, is based on taking the ratio of equation (2.5) at two positions \( x_1 \) and \( x_2 \) \((x_1 < x_2)\). The logarithm of the spectral ratio is:

\[ \ln\left| \frac{A(x_2, \omega)}{A(x_1, \omega)} \right| = -\frac{(x_2 - x_1)}{2cQ} \omega = -s \omega \]  

(2.6)
which is an equation of a straight line. To apply this method to data, one estimates the slope of the ratio of the amplitude spectra recorded at positions \( x_2 \) and \( x_1 \) with respect to frequency, then equate the absolute value of the slope to \( (x_2 - x_1)/2cQ \) and solve for \( Q \).

Equation (2.6) is also used to estimate \( Q \) from reflection data (White 1992). In the 1-D convolutional model, an event \( A(t) \) can be expressed as:

\[
A(t) = s(t) * r(t) * q(t) * g(t) * f_{div}
\]  

(2.7)

where \( s(t) \) is the source wavelet, \( r(t) \) is the reflectivity series, \( q(t) \) is the attenuation impulse response, \( g(t) \) is the receiver impulse response and \( f_{div} \) is the spherical divergence factor.

Taking the ratio of (2.7) of two events \( A(t_1) \) and \( A(t_2) \) \( (t_1 < t_2) \) in the frequency domain yields:

\[
\left| \frac{A_2(\omega)}{A_1(\omega)} \right| = \left| \frac{R_2(\omega) f_{div2}}{R_1(\omega) f_{div1}} \right| \cdot \exp\left( -\frac{(t_2 - t_1)}{2Q} \omega \right).
\]  

(2.8)

If the reflectivity series is assumed to be white noise, the ratio of these series is frequency independent (Shatilo 1994). The Logarithm of the previous expression is:

\[
\ln \left| \frac{A_2(\omega)}{A_1(\omega)} \right| = \ln \left| \frac{R_2(\omega) f_{div2}}{R_1(\omega) f_{div1}} \right| - \frac{(t_2 - t_1)}{2Q} \omega.
\]  

(2.9)

Expression (2.9) represents the equation of a straight line, like (2.6), but with non-zero intercept. However, the slope gives the \( Q \) information needed if the assumption about the reflectivity holds.

2.3 Accuracy of \( Q \) estimation

It has been observed that spectral ratio estimates of large \( Q \) values contain more error than estimates of smaller \( Q \) values. This error pattern is explained by the fact that as \( Q \) increases, the logarithm of the spectral ratio approaches a horizontal line estimating the slope of which poses a numerical difficulty in the presence of noise (White 1992).

Estimates of very small \( Q \) values also contain considerable error since the definition of the
attenuation coefficient ($\alpha$) used in the spectral ratio method is valid for low loss materials ($Q > 10$). In addition to this type of error, the slope terms in equations (2.6) and (2.9) may be contaminated by propagation effects. In deriving (2.6) and (2.9), it was assumed that both waveforms share the same attenuation coefficient and that ray paths in the overburden coincide. However if the overburden does not have the same $Q$ value as the target it is possible that the estimated target $Q$ will be influenced by the overburden anelasticity.

To investigate the effect of overburden anelastic attenuation on the target $Q$ estimate, consider the earth model in figure (2-1a). The model includes a layer stack with a source at the top and a receiver directly below the source at the bottom of the stack. Within each layer, only the intrinsic attenuation impulse response is considered. According to Bickel and Natarajan (1985), the attenuation impulse response is:

$$F(\omega) = \text{Exp}[-ikx]$$

where $k = \beta - i\alpha$, $x$ is the propagation distance, $\beta$ is the wave-number ($\omega/c$) and $\alpha$ is the attenuation coefficient ($\omega/2cQ$). By expanding $k$, the impulse response becomes:

$$F(\omega) = \text{Exp}[-\alpha x] \cdot \text{Exp}[-i\beta x].$$

The first exponential in this form is the amplitude decay factor and the second exponential represents the propagation (phase) term. The spectrum of the recorded signal in figure (2-1a), $R(\omega)$, is then the multiplication of the source signal spectrum, $S(\omega)$, with a cascaded system composed of the impulse responses of all the layers. In the frequency domain, the amplitude of the recorded signal is:

$$|R(\omega)| = |S(\omega)||F_1(\omega)F_2(\omega)F_3(\omega)F_4(\omega)|$$

$$= |S(\omega)|e^{-\alpha_1(x_1 - x_0)}...e^{-\alpha_4(x_4 - x_3)}.$$
The same procedure can be used to compute the amplitudes at the top and bottom of a target layer as shown in figure (2-1b). Using equation (2.12), the amplitudes recorded at the top and bottom of layer \( t \) in figure (2-1b) are:

\[
|R_t(\omega)| = |S(\omega)| \cdot \text{Exp}[-\sum_n \alpha_n X_n] \\
|R_b(\omega)| = |S(\omega)| \cdot \text{Exp}[-\sum_n \alpha_n Y_n] \cdot \text{Exp}[-\alpha_i L]
\]  

(2.13)  
(2.14)

where \( \alpha \) is the attenuation coefficient, \( n \) is the overburden layer index, \( t \) is the target layer index, \( X_n \) is the length of top reflection ray segment in the \( n^{th} \) layer, \( Y_n \) is the length of the bottom reflection ray segment in the \( n^{th} \) layer and \( L \) is the travel path through the target. Normalizing the bottom amplitude spectrum by that of the top and taking the natural logarithm of the spectral ratio yields:

\[
\ln \left| \frac{R_b(\omega)}{R_t(\omega)} \right| = -[\alpha_i L - \sum_n \alpha_n (X_n - Y_n)].
\]

(2.15)

When the attenuation coefficient is written in terms of \( Q \), the ratio is:

\[
\ln \left| \frac{R_b(\omega)}{R_t(\omega)} \right| = -\left[ \frac{L}{2c_i Q_t} - \sum_n \frac{(X_n - Y_n)}{2c_n Q_n} \right] \omega.
\]

(2.16)

If \( Q \) is estimated from a zero offset trace, the difference between the ray paths in the overburden is zero; therefore, the top layer anelasticity does not introduce errors in the target \( Q \) estimate.

For non-zero offset arrivals, the ray paths in the overburden will not coincide which introduces overburden interference in target \( Q \). Equation (2.16) shows that the significance of the error term in the slope depends on a number of factors. High error is introduced in \( Q \) estimation if the target is very thin relative to its depth. Weakly attenuating target or highly attenuating overburden adversely affects the estimation accuracy. As the difference between the top and bottom reflections' ray trajectories in the overburden increases, more
error is introduced in the estimation. The difference between the ray trajectories increases as the offset increases. In addition, the velocity distribution in and above the target controls the ray geometry in the overburden. For positive velocity gradient, $X_n$ is greater than $Y_n$ which suggests that $Q$ increases as the offset increases.

Although this error analysis is carried out on transmitted waves, the conclusions still apply to reflected waves under the assumption that the overburden attenuation is homogenous. Under this assumption, doubling the arrival times for the top and bottom waves will extend the results to estimating $Q$ from reflected data. Ignoring reflectivity induced amplitude losses in this analysis is justified by the fact that body wave dispersion is weak so that reflectivity is almost independent of frequency. Also, the reflectivity sequence in the propagation model is assumed to be white noise, and so the ratio of the top and bottom reflectivities are frequency independent. In this case, the reflectivity introduces an intercept to the spectral ratio (equation (2.9)), but the slope, which contains $Q$ information, is not affected.
Figure (2-1): (a) The earth model used to evaluate the amplitude of a wave traveling through a series of anelastic layers. $S(\omega)$ is the source amplitude spectrum and $R(\omega)$ is the received amplitude spectrum to evaluate. (b) The propagation model used to evaluate the effects of anelasticity in the overburden layers 1 to $n$ on the estimated $Q$ value of the target, $t$, using the amplitude spectra at the top, $R_t(\omega)$, and bottom, $R_b(\omega)$, of the target. $X_n$ and $Y_n$ are the lengths of the top and bottom ray segments in any layer, $n$, in the overburden. $L$ is the travel distance in the target.
Chapter 3: Q Estimation from Synthetic Data

3.1 Introduction

The goal of this section is to estimate Q values from synthetic seismic data and to investigate the effect of the overburden anelasticity on the accuracy of Q estimates. In addition to constant Q estimation, the spectral ratio method is used to estimate arbitrary frequency dependent Q models from synthetic seismograms. The discrete wave-number (Bouchon and Aki 1977b) method is used to generate synthetic seismograms for the earth model in figure (3-1a). The discrete wave-number method is utilized in this study due to the fast run time and the ease of incorporating attenuation in the simulation.

3.2 Test procedure

For each simulation, synthetics are generated for a point source using a Ricker wavelet (Sheriff 1991) with center frequencies ranging from 20 Hz to 100 Hz and the results were summed up in the time domain. This has the effect of widening the source spectrum producing more accurate and reliable Q estimates than using a single narrow band source wavelet. Vertical displacement is calculated at 0 m, 600 m and 1200 m offsets corresponding to 0°, 17° and 34° incidence angles at the target level respectively. Q is estimated from the top and bottom P-wave reflections recorded at these offsets.

The target top and bottom P-wave reflections for the offsets of interest are windowed. The RMS velocity is calculated to the top and bottom of the target and then the NMO equation is used to locate each event. The window length is chosen to accommodate the lowest source frequency, 20 Hz, and reduce interference between events. For each experiment, the window size is kept constant to enable result comparison. A Hanning
The window is used to reduce edge effects in the frequency domain. The window is padded with zeros up to 256 points to sufficiently sample the amplitude spectrum of the data.

The amplitude spectrum of the top and bottom P-wave reflections for the 3 offsets of interest is calculated, figure (3-2). For each offset, the bottom power spectrum is divided by that of the top, figure (3-3). The slope of the log of the spectral ratio against frequency is computed using least squares fitting and is, then, equated to the slope term in equation (2.9) to estimate Q at every offset.

The two way travel time through the target at each offset is needed in the slope factor in equation (2.9) to estimate Q. At zero offset, the travel time through the target is the difference between the top and bottom reflections NMO times. For non-normal incidence, ray tracing is used to compute the travel time through the target. Due to the simple geometry of the model, fast 2 point ray tracing is used to calculate the travel time through the target zone for all offsets.

The goal of the ray tracing algorithm is to calculate the takeoff angle (i₂) of the ray that passes through the target zone and reaches the surface at the offset, x, Q is estimated at, figure (3-4). The equation the ray tracer solves is:

\[
x = 2d_1 \frac{\psi}{\sqrt{1-\psi^2}} + 2 \sum_{k=2}^{n} d_k \frac{V_k \psi}{\sqrt{1 - \left(\frac{V_k}{V_1}\right)^2 \psi^2}}
\]

(3.1)

where \(\psi\) is \(\sin i₂\) which is the unknown, \(n\) is the target layer number, \(d_k\) is the thickness of \(k^{th}\) layer, \(V_k\) is the P-wave velocity in \(k^{th}\) layer and \(x\) is the offset of the trace from which Q is estimated. This equation is solved for \(\psi\) using Newton’s method and then for \(i₂\) by taking the inverse sin of \(\psi\). The root finder converges after 15 iterations and the computed offset differs from the true offset by no more the \(10^{-4}\) meters. Using Snell’s law, the
transmission angle ($\theta$) through the target layer is calculated and the 2-way travel time

through the target is: \[ 2 \frac{d_{\text{target}}}{V_{\text{target}} \cos(\theta)}. \] \hspace{1cm} (3.2)

Now, all the terms in the slope factor in equation (2.9) can be found and inversion for Q can be performed.

3.3 Constant Q estimation test

This test is performed by fixing the target Q value and varying the attenuation in the overburden. The attenuation values obtained at the 3 offsets are compared to the estimates in the case of a non-attenuating overburden. In the first study, the target Q is held constant at 40 while the overburden Q values vary from infinity to 30. The Q estimates for this test are shown in figure (3-5). The first observation in the results is that the zero offset estimate in the case of non-attenuating overburden differs by 25% from the true Q. This error can be attributed to estimating Q from a band limited wavelet. All the zero offset estimates converge to the same Q value indicating that overburden attenuation does not affect zero offset Q estimates as predicted by equation (2.16). Also, as equation (2.16) indicates, Q estimates increase with offset showing more drift from the zero offset values as overburden attenuation increases. To confirm that this pattern is due to overburden attenuation, the Q estimates are compared with the Q drift due to overburden attenuation alone as predicted by equation (2.16). The comparison is shown in figure (3-6) and shows a good match for overburden Q values from 200 to 50 which confirms that the deviation from the zero offset Q is truly due to overburden interference. In the case of Q=30 in the top layer, the target Q estimates show more drift than predicted by theory, and, the value estimated at the near offset trace is different from the other estimates. This pattern is due to instabilities in the spectral ratio at high frequencies caused by the diminishing bandwidth of
the top reflection, which is the input to the estimation, as a result of the high attenuation in the overburden. These instabilities in the spectral ratio take the form of oscillations at high frequency as depicted in figure (3-7). The oscillations in the spectral ratio adversely affect the slope estimation leading, in this case, to high $Q$ values especially for far offset estimates.

The same test is performed with a target $Q$ of 10 and overburden $Q$ values of infinity, 200, 80, 40 and 20. The estimates, which are shown in figure (3-8), share some common characteristics with those for the previous test. For example, the estimates for overburden $Q$ of 20 show deviation from the rest of the estimates due to the same kind of instabilities in the spectral ratio, figure (3-10). However, the high attenuation of the target in this test increased the slope of the spectral ratio so that the slope is less affected by the oscillations than the slope in the case of the weakly attenuating target. Also, the $Q$ estimates increase with offset as in the previous case. Yet, the drift from the zero offset value approximately matches the analytical drift only in the case of the overburden $Q$ of 20, figure (3-9). The maximum drift is about 30% of the true $Q$ value when overburden $Q$ is 20 as opposed to 100% drift for overburden $Q$ of 30 in the previous case. This observation indicates that the attenuation of the target is the dominant factor controlling the drift due to overburden anelastic interference. As the target attenuation increases ($Q<30$), the anelasticity of the overburden becomes a secondary process and other types of errors, such as limitations in the bandwidth of the source and low S/N ratio in the data, dominate the error in $Q$.

3.4 Frequency dependent $Q$ estimation

In this test, the feasibility of estimating frequency dependent $Q$ functions using the spectral ratio method is investigated. Velocity dispersion (Aki and Richards 1980,
Kjartansson 1979) is expected to be a second order process and is not taken into account. Campillo et al. (1985) used a frequency dependent Q model without modifying the phase velocity dispersion to successfully match synthetics to seismic data.

The Q models used in this test are of the form:

\[ Q(f) = C - A \cdot \exp[-\left(\frac{f-f_c}{\sigma}\right)^2] \]  

(3.3)

where C is a constant Q representing the background matrix attenuation, A is the magnitude of the drop in Q due to some form of fluid motion caused by the passing wave which has the effect of increasing attenuation as the frequency approaches \( f_c \). The table below lists the C, A, \( \sigma \) and \( f_c \) parameters used for the different models tested in this section:

<table>
<thead>
<tr>
<th>Model</th>
<th>C</th>
<th>A</th>
<th>( \sigma ) (Hz)</th>
<th>( f_c ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>100</td>
<td>90</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>Model 2</td>
<td>80</td>
<td>60</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>Model 3</td>
<td>200</td>
<td>160</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>Model 4</td>
<td>20</td>
<td>10</td>
<td>10</td>
<td>50</td>
</tr>
</tbody>
</table>

Model 1 shows a drop in Q from relatively non-attenuating background (Q=100) to a Q of 10 at 50 Hz. In model 2, the magnitude of the drop is lower and the background attenuation is higher relative to the fluid motion induced attenuation. Model 3 represents a case when the Q drop from the background level is huge and the fluid attenuation is relatively weak. In contrast to model 2, model 4 shows very small change between background and fluid attenuation when both attenuations are high.

The first step in Q estimation is to convert the spectral ratios at each offset, figure (3-11) to Q values using the slope terms in equation (2.9) at each frequency. These Q values are shown as ‘x’ symbols in figures (3-12) to (3-15). Some Q values are not shown in the figures because they are unrealistically high or negative. These instabilities
are associated with low background attenuation, low frequencies and zero offset Q estimates. This correlation is reasonable because for high Q values, attenuation is underdetermined at low frequencies and small travel paths within the target. The values that are not shown in the figures were not used in the later inversion.

The last step in inverting for the Q models is to fit a function which represent a constant background Q and a Gaussian drop in Q centered around some frequency through the point Q values. This procedure represents a non-linear inversion problem. The problem is linearized around an initial guess and solved iteratively (Menke 1989).

Applying this approach to the model at hand produces the following system of equations:

\[ G_n \cdot \Delta m_{n+1} = d - g(m_n^{est}) \]  (3.4 a)

\[ m_n^{est+1} = m_n^{est} + \Delta m_{n+1} \]  (3.4 b)

where \( G_n \) is a matrix of the derivative of the Q model with respect to the model parameters evaluated at the \( n^{th} \) guess of the model parameters, \( d \) is the data which represent Q values at a given offset, \( m_n^{est} \) is the current guess of the model parameters, \( \Delta m_{n+1} \) is the perturbation to the current guess to compute \( m_n^{est+1} \) which is the guess for the n+1 iteration. Figures (3-12) to (3-15) show the inversion results, using the equations above, compared to the actual models used in the simulation. The inversion results show that for all the models, the far offset results produced very small Q values and broad Q function. This loss of resolution is caused by the fact that at far offset, the reflectivity series and the divergence of the waves introduces a significant intercept, as predicted by equation (2.9), which lead in this case to underestimating Q. For the rest of the offsets the inversion results predicted the center frequency of the different models within 3 Hz and the standard deviation of the curves within 5 Hz. In model 3, the inversion result contained greater error in the constant background Q estimate than other results due to the high C value in model 3 (Q=200).
The inversion procedure succeeded in resolving the very small increase in attenuation around 50 Hz in model 4.

3.5 Conclusion

It is found that the near offset constant Q estimates always underestimate the actual Q of the target. The narrow band of the source signal is suspected of causing such systematic bias in the estimates, however, this behavior is not investigated in this chapter. Estimating constant Q from synthetic seismograms confirms equation (2.16) which predicts that the Q estimates increase with offset due to anelasticity in the overburden. However, it is found that the effects of the overburden are significant only for weakly attenuating targets (Q>30). As the attenuation of the target increases, overburden anelasticity interference in the target Q becomes a second order effect. As the overburden attenuation increases, the bandwidth of the reflections used in the inversion decreases leading to errors in estimated Q.

The estimation of frequency dependent Q functions from top and bottom reflection spectral ratios is also investigated. The models used in this study are in the form of a constant Q value representing the background rock matrix attenuation plus a Gaussian drop in attenuation around a given frequency which represent a phenomenon similar to fluid resonance due to the passing wave. The inversion results, using an iterative scheme, match the actual models with good accuracy for the zero and 600m offsets. The attenuation center frequency estimated is within 3 Hz and the standard deviation of the Gaussian Q function estimate is within 5 Hz. The constant matrix Q results show the expected high error for high Q values. However, the maximum error in the constant Q value for all the models is no more than 25%. The inversion results for the 1200m offset underestimates the actual Q values by as much as 50%. Also, the standard deviation of the Gaussian drop
in Q is larger than the actual value with a maximum error of about 300%. The increasing reflectivity induced amplitude change is the primary reason for the errors in Q in the far offset estimates. The reflectivity changes increased the significance of the intercept in the spectral ratio, as shown in equation (2.9), which led to error in Q estimates and the loss of resolution in the spectral ratio.
Figure (3-1): (a) the earth model used as input for the discrete wave-number simulator. The target thickness in 1.75 λ. (b) A sample output for the model in (a). The first and second events are the top and bottom P-wave reflections of the target. The third event is a converted S-wave from the top of the target.
Figure (3-2): The amplitude spectra for the top and bottom reflections. In this case, target $Q=10$. 
Figure (3-3): The amplitude spectral ratio of the bottom to the top reflection for target $Q=10$. 
Figure (3-4): The ray tracer is based on solving for the takeoff angles $i_1$ and $i_2$ that will allow the top and bottom reflections to emerge at the same offset given the velocity and the structure of the subsurface.
Figure (3-5): The target $Q$ is 40. The $Q$ estimates increase with offset and converge to the zero offset $Q$ value for the case of a non-attenuating overburden.
Figure (3-6): The measured and analytical Q drift from zero offset value for target Q of 40. The analytical drift is shifted to the zero offset value to allow for direct comparison between measured and calculated drift. The error in the measured Q value in the case of non-attenuating overburden is induced by the limited bandwidth of the source. For attenuating overburden (Q=30), instabilities in the inversion may arise especially for far offset estimates.
Figure (3-7): The spectral ratios at the three offsets of interest in the case when the target Q is 40 and overburden Q=30. The oscillation in the spectral ratio, especially in the far offset spectral ratio, is caused by the diminishing band width of the reflection as a result of propagation in the overburden. The ‘+’ mark is the spectral ratio and the solid curve is the best fit line.
Figure (3-8): The target Q is 10. The Q estimates increase with offset and converge to the zero offset Q value for the case of a non-attenuating overburden.
Figure (3-9): The measured and analytical Q drift from the zero offset value for target Q of 10. The analytical drift is shifted to the zero offset value to allow for direct comparison between measured and calculated drift. The error in the measured Q value in the case with a non-attenuating overburden is induced by the limited bandwidth of the source.
Figure (3-10): The instability in the spectral ratio when target Q is 10 does not affect the Q estimate as severely as when the target Q is 40 since the slope now is higher so that similar deviations in the ratio will not affect the overall slope of the ratio. The ‘+’ mark is the spectral ratio and the solid curve is the best fit line.
Figure (3-11): The spectral ratio for a frequency dependent Q function.
Figure (3-12): The results for Q model 1 at three offsets. The numbers in the upper right corner of each plot represent the center frequency, the Q value at the center frequency and the standard deviation of the inverted model, respectively.
Figure (3-13): The results for Q model 2 at three offsets. The numbers in the upper right corner of each plot represent the center frequency, the Q value at the center frequency and the standard deviation of the inverted model, respectively.
Figure (3-14): The results for Q model 3 at three offsets. The numbers in the upper right corner of each plot represent the center frequency, the Q value at the center frequency and the standard deviation of the inverted model, respectively.
Figure (3-15): The results for Q model 4 at three offsets. The numbers in the upper right corner of each plot represent the center frequency, the Q value at the center frequency and the standard deviation of the inverted model, respectively.
Chapter 4: Q Estimation from Ultrasonic Experiment in a Water Tank

4.1 Introduction

The goal of this chapter is to estimate Q of three slabs of Lucite, rubber and Berea sandstone in ultrasonic water tank lab experiment using top and bottom P-wave reflections from the target. Such a technique was used by Best et al. (1994), Klimentos and McCann (1990) and Winkler and Plona (1982) to estimate Q from cores. Their technique employed a single transducer used as both source and receiver. The above experiments were run at the mega Hertz frequency range. In this experiment, however, 2 transducers, one as a source and the other as a receiver, are used to acquire a shotgather over the target layer so that Q is estimated from non-zero offset reflections. Also, the frequency range is in the kilo Hertz range so that the dimensions and the depth of the target are closer to those encountered in seismic exploration.

4.2 Experimental setup

In this experiment, three shotgathers were acquired for three different materials each submerged in a water tank. The source and receiver were positioned at the water surface as shown in figure (4-1). Each target block was placed at a depth which separated the top and bottom reflections form the strong water bottom multiples. The table below lists the different materials used and some of their relevant properties:
<table>
<thead>
<tr>
<th></th>
<th>P-wave Velocity (m/s)</th>
<th>S-wave Velocity (m/s)</th>
<th>Density (kg/m³)</th>
<th>Thickness (m)</th>
<th>Thickness (λ)</th>
<th>Actual Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water (overburden)</td>
<td>1500</td>
<td>0</td>
<td>1000</td>
<td>0.15</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Lucite</td>
<td>2700</td>
<td>1290</td>
<td>1180</td>
<td>0.1</td>
<td>9</td>
<td>50</td>
</tr>
<tr>
<td>Rubber</td>
<td>1600</td>
<td>800</td>
<td>1180</td>
<td>0.025</td>
<td>4</td>
<td>5-10</td>
</tr>
<tr>
<td>Berea s. s.</td>
<td>4400</td>
<td>1950</td>
<td>2080</td>
<td>0.1</td>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>

All the blocks have the same length and width (7.0 cm x 20 cm). Relative to the wavelength, all the targets used in the experiment are considered thick.

The source used is a Panametrics (V303, 1 MHz) transducer and the receiver is a B&K (8103) hydrophone. The source transducer is excited with a 100 Hz square function. Since the maximum recording time for each target is 5% of the period of the square function, then the source signal basically is a step function. The data is low pass filtered and then recorded. The recorded source wavelet and its amplitude spectrum are shown in figure (4-2). The source center frequency is around 250 kHz and the bandwidth is rather narrow. However, obtaining constant Q values from such a spectrum is successful. The source waveform has a long tail caused by internal reflections within the piezoelectric crystal in the transducer.

For each target, traces are recorded from 2 cm to 16 cm offset at a 2 cm trace interval. This offset distribution represents a range from 4° to 28° incidence angles at the level of the top of the target. The data sets collected for Lucite, rubber and Berea sandstone are shown in figure (4-3). In the case of the sandstone, the bottom reflections are much smaller than the top reflections due to the small transmission coefficient caused by the high velocity contrast between the water and the target. The travel time in the rubber target is small. However, the top and bottom reflections are resolvable at all offsets and the small time thickness causes no problem in the Q estimation stage.
4.3 Data analysis

Processing the data starts with windowing the top and bottom reflections. Using the velocities in the previous table, the NMO equations for the top and bottom are used to locate the events. These events are then windowed and a cosine taper is applied at the beginning and end of the window to reduce edge effects in the frequency domain. The whole source waveform is used in Q estimation because both the main pulse and its tail pass through the target and should produce consistent Q values. Using the first cycle of the source wavelet artificially smooths the spectral ratio. However, the estimates obtained by windowing the whole waveform or the first cycle differ by no more than 20%. For each offset, the amplitude spectrum of the bottom reflection is normalized by the amplitude of the top reflection and the slope term in equation (2.9) is used to estimate Q. The travel time through the target, needed to calculate Q, is estimated by a ray tracer which solves for the top and bottom ray geometries for both events to emerge at the offset of interest. However, estimating Q at this stage produces unrealistic and even negative values. The solid curve in figure (4-4) shows that the slope of the spectral ratio becomes positive at high frequencies and far offsets. This behavior implies that the top reflection is getting progressively weaker than the bottom reflection at far offsets. The radiation pattern of the source is believed to be the cause of this problem for two reasons. The first reason is that the inversion in the sign of the slope of the spectral ratio occurs at high frequencies, relative to the source center frequency, where the radiation pattern is naturally narrower than at low frequencies. The second reason is that due to the velocity increase at the top of the target, the takeoff angle of the top reflection ray is larger than the takeoff angle of the bottom reflection ray which weakens the amplitude spectrum of the top reflection relative to the bottom reflection at large offsets. Due to this effect, the top and bottom amplitude spectra have to be corrected for the source radiation pattern before attempting to estimate Q.
4.4 Source radiation pattern correction

The amplitude spectra of the reflections are corrected for the source radiation pattern (Tang et al. 1988) empirically. To do so, a new data set is acquired in which the source is positioned at the water surface and the receiver was placed at the depth of the top of the target. The receiver is then moved from directly under the source to 19 cm offset with the direct arrival being recorded at 1 cm trace interval, see figure (4-5). The direct arrivals sample the source radiation pattern from $0^\circ$ to $52^\circ$ at an increment of $2^\circ$. After correcting for divergence, the amplitude spectra for each direct arrival is computed and the amplitude drop for each source frequency component as a function of takeoff angle is normalized by the amplitude at $0^\circ$. Figure (4-6) shows the raw and a smoothed version of the normalized amplitude drop at 120 kHz, 270 kHz and 390 kHz. It is obvious from the figure that the amplitude loss is most significant at high frequencies. The smoothed curves are used to build the radiation pattern correction function for all the significant source frequencies as shown in figure (4-7). The correction of the amplitude spectrum of an event starts with calculating the takeoff angle from the source. This angle is used to find the associated correction value for each frequency, then the amplitude spectrum of the event is multiplied by the correction function. Applying this correction significantly improves Q estimates especially for far offsets. Figure (4-4) shows the difference between the spectral ratio before and after the radiation correction.

4.5 Estimated Q values

The final Q estimates for the different targets as a function of offset are shown in figure (4-8). According to Toksöz et al. (1979), the actual values for Lucite and Berea Sandstone are 50 and 20 respectively. Rubber is known to be a highly attenuating material with Q between 5 and 10. The results of the estimation are summarized in the following table:
<table>
<thead>
<tr>
<th>Target</th>
<th>Estimated Q</th>
<th>90% Confidence interval</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Lucite</td>
<td>50</td>
<td>40</td>
<td>±5</td>
</tr>
<tr>
<td>Rubber</td>
<td>5-10</td>
<td>17</td>
<td>±2</td>
</tr>
<tr>
<td>Berea Sandstone</td>
<td>20</td>
<td>23</td>
<td>±7</td>
</tr>
</tbody>
</table>

The near offset estimate for Lucite is 40. It is 25% different from the actual Lucite Q. The near offset Q for Berea sandstone is 23 which is 15% different from the actual value. These two results show the pattern of increasing error in Q estimate as attenuation of the target decreases. The Q value of rubber is 17 and since rubber is a highly attenuating material with Q less than 10, this estimate contains significant error. However, the spectral ratio method cannot measure low Q values because it is based on the definition of the attenuation coefficient for low loss materials (Q>10) which causes error in the estimate.

Error bounds for each Q value at each offset is calculated by computing a 90% confidence interval on the slope of the spectral ratio. In general, the magnitude of the error bars increases with offset. This pattern is caused by increasing the noise level in the spectral ratio due to the process of radiation pattern correction. Rubber Q estimates, on the other hand, show very narrow error bars running from ±2 to ±5 since it required the least radiation pattern correction. The far offset reflections for the rubber target did not need strong radiation correction because the takeoff angles of the top and bottom reflecting rays are very close to each other due to similarity between P-wave velocities in water and rubber. The error bounds on the Berea sandstone Q estimates are large at all offset, running from ±7 to ±15. This reduction in the certainty in the slope of the spectral ratio is caused, in addition to the effect of source radiation correction, by noise contamination due to the extremely low transmission coefficient at the water-sandstone interface. The error bars in the case of Lucite Q estimates, which change from ±5 to ±7, show an intermediate
behavior between rubber and the sandstone since the radiation pattern correction is more
significant than in rubber and the transmission coefficient at the water-Lucite interface is
higher than that of the water-sandstone interface.

The estimates from the different offsets show no significant drift from the near
offset value due to the extremely low attenuation of water, which supports equation (2.16)
predicts. The residual drift in the Q estimates of Lucite (40-44) and rubber (17-20) is
caused by inaccuracies in the source radiation pattern correction. Q estimates for the
sandstone show more drift (23-32) due to the low S/N ratio at the far offsets.

4.6 Conclusion

The goal of this lab test is to investigate the accuracy of estimating Q from P-wave
reflection data on a scale close to that encountered in exploration seismics. So, Q is
estimated from non-zero offset traces and the wavelength of the waves used gives the
target both a depth and a thickness comparable to real earth targets. The targets used are
slabs of Lucite, rubber and Berea sandstone. It is found that the radiation pattern of the
source introduces instabilities in Q estimation. To correct the data for the source radiation
pattern, direct arrivals were recorded at the target depth for takeoff angles from 0 to 52° and
the amplitude of the wave field is interpolated in between these angles. The interpolated
amplitude is used to correct the amplitude spectrum of a given reflection using its takeoff
angle which is calculated using the ray tracer described in chapter 2.

The near offset estimate for Lucite is 40 which contains 25% error from the actual
value. Q for the sandstone is 23 which is 15% off from the actual value of 20. This
behavior shows the well known decrease in Q estimation accuracy as actual Q increases.
The estimate of Q for rubber is 17 and contains more error than other estimates due to the
fact that the spectral ratio method uses the definition of the attenuation coefficient for low
loss materials (Q>10) while rubber is a highly attenuating material with actual Q less than 10. Q estimates for rubber and Lucite show minor drift with offset due to inaccuracies involved in the radiation pattern correction. The negligible deviation from the near offset value is consistent with equation (2.16) since the attenuation of water is low.

The error bounds show an increase with offset caused by an increasing noise level in the spectral ratio due to the radiation pattern correction at the high frequencies. In the case of the sandstone, the small transmission coefficient at the water-sandstone interface reduced the S/N ratio for all the offsets which led to increasing the error bounds more than the other targets.
Figure (4-1): Water tank experiment setup.
Figure (4-2): The basic source wavelet and its amplitude spectrum.
Figure (4-3): The three datasets acquired for the three target materials used in the water tank experiment.
Figure (4-4): A comparison between the spectral ratio before (solid curve) and after (dotted curve) radiation pattern correction.
Figure (4-5): The dataset acquired by fixing the source position at the water surface and moving the receiver from 0 cm to 19 cm offset at the depth of the target. The purpose of acquiring this dataset is to correct for the directivity of the source.
Figure (4-6): The amplitude drop for three selected frequencies due to the radiation pattern of the source calculated from the amplitude spectra of the direct arrivals shown in figure (19). The drop is corrected for divergence.
Figure (4-7): The correction function used to adjust the amplitude spectra for reflected events. The correction will proceed by calculating the takeoff angle of the reflection and multiplying its amplitude spectrum by the corresponding factor at every frequency.
Figure (4-8): Final Q estimates for each target material.
Chapter 5: Conclusion

The goal of this study is to investigate the accuracy of estimating $Q$ using the spectral ratio method applied to the top and bottom P-wave reflections of a target horizon. The effects of the anelasticity of the overburden is investigated analytically and, then verified using synthetic data. In addition, synthetic data are used to estimate frequency dependent $Q$ functions. For another test, an ultrasonic water tank experiment is performed to estimate constant $Q$ values for Lucite, rubber and Berea sandstone. According to these studies, $Q$ values can be estimated with reasonable accuracy given that the target horizon has a thickness of greater than one wavelength and a $Q$ value of less than about 100.

Factors that affect the accuracy of estimating target $Q$ include the anelasticity of the overburden. The analytical study at the beginning of this paper shows that the significance of the error in target $Q$ caused by overburden anelasticity depends on a number of factors. High error is introduced in $Q$ estimation if the target is very thin relative to its depth. Weakly attenuating target or highly attenuating overburden adversely affects the estimation accuracy. As the difference between the top and bottom reflections’ ray trajectories in the overburden increases, more error is introduced in the estimation. The difference between the ray trajectories is as the offset increases. In addition, the velocity distribution in and above the target controls the ray geometry in the overburden. For zero offset traces, the difference between the ray paths in the overburden is zero; therefore, the top layer anelasticity does not introduce errors in the target $Q$ estimate. For non-zero offset reflections, the ray paths in the overburden will not coincide which introduces overburden interference in target $Q$. The analytic expression suggests that $Q$ estimates increase with offset due to overburden anelasticity.
Estimating constant Q values from synthetic data shows that there are some unexpected errors in the estimated target Q even at zero offset. The estimated value is generally lower than the actual Q. This may be due to the narrow band nature of the source impulse. As equation (2.16) predicts, Q estimates increase with offset due to anelasticity in the overburden. However, it is found that the effects of the overburden are significant only for weakly attenuating targets (Q>30). As the attenuation of the target increases, overburden anelasticity interference becomes a second order effect. Also, the Q estimates for all the different attenuations in the top layer converge to the target Q value for non-attenuating overburden which again supports equation (2.16). As the overburden attenuation increases, the bandwidth of the reflections used in the inversion decreases introducing errors in estimated Q values.

Estimation of frequency dependent Q functions from the spectral ratio of the top and bottom reflection is investigated in this study. The Q models used in the simulation are in the form of a constant Q value, representing the attenuation of the background rock matrix, plus a Gaussian drop in Q around a given frequency which represents attenuation due to fluid motion caused by the passing wave. The inversion results, using an iterative scheme, match the actual models with good accuracy for the zero and 600 m offsets. The attenuation center frequency was estimated within 3 Hz and the standard deviation of the Gaussian Q function was estimated within 5 Hz. The constant matrix Q results showed the expected high error for high Q values. The inversion results for the 1200 m offset underestimated the actual Q values by as much as 50%. Also, the standard deviation of the Gaussian drop in Q was larger than the actual value with a maximum error of about 300%. The increasing reflectivity induced amplitude change is the primary reason for the errors in Q in the far offset estimates. The reflectivity increased the significance of the intercept in
the spectral ratio, as shown in equation (2.9), which led to the errors in the $Q$ estimates and the loss of resolution in the spectral ratio at far offsets.

The goal of the ultrasonic lab experiment was to investigate the accuracy of estimating $Q$ from P-wave reflection data on a scale close to that encountered in exploration seismics. So, $Q$ is estimated from non-zero offset traces and the wavelength of the waves used gives the target both a depth and a thickness comparable to real earth targets. The targets used are slabs of Lucite, rubber and Berea sandstone. It was found that the radiation pattern of the source introduced instabilities in $Q$ estimation. To correct the data for the source radiation pattern, direct arrivals were recorded at the target depth for takeoff angles from 0 to 52°. The amplitude spectrum of the recorded wave field at the level of the top of the target is used as a reference to correct the amplitude spectrum of the reflections used in $Q$ estimation.

We compare the $Q$ estimates for Lucite, sandstone and rubber with values available from the literature for these materials. Comparisons are good given that the $Q$ values of the actual samples used in the experiment were determined independently and could be different from published values. $Q$ estimates for rubber and Lucite show minor drift with offset due to inaccuracies involved in the radiation pattern correction. The negligible deviation from the near offset value is expected since the attenuation of water is low.

An important part of this method of determining interval $Q$ is to test its applicability to real earth data. This is beyond the scope of this thesis and remains to be an important goal for future work.
References


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