Mechanical Design of a Quadruped Robot
by
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Abstract

This thesis presents the mechanical design and fabrication of the Super Mini Cheetah (SMC) robot, a small (9kg) quadruped that is capable of jumping, bounding and trotting. The robot is designed using commercially available components and rapid prototyping methods, resulting in a low-cost, replicable and modifiable platform capable of force, position, or impedance control of each limb. The mechanical system consists of an aluminum torso frame and 3D printed legs. The design of the legs is based on kinematic calculations which determined the lengths of the linkages, force generating capability and the available workspace.

The detailed design of the leg components was determined by estimating of the loading conditions required for dynamic locomotion. The design of the mechanical system was refined using FEA and bench level calculations. This thesis documents drop tests and hopping experiments that were performed with the first quadruped prototype.

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Chapter 1

Introduction

1.1 Motivation

We present a new quadrupedal robot for dynamic locomotion, the Super Mini Cheetah (SMC). The robot is designed using commercial-off-the -shelf components and rapid prototyping methods. It can perform dynamically stable jumping, bounding, turning in place, and braking using only foot-contact detection. To the best of our knowledge it is the first small, power autonomous quadruped to perform such a wide range of behaviors [1].

The MIT Cheetah has made significant progress in the world of robot legged locomotion. It presents advanced design solutions in both controls and mechanical structure for this class of robots making it one of the only well known robots that can perform dynamic maneuvers. However, its custom motors and components make it an expensive research platform. The cost of production and maintenance of such a robot makes it less accessible even though the learning opportunities for gait control are high.

The goal of the Super Mini Cheetah (SMC) robot, shown in its current form in Figure 1-1 and 1-2, is to provide a more accessible platform at a fraction of the cost and the weight of the MIT Cheetah while maintaining many of the dynamic characteristics. The mechanical system addressed accessibility of the design in not only its size and weight, but also in the use of rapid prototyping methods like 3D
printing.

Figure 1-1: Current SMC prototype with some of its features labeled.

Figure 1-2: Different views of the SMC robot; a) Top view; b) Side view and c) Front View.

1.2 Scope of Thesis

This thesis describes the design, manufacture and assembly of the mechanical components of the SMC robot. These components include the 3D printed links and an aluminum torso frame. The robot will be used in ongoing research on the control
of legged locomotion. The Author's contribution was to develop the initial detailed design of the quadruped's parts given a specification of the target motor, required limb geometry, and expected loading.

An individual leg module is a planar two degree of freedom linkage. The SMC quadruped is made of four identical leg modules. The robot described here is the first functional prototype. Some decisions made during the design process were conservative.

1.3 Organization of Thesis

This thesis is organized as follows;

- **Chapter 2** discusses the functional requirements, specifications and resulting layout of the machine.

- **Chapter 3** covers the detailed design of the leg module components divided into sections which are the Shins, Femurs and Shoulder. Each section will discuss the bench level calculations, conceptual design and loading conditions.

- **Chapter 4** covers the design of the quadruped torso components.

- **Chapter 5** outlines the experiments done to check the performance of the leg design.

- **Chapter 6** summarizes the fulfilled objectives of the final design and suggested improvements for future work.
Chapter 2

System Overview

Our overarching goal was to create a simple, replicable and hence more accessible robot than existing legged locomotion robots. In order to achieve adequate dynamic performance to do dynamic experiments, we first selected motors which appeared to have adequate torque density, inertia, friction and load capacity. We used the total mass of the motors to estimate the machine’s mass and calculate the required force and motion capabilities. More on this highlevel design process can be found in [1]. The force and range of motion requirement drove the selection of the geometry discussed in section 2.1. Finally, I used the linkage geometry, required force and range of motion to inform the functional requirements of the structural system (Section 2.2).

2.1 Geometry and Layout

Force and motion output at the foot as a result of torque and motion at the motors are dictated by the layout and geometry (Figure 2-1 and Figure 2-2). Two common configurations for planar legs are the series and parallel configurations. Figure 2-1a shows the series configuration; the knee motor is downstream of the hip motor. Examples of this can be seen in the MIT Cheetah and previous prototypes of the mini cheetah. The parallel configuration shown in Figure 2-1b has the motors mounted in the same enclosure parallel with respect to the structure. A good example of the parallel layout is in the ATRIAS robot.
The configuration chosen for the SMC leg prototype was the parallel motor layout. In this layout, both motors can contribute to the vertical (y) and horizontal (x) ground reaction forces (Figure 2-1a). This layout also enables the stators of both motors to be static with respect to the torso. On the other hand, the series layout would have the knee motor hanging off a limb, increasing the inertia at that joint. Figure 2-2 visualizes some the leg configurations that can be achieved with the parallel layout.

Following selection of the layout, we determined the link lengths driven by the
estimated requirements for peak force generation within a desired workspace. We calculated the maximum force generated for a variety of leg configurations as a function of peak motor torque. A visual representation of this can be seen in Fig 2-3. A maximum motor torque of 9 Nm was used to calculate maximum vertical forces [1]. Figure 2-3 shows an 8 cm range over which the leg can generate adequate ground reaction forces. We were then able to generate dimensions of the leg and its maximum range of motion between the links (Figure 2-4).

Figure 2-3: The predicted workspace of the SMC leg prototype generated by calculating the Jacobian for each leg configuration. The x and y units correspond to the position of the foot with respect to the centroid of the motor enclosure mounted on the torso [1].

Figure 2-4: SMC Leg design showing lengths of linkages. The dimensions are: L0 = 60, L1 = 60, L2 = 145 mm. [1].
2.2 Functional Requirements

Since this robot is to be used for testing and experimentation, it is important that its mechanical system satisfies three major parameters; Adjustability, Modularity and Expandability.

In terms of adjustability, properties like the leg geometry, mass and layout of the machine were considered in such a way that changes to the mechanics of the robot, for example, change in motors, will not limit/interfere with the implementation of the control strategies. Modularity allows for quick repairs and in some cases speeds up the replacement of components that have been damaged or the insertion of parts in order to try new design concepts. Satisfaction of this parameter also reduced the number of parts that needed to be designed. Therefore the analysis work was simplified as I only needed to implement loading conditions on one of the four identical parts. Lastly, the mechanical system needed to allow for expansion. In particular, the addition of sensors and batteries to power the robot autonomously.

Other than the above, the robot had to be light weight so that it could be carried from the programming station to the track. The initial goal for the weight, 8-9 kg was set using the weight of the motors which are the heaviest part of the robot. It is important to note that the MIT Cheetah currently stands at 33kg; more than 3 times the current weight of the SMC robot at 9.5kg. Each motor weighs about 0.6kg. For four legs, a total of eight motors would weigh about 4.8kg. The rest of the mechanical system had to be as light as possible in order to remain close to the goal weight (a budget of about 4.2kg). This guided the selection of ABS plastic and aluminum as the materials for the leg module and torso frame respectively.

Manufacturing advantages, for example access to rapid prototyping and hobby scale machines, were also taken into account with the selection of the materials. 3D printed legs had to be sturdy enough to carry the distributed weight of the machine as well as withstand impact forces with the ground during testing which was set to 3 times the weight of the machine(270 N). Changes to the leg module could easily be made by simply printing a new part when parameters were changed or printing more
parts when scaling up from one to four legs.

In addition to being light weight, the 3D printed plastic parts should not interfere with the expected output of the control system at the foot. This can be achieved by minimizing the inertia and friction losses at the joints. In the following Chapters, I will discuss the detailed process of creating the three dimensional links from the two dimensional layout and requirements presented in this chapter.
Chapter 3

Leg Module Design

The robot leg must be strong enough to carry the body weight and withstand impact loads when the robot foot hits the ground. This chapter covers the detailed design of the components that make up a single leg module. These include the Shoulder, Femurs and Shins shown in Figure 3-1. The design is guided by the strength of the parts under different loading conditions. The shins are the load bearing links; this chapter covers their design process in more detail than the rest of the components.

Figure 3-1: Leg module with labeled major components
One factor that affects the design of the leg is the material used. As mentioned in section 2.2, the legs needed to be light-weight in order to stay within the budgeted machine weight. We considered materials like 3D printed ABS plastic and aluminum because of the ease of manufacturing. The strength of 3D printed parts is nowhere near the strength of machined aluminum parts (Table 2-1). However, aluminum was undesirable because it is heavy. 3D printed parts can be modeled to achieve more complex shapes as seen in the following sections. 3D printers also have software that gives the user the ability to set the density and orient parts to get maximum strength. The calculations and analysis of the parts described in this chapter are assuming that the part is made up of the ideal solid material. We used a Stratasys Dimension 3D printer where a density setting of solid implies that the part has a very thick shell and a very dense fill pattern (Figure 3-2). The ABS plastic used has a yield stress of $42.5\, MPa$ and a modulus of elasticity, $E = 2\, GPa$.

<table>
<thead>
<tr>
<th>Material</th>
<th>Strength to weight ratio [KN-m/kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D printed ABS plastic</td>
<td>80</td>
</tr>
<tr>
<td>Aluminum Alloy</td>
<td>214</td>
</tr>
</tbody>
</table>

Table 3.1: Table showing strength to weight ratio of different materials.

Figure 3-2: Left to right; Solid, Sparse Low Density and Sparse High Density [2]
3.1 Shins

The shin is the part of the robot leg that makes direct contact with the ground. The provided geometry (Figure 2-1 and 2-2) indicates that the bottom rotational joint would make contact with the ground. This is not ideal because the joints are a point of stress concentration. Therefore the two shins were designed differently as the external and internal shins. The external shin has a foot that is slightly larger to make direct contact with the ground. The internal shin sits within the external and does not make contact with the ground as shown in Figure 3-3. The impact loads at the foot were assumed to be three times the static weight that is 270N.

![Figure 3-3: Interface between internal shin and external shin](image)

### 3.1.1 Initial Cross Section Selection of the Shins.

I modeled the structure of the shins as a beam in bending with a load acting on its end. The external shin has a thicker foot (Figure 3-3) to assist with impact which I modeled separately. The approach to choosing the appropriate cross section area and foot dimensions is similar to that outlined in [3]. First, I identified the worst case loading condition for a maximum bending load with the links arranged as shown in
Figure 3-4. The femurs are made to be almost parallel with the ground so that the angle between the external and internal shin is $75.6^\circ$ ($\theta = 37.8^\circ$).

![Figure 3-4: Leg orientation and Free Body Diagram of External Shin](image)

I will start by discussing the cross section of the beam in bending. For reference the shin length, $L_s = 145mm$ and the upper joint to the femur is assumed to be fixed at the moment of impact (Figure 3-4). The required safety factor must be two. The expected vertical load $F_v = 270N$. Thus the minimum section modulus, $z$, can be calculated. It relates the applied moment $M$ to the maximum tensile stress of the material such that

$$
\sigma = \frac{M}{z}
$$

(3.1.1)

where

$$
M = FL_s
$$

(3.1.2)

and

$$
F = F_v \sin(37.8^\circ)
$$

(3.1.3)

Given that, ABS plastic has a tensile yield stress $\sigma_y = 42.5MPa$ and the factor of safety is two, the maximum tensile stress, $\sigma_{max} = 21.25MPa$. Using equations 3.1.1-3.1.3, I calculated the minimum section modulus as $z = 1.13 \times 10^{-6}m^3$. Section
modulus, \( z \), is the ratio of the area moment of inertia, \( I \), to the distance from the centroid and is therefore a function of the dimensions in the cross section area.

From previous designs, I determined that the maximum outer dimension of each link should be 2.5 cm. This dimension allowed for a suitably lightweight and stiff link with enough lateral clearance within the workspace. The next step was to determine a suitable boundary cross section. I evaluated a beam of uniform rectangular cross section whose section modulus, \( z_{rect} \), is given by

\[
z_{rect} = \frac{bh^2}{6}
\]

where \( b \) is the thickness and \( h \) is the width of the shin. In order to determine the optimum cross section, I needed the weight per length of the section. All the links would be made of ABS plastic with the same length so only the cross section area would be needed.

\[
A_{rect} = bh
\]

Using Equation 3.1.4 and 3.1.5, I created a design tool that calculated different combinations of \( h \) and \( b \) within the range of 0.5cm to 2.5cm with an increase of 0.5 cm. I obtained multiple values for the modulus and plotted the results as shown below.

Figure 3-5: Graph of section modulus vs the initial rectangular cross section area of the shin
As a consequence of the data shown in Figure 3-5, a rectangular section of \( h = 2.5\text{cm} \) and \( b = 1.5\text{cm} \) were chosen for the design. This combination performed best because satisfied minimum section modulus \( (z_{rect} = 1.56 \times 10^6 \text{m}^3) \) while reducing the weight \( (A_{rect} = 3.75 \times 10^{-4}) \).

### 3.1.2 Weight Optimization of the Shins.

Rapid prototyping techniques like 3D printing employed in this project provide the user with the opportunity to manufacture parts with complex shapes. Taking advantage of this, pockets of excess material were removed from the shin to leave behind an I-beam cross section with reduced weight (Figure 3-6). I-beam sections are usually stiffer than regular rectangular cross sections of similar section modulus calculated from

\[
z_l = \frac{bh^3 + (b - t)d^3}{6h}
\]

\[
A_l = 2bx + dt
\]

Where the dimensions \( b \) and \( h \) are as described in section 3.1.1 for a rectangle while the rest of the dimensions thicknesses, \( t \) and \( x \), and width of the pocket, \( d \), are as shown in Figure 3-6.

![Figure 3-6: Major dimensions for an I-beam cross section](image-url)
Keeping \( b = 1.5\text{cm} \) and \( h = 2.5\text{cm} \), I created a new design tool that calculated different combinations of \( t \) and \( x \). \( d \) is calculated from \( d = h - 2x \). I obtained multiple values for the section modulus and cross section area shown in the graph below.

![Graph of section modulus vs the I-beam cross section area for the shin with pockets of material removed](image)

**Figure 3-7:** Graph of section modulus vs the I-beam cross section area for the shin with pockets of material removed.

<table>
<thead>
<tr>
<th>Value</th>
<th>External Shin</th>
<th>Internal Shin</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>1.50</td>
<td>1.50</td>
<td>cm</td>
</tr>
<tr>
<td>( h )</td>
<td>2.50</td>
<td>2.50</td>
<td>cm</td>
</tr>
<tr>
<td>( x )</td>
<td>0.55</td>
<td>0.55</td>
<td>cm</td>
</tr>
<tr>
<td>( t )</td>
<td>0.85</td>
<td>0.75</td>
<td>cm</td>
</tr>
<tr>
<td>( A_t )</td>
<td>(2.84 \times 10^{-4})</td>
<td>(2.70 \times 10^{-4})</td>
<td>(m^2)</td>
</tr>
<tr>
<td>( z_t )</td>
<td>(1.44 \times 10^{-6})</td>
<td>(1.42 \times 10^{-6})</td>
<td>(m^3)</td>
</tr>
</tbody>
</table>

**Table 3.2:** Selected values for the internal and external shins' cross section.

Note that because the outside dimensions, \( b \) and \( h \), remain the same, the section modulus and cross section area will always be less than that of the rectangular beam.
of the same dimension. Increasing the $x$ dimension has a higher effect on the change in section modulus. A relatively high value of $t$ was therefore be selected. Table 3.2 shows the selected values for the external and internal shin. I selected a relatively lighter internal shin because it is required to fit within the external shin.

### 3.1.3 Design of the Foot.

The conceptual design of the foot at the end of the external shin was motivated by the need to create an ideal contact with the ground. The idea was to add material concentrically to the endpoint of the external shin to increase its strength. Therefore the foot is a cylinder making theoretical line contact with the ground. The minimum outer diameter would be the width of the shins ($D_o = 2.5cm$). Figure 3-8 shows the cross section of the foot on the left and a simplified drawing on the right.

![Diagram of foot showing dimensions](image)

**Figure 3-8:** Section of the foot showing important dimensions

Analysis was done on the areas $A$ and $B$ for shear failure. Area $A$ and $B$ are evaluated as

\[
AreaA = t_A x
\]

\[
AreaB = t_B \Delta d
\]
where $t_A$ is the gap in which the internal shin fits, $\Delta d$ is the difference between the foot diameter and the hole diameter (that is the thickness of material below the joint). Because the foot makes direct contact, the aim was to achieve a minimum factor of safety of four for Area A.

$$\tau_A = \frac{F}{t_Ax} \quad (3.1.10)$$

Using a similar procedure as in Section 3.1.2, I used different values of $x$ and $t_A$ until the required factor of safety was reached. The minimum value for $t_A$ is the width of the internal shin (7.5 cm) plus a certain clearance to allow for movement. In this case the clearance was 1.875 mm.

Once a value for $x$ was selected, it was easy to estimate Area B. $\Delta d$ is dependent on $x$ and the bottom clearance which is estimated as 5mm. Shear in Area B was calculated as

$$\tau_B = \frac{F}{t_B\Delta d} \quad (3.1.11)$$

Figure 3-9: Outer view of the foot showing a cylinder of diameter $D$ and length $t$.

### 3.2 Femurs

The femurs connect to the motor through shaft collars at one end and form a knee joint with the shins on the other end (Figure 3-10). The collars are 25mm OD clamp-on shaft collars that are clamped directly onto the motor shaft using a set screw or
key way; these and the bolt circle the femur fastens to were machined after purchase of the collars. I chose the design shown below based on the size of the shaft collar, bolt circle diameter and width of the shins. Note that the motor shaft determined the size of the shaft collar available for purchase.

Figure 3-10: Femurs with different shaft collars for the set screw (left) and key way (right) mechanisms. The cylindrical motor shaft fits in the hole at the center of the shaft collar and femurs. The 3D printed femurs were designed to be identical except for the key way feature. Bolt circle is the same for both femurs.

The material of the femur is ABS plastic while the bolt and shaft collar are made of steel. I selected the size of the bolt from previous designs as well as the space available on the shaft collar. An $M3 \times 0.5$ screw was used which was threaded into the shaft collar (Figure 3-11).

Repeated failure of the bolted joint during experimentation raised the need to carry out bench level calculations like those outlined in [5] for different modes of shear failure. An example of this failure is photographed in Figure 3-12. An impact load on the foot would produce a force at the end of the femur. This results in shear forces and a moment at the bolt circle.
Figure 3-11: A cross section view showing the femur-shaft collar-motor shaft interface and its location relative to the rest of the leg module and body.

Figure 3-12: Failure of the femur bolted joint. Top; image at the moment of failure showing intact steel components. Bottom; image taken after plastic debris has been cleared from the joint.
The first step was to determine the effective stiffness of the bolted joint. Next I estimated the stress at each bolt and its factor of safety. Figure 3-13 shows the geometry of the bolted joint, including the pressure cone of the bolt. Note that a steel washer is included in the geometry to distribute the load in the ABS plastic where the bolt head is located.

### 3.2.1 Femur Bolted Joint Stiffness

To find the stiffness of a bolted joint, one must evaluate the effective stiffness of the clamped members and the bolt individually. Starting with the bolt, one must know the engaged threaded length, \( l_t \), the bolt modulus which is approximated as the modulus of steel, \( E_{steel} \), and the tensile area, \( A_t \). [6] gives the tensile area as

\[
A_t = \frac{\pi}{4} \left( \frac{d_p + d_r}{2} \right)^2
\]  

(3.2.1)

where \( d_p \) is the pitch diameter and \( d_r \) is the minor diameter of the bolts. The engaged threaded length is approximated in [5] to be the same as the grip length, \( l \), of the bolted joint geometry.

\[
l_t = l = h + \frac{d}{2}
\]  

(3.2.2)

where \( h \) is the sum of the thicknesses of the washer and ABS material. Using the length, area and modulus, the bolt stiffness is calculated as follows

\[
K_b = \frac{A_t E_{steel}}{l_t}
\]  

(3.2.3)

The next step is to find the stiffness of the clamped members which include the ABS plastic and shaft collar. I approximated the stiffness for material within a 30 degree cone as shown in Figure 3-13. According to [5], the stiffness of a frustrum of material of thickness, \( t \), inner bolt diameter, \( d \), and minor diameter, \( D \), is approxi-
mated as

$$K = \frac{\pi Ed \tan(\alpha)}{\ln \left(\frac{2\tan(\alpha)+D-d}{2\tan(\alpha)+D+d}\right)}$$  \hspace{1cm} (3.2.4)$$

where $E$ is the modulus and $\alpha$ is the cone angle. Since there are two different materials, ABS and steel, I calculated their stiffnesses separately. In this geometry, the cone of pressure will have its maximum diameter somewhere in the femur, which means that the lower frustum has a component in the femur. The member stiffness is divided into three parts: the upper frustum of thickness $l/2$ in the femur, the portion of the lower frustum of thickness $(\Delta t = h - l/2)$ in the femur and the portion of the lower frustum of thickness $(x = l - h)$ in the shaft collar. These three stiffnesses must be combined in series to give the total member stiffness as follows

$$K_m = \frac{1}{K_{ABSU}} + \frac{1}{K_{ABSL}} + \frac{1}{K_{STEEL}}$$  \hspace{1cm} (3.2.5)$$

In order to find the effective stiffness of the system, consider a case where the preloaded joint is subjected to a tensile load. Both the material and the bolt would
deform the same amount. This places \( K_m \) and \( K_b \) in parallel

\[
K_{eff} = K_m + K_b
\]  

Table 3.3 shows the parameters and results for a joint using a \( M3 \times 0.5 \) bolt.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Label</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolt Diameter</td>
<td>( b )</td>
<td>( 3.00 \times 10^{-3} )</td>
<td>( m )</td>
</tr>
<tr>
<td>Bolt Tensile Area</td>
<td>( A_t )</td>
<td>( 5.62 \times 10^{-6} )</td>
<td>( m^2 )</td>
</tr>
<tr>
<td>Washer Diameter</td>
<td>( d_w )</td>
<td>( 6.00 \times 10^{-3} )</td>
<td>( m )</td>
</tr>
<tr>
<td>Washer Thickness</td>
<td>( t_w )</td>
<td>( 6.00 \times 10^{-4} )</td>
<td>( m )</td>
</tr>
<tr>
<td>ABS Material Thickness</td>
<td>( t_A )</td>
<td>( 7.00 \times 10^{-3} )</td>
<td>( m )</td>
</tr>
<tr>
<td>Shaft Collar Thickness</td>
<td>( t_S )</td>
<td>( 9.00 \times 10^{-3} )</td>
<td>( m )</td>
</tr>
<tr>
<td>Steel Modulus</td>
<td>( E_{steel} )</td>
<td>( 2.10 \times 10^{11} )</td>
<td>( Nm^{-2} )</td>
</tr>
<tr>
<td>ABS Plastic Modulus</td>
<td>( E_{ABS} )</td>
<td>( 2.00 \times 10^{9} )</td>
<td>( Nm^{-2} )</td>
</tr>
<tr>
<td>Grip Length</td>
<td>( l )</td>
<td>( 9.10 \times 10^{-3} )</td>
<td>( m )</td>
</tr>
<tr>
<td>Threaded Length</td>
<td>( l_t )</td>
<td>( 9.10 \times 10^{-3} )</td>
<td>( m )</td>
</tr>
<tr>
<td>Material Stiffness</td>
<td>( K_m )</td>
<td>( 1.71 \times 10^{7} )</td>
<td>( Nm^{-1} )</td>
</tr>
<tr>
<td>Bolt Stiffness</td>
<td>( K_b )</td>
<td>( 1.30 \times 10^{8} )</td>
<td>( Nm^{-1} )</td>
</tr>
</tbody>
</table>

Table 3.3: Parameters for Calculating Femur Bolted Joint Stiffness

Note that the material stiffness is an order of magnitude less than the stiffness of the bolt and two orders of magnitude less than the shaft collar stiffness (\( 4.08 \times 10^9 Nm^{-1} \)). This means that we expect bearing of bolts on the members, which is when the bolt presses against the channel (Figure 3-14)

### 3.2.2 Femur Bolted Joint in Shear

This section gives a summary of the shear stress calculations at the motor shaft due to loading at the foot. For the femur bolted joint, I concentrated on the failure modes in Figure 3-14 (a) and (e). Figure 3-15 shows the bolt circle that connects the femur to the shaft collar and the primary and secondary shear forces due to a force, \( F \), at the end.
Figure 3-14: Modes of failure in shear loading of a bolted or riveted connection: (a) shear loading (b) bending of rivet; (c) shear of rivet; (d) tensile failure of members; (e) bearing of rivet on members or bearing of members on rivet; (f) shear tear-out; (g) tensile tear-out [5].

Figure 3-15: Femur free body diagrams showing resulting forces on the bolt circle. F’ is the primary shear force due to the reaction force, V and F” is the secondary shear force due to the reaction Moment, M.
According to [5], the primary shear force per bolt is given by

\[ F' = \frac{F}{4} \]  
(3.2.7)

The secondary shear force is a function of the reaction moment, \( M \) and is given by

\[ F'' = \frac{M}{4r} \]  
(3.2.8)

where \( r \) is the radius of the bolt circle and \( M = FL_f \). The resultant force per bolt can be calculated as

\[ F_{A,C} = \sqrt{F'^2 + F''^2} \]  
(3.2.9)

\[ F_B = F' + F'' \]  
(3.2.10)

\[ F_D = F'' - F' \]  
(3.2.11)

This shear force acts on the threaded portion of the bolt and the joint will tend to shear across the minor diameter. \( B \) has the highest resultant force and is used to calculate the maximum shear. Therefore shear stress area is \( A_s = 1.08 \text{mm}^2 \) and shear stress is

\[ \tau = F_B \frac{A_s}{A_s} \]  
(3.2.12)

As mentioned in 3.2.1, bearing stress is due to the bolt pressing against the channel. Bearing area, \( A_b = t_A d \) where \( t_A \) is the thickness of the ABS material and \( d \) is the bolt diameter. Thus the bearing stress is

\[ \sigma = -\frac{F_B}{A_b} \]  
(3.2.13)

Using the equations 3.2.7 to 3.2.13, I was able to calculate values of shear and bearing stress as well as the corresponding factors of safety given that the shear strength of ABS plastic is 21.25 MPa. These values are shown in Table 3.4.

Note the very low factor of safety of the shear stress. This is limited by the radius of the bolt circle which is in turn limited by the dimensions of the shaft collar. The
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Label</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Femur</td>
<td>$L_f$</td>
<td>0.06</td>
<td>m</td>
</tr>
<tr>
<td>Bolt Circle Radius</td>
<td>$r$</td>
<td>$9.50 \times 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>Applied Force</td>
<td>$F$</td>
<td>$6.00 \times 10^{-3}$</td>
<td>N</td>
</tr>
<tr>
<td>Reaction Moment</td>
<td>$M$</td>
<td>8.4</td>
<td>Nm</td>
</tr>
<tr>
<td>Primary Shear Force</td>
<td>$F'$</td>
<td>35</td>
<td>N</td>
</tr>
<tr>
<td>Force per Bolt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary Shear Force</td>
<td>$F''$</td>
<td>221</td>
<td>N</td>
</tr>
<tr>
<td>Force per Bolt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear Stress Area</td>
<td>$A_s$</td>
<td>$1.08 \times 10^{-6}$</td>
<td>$m^2$</td>
</tr>
<tr>
<td>Shear Stress</td>
<td>$\tau$</td>
<td>$2.37 \times 10^8$</td>
<td>Pa</td>
</tr>
<tr>
<td>FOS</td>
<td></td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Bearing Stress</td>
<td>$\sigma$</td>
<td>$1.22 \times 10^7$</td>
<td>Pa</td>
</tr>
<tr>
<td>FOS</td>
<td></td>
<td>1.74</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: Calculating Shear and Bearing Stress at the Femur bolted joint.

The current size of the bolt circle is the maximum value for a 25 mm shaft collar which is also the largest size that can be bought for this type. I would suggest changing the material of the femur to increase its stiffness even though this would greatly increase the weight. For example changing the material to Aluminum 6061 alloy would increase the weight of the femur by 2.6 times its original weight in 3D printed ABS plastic.

### 3.3 Shoulder

![Figure 3-16: From Left to Right; Front view and Side view of Shoulder](image)
The purpose of the shoulder is to act as an enclosure to the motors and connect the leg assembly to the torso. The design of this part was guided by the size of the motor and position of the femur. This design was also based on previous versions of the mini-cheetah. However, I made adjustments to give clearance for rotation of the femur and a ledge was added on each side to provide a hard stop for the femur. Figure 3-16 shows two different design views of the shoulder. Figure 3-11 also gives a good example of how the interface between the motor, shoulder and the femur would look like.

Failure in this part has been along the layers of ABS plastic relative to the load. This was rectified by re-orienting the part in the 3D printer and adding rounded corners to reduce stress concentration.

3.4 Design for Manufacturing and Assembly

In this section, I will discuss the additional steps taken to improve the quality of the above parts and the assembly of the leg. First is to consider the interaction between the femurs and shins. As mentioned in Section 2.2, a requirement of the mechanical system is that it does not interfere with the expected output of the control system. One way to ensure this is to minimize inertia and friction losses at the joints. This was achieved by addition of flanged bushings with shoulder screws at the three rotary joints. A cross section of the knee joint is shown in Figure 3-17. The bushings are press fit into the femurs so that the flange face is in contact with the shin. A shoulder screw tightened with a nut was used to make sure that the joint does not separate during repeated impact.

As discussed in Section 2.2, rapid prototyping provides an opportunity to improve the lifetime of the parts. Features like sharp corners and rapid changes in cross section area create locations for stress concentration. These are rectified by rounding off the corners which makes this unique to what can be achieved if the parts were machined. A good example of this can be seen in the internal shin.

One important characteristic of parts manufactured using 3D printers is shrinkage
in certain features. For example holes modeled into the part model shrink by 0.2\textit{mm} in the actual part. This is due to the shrinkage of the plastic parts as they cool down after removal from the printer. Therefore parts made with holes in them are modeled with slightly larger hole diameters. However, this criteria is different when a free fit is needed compared to a press fit. For a free fit, I modeled the holes to be 0.2\textit{mm} larger than the actual diameter while for a press-fit, I modeled the holes to be only 0.1\textit{mm} larger than the desired diameter.

Lastly, I added rubber stops and foam tape to parts of plastic that would make direct impact with the ground or each other. These features include the bottom of the foot and hard stops on the shins and femurs seen in Figure 3-18.
Figure 3-18: Leg module with rubber stops
Chapter 4

Quadruped Design

After testing with a single leg and biped structures was successful, the next step was to scale up to a full quadruped assembly. The mechanical assembly is made up of the Aluminum torso frame and the four leg modules. This chapter covers the design of the metal frame torso only. More information on the design of the leg module can be found in Chapter 3. The frame needed to be able to withstand the weight of the robot as well resultant forces from impact on the ground. I chose the initial layout of the frame shown in Figure 4-1 according to the relative positions of the leg modules.

![Figure 4-1: Initial frame layout](image)

The parts labeled with the same letter are identical and therefore I did analysis on one of each type. I selected the width between the left and right legs, length of B, to be the same as the length of one motor. For beam A, I selected the length between the legs on one side (right or left) to be the minimum distance between the travel limits of each leg in the planar direction plus a clearance (effective workspace).
For beam A, I chose an Aluminum square one inch L-bar as the cross section area. Using equation 3.1.1, the maximum stress in the frame, $\sigma_{max} = 5.81 MPa$. I calculated the section modulus using equations 4.0.1 to 4.0.2.

\[
z = \frac{I}{y} \quad (4.0.1)
\]

where

\[
y = \frac{a^2 + at - t^2}{2(a - t)} \quad (4.0.2)
\]

and

\[
I = \frac{[ty^3 + a(a - y)^3 - (a - t)(a - y - t)^3]}{3} \quad (4.0.3)
\]

In the above equations, $t = 1/8in$ is the uniform thickness and $a = 1 in$ is the width of the L-bar. I obtained a factor of safety greater than 10 given that the yield stress of Aluminum is 276 MPa.

Bars labeled B in Figure 4-1, that connect the two planar halves are a circular Aluminum rods with threaded holes on each end for bolted joints. Similar to the shins, I determined the appropriate diameter of the rod which would in turn determine the
diameter of the screw to be used. Using Equation 3.1.1 for a length of $B = 6$ in, the minimum section modulus is $z_{\text{min}} = 7.45 \times 10^{-8} m^3$, where the force is $F = 67.5 N$ and the maximum stress is $138 MPa$ for a factor of safety of two. The section modulus of a circular rod of diameter $d$ can be calculated from

$$z_{\text{circ}} = \frac{\pi d^4}{32}$$

The final diameter selected that satisfied the minimum section modulus was a 3/8 inch rod.

The frame also protects the motors from impact with the ground in the event that the robot tilted to one side. I achieved this by adding a horizontal beam that extended past the length of the motor. However, to ensure that the added guards made contact with the ground before the motors, they needed to be placed in a horizontal plane below the motors. Therefore, I added a vertical beam between the aluminum side bars (Beams A) and the horizontal beams. Figure 4-3 shows the final version of the torso without the leg modules while Figure 4-6 shows the full quadruped mechanical assembly.

![Figure 4-3: Full frame with added side guards](image)

In addition to bench level analysis described above, I also carried out Finite Element Analysis on the load bearing bars (Beams A and B) to show that they could withstand the load. These can be seen in figures 4-4 and 4-5.
Figure 4-4: FEA on Beam A showing that the maximum stress on the beam is not as high as the yield strength of Aluminum.

Figure 4-5: FEA on Beam B showing that the maximum stress on the beam is not as high as the yield strength of Aluminum.
Figure 4-6: CAD of Final assembly of quadruped
Chapter 5

Design Evaluation

The final stage in this design process was to test the performance of the system, specifically, the leg module. This was done by doing an experiment to compare the force output of one leg to the input commanded at the motor encoder.

5.1 Set Up and Initial Results

![Experiment set up for force testing](image)

We anchored an ATI 6-axis load cell to the floor and a metallic block structure next to it to raise the ground height of the other legs so that the machine hopped on level ground. We collected data from the load cell and the microcontrollers for
comparison. We performed two types of tests on a single leg; a hop test and a drop test. A hop test involved feedforward force profile and impedance control while a drop test consisted of only impedance control. Figure 5-2 shows a force response measured by the load cell during hopping while figure 5-3 shows the kinematics of the motors logged by the encoders during the same stance event.

![Figure 5-2: Load cell data showing ground reaction during hopping](image)

![Figure 5-3: Hopping state information and angle of one motor logged by the microcontroller](image)
Data from the microcontroller contains less high-frequency oscillations than the load cell data over the course of impact. This suggests that a high oscillatory mode exists in the electromechanical structure that is causing the foot to bounce at such a frequency. These high frequency modes were calculated to be within a range of 30-50Hz. Drop tests performed showed that the high frequency mode must be due to intrinsic impedance in the electromechanical system and not in the force controller.

Figure 5-4: Components of the leg that may contribute to the observed 30-50 Hz resonance in the ground reaction profiles

The candidate causes as shown in Figure 5-4 would be;

- Resonance modes of the load cell
- Material at the foot-ground interface
- The shin structure
• The shin-femur joint
• The motor gear box
• The shoulder
• PSoC-side control
• Current controller list

5.2 Further Investigation

The next few sections explore the above causes in an effort to eliminate them and narrow down the cause. The scope of this thesis only covers changes or experiments done to parts of the mechanical system as well as the load cell.

5.2.1 Resonance modes of the load cell

The purpose of this test is to compare the resonant modes of the load cell with those of the initial results shown in Figure 5. We dropped a two pound weight onto the load cell and the data logged is shown in Figure 5-5.

![Graph](attachment:5-5.png)

Figure 5-5: Load Cell data from dropping a 2 lbs weight
The resonance recorded from the above data was a lot higher than the observed resonance. In addition, specifications of the load cell suggest that it has a bandwidth greater than 1.5 kHz, therefore eliminating it from the list of causes.

5.2.2 Material at the foot-ground interface

The tests performed for this interface involved removing the soft rubber footpad so that the ABS plastic of the shin made contact with the load cell. Figure 5-6 shows data logged from a drop test without the foot pad while Figure 5-7 shows data logged from a drop test with the foot pad still on.

![Figure 5-6: Load cell data from a drop test without rubber foot pad](image)

Data from the leg without the foot pad showed large oscillations (40 ms apart) at the beginning of impact which are evidently less in the load cell data with the foot pad. Therefore, the foot pad reduces leg bounce at the start of impact. However, these oscillations do not explain the data in Figure 5-2 as the first impact mode remains even when the foot pad was removed.
5.2.3 Vibrations in the Shoulder

In the high speed video, we observed that the cantilever of the shoulder was resonating in the vertical direction. In order to determine the resonant frequency of the shoulder, we first estimated the stiffness of the shoulder by hanging a 2 pound weight at the end and measuring its deflection in a high speed photograph (Figure 5-8). The measured deflection was $\delta = 2mm$, which produced a torque, $T = 1.24Nm$ at the end of the shoulder and an angular deflection of $\theta = 0.013rads$. We calculated the stiffness of the shoulder in the vertical direction as $6.67 \times 10^{-4} N/m$. We estimated the resonant frequency from

$$\omega = \sqrt{\frac{K}{m}} \quad (5.2.1)$$

We estimated the hung mass, $m$, from the mass of the motors and the shoulder, giving an approximate for the Shoulder resonant frequency at over 200Hz. This is way above the frequency observed in the initial data.
5.2.4 Shin Structure

The purpose of this test was to investigate whether the shin and therefore the foot were too heavy. Using the design tool described in section 3.1, I was able to design and manufacture a lighter shin at about half the weight of the original shin. Data collected from the load cell for both a drop and hop test is shown in Figure 5-9 and 5-10.
The data from the lighter showed the same high impact at the beginning. However, the data also settles to the steady value faster than the heavier leg (Compare Figure 5-9 for the smaller leg to Figure 5-7 for the original). Foot bounce in the lighter leg was also less visible than in the heavier leg. This shows that a lighter leg might prove to be an improvement in the performance of the leg modules.

Figure 5-10: Load cell data from a hop test with a lighter leg
Chapter 6

Summary and Conclusion

This chapter summarizes the fulfilled objectives of the final design, weight distribution and suggestions for improvement in future work.

6.1 Suggested Improvements and Future work

The shoulder design is based on previous designs of similar robots; however the cantilever design is not favorable for repeated impact due to the vibrations seen in the high speed videos of the experiments, Chapter 5; Design Evaluation. The stiffness of this part can be greatly increased by reducing the length of the cantilever. Figure 6-1 shows a concept sketch for the suggested design change in the shoulder. During experimentation, we observed that the Aluminum L-bar, Beam A, can act as a hard stop instead of the shoulder.

The conclusion of Chapter 5 showed that lighter shin reduced the foot bounce on impact. Further investigation should be done to reduce the weight of the foot but still maintain a relatively good stiffness. On the other hand, change in the material of the femur would be suggested in order to improve its strength of the bolted joint. One solution is to wrap the plastic parts in carbon fiber; however, this would make the manufacturing process complicated.
6.2 Final Design

The final design produced by this thesis is shown at the conclusion of Chapter 4: Quadruped Design. The design satisfies the three main functional requirements summarized in Chapter 2. Each of the leg modules is identical except for parts that carry different motors. Here the only difference is the diameter of the hole that carries the motor and the shaft diameter in the femur shown in Figure 3-10. The geometric design of the parts is the same. Adjustability is also shown in the design of the smaller leg (Chapter 5, Section 5.2.4). The design tools created for the shin and foot made it easy to change the dimensions of the shin and observe the effects on the section modulus and factor of safety for use in experimentation in Chapter 5. Design tools were created for each part that was designed and will prove useful in order to make future adjustments.

The machine was designed to meet a weight budget of 8-9 kg. A distribution of the modeled weight is shown in Figure 6-1. Note that the modeled weight did not account for the additional wiring present on the current machine. The current weight of the machine is measured to be 9.5 kg.

The 3D printed leg modules have proved to be easy to assemble and disassemble over the course of experimentation. The shape of the foot made it conducive for the addition of contact switches. The open frame design is also conducive for repairs and assembly. In the end, the mechanical system is a promising start and meets most of the design specifications.
Figure 6-2: Modeled Weight Distribution of SMC robot
Bibliography


