DESIGN OF A CASCADE TEST RIG

by

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1.0 Summary

To improve the design of axial compressors, and of turbines, it is desirable to study the relative motion of the fluid and the compressor or turbine blades at high subsonic Mach numbers. A practical approach to this problem is the study of air flow around an airfoil section in cascade, where the cascade is designed to represent a continuous series of similar airfoils. To make these relatively simple tests significant, it is necessary to minimise the effect of aspect ratio and boundary layer.

An attempt has been made to design a high speed cascade, operating in a closed circuit at approach Mach numbers up to 0.95 to conform to the above conditions. It was found that there were so many unknown factors involved in such a design, which could not be satisfactorily analyzed by reference to previous work in the field, that the high speed project was temporarily set aside.

A simple low speed cascade, operating at $M=0.15$ (170 ft/sec), has been designed, and is under construction. This low speed cascade, it is hoped, will give the information on aspect ratio, boundary layer, and guiding walls necessary to build a high speed cascade. It is not thought that the low speed tests will give a complete answer on how to build a high speed rig, but much useful information should be forthcoming on all points except compressibility effects.

Methods of design for the low and high speed tunnels are presented. Drawings of the low speed tunnel are included, as well as some proposals of mechanical design for the high speed rig.
2.0 Introduction

2.1 Definition and Nomenclature of a Cascade

A cascade test rig is a wind tunnel, or water channel, in which flow can be studied past a cascade, or parallel arrangement of airfoils or other cylindrical shapes. The arrangement should be such that conditions near the mid-span of the centre blade are similar to conditions which would exist if an infinite number of foils, of infinite aspect ratio, were used. This is known as an infinite cascade. Thus, the study of the flow near the centre of such a cascade is essentially two dimensional, only measurements in a plane perpendicular to the span of the foils being significant.

Fig. 1a shows a typical compressor cascade, in longitudinal sections, while Fig. 1b is a turbine cascade. The nomenclature as defined by Fig. 1 is that standardized in England. It is used here because there is no standard American system. Many of the useful references on cascades use this nomenclature: The principal angles are measured from the normal to the plane of the cascade.

Note that the stagger is equal to $-\gamma$ for a compressor and $+\gamma$ for a turbine, where $\gamma$ is the counterclockwise rotation of the camber line from the normal. Care must be taken in comparing information from different sources to check the nomenclature.

For example, Marcinowski (Ref.1) uses a stagger which is the conjugate of $\gamma$. Shimoyama (Ref.2) measures the stagger similarly, but from the tangent instead of the chord, while the
N.A.C.A. defines stagger as the angle between the perpendicular to the cascade and the entering air. United Aircraft call the stagger, $\beta_1^*$, the angle between the plane of the cascade and the tangent to the camber line at inlet. There is an equal confusion with respect to the definition of other terms.

With reference to Fig. 1, in the British system of nomenclature a blade section is defined by:

- $\Theta$ - Camber
  - Type of camber line (Eg. Circular Arc)
- $c$ - Chord
- $t/c$ - Thickness:chord
  - Type of airfoil (Eg: "N.A.C.A. Family", Ref. 6)

When the blade is set up in cascade, the cascade is fully limited by:

- Blade section
- $\gamma$ or $\delta$ - Stagger
- $s/c$ - Pitch:chord (reciprocal of "solidity")
- $L/c$ - Aspect ratio
- $n$ - Number of blades

This cascade may then be subjected to any condition of air flow, which can be defined by:

- $\alpha$ - Angle of incidence
- $M$ - Mach number of entering air, relative to cascade
- $R$ - Reynolds number ($\frac{\rho w_1 L}{\mu}$)

The influence of the cascade on this air flow may be described by the airfoil geometry, with the addition of:

- $\delta$ - Deviation (Measured at many points in a plane after the cascade and parallel to the plane of the cascade)
- $p$ - Stagnation Pressure ($\ldots$)
An angle implicitly described in the above information, and often referred to, is the angle through which the velocity vector of the free air stream is turned:

\( \epsilon \) - Deflection

The effect of the air stream on a blade element in cascade is a force, which may be determined in magnitude and direction either by direct measurement or by integrating normal pressure and friction forces over the blade profile. The direct measurement is seldom made, due to experimental difficulties. Normal pressures may be obtained from pressure tappings, friction forces from integration of the losses calculated from the measurements mentioned above. Alternatively, tangential forces may be calculated from momentum considerations, and axial forces from change of pressure.

2.2 Definition of Symbols

Note: Symbols describing a cascade, and the passage of the air through it, are defined in the previous section, and on Fig.1.

- **A** - Cross sectional area, sq.ft.
- **a** - sonic velocity, ft./sec.
- **a_t** - throat area of ejector, sq.ft.
- **C_d** - discharge of coefficient
- **D** - diameter, or hydraulic mean diameter, \((4A/\text{perimeter})\), ft.
- **F** - force, lb.
- **g** - gravitational constant, 32.2 ft./sec.\(^2\)
- **H** - pressure, inches of water
- **h** - pressure, feet of fluid
- **J** - mechanical equivalent of heat
- **K** - a constant
k - ratio of specific heats
L - length, or span
M - Mach number
N - speed of rotation, rpm
p - pressure, lb./sq.in.
Q - discharge, lb./sec.
q - discharge, cu.ft./min.
R - Reynolds number, or gas constant
T - temperature, F abs.; or time, sec.
V - velocity, ft./sec.
w - relative velocity ft./sec.; or load on a beam, lb./ft.
x - length of equivalent flat plate, ft.
X - axial load, lb./inch of blade span
Y - tangential load, lb./inch of blade span
θ - thickness of boundary layer, ft. or deflection of a beam, inches
ρ - density, lb./cu.ft.
μ - viscosity, lb./sec.ft.

Subscripts:
b - boundary layer
c - corrected
f - friction
t - stagnation, or total

2.3 **Reasons for Cascade Testing.**

Before discussing the problems of building a test rig, or cascade wind tunnel, we should explain why we want such an apparatus. The flow in a cascade does not closely resemble the flow through any rotating machinery, which is three dimensional. Effects present in rotating machinery but not suitably observable in a
cascade include:

- Centrifugal force
- Flow parallel to blade span
- Boundary layer
- Relative motion of parts
- End losses
- Blade twist
- Varying solidity along span

The following can be directly measured in a cascade tunnel:

- Pressures before and after turning
- Velocities before and after turning
- Direction of streamlines before and after turning
- Pressure distribution over blades
- Flow patterns through cascade (Interferometer)
- Occurrence of shock
- Occurrence of separation

From this data we can obtain for a blade element at the centre of the span of a blade in the middle of the cascade:

- Forces on blade element
- Losses
- Maximum turning angle (Deflection, Fig. 1)

It is beyond the scope of this thesis to discuss the correlations of tests of an infinite cascade with compressor or turbine design. However, it is obvious that empirical comparisons can be drawn which will be useful in designing compressor blades. In addition, a mathematical analysis should make it possible to apply three dimensional corrections.
to the two dimensional case.

The larger the turning angle for a compressor cascade, the greater will be the pressure rise per blade row, other things being equal. The same can be said about Mach number. The cascade study of these problems in two dimensions should be very useful if comparative tests are made with rotating machinery.

It is thought that the shape of the pressure distribution over the blade profile has an important effect on maximum turning angle and maximum inlet Mach number which can be used without excessive loss occurring. Information from the two dimensional cascade should be useful in this respect if properly interpreted.

A single stage rotating cascade is to be built as a separate project. By testing the same blade profiles in the two rigs it should be possible to determine the significance of stationary cascade tests.

2.4 Control of Variables.

All of the defining terms used in paragraph 2.1 may be considered as variables. The obvious way of presenting useful data takes the form of testing a "cascade" as a unit. This involves variation only of Mach number, Reynolds number, and angle of incidence, i. Mach number and Reynolds number should be easily variable in infinitesimal steps, through a range, but varying i, or $\alpha_1$, requires complex mechanical manipulation, where the complete cascade must be moved relative to the angle of entering air.
When a series of tests has been made on the cascade, giving a complete picture of the effect of the above three variables in a given range, then the quantities defining the cascade may themselves be varied. Thus, the stagger angle may next be varied. This requires a series of tests as described in the last paragraph, for each new value of the stagger.

If the tests of the last two paragraphs were now to be repeated for each of a series of different pitch:chord ratios, a complete picture of the behavior of one blade section in cascade would be obtained, assuming that the effect of aspect ratio is not a function of the type of blade.

Of course all of the above would have to be repeated for each blade of a family under investigation, and it must all be done over again if another family of airfoils is tested.

The number of readings to be taken obviously is limited only by the intervals between points.

If the order set down above is followed, the only way to determine the effect of varying, say pitch:chord ratio at fixed values of the other variables, is to cross plot many tests each at fixed pitch:chord ratio. At first glance it looks as though the test apparatus should have push button, or crank turning, control of all the variables, but if this type of systematic complete investigation is to be done, and no other seems to be worth while for obtaining basic data, each step is a long job in itself, and time taken to change the fixtures will be short in comparison with time to record each test.
There are several alternatives which should not be lightly dismissed, especially if an automatic transversing and recording gear is used. (The question of automatic recording will not be followed up in this thesis, but it should be reconsidered before the high speed tunnel is built.) The most obvious way to take measurements is to vary the Mach number at several pressure levels or temperatures. Reynolds number will not remain independent of Mach number in this way, but the results may be interpreted. This series may be repeated at intervals of \( \alpha_1 \), and so on as above.

It may be more convenient to vary stagger, at fixed values of \( \alpha_1 \), thus varying incidence and stagger simultaneously. By plotting several such series, at different values of \( \alpha_1 \), the same results may be obtained as by the other method.

2.5 High Speed Testing

Seeing that the amount of work per stage of a compressor or turbine increases with the relative speeds of gas and blades, as well as with the absolute velocity of the blades, designers push these speeds as high as possible, only to find efficiencies falling off as Mach numbers reach extremes. To investigate the phenomena in this region, and to try to push the limits up, information is wanted at high Mach numbers. A high speed cascade seems to be the only way to get systematic information. It is desirable then, to build and run a rig for cascade testing at Mach numbers as close to unity as possible.

2.6 Unknown Effects

The design of a high speed cascade tunnel was undertaken, but it finally became apparent that it was as much guess work as
design. The principal questions to which satisfactory answers could not be found in the literature were:

How many blades are necessary to simulate an "infinite" cascade?

What is the aspect ratio of an "infinite" cascade, or what correction should be applied for a finite aspect ratio?

What effect has the boundary layer on the direction of entering airstream, and how can this effect be controlled?

Should a cascade discharge into free air space, or should the discharge be guided by walls? If a free air space is desirable, how big must it be so that recirculation will not affect the airstream?

How close to the air stream can walls be without affecting it?

What shape is suitable at the ends of a cascade?

Can the discharge be diffused without affecting exit conditions?

2.7 Decision to Make Low Speed Cascade.

In view of these difficulties, it was decided to investigate the unknown effects separately, rather than build an expensive rig only to find out that the results from it could not be interpreted because of the unknowns. This decision was prompted by the fact that the rig had grown to some twenty tons on the drawing board in an effort to minimize the unknowns.

The best way to investigate the largest number of these effects quickly is to build a cascade in which all the unknown factors can be varied, and their effect on the airstream observed. Such a cascade can be relatively simple, for there is no need to provide for varying Mach or Reynolds numbers, angle of
incidence, stagger, pitch:chord ratio, or blade section. Factors which should be controlled are: number of blades, aspect ratio, boundary layer, shape and position of exit walls, and the geometry of the surrounding space.

Although all of these factors will probably have a different effect at high Mach numbers than at low Mach numbers, a great deal can be learned about their influence at low Mach numbers, which will be applicable to a high speed test rig. At least lower limits can be put upon the number of blades and aspect ratio. A low speed rig is much cheaper to build than a high speed one, and results can be expected in a reasonable time if a simple low speed rig is built. Such a rig should be large enough to give reasonable Reynolds numbers.
3.0 Design of Low Speed Cascade Test Rig.

3.1 Layout

When it was first decided to build a low speed cascade the proposal was to fasten an accelerating nozzle onto a large ventilating duct, add a cascade of sheet metal turning vanes, and do a series of experiments on varying aspect ratio, boundary layer suction, and walls after the cascade. It soon became apparent that a makeshift rig would not give results of the order of accuracy required, so the apparatus outlined in fig. 2 was designed. The space available to set up this cascade rig, one of the gas turbine laboratory engine test cells, is the same order of size as the apparatus. Care has been taken to raise the cascade as far as practical above the floor, to prevent interference with the issuing air stream, but it is not certain that re-circulation from the walls will have a negligible effect on the direction of the stream. With this in mind, the apparatus is being built so that it is fairly easily disassembled and may be taken out of doors in the summer.

3.11 Cascade

The cascade itself was the unit around which this rig was designed. Its major dimensions evolved from the size of the ventilating duct of the original scheme. These dimensions seemed reasonable so they were kept.

The following were the basic considerations:

Blade chord: large enough so that the Reynolds number would be of a similar order for the low speed as for the high speed cascade.

Aspect ratio: as large as possible while keeping
blades stiff enough to hold their alignment to close limits. Aspect ratio to be variable by means of walls, or baffles, sliding along the blades.

Number of blades - arbitrarily large.
Blade section - any easily manufactured section.

The principal criterion was that each blade must be the same as every other, as near as reasonably possible.

Pitch:chord ratio - fixed at any representative value.
Stagger - fixed at any representative value.
Incidence - fixed at a value well away from stalling.

The blade section is shown in fig.3. Its exact dimensions were determined by manufacturing considerations and material available. 6in. O.D. Shelby tubing, 1/8 in. thick was the basis. By taking 45 deg. sections and milling off the trailing edge tangent to the inner surface, a blade profile of 2\(\frac{1}{2}\) in. chord and 35 deg. camber resulted. This method also set the number of blades, for two lengths of tubing give 16 blades, which is considered enough. When a machining mistake resulted in spoiling one set of blades, it made available two more blades to use as guide walls at the ends of the cascade.

The calculations of Appendix 1 show that this blade will be amply stiff, unless there are vibration difficulties.

The remainder of the mechanical detail of the cascade itself, as shown in fig. 4, is self explanatory. Provision is made for sucking off boundary layer on all four sides. The amount of the stream sucked off may be varied from zero to the maximum calculated thickness of boundary layer.
3.12 Blower

The choice of blower was a matter of accepting what was available, with characteristics within certain limits. These limits were set by the following calculation, to relate the cascade area to two quantities generally quoted by blower manufacturers: \( q \), cu.ft. per minute, and \( H \), in.H\(_2\)O, outlet total pressure of blower.

\( H \) is usually quoted as the difference in inlet and outlet total pressures, which is equivalent to the outlet total gauge pressure. Some of the following assumptions are based on the fact that velocities will be very low in the settling tank. (Subscripts refer to Figs. 1 & 2)

Assume:

Total Pressure after cascade equals total pressure after blower. (A correction for losses is applied later.)

Incompressible flow. (This is justified with the small changes in absolute pressure involved)

\( \alpha_1 = 45 \text{ deg.} \) (Design value, corresponds approximately to \( \alpha = 30 \text{ deg} \) and \( \phi = -5 \text{ deg} \))

\( \alpha_2 = 25 \text{ deg.} \) (Probably within \( \pm 10 \text{ deg} \))

16 blades at 1.83 in. pitch, 16 in. long

Then: \( w_2 \) (Fig.1) = \( V_6 \) (Fig.2) \( \cdot \sqrt{2gh} = \sqrt{2gH} \frac{62.4}{12} \cdot 0.076 \)

\( A_5 = A_6 \), in plane of cascade, \( \cdot 1.83 \times 17 \times 16 = 500 \text{ sq in} \)

\( \dot{V}_{a6} \) (axial velocity) = \( V_6 \cos \alpha_1 = 0.91 V_6 \)

So \( q = 60 \times \frac{500}{144} \times 0.91 \sqrt{\frac{2g}{12} \times \frac{62.4}{0.076} \sqrt{H}} \)

\( = 12,500 \sqrt{H} \) cfm (1)

E.g. at 3 in. water, \( q = 12,500\sqrt{3} = 21,700 \) cfm.
This gives a characteristic curve for the cascade, as shown on Fig. 5. On this same figure we can plot the characteristic of any compressor to determine the operating point.

The principal inaccuracy in the above analysis is the assumption that $\alpha_2 = 25$ deg, for $\alpha_2$ will not be known until measured. However, suppose it were 10 deg out in either direction, $\cos 15 = .95$ and $\cos 35 = .82$, so the cascade characteristic might be displaced 4% to the right or 10% to the left. This would not appreciably affect the choice of a compressor.

If a different cascade were used, on the same accelerating nozzle, possibly with different $\alpha_1$ and $\alpha_2$, it would have a different characteristic, but if the compressor characteristic is not too steep in this region it is likely that an operating point can be found.

The outlet velocity $w_2$, and indirectly the approach Mach number, will depend on the value of $H$ at the operating point, but any reasonable value will be satisfactory, for the Mach number is not critical at these low speeds.

An axial flow fan with A.C. motor has been obtained. It was built for ship ventilation. Its characteristics, according to the Navy Bureau of Ships, are shown plotted on fig. 5. An axial type fan was preferred because the velocity distribution at the discharge of such a fan is far superior to that obtainable with a centrifugal fan, thus lessening the settling chamber problem.

If the ordinate $H$ of fig. 5 is taken as the pressure across the nozzle, a correction should be made for losses between fan exit and the nozzle entrance. Assume all the velocity head from the
fan is dissipated. Assume further, looking ahead, that the settling chamber is 7 ft X 9 ft in cross section, and contains five screens, each of which causes a loss of one velocity head. (This last term is so small that it doesn't matter whether a correction is made of \( \frac{1}{2} \) velocity head or \( 1 \frac{1}{2} \) heads.

Velocity in fan exit is dissipated. Assume further, looking ahead, that the settling chamber is 7 ft X 9 ft in cross section, and contains five screens, each of which causes a loss of one velocity head. (This last term is so small that it doesn't matter whether a correction is made of \( \frac{1}{2} \) velocity head or \( 1 \frac{1}{2} \) heads.

\[
\text{Velocity in fan exit} = \frac{25000}{\text{Area}} \times \frac{1}{60} = 39.5 \text{ ft/sec.}
\]

\[
\text{Velocity in settling chamber} = \frac{25000}{60 \times 63} = 6.8 \text{ ft/sec.}
\]

Losses with 5 screens:

\[
\left( \frac{39.5 - 6.8}{2g} \right)^2 \times 12 \times \frac{0.76}{62.4} + \frac{5}{2} \times \frac{6.8^2}{2g} \times 12 \times \frac{0.76}{62.4}
\]

\[
= 0.27 \text{ in water.}
\]

After applying this correction to get a new compressor characteristic, we investigate the flow conditions at the cascade: The operating point is at 4.0 in. water, equivalent to a discharge velocity \( w_2 = 132 \text{ ft/sec} \).

This corresponds to \( w_1 = 132 \frac{\cos 25}{\cos 45} = 170 \text{ ft/sec} \)

\[
M_4 = \frac{170}{49/530} = 0.150, \text{ and}
\]

\[
R = \frac{\frac{1}{\mu_1} \frac{w_1}{c}}{120 \times 10^{-7}} = \frac{0.76 \times 170}{2.25} \times \frac{2.25}{12} = 200,000
\]

3.13 Settling Chamber

The settling chamber between the fan and accelerator is for the purpose of removing all large turbulence, to supply the nozzle with a smooth stream of air containing only such small eddies as will become negligible size when the stream is accelerated.

The design is based entirely on empirical consideration.
Dr. Tsien was of great help in this case, drawing on his experience with similar problems to make the following recommendations:

The size of the settling chamber should be about twice that of the nozzle entry on each side or the rectangle. This is an area reduction of 4:1 to accelerator entrance, or $4 \times 2.5^2 = 25:1$ to the nozzle.

Screens are the most effective way of getting an even velocity distribution. Three screens may be enough, but possibly five will be required. Metal screens should be used, cloth screens pick up dirt and so become a disturbing influence instead of evening the flow. Joints in screens are bad, and should be staggered if they can not be eliminated. 1/16 in. mesh is suggested.

Spacing between screens should be about 40-50 times the mesh. Between the last screen and the entrance to nozzle there should be 200-300 mesh diameters.

It is not necessary to diffuse after the fan, if one fan diameter is left between it and the first screen.

Following these recommendations, a settling chamber 9 ft wide, 7 ft high and 6 ft long, containing 5 screens of 1/16 in. mesh, was arrived at. It was necessary to go to a rather heavy construction because the inside could contain no braces, and even the small pressure of 4.2 in. water is enough to produce considerable distortion when acting on such large areas.

2 inch X 6 inch timbers on 17.5 in. centres were chosen
as the load carrying members. Neglecting stiffness of the wall material, we find that the maximum deflection, at the center of the top or bottom, is given by

\[ \delta = \frac{5}{384} \frac{wL^4}{EI} \]

\( \approx 0.10 \text{ in.}, \) an allowable figure assuming: pressure = 4.2 in. water

\( E \) for wood = \( 12 \times 10^5 \) lb/sq in.

Ends are not restrained

If it is found desirable to increase the velocity in the cascade, slightly, an elliptical disc will be placed in front of the blower discharge. Such a disc will act as a diffuser, preserving some velocity head otherwise lost. It would be elliptical to fit in the chamber. An increase of 0.15 in. static pressure might be expected.

3.14 Accelerating Nozzle

No reference has been found in the literature to the design of a rectangular accelerating nozzle in three dimensions, starting from a flat wall.

There are several ways in which this problem might be attacked. One could start with a circular section and build a convergent nozzle according to established practice (Ref.13), then use a transition section to the rectangular shape required. 22 in. x 16 in. Alternatively, the initial section could be square or rectangular, and the corners could be rounded, cut off, or 90 deg angles.
On the advice of Dr. Tsien, it was decided to start with a rectangular section of dimensions proportional to the final nozzle, to keep square corners all the way, and to use an elliptical profile similar to the ASME family of circular nozzle profiles (Ref 13). The straight part after the convergent nozzle, which allows the velocity profile to even out, has been reduced to about half that specified for the ASME nozzle.

A rectangular nozzle cannot be built with all four sides converging according to the ASME design unless the rectangle is a square, for the length must be the same for all sides. The profile of the centre line of the small sides of the nozzle, because it should be longer than a small diameter nozzle, was designed in accordance with the ASME profile, and the large sides were fared in with a quarter ellipse. (Fig. 6)

Tests will be done on this nozzle without the cascade. If not an even velocity profile is obtained, the nozzle will have to be modified.

It is expected that there will be an appreciable boundary layer, especially in the corners, so means will be provided to remove it just before the cascade.

A method of estimating the boundary layer thickness is given below, but this is an estimate at best. To be on the safe side, arrangement is to be made to withdraw 3/4 in. all around, just before the cascade. This effectively reduces the area discharging the flow from the main blower by 15%, and will give a
different cascade characteristic on fig.3.

The following notes on the boundary layer in the accelerating nozzle are based on a discussion with Dr. Tsien:

Flat plate formulae for boundary layer development will give no indication of the conditions in a converging nozzle. As the air accelerates in the nozzle, boundary layer build up is arrested. At the end of the converging section the air near the wall, just outside the boundary layer, is moving faster than the mean air stream. The straight section on the ASME elliptical nozzle is to allow the velocity profile to even out. Dr. Tsien suggests a straight section 6 in. long on our nozzle, which is shorter than the 13 in. specified by ASME. He estimates that we will have a 1/8 in. boundary layer at this point. There has been no method developed for calculating the correct profile for, or the boundary layer in a rectangular nozzle. After the nozzle, boundary layer will develop approximately according to flat plate formulae, except in the corners.

We shall probably have laminar flow, except on the long side of the 45 deg transition, ending at section 4,(Fig.2) where the boundary layer will probably be turbulent.

There is no analysis available which will put a value on the thickness of boundary layer in a converging nozzle, but it has been suggested by Prof. E.S. Taylor and Prof. Neumann that a fair estimate should be obtainable by considering the discharge coefficient in the formula

\[ Q = C_d \rho A V \]  

If there were no boundary layer, and a uniform discharge
velocity \( V \), \[ Q = \frac{\rho A V}{C_d} \]

so \( C_d \) is a measure of relative boundary layer area. Ref. 14, page 100, gives experimental values of \( C_d \) for axially symmetrical venturi of various sizes over a large range of Reynolds number. In order to use this chart we must estimate the size of nozzle and Reynolds number. It may be pardoned if we base our "estimate" on calculations which will appear later.

Assume boundary layer 1/8 in. Since the definition of boundary layer thickness is arbitrary, assume \( V_b = \frac{2V}{3} \)

Then for 22 in. x 16 in. nozzle, \((21.75 \times 15.75 V) + (1/8 x 76 x 2V/3) = 22 \times 16 C_dV\), whence \( C_d = 0.990 \)

Take \( R = \frac{4Dv}{\mu} \), where \( D \) is the hydraulic mean diameter, or \( R = 1.47 \times 10^6 \). At \( R > 10^5 \), Ref. 14 shows \( C_d = 0.988 \) for a 16 in. nozzle. The agreement on \( C_d \) shows that the estimation of 1/8 in. is reasonable.

To get a rough idea of the boundary layer thickness on the longest side of the nozzle (point A, Fig. 2), we can make use of two formulae from Ref. 15:

For laminar flow \( \frac{\delta}{x} = \frac{5}{\sqrt{R}} \) \hspace{1cm} (3)

For turbulent flow \( \frac{\delta}{x} = \frac{277}{\sqrt{R}} \) \hspace{1cm} (4)

where \( R = \frac{2Vx}{\mu} = \frac{0.076 \times 170x}{120 \times 10^7} = 1.07 \times 10^6 x \)

\( x = \) distance from leading edge, feet

\( \delta = \) boundary layer thickness, feet
The equivalent length of flat plate before the nozzle is given by
\[
\frac{1}{8x} \times 12x = \frac{5}{(1.07 \times 10^6x)^{\frac{1}{2}}}
\]
or
\[x = 4.65,\text{ assuming laminar flow.}\]
The long wall will extend 1.83 ft beyond Section 4, Fig. 2, so
\[
\frac{6}{4.65 + 1.83} = \frac{5}{(1.07 \times 10^6 \times 6.48)^{\frac{1}{2}}}
\]
\[\therefore \delta = 0.0123 \text{ ft} = 0.148 \text{ in.}\]

If we assume that the boundary layer becomes turbulent at the end of the symmetrical part of the nozzle (section 4, Fig. 2) and apply equation (4) to the flow for 1.83 additional feet, we get
\[\delta = 0.447 + 0.125 = 0.572 \text{ in.}\]

This is a very loose application of equations (3) and (4), and must be considered as only a directed guess. Considering that the square corners of the duct will aggravate boundary layer conditions, it will be safer to accept the greater value of 0.57 in, rather than 0.15 in. as obtained from the laminar flow assumption.

The boundary layer just before the cascade then, at section 4 of Fig. 2, will be assumed .125 in. thick on the short side, .57 in. thick on the long side, varying between these limits on top and bottom, and of unknown thickness in the corners.

3.2 Boundary Layer Control

In paragraph 3.14 an estimation was made of boundary
layer thickness. Fig. 4 shows how provision has been made for the removing of this layer. In order to keep the tunnel as symmetrical as possible, it will be possible to draw off an equal thickness (3/4 in.) all around the cascade. Alternatively, the boundary layer suction slots can be streamlined into the walls, so the tunnel can operate will full boundary layer present.

To control the amount of air drawn from each side, several schemes could be used. A large fan, with an ample pressure ratio, could be manifolded to four separate pipes, to the four suction chambers. A valve in each pipe would control the amount of air drawn off. With this scheme, however, an adjustment on any one pipe would cause changes in the other three, in the opposite direction. This would make control very difficult.

Another proposal was to use an ejector on each side. It was hoped that this would cut down the size of the unit, but calculations showed that large ejectors would be required. Besides, the noise from the four ejectors would make work around the rig more difficult and unpleasant. Calculations on the ejector scheme appear in Appendix 2.

The final plan was to use four small fans, or centrifugal blowers, to draw off the four separate boundary layers. Calculations showing the fan characteristics required are in Appendix 3, together with some notes on the diameter of ducting to use with the fans.

3.3 Instrumentation

The instrumentation of the low speed tunnel is very
simple. If the boundary layer in the approach section could be neglected, measurements of total head and direction of air the stream after/cascade would suffice to show the effects of varying aspect ratio, proximity of walls, etc. To observe the effect on the approach stream of boundary layer suction, it is necessary to add a means of taking static pressure or yaw readings at several points before the cascade.

The combined yawmeter and total pressure tube is a simple instrument which has been used in almost identical form in several laboratories. (Ref. 6) The yawmeter is of the claw type, where two total pressure/approach a point from opposite sides. When the same pressure is recorded on both tubes the instrument is heading directly into the stream. Of course it must be calibrated with an air stream of known direction, or by traversing an unknown stream twice, once with the instrument inverted. The true air stream direction must be the mean of the two readings.

The advantage of the claw type over ball or disc type yawmeters is that the two arms may be brought very close to each other, thus measuring the yaw at a point, not averaging it over an area or along a line.

The traversing gear needs little explaining, being a simple mechanical device for carrying the yaw-pressure head to any point in front of the cascade. One inch overlap has been allowed on every side, to observe the divergent air stream. Vernier adjustments make it possible to record the position of the head within 0.001 in., and its angle within 0.1 deg. The
lead screws have a pitch of 0.1 in. and a half nut enables rapid movement to any position. The position is shown by scales engraved on the guide rods, and by the micrometer head readings.

The calculations of Appendix 4 show that the maximum deflection of the head, due to the air stream, will be .034 in. when extended completely across the cascade. This deflection is in the direction of stream lines, so it is not important. At the center of the cascade, deflection will be only .002 in.
4.0 Design of High Speed Cascade Test Rig

4.1 Problem

To design a cascade test rig in which the flow of air about a cascade of airfoils can be studied under different conditions of stagger, pitch chord ratio, angle of incidence and Reynolds number, at inlet Mach numbers up to that corresponding to choking in the cascade. In other words, the inlet Mach number is to be pushed as high as is possible with a conventional convergent accelerating nozzle.

4.2 Limitations

The work reported here was the design of a cascade tunnel to fulfill the following conditions:

a. To use a closed circuit. This was dictated by the necessity to suppress noise, to permit variation of Reynolds number by changing the pressure level, to limit required power input, and to ensure clean, dry air. Clean air is a necessity when using an interferometer, in order to protect the glass walls. Dry air is a requirement of the super sonic tunnel in the same circuit, and is also required to prevent snow from forming in the cascade nozzle.

b. To make measurements by pressure readings and by interferometer. The interferometer is a very useful instrument for studying gas flow, but not enough is known about interpreting the results to eliminate pressure tappings and yaw measurements.

c. To use the air supply from a given compressor. The compressor was chosen with characteristics suitable for a super sonic wind tunnel, so the cascade had to be designed to make the best use of these same characteristics.
d. **To use the maximum pressure range available**, governed at the lower limit by the steam ejector evacuating the system, and at the upper limit, for a cascade of reasonable dimensions, by the power available to run the compressor.

e. **The power available**, in turn, was limited by the motor characteristics, gear ratio, and maximum safe speed of motor.

4.3 **Circuit**

At this point we should describe the proposed circuit. This is shown in Fig.7.

A D.C. motor, the characteristics of which are shown in Fig.8 (forced cooling will be used), drives a five stage centrifugal compressor through a 3.85:1 gear ratio. The compressor discharge passes through three aircraft type coolers in series, the cooling medium being mains water, in counterflow.

In normal operation the air can flow either through the cascade, 3-4-5-6, or through a super sonic wind tunnel, selection being made by valves in the 24 in. air ducts.

There will probably be an auxiliary pipe 4-7 for sucking off boundary layer before the cascade. The necessary pressure drop for this can be supplied by partially closing valve 7. This allows re-circulation of boundary layer air, obviating the necessity of drying make-up air.

The closed circuit is completed at O, the compressor inlet.

The air in the system can be dried by circulating it through the drier, under the action of the main compressor. There is also
A filter in this line, and an auxiliary cooler to improve drier performance.

A steam ejector will be capable of lowering the discharge pressure, $p_1$, to 0.1 atmospheres.

A compressed air supply may be used to supercharge the circuit up to 3 atmospheres intake pressure, $p_0$.

The air ducting will be carefully manufactured 24 in. steel pipe, smooth inside. Elbows will have turning vanes, except side outlet elbows. Valves will be streamlined butterfly valves.

The general arrangement of the apparatus and piping details have been fixed by space available and by the supersonic wind tunnel, and will not form part of this thesis.

The size of the piping will be such that the velocity head will be negligible (between 50 and 100 ft/sec), so the total pressure and temperature will be substantially equal to the static temperature and pressure at all points of the circuit except sections 4 to 6, fig. 7. The symbols $p_t$ and $T_t$ will be used in the following analysis rather indiscriminately with $p$ and $T$, except at 4 to 6, although an attempt will be made to use the symbol most appropriate to the discussion. Change from total to static may be made, however, without further explanation that they are equal.

4.31 Compressor Characteristics

The first step in determining the size of the cascade
The working section was an analysis of the compressor characteristics to determine volume flow, or cross section of air stream at any velocity. The "expected characteristics" were supplied by the manufacturer, as shown in Figs. 9a, 9b, 10a, and 10b. The curves include a plot of bhp, discharge temperature, discharge pressure, and pressure ratio, against mass flow at two fixed compressor speeds (Figs. 9); and cross plots at two fixed values of inlet pressure and pressure ratio, of bhp, rpm, and discharge temperature against mass flow (Figs. 10).

The compressor curves were reduced to a more convenient form, Fig. 11a, b, and c. This manner of plotting compressor characteristics is standard in British gas turbine work (Ref. 3, page 433, L.J. Cheshire). It is based on dimensional analysis, but to make truly dimensionless groups a representative dimension of the compressor would have to be included in the corrected flow, rpm, and hp. The constants $\sqrt{\frac{520}{14.7}}$ are included for comparison with actual values at 60°F and one atmosphere. By use of these parameters, operating conditions at any inlet condition may be immediately predicted from any test point, or design point.

Corrected values used are as follows:

\begin{align*}
Q_c &= \frac{Q}{\sqrt{\frac{p_{\text{tot}}}{520}}} \quad \text{lbs/sec.} \quad (5) \\
N_c &= \frac{N}{\sqrt{\frac{520}{p_{\text{tot}}}}} \quad \text{R.P.M.} \quad (6) \\
\frac{(p_{\text{lt}})_c}{(p_{\text{tot}})_c} &= \frac{p_{\text{lt}}}{p_{\text{tot}}} \quad \text{Discharge pressure} \quad (7)
\end{align*}
Discharge Temperature

\[ T_{lt} = T_{lt} \frac{520}{T_{ot}} \]

Horse power

\[ h_{pc} = \frac{h_p}{14.7 \sqrt{\frac{T_{ot}}{520}}} \]

The "non-dimensionality" of these quantities is demonstrated in Appendix 5.

From figures 9, characteristics were plotted for corrected values of rpm, giving two lines on each of figures 11 a, b, and c which could help in interpolation and extrapolation.

The suppliers' curves all stop at values of pressure ratio above, and mass flow below, expected values for the cascade rig. They are prepared to guarantee only one point, at pressure ratio 3.0, which is far from the operating range of the cascade. Extrapolation on the curves is questionable, so the following design quantities must be checked after testing the compressor over the full range. In fact, the figures below should be considered only as a rough estimate of size, and as a means of developing a method of design. Construction should not be started until the compressor has been tested, but the following calculations should give an order of magnitude of the tunnel, around which some mechanical details can be designed.

4.32 Cooler

It is evident that flow in the circuit of fig. 7 will be affected by the temperature drop and pressure drop in the cooler.

The following information for the cooler is taken from the...
specification on which it was ordered:

\[
\begin{align*}
\text{Air mass flow } Q &= 7 \text{ lb/second} \\
\text{Inlet pressure} &= 16 \text{ psia} \\
\text{Inlet temp.} &= 400 \text{ F } \\
\text{Outlet temp.} &= 100 \text{ F } \\
\text{Press. drop} &= 6 \text{ in. H}_2\text{O}
\end{align*}
\]

hence \( \rho_1 = 0.0503 \text{ lb/cu ft} \)

Performance under other inlet conditions may be calculated from the following relationships:

\[
\text{Thermal ratio } \frac{T_1 - T_2}{T_1 - T_{\text{water in.}}} = K \text{ (Ref. 17)} \tag{10}
\]

\[
\Delta p_{12} = \text{const. } x \frac{Q^2}{\rho_1} = 0.000222 \frac{Q^2}{\rho_1} \tag{11}
\]

where \( Q \) = Air flow, lb/sec  
\( \Delta p \) = pressure loss, lb/sq.in.  
\( \rho_1 \) = density at cooler inlet, lbs/cu ft

Equation (11) can be derived from the equation

\[
\frac{\Delta p}{\rho_1 Q^2 A^2} = K_1 + K_2 \left( \frac{\rho_1}{\rho_2} - 1 \right)
\]

and a curve which shows that \( K_2 \) is small for straight through flow. Both are on page 471 of Ref. 3.

From the specification data, and the fact that the maximum temperature of Cambridge water is 70 F (Ref. 18), the Thermal ratio becomes

\[
\frac{860 - 560}{860 - 530} = 0.91
\]
Heat conduction through the walls of the circuit of fig. 7 may be considered negligible, so

\[ T_{2t} = T_{ot} \]

\[ \frac{T_1 - T_0}{T_1 - 530} = 0.91, \text{ and since} \]

\[ T_1 = T_{1o} \left( \frac{T_0}{520} \right), \quad (8) \]

\[ T_0 \left( 1 - \frac{0.09 T_{1o}}{520} \right) = 482 \quad (12) \]

For any operating point, \( Q_0 \), \( T_{1o} \), and \( p_{1t} \) are known, so \( T_{ot} \) can be found from (12), and \( T_{1t} \) from (8). If either \( p_1 \) or \( p_0 \) is known or assumed, the other can be found, so \( p_1 \) can be calculated from the equation of state and \( Q \) from (5). Now \( \Delta p_{12} \) can be calculated from (11). A sample calculation appears below in section 4.51.

4.33 Accelerating Nozzle

In order to design an accelerator which will make the best use of the compressor characteristics, giving the maximum size of test section at the required Mach number, it is necessary to find a relation between the area \( A_4 \), Mach number \( M_4 \), and the air flow variables, \( Q \), \( p_0 \), and \( T_0 \).

For expansion in a short nozzle, it is justifiable to assume isentropic flow. Consider flow from 3-4, fig. 7.

\[ \frac{p_{3t}}{p_4} = \left( \frac{T_{3t}}{T_4} \right)^{\frac{k}{k-1}} \]

\[ T_{3t} = T_{4t} = T_4 + \frac{V_4^2}{2gJC_p} \]
Also, since \(a_4^2 = k \rho R \frac{k-1}{k} = \frac{R}{\gamma p}, \) and \(V_4 = M_4 a_4\)

\[
\frac{T_{3t}}{T_4} = \left(\frac{P_{3t}}{P_4}\right)^{\frac{k-1}{k}} = 1 + \frac{V_4^2}{2gJc_p} \frac{\rho R}{\gamma p a_4^2} = 1 + \frac{k-1}{2} M_4^2
\]

(13)

Also, 

\[
Q = \rho_4 A_4 V_4
= \frac{144}{R} \frac{P_4}{\sqrt{T_4}} A_4 M_4 \sqrt{\frac{k \rho R}{T_4}}
= \frac{144}{R} \frac{P_4}{\sqrt{T_4}} A_4 M_4 \sqrt{\frac{k \rho R}{T_4}}

But:

\[
P_4 = P_{3t} / (1 + \frac{k-1}{2} M_4^2) \frac{k}{(1 + \frac{k-1}{2} M_4^2)^{(k-1)/2}} \frac{k-1}{2}\]

(13)

So

\[
Q = \frac{144}{R} \frac{P_4}{\sqrt{T_4}} A_4 M_4 \sqrt{\frac{k \rho R}{T_4}} \frac{P_{3t} \frac{k}{(1 + \frac{k-1}{2} M_4^2)^{(k-1)/2}}}{(1 + \frac{k-1}{2} M_4^2)^{(k-1)/2}}
\]

or

\[
\frac{Q \sqrt{T_{3t}}}{A_4 P_3t} = \frac{144}{R} \frac{M_4}{\sqrt{\frac{k \rho R}{T_4}}} \frac{k+1}{(1 + \frac{k-1}{2} M_4^2)^{(k-1)/2}}
\]

\[k = 1.4, \quad R = 53.3 \text{ for air, and } g = 32.2\]

So

\[
\frac{Q \sqrt{T_{3t}}}{A_4 P_3t} = \frac{132.5 M_4^2}{(1 + 0.2 M_4^2)^3}
\]

This is a general expression for the Mach number in a nozzle of given area, carrying a given mass flow of air under given inlet conditions.

Now, \(T_{3t} = T_{ot}\),

So

\[
\frac{Q \sqrt{T_{ot}}}{A_4 P_3t} = \frac{132.5 M_4^2}{(1 + 0.2 M_4^2)^3}
\]

(14)

A calculation similar to that for the low speed cascade shows that boundary layer thickness, at the cascade, on the short
wall will be about .08 in., while that on the long wall will be about .33 in. The calculation was done assuming $M_4 = .95$ and a turbulent boundary layer. (Appendix 7).

The boundary layer will reduce the effective area of the cascade slightly, but this is a further factor of safety on the capacity of the compressor. In view of the uncertainty of the boundary layer calculations, it is better not to increase the nozzle area, especially as this would require divergent walls after the throat, diverging at the same rate that the boundary layer grows.

It should be noted that the above boundary layer calculations may give an idea of the boundary layer thickness at the middle of any side, but give no measure of the effect in the corners of the rectangular duct. All that can be said is that the boundary layer will be much thicker in the corner.

No attempt will be made at this stage to do a detailed design of the accelerator, for its form will depend on the major decisions on the general form of the high speed tunnel.

It should be put on record, however, that design of a rectangular accelerator having the low speed end protruding into a plenum chamber, or the atmosphere, is an extremely complicated problem. Mr. Khurt of United Aircraft has pointed out that they required six months of testing before a satisfactory shape was obtained. (Ref. 20).

If the accelerator can be started from a hole in a flat wall as in the low speed tunnel, or as the necking down of a pipe, the problem is much simpler, for much of the trouble
with the "gramophone horn" idea seems to be that even low speed air breaks away at the entrance lip and causes rapid growth of boundary layer, with consequent Mach number trouble before the throat. United Aircraft engineers found that they had to put an exaggerated scroll on the lip to achieve success.

4.34 Pipe Friction Losses

Pipe friction losses are negligible in comparison with cooler pressure loss and other doubtful factors. Therefore, no correction for pipe friction is necessary in preliminary design. Calculations showing the order of pressure drop in the pipes are shown in Appendix 8.

4.35 Cascade

4.35.1 Pressure Change

To consider the effect of the cascade on the air flow, it is assumed that the cascade discharges into a plenum chamber where all the velocity energy is dissipated without pressure recovery. If low speed cascade tests show that useful results may be obtained with diffusion after the cascade, and if a practical method of doing this is devised, a much lower compressor pressure ratio can be used. It will appear below that this would result in a larger nozzle, but the following calculations will assume no recovery after the cascade, for at present there is no justification for believing that diffusion would not distort the results.
Consider first a compressor cascade. The pressure rise in such a cascade may be estimated from facts known about compressors now in operation. A conservative industrial design uses 24 stages for a 4:1 compression ratio. This is a rise per stage, assuming equal distribution, of \(1.06/1 \times 2^{14} = 4\). If an equal pressure rise is assumed in rotor and stator, the pressure rise per blade row, \(\frac{P_6}{P_5}\), is \(\sqrt{1.06} = 1.03\). At the other extreme, a high performance compressor might give a pressure ratio of 5:1 in 6 stages, or \(1.144/1\) per blade row on the above assumption. In the hope that the better values may be obtained in the cascade, let us assume the maximum will be \(1.2/1\) per blade row (\(1.144\) per stage). It will appear below that the design of the rig will not limit the upper value of obtainable pressure rise, so if ways are found to improve on \(1.144/1\) per stage, they may be tested in this rig, unless they require supersonic flow.

Now consider a turbine cascade. Turbine cascades generally have accelerating passages, except in the case of the rotor of an impulse turbine, or the blade root section of a high performance constant circulation, or free vortex, turbine rotor, where pressure is constant through the cascade.

It is easy to calculate the maximum pressure drop

\[
\frac{P_{3t}}{P_6} = \frac{P_{3t}}{P_7} \times \frac{P_{3t}}{P_{ot}}
\]

(assuming no recovery after cascade, small pipe losses, and small velocity \(V_0\))
If it is assumed, only for purposes of discussion, that the Mach number at exit from the cascade, \( M_6 = 1 \), then

\[
\frac{p_{3t}}{p_6} = 1.89 \approx \frac{p_{3t}}{P_{ot}}
\]

Some turbine nozzles operate at a slightly higher Mach number but this assumption will do for design purposes, for it will be shown that an approach section designed for a compressor cascade is conservative for a turbine. It will appear below that the maximum pressure ratio \( p_1/p_0 \) for a compressor cascade is 2.0, when allowance is made for boundary layer suction. This leaves a generous factor for losses when a turbine cascade is substituted requiring \( p_{3t}/p_6 = 1.89 \).

Equation (16) will show that either \( \frac{p_{3t}}{p_6} \) or \( \frac{p_{3t}}{P_{4t}} \) may be used to find an operating point if an assumption is made of \( \frac{p_4}{p_6} \).

The above discussion is of interest only if a tunnel without boundary layer suction is to be designed, or if a separate system is used to withdraw the boundary air. Equation (16b) will show that the ratio \( \frac{p_4}{p_6} \) does not come into the calculations when the line 5-6-7, fig. 7, is throttled sufficiently to allow the boundary layer to be drawn through 4-7 by the main compressor.

4.352 Mach Number

The characteristic Mach number of a compressor or turbine stage, or of a cascade, is the Mach number of the approaching air stream, relative to the blades. As the velocity
of the approaching stream is increased to a limit, choking may occur in the throat of the cascade at an approach Mach number, depending on the type of cascade under test and on other factors, such as the boundary layer in the approach channel.

Experience with axial compressor cascades has shown that a Mach number up to .80 is practical without a serious effect on efficiency (Prof. C. R. Soderberg). It is hoped that this can be increased to .90. One of the objects of the high speed cascade rig is to raise this limit as high as possible. In order to do this, it will be necessary to push conditions past the optimum point to study the losses under bad conditions, with a view to analyzing them and making improvements. With this in mind, it will not be unreasonable to set the maximum Mach number for compressor testing at the high value of .95.

Calculation (as in section 4.51) shows that if the design Mach number were set at .90 the resulting tunnel would have an area only about 3% greater than for .95.

Once the apparatus has been constructed, there will be only one operating point on Fig. 11, and hence only one value of rpm, for any given value of \( M_4 \). (Reference to equations 15, 16b and 18 in section 4.4 will demonstrate this.) Therefore, there will be no way of increasing the Mach number above the maximum design value except by overspeeding the compressor. It is advisable, therefore, to design for such an extreme value as .95, with the expectation of running the compressor at a more conservative speed for most tests.
To estimate $M_4 \max.$ for a turbine cascade, consider rotor and stator separately.

A convenient way of arriving at $M_4 \max.$ for a stator is by use of the equation

$$\frac{A_6}{A_4} = M_4 \left[ \frac{k+1}{2(k-1)} \right]$$

(15)

This equation is derived in Appendix 9. It assumes, as before, only for purposes of discussion, sonic speed at section 6. $A_6$ is the area normal to the outlet velocity, $a_6$.

Reference to fig. Ib shows that $\frac{A_6}{A_4} = \frac{\cos \alpha_2}{\cos \alpha_1}$ and $M_4 \max.$ will occur when this ratio is a maximum. Study of several turbine designs shows values of $\frac{\cos \alpha_2}{\cos \alpha_1}$ up to .54 for stator cascades. Solution for $M_4$ gives $M_4 \max. (\text{stator}) \approx .35$.

The ratio of $\frac{\cos \alpha_2}{\cos \alpha_1}$ for turbine rotor cascades may be as high as unity, for an impulse turbine, but in this case both $M_4$ and $M_5$ must be less than unity. This can be seen from the velocity triangle (Fig. 12), which shows that $w_1$, the velocity relative to rotor at rotor inlet, must be less than $c_1$, the absolute velocity at rotor inlet. This may be expressed by the ratio

$$w_1 = c_1 \sqrt{1 - \xi} \quad \text{where}$$

$$\xi = V_1 \left( 2 \cos \alpha_1 - V_1 \right)$$

and

$$V_1 = u/c_1$$

but rather than set arbitrary values for $\alpha_1$ and $u$, we can go to the literature for maximum values of $M_4$. 
Reeman (Ref. 3, page 500) shows $M_4$ varying from 0.45 at tip to 0.8 at root for a high performance single stage turbine. At the tip the outlet Mach number, $M_6$, may be near unity, but at the base it will not be higher than $M_4$. The compressor pressure ratio, then, will be lower when the inlet Mach number is high, so it again appears that a tunnel designed for testing compressor cascades will be satisfactory for testing turbine cascades.

4.4 The Operating Line

Once the compressor, accelerator, and cascade characteristics have been determined, the size of the accelerator can be set, and an operating line, where the characteristics are all satisfied, may be plotted, e.g., fig. 11. If the cascade is short-circuited by a boundary layer control device, necessitating use of valve 7, fig. 7, as a throttle, the cascade characteristics will not influence the operating line.

Consider first the case where no allowance is made for boundary layer suction. When the area $A_4$ has been fixed, an operating point on the compressor characteristics, $(Q_c$ and $P_1/P_0$), may be found by assuming two quantities, $M_4$ and $P_6/P_4$, or $M_4$ and $M_6$, and considering pressure loss ratio in cooler and pipes constant, by use of the following equations:

$$\frac{P_{\text{at}}}{P_4} = \left[1 + \frac{2M_4^2}{3.5}\right]$$

$$\text{(13)}$$

$$\frac{P_{\text{at}}}{P_6} = \left[1 + \frac{2M_6^2}{3.5}\right]$$
\[
\frac{P_1}{P_0} = \frac{P_6}{P_0} \frac{P_1}{P_3} \frac{P_{3t}}{P_4} \frac{P_4}{P_6} = \frac{1}{1.955} \frac{P_{3t}}{P_4} \frac{P_4}{P_6} \tag{16a}
\]

(The assumption that \( P_6/P_0 = 1 \) and \( P_3/P_1 = 0.955 \) are checked in Appendices \( G, 6 \) and \( 10 \).)

\[
Q_c = \frac{Q}{\sqrt{T_{ot}}} = \frac{14.7}{\sqrt{520}} \frac{P_{3t}}{P_1} \frac{P_1}{P_0} = 74.04 \frac{1 + 0.2\mu_i^2}{(1 + 0.2\mu_i^2)^3} \frac{P_1}{P_0} \tag{17}
\]

If \( A_4 = 0.292 \) sq ft (see section 4.51),

\[
Q_c = 21.6 \frac{M_i}{(1 + 0.2\mu_i^2)^3} \frac{P_1}{P_0} \tag{18}
\]

Of course \( \frac{P_6}{P_4} \) is a function of all the other variables besides the form of the cascade, for this is precisely one of the unknowns which the tests will be designed to determine, but it should be possible to make a good estimate of the maximum recovery in any cascade, and thus set the design point.

If the main compressor is used to draw off the boundary layer, \( \frac{P_4}{P_0} \) \( 1 \) and equation (16) would become

\[
\frac{P_1}{P_0} > \frac{P_{3t}}{P_4}
\]

\[
= \frac{P_1}{P_{3t}} \frac{P_4}{P_0} \frac{P_{3t}}{P_4}
\]

to allow for pressure drop from \( h = 0 \), fig. 7, say \( P_4/P_0 = 1.05 \), obtained by setting an arbitrary value (0.6 psi at full load) on the pressure drop in boundary layer suction ducts and main ducts (App. 8).

\[
\therefore \frac{P_1}{P_0} = \frac{1}{1.955} \times 1.05 \frac{P_{3t}}{P_4} \tag{16b}
\]

Sample calculations, using the above method, will be found in App. 10.
4.5 Determining Principal Dimensions

4.51 Accelerator Discharge Area

If boundary layer suction is to be done with the main compressor, and this will be assumed, the discharge nozzle will be smaller than could be accommodated with a smaller compressor pressure ratio. Fig. 11a shows that for a given rpm the mass flow falls with rising pressure ratio, and equation (17) shows that for a given Mach number, the area $A_4$ must diminish under both these conditions. The following calculations, therefore, will give a conservative value of area if boundary layer suction is not used.

Design for the maximum $M_4$, at maximum compressor speed and maximum power, allowing for boundary layer suction:

If $M_4 = 0.95$,

$$\frac{P_{3t}}{P_4} = \left(1 + 0.2 M_4^2\right)^{3.5} \tag{13}$$

$$= 1.768$$

$$\frac{P_1}{P_0} = 1.12 \frac{P_{3t}}{P_4} \tag{16b}$$

$$= 2.0$$

(This requires that the boundary layer suction ducting should have a pressure drop $\approx 0.6$ psi.)

Plot $\frac{P_1}{P_0} = 2$ on Fig. 11a. To find $N_c$, assume that $T_{0t} = 550$ Fabs. This must be checked later.

Then $N_c = \frac{5000}{\sqrt{\frac{550}{520}}} = 4860$ rpm. This is also plotted on Fig. 11.
Hence, \( Q_c = 24 \text{ lb/sec} \) (Fig. 11a)
\( T_{1c} = 730 \text{ Fabs.} \) (Fig. 11b)
\( hpc = 1900 \text{ hp} \) (Fig. 11c)

and \( A_4 = 0.292 \text{ sq ft} \) (17)

Then \( T_{ot} = \frac{482}{(1 - \frac{0.09T_{1c}}{520})} \) (12)

\[ = 552, \text{ a good check.} \]

hp available at full speed

\[ = \text{Motor hp} \times \text{gear efficiency} \]

\[ = 1300 \times 0.97 \]

\[ = 1260 \text{ hp} \]

Solve equation (9) for \( P_0 \):

\[ 1260 = 1900 \frac{P_0}{14.7} \sqrt[520]{T_o}, \text{ so } P_0 = 9.47 \text{ psia.} \]

To check assumptions on pressure loss used in the analysis of section 4.33, assume:

- Velocity head in conduits negligible.
- Boundary layer in nozzle to be neglected.

Assumptions above were:

- \( N = 5000 \text{ rpm} \) (given value)
- \( M_4 = 0.95 \) (given value)
- \( T_0 = 550 \text{ Fabs} \) (checked above)
- \( \frac{P_7}{P_o} = 1 \) (See appendix 8)
- \( \frac{P_3}{P_1} = 0.955 \)

\( \Delta P_{4-7} = 0.6 \text{ psi}, \text{ at the flow corresponding to maximum } M_4 \text{ and maximum hp}. \) (Ducts should be designed for this value.)

This leaves only the assumption of \( p_3/p_1 \) to be checked.
Solve for

\[ P_1 = P_0 \cdot \frac{P_1}{P_0} = 9.47 \times 2 = 18.94 \quad (7) \]

\[ T_1 = T_{1c} \cdot \frac{T_o}{520} = 730 \cdot \frac{552}{520} = 775 \quad (8) \]

Hence

\[ \rho_1 = \frac{18.94 \times 144}{53.3 \times 775} = 0.066 \text{ cu ft/lb} \]

\[ Q = Q_c \frac{P_o}{14.7 \sqrt{T_o}} \]

\[ = 24 \times \frac{9.47}{14.7} \times \frac{1}{1.03} = 14.91 \text{ lb/sec} \]

\[ \therefore \Delta P_{1-2} = 0.000222 \frac{Q^2}{1} \]

\[ = 0.000222 \times \frac{14.91^2}{0.066} = 0.747 \text{ psi} \]

or \[ \Delta P_{1-3} = 0.78 \text{ psi} \] (Appendix E)

\[ \frac{P_3}{P_1} = \frac{18.16}{18.94} = 0.959 \]

It is now apparent that the original estimate of \( \frac{P_3}{P_1} \) was too low, but by such a small amount that it is not necessary to correct at this time.

A similar calculation, without allowance for boundary layer suction, gives a nozzle area of 0.311 sq ft.

With \( A_4 \) fixed at 0.292 sq ft, equation (18) may be used in conjunction with (13) and (16b) to plot an operating line on fig. 11, for various values of \( \dot{M}_4 \). This calculation appears in appendix 10.

Similar calculations were done for the case of no boundary layer suction and two different values of \( p_6/p_4 \). They are plotted on fig. 11a, but are not of sufficient interest to record the calculations.
4.52 Chord, Pitch:chord, Aspect ratio

In the absence of the sort of information which the low speed cascade is intended to give, it must be assumed that the nearest approach to an "infinite" cascade will be obtained by making aspect ratio and number of blades as close to infinity as possible.

In a channel of given area, this would be achieved by using the smallest possible chord, but consideration of Reynolds number, blade stiffness, and the necessity of putting pressure taps on the blade surface forces a decision on minimum chord. This has somewhat arbitrarily been set at 1.5 in.

Considering the number of blades in the cascade, the worst case occurs for a turbine cascade with a $\alpha_1 = 0^\circ$. At any other value of $\alpha_1$, the length along the cascade, which is to be divided into a certain number of pitch lengths, is greater than the width of the accelerator nozzle. The maximum value of pitch:chord ratio likely to be tested for such a cascade is about 1. The least number of blades used seems generally to have been 5, so let us assume this value until the low speed results are available, remembering that most compressor testing will be done with $\alpha_1$ around $50^\circ$ so that the minimum number of blades will be increased by a factor $\frac{\cos 0}{\cos \alpha_1} \approx 1.55$. Combining a chord of 1.5, a pitch:chord ratio of 1, $\alpha_1 = 0$, and 5 blades, we get a nozzle width

$$6 \times 1 \times 1.5 = 9 \text{ in.}$$

This, together with $A_4 = .292 \text{ sq ft} = 42 \text{ sq in.}$, sets the height of the blade at $\frac{42}{9} = 4.67 \text{ in.}$, which is equivalent
to an aspect ratio of 3.11. Such a blade appears adequate from a stiffness point of view.

The above example is to show how the dimensions may be arrived at when the low speed rig results are available to build upon. Even then, such figures need not be binding. For instance, if it were desired to do tests on very large pitch:chord ratios, or very large chords, it would be advisable to confine the tests to large values of $\alpha_1$. Probably the great majority of testing will be done with a certain chord size; it appears that with chords of 1.5 in. this tunnel will be satisfactory.

If the Reynolds number $\left( \frac{\rho_4 v_4 C}{\mu_4} \right)$ is calculated, using a blade chord 1.5 in., at the extremes of Mach number and pressure level it is found to range from about 40,000 to 420,000 so the choice of $R = 200,000$ for the low speed cascade seems suitable.
4.6 Mechanical Details

A study of the literature reveals no cascade tunnel which would fit the stipulations laid down in section 4.2. There are no flexible rigs using an interferometer, no large ones running at high Mach number, none with closed circuits. Ref. 9 describes an attractive one, but at large values of $\alpha_1$ the swinging walls approach each other so closely that there is almost no channel left.

There are two drawbacks to systems using a cascade mounted on a rotatable disc: if the full channel width is to be used at large $\alpha_1$, the disc becomes very large (the diameter is equal to channel width/$\cos \alpha_1$); and the air stream cannot be allowed to expand in the plane perpendicular to the disc until it gets past the disc edge, thus causing further contraction and distortion of the stream due to boundary layer build-up.

The requirement of using an interferometer on the centre of the cascade limits the form the tunnel can take. For instance, an attractive scheme is to have a cascade swinging about one end, on a fixed inlet nozzle. The interferometer could possibly be made to swing with the cascade, but even that is not admissible when a closed circuit is used. That would require the cascade and interferometer to be enclosed in a plenum chamber.

Fig. 13 shows how it has been proposed to meet all the requirements. This is not to be considered a working drawing, for no provision has been made for boundary layer removal, and the other details await the low speed tests. The basic unit is
the cascade assembly, which must be fixed in space, and accessible to the interferometer. The blades may be set in a fixed position, by use of jig drilled holes for each stagger and each pitch:chord ratio; or the stagger might be continuously variable by means of pins in two separate slides at each end of the blades, a fixed one holding the blade leading edges and a sliding one swinging the trailing edges together.

Building onto the cascade assembly, an accelerating nozzle is added. It is proposed to use a series of fixed nozzles. The simplest way to have a swinging or replaceable inlet in a closed circuit is to enclose the nozzle in a large plenum chamber, fastening onto the cascade assembly. It is proposed to vary $\alpha_1$ from 0 to 60 degrees with one series of nozzles, and from 60 to 70 with another series of half the width. Half width nozzles are necessary at the end of the range in order to keep the cascade length within reason; an ample number of blades could still be accommodated.

The form of the outlet from the cascade is a function of the range of $\alpha_2$. A cascade to be used for all types of blading may vary from 75 degrees one side, for a turbine stator, to 55 degrees on the other, for a compressor. These are the extreme values taken from a study of all the references. To these angles must be added 15 degrees, half the angle of expansion of a free air stream (Ref. 26). Again remembering the requirement of a closed circuit, the best way to accommodate a large range, especially if expansion is to be allowed in two planes, is to discharge into a plenum chamber. Even if guide walls are used to diffuse the leaving air stream, the convenient
way to do it is inside a plenum chamber. Fig. 13 shows how it is proposed to fix the cascade between two plenum chambers.

The size of the inlet plenum chamber will be governed by the size of the accelerating nozzle, and the amount of clearance required before its inlet. The size of the exit plenum chamber is still unknown. Rig tests, described in appendix 11, show that there may be serious re-circulation trouble unless the plenum chamber is larger than could be entertained for this project - something of the order of a fair sized building may be needed. The low speed tests may throw more light on this problem. The answer may be to build a swinging diffuser after the cascade, still enclosed in a plenum chamber.

The plenum chambers, and the cascade assembly, must be of massive construction, to stand the atmospheric pressure when running at low pressures. The calculations will not be presented here for it is still doubtful whether the apparatus will take this form, and they are straightforward calculations following any good strength of materials book. The complexity of the calculations is a function of the assumptions made, which can be generous.

The inlet plenum chamber can be made from the shell of a ship's condenser, and possibly the outlet can be similarly handled. Surplus Navy condensers should be obtainable without cost, and they are conveniently designed to withstand the same vacuum which will be used here. When choosing condensers for this job, care should be taken to see that they can go through a door 8' x 15'.
5.0 References


7 MacPhail. Experiments on Turning Vanes at an Expansion. R. & M. 1876.


17 Allen, Walker & Fraser. Heating and Air Conditioning.


References, cont'd.


21 Pratt & Whitney. Memo No. 38 from C. T. Wang on Wind Tunnel Wall Shape.

22 Pratt & Whitney. Report No. 77 from George F. Carrier on Cascade Tunnel Inlet Investigation.


24 B. F. Sturtevant Co. Catalog No. 500.

25 Walworth Co. Catalog No. 42.

Appendix 1. Deflection of Blades, Low Speed Cascade.

Neglect friction forces.

Then, tangential loading, per inch of blade length,

\[ Y = \frac{w_x}{s(w_{u1} - w_{u2})} \]  
(Subscripts refer to Fig. 12)

and axial loading,

\[ X = s(p_2 - p_1) \]
\[ = s \frac{\rho(w_{u1}^2 - w_{u2}^2)}{2g} \]

Now

\[ l = 16 \text{ in.} \]
\[ s = 1.83 \text{ in.} \]
\[ w_1 = 170 \text{ ft/sec} \]
\[ w_x = 170 \sin \alpha_1 = 170 \sin 45 = 120 \text{ ft/sec} \]
\[ w_{u1} = 170 \cos \alpha_1 = 120 \text{ ft/sec} \]

Assume \( \alpha_2 = 25^\circ \)

\[ w_{u2} = w_x \tan 25 = 56 \text{ ft/sec} \]

\[ \therefore Y = 120 \times \frac{0.076}{32.2} \times \frac{1.83}{12} \times \frac{1}{2}(120 - 56) \]
\[ = 0.307 \text{ lbs/in.} \]

\[ X = \frac{1.83}{12} \times \frac{0.076}{32.2} \times \frac{1}{12}(120^2 \times 56^2) \]
\[ = 0.225 \text{ lbs/in.} \]

Deflection of uniformly loaded beam, free ends:  
(i.e., case of maximum deflection)

\[ S = \frac{5}{384} \frac{w^4}{E I} \]

Tangentially

\[ w = 0.307 \text{ lb/in.} \]

\[ L = 16 \text{ in. = 1.333 ft} \]
I \approx .0169 \text{ in.}^4 \quad \text{(by graphical means)}

\[ S = \frac{5}{384} \times \frac{16^4}{30 \times 10^6} \times .307 \times \frac{1}{.0169} \]

\[ = .000517 \text{ in.} \]

Axially

w = .225 \text{ lb/in.}

I \approx .0505 \text{ in.}^4

\[ S = 28.5 \times 10^{-6} \times \frac{.225}{.0505} = .000127 \text{ in.} \]

Appendix 2. Ejectors for Boundary Layer Control.

The worst condition that the boundary layer suction might have to meet is the possibility that there will be a boundary layer in the corner, but practically none along the sides. If the control slots were fully open under these conditions they would have to handle the full air stream velocity over most of their area. The maximum flow in this case would be given by:

Sides. \( q = 170 \times \frac{.75}{12} \times \frac{16}{12} \times 60 = 850 \text{ cu ft/min} \)

\[ = 1.08 \text{ lb/sec} \]

Top and bottom. \( q = 850 \times \frac{22}{16} = 1170 \text{ cu ft/min} \)

\[ = 1.48 \text{ lb/sec} \]

The pressure against which the boundary layer removal must take place can be conservatively estimated by assuming that all the velocity head in this air stream will be dissipated in the removal slots. The static pressure before the cascade must be below atmospheric by the velocity head:

\[ H = \frac{v^2}{2g} \times 12 \times \frac{.076}{62.4} - 4.2 \approx 2.35 \text{ in. water} \]
If an ejector were used, there would be practically no piping, so assume the total head to be 2.4 in. and calculate the size of ejector needed to handle 1.5 lb/sec and 1.1 lb/sec against this head.

The ejector calculations are based on Ref. 23. By a fortunate coincidence, the authors have plotted test results using the same primary air pressure which is available for this apparatus, 100 psia.

Fig. 12 of the above reference gives the ratio of secondary to primary air as 7.8, to eject air against a pressure of 4.2 in. water when $p_1/p_2 = 6.82$. Subscripts refer to the accompanying sketch.

Hence, primary flow $W' = 1.5/7.8 = .192$ lb/sec, and $1.1/7.8 = .141$ lb/sec

Total primary flow is $2(.192 + .141) = .666$ lb/sec

The capacity of the compressor is only 500 cu ft of free air per minute, or .633 lb/sec, but we shall carry the calculation through to determine the size the ejectors would have to be.
Considering the larger ejector, total flow at 1 or 2 is \( 0.192 \times 1.5 = 1.69 \text{ lb/sec} \). Extrapolation on Fig. 8 gives the ratio of mixing tube area to primary throat area as 160. To get primary throat area, \( a_t \), consider a choking nozzle in which the flow is given by:

\[
\frac{W'}{a_t} = \frac{76.5 p_1}{\sqrt{T_1}}.
\]

(This relationship can be demonstrated similarly to the analysis of Appendix 9.) Here \( a_t \) is in square feet, \( W' \) in lb/sec, and \( p_1 \) in lb/sq in. Hence,

\[
a_t = \frac{0.192}{76.5} \times \frac{530}{100} \times 144 = 0.0831 \text{ sq in.}
\]

Therefore, mixing tube area = 160 x 0.083 = 13.3 sq in.

and the diameter = 4.1 in. Assuming that length is seven times the diameter, length = 29 in.

Thus, there is nothing very neat about the scheme. Add to this the facts that the compressor available is not quite big enough, and the noise would be objectionable, and there is a good case for an alternative means of removing the boundary layer.

**Appendix 3. Fans for Boundary Layer Control.**

Appendix 2 showed that it will be required to remove 850 cu ft/min at each end of the cascade, and 1170 cu ft/min at top and bottom, all against a pressure of 2.35 in. water. To this head must be added the pressure drop in the ducting to fans. Assume that there will be 5 ft of ducting to each fan. Ref. 24, page 137, shows that a 5 in. pipe will be required to keep the pressure loss down to 0.5 in. for a flow of 1000 c.f.m.
This is a reasonable size, so consider delivery to be against 3.0 in. total for the large delivery, and 2.7 in. for the ends, remembering that the resistance of the pipe will vary as the flow squared.

The fan requirements may be summed up, then, as follows:

2 to handle 1170 c.f.m. against 3.0 in. water
2 " " 850 " " 2.7 in. 

A survey of fans around the gas turbine and Sloan laboratories which might be useful for such a purpose is recorded below. Some are obviously not suitable, but they are included to record what is known about them. All are single entry centrifugal fans. The characteristics are not well known in any case, so numbers 3, 4, 5, 6 and 7 of the following table will be tested.

Table of Some Centrifugal Fans in Gas Turbine and Sloan Laboratories.

(1) Main ventilating fan, gas turbine lab.  
Approx. 24,000 c.f.m. @ 3 in. water

(2) Sloan Lab. blower, not in use  
1/2 in. to 12 in. water  
12,000 to 15,000 c.f.m. (14,520 c.f.m. @ 8 in. H₂O)  
1776 rpm  40 hp

(3) Ventilating fan from old chemistry lab., gas turbine lab.  
1/2 hp Belt drive  10 in. intake  
Performance unknown

(4) Plating tank fan. Gas turbine lab.  
M.I.T. No. 354-15 D.I.C.  
Buffalo Type LL Size 2 1/4 Class 1  
Shop order AA-5550  
G.E. Motor Type RCP  
1140 rpm  1 ph  1/4 hp  115-230V  
Performance unknown
In Sloan Lab. Cell No. 6 (Back one)
Sturtevant Planovane Exhauster
#352665 Size 35 Design 3
2930 c.f.m. @ 7 in. water
4.52 hp

Sloan Lab. portable blowers - 2 similar
"M.V. Blower Size D" Direct drive

3/4 hp at 1725 rpm
8 3/4 in. inlet No. 113943
1180 c.f.m. at free delivery
955 c.f.m. at 1 in. water

*Note: One has motor trouble

Sloan Lab. portable blower
Sturtevant #0 Design 3
1 hp 3450 rpm
757 c.f.m. at 1 in. water
592 c.f.m. at 3 in. water

Other fans which may be suitable, to be found in reference 21, are as follows:

Design 7, Size 60
Design 8, Size 60 R 16 page 50
Design 6, Size 2 12 in. fans pp 46 & 48

Appendix 4. Deflection of Pressure Head.

To obtain the load on the pressure head, consider it all as a circular cylinder, with axis perpendicular to the flow. The velocity of the air stream at exit will be given by:

\[ \frac{V^2}{2g} = \frac{4.2 \times 62.4}{12 \times .076} \text{, or } V = 136 \text{ ft/sec.} \]

\[ R = \frac{\rho V D}{\mu} = 26,500 \]

Ref. 15, page 247, gives \( C_d = 1.20 \) if \( R \) is between \( 10^4 \) and \( 10^5 \).
\[ P = \frac{C_d A \rho v^2}{2}, \text{ or the drag per foot of a 3/8 in. rod} \]
\[ = 1.20 \times \frac{.375}{12} \times .0025 \times \frac{136^2}{2} \]
\[ = 0.87 \text{ lb/ft} \]

Note the reduction in drag obtainable by use of a streamline strut: \( C_d = .07 \), so \( P = .05 \text{ lb/ft} \). However, it is thought that a streamline strut would be less stable laterally, and such a small size would have to be made up specially.

The 3/8 in. rod has a hole .157 in. dia. through the centre. The moment of inertia of the section is .00093 in.\(^4\). Consider a beam of this section, cantilevered 18 in. and carrying a uniform load of .87 lb/ft. The deflection at the end is:

\[ \delta = \frac{wL^4}{8EI}, \text{ or } \delta = .034 \text{ in.} \]

If the arm is extended only to the centre of the cascade,
\[ l = 9 \text{ in.} \]
and
\[ \delta = .002 \text{ in.} \]

Appendix 5. "Non-Dimensional" Compressor Characteristics.

\[ Q = \frac{M}{T'} \]
\[ T = \frac{L^2}{T'^2} \]
\[ P = ML \left( \frac{1}{T'^2} \right) \]
\[ N = \frac{1}{T'} \]
\[ \text{hp} = L \left( \frac{ML}{T'^2} \right) \frac{1}{T'} \]

Note: \( T' = \text{time} \)
\( T = \text{temperature} \)

which has dimensions \( \frac{L^2}{T'^2} \)

according to kinetic theory.
\[ Q_c = \frac{Q}{\sqrt{\frac{520}{\text{or} 14.7}}} \left( \frac{1}{D^2} \right) = \frac{M}{T' \cdot L \cdot \frac{\text{LT}^2}{M}} \left( \frac{1}{L^2} \right) \]

\[ N_c = \frac{N}{\sqrt{\frac{\text{or} 520}{\text{Tot}}} (\text{D})} = \frac{1}{T' \cdot L} \]

\[ \text{Pitc} = \frac{\text{Pit} \cdot 14.7}{\text{Pot}} = \frac{M}{L \cdot \frac{\text{LT}^2}{M}} \]

\[ \left( \frac{\text{Pit}}{\text{Pot}} \right)_c = \left( \frac{\text{Pit} \cdot 14.7}{\text{Pot}} \right)(\frac{1}{\text{Pot} \cdot 14.7}) = \frac{\text{Pit}}{\text{Pot}} \]

\[ \text{Titc} = \frac{\text{Tit} \cdot 520}{\text{Tot}} \]

\[ \text{hp}_c = \frac{\text{hp}}{\sqrt{\frac{\text{Tot}}{14.7} \sqrt{520}}} \left( \frac{1}{D^2} \right) = \frac{M^2 \text{LT}^2}{T' \cdot 3 \cdot M} \cdot \frac{T'}{L} \left( \frac{1}{L^2} \right) \]

Note that 14.7 and \( \sqrt{520} \) are dimensionless numbers to bring the corrected quantities to the same order of size as the actual quantities. At one atmosphere and 60°F the corrected value is equal to the actual value. The size "D" of the compressor is omitted because if one compressor is being considered it is unnecessary. If compressors of different size are being compared it is usual to compare performance under standard conditions, but we are seldom, though sometimes, interested in what a compressor would do if it were some standard size.
Appendix 6. Pressure Drop in Cooler.

The pressure drop through the cooler is expressed by equation (11): \[ \Delta P_{1-2} = (\text{a constant}) \frac{Q^2}{\rho_1} \]. See section 4.32 and ref. 3.

The constant can be evaluated from the specifications of the cooler: when \( \Delta p = 6 \) in. water, \( Q = 7 \) lb/sec and

\[ p_1 = 16 \text{ psia} \]
\[ T_1 = 400 \text{ F} \]

hence,

\[ \rho_1 = 0.0502 \text{ lb/ft}^3 \]

If \( p \) is to be in \( \text{lb/in}^2 \), the constant must be

\[ \frac{6}{12} \times \frac{62.4}{144} \times \frac{0.0502}{49} = 0.000222 \]

so the equation becomes:

\[ \Delta P_{1-2} = 0.000222 \frac{Q^2}{\rho_1} \]

Appendix 7. Boundary Layer Thickness, High Speed Accelerating Nozzle.

At the Accelerator throat, assume:

\[ M_4 = 0.95 \]
\[ p_1 = 18.9 \text{ psia} \]
\[ p_3 = 18.2 \text{ psia} \]
\[ T_2 = T_3 = T_0 = 550 \text{ F} \text{ abs} \]

Cross section = 9 in x 5 in

\[ V_{b.l.} = \frac{2V}{3} \]

\[ \alpha_1 = 60 \]
Then \( D = (4 \times 9 \times 5)/28 = 6.4 \text{ in.} \)

\[
P_4 = \frac{18.2}{\left(1 + .2M_4^2\right)^{3.5}} = 10.2
\]

(13)

\[
T_4 = \frac{T_3}{1 + .2M_4^2} = \frac{550}{1.18} = 466
\]

(13)

\[
\rho = \frac{P_4}{RT_4} = .0618
\]

\[
V = .95 \times 49 \sqrt{466} = 1000 \text{ ft/sec}
\]

\[
R = \frac{\rho V D}{\mu} = \frac{.062 \times 1000 \times 6.4}{182 \times 10^{-7} 12} = 1.80 \times 10^6
\]

For a 6 in. nozzle, and \( R \gtrsim 10^6 \), \( C_d = .984 \) (Ref. 13).

Then \( (9-2\delta)(5-2\delta) V + 6(28 - 46) \frac{2V}{3} = .984 \times 45 \times V \)

or \( \delta = .077 \text{ in.} \)

Assuming turbulent flow, length of equivalent straight plate, \( x \), is obtained from (4).

\[
R = 1.80 \times 10^6 \times \frac{12x}{6.4} = 3.38 \times 10^6 x
\]

\[
\frac{.077}{12x} = \frac{.377}{(33.8 \times 10^5 x)} 1/5
\]

\( x = 0.263 \text{ ft} \)

Additional length, long side = \( \frac{9}{12 \cot 60^\circ} = 1.3 \text{ ft} \)

Total length = 1.56 ft

\[
\delta \approx \frac{.377}{(33.8 \times 10^5 \times 1.56)^{1/5}}
\]

\( \delta = .32 \text{ in.} \)
That is, the boundary layer will vary from .08 in. on the short side of the asymmetric nozzle to .32 in. on the long side. The method of analysis is explained above in section 3.14. As mentioned there, it is an approximation probably erring on the large side, but not allowing for worse conditions in the corners.

Appendix 8. Pipe Friction Losses.

To calculate the head loss in the piping, use is made of a formula from ref. 15:

\[ h_f = f \frac{L v^2}{D \sqrt{g}} \]

where \( f \) is a function of Reynolds number, as determined by fig. 107 of the above reference.

To determine \( V \) and \( R \), before and after the cascade, use the design figures of section 4.5.1. Consider the short section between the compressor and cooler as part of the piping after the cooler.

Before the cascade, using the same units as above,

\[ P_3 = 18.16 \]
\[ T_3 = T_0 = 550, \text{ so } \rho = 193 \times 10^{-7} \]
\[ \rho_3 = .0892, \text{ and since } Q = AV, \]
\[ v = 53.2 \]

Hence,
\[ R = \frac{.0892 \times 53.2 \times 2}{193 \times 10^{-7}} = 4.92 \times 10^5 \]

After the cascade

\[ P_0 = 9.47 \]
\[ T_0 = 550 \]
\[ \rho_0 = .0465, \text{ so } \]
\[ V = 102, \text{ and } R \text{ is the same as above: } \]
\[ R = 4.92 \times 10^5 \]
Therefore, \( f \) is the same before and after the cascade:

\[ f = 0.02 \]

A value must be put on all the pipe fittings in the circuit. This is most convenient if each fitting is expressed as an equivalent length of pipe. The following arbitrary values will be reasonably close, especially if it is considered that the total correction for friction is very small.

<table>
<thead>
<tr>
<th>Fitting</th>
<th>Equivalent Length</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diameters</td>
</tr>
<tr>
<td>90 elbows with turning vanes</td>
<td>2</td>
</tr>
<tr>
<td>Expansion bellows</td>
<td>5</td>
</tr>
<tr>
<td>Cross connection banjo</td>
<td>3</td>
</tr>
<tr>
<td>Butterfly valve (streamlined)</td>
<td>1</td>
</tr>
<tr>
<td>Side outlet elbows†</td>
<td>50†</td>
</tr>
</tbody>
</table>

† This figure is obtained by extrapolation of information on pipe fittings on page 432 of ref. 25, assuming that side outlet elbows will have approximately the same resistance as a side outlet T, that a correction factor of 1.33 may be used for air flow, and a factor of 0.75 for flush fittings.

Using these values, a study of the piping layout shows that the equivalent length, compressor to first plenum chamber, less cooler, is 185 ft, and from valve 7 to the compressor inlet is 209 ft (equivalent). No allowance has been made for friction in the first plenum chamber, but this will be very small if the air is diffused at inlet.
Now we can obtain the head loss, before cascade,

\[ h_f = 0.02 \times \frac{185}{2} \times \frac{53.2^2}{64.4} = 81.3 \text{ ft of air} \]

\[ = 81.3 \times \frac{0.0892}{144} = 0.0503 \text{ lb/sq in.} \]

and after the cascade,

\[ h_f = 0.02 \times \frac{209}{2} \times \frac{102^2}{64.4} = 338 \text{ ft} \]

\[ = 338 \times \frac{0.0465}{144} = 0.109 \text{ lb/sq in.} \]


Consider a turbine stator as a series of convergent nozzles whose total area normal to the issuing velocity is \( A_6 \). Assume that the extreme case is when the nozzle is choking, or \( M_6 = 1 \). Let the Mach number at entry be subsonic, \( M_4 \), and the area be \( A_4 \). Let stagnation temperatures be indicated by \( T_t \), while \( T \) represents stream temperature. The sonic velocity will be denoted by \( a \), velocity in general by \( V \), and specific volume by \( v \).

Noting that \( a^2 = kgRT_4 \) and \( C_p = \frac{Rk}{k-1} \)

\[ T_{4t} = T_4 + \frac{V_4}{2gJC_p} = T_4 \left[ 1 + \frac{V_4(k-1)}{2gJRkT_4} \right] = T_4 \left[ 1 + \frac{k-1}{2} M^2_4 \right] \]

\[ T_{6t} = T_6 + \frac{a_6^2}{2gJC_p} = T_6 + \frac{kgRT_6}{2gJC_p} \]

But \( T_{6t} = T_{4t} \) and \( \frac{R}{JC_p} = \frac{k-1}{k} \)

So \( T_{4t} = T_6 \left( \frac{1 + \frac{k}{2}}{2} \right) \)

\[ V_6^2 = a_6^2 = kgRT_6 = \frac{2gkRT_4t}{k+1} \]

\[ = \frac{2gkRT_4}{k+1} \left[ 1 + \frac{k-1}{2} M^2_4 \right] \]
So \( V_6 = a_6 = a_4 \sqrt{\frac{2}{k+1} (1 + \frac{k-1}{2} M_4^2)} \)

By continuity, \( A_4 V_4 = A_6 V_6 \), or \( \frac{A_6}{A_4} = \frac{V_6}{V_4} \).

For an adiabatic process,

\[
\frac{V_6}{V_4} = \left( \frac{T_4}{T_5} \right)^{\frac{1}{k-1}} = \left( \frac{T_4}{T_4} \right)^{1-\frac{1}{k-1}} = \left( \frac{1}{1+\frac{k-1}{2}} M_4^2 \right) \left( \frac{k+1}{2} \right) \frac{1}{k-1}
\]

\[
\frac{\sqrt{\frac{2}{k+1} (1 + \frac{k-1}{2} M_4^2)}}{a_4 \sqrt{\frac{2}{k+1} (1 + \frac{k-1}{2} M_4^2)}} = \frac{V_4}{a_4 \sqrt{\frac{2}{k+1} (1 + \frac{k-1}{2} M_4^2)}} = \frac{1}{k-1}
\]

\[
\frac{A_6}{A_4} = \left[ \frac{\frac{k+1}{2}}{1 + \frac{k-1}{2} M_4^2} \right] = \left[ \frac{\frac{k+1}{2}}{1 + \frac{k-1}{2} M_4^2} \right] \frac{1}{2(k-1)}
\]

(Appendix 10. Operating Line Calculations.

With the area of the accelerating nozzle throat, \( A_4 \), set at .292 sq ft, any point on the operating line may be obtained from the Mach number required, \( M_4 \), by use of three equations: \( (15) \)
\[ \frac{p_3}{p_4} = (1 + 0.2M_4^2)^{3.5} \]  
\[ \frac{p_1}{p_0} = 1.12 \frac{p_3}{p_4} \]  
\[ Q_c = 21.6 \frac{M_4}{(1 + 0.2M_4^2)^3} \]

Equation (16b) assumes that \( \frac{p_3}{p_1} = 0.955 \), so that assumption should be checked, and equation (16b) modified accordingly, when mass flow and density entering cooler are known, as well as the pressure at that point. Any correction will be of small order, so two approximations should give the answer.

The following table is the result of solution of the equations for various values of \( M_4 \):

<table>
<thead>
<tr>
<th>( M_4 )</th>
<th>( \frac{p_1}{p_0} )</th>
<th>( Q_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>2</td>
<td>24.9*</td>
</tr>
<tr>
<td>0.80</td>
<td>1.706</td>
<td>20.6</td>
</tr>
<tr>
<td>0.60</td>
<td>1.428</td>
<td>15.0</td>
</tr>
<tr>
<td>0.50</td>
<td>1.329</td>
<td>12.4</td>
</tr>
</tbody>
</table>

These values can be used to plot the operating line on fig. 11a, and thence on 11b and 11c.

The pressure ratio across the cooler has been checked for \( M_4 = 0.5 \), but this required such wild extrapolation on the curves that the result is not reliable. The method follows:

At \( \frac{p_1}{p_0} = 1.329 \), and \( Q_c = 12.4 \), \( N_c = 2600 \), \( T_{1c} = 570 \) and \( hp_c = 275(\times 100) \).
Then from (12), $T_0 = 535$. From (6), $N = 2640$, and from fig. 8, maximum hp = 664. The maximum value of $p_0$ for this Mach number is then obtained from (9), $p_0 = 35$ psia.

Now discharge conditions can be found from the pressure ratio and equation (8): $p_1 = 46.5$, $T_1 = 586$, $P_1 = .2145$. From (5), $Q = 29.1$, so

$$\Delta P_{1-2} = .000222 \times \frac{874}{2145} = .905 \text{ psi},$$

and

$$\frac{P_2}{P_1} = \frac{46.5 - .905}{46.5} = .982, \text{ this should be } .955,$$

and if the values taken from fig. 11 were reliable, it would be necessary to amend equation (16b) by the factor $0.955 / 0.982$.

Appendix II. Model Experiment on Re-circulation.

The penetration of an air jet of given velocity is proportional to the diameter of the jet (Ref. 25). To get an idea of the re-circulation to be expected in a plenum chamber after a cascade, a 1/12 scale model of the exit from the cascade and the plenum chamber was constructed, using a large glass jar. Wool tufts on wires stretched across the bottle showed the condition of air flow.

The results obtained were purely qualitative, but quite definite. Even at low air velocities there was re-circulation of a nature which would deflect the issuing jet. This test showed that it would be necessary to experiment with guiding and deflecting walls. These experiments can best be carried out on the low speed cascade.
An interesting sidelight of this experiment is the fact that when some water accidentally got into the bottle, the pattern of the water vortices showed much better than the wool tufts which way the air was flowing.


Due to the following requirements for holding blades between the interferometer windows, considerable experimentation was done.

The setting of the blades must be variable, from one test to the next, both in angle and pitch, so dowling into holes in the glass was out of the question.

Any method used should leave nothing which would appear in the interferometer picture, projecting beyond the contour of the blade.

The aerodynamic load on a 1.5 in. chord blade was calculated to be about 40 lbs maximum (see method of appendix 1), with serious vibration difficulties expected at stalling. This demands a shear strength of 20 lbs, or about 100 psi at each end of each blade, plus an ample factor of safety to allow for vibration.

The first line of attack was to look for an adhesive which would hold the blades to the glass. The low shear strength required made this look quite feasible, but it is doubtful whether all traces of the adhesive could be dissolved off when changing the blade settings. A slight residue would affect the optical properties of the windows.
Most adhesives which might be of use require a layer to be applied to each surface. The excess on the glass could be washed away when the blades were set in place, but it would be a very delicate job, especially if the glass could not be rubbed. The blades would have the ends chamfered so that there would be no chance of adhesive appearing in the photographs. The chamfer would have little effect on the aerodynamic properties, for the interferometer averages the flow over the blade length. In any case, the chamfer would be in a relatively stagnant boundary layer.

Inquiries among adhesives experts indicated that all adhesives used for metal to glass are thermal setting plastics. This would mean cooking the windows, assembled in a jig with the blades, at about 200 to 300°F. The idea of heating the plate glass was extremely distasteful, but it seemed probable that sufficient strength could be obtained without heat.

Some tests were done with a Pliobond adhesive to see what order of strength could be obtained without heat, and to study the technique. Test pieces used were brass nuts of about the same area as a blade end, filed flat and smooth. These were clamped to plate glass, with adhesive between, and allowed to set. After setting, the test piece was pulled off the glass by a measured shear force. The first test gave a shear strength of about 130 psi, but no variation of the technique gave better results.
These tests were extremely rough. The strength figures are inconclusive, but they did show that the problem of getting a good bond without spreading glue all over the glass is very difficult. It is still felt that, if necessary, a technique might be developed in this way, but any adhesive method requires washing the windows with solvents, and this may not be admissible. For schlieren tests, where the glass is less critical, it might work.

Project Adhesive was abandoned, temporarily at least, because a better method presented itself.

Tentative calculations showed that, when the apparatus was running at sub atmospheric pressures, if the blades were glued in they would be supporting the full load of the atmosphere trying to deflect the sides of the cascade. Even assuming that the other blades in the cascade, those not between the windows, would be tightly fitted and would do their share in preventing wall deflection, this left a length of cascade about 5 1/2 in. supported by the blades on the windows. This meant that each blade would have an end load of about 250 lbs, or over 1200 psi.

This suggested two things: First, that the pressure might cause enough distortion of the glass to influence the interferometer; second, with this end load, an adhesive would not be necessary to hold the blades in place, for a pad of rubber would serve the same purpose. The rubber pad could be preloaded enough to hold the blade in, without going to such extreme forces as 250 lbs per blade, then when the section was evacuated, a small deflection of the walls, say .001 in., would only compress the rubber slightly more and would not greatly increase the load on the glass.
Another method which might be used is to put two spring loaded, rubber tipped, plungers in one end of each blade. In this way the end load would be almost independent of small wall deflections, but the small blade section would make it a difficult mechanical job, and a small area of contact might cause too high a stress concentration in the window.

It is possible that in order to get equal loading on all blades, a two piece blade with a spring loaded section on one end might have to be used.

Work was started to develop a technique of fixing a layer of rubber on each end of the blades in contact with the windows. This problem has been solved, but it has come to light that different rubbers have very different coefficients of friction with glass and the best rubber to fasten to the metal has a poor coefficient of friction.

The following tests were made, again using a brass nut and plate glass. An axial load was applied, holding the rubber coated face against the glass in a drill vice, by means of a calibrated spring. A shear force was then exerted, using another calibrated spring pushed down in a drill press, until slipping was observed. No figures are included in the results because the technique was very crude.

<table>
<thead>
<tr>
<th>Coating on brass:</th>
<th>Coefficient of Friction</th>
<th>Method of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowles Rubber Co.</td>
<td>High</td>
<td>Rubber sheet peeled off cement, which stuck very firmly to brass.</td>
</tr>
<tr>
<td>Utility Cement #503</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and sheet of .006&quot; pure gum rubber.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1 coat on each surface, dried, another coat on each, clamped together when tacky. Dried 24 hours.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Coating on brass: | Coefficient of Friction | Method of Failure |
---|---|---|
Same as above, With 32 gage rubber sheeting | | Rubber peeled off easily |
Utility Cement #503 without rubber sheet | Low | Cement smears on glass, never sets very hard |
Cork sawdust embedded in Cement #503 | Very low | Sliding |
Art gum dust embedded in Cement #503 | Very low | Art gum particles shear off with almost no force |
Goodrich Sole & Heel. Rubber cement, with various combinations of other rubber. | | Cement did not dry, even when baked at 200°F. |

On reading a report on the operation of a small, high speed tunnel, it was noticed that stainless steel blades, anchored at only one edge, but of small length:chord ratio, had broken off due to vibration when the cascade stalled at high Mach number. This suggested that there may be grave difficulties with holding the blades. Furthermore, it was learned that the interferometer windows were very delicate, and that any pressure, even rubber padded, would be undesirable. Further, any slight deposits would disturb the optical properties, so it has been tentatively decided to isolate the windows from the blades as much as possible.
Fig. 14, showing how it is proposed to do this, is largely self explanatory. The chief drawback is that only one passage between the blades can be completely visible. If the supporting members are well streamlined they should not have very much effect on the flow, especially if the aspect ratio is great. An advantage is that the leading and trailing edges of the blades can be supported on separate pieces, and the stagger can be varied by their relative movement. Unless this sliding feature is incorporated, the surfaces in contact with the glass should have a rubber coating to prevent scratching and to damp vibration. If one of the members is to slide, the fixed one should be rubber covered and the moving one felt covered.

If the leading and trailing edges are supported on separate slides it will be necessary to leave a small gap for one set of pins to move normal to the cascade when stagger is altered. Low speed cascade tests can be used to determine the seriousness of such a gap. This method would not be practical with the interferometer unless the blade section were large enough to contain a sliding fastening pin.

Another way of altering stagger continuously is to mount each blade on a disc at each end, with linkage to rotate the discs together. This method is limited to use without an interferometer, and has a further structural limitation in that the discs must be smaller than the blade pitch. If the blades are rotated about the leading edge, as would be desirable, the trailing edges must overhang past the edge of the disc by more than half of their chord length.
i = incidence
ξ = deviation
αᵢ = blade inlet angle
αᵥ = air inlet angle
αᵢᵢ = air outlet angle
αᵢᵢᵢ = blade outlet angle
ε = deflection = αᵢ - αᵢᵢ

s = pitch
c = chord
-γ = stagger
θ = camber
Xᵢ = "inlet angle
Xᵢᵢ = "outlet angle
t = max. blade thickness
h = blade height, normal to plane of sketch

FIG. 1a
CASCADE DIAGRAM - COMPRESSOR
Nomenclature as for Fig. la, except

\( \gamma = \) stagger.
CASCADE CHARACTERISTICS

$A_s = 500$ sq. in.

$\alpha_1 = 45^\circ$

No boundary layer suction $\alpha_s = 25^\circ$

" " " " $\alpha_s = 15^\circ$

" " " " $\alpha_s = 35^\circ$

$15\%$ " " " " $\alpha_s = 25^\circ$

CHARACTERISTICS OF
U.S. Navy Standard Axial Fan
Size No. A-30-A
(440 V, 3 Ph, 60 cycle
25 hp, 1200/900 rpm)

Figure 5 LOW SPEED CASCADE RIG

CASCADE & FLOWER CHARACTERISTIC CURVES

Air Flow, $q$, cfm $\times 10^{-5}$

Operating Point
No Losses

0.27 in. Loss

No Losses
Elliptical Profile

\[ \frac{3}{4} \times \frac{3}{4} \times \frac{1}{8} \text{ ANGLES} \]

Finish Smooth Inside
16 Ga. Sheet Steel or
Other Suitable Mat'l.

Low Speed Cascade Rig
Accelerating Nozzle

Scale: 1" = 12"

Fig. 6
HIGH SPEED CASCADE RIG
SCHEMATIC LAYOUT

FIG. 7
D.C. MOTOR CHARACTERISTICS
(GEAR RATIO = 5.85)
(GEAR EFFICIENCY = .97)
MAXIMUM SAFE SPEED 1500 RPM

Forced Cooling
Self-Cooling

H.P.

R.P.M.
HORSEPOWER VS. MASS FLOW
(CORRECTED VALUES)

Operating Line

$N_o = 4530$ RPM

$N_o = 3625$ RPM

$P.R. = 3.0$

$P.R. = 1.5$

$(H.P.)_o$
TURBINE ROTOR VELOCITY TRIANGLE

"\( \alpha_2 \)" STATOR

"\( \alpha \)" NEXT STATOR

TYPICAL TURBINE VELOCITY TRIANGLE

FIG. 12
METHOD OF ATTACHMENT
WITHOUT INTERFEROMETER

SECTION A A

METHOD OF HOLDING BLADES
FOR USE WITH INTERFEROMETER

SCALE: FULL SIZE
JAN 12/47

Fig. 14
4 1/2 CSK SCREWS ON 1 ½ IN. P.D.

LOW SPEED CASCAD
MICROMETER HEAD FOR TRAVERSING GEAR

SCALE: 1 IN. = 10 DIVISIONS

100 DIVISIONS

3 FOR VERTICAL SCREW
5 FOR HORIZONTAL

10 THREADS/IN.