Essays in Corporate Finance and Macroeconomics

by

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Abstract

Chapter One
Chapter One contains an introduction and overview of the thesis.

Chapter Two
Macroeconomic models of credit market imperfections have been offered as a theory for how common shocks to the balance sheets of credit constrained firms are amplified through changes in the value of collateral and transmitted as fluctuations in output. This paper clarifies and extends these models by first showing that they are not robust to the introduction of markets which allow these firms to insure against common shocks. A theory of limited hedging is then proposed in which the supply of hedging available in the economy is constrained by the quantity and value of physical collateral. We find that the constraint introduces a skewed response of the economy to shocks. While the constraint may not affect activity in many states of the world, if shocks are sufficiently adverse, the constraint binds and financial market imperfections amplify the downturn.

Chapter Three
This chapter studies the efficiency of asset redeployment in a general equilibrium model with agency conflicts between firms and investors. It shows that debt is an optimal mechanism because it promotes destruction of bad firms and redeployment of their assets in good firms. The key assumption generating this result is a positive correlation between the cash flow and productivity of firms. The socially optimal leverage ratio features over-leveraging: firms should take on more leverage than the value of their assets. Privately, on the other hand, firms choose to take on leverage equal to the value of their assets. Finally, we show that weak bankruptcy penalties and over-leveraging may lead to financial fragility. Thus public policy concerns about over-leveraging should reflect an analysis of both leverage ratios and the strength of
bankruptcy penalties.

Chapter Four

We explore the implications of liquidity of financial contracts on both security design as well as intermediary design. Liquidity is valuable when investors have needs that may require them to sell financial contracts. However, if potential buyers have the ability to purchase information pertaining to the payoff of these contracts, they may have an incentive to do so, thereby capturing a larger portion of the gains from trade. The benefit of this rent-seeking behavior is that information useful for monitoring is generated (market monitoring). The crucial question is whether this behavior is ex-ante valuable or not and on this turns the choice of securit and intermediary design. The main result regards covariation in these choices. On the one hand, if rent-seeking behavior results in overproduction of information, an intermediary will shut down incentives to rent-seek by investing in multiple projects, but restricting investors to trade in the bundle of all projects. Thus, bank monitoring and security bundling should be observed together. On the other hand, when rent-seeking is valuable, the model predicts market monitoring and trade in individual securities.

Thesis Supervisor: Bengt R. Holmstrom
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I owe large and uncollateralized debts to many people.

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Chapter 1

Introduction

This thesis is a theoretical investigation into the implications of credit market imperfections for macroeconomics. The focus in corporate finance on agency conflicts between lenders and borrowers has delivered a number of rich and interesting insights for understanding the microeconomics of firm level borrowing and investment. This thesis asks what the macro implications of these agency conflicts are. All three chapters are unified in their presentation of general equilibrium models of economies in which there are agency conflicts between borrowers and lenders.

The first chapter investigates a mechanism through which shocks to collateral values are amplified and transmitted as fluctuations in output. That collateral values and output are correlated over the business cycle is a well established macroeconomic fact. However, the behavior of asset prices and output, especially at the time of financial crises have led many authors to suggest that falling collateral values can cause a large fall in output. While this line of thinking dates back at least to Irving Fisher’s (1933) analysis of the Great Depression, the mechanisms through which this occurs are less well understood.

This chapter begins by studying in some detail the amplification mechanism proposed by Kiyotaki and Moore(1997). Their mechanism highlights the fragility of leveraged balance sheets when loans are secured by collateral (which is necessary because of agency conflicts between lenders and borrowers). Falling collateral values lead to default and feeds back into lower collateral values. This cycle is reinforced
when another wave of defaults occurs due to the lower collateral values. I adapt their model by retaining credit market constraints, but introduce insurance markets. I show that the amplifying mechanism disappears. Thus, their mechanism crucially depends on either the lack of indexation of debt contracts or the lack of availability of insurance.

I then introduce a supply constraint into the model on the availability of insurance. Insurance must be collateralized by its providers in order to be credible. This limits the aggregate amount of insurance provided by financial intermediaries and links its provision to the value of collateral. The interpretation of this is that the capital of banks and insurance companies is finite and depends on the value of collateral. It is fluctuations in the value of this capital which leads to amplification. I show that a number of the stylized facts surrounding a “credit crunch” are preserved by the model.

The second chapter studies the macroeconomic implications of leverage. Firms that take on debt face the possibility of bankruptcy and seizure of assets. These assets must then be redeployed in other firms. By studying the efficiency of this redeployment, I seek to understand the costs and benefits of leverage.

Redeployment of assets occurs in any industry in which there is stochastic variation in the productivity of firms. That is, firms with high marginal product of using assets will purchase the assets of those with low marginal product. When there are credit market imperfections this process of redeployment is impaired and can be enhanced through the use of long term contracts. I first show that debt can be an optimal contract to achieve second best optimal asset redeployment allocations. The key assumption needed to generate this result is that there is a positive correlation between firm level cash flow and the productivity of asset use.

This framework is then used to answer questions about leverage. Much of the public policy concerns in the '80's were over the increased leverage resulting from recapitalization. I first show that the private economy will tend to underleverage relative to the first best, and thus some of this concern may be misplaced. However, this result depends crucially on the existence of strong bankruptcy penalties. When
bankruptcy penalties are weak, over-leveraging may lead to fragility in the sense that firms, seeing that the collateral is worth little, default, resulting in lower collateral values and output.

The results are interesting for two reasons. First, the model provides a clearly articulated general equilibrium view of the benefits of leverage. While, the corporate finance literature (Jensen(1986)), has alluded to these benefits, a formal presentation of this view has been lacking. In this light, the main result is that the efficiency of leverage depends on the strength of bankruptcy penalties. The paper is also interesting in its relation to the labor market literature on "churn." If one thinks of the assets as a fixed supply of skilled labor, the contribution of the paper is in recognizing that the process of churn will be affected by financial structure.

The last chapter in the thesis studies a more institutional question. Banks and stock markets are the principal mechanisms for resource allocation in most economies. Each of these mechanisms is characterized by a number of institutional features. In the case of banks, lending via loan contracts, illiquidity of individual loans with respect to the bank's claim holders, and monitoring delegated to the bank. In the case of stocks, these are equity contracts, liquidity of individual securities, and monitoring accomplished via trade in these securities. In this chapter, I present a model which explains why these features of banks and stock markets covary. I show that features of banks and markets arise from a joint consideration of the liquidity and informational needs of the economy. The study is interesting for both its results and its approach. I take a different approach than usual in that what I am most interested in understanding is the covariation of features associated with banks and markets rather than in making a welfare comparison of bank and markets.
Chapter 2

Collateral Constraints and the Credit Channel

The correlation between collateral values and output over the business cycle has prompted researchers to study conditions under which changes in the value of collateral cause fluctuations in output. Much of the resulting literature has focused on a credit channel – the role of credit market imperfections in transmitting and amplifying shocks to the balance sheets of borrowers. Under the assumption that credit markets are imperfect, this research shows that a shock to the net worth of a credit constrained firm will change the firm’s production decisions and is transmitted as a shock to output (see Bernanke and Gertler(1989)). Further research builds on this idea by describing an amplification mechanism through changes in collateral values (see Kiyotaki and Moore(1997) and Shleifer and Vishny(1993)). When a physical asset, as an input into production, is held as collateral against borrowing, a small common shock to the net worth of credit constrained firms may feed back into a general equilibrium change in the value of collateral. This general equilibrium effect results in an amplified shock to net worth and output.\footnote{This is akin to an old idea that buying stocks on margin leads to more volatile stock prices. If stock prices fall, margin calls force liquidation driving prices down further. See also Fisher(1933) for an early analysis of the Great Depression.} While these models deliver amplification effects, they raise theoretical questions because they assume that
borrowers cannot hedge against common shocks. This is troubling for two reasons. First, there is a growing literature describing the importance of firm level hedging (see Myers and Majluf(1984) and Froot, Scharfstein, and Stein(1993)). Second, this literature is accompanied by the explosive growth in hedging instruments available to corporate risk managers.

This paper begins by reconsidering the amplification mechanism of the credit channel in the presence of markets that allow firms to insure against common shocks. The specific model we focus on is a variation of that proposed by Kiyotaki and Moore(1997). Our paper shows that amplification effects are not robust to the introduction of insurance markets. Thus, a model of the amplification mechanism must incorporate a theory of limited hedging. An approach that would be consistent with the Kiyotaki and Moore analysis is to assume that firms are myopic in anticipating shocks and hence do not take steps to insure against them. This paper takes a different approach by focusing on the suppliers of insurance. We note that insurance, like credit, is a contract subject to credibility concerns; in particular, the credibility of the provider of insurance. We posit that credibility requires the holding of collateral and this limits the depth of insurance markets to the value and quantity of collateral. The collateral constraint restores the amplification mechanism.

While the introduction of credibility concerns on the supply side of finance is not new, our modeling of it differs from much of the existing literature on financial intermediation. Other papers, which focus on the effects of shocks to the capital of banks and other financial intermediaries, also assume that intermediaries cannot hedge against aggregate shocks. Insurance markets in our model are open to both intermediaries and firms, thus the model is free of this criticism.

An example will help to fix ideas regarding the collateral constraint. An insurance

---

2The question of robustness to insurance is raised by Kiyotaki and Moore in the conclusions of their paper. However, since agents in their model do not anticipate shocks, they are not able to address the question. In the model we construct, shocks are fully anticipated; thus, we are able to clarify this issue.

3The literature on capital constraints in banking is large. See for example, Bernanke(1983), Bernanke and Blinder(1988), Kashyap and Stein(1994), and Holmstrom and Tirole(1997). There is also a growing literature on capacity constraints in the insurance industry. See Gron and Lucas(1993), Froot and O’Connell(1997).
company has long term obligations to its policy holders. Against this, an insurance company typically holds corporate bonds and mortgages as collateral. In the US, most insurance companies are sufficiently collateralized that they are highly credible (for example, most are AAA rated). Now, consider the following experiment. Suppose that there was only one insurance company in the economy and that it held all of the collateral in the economy - i.e. all of the corporate bonds and mortgages. Take a state of the world, say \( z \), and suppose that the value of all of this collateral in this state is \( V(z) \). Then, insurance claims sold against this state of the world, by assumption of the collateral constraint, cannot exceed \( V(z) \). \( V(z) \) is referred to as the aggregate collateral constraint.

Amplification effects arise only when the collateral constraint is the factor limiting insurance use. Take a "bad" state of the world - a notion that will be made precise in the model. For emphasis, call this a depression and note that \( V(z) \) is low in this state. Insurance against a depression is limited to \( V(z) \). The supply of insurance may be limited in precisely the states in which it is most desired. In addition to restoring amplification effects, this channel also generates implications for equilibrium interest rates, hedging, and the volatility of asset prices that are in accord with empirical findings.

As amplification arises only if the aggregate collateral constraint binds, we ask what factors determine the supply of collateral. The supply of collateral is typically endogenous to an economy. The example shows that collateral is a necessary input into the production function of an insurance company. If there were no corporate bonds or mortgages in the economy, the insurance company would not be able to operate. The insurance company typically holds financial collateral which are claims on other firms or other assets in the economy. Financial collateral, in the economy considered in this paper, is limited to claims on land (physical collateral), which is valuable because it is a physical input into the production function of farmers. \( V(z) \) in this case is the value of the aggregate stock of land and is dependent on the operation of farmers. For instance, if farmers collectively forget how to use fertilizer, land is less productive and \( V(z) \) will fall, constraining the supply of insurance. This
insurance may be demanded by farmers, but may also be demanded by other sectors of the economy. We define farmers to be *collateral creators*, because as land is an input into the production function of farmers, their actions result in changes to the aggregate collateral constraint. Sectors that use financial collateral as an input into production are defined as *collateral users* because they depend on outside collateral for their operation. This identification suggests natural measures for the aggregate collateral constraint.

The theoretical ideas in this paper are related to the literature on contractual restrictions to risk sharing arrangements (Townsend(1979), Diamond and Dybvig(1983), Dubey, Geanakoplos and Zame(1995), and Holmstrom and Tirole(1998), among others). The introduction of the collateral constraint into the operation of insurers is similar to ideas in Holmstrom and Tirole. Their paper presents a model of firms that demand liquid assets to guard against a stochastic liquidity shock (what we call insurance). Liquid assets are claims on these same firms. Holmstrom and Tirole explore whether the private supply of liquid assets will be sufficient for liquidity demand, concluding that it may not be and hence motivating a role for the state in the creation of these assets. We differ from Holmstrom and Tirole in two ways. First, liquid assets in this paper are identified with collateral (land). Second, this paper mainly explores implications of collateral scarcity for the operation of the private sector.

In its focus on collateral as a device to increase the efficiency of risk sharing arrangement, this paper is similar to that of Dubey, Geanakoplos and Zame(1995). That paper begins by presenting a general framework in which collateral is required to back Arrow-Debreu insurance claims. The authors then use this framework to study the economics of the mortgage backed securities market. While this paper uses a similar framework, it is different in that its focus is on the macroeconomics of collateral. We study the consequences of changes in collateral values for firm level investment and output.

The paper is arranged as follows. The next section lays out the basic model and its assumptions. Sections 2.1 and 2.2 studies an economy in which firms are credit constrained but are allowed to hedge against aggregate shocks. We show that that
they will have a desire to do so, and that doing so eliminates amplification effects. Section 2.3 introduces collateral constraints into the supply of hedging. Sections 3 and 4 describe the effects of this constraint on the economy. Section 5 considers a simplification of the model to demonstrate that the collateral constraint reintroduces amplification effects. Section 6 contains conclusions.

2.1 The Model

The Economy

The economy has three dates, \( t = 0, 1, 2 \). There are two goods, one perishable ("corn") and the other durable ("land"). As will be described, the durable good is valued only as an input into production, and has no consumption value. It is in fixed supply, \( \tilde{K} \), and does not depreciate.

Firms

There are three types of firms, labeled B, F and H. There is a continuum of unit measure of each type. Firms are run by entrepreneur/managers who choose corn, \( (c_0, c_1, c_2) \) to maximize,\(^4\)

\[
U = E[c_0 + c_1 + c_2], \quad c_t \geq 0
\]  

(2.1)

\( c_t \) is the dividends of the firm at date \( t \).\(^5\)

Production Opportunities

Firms are characterized by their endowments and their production functions. We refer to \( F \) as the farming sector. \( F \) has access to a production technology at dates 0 and 1 which allows it to produce corn with an input of land. An input of \( k_0 \) units of land at date 0, yields \( zf(k_0) \) units of corn at date 1. An input of \( k_1 \) units of land at date 1 yields \( f(k_1) \) units of corn at date 2. While date 1 production is non-

\(^4\)The consumption good is corn rather than money for the simple reason that money is a financial asset that can also serve as collateral. For the sake of clarity it is essential to define consumption over real goods.

\(^5\)The use of linear preferences, in addition to simplifying the exposition, will highlight the demand for long term contracting as being production rather than preference related.
stochastic, date 0 production is stochastic. $z$ is a random variable which is realized after date 0 investment is made and is a common shock affecting all F firms. It is a discrete random variable taking on values in a finite set, $z \in \mathcal{Z} \equiv \{z_1, z_2, \ldots, z_N\}$, with corresponding probabilities $\pi(z)$. The shock is normalized so that $E[z] = 1$. With a slight abuse of terminology, $z$ will be referred to as both the state of the world and as the aggregate shock.

B also uses land in production. It produces via a non-stochastic technology, $b(k)$

The two technologies are as follows,

$$f'(\frac{\bar{K}}{2}) = \rho, \quad f(0) = 0, f'(k) > 0, f''(k) < 0, \quad (2.2)$$

$$b'(\frac{\bar{K}}{2}) = \rho, \quad b(0) = 0, b'(k) > 0, b''(k) < 0, \quad \forall k \quad (2.3)$$

$$\rho > 1$$

Both functions have continuous first and second derivatives. There is decreasing returns to scale with marginal product equalized when sharing land evenly.\(^6\)

H firms have access to a production technology yielding $zh(C_0)$ at date 1 and $h(C_1)$ at date 2. The production shock, $z$, is common to both H and F sectors.\(^7\)\(^8\) $h(C_t)$ uses only corn as input (and produces corn as output). $h(C_t)$ is strictly increasing and concave, satisfying $h(0) = 0, h'(C) > 0, h''(C) < 0$.

The technology specification is in contrast to that of the farming sector. H firms use no physical capital in production, but pay wages in units of corn to secure "human capital." We shall refer to H as consultants.

\(^6\)Decreasing returns to scale is meant to capture the idea that there are some fixed factors of production in the short run.

\(^7\)The shock is common to both H and F, and has an effect on the aggregate endowment of corn. However, because B has a large endowment of corn at each date, its effect on output is small. However, when we introduce credit constraints, these shocks can have a large effect on output. Thus the important point to note is that the facet of common shocks that we study in this paper is their effect on the distribution of resources.

\(^8\)We have studied a variety of other shock structures including: cases in which the shocks include both idiosyncratic and aggregate components; cases in which $H$ and $F$ have uncorrelated shocks; and a case in which shocks occur at both dates and are correlated across time. While interesting in their own right, the results from these cases do not affect the substance of what is being presented in this paper. Details are available upon request.
Endowments

The basic problem is a borrowing/lending relationship. On the one hand, B is endowed with a large amount of corn at all three dates (exogenous to the specification) and all of the stock of land, \( \bar{K} \). On the other hand, F and H only have a small date 0 endowment of corn, \( w^F_0 \) and \( w^H_0 \), respectively. B will be the source of credit in this economy and we refer to it as the banking sector.

E0: The Benchmark Economy

In the economy described so far, the assumptions behind Modigliani-Miller hold. F and H can borrow from B to finance any positive net present value investment. Thus, consumption and production decisions can be treated separately and the equilibrium is fairly easy to characterize. However, we shall go through the individual agent optimization problems. The purpose in doing this is to introduce the notation for prices and quantities that will be used in the rest of the paper.

Let \( X^a \) be the vector of consumption and investment choices, and \( \theta^a \) be a security plan for each of \( a \in \{ F, B \} \). Then,

\[
X^a = (c^a_0, c^a_1(z), c^a_2(z), k^a_0, k^a_1(z)) \quad (2.4)
\]

\[
\theta^a = (\theta^a_0(z), \theta^a_1(z)) \quad (2.5)
\]

Define \( P \) as the equilibrium price vector: \( P = (u_0, u_1(z), \phi_0(z), \phi_1(z)) \). \( u_0 \) and \( u_1(z) \) are the rental price of land at date 0 and in each state \( z \) at date 1. The rental prices are expressed as relative to the price of corn in that date and state. Corn at date 0 is taken to be the numeraire, and its price is set to one. \( \phi_0(z) \) is the date 0 price of contingent claim that pays one unit of corn at date 1 in state \( z \). \( \phi_1(z) \) is the date 0 price of a claim that pays one unit of corn at date 2 in state \( z \).

Notice that a price for ownership of land has not been specified. It is unnecessary to do so because there is a market for rental of land. Since the only use of land is as a productive input, and ownership carries no special rights, specifying a rental market is sufficient.
The optimization problem for \( F(\text{similar for B}) \) is,

\[
\max_{\{X^f \in \mathbb{R}_+^{2N^2 + 2}, \theta^f \in \mathbb{R}^{2N}\}} E[c_0^f + c_1^f(z) + c_2^f(z)] \\
\text{s.t.} \quad c_0^f + u_0 k_0^f + \sum_{z \in Z} (\phi_0(z)\theta_0^f(z) + \phi_1(z)\theta_1^f(z)) \leq w_0^f \\
\quad c_1^f(z) + u_1 k_1^f(z) \leq z f(k_0^f) + \theta_1^f(z) \\
\quad c_2^f(z) \leq f(k_1^f(z)) + \theta_1^f(z)
\]

The first constraint is the date 0 budget constraint for \( F \). \( F \) purchases date 0 corn, rents land, and purchases financial securities. At date 1 the harvest from date 0 plus the payoff on financial securities becomes the resources for date 1 consumption and production. At date 2, there is no further production, so all proceeds from investment are consumed as dividends.

\( H \)'s problem is different only in that the input into production is corn rather than land. Define,

\[
X^h = (c_0^h, c_1^h(z), c_2^h(z), C_0^h, C_1^h(z)) \quad (2.6)
\]

\[
\theta^h = (\theta_0^h(z), \theta_1^h(z)) \quad (2.7)
\]

\( H \) solves,

\[
\max_{\{X^h \in \mathbb{R}_+^{2N^2 + 2}, \theta^h \in \mathbb{R}^{2N}\}} E[c_0^h + c_1^h(z) + c_2^h(z)] \\
\text{s.t.} \quad c_0^h + C_0^h + \sum_{z \in Z} (\phi_0(z)\theta_0^h(z) + \phi_1(z)\theta_1^h(z)) \leq w_0^h \\
\quad c_1^h(z) + C_1^h(z) \leq z h(C_0^h) + \theta_0^h(z) \\
\quad c_2^h(z) \leq h(C_1^h(z)) + \theta_1^h(z)
\]

In equilibrium, markets for land use and financial securities must clear,

\[
\begin{align*}
k_0^f + k_0^b &= \bar{K} \quad (2.8) \\
k_0^f(z) + k_1^f(z) &= \bar{K} \quad (2.9)
\end{align*}
\]
\[
\sum_{a \in \{F,B,H\}} \theta^a_0(z) = 0 \tag{2.10}
\]
\[
\sum_{a \in \{F,B,H\}} \theta^a_1(z) = 0 \tag{2.11}
\]

**Definition 2.1** The competitive equilibrium of E0 consists of consumption and investment choices, \(X = (X^f, X^b, X^h)\), a security plan, \(\theta = (\theta^f, \theta^b, \theta^h)\) and prices \(P\). \(X\) and \(\theta\) are optimal choices for each agent given \(P\), and given \(X\) and \(\theta\), the market clearing conditions, (8), (9), (10) and (11) hold at \(P\).

Since preferences are linear and utility transferable, the equilibrium is easily characterized. The first best consists of date 0 and date 1 production choices which maximize expected output.\(^9\)

\[
\max_{\{k^f, k^b, C^h\}} f(k^f) + b(k^b) + h(C^h) - C^h \tag{2.12}
\]
\[
\text{s.t.} \quad k^f + k^b = K \tag{2.13}
\]

**Proposition 2.1** In the first best, \(B\) and \(F\) each produce with exactly half the land. \(k^{f*} = k^{b*} = \bar{K}/2\). \(H\) sets \(C^{h*} = h^{-1}(1)\). Rental prices and security prices are,

\[
\begin{align*}
\theta_0 &= u_1(z) = \rho \\
\phi_0(z) &= \phi_1(z) = \pi(z)
\end{align*}
\]

The marginal product of land use is equalized when sharing land equally. At this point the rental on land, its one period use-value, must be equal to the marginal product of \(\rho\). Since all firms are risk neutral and there is no discounting, the prices of financial securities are equal to the probabilities.

We can also define a purchase price of land as \(q_t\). At date 0, a firm that purchases land enjoys the benefit of the rental stream for two periods, therefore \(q_t = 2\rho\). At date 1, since there is no production at date 2, purchase and rental are equivalent and \(u_1\) equals \(q_t\).

\(^9\)In writing this optimization, we have also used the fact that \(B\) has a large endowment of corn at each date, and has linear preferences with no discounting.
Neither the uncertainty in production nor the distribution of initial endowments affect investment and prices. Uncertainty is irrelevant as firms are risk neutral. Initial endowments do not matter as firms can always raise funds for a positive net present value investment. Finally, collateral has no use in this case.

Contracting Assumptions

In E0, both F and H borrow from B to finance production. They short sell financial securities ($\theta^f, \theta^h < 0$) and use these proceeds to purchase inputs into production. This section introduces contracting assumptions that restrict the firms from short selling financial securities. These assumptions also create a role for collateral.

Assumption 2.1 (Incomplete Contracts): Contracts cannot be written contingent on output.\(^{10}\)

Assumption 2.2 (Collateral): Ownership of land is contractible.

Contracts are incomplete and property rights (ownership) are the only enforceable long-term contract. The simplest interpretation of the assumptions is that output ("cash flow") can be easily diverted to fund an entrepreneur's favorite private benefits. This is an extreme form of moral hazard. However, the physical assets cannot be stolen or altered in any way.

The first is a stark assumption that implies that none of output can be promised to outsiders. With just this assumption, a borrower can never seek outside funds for investment as it cannot commit to repay these funds. This means that prices and investment levels of a firm will be a function of initial endowments. This is a common theme in most asymmetric information based corporate finance models.\(^{11}\)

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\(^{10}\)See Hart-Moore(1997) for a justification of contractual incompleteness. In this context, one can motivate the assumption two ways. If output is stochastic and not verifiable, an entrepreneur can always claim that no output was realized. Alternatively, suppose the entrepreneur exerts effort continuously towards output. However, a cessation of efforts will result in all output perishing. Then, an entrepreneur who has pledged some output will always renegotiate this promise down, using quitting as a threat point.

\(^{11}\)see Hubbard(1995) for a survey and exposition of these ideas as applied to firm level investment.
The first assumption also creates a need for collateral. The second assumption identifies this collateral to be land. For instance, a farmer who seeks funds to purchase land may take out a collateralized mortgage. In this case land will be the collateral to make the farmer’s promise of repayment credible.

**The Contract between B and F**

F is short of funds to control land for production and will sign a contract at date 0 with B in order to secure enough land. Define the following,

- (i) $r_0, r_1(z), r_2(z)$ are state contingent transfers (corn) from F to B at each date and,

- (ii) $k_0, k_1(z)$ are F’s production scale in units of land.

A contract consists of $(r_0, k_0, S)$. $S : \mathbb{Z} \to \mathbb{R}_+^3$ is defined by $S(z) = (r_1(z), k_1(z), r_2(z))$. $S$ is a function that maps outcomes to allocations of corn and land. The aggregate shock, $z$, is assumed to be verifiable and contractible.\(^{12}\)

A contract, $(r_0, k_0, S)$ is *feasible* if it satisfies the restrictions imposed by Assumption 1 and Assumption 2 and satisfies B’s breakeven constraint,

$$r_0 + \sum \phi_0(z)r_1(z) + \sum \phi_1(z)r_2(z) = k_0q_0 + \sum \phi_0(z)u_1(z)k_1(z)$$

**Lemma 2.1** Consider a feasible contract, $(r_0, k_0, S)$. Define a financial security with date 1 state contingent payoffs given by,

$$\theta(z) = (u_1(z)k_1(z) - r_1(z))1_{\{u_1(z)k_1(z) > r_1(z)\}} \geq 0$$ \hspace{1cm} (2.14)

Any feasible contract can be implemented by F renting $k_0$ units of land at date 0 and saving the balance by purchasing this financial security.

Proof: see Appendix

---

\(^{12}\)Without a further caveat, this would mean that the aggregate component of output should be contractible since F is assured to have at least $zf(k_0)$. Assume that with a small probability, $\delta$, production fails completely and all output is lost. Then, output is not contractible, but initial contracts can be written contingent on the aggregate state.
A contract consists of primitives \((r_0, k_0, S)\). There are many possible implementations of these primitives, ranging from rental of land plus hedging contracts to purchase of land plus borrowing. The lemma states that, given feasibility conditions, all of these can be restated in terms of rental of land, \(k_0\), and financial investment, \(\theta(z)\). The first part of this, land rental, is just a spot transaction. The need for a long term contract stems from the second part, the desire to save. Note that we have not yet described what an optimal contract is, we shall do so in the next section. Compared to economy E0, the main restriction imposed by the contracting assumptions is that \(\theta(z) \geq 0\).

**Remark**: A collateralized debt contract with face amount, \(D\), on \(k_0\) units of land can be represented as, \(\theta(z) = \max[k_0 u_1(z) - D, 0]\), and a rental of \(k_0\) units of land. In states of the world in which the collateral is worth less than the face of debt \(D\), \(F\) defaults on the debt and gives up the collateral. This implies that \(\theta(z) = 0\). In states of the world in which the collateral is worth more than the face, the borrower does not default and is left with the excess of collateral value over face, \(k_0 u_1(z) - D\). Kiyotaki and Moore (1997) derive the collateralized debt contract as optimal in their setting.

The main substantial difference in our assumptions vis-a-vis Kiyotaki and Moore is that we assume that while a firm's holding of corn is not verifiable, the aggregate state is verifiable. Thus, contracts can be written contingent on the aggregate state.

The justification for this is two-fold. First, it seems reasonable to treat the aggregate state as verifiable. More compellingly, (as will be shown) the date 1 price of land, in equilibrium, will reflect the aggregate state. Thus if the price of land is verifiable, the aggregate state becomes verifiable.

**The Contract between B and H**

A similar exercise for the contract between B and H shows that the only room for a long term contract is in terms of savings.

**Lemma 2.2** Any feasible contract between \(B\) and \(H\) can be implemented by \(H\) purchasing \(C_0\) units of corn at date 0 and saving the balance by purchasing a financial security with payoffs \(\theta(z) \geq 0\).
The contracting assumption: impose restrictions on the choices of the firms at date 0. They both involve some productive investment \((k_0 \text{ and } C_0)\), and some financial investment, \(\theta(z) > 0\).

### 2.1.1 E1: Credit Constraints

The contracting assumptions imply that F and H cannot short sell financial securities and therefore face credit constraints. This section analyzes the general equilibrium when F and H face credit constraints. This will clarify the role of the financial investment, \(\theta(z)\) and the form that it takes. We will interpret \(\theta(z)\) as a hedge contract that firms F and H enter into with B.

**Control Problem for F**

At date 0, F chooses an amount of productive investment, \(k^f_0\), and financial investment \(\theta^f(z)\), out of initial resources \(w^f_0\). These choices, and the realization of the production shock will result in F beginning date 1 with resources \(w_1\).

\[
w_1 = zf(k^f_0) + \theta^f(z)
\]

Again, a productive investment choice is made resulting in output at date 2. F's program is,

\[
(E1F) \quad \max_{\{c_0^f, k_0^f, \theta^f(z), k_1^f(z), c_1^f(z)\}} \quad E_z [c_0^f + c_1^f(z) + f(k_1^f(z))]
\]

\[
s.t. \quad c_0^f + u_0k_0^f + \sum \phi(z)\theta^f(z) \leq w_0^f
\]

\[
q_1k_1^f(z) + c_1^f(z) \leq zf(k_0^f) + \theta^f(z)
\]

\[
c_0^f, c_1^f(z), \theta^f(z) \geq 0
\]

\(f(k_1^f(z))\) has been substituted in for \(c_2^f(z)\). Otherwise the objective is exactly as defined in E0. Constraints are altered in only one substantial way. \(\theta^f(z)\) is restricted to be non-negative. This implies that F firms face a sequential budget constraint. The first constraint is the date 0 budget constraint. F divides resources between productive investment and financial securities. The second constraint is that at date
1 in each state of the world, consumption plus productive investment must equal resources from date 0 investment plus security holdings.

To solve this program, work backwards from date 1. At date 1 in state \( z \), given resources \( w_1 \), \( F \) chooses an amount of land to rent (or equivalently, to purchase) out of this wealth to solve,

\[
\max_{\{k_1(z)\}} \quad w_1 - u_1(z)k_1(z) + f(k_1(z))
\]

\[
s.t \quad u_1(z)k_1(z) \leq w_1
\]

This can be broken down into two regions depending on whether the firm is wealth constrained. Assume the constraint does not bind, then the solution is given by \( k_1(z) = f^{-1}(u_1) \). We verify later that, in equilibrium, \( u_1(z) \) is always less than \( \rho \) so that \( k_1(z) < \frac{K}{2} \). The constraint will bind if \( f^{-1}(u_1(z))u_1(z) > w_1 \). Call this critical value \( w_1(z)^* \) and write the date 1 land demand and \( F \)'s value function over \( w_1 \) as,

\[
k_1(z) = f^{-1}(u_1(z)) - \frac{1}{u_1(z)}(w_1(z)^* - w_1)^+
\]

\[
J^*(w_1) = w_1 + f\left(\frac{w_1(z)^*}{u_1(z)}\right) - w_1(z)^* - \left(f\left(\frac{w_1(z)^*}{u_1(z)}\right) - w_1(z)^* - (f\left(\frac{w_1(z)^*}{u_1(z)}\right) - w_1)\right)^+
\]

where, the notation \((X)^+\) denotes \(\text{Max}[X,0] \). This expression allows for a simple interpretation. In a world without contracting problems, \( F \) would be risk neutral - this is the first part of the expression for the value function. The contracting problem introduces the second term - the firm is short levered "put" options on \( w_1 \) struck at \( w_1(z)^* \). Note that \( J \) is concave since \( f \) is concave. \( F \) is effectively risk averse at date 0 with respect to date 1 wealth. Because of decreasing returns in the reinvestment technology, low wealth results in foregoing high marginal product investments. This risk averse behavior arises due to the interaction of contracting considerations and the dynamic investment problem.\(^{13}\)

\(^{13}\)See Froot, Scharfstein, and Stein(1993) for a similar discussion. See Gross(1995) for theory and test of this sort of behavior. Also see Grossman and Vila(1992) for an analysis in the context of a wealth constrained investor.
The date 0 control problem is ($c_0^f$ must be 0 since a little algebra shows that $J'(\cdot) \geq 1$),

$$\max_{\{k_0^f, \theta^f(z)\}} E_z[J^z(zf(k_0^f) + \theta^f(z))]$$

$$\text{s.t} \quad \sum \phi(z)\theta^f(z) + u_0k_0^f \leq w_0^f$$

$$\theta^f(z) \geq 0$$

G chooses a pattern of payments $\theta^f(z)$ that best matches its value function. Because $J^z(\cdot)$ is concave and production is risky, there is scope for insurance. In particular, F will choose to guard against low realizations of $z$, as the following proposition shows.

**Proposition 2.2** In economy $E1$, F’s optimal financial security provides insurance against low realizations of $z$ and takes the form of put options.\(^4\)

Proof: see Appendix.

Let $\theta^f(z)$ be the payoff of the optimal financial security in state $z$. Then,

$$\theta^f(z) = \text{Max}[\theta^f(z^*) + f(k_0^f)(z^* - z), 0] \quad (2.15)$$

The strike of the put option is $z^*$, and the payments increase linearly in the difference, $z^* - z$. Basically, F would like to protect against having too little resources available for investment at date 1. Hedging means setting the shadow value of these resources equal in all states, or equalizing these resources themselves. Transfers required to accomplish this turn out to be a linear function of the state, $z$. However, above some critical level, F would like to commit to paying out funds. As this cannot be done (since F’s promise of payment is not credible), payments in these states are zero. This gives the strike of the put option.

**Remark:** A debt contract will have payments, $\theta^f(z) = \text{Max}[k_0^f u_1(z) - D, 0]$. In other words, payments are highest in the best states. This is the opposite of what an optimal choice of $\theta^f(z)$ will dictate.

---

\(^4\)Clearly the prediction of put options is due to the specifics of this model and the desire for insurance therein.
Control Problem for H

Sector H's problem is very similar to F's except that the input into H's production is corn rather than land. H solves,

\[
(E1H) \quad \max_{\{c_0^h, c_1^h, \phi^h(z), C_1^h(z), c_t^h(z)\}} \quad E_z[c_0^h + c_1^h(z) + h(C_1^h(z))] \\
\text{s.t.} \quad c_0^h + C_0^h + \sum \phi(z)\theta^h(z) \leq w_0^h \\
C_1^h(z) + c_1^h(z) \leq zh(C_0^h) + \theta^h(z) \\
c_0^h(z), c_1^h(z), \theta^h(z) \geq 0
\]

It is straightforward to show that H will also have a demand for insurance and that this will take the form of put options.

\[
\theta^h(z) = \max[\theta^h(z^h) + f(k_0^b)(z^h - z), 0]
\]

(2.16)

Control Problem for B

B is not constrained in investment choices at any date. The optimization is,

\[
(E1B) \quad \max_{\{c_0^b, \phi^b(z), k_0^b, c_t^b(z)\}} \quad E[c_0^b + c_1^b(z) + b(k_0^b(z))] \\
\text{s.t.} \quad c_0^b + \sum \phi(z)(c_1^b(z) + \theta^b(z)) + u_0k_0^b + u_1(z)k_1^b(z) \leq b(k_0^b)
\]

Note that we have not imposed non-negativity on \( c_t \). This is a simple way of representing that B is not wealth constrained. Because B is not constrained, optimality for land use means setting the marginal product of land equal to its rental price,

\[
b'(k_t^b) = u_t
\]

Equilibrium

The equilibrium conditions are market clearing for land at date 0 and in each state
\( z \) at date 1 and market clearing for financial securities.

\[
\begin{align*}
k_0^f + k_0^b &= \tilde{K} \\
k_1^f(z) + k_1^b(z) &= K \\
\theta^f(z) + \theta^h(z) &= 0
\end{align*}
\] (2.17) (2.18) (2.19)

Definition 2.2 Equilibrium in E1 consists of \( (X, \theta, P) \): consumption and investment choices, \( X = ((c_0^j, c_1^j(z), c_2^j(z), k_0^j, k_1^j(z))_{j=b,F}, c_0^h, c_1^h, C_0^h, C_1^h) \), a security plan, \( \theta = (\theta^h(z), \theta^f(z), \theta^h(z)) \) and prices, \( P = (u_0, u_1(z), \phi(z)) \). \( X \) and \( \theta \) are solutions to E1F, E1H and E1B, given \( P \), and given \( X \) and \( \theta \), (19), (20) and (21) are satisfied at \( P \).

Lemma 2.3 An equilibrium of E1 exists under the maintained assumptions.\(^{15}\)

Proof: see Appendix.

Closing the model in general equilibrium allows us to characterize land prices and the prices of financial securities. It is easy to see that we must have,

\[
\phi(z) = \pi(z)
\]

\( F \) and \( H \) purchase a hedge which pays off in bad states of the world. \( B \) provides this hedge at an actuarially fair price. In a standard general equilibrium model of insurance (for example, Lucas (1978)), insurance against states in which the aggregate endowment is low carries a premium. The key distinction is that, while a low realization of \( z \) does result in a low aggregate endowment of corn at date 1, since \( B \) is risk neutral and is assumed to have a large date 1 endowment of corn, the price of insurance is actuarially fair.

Proposition 2.3 Land rentals in economy E1 are as follows: in states where \( z < z^* \), the equilibrium rental price of land, \( u_1(z) \) is constant. For \( z > z^* \), \( u_1(z) \) is increasing in \( z \).

\(^{15}\)Unfortunately, results on uniqueness of equilibrium have not been established.
Proof: see Appendix

Since resources in the hedging states are equalized, the date 1 demand for land from F in these states is equalized. This implies that the price of land in these states is constant. High realizations of $z$ are high realizations of date 0 investment. Thus, F is less constrained and is able to use more land at date 1. Since F is more productive with the land, rental and land prices rise. Hedging insures against swings in the distribution of wealth. As it is this distribution which affects asset prices, the volatility of prices is reduced through the holding of financial securities.

2.1.2 Amplification and Hedging

The main difference between E1 and E0 is that F and H firms are restricted from taking short positions in financial securities. In E0, F and H short sold output from production to finance this production. At first glance it seems surprising that when short sales are prohibited, firms actually may go long financial securities. This arises because of the dynamic investment problem. Production at date 1 is dependent on output from date 0. If output from date 0 production was very low (suppose $z = 0$), the firm will have to shut down date 1 production. As this can be very costly, the firm will seek to hold insurance that buffers against a low realization of $z$.\footnote{The demand for insurance by F would arise even if production was constant returns to scale. Suppose $f(k) = pk$, then, $J^z(w) = u_1(z)$. The marginal valuation depends inversely on $u_1(z)$. Since it is the marginal value that must be hedged, differences in prices across two states will lead to a firm shifting all of its hedge to the high marginal value state. In equilibrium, this will lead to equal prices in hedging states and a hedge security that resembles a put option.}

The restriction on short sales also leads to a violation of the conditions necessary for Modigliani-Miller to hold. In particular, production choices and land prices in E1 are functions of the shock $z$ and the endowments of the F and H firms. Output is a function of the distribution of wealth. This is the basic mechanism of credit channel models, as presented, for example, by Bernanke and Gertler(1989). In E1, however, there is no additional amplification mechanism at work. Collateral values and output are correlated, but there is no feedback of changes in collateral values into output.

Let us contrast these results with the amplification mechanism that Kiyotaki and
Moore (1997) illustrate. By their assumptions, the aggregate shock is not verifiable and firms are forced to borrow via debt contracts. Under these restrictions, \( \theta(z) \) would be \( \max[k_0^1u_1(z) - D, 0] \). A firm, after the date 0 production shock, is left at date 1 with resources \( zf(k_0^1) + \theta(z) \). The collateral amplification mechanism of Kiyotaki and Moore is that in states in which \( z \) is high, not only is \( zf(k_0^1) \) high, but also \( \theta(z) \) is high. In their infinite horizon model they show that the second term can be quite large. Thus a shock \( (z) \) to the balance sheet of a firm feeds back via a general equilibrium change in the value of collateral as an amplified shock to the firm’s balance sheet.

As we have shown, the crucial restriction in arriving at this result is that the aggregate shock cannot be verifiable. If contracts can be written contingent on this shock, firms will enter into hedge contracts that will undo any amplification effects.\(^{17}\) In fact, \( \theta(z) \) will resemble put options whose payoffs are highest in low \( z \) states. This not only eliminates amplification effects, it also dampens the effect of shifts in the distribution of wealth on activity.

### 2.1.3 E2: Collateral Constraints

Economy E1 illustrates that credit constrained firms will have a desire to hedge changes in their production opportunities. This is manifest as a desire to hold a particular financial security. When the choice of security is optimized, amplification effects or feedback effects from land prices to production are broken.

Two important issues have been overlooked. First, how are financial assets synthesized? Second, how does this relate to the optimal long term contract between B and F?

A financial asset is a promise of payment in state \( z \in \mathcal{Z} \) at date 1. However, promises are not enforceable. They must be collateralized by physical assets, which can be seized in the event of non-payment.\(^ {18}\) This is a direct implication of Assump-

\(^{17}\) If shocks were not common across all F firms, there would be no general equilibrium change in collateral values. As such hedging, while it would still be desirable, would not affect amplification.

\(^{18}\) This is similar to the treatment in Dubey, Geanakoplos, and Zame (1996).
tion 2. B’s holdings of corn at date 1 are not verifiable. The only enforceable contracts are property rights over physical assets. A financial asset is a long term contract. Its seller promises to make a payment of corn in the future. These are secured through the holding of land as collateral.

One can look at this in two ways. The financial asset can be thought of simply as a derivative security. B acts as an investment bank by purchasing land and then issuing a land-backed security against this. Alternatively, one can think of the financial asset as part of the long term contract between B and F. F demands a particular pattern of payments (θ^f(z)) from B in the optimal contract. We can interpret B as an insurance company that makes state contingent promises which are backed by the collateral of land.

A state z Arrow-Debreu claim is a financial asset promising one unit of corn in state z. It must be backed by an amount of collateral, L (land), that is required to be held by a seller of the claim.\textsuperscript{19} Non-performance on an asset promise constitutes default and results in seizure of collateral by the owner of the financial asset. Thus in any state z, the actual delivery is given by d(z) = min(1, Lq_1(z)).

The description of the financial market has the following implication: while a full set of Arrow-Debreu claims are traded, the supply of these claims is limited by collateral considerations. In this sense, the financial market has limited depth.

There are two sets of equilibrium conditions: market clearing for land at date 0 and in each state z at date 1, and market clearing for financial securities at date 0.

\begin{align*}
k_0^f + k_0^b &= \bar{K} \quad & (2.20) \\
k_1^f(z) + k_1^b(z) &= \bar{K} \quad & (2.21) \\
θ^b(z) + θ^f(z) + θ^b(z) &= 0 \quad & (2.22)
\end{align*}

In case E1, it was assumed that B was unconstrained in its sale of financial assets. Market clearing for financial assets and the introduction of collateral constraints in

\textsuperscript{19}Collateral is held by the seller of the asset in this model. That is, if there are any benefits from using the asset, they accrue to the seller. Dubey, et al. consider more general asset structures.
this market, requires that,

$$\theta^h(z) + \theta^l(z) = -\theta^b(z) \leq Lq_1(z)$$

In other words, B will have to be the provider of insurance to F and H. However, the sale of insurance must be backed by holdings of collateral L. Since there is no cost of increasing L, B may as well pledge all of its land as collateral, and since B is wealthy, assume without loss of generality that B owns all of the land in the economy. Thus, rewrite the collateral constraint as,

$$\theta^h(z) + \theta^l(z) = -\theta^b(z) \leq \bar{K} q_1(z)$$

In this expression, the price of land, $q_1(z)$, rather than the rental on land, $u_1(z)$, has been used purely for expositional purposes. In fact, $q_1(z) = u_1(z)$, since land has no use after date 2. If the constraint binds, a premium is introduced into the price of collateralized insurance. Call this premium $\eta(z) = \frac{\phi(z)}{\pi(z)}$, and write the market clearing condition as,

$$\theta^h(z, \eta(z)) + \theta^l(z, \eta(z)) \leq \bar{K} q_1(z) \quad (2.23)$$

**Definition 2.3** The equilibrium of E2 consists of choices, $X = ((c^j_{it})_{i=0,1,2}, (k^b_t, k^l_t, C_t)_{t=0,1})$, a security plan, $\theta = (\theta^l)^{j=b,f,h}$ and prices $P = (u_0, (u_1(z), \eta(z))_{z \in Z})$. Choices are optimal given prices, and given choices markets clear at these prices.

**Lemma 2.4** An equilibrium of E2 exists.

Proof: see Appendix.

If $\eta(z) > 1$, insurance against state $z$ carries a premium. Since the quantity of collateral is fixed at $\bar{K}$, the supply of insurance will be determined by the value of collateral, $q_1(z)$. When this is low, the supply is constrained and insurance may carry a premium. Note that the collateral constraint may bind for only a small number of states. There is a non-linearity in insurance provision that may imply that only
with a low probability will the economy get hit by a sufficiently negative shock that the constraint binds. But it is precisely in these low \( z \) states that the availability of insurance is constrained.

The premium on insurance against aggregate shocks is analogous to one in the usual pure-exchange consumption based models (see Lucas(1978)). In these, insurance against states of the world in which the aggregate endowment is low is expensive because insurance is constrained by the endowment. However, in our case there is no constraint on the endowment, the premium on insurance arises purely through the introduction of the collateral constraint.

**Definition 2.4** The economy is collateral sufficient if \( \eta(z) = 1, \forall z \)

The question of sufficiency is a determination at equilibrium prices if the supply of financial assets exceeds the demand for them. In short, is there enough collateral in the economy to satisfy insurance demand. There is no reason to expect this to hold. However, it is worth noting that in one special case the economy is sufficient.

**Proposition 2.4** Suppose \( \theta^h(z) = 0 \), then economy E2 is collateral sufficient.

**Proof**: 

The collateral constraint is,

\[ \theta^I(z) \leq \bar{K}u_1(z) \quad (2.24) \]

The LHS is the demand in state \( z \), and the RHS is the aggregate supply of financial assets. Both sides are determined in equilibrium. In any state, optimal production requires that \( F \) use exactly half the land, \( k^f^* = \bar{K}/2 \). Now, suppose that \( C = \bar{K} \) (this is all of B’s land) and there is default. \( F \) would be left with \( k^f = \bar{K} \), which is greater than \( k^f^* \) and would clearly be sub-optimal. At date 0, \( F \)'s demand for insurance is a desire to protect against having too few resources at date 1. If, in a state of world with default, \( F \) is left with more land than it needs it can clearly do better by cutting back on insurance purchase in that state and spending this on insurance in

32
another state. Thus, there can never be default as $F$ would never demand an amount of insurance that is not possible to provide.

2.2 Collateral Creation and Collateral Use

The key to the self-sufficiency result in the previous section is understanding the demand for hedging. Farmers wish to ensure that they will have access to sufficient inputs into production at date 1. But the only input into production is land which serves as collateral. Hence, there is always sufficient collateral to satisfy hedging demand.

Contrast this to a sector where the input into production is not collateralizable. A management consulting firm's main input into production is the human capital of its consultants. Securing their services means holding resources to pay the wages of these consultants. Now, in principle, if the human capital of the consultants was collateralizable - i.e. if they could be bought and sold in advance - the sector would be collateral sufficient. However, as this is not the case, they are a collateral user.

We can generalize the two examples as follows. Consider a sector of firms with a strictly concave production function given by $f(K^c, K^u)$. $K^c$ is a vector of inputs into production which are collateralizable, $K^u$ is a vector of uncollateralizable inputs. Denote the aggregate supply of these inputs as $\bar{K}^c$ and $\bar{K}^u$. The inputs could be physical assets (land) and human capital (labor), but may include more. For instance, they may be intermediate goods that must be purchased from a supplier.

Consider the problem that this firm faces in a state of the world, $z$, at date 1. Denote the prices of inputs in this state by $q^c_z$ and $q^u_z$.

A firm with resources of $w$ (at date 1) solves,

$$J^z(w) = \max_{\{K^c, K^u\}} f(K^c, K^u) + w - q^c_z K^c - q^u_z K^u$$

$$s.t. \quad q^c_z K^c + q^u_z K^u \leq w$$

Given our assumptions, it is easy to show that $J^z(w)$ is concave in $w$. When
the resource constraint does not bind, the solution is given by \( K^{c*}, K^{u*} \). Thus the constraint binds if,

\[
w < w^* = q^w_cK^{c*} + q^u_cK^{u*}
\]  

(2.25)

Since \( J \) is concave, if date 0 actions result in uncertainty in \( w \), the firm will have a demand for insurance. The actual amount of insurance purchased will depend on the specifics of the problem (i.e. uncertainty, curvature of \( J \)). However, an upper bound for the amount of insurance desired is \( w^* \). Then, the question of collateral sufficiency is represented by the inequality,

\[
q^w_cK^{c*} + q^u_cK^{u*} \leq q^w_c\bar{K}^c
\]

The LHS is the maximum amount of collateral used in this sector, the RHS is the supply of collateral. We can rewrite this as,

\[
q^u_cK^{u*} \leq q^w_c(\bar{K}^c - K^{c*})
\]  

(2.26)

The sector uses its collateralizable assets to support demand arising from the un-collateralizable assets. It is possible that if there are enough collateralizable assets within it, the sector will be collateral sufficient.

In the incomplete contracts/property rights framework adopted in this paper, physical capital is collateral, human capital is not. The consulting sector is an example of one in which there are no physical assets but there is a need for collateral because of the human capital input. In the farming example, all assets are collateralizable and the sector is collateral sufficient.

**Definition 2.5** A **collateral creating sector** is one in which the [productive] value of collateralizable inputs into production exceeds its demand for collateral. A **collateral using sector** is one in which there is a deficit.

A collateral using sector requires collateral that it does not create for its opera-
tion. These instruments are supplied by a collateral creating sector. Thus, a market emerges which links these two sectors.

2.2.1 E2: Efficiency and Leveraging

The market for collateral is represented in the model by the financial market. Firms' collateral use is their demand for financial assets (AD insurance claims). To investigate the properties of this market, return to the two sector version of the model, with one sector having purely collateralizable inputs (land), and the other having only uncollateralizable inputs (wages).

**Definition 2.6** Suppose markets clear competitively at date 1. Fix the date 0 production choices, \((k^b_0, k^f_0, C_0)\), and consider only date 0 choices over financial securities, \(\theta\). The competitive equilibrium is constrained financially efficient (CFE) if there is no perturbation to the security holdings, \(d\theta\), that is feasible and results in a Pareto improvement.

**Proposition 2.5** The competitive equilibrium is CFE as long as either,

- the economy is collateral sufficient \((\eta(z) = 1 \ \forall z)\), or
- the F sector is not credit constrained \((\theta^f(z) = 0, q_1(z) = \rho)\).

If both conditions are violated, the equilibrium is constrained financially inefficient.

Proof: see Appendix.

The appropriate notion of efficiency is constrained efficiency - can a central planner subject to the same trading restrictions as the agents reallocate portfolios to make everyone better off. In the first two cases, efficiency is not surprising. Agents face a short sale constraint, as they cannot borrow from the future. Forced to reallocate portfolios along the same dimensions as agents, a central planner cannot improve allocations either.

In the third case, prices become important. The inefficiency is caused by an externality. Insurance use by the F sector affects the wealth distribution and asset
prices. In particular, increased use of insurance raises the value of collateral. This relaxes the aggregate collateral constraint, allowing for more insurance to all sectors of the economy. The farmer's action therefore creates a pecuniary externality on others.

The two cases in which efficiency is restored highlight this. When the aggregate collateral constraint does not bind, relaxing it has no effect. If the farming sector is not credit constrained, asset prices are constant. Thus, even if the aggregate collateral constraint binds, a farmer's actions do not get transmitted to the rest of the economy.

The collateral users- the H sector- leverage the collateral from the collateral creators- the F sector -, and are better off because of this. This highlights the importance of physical collateral and its counterpart, financial assets. The entire economy values their existence, but their existence is a byproduct of the operation of a subset of the economy.

2.3 Interpretation: Collateral and Liquidity

In an Arrow-Debreu world of perfect markets, there would be an AD claim traded on the output of a firm in each state of the world. In such a world, firms would finance all investment by simply selling off output. Since all claims would be liquid, investment would be efficient. Introducing agency problems into this setting imposes a restriction on which claims are liquid. For instance, with both moral hazard and adverse selection, firms are forced to retain a portion of output (for incentive reasons), thus not all claims on output are liquid.

In the economy described in this paper, no claim on output is liquid. However, inputs into production may have liquid claims issued against them. In particular, physical capital is liquid and human capital is not. This too is a restriction on the set of financial claims that are liquid. The paper has studied how a limited set of these claims satisfy the needs of the economy. The efficiency result can be restated as, when liquidity is valuable, but is created by a subset of the economy, this subset should be treated favorably.
In practice, leveraging of scarce collateral is accomplished through the use of financial assets. The farmer, through his use of land, gives rise to the value of land as collateral. An insurance company, using as collateral the financial mortgage on this land, writes insurance contracts. Financial assets sever the link between the farmer and the use of his land as collateral. In the same way that corporate governance structure allows for the separation of ownership and control of a firm, financial assets allow for the separation of creation and use of collateral.

Recognizing this link between liquid financial assets and collateral allows for a broader interpretation of the model. Theoretically, the model is of interest because it studies how a subset of liquid claims may satisfy the liquidity needs of an economy. Interpreting this in practice requires that the specification of the subset by accurate. We have cut through this issue by assuming only claims on physical assets are liquid. While plausible as an assumption, it does not capture all of the reality of liquid assets. Thus, a less ambitious read of our results is in the identification of liquidity in terms of collateral creators and users.

There are actually two levels of illiquidity introduced into the model. The first is the physical/human capital split that has been alluded to. The second is that the value of physical capital is endogenous. If the F sector was unconstrained in production, the value of land would be $\rho$. However, in a second best world it is generally $q_1 < \rho$. Poor asset distribution creates illiquidity in the amount of $(\rho - q_1)\bar{K}$.

The measure of aggregate liquidity in the model is $q_1\bar{K}$, which is the amount of collateral in the F sector. In Lemma 1, we interpreted F's productive investment in terms of rental of land. This can be equivalently stated as a borrowing with a down-payment (the rental), against the collateral of land, plus hedging contracts. Then the amount of borrowing will correspond to the value of collateral, and therefore serve as a measure of aggregate liquidity. This suggests a simple computation in putting the model to practice. The aggregate amount of credit (bank plus private) is the aggregate collateral constraint.

The collateral created by a sector is the value of credit outstanding to that sector. Collateral use is more tricky. Collateralized insurance is used to ensure that the firm
has enough cash (arising from operations and payoffs on insurance) to pay its ongoing expenses (wages, capital rental, investment, debt service, etc.) - see equation (27). Two factors are involved: the variance of cash from operations and the cost of falling short of meeting expenses (the curvature of the production function). The first is directly measurable, the second is harder. High fixed costs of operation give rise to curvature. We also may be able to glean it from industry studies - what is the cost in output for a particular industry if it is forced to layoff employees or close plants. Almost any research intensive industry would fit the description of a collateral user. That is, pharmaceuticals or high-technology. On the other hand, a collateral creating sector may be one that uses a natural resource as its main input into production. Mining and agriculture are examples. Finally, an alternative way to estimate collateral use is to look directly at hedging arrangements by firms. However, even given such disaggregated data, this may be hard to determine. Insurance arrangements include holding of cash (buffer stocking), or holding of insurance contracts or options, but they may be more. For instance, a credit line taken from a bank is insurance. A contract signed with a supplier to lock in the price of a raw material is also insurance. As it is hard to measure these items, the first approach may prove more fruitful.

2.4 Amplification and Collateral Crises

The introduction of the collateral constraint on the supply of insurance reinstates amplification effects into the model. This is most easily seen by considering a special case of the previous model. We simplify by reducing the uncertainty to take on one of two values. Second, we specialize the model by focusing on a case in which the externality of the previous section generates two equilibria, a low collateral value and high collateral value equilibrium. We can then compare equilibrium prices and quantities across these equilibria to generate implications for the effect of a change in collateral values on the economy.

Output in the farming sector($F$) and the consulting sector($H$) is subject to one of two aggregate shocks. In the boom, which occurs with probability $\pi$, the aggregate
shock, $z_H$, equals $\frac{1}{\pi}$. In the recession, the aggregate shock $z_L$ is zero (probability $1 - \pi$).

An F firm solves the following control problem at date 0,

$$\max_{(k_0^f, \theta_H^f, \theta_L^f)} \pi J^H \left( \frac{f(k_0^f)}{\pi} + \theta_H + (1 - \pi)J^L(\theta_L) \right)$$

$$s.t. \quad k_0^f u_0 + \theta_H^f \phi_H + \theta_L^f \phi_L = w_0$$

As before, $J^z(\cdot)$ is the value function over date 1 resources. One further simplifying assumption is made. In the boom, F firms are not credit constrained, while in the recession they are. That is, $\frac{f(k_0^f)}{\pi} > \rho \frac{K}{2} > \theta_L^f$. From the previous sections, it is immediate that $\theta_H^f = 0, k_H^f = \frac{K}{2}$. Since F is not credit constrained, the value function, $J^H(w)$ is linear in $w$. Rewriting F’s problem,

$$\max_{(k_0^f, k_L^f)} \pi f(k_0^f) + (1 - \pi)f(k_L^f)$$

$$s.t. \quad k_0^f u_0 + k_L^f q_L \phi_L = w_0$$

where, $\theta_L^f = q_L k_L$. At the optimum, demand for land is decreasing in both the price of land as well as in the collateral premium,

$$k_L^f = f'^{-1}(\lambda q_L \eta_L) \quad (2.27)$$

$$k_0^f = f'^{-1}(\lambda u_0) \quad (2.28)$$

H’s problem is exactly as previously discussed. For the current purpose, the only salient feature is that H’s demand for insurance can be written as a decreasing function of the collateral premium,

$$\theta^b_H(\eta_z), \quad \theta^b_H(\eta_z) < 0 \quad (2.29)$$

Finally, from B’s problem,

$$k_0^b = b'^{-1}(u_0), \quad k_z^b = b'^{-1}(u_1(z)) \quad (2.30)$$

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There are two markets that clear, land in all three states, and insurance in the date 1 states.

\[ k_i^l + k_i^b = \bar{K} \]
\[ \theta_i^b + \theta_i^l \leq u_1(z)\bar{K} \]

Since, F is not credit constrained in the boom, \( q_H = \rho \). That is land prices are high and at the first best levels. The state leading to a collateral crisis is \( z_L \). The collateral constraint has been assumed to bind in this state. The two market clearing conditions are,

\[ f'^{-1}(\lambda q_L \eta_L) + b'^{-1}(q_L) = \bar{K} \]
\[ \theta_L^b(\eta_L) = q_L(\bar{K} - f'^{-1}(\lambda q_L \eta_L)) = q_L b'^{-1}(q_L) \]  \hspace{1cm} (2.31) \hspace{1cm} (2.32)

Graphically, market clearing for land is as above. It is clear that as \( \eta_L \) rises, \( q_L \) falls. Thus, shifts in \( \eta_L \) trace out points along B’s demand for land. Netting out F’s demand for insurance, the supply of insurance to the H sector is equal to the value of B’s land, or its pledgeable collateral. Let, \( C_L(\eta_L) = q_L k_L^b \) be this supply.

\[ \frac{dC_L}{d\eta_L} = \frac{d}{dq_L}(q_L k_L^b(q_L)) \frac{dq_L}{d\eta_L} \]

The second part of this expression is negative. The first part may be positive or
negative depending on the elasticity of B’s demand for land with respect to the land price,

\[ \frac{d}{dq_L}(q_L k_L^b(q_L)) = k_L^b(1 + \epsilon_L^b) \]

Take the special case in which, \(-k \frac{b''(k)}{b'(k)} > 1\). Then, it is easy to show that \(\epsilon_L^b > -1\). In this case, the supply curve for insurance is downward sloping in its price. It is clearly possible to have multiple equilibria, the exact number of which will require a finer specification of the problem. Focus on a situation in which there are two equilibria with \(\bar{q}_L > q_L\) and \(\bar{\eta}_L < \eta_L\). The comparison across these equilibria generates implications for the effect of a fall in collateral values on the economy, referred to as a collateral crisis.

- There is a large fall in asset prices from date 0 to the recession state of date 1. Financial activity can exacerbate this downturn

\[ q_0 - q_L = u_0 + \pi \rho - q_L(1 - (1 - \pi)\eta_L) > q_0 - \bar{q}_L > 0 \]  
 \([2.33]\)

- The onset of the crisis (at date 0) creates a scramble for collateralizable assets that pushes up the collateral premium, \(\eta_L > \bar{\eta}_L\). This is the flight to quality associated with financial crises. Even in non-crises times, short term Treasury Bills, which are among the most actively used financial collateral, carry a sig-
nificant premium over similar financial assets which are not used as collateral. An empirical regularity in financial crises is a widening of the return spread between safe and risky assets (see, for example, Mishkin (1993)). As the former is often used as collateral, this fact is consistent with the model.

- The fall in asset prices causes a reduction in the supply of insurance. This should translate to lower volume of hedges (Θ) by both F and H. In practice this may be manifest as a loss of collateral leading to downgrades in the credibility of financial institutions and a drop in the volume of services provided by these institutions.

- The decrease in hedging contracts entered into by firms at date 0 will mean that the wealth distribution at date 1 will be more dissimilar across the low and high states. This will imply that the volatility of land prices rises. Basically, \( \rho - \bar{q}_L > \rho - \bar{q}_L \).

- The crisis is caused by a shock in the farming sector. This shock is transmitted via the financial market to the consulting sector (since \( \eta_L \) rises). Thus shocks to collateral creating sectors should affect output more strongly than shocks to other sectors.

The crisis can only occur in this model when the aggregate collateral constraint binds. Two alternatives suggest themselves to restore financial health. First, if a government can relax the constraint by creating collateral in the economy, it will. Alternatively, if a government redistributes the limited insurance towards the firms who have a large effect on the price of collateral (farmers in this case), stability in land prices is restored. Actions to stabilize land prices and the collateral creators are the best ones. Thus, the model suggests a combination of industrial and stabilization policy during a crisis.

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20This line of thinking leads to Holmstrom-Tirole’s interpretation of government bonds as tools to restore liquidity.
2.5 Conclusion

This paper has presented a model which highlights the role of collateral in backing insurance contracts. Risk sharing is limited by the quantity and value of collateral. Thus, unlike other models of market incompleteness (for example, Allen and Gale(1991)), the model endogenizes the depth of insurance markets.

The paper has explored how the collateral constraint affects activity. The main contribution of the paper has been in highlighting an insurance channel. Models that only focus on credit market imperfections to explain how changes in collateral values affect output are incomplete because they are not robust to the introduction of insurance markets. When the supply of insurance is constrained by collateral, amplification effects are reintroduced. We have shown that the collateral constraint causes a skewed response of the economy to aggregate shocks. The constraint may affect activity in only a small number of sufficiently adverse states of the world. In these, the collateral constraint causes shocks to be amplified.
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Chapter 3

Default and Redeployment

The dynamic process through which factors of production are allocated to and then away from production units has long held the interest of economists. Schumpeter (1942) has labeled this process creative destruction, recognizing that the impetus for this process is the creation of more productive units. The micro-dynamics of labor markets, a topic in which there has been an upsurge of academic interest, is a reflection of this process (see Mortenson and Pissarides (1994), Davis and Haltiwanger (1992)). Similarly, studies of creative destruction over the business cycle also reflect this interest (see Caballero and Hammour (1994a, 1994b)).

This paper takes as given an industry’s natural need to create and destroy production units, and asks how financial structure can alter the reallocation of factors of production. We consider an industry with a fixed supply of factors of production ("assets"), and study how the efficiency of the dynamic reallocation process of these factors can be enhanced through the use of financial structure.

The focus of the paper is on debt contracts and leverage. Jensen (1986, 1993) has argued that high leverage ratios are necessary in order to curtail the tendency of managers to over-invest. At the aggregate level, efficiency is promoted because managers pay out cash to investors so that it may be redeployed by investors in firms where investment is most profitable. This paper formally investigates the efficiency of redeployment by presenting a general equilibrium model of firm finance when there are agency conflicts between firms and investors. There are two considerations that
arise in the study. First, when there are agency conflicts between firms and investors, the flow of resources is not only hampered between firms and investors, but also between investors and firms. Thus, redeployment by investors is not a frictionless task. Second, in a general equilibrium with credit constraints prices will not reflect marginal products. \( q \)'s are not a benchmark for over or under investment. Instead we use a relative benchmark. A firm is over-investing if it's productivity using assets is lower than that of another firm in the industry.

Firms with high leverage have to generate cash to meet debt service. Falling short will not only curtail investment, but will also cause disinvestment. At the extreme, a firm that fails to service its debt goes bankrupt, laying off its workers and selling its capital assets. These factors of production flow elsewhere. To the extent that factors of production are industry specific, they are redeployed in other firms in the industry (as pointed out by Shleifer and Vishny (1992)). Firms purchase these assets with cash. If firms are credit constrained, the buyers of assets will be those in the industry with the most cash. Redeployment is efficient if the buyers of these assets are the firms who are most productive. Thus, leverage promotes efficiency if cash flow and the productivity of investment are positively correlated. We study a world in which this correlation holds. Debt serves a sorting role - it bankrupts bad firms and allows for their assets to be redeployed in good firms. In a partial equilibrium setting, Hart and Moore (1997) show that debt is an optimal contract when this correlation holds. This paper focuses on the efficiency of asset redeployment in a general equilibrium counterpart of their analysis.

Firm usually engage in either green-field investment or asset purchase from other firms. We focus on the latter. Investment is efficient if a limited supply of assets is being deployed efficiently within the firms of an industry. Thus, the analysis is applicable to industries in which asset resale is common - Airlines, for example. Asset resale can take the form of liquidation in or near bankruptcy, or can occur through the sale of a whole firm as in mergers or takeovers. The analysis is also appropriate when thinking about labor as the relevant "asset." Movement of workers between firms in an industry is an ongoing process and not as discrete as bankruptcy.
Leverage and debt promote efficiency by liquidating bad firms and allowing their assets to be redeployed in good firms. The macroeconomic literature on labor markets emphasizes "churn" - jobs are continually being created and destroyed within an industry (see Davis and Haltiwanger (1992)). Cross sectional variation in marginal productivity creates the need for reallocation of labor and drives the phenomena of churn. The literature studies the efficiency of reallocating labor when there are search frictions. We focus on free cash flow problems in studying how financial structure (leverage) affects churn. Our contribution to this literature is in investigating this link.

We first present a model to formalize the sorting role of debt. When there is a positive correlation between cash flows and investment opportunities, an optimal redeployment mechanism will transfer assets from the bad firms to the good firms in an industry. In the presence of credit constraints, bankruptcy penalties are necessary in order to induce this transfer. We show that the optimal mechanism can be interpreted as a debt contract. Over-leveraging, or taking on more debt than the value of assets, promotes investment efficiency. The model is used to understand the costs and benefits of leverage. We study the private incentive to recapitalize, or take on more leverage. Private incentives differ from social incentives in that firms will tend to leverage exactly up to the value of their assets, or under-leverage relative to the social optimum. The model articulates one rationale for why public policy concerns over the increases in leverage ratios in the corporate sector is misplaced. However this point is qualified as we demonstrate that if bankruptcy penalties are weak, over-leveraging can give rise to multiple equilibria leading to financial fragility. Firms, conjecturing that their collateral is worth less than their debts and expecting weak bankruptcy penalties, default. Default drives down the value of assets and fulfills their conjectures. The capacity of an industry to take on leverage will depend on the toughness of bankruptcy penalties. This points out, on the one hand, the necessity for tough bankruptcy penalties, and on the other, the limit on leverage in the presence of weak penalties.

Our work is most closely related to the paper of Shleifer and Vishny (1992) on
industry debt capacity. However, our analysis is more concerned with efficiency of asset reallocation than with debt capacity. In addition, while Shleifer-Vishny focus on industry wide shocks to cash flow, the important factor in our analysis is cross sectional variation across firms in the industry. Their point is that when all firms in an industry do poorly, the assets of any potential seller will fetch very little as the buyers have little cash to pay for these assets. Assets are more illiquid during recessions; firms are less willing to sell assets in these times. Our model emphasizes that idiosyncratic variation is quite important. Asset sales within an industry are simply a transfer of assets from one firm to another. While transfer prices may be low during a recession, efficiency simply requires transfer from bad to good firms, not high prices. Thus, illiquidity may not be socially costly. As in Shleifer-Vishny, our model also has the feature that bad firms are less willing to sell their more illiquid assets in recessions. Liquidation of these assets is privately inefficient. This is all the more reason to have high leverage and a tough bankruptcy code, because it forces liquidation and transfer.

Empirically, the connection between leverage and real investment decisions of firms is well documented. Whited (1994) and Cantor (1990) find a greater sensitivity of investment to earnings or cash flows in highly leveraged firms. Sharpe (1994) finds that the employment of highly leveraged firms is more sensitive to downturns in sales. Brown, James and Ryngaert (1992) find that, among firms experiencing financial distress, those with higher leverage suffer the largest declines in investment and employment. An open empirical question is the relation between industry leverage and measures of job reallocation. This is clearly a relevant inquiry for the analysis conducted in this paper. The literature on leverage and asset sales also provides useful insights. Ofek (1994) finds that high leverage increases the probability of asset sales being used to pay debt. Brown, James and Mooradian (1994) find that asset sales of firms in default where the proceeds are paid to creditors typically benefit creditors at the expense of shareholders. This is in line with our idea that liquidation is a forced transfer away from firms. Asquith, Gertner, and Scharfstein (1992) show that much of the proceeds of asset sales are used to pay off senior private debt. Additionally,
they find that firms with more secured creditors are more likely to enter bankruptcy than engage in restructuring. Again, the model in this paper is one of secured credit and liquidation, which makes their evidence applicable.

The layout of this paper is as follows. The next section presents the model and its assumption. We then analyze the model in sections 3, 4 and 5. Section 5 presents the optimal reallocation mechanism and interprets it in terms of debt and bankruptcy. We then study incentives to recapitalize and show how high leverage may lead to fragility. Section 8 contains conclusions.

### 3.1 The Model

Consider an economy of three periods \( t = 0, 1, 2 \) and a single good ("money"). There is a continuum of lenders in the economy with VNM preferences over money,

\[
U^l = E[c_0 + c_1 + c_2] \quad c_l \geq 0
\]

Lenders have an initial endowment of \( e_0 \) which is assumed to be large.

The productive side of the economy will be represented by a single industry. The industry is made up of a continuum of firms of unit measure. Each firm is run by an entrepreneur/manager with an endowment consisting of money at date 0 of \( w_0 \). The manager has preferences over the dividends of the firm,

\[
U^f = E[c_0 + c_1 + c_2] \quad c_t \geq 0
\]

Firms have access to a production function as follows. An investment of \( k_0 \) at date 0 creates physical capital of \( k_0 \) units immediately. Capital can only be created at date 0, its creation is irreversible (no salvage value) and it does not depreciate. Capital, when operated on by firms, produces output at each of dates 1 and 2. Capital is valuable at date 1 because it still produces output at date 2, however at date 2 capital is worthless.

Return on capital varies across firms depending on their types. While ex-ante
identical, firms are realized to be one of types $\theta \in \{L, M, H\}$ at date 1 with probabilities $\pi \in \{\pi_l, \pi_m, \pi_h\}$. This uncertainty is realized independently across firms, so that the population mirrors the underlying probability distribution. The capital produces type dependent revenues of $r k_0$ at date 1, where $r \in \{r_l, r_m, r_h\}$.

Firms at date 1 also have the opportunity to expand (or contract) production. Doing this requires the firm to purchase additional capital in the market. A firm that scales its production up to $k_1$ units at date 1 receives flows at date 2 of $\bar{v} k_1 g(k_1/k_0)$. While these flows can be some combination of private benefits and cash flows, the important thing is that they be non-transferable. The informational assumptions we make below will ensure this. Let $v = E[\bar{v} \mid \theta]$. Then, $v \in \{v_l, v_m, v_h\}$.

Firms face a capacity constraint limiting expansion. The function $g(k_1/k_0)$ is less than one and decreasing in its argument. Define $I_1 = k_1/k_0$. The following assumptions are made on $g(\cdot)$,

$$
\begin{align*}
g(I_1) &< 1 \quad \forall I_1 > 1 \\
g(I_1) & = 1 \quad \forall I_1 \leq 1 \\
\frac{\partial}{\partial I_1} (I_1 g(I_1)) & = g(I_1)(1 + I_1 \frac{g'(I_1)}{g(I_1)}) = g(I_1)(1 - \eta(I_1)) > 0 \\
\frac{\partial^2}{\partial I_1^2} (I_1 g(I_1)) & < 0
\end{align*}
$$

The third assumption simply says that the marginal product of investing at date 1 is always positive. The last means that there is decreasing returns in reinvestment.

The important things about capital are that it is specific to the industry and is in limited supply at date 1. At date 0, firms create the capital, and at date 1 the capital is simply transferred via asset sales between firms in the industry. One can also think of the capital good as industry specific labor. At date 0, firms hire and train $k_0$ workers. At date 1, these workers are hired away from some firms and fired from others. The transfer price can be thought of as the wage rate.

We assume that cash flows and investment opportunities are increasing in the type (and are therefore correlated). Second, it is assumed that $v \geq r$ for all types. The
returns to the date 0 investment in capital are simply the date 1 and date 2 flows of \( r \) and \( v \). We make the following assumptions on returns,

\[
E[r] < 1 \\
\pi_l r_h - r_m < \frac{\pi_l}{\pi_h} \\
E[r + v] > 1 \\
r_h, r_m, v_h, v_m > 1 \\
r_l, v_l = 0
\]

The first inequality will be motivated shortly, but will imply that the firm needs to contribute its own resources in order to produce. The next three imply that both date 0 investment and date 1 reinvestment are sufficiently profitable. The last just simplifies the arithmetic of the model. Finally we make two technical assumptions that are needed to guarantee an interior solution to the problem.

\[
v_h g(1/\pi_h)(1 - \eta(1/\pi_h)) < v_m \\
v_h g(r_h - r_m + 1)(1 - \eta(r_h - r_m + 1)) > 1
\]

The type of a firm at date 1 is private information of the firm and unobservable to lenders. Similarly, the amount of cash held by a firm at both dates 1 and 2 is also unobservable to lenders. The ex-post information asymmetry is similar to Townsend’s (1979) costly state verification setting. However, in our case, verification costs are infinite leading to an extreme form of moral hazard. At date 0, firm type is unknown to both firms and lenders. We will shortly show that these assumptions imply that the optimal contract is a debt contract (see Hart-Moore (1997)). In addition, because of the information structure, there will be no signaling mechanisms in debt choice at date 0 (see Ross (1977)).

Lenders have two sources of security for their loans. First, they can seize assets (capital), in case of non-repayment. Thus, the capital serves as collateral. Second, they can exclude firms from the capital market at date 1. This threat of exclusion
is a bankruptcy penalty that is also a device to force repayments from firms. The combination of these devices constitutes the threat of lenders in bankruptcy.

Finally, it is assumed that there is no capital specificity. That is, capital lost from one firm can be redeployed in another firm with no deadweight costs.

### 3.2 Equilibrium with No Bankruptcy Penalties

As a benchmark, consider equilibrium in an economy in which lenders have no recourse but to seize assets in bankruptcy. They cannot exclude lenders from future participation because of default.

The price of capital at date 1 will be denoted $\phi$. It is possible to show that the only contract that firms will sign is a debt contract collateralized by the assets of a firm (see Krishnamurthy (1998) for a proof). Firms purchase $k_0$ units of capital at date 0 by borrowing $F$ and contributing $w_0$ of their own resources.

$$k_0 = F + w_0$$

This yields output at date 1 of $r k_0$. A firm's resources per unit of capital at date 1 are $k_0 (r + \theta + \phi)$. The firm must make a repayment out of this of $F$. Let $f = \frac{F}{k_0}$ be the per capital face value of the debts. Then, the firm makes a payment as long as,

$$f \leq \phi$$

If $f > \phi$, the firm will always default on its obligation of $f$ letting lenders seize the assets, as it can simply go into the market to replace these assets at a cost of $\phi < f$. This is the key logic of no bankruptcy penalties. Lenders cannot exclude firms from purchasing assets from the market if they default on their debt obligation. Given this, the amount of investment at date 0 must be,

$$k_0 = \frac{w_0}{1 - f}$$

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Our assumptions will ensure that $f < E[r] < 1$, so that the multiplier is positive.

Given a face of debt of $f$, a firm of type $\theta$ has resources after payment of its debt of,

$$w_1 = r(\theta) + \phi - f$$

Its date 1 reinvestment decision solves,

$$\max_{k_1} \{ v(\theta)k_1g(k_1/k_0) \text{ s.t. } \phi k_1 \leq k_0w_1 \}$$

The firm cannot purchase more capital than it has resources for. This is the date 1 credit constraint. We can rewrite this problem as,

$$\max_{I_1} \{ v(\theta)I_1g(I_1) \text{ s.t. } \phi I_1 \leq r(\theta) + \phi - f \}$$

where, $I_1$ is the fraction that production is scaled up or down at date 1. The unconstrained solution to this problem, $\bar{I}_1$, is,

$$g(\bar{I}_1)(1 - \eta(\bar{I}_1)) = \phi/v(\theta)$$

We shall assume throughout that the resource constraint binds for both M and H types of firms. That is,

$$I_1 = \frac{r(\theta) + \phi - f}{\phi} < \bar{I}_1$$

Then at date 0 the program for a firm is,

$$\max \frac{w_0}{1-f} \left( \pi_h v_h I_1^h g(I_1^h) + \pi_m v_m I_1^m g(I_1^m) \right)$$

s.t. $f \leq \phi$

$$I_1^h = \frac{r_h + \phi - f}{\phi}$$

$$I_1^m = \frac{r_m + \phi - f}{\phi}$$
Differentiating the objective with respect to $f$ and simplifying gives,

$$\frac{1}{(1-f)\phi} \left( \pi_h v_h g(I_h^h)(r_h + \phi - 1 + \eta(I_h^h)(1-f)) + \pi_m v_m g(I_m^m)(r_m + \phi - 1 + \eta(I_m^m)(1-f)) \right)$$

since $f < \phi < 1$ and $r_m, r_h > 1$, this is greater than,

$$\frac{1}{(1-f)\phi} \left( \pi_h v_h g(I_h^h)(r_h - 1) + \pi_m v_m g(I_m^m)(r_m - 1) \right) > 0$$

Thus, it must be that $f = \phi$. The debt capacity of each firm in this industry is exactly equal to the value of capital at date 1. Our assumptions ensure that investment is sufficiently profitable that each firm will want to borrow fully against its debt capacity to invest at date 0.

Equilibrium for capital at date 1 is,

$$\pi_h I^h_1 + \pi_m I^m_1 = 1$$

since,

$$I^h_1 = \frac{r_h}{\phi}$$

$$I^m_1 = \frac{r_m}{\phi}$$

market clearing results in,

$$\phi = E[r] < 1$$

We summarize these results as follows,

**Proposition 3.1** When lenders have no recourse in default other than to seize the borrower's assets, firms borrow up to their debt capacity, $f = \phi = E[r]$. The equilibrium reinvestment decisions at date 1 have both M and H firms scaling up production, while L firms drop out.

$$I^h_1 = \frac{r_h}{E[r]} > 1$$
\[ I_1^m = \frac{r_m}{E[r]} > 1 \]

Reinvestment is dependent on cash holdings. Note also that since \( \phi < 1 \) and \( v_h, v_m > 1 \), the internal q's of both M and H firms are greater than one in this economy. This is a general feature of equilibrium with credit constraints. If there were no credit constraints, H firms would bid up assets so that their prices would reflect their marginal products. With respect to lenders, it seems that both M and H firms are under-investing. However, we show in the next section that with respect to resource allocation, M firms are over-investing and H firms under-investing.

### 3.3 Optimal Reallocation

Because firms face credit constraints at date 1, their investment decisions will reflect their cash from date 0 operations. Allocation of capital at date 1 depends purely on relative cash holdings, and does not reflect marginal products. A reallocation of capital that was based on marginal products would solve,

\[
\begin{align*}
\max & \quad k_0(\pi_h v_h I_1^h g(I_1^h) + \pi_m v_m I_1^m g(I_1^m)) \\
\text{s.t.} & \quad I_1^h \pi_h + I_1^m \pi_m = 1
\end{align*}
\]

Since \( v_h > v_m \), any reallocation will involve a transfer of assets away from M firms and to H firms. Thus it must be that \( I_1^m < 1 < I_1^h \), implying that \( g(I_1^m) = 1 \). Rewriting,

\[
\begin{align*}
\max & \quad k_0(\pi_h v_h I_1^h g(I_1^h) + \pi_m v_m I_1^m I_1^h) \\
\text{s.t.} & \quad I_1^h \pi_h + I_1^m \pi_m = 1
\end{align*}
\]

which gives the FOC,\(^1\)

\[ g(I_1^h) \left[ 1 + I_1^h \frac{g'(I_1^h)}{g(I_1^h)} \right] = v_m/v_h \]

\(^1\)Checking the SOC ensures that this is necessary and sufficient for a unique optimum.
or,
\[ g(I_1^h)(1 - \eta(I_1^h)) = \frac{v_m}{v_h} \]

Denote the optimal reallocation as \( I_1^{*h} \) and \( I_1^{*m} \). The competitive equilibrium reallocation will not be optimal because both \( I_1^h \) and \( I_1^m \) are greater than one. Whereas, an optimal reallocation would have \( I_1^{*h} > 1 > I_1^{*m} \). Since the H firms have higher marginal product, they should use more capital at date 1. This leaves room for a long term contract. If firms sign an ex-ante contract committing them to transfer capital from M firms to H firms at date 1, the allocation would be Pareto superior to the competitive equilibrium.

3.4 The Optimal Mechanism with Bankruptcy Penalties

We now introduce bankruptcy penalties. If a firm defaults on a contract at date 1, we assume that it can be excluded from further participation in the capital market. In the context of the model, this means that any firm that has defaulted (even one with cash) cannot enter the market to purchase assets. The interpretation is that in reality a firm combines its cash with resources that lenders provide in purchasing capital. Exclusion means they are denied access to credit. This, in turn, restricts their purchase of capital assets. The threat of exclusion gives firms the ability to enforce a contract with reallocation. We shall ask whether or not the optimal reallocation can be achieved.

A reallocation mechanism at date 1 will consist of \((t_1(\theta), I_1(\theta))\); \(t_1(\theta)\) is a transfer away from a firm of type \( \theta \) and \( I_1(\theta) \) is the investment allocation of that firm. Since firm types are private information, an optimal mechanism must elicit these types. The revelation principle allows us to restrict attention to truth-telling mechanisms; ones in which a firm of type \( \theta \) chooses the allocation corresponding to type \( \theta \).
An optimal mechanism solves,

\[
\begin{align*}
\max & \quad k_0(\pi_h v_h I_1^h g(I_1^h) + \pi_m v_m I_1^m g(I_1^m)) \\
\text{s.t.} \ (ARC) & \quad I_1^h \pi_h + I_1^m \pi_m = 1 \\
\ (BC) & \quad k_0 \leq w_0 + k_0 \pi_h t_1^h + k_0 \pi_m t_1^m \\
\ (RCH) & \quad t_1^h \leq r_h \\
\ (RCM) & \quad t_1^m \leq r_m \\
\ (IRH) & \quad v_h I_1^h g(I_1^h) + r_h - t_1^h \geq 0 \\
\ (IRM) & \quad v_m I_1^m g(I_1^m) + r_m - t_1^m \geq 0 \\
\ (ICHL) & \quad v_h I_1^h g(I_1^h) - t_1^h \geq 0 \\
\ (ICML) & \quad v_m I_1^m g(I_1^m) - t_1^m \geq 0 \\
\ (ICHM) & \quad v_h I_1^h g(I_1^h) - t_1^h \geq v_h I_1^m g(I_1^m) - t_1^m \\
\ (ICMH) & \quad v_m I_1^m g(I_1^m) - t_1^m \geq v_m I_1^h g(I_1^h) - t_1^h
\end{align*}
\]

The final constraint, which is ICMH, is only needed if \( t_1^h < r_m \). Because of credit constraints, the constraint will not apply if M does not have the funds to purchase H's allocation.

The first two constraints are just the date 1 reallocation resource constraint and the date 0 budget constraint. RCH and RCM are the date 1 credit constraints for H and M - they must be able to purchase their allocation out of their cash flows. IRH and IRM are date 1 individual rationality constraints for M and H. Note that they never bind, since RCH and RCM guarantee that \( r < t \).

We have suppressed the null allocation of \((0, 0)\) which must be offered for L to take. Since M and H can always take this allocation, the ICHL and ICML constraints must be satisfied. The final two constraints are the IC constraints requiring H not to take M's allocation and vice versa.

Bankruptcy penalties are important because they restrict the options of the firms at date 1. Firms can only choose one of the allocations, \((t_1(\theta), I_1(\theta))\). If they opt out, they keep their cash of \( r(\theta) \) but are not allowed to purchase any capital. We begin
by asking under what conditions \((I_1^*, I_1^{\ast m})\) can be implemented.

**Lemma 3.1** In implementing \((I_1^*, I_1^{\ast m})\), ARC, BC, RCM, RCH all bind. IRH and IRM never bind.

It is clear that the budget constraint binds. This is just a statement that investment is profitable. Likewise, RCM and RCH must bind, otherwise firms would not be using all of their borrowing capacity. Thus, \(\iota_1^h = \iota_1^h\) and \(\iota_1^m = \iota_1^m\). Given this it is easy to see that IRH and IRM never bind.

**Lemma 3.2** ICMH and ICHM do not bind. Given ICML, ICHL will never bind.

ICHM can be written,

\[
v_h I_1^{*h} g(I_1^{*h}) - r_h \geq v_h I_1^{*m} g(I_1^{*m}) - r_m
\]

or,

\[
v_h (I_1^{*h} g(I_1^{*h}) - I_1^{*m} g(I_1^{*m})) \geq r_h - r_m
\]

\[
v_h \int_{I_1^{*m}}^{I_1^{*h}} dI g(I) \frac{dI}{dI} \geq r_h - r_m
\]

\[
v_h \int_{I_1^{*m}}^{I_1^{*h}} g(I)(1 - \eta(I)) dI \geq r_h - r_m
\]

Now from the FOC for optimal reallocation we have that,

\[
\frac{g(I_1^{*h})(1 - \eta(I_1^{*h}))}{v_m/v_h} = \frac{v_m}{v_h}
\]

Since \(I g(I)\) is concave, it must be true that for \(I < I_1^{*h}\),

\[
\frac{g(I)(1 - \eta(I))}{v_m/v_h} \geq \frac{v_m}{v_h}
\]

rewriting the constraint,

\[
v_h \int_{I_1^{*m}}^{I_1^{*h}} v_m/v_h dI \geq r_h - r_m
\]
\[ v_m(I^{*h} - I^{*m}) \geq r_h - r_m \]
\[ v_m I^{*m}(I^{*h}/I^{*m} - 1) \geq r_m(r_h/r_m - 1) \]

Now we also know that,
\[ I^{*h}/I^{*m} > r_h/r_m \]
thus the ICHM is,
\[ v_m I^{*m} - r_m \geq 0 \]
which is simply IRM and never binds.

Consider next ICHL,
\[ v_h I^{*h} g(I^{*h}) - r_h \geq 0 \]
but given ICHM,
\[ v_h I^{*h} g(I^{*h}) - r_h \geq v_h I^{*m} g(I^{*m}) - r_m \]
\[ > v_m I^{*m} g(I^{*m}) - r_m \]
\[ > 0 \]
where the last step follows from ICML.

ICMH does not bind since M does not have resources to pay for H’s allocation when \( t^h \geq r_m \). ICHM does not bind because at the optimum, the allocation is distorted to favor H firms. Thus, H firms prefer their allocations to the M firms’. ICHL does not bind, because it is subsumed by ICML. That is if M firms do not choose L firm’s allocations, H firms whose allocations are preferred to M firms also do not choose L’s allocation.

Lemma 1 and Lemma 2 allow us to rewrite the program as,

\[ \max \quad k_0(\pi_h v_h I^{*h} g(I^h) + \pi_m v_m I^{*m} g(I^m)) \]
\[ \text{s.t. (ARC)} \quad I^h \pi_h + I^m \pi_m = 1 \]
\[ (BC) \quad k_0 = w_0 + k_0 \pi_h r_h + k_0 \pi_m r_m \]
\[(ICML) \quad v_m I_1^m g(I_1^m) - r_m \geq 0\]

**Lemma 3.3** The borrowing capacity of firms is, \( k_0 = \frac{\text{sup}}{1 - E[r]} \). It is dependent only on the expected cash flows in the industry at date 1, and is independent of any date 1 reallocation.

This follows from solving BC for \( k_0 \). We then have the following,

**Proposition 3.2** The optimal reallocation, \((I_1^{*h}, I_1^{*m})\), is implementable as long as,

\[v_m I_1^{*m} g(I_1^{*m}) - r_m \geq 0\]

For now let us assume that this condition holds. We turn next to the implementation of this allocation via financial contracts.

**Proposition 3.3** The optimal allocation can be implemented by firms issuing a debt contract with face,

\[f = r_h - (r_h - r_m) \frac{I_1^{*h} - 1}{I_1^{*h} - I_1^{*m}}\]

The proceeds from the debt issue are \( k_0 E[r] \) and are used to purchase capital at date 0 of \( \frac{\text{sup}}{1 - E[r]} \). A firm that makes the repayment of \( f \), keeps the excess of its cash flow over face. This excess may be used to purchase additional capital in the marketplace. A firm whose cash flow falls short of \( f \), must liquidate a portion of its capital, \( k_0 \), in order to make up the shortfall. Default on the debt contract results in bankruptcy, and exclusion from the capital market.

Two comments are in order. First the interpretation of the implementation as debt contracts is not unique. Second, to implement the allocation firms must be required to choose \( f \) at date 0. While markets can clear competitively at date 1, the date 0 leverage decision may not be privately optimal. Section 6 returns to this issue.

H firms are left with resources of \( r_h + \phi - f \) after making their debt payment. As they are assumed to be credit constrained, all of this will be used to purchase capital.
Given a market clearing price of $\phi$,

\[ I_1^{*h} = 1 + \frac{r_h - f}{\phi} \]

M firms must liquidate capital in order to cover their debt service. This leaves them with,

\[ I_1^{*m} = 1 + \frac{r_m - f}{\phi} \]

combining the above expressions, we find that

\[ I_1^{*h} = I_1^{*m} + \frac{r_h - r_m}{\phi} \]

and solving for $\phi$,

\[ \phi = \frac{r_h - r_m}{I_1^{*h} - I_1^{*m}} \]

This is the market clearing price that will prevail at date 1. We can next solve for the choice of $f$ which ensures the allocation,

\[ f = r_h - \phi(I_1^{*h} - 1) = r_h - (r_h - r_m)\frac{I_1^{*h} - 1}{I_1^{*h} - I_1^{*m}} \]

Since $I_1^{*m} < 1$, we have that $r_h > f > r_m$. This has two implications. Since, $\nu_m > 1 > \phi$, it must be that M is cash constrained in its investment policy.\(^2\) Likewise,

\(^2\)We show below that $\phi < 1$. The expression for market clearing can also be written as,

\[ \pi_h \frac{r_h - f}{\phi} + \pi_m \frac{r_m - f}{\phi} = \pi_l \]

or,

\[ \phi = \frac{1}{\pi_l} (E[r] - f(\pi_m + \pi_h)) \leq (r_h - r_m) \frac{\pi_h}{\pi_l} < 1 \]

Next consider M’s resource constraint. In order for this allocation to be feasible M must be able to afford this repayment,

\[ f \leq r_m + \phi \]

or,

\[ \frac{r_h - r_m}{I_1^{*h}} < 1 \]

which is equivalent to,

\[ \frac{r_h - r_m}{I_1^{*h}} < \frac{r_h - r_m}{I_1^{*h} - I_1^{*m}} = \phi < 1 \]
our technical assumptions ensure that H is also cash constrained in investment.

H can afford the repayment of \( f < r_h \) and always makes the payment. M has a choice of making the repayment or defaulting and being excluded from the market. On the one hand making the repayment requires liquidation of some capital, so that M is left with \( I_1^m \). On the hand, defaulting allows M to keep its cash since cash flow is not observable. M will make the repayment of \( f \) as long as,

\[
v_m I_1^m \geq r_m
\]

which is simply the IRML constraint.

There are two ways to transfer resources into the hands of firms in order for them to take advantage of good investment opportunities. The partial equilibrium solution is to simply lend cash to all good firms. In the context of the model, if there were no credit constraints at date 1, resource allocation at date 1 would be optimal. However, when there are credit constraints, the amount of cash transfer may be insufficient to harness all good investment opportunities. The general equilibrium solution is to lower capital prices so that firms with excess cash can purchase more capital with their limited cash. Capital is the input into production at date 1, not cash. Holding cash levels in firms constant, lowering capital prices allows for more transfer of assets to firms with cash. In general equilibrium, liquidation is valuable because it frees up assets from bad firms and allows them to be transferred to good firms. Thus, high levels of debt not only forces M firms to pay out their cash, it also forces them to liquidate some of their capital \((I_1^m < 1)\). H firms who have more cash are then able to purchase these assets cheaply. Debt serves a sorting role in liquidating bad firms and transferring their assets to good firms. The key assumption that is exploited in the model is the positive correlation between cash flows and investment opportunities.

The price of capital is determined along the cash margin. Because firms are liquidity constrained, prices do not reflect marginal product, they reflect the amount of cash held by firms. The general point is that when there are credit constraints,
private efficiency and social efficiency of asset allocation are not aligned. M firms are
ewser operators of capital than H firms, but with respect to capital prices both have
an internal q of greater than one. A choice of \((f, \phi)\) ensures that reallocation is done
to reflect marginal products. The role of prices in the allocation is quite different
from the usual.

Liquidation is privately inefficient for the M firms since \(\phi < v_m\). That is M firms
do not want to sell assets, they are forced to do so because their cash flows are low.
With respect to Jensen’s (1986) free cash flow theory, debt works on two margins.
First, it extracts cash from all firms. But more importantly, it extracts assets from
the M firms because their assets can be deployed more efficiently elsewhere.

Because liquidation is privately inefficient, firms and lenders may try to renegotiate
their way out of liquidation. The model suggests that it is important to commit to a
tough liquidation policy. In this way the model is similar to models of debt such as
Bolton and Scharfstein (1996). However, in that case, the importance of commitment
to a tough ex-post liquidation policy is so that a entrepreneur will have incentives
to put in effort ex-ante. In our case, the liquidation policy ensures a better ex-post
reallocation of resources. In other words, liquidation results in an externality.

3.4.1 Asset Specificity

The analysis has thus far ignored asset specificity, which in reality, is a large part of
a firm’s specific capital (organizational, etc.). When a firm is liquidated, this capital
is lost. Suppose that liquidation results in a deadweight cost of \(\lambda\) per unit of capital.
Assuming that \(I^m_i\) is still less than one, the allocation program is,

\[
\max \quad k_0(\pi_h v_h I^h_i g(I^h_i) + \pi_m v_m I^m_i - \pi_m \lambda (1 - I^m_i))
\]

\[
s.t. \quad I^h_i \pi_h + I^m_i \pi_m = 1
\]

\[
k_0 = w_0 + k_0 \pi_h r_h + k_0 \pi_m r_m
\]

\[
v_m I^m_i g(I^m_i) - r_m \geq 0
\]
which is equivalent to,

\[
\begin{align*}
\max & \quad k_0 (\pi_h v_h I^h_1 g(I^h_1) + \pi_m (v_m + \lambda) I^m_1) \\
\text{s.t.} & \quad I^h_1 \pi_h + I^m_1 \pi_m = 1 \\
& \quad k_0 = w_0 + k_0 \pi_h \tau_h + k_0 \pi_m \tau_m \\
& \quad v_m I^m_1 g(I^m_1) - \tau_m \geq 0
\end{align*}
\]

Conceptually the problem is no different than before. The \( \lambda \) introduces a shadow cost of liquidation, which enters as an increase in \( v_m \), the M firms valuation of assets. From the FOC for the optimum, it is clear that the introduction of \( \lambda \) simply increases \( I^m_1 \) closer to one.

### 3.5 Decentralized Equilibrium and Recapitalization

The previous sections have motivated an efficient role for debt in terms of its capacity to sort between good and bad firms. In part, this is the general equilibrium formalization of Jensen's (1986) ideas about debt and efficiency.

The central question regarding debt that has interested policymakers and academics is whether or not firms take on too much leverage. The '80s saw a record amount of mergers, acquisitions, leveraged buyouts and share repurchases that left the corporate sector highly leveraged. Most of the activity was not so much issuance of debt for new investment, as much as issuance of debt in order to retire equity. In short, recapitalization has been the theme of the past 20 years.\(^3\)

In this section we ask how private incentives to recapitalize differ from social incentives. Leverage in our formulation corresponds to \( f \). Changes in \( f \) can have an effect on both the amount of date 0 investment as well as on the date 1 debt service requirement. As we are interested in the effects of recapitalization, the problem is

\(^3\)see Bernanke and Campbell (1988), and Bernanke, Campbell, and Whited (1990) for a detailed accounting of changes in leverage over the past 20 years.
formulated to abstract away from ex-ante effects on $k_0$. Lemma 4 below show that changes in $f$ have no effect on industry debt capacity and hence no effect on $k_0$. This allows us to focus purely on the incentive to recapitalize.

The only equilibrium price is that of capital at date 1. Given this, firms choose $k_0$, $f$ and $k_1$. At date 1, after making the debt repayment each type of borrower has resources (per unit of capital),

$$w_1(\theta) = r(\theta) + \phi - f$$

Out of this, a firm may choose to expand capacity, so that $k_1 > k_0$. Since $\phi < 1$, and both $v_h, v_m > 1$, it must be that both M and H firms will prefer to expand capacity. L firms do not since $v_l = 0 < \phi$. Then for M and H firms, $I_1$ solves,

$$\max_{I_1} I_1(v(\theta)g(I_1) - \phi)$$

$$s.t. \quad \phi I_1 \leq \max[r(\theta) + \phi - f, 0] = w_1(\theta)$$

A firm is credit constrained at date 1 if the constraint binds. That is, capacity expansion is limited by the fact that a firm has limited cash flow. We shall continue to assume that both M and H firms are credit constrained. Again, the assumptions we have made will ensure that equilibrium is such that this is true. In this case,

$$I_1(\theta) = \frac{w_1(\theta)}{\phi}$$

A firm that over-leverages is one that borrows more than the value of its assets ($f > \phi$). As was shown in section 3, in the absence of bankruptcy penalties, it must be that $f \leq \phi$. Otherwise, a firm will always walk away from its debts, letting lenders seize the collateral worth $\phi$, since it can replace these in the market for $\phi$ and keep the excess of $f - \phi$. Bankruptcy penalties allows firms to borrow more than the value of assets.
Consider debt with face $f$. M liquidates assets and repays this debt as long as,

$$v_m \frac{r_m + \phi - f}{\phi} \geq r_m$$

Repayment to the lenders will depend on the face of debt $f$. Let $E[R(f)]$ be the expected repayment to a lender. When $r_m + \phi \geq f$, the repayment to lenders is,

$$E[R(f)] = \pi_l\phi + (\pi_m + \pi_h)f$$

The L firm goes bankrupt and its assets are seized, while the M and H firms have enough resources to make repayments. On the other hand, if $f < \phi$ then $E[R(f)] = f$.

Date 1 investment choices of the firm are,

$$I^h_1 = \frac{r_h + \phi - f}{\phi}$$

$$I^m_1 = \frac{r_m + \phi - f}{\phi}$$

Before writing the date 0 program for the firm, consider the date 1 market clearing condition for capital.

$$\pi_h I^h_1 + \pi_m I^m_1 = 1$$

**Lemma 3.4** In any equilibrium in which $f \geq \phi$, and both M and H firm are credit constrained at date 1, we must have that $E[R(f)] = E[r]$. The amount of funds raised by firms is constant at $k_0 E[r]$. The amount of date 0 investment is $\frac{w_0}{1-E[r]}$ and is invariant to the choice of $f$. Debt capacity depends only on the expected cash flows of the industry.

The market clearing condition is,

$$\pi_h I^h_1 + \pi_m I^m_1 = 1$$

$$\pi_h(r_h + \phi - f) + \pi_m(r_m + \phi - f) = \phi$$

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rewriting, we find that,

\[
E[r] - f(\pi_m + \pi_h) = \phi \pi_l \\
f(\pi_m + \pi_h) + \phi \pi_l = E[r] \\
E[R(f)] = E[r]
\]

At date 0, the firms total resources available for investment are \(k_0E[r]\) plus \(w_0\). Solving, we find that the amount of date 0 investment is constant at \(\frac{w_0}{1-E[r]}\).

The intuition for this result is straightforward. Lenders receive \(f\) from both M and H firms. They receive collateral from the defaulting L firms. This collateral is then resold to both M and H firms. Since it is profitable for M and H firms to purchase all of the capital (i.e. the credit constraint binds), lenders end up receiving the remaining cash of M and H firms as the purchase price for the assets. The interesting point of this lemma is that it ties debt capacity directly to the profitability of the entire industry. An industry with low cash flows must have low \(E[r]\) and therefore less debt capacity. This is the heart of Shleifer and Vishny’s analysis of industry debt capacity.

Lemma 4 allows us to focus purely on the effect of recapitalization by deriving incentives to change \(f\), given that \(k_0 = \frac{w_0}{1-E[r]}\). The program for a firm is,

\[
\text{max } s(f) = \pi_h v_h I_1^h g(I_1^h) + \pi_m v_m I_1^m g(I_1^m) \\
\text{s.t. } f \geq \phi \\
I_1^h = \frac{r_h + \phi - f}{\phi} \\
I_1^m = \frac{r_m + \phi - f}{\phi}
\]

Differentiating with respect to \(f\),

\[
\frac{\partial s(f)}{\partial f} = -\frac{1}{\phi} (\pi_h v_h g(I_1^h)(1 - \eta(I_1^h)) + \pi_m v_m g(I_1^m)(1 - \eta(I_1^m))) < 0
\]

Which yields the corner solution that \(f = \phi\).

**Proposition 3.4** Firms in the decentralized economy set \(f\) equal to the value of their
assets. They underleverage relative to the social optimum.

The private incentive to leverage departs from social incentives because firms do not take into account the effect of $f$ on $\phi$. Raising $f$ above one seems like it reduces resources available for investment at date 1 as it increases the chance of bankruptcy. In equilibrium, bankruptcy at date 1 is just a transfer of resources from one firm to another. It only reallocates assets; it does not destroy investment opportunities. Lowering $\phi$ allows good firms to take on more investment at date 1. As private incentives do not take this into account, firms choose too little leverage in the decentralized equilibrium. The model therefore provides a rationale for tax-advantaged debt financing.

3.6 Weak Bankruptcy Penalties and Financial Fragility

The decision to exclude $M$ is privately inefficient. If lenders could not commit to a tough bankruptcy rule, firms would always default, and renegotiate down their debts. In practice, renegotiation is affected by both the identity of the lenders and the bankruptcy process and law. As pointed out by Bolton and Scharfstein (1996), banks are likely to be easier to renegotiate with than markets. The law can affect bankruptcy penalties to the extent that credit history is required to be public information.

The model has assumed a very tough bankruptcy process by allowing lenders to commit to excluding defaulting firms from the capital market. We consider in this section the effect of weaker bankruptcy penalties. A $p$-exclusion rule specifies that with probability $p \in [0, 1]$, a defaulting firm will be excluded from the capital market. $p$ parameterizes the toughness of bankruptcy. So far the analysis has taken $p$ to be one.

Return to the Pareto problem of section 5. When $p < 1$, the IC constraint for repayment by $M$ firms becomes,

$$\frac{u_m I^m_i}{r_m} \geq p + (1 - p) \frac{u_m}{\phi}$$
Note that $\phi$ enters into this expression when $p < 1$.

Consider an allocation in which the IC constraint binds,

$$\frac{v_m}{r_m} I_1^m = p + (1 - p) \frac{v_m}{\phi}$$

The fact that $\phi$ enters into this expression creates the possibility of multiple Pareto rankable Nash equilibria. Suppose that $M$ firms conjecture a price of $\hat{\phi} < \phi$. Clearly the IC constraint dictates that they will default. Write the demand for capital in this default equilibrium as,

$$\hat{D}(\hat{\phi}) = \pi_h \hat{I}_1^h + (1 - p) \pi_m \hat{I}_1^m$$

where,

$$\hat{I}_1^m = \frac{r_m}{\hat{\phi}}$$

Now if firms did not default, demand for capital would be,

$$D(\phi) = \pi_h I_1^h + \pi_m I_1^m$$

Then, $\hat{\phi} < \phi \Leftrightarrow \hat{D}(\phi) < D(\phi)$. Note that at $\phi = \hat{\phi}$ we have that $I_1^h = \hat{I}_1^h$. Thus the conjecture is validated if,

$$\hat{D}(\phi) < D(\phi)$$

$$\frac{r_m}{\phi} (1 - p) < I_1^m = p \frac{r_m}{v_m} + (1 - p) \frac{r_m}{\phi}$$

which is always true.

**Proposition 3.5** For any $p < 1$, when the IC constraint for asset transfer away from $M$ firms binds, equilibrium at date 1 is fragile in the sense that $M$ firms may conjecture lower asset prices and default.

Note that the economy could avoid fragility by setting lower leverage ratios. Sup-
pose we rewrite the IC constraint by replacing $\phi$ with $\hat{\phi}$,

$$\frac{\nu_m I_1^m}{r_m} \geq p + (1 - p) \frac{\nu_m}{\hat{\phi}} > p + (1 - p) \frac{\nu_m}{\phi}$$

This will clearly avoid fragility, because there will be slack in the IC constraint at a price of $\phi$. But, in turn, leaving the IC constraint slack means increasing $I_1^m$ and is Pareto inferior to the no-default equilibrium of before.

High leverage ratios create the possibility of a fall in asset prices if default is triggered. This feature of the model is shared by a number of other macro models of debt (see Fisher (1933), Shleifer and Vishny (1992), Lamont (1995), Kiyotaki and Moore (1997)). The mechanism in this model is that firms, seeing that their collateral is worth much less than the face value of their debts, choose to default. Default pushes down asset prices. Firms, then hope to purchase assets more cheaply at these new levels. However, the resulting equilibrium is Pareto inferior to the no-default one as many productive firms are excluded from repurchasing assets.

It is the combination of high leverage ratios and weak bankruptcy rules that create fragility. The model suggests why bankruptcy rules should be strong. On the other hand, it points out that in the presence of weak bankruptcy rules, higher leverage ratios should be avoided.

### 3.7 Conclusion

This paper had two main contributions. First, we formalized the idea that debt and leverage promote efficient sorting and redeployment by bankrupting bad firms and transferring their assets to good firms. To do this, we presented a general equilibrium model of asset redeployment when there are agency conflicts between lenders and firms. Second, we used the model to ask questions about the efficiency of leverage. We showed that private incentives to recapitalize will leave firms with less leverage than is socially optimal. Thus, concerns about too much leverage in corporate America may be misplaced. This statement was qualified in the last section where we showed
that the combination of weak bankruptcy penalties and high leverage may lead to financial fragility.
3.8 References


Chapter 4

Why are Individual Bank Loans Illiquid?

Why are individual bank loans illiquid? This paper is motivated by the following observation: a loan made to a business enterprise by a bank is not directly tradable. Investors in the bank (deposit and equity holders) can increase their share in the business loan only via an increase in the share of all of the bank's loans. We take this to be a key distinction between intermediated and direct financial arrangements. Direct finance entails that firms issue securities that investors hold and trade. Under intermediation, firms issue securities that are held by an intermediary, which in turn issues claims on itself to investors. That is, banks make loans to firms, and restrict investors to holding and trading a basket of all loans (via deposits and equity).¹ This paper shows that loan illiquidity is an essential feature of a bank.

Diamond(1984) considers the question of banks from the standpoint of efficient information production. Information ("monitoring") is necessary for investments to provide returns. However, if this information is costly, investors would seek arrangements to minimize expenditures on information. In his model, direct finance involves a duplication of this cost across investors. While delegation of monitoring to a single investor does reduce the cost duplication, it introduces an agency cost of monitoring

¹While securitization of a banks loans does increase tradability, trade is still restricted to a large pool of the banks loans.
the monitor. Diamond shows that an intermediary arrangement, through diversification, can reduce this agency cost.

The question that Diamond raises can be restated as one of designing a mechanism to communicate information. As information is both costly and a public good we are forced to ask, who will acquire information, how will they truthfully communicate it, and on what basis will they be compensated for their efforts?

There is a large literature on asset pricing which emphasizes the role of securities markets in generating and aggregating information (seminal papers include Grossman(1976) and Grossman and Stiglitz(1980)). Information enables traders to earn superior returns, thus they have incentives to acquire it. Trading activity reveals some of this information, therefore prices become informative. The price mechanism is a communication device which encourages information production.

This paper reexamines the bank question in an economy in which a liquid securities market is a viable alternative. Liquidity has two implications:

- Consumption demand: as part of a portfolio, a liquid security is valuable as it can be easily traded for current consumption.

- Information: a liquid market for a security produces information pertaining to the payoff of the security.

In a setting very similar to that of Diamond, the paper studies which of intermediated or direct finance satisfies these needs more efficiently. Unlike Diamond's model, the alternative to intermediated finance is market finance in which monitoring is accomplished via speculation. The main result regards covariation in the characteristics of arrangements that jointly satisfy liquidity and informational needs. When bank monitoring is more efficient, security bundling should be observed. It is shown that in these cases banks will lend via debt contracts. As a counterpoint, a case is presented in which speculative monitoring is more efficient. Here, it is shown that trade in individual securities is optimal.

The intuition behind bundling in the first result is as follows. Incentives to gather information in an asset market are related to the amount of non-informational trade
(see Grossman and Stiglitz(1980)). In the model presented here there is consumption demand for liquidating the asset, which implies that there will always be incentives to acquire information. Thus, in cases where banks are a superior monitor (i.e. market information is duplicated), they will seek to shut off motives for informed speculation. They accomplish this via bundling. Take a simple example. If a bank satisfies consumption demands of its investors by agreeing to buy each of their financial assets at a fixed price, an inefficiency would arise. Investors would have an incentive to gather information about the payoff of these assets, selling selectively against the fixed price. This has two effects. First, the bank loses money because of adverse selection. Second, if information is costly, the economy overspends resources on information gathering. Now, if instead the bank provides liquidity only on the bundle of all financial assets, investors incentives to gather information are muted. Any information they do gain will only correspond to a small part of what they trade, and thus will not provide much speculative profits.

The idea of bundling to reduce adverse selection is pointed out in a securitization context by both Gorton and Pennacchi(1992) and Subrahmanyam(1991). Gorton and Pennacchi consider an economy in which investors’ only need is liquidity and show that bundling enhances the welfare of liquidity traders. However, the fact that this also mutes the reward to speculative trade has no consequences in their model. Subrahmanyam does consider the effect of bundling on the informativeness of prices, but as a value to information is not modeled, welfare consequences of bundling cannot be studied. In contrast, the economy we consider has both liquidity as well as informational needs. Our main result is in describing the covariation in the characteristics of arrangements that jointly satisfy these needs.

The rationale for financial intermediaries as the creators of liquid securities is pointed out by Gorton and Pennacchi(1993). Their paper shows that a bank, by issuing riskless debt (deposits) against its assets, creates liquid securities that enhances the welfare of liquidity traders. However, their paper does not consider the effect of this securitization on incentives to collect information as information is not valuable in their economy.
The paper is also related to a recent literature on the effects of market liquidity on monitoring incentives (see Holmstrom and Tirole(1993), Bolton and von Thadden(1995), Kahn and Winton(1995), Maug(1996)). As in our model, these papers consider a world in which information is valuable for monitoring a firm and liquidity is important for investors. The question that is raised is how changes in ownership structure (public/private, large blocks/small blocks) affect the liquidity of the market and monitoring incentives. In contrast, we focus on the question of direct versus indirect financing.

The layout of the paper is as follows: we start by presenting the model and its assumptions; sections 3.1 and 3.2 illustrate the financing problem of a single project; sections 3.3 and 3.4 present the basic adverse selection problem; section 3.5 introduces the bundling solution, and interprets it in a banking context; section 4 extends this idea to describe market finance; and we end with a discussion and conclusions.

4.1 The model

Agents

The economy lasts three periods, \( t \in \{0, 1, 2\} \). There are three agents, two investors (I) and an entrepreneur (E). Investors have endowments in units of the single consumption good.

\[
e^i = (1, W_1)
\]

That is, a date 0 endowment of one unit, and a date 1 endowment of \( W_1 \). The latter is assumed to be large and its source is exogenous to the specification.

Preferences are over consumption at both dates 1 and 2, \( U(c_1, c_2) \), and are similar to Diamond-Dybvig(1983). At date 1, investors receive a correlated shock that results in preferences:

\[
U^* = c_1 + \Gamma c_2
\]
\[ U^b = c_1 + c_2 \]

where, \( \Gamma < 1 \). The superscripts are used to refer to one as a “buyer” and one as a “seller”. This terminology will become clear later. The important thing to note is that, while ex-ante identical, investors preference over date 2 consumption will diverge at date 1. Ex-ante, these preference can be represented as:

\[ U^i = c_1 + \gamma_0 c_2 \]
\[ \gamma_0 = \frac{\Gamma + 1}{2} < 1 \]

The entrepreneur has no initial wealth and preferences only over date 2 consumption:

\[ U^e = c_2 \]

**Technologies**

There are two investment opportunities in the economy. First, a storage technology is available at both date 1 and date 2. The technology converts one unit of consumption today into one unit tomorrow. Additionally, E has access to an investment project described as follows:

- The project is indivisible and requires two units of consumption at date 0 to realize output of \( 2x \) at date 2.\(^2\)

- \( x \) is a random variable. \( x \in [0, \bar{X}] \), with p.d.f. given by \( f(x) \). \( f(x) \) satisfies the Monotone Hazard Rate Condition (MHRC). That is, \( \frac{1-F(x)}{f(x)} \) is monotone decreasing.

- Uncertainty regarding \( x \) is resolved at date 1. However, the project cannot be liquidated until date 2.

- There is an ex-post information asymmetry between entrepreneur and investors. Entrepreneur observes output for free, while investors have to “monitor” if they

\(^2\)Scaling by two saves carrying a half in later calculations.
wish to learn output.

- The monitoring technology provides a perfect signal of $x$. It is available at both dates 1 and 2, at cost $k_1, k_2$, respectively. For most of the paper it is assumed that $k_1 \geq k_2$. That is monitoring early may be more costly.

- If the project is to be undertaken, $E$ must borrow all of the funds from $I$.

- As $E$ is essential to the investment project, it is assumed that $E$ has the bargaining power in specifying terms of the financial contract with investors.

Absent date 1, the setup is similar to Gale and Hellwig(1985) and Townsend(1979)'s costly state verification framework. Date 1 has been introduced to study the effect of tradability. Since investors' valuation of date 2 consumption diverges, there is reason to trade.

4.2 Analysis

4.2.1 Autarky: Non-tradable financial contract

(A1) Monitoring at date 2 is observable and verifiable.

This assumption ensures that monitoring at date 2 can be delegated without cost. That is, investors can sign a contract which specifies that one of them will do the monitoring and be reimbursed the cost. Since monitoring is perfectly contractible, delegation is achieved costlessly. While not realistic, this assumption is made to highlight a different set of issues than Diamond(1984).

Optimal Contract

The optimal financing contract will specify a fixed repayment amount, $R$. If output is greater than $R$, $E$ repays exactly $R$ to the investors. If output is less than $R$, investors monitor and seize all output.\(^4\)

\(^3\)This assumption can easily be relaxed.
\(^4\)We assume that stochastic verification schemes are not possible.
For the project to be undertaken, each of the investors must contribute their date 0 endowments. Assume that there are two financial contracts issued, one to each investor, define \( g(x, R) \) as the per share output, and normalize \( R \) to be the per share repayment. Additionally, let \( \phi(x) \) be the indicator function of monitoring. Then,

\[
g(x, R) = \text{Min}(x, R) \\
\phi(x) = 1_{x<R}
\]

Proof: see Gale and Hellwig(1985).

Ex-ante, investors' valuation of date 2 consumption (relative to date 1) is \( \gamma_0 \). A contract is feasible if it satisfies investors individual rationality constraint,

\[
\gamma_0 E_x[g(x, R) - \frac{k_2}{2} \phi(x)] \geq 1
\]

E solves the following program then,

\[
\min\{R \text{ s.t. } \gamma_0 E_x[g(x, R) - \frac{k_2}{2} \phi(x)] \geq 1\}
\]

Since \( x \) is bounded above by \( \bar{X} \), the maximum repayment that E can promise is \( \bar{X} \). Thus, this program has a solution if,

\[
\gamma_0 E_x[x - \frac{k_2}{2}] \geq 1
\]

Rewriting, we arrive at,

**Proposition 4.1** When the financing contract is non-tradable, the project is undertaken if,

\[
E_x[x] \geq \frac{1}{\gamma_0} + \frac{k_2}{2} = X_A
\]

There are two terms in this expression. First, \( \frac{1}{\gamma_0} \) is the date 2 valuation of the one unit of date 0 investment. Second, \( \frac{k_2}{2} \) is the cost of monitoring. In order for the investment to be made, the expected return on the project must exceed the sum of these costs.
4.2.2 Tradable financial contract, \( k_1 = \infty \)

Next consider a case in which monitoring is not possible at date 1, however agents can re-trade the financial contract. In the previous example, there are unrealized gains from trade at date 1. Suppose both agents agree to trade shares at date 1 at a price \( P \) that is mutually agreeable. Then \( P \) must satisfy,

\[
\Gamma v^s \leq P \leq v^b
\]

where, \( v^s \) and \( v^b \) are the seller and buyer's valuation of the share.

\[
v^s = E_x[\Gamma g(x, R)]
\]
\[
v^b = E_x[g(x, R)]
\]

As investors contribute equally toward the project, they hold one share each. An investor has one of two possibilities at date 1 (after trade),

\[
c_1^s = P + W_1 - E_x[\frac{k_2}{2}\phi(x)]
\]
\[
c_2^s = 0
\]

or,

\[
c_1^b = 0
\]
\[
c_2^b = 2g(x, R) + W_1 - P - \frac{k_2}{2}\phi(x)
\]

Date 1 utilities are,

\[
U^s = P + W_1 - E_x[\frac{k_2}{2}\phi(x)]
\]
\[
U^b = E_x[2g(x, R) - \frac{k_2}{2}\phi(x) + W_1 - P]
\]
Since an investor is equally likely to be a seller or buyer, ex-ante utility is,

\[
U^i = \frac{1}{2} E_x[P + W_1 + 2g(x, R) - \frac{k_2}{2} \phi(x) + W_1 - P] \\
= W_1 + E_x[g(x, R) - \frac{k_2}{2} \phi(x)]
\]

The participation constraint for investors is,

\[
E_x[g(x, R) - \frac{k_2}{2} \phi(x)] \geq 1
\]

**Proposition 4.2** When the financing contract can be traded at date 1, but \(k_1 = \infty\), the project is undertaken if,

\[
E_x[x] \geq 1 + \frac{k_2}{2} = X_T < X_A
\]

In other words, because investors can realize gains from trade, financing will be less costly and the project is more easily undertaken.

### 4.2.3 Tradable financial contract, \(k_1\) finite

Allowing early information acquisition can have two effects. First, the debt contract derived in the previous sections need not be optimal. For now, ignore this possibility, and fix the financial contract to be debt. It will be clear by the end of section 3.5 that this oversight is unimportant. Second, the ability to acquire information at date 1 affects trade between investors. We focus on this and show that private incentives to acquire information diverge from social optimality in this case.

**\(A2\)** Information acquisition at date 1 is observable but not verifiable.

Date 2 information acquisition is interpreted as “auditing the books” of the Entrepreneur. Date 1 information is gained via a surreptitious inspection of the firm, thus it is non-contractible. However, it is assumed that both types of monitoring are substitutes in getting payments from the entrepreneur. In other words, acquiring information at date 1 saves the investors the cost of acquiring at date 2.
At date 1 one investor will need liquidity, which the other will provide. The interaction at date 1 is interpreted as the reduced form for a security market. Trade in this market results in allocations and an informative price. The game specified below characterizes the allocation; the fact that date 1 and date 2 monitoring are substitutes captures the fact that prices will be informative. The main feature of the game we specify is that it represents trade under asymmetric information.

Information acquisition is modeled as a simultaneous move game as follows: ⁵

There are four subgames in which trade occurs. Trade is modeled as a three stage game:

• Stage 1: Nature moves. Bargaining power is assigned with equal probability to either buyer or seller.

• Stage 2: The party with the bargaining power offers contracts to the other party.

• Stage 3: The offer is accepted or declined.

Assumption A2 allows us to separate the information acquisition game from the trading game and thereby simplifies the analysis. ⁶

---

⁵It is assumed that agents make information acquisition decisions after realizing their types. Switching this so that information acquisition is prior to type realization will not materially affect the outcome.

⁶If information acquisition was not observable, the strategy spaces of each agent would be (acquisition decision) X (bargaining decision). This is a large strategy space, and needlessly complicates
Analysis of the trading game in each of the four cases is in the Appendix. Here, we go through a simpler version in which \( \Gamma \) equals zero. Define the following items.

\[
T = v^b - v^s = (1 - \Gamma)E_x[g(x, R)] \\
\hat{\phi} = E_x[\phi(x)] \\
\hat{g} = E_x[g(x, R)]
\]

\( T \) represents the unconditional (on the realization of \( x \)) gains from trade. \( \hat{\phi} \) and \( \hat{g} \) are the [unconditional] probability of monitoring and mean return.

In stage 2, one of the parties makes an offer to the other, which may then be accepted or declined by the other party in stage 3. In the case of symmetric information, trade is easy to characterize. When there is asymmetric information (either (U,I) or (I,U)), the party making the offer is either in a signalling or screening setting. Because of linearity in buyer and seller valuations, the optimal scheme in both cases is to offer a fixed price. The price that is offered is denoted as \( q \) or \( p \) depending on which subgame we are in.

Let \( \pi_b, \pi_s \) be the unconditional expectations of trade relevant payoffs. Then in each subgame, these payoffs are,

- (U,U):

\[
\pi_b = \frac{1}{2}(T - k_2\hat{\phi}) \\
\pi_s = \frac{1}{2}(T - k_2\hat{\phi})
\]

- (I,I):

\[
\pi_b = \frac{T}{2} - k_1
\]

the problem. What we have in mind is that agents independently choose to acquire information. They then meet in a room to decide terms of trade. But in this meeting room, in bargaining over the price, each party knows whether the other is informed or not. This assumption will be relaxed a little later, so that information acquisition is not observable.
\[ \pi_s = \frac{T}{2} - k_1 \]

- (I,U): Two cases, depending on who has bargaining power,
  - B: sets \( q_b = 0 \), then \( \pi_b = T - k_1, \pi_s = 0. \)
  - S: \( q_s \) such that, \( \pi'_s(q_s) = 0 \), where \( \pi(q_s) = E[q_s 1_{g(x,R) > q_s}] \). Then, \( q_s \in [0, R] \), and \( \pi_b(q_s) = E[(g(x,R) - q_s) 1_{g(x,R) > q_s}] - \frac{k_1}{2} \). Note that the first part of this expression is always positive.

- (U,I)
  - B: sets \( p_b = 0 \), \( \pi_b = T, \pi_s = -k_1. \)
  - S: \( p_s = E[g(x,R)] \), at which price S always sells the security. \( \pi_s = T - k_1, \pi_b = 0. \)

In normal form these payoffs can be written as,

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>S</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( \frac{T}{2} - k_1, \frac{T}{2} - k_1 )</td>
<td>( \frac{T}{2} - k_1 + \frac{\pi_b(q_s)}{2}, \frac{\pi_s(q_s)}{2} )</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>( \frac{T}{2}, \frac{T}{2} - k_1 )</td>
<td>( T - k_2 \frac{\phi}{2}, \frac{T}{2} - k_2 \frac{\phi}{2} )</td>
<td></td>
</tr>
</tbody>
</table>

It is clear that (U,U) is efficient, but this may not be an equilibrium. B has an incentive to deviate from (U,U) as long as \( \pi_b(q_s)/2 > k_1 - k_2 \frac{\phi}{2} \). If this condition is met, the equilibrium will always be a mixed strategy in which both players place weight on being informed at date 1. Since early information acquisition is inefficient, the equilibrium will generally be inefficient.

There is a divergence between private and social incentives to gather information. The social value of information is that monitoring is necessary to ensure repayment
by the entrepreneur. However, in the CSV setting, this monitoring should only be
done at date 2 and contingent on the report of the entrepreneur (i.e. if E declares
bankruptcy). Private incentives to acquire information are in gathering a larger share
of the gains from trade. Since these are not aligned, there can be over-acquisition of
information in equilibrium.

4.2.4 Option to Trade

The inefficiency is created by the investors' inability to commit to not acquiring
information at date 1. Given this, investors will seek commitment devices to disable
incentives to speculate. Before exploring this avenue, it is helpful to reinterpret the
investors date 1 incentive as an option to trade. The result is apparent under the
following assumption.

(A2') Information acquisition at date 1 is not observable.

Since we are only concerned with the question of whether or not (U,U) is an
equilibrium, the details of the trading game of the last section need not be analyzed.
If (U,U) is an equilibrium, then the price of date 2 trade can be prespecified. Call
this price, \( P \).

\[ \Gamma \hat{g} \leq P \leq \hat{g} \]

Both parties have the option to trade at this price. When parties are uninformed, this
option is worthless, as trade always occurs. However, if one party becomes informed,
he will choose to accept only a fraction of trades. Consider B's (similar for S) incentive
to deviate.

If B becomes informed,

\[ \pi^i_b = E[Max(g(x, R) - P, 0)] - k_1 \]
If B remains uninformed,

\[ \pi^u_b = E[g(x, R) - P] - \frac{k_2}{2} \hat{\theta} \]

the gain to becoming informed is,

\[ \Delta \pi_b = E[\max(P - g(x, R), 0)] - k_1 + \frac{k_2}{2} \hat{\theta} \]

This must be positive for B to become informed.

**Proposition 4.3** Suppose investors agree ex-ante to trade at date 1 at a price \( P \in [\Gamma g, \bar{g}] \). Then, B does not become informed as long as,

\[ E[\max(P - g(x, R), 0)] < k_1 - \frac{k_2}{2} \hat{\theta} \]

Similarly, S remains uninformed if,

\[ E[\max(\Gamma g(x, R) - P, 0)] < k_1 - \frac{k_2}{2} \hat{\theta} \]

If either condition is violated \((U, U)\) cannot be an equilibrium.

### 4.2.5 Multiple Firms and Bundling

With multiple projects, the distinction between direct and intermediated finance emerges. Direct finance allows trade in every contract, while intermediated finance restricts trade to a bundle of all contracts. We show that bundling of contracts eliminates the incentive to acquire information at date 1.

There are \( N \) firms, indexed by \( i = 1 \ldots N \). Likewise there are \( 2N \) investors, each exactly as previously described. The payoff of a single firm is \( x_i \), which is i.i.d. across all firms. \( x_i \in [0, \bar{X}] \) with p.d.f. given by \( f(x) \). The payoff to the security of firm \( i \) is \( g(x_i, R) = \min(x_i, R) \). Since firms are identical, \( R \) will be the same for all firms.
Consider a bundled security with payoff given by,

\[ y_n = \sum_{i=1}^{n} g(x_i, R) \]

**Proposition 4.4** Incentives to become informed are muted by bundling. In the set of combinations of security bundles, the one that best accomplishes this will be that which includes all securities. Second, for a sufficiently large number of securities, \((U,U)\) will be the only equilibrium to the information acquisition game.

Proof:

Consider \(B\), facing trade in two securities with payoffs, \(y_n\) and \(g(x_{n+1}, R)\). To prove the first part of the statement, we show that incentives to acquire information are lower in the case where the securities are bundled than when they are separate. Given this result, by induction, one can conclude that the optimal security pays \(y_N\). Suppose, \(B\) chooses to become informed about \(m\) of the \(n\) firms. In the unbundled case,

\[
\Delta \pi^u_b = E[Max(P - g(x_{n+1}, R), 0)] - k_1 + \frac{k_2}{2} \phi + \frac{E[Max(mP - y_m - (n-m)(\hat{g} - p), 0)]}{m} - mk_1 + m\frac{k_2}{2} \phi
\]

bundled,

\[
\Delta \pi^b_b = E[Max((m + 1)P - y_{m+1} - (n-m)(\hat{g} - p), 0)] - (m + 1)k_1 + (m + 1)\frac{k_2}{2} \phi
\]

By Jensen’s inequality, \(^7\)

\[
E[Max(P - g(x_{n+1}, R), 0)] + E[Max(mP - y_m - (n-m)(\hat{g} - p), 0)] > E[Max((m + 1)P - y_{m+1} - (n-m)(\hat{g} - p), 0)]
\]

\(^7\)Alternatively, this is the option pricing result that the option on a sum is worth less than the sum of options.
which implies that $\Delta \pi^b < \Delta \pi^u$.

The second part of the proposition is established in two steps. First, define,

$$
\Delta \pi_b(m, n) = E[Max(mP - y_m - (n - m)(\hat{g} - p), 0)] - mk_1 + m\frac{k_2}{2}\phi
$$

Note that, $\Delta \pi_b(m, n) > \Delta \pi_b(m, n + 1)$. Let $m(n)$ be B's optimal choice of $m$. Then, for $m(n + 1) \neq n + 1$,

$$
\Delta \pi_b(m(n), n) > \Delta \pi_b(m(n + 1), n) > \Delta \pi_b(m(n + 1), n + 1)
$$

The first inequality is simply the condition for the optimum. The second is from the previous observation. If $m(n + 1) = n + 1$, we have

$$
\Delta \pi_b(n, n) > \Delta \pi_b(n + 1, n + 1)
$$

Thus,

$$
\Delta \pi_b(m(n), n) > \Delta \pi_b(m(n + 1), n + 1)
$$

The incentive to become informed is monotone decreasing in $n$. The next step is to prove that this incentive goes to zero. As $n \to \infty$, either $n - m \to \infty$ or $n - m \to c(n)$, where $c(n)$ is bounded above be $\bar{c}$. In the first case, $\Delta \pi_b(m, n) \to 0$, since $\hat{g} - p > 0$. In the second case, it must be that $m \to \infty$. Then, by the central limit theorem, $\frac{y_m}{m} \overset{P}{\to} \hat{g}$.

$$
\Delta \pi_b(m, n) < (E[Max(P - \frac{y_m}{m}, 0)] - k_1 + \frac{k_2}{2}\phi)m
$$

$$
< (Max(P - \hat{g}, 0) - k_1 + \frac{k_2}{2}\phi)m
$$

$$
< 0
$$

Therefore, it must be the case that for sufficiently large $n$, $\Delta \pi_b(m(n), n) < 0$. QED.

The result is driven by two forces that can be illustrated by considering two
extreme cases. First, suppose the investor collected information about all of the asset payoffs. Because of diversification, the uncertainty in the aggregate payoff is proportionately less than in the individual payoffs. Since the profits from trading are directly due to this uncertainty, profits fall. That is, this case is not equivalent to trading in the individual asset repeated many times. At the other extreme, suppose the investor collected information about the payoff of only one of the assets. Because of bundling, the investor is forced to link his trading of that asset with the trading of the entire security basket. This decreases instances in which his trade is changed by the realization of the information. Therefore, information is less valuable.

We interpret the bundling solution as a bank. Loans are made at date 0. A fixed price, $P$, is posted at date 1 at which liquidity on these loans is provided to the early consumer. However, liquidity is only granted on the bundle of all loans.

**Proposition 4.5** Under intermediated finance, projects are undertaken if,

$$E_x[x] \geq 1 + \frac{k_2}{2} = X_T$$

We summarize the analysis by noting our main result, bank monitoring, security bundling, and debt contracts should be observed together.

### 4.3 Bank Monitoring versus Market Monitoring

The previous examples have shown two things. First, full bundling is the efficient solution to the needs of the economy. Second, the financial contract signed between investors and entrepreneur will resemble a debt contract.

The analysis is somewhat slanted. There are two needs of the economy, information production and the ability to liquidate financial assets. In both of these cases, the assumptions have led to a bank dominating a securities market. It is instructive to consider why this occurs.

Private incentives to produce information in a securities market may not be aligned with social incentives (see Hirshleifer(1971), for example). This is clearly true in this
model of trade. The private incentive to monitor is a desire to rent-seek – to capture a larger portion of the gains from trade. Social incentives are to monitor in order to receive repayment from the entrepreneur. In the CSV model, this is restricted to the bankrupt states of the world. That is, the debt contract is optimal because it is informationally less intensive as monitoring need only be done if the entrepreneur does not make the promised repayment. This wedge between private and social incentives is increased if one also considers that adverse selection will result in some unrealized gains from trade. In this way, our model of the securities market creates incentives for overproduction of information.  

Contrast this with the description of bank monitoring. In the model, date 2 monitoring, which is identified with the bank, is no more costly than date 1, or market monitoring. However, this may not be realistic. Despite Diamond’s point about agency costs of delegation, one might well believe that there are significant costs to delegated monitoring. Alternatively, one may argue that a market is a low cost information producer as it provides incentives for diverse agents, each of who naturally arrives at a piece of information to communicate that information.  

---

8 This result is not specific to our description of the trading game. Consider the Walrasian rational expectations model of Grossman and Stiglitz(1980). Ex-ante identical risk averse agents trade competitively to absorb a random supply of a risky asset (the stock). Agents have access to a costly information technology which will provide them a signal of which state is likely to occur. An agent who is informed will weight his portfolio holdings to reflect that information. However, because of this the final allocation will not be equal across agents. From a welfare standpoint, full risk sharing, or equal stock holdings, would be Pareto optimal. The equilibrium is suboptimal in two ways. Adverse selection results in distorted allocations. This is compounded by the fact that agents pay a cost to become informed and increase adverse selection. Information acquisition is socially wasteful in this case, and banning (or taxing) it would result in an ex-ante Pareto gain.

More generally, we might think of altering the setup to include some production. In this case, the information in prices would enable the economy to better choose production plans (as in Grossman(1976)). Even here it is unclear whether a socially optimal amount of information would be produced. The problem is that information acquisition by an agent involves two externalities. One is a negative externality on all other market participants (speculative externality). The other is a positive externality which helps guide resource allocation (productive externality - I owe this terminology to a lecture by Martin Hellwig). There is no reason to expect these two externalities to offset. The overproduction result typically arises in a setting where the latter is missing.

9 Alternatively, Holmstrom and Tirole(1993) point out that the real value of market information may be its integrity. A liquid stock provides an informative price which may be an essential tool in writing contracts to incentivize management. That is, markets convert “soft” information into contractible, or “hard”, information.
4.3.1 Direct Finance

Much of the recent literature on market monitoring (Holmstrom and Tirole(1993), Bolton and von Thadden(1995), Kahn and Winton(1995), Maug(1996)) studies how the liquidity of the market affects incentives to acquire information. In particular, the reward to speculative trade is a transfer from those who need liquidity to speculators. We can investigate the implications of this by considering a specific case of the model.

Suppose that date 1 information acquisition is cheaper than date 2, \( k_1 < k_2 \). However, just this assumption is insufficient to guarantee that market monitoring is more efficient than bank monitoring because adverse selection in the market results in a loss in the gains from trade. Consider the game in section 3.3 in which \( \Gamma = 0 \), and suppose that parameter values dictate \((I, U)\) as the efficient solution. In this case, the payoffs to buyer and seller are,

\[
(\pi_b, \pi_s) = \left( \frac{T}{2} - k_1 + \frac{\pi_b(q_s)}{2}, \frac{\pi_s(q_s)}{2} \right)
\]

Since \( \pi_b(q_s) + \pi_s(q_s) < \frac{T}{2} \), not all gains from trade are realized. The benefit of this equilibrium is that information acquisition at date 1 is cheaper and is only done by the buyer. However, because date 1 monitoring is non-contractible, the cost \( k_1 \) is borne privately by the buyer. His reward for this is a larger share of the surplus in trade. B will be willing to become informed if,

\[
\pi_b(q_s)/2 > k_1 - k_2 \frac{\phi}{2}
\]

In this case, market structure will be such as to provide maximal incentives for information acquisition, and to maximize,

\[
\pi_b(q_s) = E_x[(g(x, R) - q_s)1_{g(x, R) > q_s}]
\]

B’s incentive to become informed is that he gets an option to choose to trade with S
at $q_s$. $q_s$ solves $\pi_s'(q_s) = 0$, where,

$$\pi_s(q_s) = E[q_s g(x_R) > q_s]$$

Incentives to become informed are affected by two factors: the bundling of securities, and the form of these securities. Consider the latter first.

The form of the debt contract in the CSV setting is to minimize ex-post information gathering costs. When information is always acquired, CSV cannot speak to the form of contracts, and it is indeterminate due to date 2 contracting considerations. However, the form of the security does affect incentives to acquire information. Suppose, we restrict ourselves to securities of the form $g(x, R) = \alpha \min(x, R)$. Then, the security choice solves,

$$\max_{\alpha, R} E_x[(g(x, R) - q_s) 1_{g(x, R) > q_s}]$$
$$s.t. \quad q_s = \arg\max_q E[q_s g(x_R) > q_s]$$
$$E_x[g(x, R)] = C$$

where, $C$ is a constant chosen to satisfy investors' individual rationality constraint. We show in the Appendix, that the solution to this program yields $R = \bar{X}$.

Given this, return to the question of bundling, and ask whether the bundled security, $Y_N$, provides better information incentives than individual securities. The logic now is exactly as in section 3.5. Since the option on the sum is worth less than the sum of options, individual security trade maximizes B's option to trade. This demonstrates that market monitoring and trade in individual securities should be observed together.

### 4.3.2 Coexistence

The question of which of direct or indirect finance is appropriate for a firm will be dependent on their characteristics, particularly $k_1$ versus $k_2$. Thus, as long as their is a distribution of firms over these parameter values, the model predicts a coexistence
of market and bank finance. A thornier issue is the coexistence of bank and market finance for a single firm. As it stands, there is nothing the model can say about this, and we leave it as a question for future research.

4.4 Conclusion

A comparison of banks and stock markets is of great relevance today given the transition in financial systems occurring both in Eastern Europe and in Asia. The academic literature has addressed this by studying the question of financial system design (see Allen(1992), Allen and Gale(1995), Boot and Thakor(1997)). By making a welfare comparison of banks and markets this literature seeks to identify characteristics of an economy that are more suited to one or the other system. This paper, in contrast, has avoided making a welfare comparison of banks and stock markets. Its less ambitious goal has been to identify factors leading to the covariation in features associated with banks and stock markets. The policy relevance of this analysis is in pointing out that regardless of the choice between banks and markets, conditional on this choice, a number of characteristics of the financial system should follow.

In the banking context, the question posed is, "Why are individual bank loans illiquid?" Equally, in the stock market context, the question could be, "Why is common stock liquid?" The main result of the analysis is that bank monitoring, bundling of loans, and debt securities should be observed together. We arrive at this conclusion by considering how security design and intermediary design can jointly solve the liquidity and informational needs of an economy. We show that when banks are a cheaper gatherer of information than markets, they will bundle securities to reduce a markets desire to speculate.

This paper also adds to the literature on the efficiencies of debt contracts. Debt is often seen as a less informationally intensive security (see Gale and Hellwig(1985)). However, allowing debt to be tradable creates some tension in this aspect. As tradability encourages information production by the market, the low information intensiveness benefits of debt are lost. Thus, we have argued that debt contracts should
be held as part of bundled portfolio, as in a bank. Two predictions that follow from this logic is that (1) they also be held by a single agent unlikely to need liquidity, as in private placements to pension funds or insurance companies, or (2) not be subject to adverse selection in trade. To clarify this last point, while it is true that there is a large amount of publicly traded debt, much of this is free of adverse selection. Most companies that have public debt also have public equity. Thus, the information that is embedded in the price of traded equity is useful in reducing adverse selection in trade among holders of debt.
4.5 References


Appendix A

Proofs for Chapter Two

Proof of Lemma 1

G and B will sign an initial contract that specifies, (1) transfers, \( r_0, r_1(z), r_2(z) \) from F to B at each date and (2) investment levels \( k_0, k_1(z) \) in land.

Since land is worthless at date 2, assumption 1 implies that F cannot commit to make payments at this date. Hence,

\[ r_2(z) = 0 \]

G is assumed to be "poor," but has high marginal product for low levels of production. This means that F will always find it profitable to invest all of its initial wealth in the contract. Therefore,

\[ r_0 = w_0 \]

At date 1, resources available to F are,

\[ zf(k_0) + u_1(z)k_1(z) \]

This is the sum of output from date 0 production and the value of land under its control. Out of this F can choose to make the payment of \( r_1(z) \) to B. If F chooses to default, B seizes the collateral and receives an effective payment of \( u_1(z)k_1(z) \). In
this case, F walks away with output $\tilde{a}f(k_0)$. If F does not default, the contractual payment of $r_1(z)$ is made, and control of $k_1(z)$ units of land is retained. Let $m(z)$ be the probability measure over events in which default occurs.

As there is a continuum of B, lenders are competitive and make zero profits. The breakeven constraint for lenders can be written as,

$$w_0 + \sum \phi_0(z)[m(z)u_1(z)k_1(z) + (1 - m(z))r_1(z)] = q_0k_0 + \sum \phi_0(z)[u_1(z)(k_1(z) - k_0)]$$

The LHS of this expression is expected receipts by B, and the RHS is the value of physical assets that are transferred to F. This can be rewritten as,

$$w_0 = u_0k_0 + \sum \phi_0(z)[(1 - m(z))(u_1(z)k_1(z) - r_1(z))] \quad (A.1)$$

where, $u_0 = q_0 - \sum \phi_0(z)u_1(z)$ is the date 0, one period rental on land. This expression allows for a very simple interpretation of the contract. $k_0$ units of land are rented initially, and an option to purchase another $k_1(z)$ units at a price of $r_1(z)$ is granted to F in state $z$. The default rule follows easily from this. The option that is granted is the option to default. If the value of the land $u_1(z)k_1(z)$ is less than the exercise price, $r_1(z)$, F will always default, and $m(z) = 1$. In the case where $u_1(z)k_1(z) > r_1(z)$, default never occurs and $m(z) = 0$.

$$w_0 = u_0k_0 + \sum \phi_0(z)[(u_1(z)k_1(z) - r_1(z))1_{u_1(z)k_1(z) > r_1(z)}]$$

Since the second expression in the RHS is non-negative, we have that $w_0 \geq u_0k_0$. In other words, not all of the initial wealth is used for date 0 rental, some of it is applied towards the date 1 payment. One can think of this simply as savings. Let, $s_0 = w_0 - u_0k_0$, be the amount saved. This is transferred to date 1 in state $z$ in the form of a subsidized purchase price on land.

Define a financial security with state contingent payoffs given by,

$$\theta(z) = (u_1(z)k_1(z) - r_1(z))1_{u_1(z)k_1(z) > r_1(z)} \quad (A.2)$$
The optimal contract between B and F can be implemented by F renting $k_0$ units of land at date 0 and saving the balance by purchasing this financial security.

At date 0, F divides its resources between rental and savings. There is no lending from B to F (in the sense of B contributing extra resources towards F's production.) This is an importance difference from other models of collateralized borrowing (for example Hart-Moore(1997) and Bolton-Scharfstein(1993)). It occurs because the asset used by one firm in production is a perfect substitute for that of another firm in this model. Thus the entrepreneur has full bargaining power and he can always let the investor seize the assets of the firm, as he can buy new and equivalent ones in the marketplace. Most other models assume some form of asset specificity which breaks this substitutability. The reason we have not done this is to shy away from firm specific effects and focus on the aggregate effects of collateral.¹

Proofs of Propositions 2 and 3

The claim is twofold: the optimal financial security is a put option, and land prices are constant for $z < z^*$ but increasing for $z > z^*$. To show this, we solve F's problem in a world with a full set of state $z$ contingent claims, but assuming a short sale constraint for F. Note from the text that $\phi(z) = \pi(z)$.

$$\max_{(k_0, \theta(z))} E_z J^z(z f(k_0) + \theta(z))$$
$$s.t \quad k_0 u_0 + E_z[\theta(z)] \leq w_0$$
$$\theta(z) \geq 0$$

The second constraint is the short sale constraint for F. The FOC's are,

$$f'(k_0) E_z [z J^z(z f(k_0) + \theta(z))] = \lambda u_0 \quad (A.3)$$
$$\pi(z) J^z(z f(k_0) + \theta(z)) = \lambda \pi(z) + \mu_z \quad (A.4)$$

¹It may also be clear from this paragraph that we have implicitly made another assumption. A firm that defaults on its obligation to one agent at date 1 walks with its output and purchases new land from another agent. We have assumed that this new purchase is unverifiable and cannot be ruled out in the original contract. Thus, there is full substitutability of assets.
Consider \( z^1, z^2 \in Z' \subseteq Z \), where \( Z' \) represents states in which the short sale constraint does not bind. In these two states,

\[
J^{z_1}(z^1 f(k_0) + \theta(z^1)) = J^{z_2}(z^2 f(k_0) + \theta(z^2))
\]

Optimality requires that the marginal product be equalized over all states where the short sale constraint does not bind.

Now, we have claimed that in equilibrium, land prices in these two states, \( q_1(z^1), q_1(z^2) \), are equal. Given this, it must be that \( J^{z_1}(w) = J^{z_2}(w) \). Then, since the marginal product function is invertible, equating marginal product must mean equating wealth.

\[
f(k_0)(z^1 - z^2) = \theta(z^2) - \theta(z^1)
\]

This implies, that if \( z < z^* \) and \( z^* \in Z' \) then, \( z \in Z' \). Call this critical value \( z^* \), then,

\[
\theta(z) = Max[\theta(z^*) + f(k_0)(z^* - z), 0]
\]

Note that this is just a put option. The budget constraint gives,

\[
k_0u_0 + Ez[\theta(z) = w_0]
\]

Combining these equations, we can solve for \( k_0 \) and \( z^* \). To complete the argument, we need to verify that in equilibrium, land prices in the two states \( z^1 \) and \( z^2 \) are indeed the same. Or more generally, that land prices for states, \( z < z^* \) are constant.

\[
k^f_z = \min[f'^{-1}(q_1(z)), \frac{w}{q_1(z)}]
\]

In the two states, \( z^1, z^2 \), the optimal hedging strategy prescribes setting wealth equal. If in addition, land prices are constant in these two states, it must be \( k^f_{z_1} = k^f_{z_2} \). Then, from the market clearing condition it is clear that land prices are in fact equal in these two states. By equating wealth in hedging states, the dependence of \( k^f_i \) is eliminated. This in turn means that land prices are equalized across these states, resulting in
decreased volatility of land prices and returns.

The last claim is that for \( z > z^* \), land prices are increasing in \( z \). First note that for these states \( \theta(z) = 0 \). The proof is by contradiction. For two states \( \hat{z} \) and \( z \), with \( \hat{z} > z \), suppose that \( \hat{q}_1 < q_1 \).

Date 1 wealth is given by,

\[
w_1 = zf(k_0)
\]

Given this wealth, \( F \) will choose a date 1 production level, \( k_1^f \) of,

\[
k_1^f = \frac{w_1}{q_1} - \left( \frac{w_1}{q_1} - f'^{-1}(q_1) \right)^+
\]

Then,

\[
\begin{align*}
\hat{k}_1^f &= \frac{\hat{z}}{\hat{q}_1} f(k_0) + L - \left( \frac{\hat{z}}{\hat{q}_1} + L - f'^{-1}(\hat{q}_1) \right)^+ \\
k_1^f &= \frac{z}{q_1} f(k_0) + L - \left( \frac{z}{q_1} + L - f'^{-1}(q_1) \right)^+
\end{align*}
\]

Since \( \frac{\hat{z}}{\hat{q}_1} > \frac{z}{q_1} \) and \( f'^{-1}(\hat{q}_1) > f'^{-1}(q_1) \), we have that \( \hat{k}_1^f \geq k_1^f \). In effect, \( F \) demands more land if its wealth is higher and land prices are lower. Market clearing in these two states are,

\[
\begin{align*}
k_1^b(\hat{q}_1) + \hat{k}_1^f &= \hat{K} \\
k_1^b(q_1) + k_1^f &= \hat{K}
\end{align*}
\]

However, as \( k_1^b(\hat{q}_1) > k_1^b(q_1) \), both of these conditions cannot hold and we have a contradiction. Therefore, prices are strictly increasing in the state.

**Proof of Proposition 5**

The competitive equilibrium has each of sectors B, F and H choosing consumption and production levels, \( X = \{X^b, X^f, X^h\} \) and a security plan \( \theta = \{\theta^b, \theta^f, \theta^h\} \), given prices, \( P = \{u_0, \{q_1(z), \eta(z)\}_{z \in Z}\} \).
Analysis of sector H is as follows. At date 1, given wealth \( w_1 \), H solves,

\[
\begin{align*}
\max_{\{C^h_1(z)\}} & \quad (w_1 - C^h_1(z)) + E_z[z h(C^h_1(z))] \\
\text{s.t.} & \quad w_1 \geq C^h_1(z)
\end{align*}
\]

The solution will be given by,

\[
\begin{align*}
C^h_1(z) & = \min(w_1, C^*_1) \\
V(w_1) & = w_1 - C^*_1 + h(C^*_1) - ((h(C^*_1) - C^*_1) - (h(w_1) - w_1))^+
\end{align*}
\]

where, \( C^*_1 = h^{-1}(1) \). At date 0,

\[
\begin{align*}
\max_{\{c^0_1(z), \theta^h(z)\}} & \quad E_z[V(z h(C^0_1) + \theta^h_0(z))] + c^h_0 \\
\text{s.t.} & \quad w^h_0 = C^h_0 + c^h_0 + E_z[\eta(z)\theta^h(z)] \\
& \quad \theta^h(z) \geq 0
\end{align*}
\]

Then H's problem can be written as,

\[
\begin{align*}
\max_{\{(C^0_0,c^0_0,\theta^h(z)) \in \mathbb{R}^{N^+ + 2}\}} & \quad c^h_0 + E_z[V(z h(C^0_1) + \theta^h(z))] \\
\text{s.t.} & \quad w^h_0 = C^h_0 + c^h_0 + E_z[\eta(z)\theta^h(z)]
\end{align*}
\]

B's problem is as follows.

\[
\begin{align*}
\max_{\{c^b_1(z), c^b_1(z), k^b_1(z), \theta^b(z)\}} & \quad c^b_0 + E_z[c^b_1(z) + b(k^b_1(z))] \\
\text{s.t.} & \quad u_0 k^b_0 + c^b_0 + E_z[\eta(z)\theta^b(z)] = 0 \\
& \quad c^b_1(z) + u_1(z)k^b_1(z) = \theta^b(z) + b(k^b_0) \\
& \quad \theta^b(z) + q_1(z)\bar{K} \geq 0 \quad \forall z
\end{align*}
\]

Notice that we have not imposed non-negativity on consumption in this case. This is equivalent to saying that B is wealthy.
Finally, the optimization problem for \( F \) is,

\[
\max_{\{c'_0, c'_1(z), k'_0, k'_1(z), \theta'(z)\}} \quad c'_0 + E_z[c'_1(z) + f(k'_1(z))]
\]

\[
s.t \quad c'_0 + u_0 k'_0 + E_z[\eta(z)(c'_1(z) + u_1(z)k'_1(z))] = w_0 + E_z[\eta(z)z f(k'_0)]
\]

\[
\theta(z) + zf(k'_0) = c'_1(z) + u_1(z)k'_1(z)
\]

\[
\theta(z), c'_1(z) \geq 0
\]

Market clearing is,

\[
c'_0 + c'_1 + c'_1 + C'_0 + C'_1(z) = w'_0 + w'_1
\]

\[
c'_0(z) + c'_1(z) + c'_1(z) + C'_1(z) = z h(C'_0) + b(k'_0) + z f(k'_0)
\]

\[
k'_0 + k'_1 = \bar{K}
\]

\[
k'_0(z) + k'_1(z) = \bar{K}
\]

\[
\theta'(z) + \theta'(z) + \theta'(z) = 0
\]

Let \((X, \theta, P)\) be a competitive equilibrium.

**Case 1:** Suppose the aggregate collateral constraint, which in this case is part of B's budget constraint, does not bind in any state. That is, suppose,

\[
\theta'(z) + u_1(z)\bar{K} > 0 \quad \forall z
\]

\[
\theta'(z) + \theta'(z) < q_1(z)\bar{K}
\]

We wish to show that the competitive equilibrium is CFE. Our proof proceeds in two steps. First, fix the date 0 production decisions, \((C'_0, k'_0, k'_1)\). Next, consider a feasible perturbation of the security holdings, \(d\theta\). We wish to show that there does not exist \(d\theta\) that results in a Pareto improvement.

WLOG, drop the short sale constraint for B. This strictly enlarges the set of feasible perturbations. Then, a feasible perturbation must satisfy,

\[
d\theta = (d\theta^h, d\theta^f, d\theta^h)
\]

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\[ d\theta^h_z \leq 0 \]
\[ d\theta^f = (d\theta^f_z, d\theta^f_h) \]
\[ d\theta^f_h \geq 0 \]

\[ d\theta^f \cdot \pi = \sum d\theta^f_z \pi_z \leq 0 \]

\[ d\theta^f_z + d\theta^h_z + d\theta^b_z = 0 \]

Since H is not consuming at date 1, it can only decrease its security holding (assuming these are greater than zero, otherwise it cannot do anything).

It is easy to show that in the competitive equilibrium F will have \( \theta_z = 0 \) for \( z > z^t \) and \( \theta_z > 0 \) for \( z < z^t \). Then, since F has a short sale constraint, the only feasible perturbations are an increase in security holdings in the high states and a decrease in the low states. Finally, B has no constraints. However, the perturbation to security holdings must net to zero in each state.

H's security holdings has no effect on date 1 land prices. Therefore, the \( d\theta^h \) has no effect on F or B at date 1. This is not so with \( d\theta^f \), since the change in wealth affects relative prices at date 1, and therefore production possibilities at date 1. We can therefore treat \( d\theta^h \) as separate from \( d\theta^f \). It is then easy to show that the perturbation to \( \theta^h \) has no effect on utilities (envelope theorem). F’s perturbation is a little harder since it triggers a change in date 1 land prices, and the proof focuses on this.

Let \((dU^b_z, dU^f_z)\) be the date 1 changes in welfare from the perturbation. For \( d\theta^f \) to result in a Pareto gain, \( dU^b \cdot \pi > 0 \) and \( dU^f \cdot \pi > 0 \).

From the competitive equilibrium, we can identify levels of the aggregate shock, \( Z^c \) and \( Z^t \), such that, for \( z > Z^c \), F is unconstrained in date 1 production, and for \( z < Z^t \), F's land use is constant (\( Z^t \) is the strike of the put option). Then, we have that, for \( z > Z^c > Z^t \),

\[ dU^f_z = d\theta^f_z \]
\[ dU^b_z = 0 \]
for $z < z^c$,

$$dU^b_z = f'(k^f_z)dk^f_z$$
$$dU^b_z = d\theta^b_z - b'(\bar{K} - k^f_z)dk^f_z$$

where, $dk^f_z$ is the change in date 1 land use that the perturbation results in.

$$dU^b = \sum_{z < z^c} d\theta^b_z \pi_z - \sum_{z < z^c} b'(\bar{K} - k^f_z)dk^f_z \pi_z$$
$$dU^f = \sum_{z > z^c} d\theta^f_z \pi_z + \sum_{z < z^c} f'(k^f_z)dk^f_z \pi_z$$

The first expression on the RHS is just a transfer from B to F. Call this $t_c \geq 0$. Then, for $d\theta^f$ to be a Pareto improvement,

$$dU^b = -t_c - \sum_{z < z^c} b'(\bar{K} - k^f_z)dk^f_z \pi_z > 0$$
$$dU^f = t_c + \sum_{z < z^c} f'(k^f_z)dk^f_z \pi_z > 0$$

For $z > z^l$, $d\theta^f_z \geq 0$ and $dk^f_z \geq 0$. For $z < z^l$, $k^f_z = k^f_z$, implying that, $f'(k^f_z) = f'(k^f_z)$ and $b'(\bar{K} - k^f_z) = b'(\bar{K} - k^f_z)$. Then we can rewrite the two inequalities as,

$$-\frac{t_c}{b'(\bar{K} - k^f_z)} - \sum_{z^l < z < z^c} \frac{b'(\bar{K} - k^f_z)}{b'(\bar{K} - k^f_z)}dk^f_z \pi_z > \sum_{z < z^l} dk^f_z \pi_z$$
$$+ \frac{t_c}{f'(k^f_z)} + \sum_{z^l < z < z^c} \frac{f'(k^f_z)}{f'(k^f_z)}dk^f_z \pi_z > \sum_{z < z^l} dk^f_z \pi_z$$

However, since $f'(k^f_z) > b'(\bar{K} - k^f_z)$, $t_c \geq 0$ and $\frac{f'(k^f_z)}{b'(\bar{K} - k^f_z)} < \frac{b'(\bar{K} - k^f_z)}{b'(\bar{K} - k^f_z)}$, these inequalities cannot be simultaneously satisfied.

**Case 2:** If F is not credit constrained, effectively the non-negativity restriction on consumption is lifted.

Since markets are assumed to clear competitively at date 1, regardless of date 0
choices,

\[ k^b_1(z) = k^f_1(z) = \frac{K}{2}, \quad q_1(z) = \rho \]

Given this, we can remove the date 1 production decision and rewrite the date 0 problem for B and F as,

\[
\begin{align*}
\max_{\{c^b_0, c^b_1(z), \theta^b(z)\}} & \quad c^b_0 + E_z[c^b_z] \\
\text{s.t.} & \quad c^b_0 + E_z[\eta(z)\theta^b(z)] = 0 \\
& \quad c^b_1(z) = \theta^b(z) \\
& \quad \theta^b(z) + \rho K \geq 0
\end{align*}
\]

and,

\[
\begin{align*}
\max_{\{c^f_0, c^f_1(z), \theta^f(z)\}} & \quad c^f_0 + E_z[c^f_z] \\
\text{s.t.} & \quad c^f_0 + E_z[\eta(z)\theta^f(z)] = w^f_0 \\
& \quad c^f_1(z) = \theta^f(z) \\
& \quad \theta^f(z) \geq 0
\end{align*}
\]

The problem is recast as a 2 period, 1 good economy. The proof of constrained efficiency is fairly standard in this case. Write H’s problem,

\[
\begin{align*}
\max_{\{(C^h_0, c^h_1(z), \theta^h(z))\in \mathbb{R}^{N+2}_+\}} & \quad c^h_0 + E_z[V(zh(C^h_0) + \theta^h(z))] \\
\text{s.t.} & \quad w^h_0 = C^h_0 + c^h_0 + E_z[\eta(z)\theta^h(z)]
\end{align*}
\]

There are \(N+1\) market clearing conditions,

\[
\begin{align*}
& \quad c^b_0 + c^f_0 + c^h_0 + C^h_0 = w^h_0 + w^f_0 \\
& \quad \theta^h(z) + \theta^b(z) + \theta^f(z) = 0
\end{align*}
\]

The competitive equilibrium consists of an allocation, \(X = (c^b_0, c^f_0, c^h_0, C^h_0)\), a security
plan, $\theta = (\theta^h(z), \theta^f(z), \theta^h(z))$, and prices $P = (\eta(z))$.

Now, suppose there exists another feasible allocation, $(\hat{X}, \hat{\theta})$, that is strictly better for all agents. Then, it must have cost more at prices $P$, or,

$$\hat{c}_0^h + E_z[\eta(z)\hat{\theta}^h(z)] > 0$$
$$\hat{c}_0^f + E_z[\eta(z)\hat{\theta}^f(z)] > \hat{w}_0^f$$
$$\hat{c}_0^h \hat{C}_0^h + E_z[\eta(z)\hat{\theta}^h(z)] > \hat{w}_0^h$$

But, summing these budget constraints and using the fact that $\sum \theta^j = 0$, we find that,

$$\hat{c}_0^h + \hat{c}_0^f + \hat{c}_0^h + \hat{C}_0^h > \hat{w}_0^f + \hat{w}_0^h$$

which is a violation of the aggregate resource constraint, and hence, a contradiction.

**Case 3:** If neither of cases 1 nor 2 applies, the allocation is inefficient. Thus, a central planner can reallocate portfolios to make all agents better off. We show this via a perturbation argument.

Consider two states $\hat{z}$ and $z$ in which the aggregate collateral constraint binds, and $F$ is using a positive amount of insurance. Suppose that prices in these states in the competitive equilibrium are (WLOG), $\hat{q}_1 > q_1$ and $\hat{\eta} < \eta$. Since, $q_1 = b'(\hat{K} - \hat{k}_1^f)$, this means that $\hat{k}_1^f > k_1^f$. Now market clearing for insurance is,

$$\hat{\theta}^h + \hat{\theta}^f = \hat{q}_1 \hat{K}$$
$$\theta^h + \theta^f = q_1 K$$

Consider transfers at market prices, $(d\theta^h, d\theta^f)$ and $(d\hat{\theta}^h, d\hat{\theta}^f)$. These must satisfy,

$$d\theta^h + d\theta^f = 0$$
$$d\hat{\theta}^h + d\hat{\theta}^f = 0$$
$$\frac{d\theta^f}{\eta} = \hat{\eta} d\hat{\theta}^f$$
Note that these transfer are feasible (since security holdings are positive for both agents in these states) and leave agents no worse off (envelope condition). Now, in each of these cases, the date 1 price of land is affected.

\[
\frac{dq}{dk^f} = -b''(\bar{K} - k^f)
\]

thus, the perturbation causes land prices to change,

\[
d\hat{q} = \frac{b''(\bar{K} - \hat{k}^f)}{b'(\bar{K} - \hat{k}^f)} d\hat{h}^h
\]

\[
dq = -\frac{b''(\bar{K} - k^f)}{b'(\bar{K} - k^f)} \frac{\hat{\eta}}{\eta} dd\hat{h}^h
\]

The value of this change in liquidity at market prices is,

\[
d\hat{h}^h \hat{\eta} \bar{K} \left( \frac{b''(\bar{K} - \hat{k}^f)}{b'(\bar{K} - \hat{k}^f)} - \frac{b''(\bar{K} - k^f)}{b'(\bar{K} - k^f)} \right)
\]

As long as this is not zero, we can choose $d\hat{h}^h$ to make this expression positive, and redistribute the additional liquidity to all agents making everyone better off.

**Proof of Existence**

We shall prove existence of equilibrium for economy E2. The proof proceeds by defining and establishing properties of relevant sets and then applying Kakutani’s fixed point theorem.

Define the price simplex,

\[
P = \{(u_0, \phi_0, (u_1(z), \phi(z))_{z \in Z}) \in \mathbb{R}^{2(N+1)}_+ : \phi_0 + u_0 + \sum (\phi(z) + u_1(z)) = 1 \}
\]

Here, $\phi_0$ is the date 0 price of corn, which is no longer the numeraire good. Note that $P$ is compact and convex.

The competitive equilibrium consists of $(x \in X, p \in P)$.

We have shown that we can write each agent’s problem as a one period static
optimization. Define the budget set for $H$,

$$B^h = \{(c^h_0, C^h_0, \theta^h(z)) : 0 \leq c^h_0 \leq \sum w^a_0, 0 \leq C^h_0 \leq \sum w^a_0, -\rho \bar{K} \leq \theta^h(z) \leq \rho \bar{K}\}$$

Holdings of financial securities, because of the collateral constraint, must be backed by land. The aggregate quantity and value of land cannot exceed $\rho \bar{K}$. $B^h$ is compact and convex. Date $0$ optimization for $H$ is

$$\Phi^h(p) = \text{argmax}_{\{(c^h_0, C^h_0, \theta^h(z)) \in B^h\}} c^h_0 + E_z[V(zh(C^h_0) + \theta^h(z))]
\text{ s.t. } \phi_0 w^h_0 = \phi_0 C^h_0 + \phi_0 c^h_0 + E_z[\eta(z)\theta^h(z)]$$

where $V(\cdot)$ is concave and continuous. Then, since the choice set is compact and convex and the objective is concave and continuous, $\Phi^h(p)$ is non-empty, convex, and upper semi-continuous.

Define the budget sets for $a \in \{F, B\}$,

$$B^a = \{(c^a_0, k^a_0, k^a_1(z), \theta^a(z)) : 0 \leq c^a_0 \leq \sum w^a_0, 0 \leq k^a_0 \leq \bar{K}, 0 \leq k^a_1(z) \leq \bar{K}, -\rho \bar{K} \leq \theta^a(z) \leq \rho \bar{K}\}$$

where, this time we have take B's date $0$ endowment to be $w^b_0$ and assumed that this is large but bounded above.

Date $0$ optimization for $F$ is,

$$\Phi^f(p) = \text{argmax}_{\{(c^f_0, k^f_0, k^f_1(z), \theta^f(z)) \in B^f\}} c^f_0 + E_z[J^f(zf(k^f_0) + \theta^f(z))]
\text{ s.t. } \phi_0 w^f_0 = u_0 k^f_0 + \phi_0 c^f_0 + E_z[\eta(z)\theta^f(z)]
\text{ s.t. } k^f_1(z) = \min\left[\frac{zf(k^f_0) + \theta^f(z)}{u_1(z)}, g^{-1}(u_1(z))\right]$$

where $J^f(\cdot)$ is concave and continuous. Then, since the choice set is compact and convex and the objective is concave and continuous, $\Phi^f(p)$ is non-empty, convex, and upper semi-continuous.
Date 0 optimization for B is,

\[
\Phi^b(p) = \arg\max_{\{(c_0^b, k_0^b, k_1^b(z), \theta^b(z)) \in B^b\}} \quad c_0^b + E_z[z b(k_0^b) + \theta^b(z)] \\
\text{s.t.} \quad \phi_0 w_0^b = u_0 k_0^b + \phi_0 c_0^b + E_z[\eta(z) \theta^b(z)] \\
k_1^b(z) = b^{-1}(u_1(z))
\]

Then, since the choice set is compact and convex and the objective is concave and continuous, \( \Phi^b(p) \) is non-empty, convex, and upper semi-continuous.

Next, define the correspondence, \( \Phi^0 : B^f \times B^h \times B^b \equiv B^A \rightarrow P \) by,

\[
\Phi^0(x) = \arg\max_{p \in P} \quad u_0(\sum_a k_0^a - \bar{K}) - \sum_z u_1(z)(\sum_a k_1^a(z) - \bar{K}) + \sum_z \phi(z) \sum_a \theta_z^a
\]

The choice set is compact and convex, the objective is continuous and concave, implying that \( \Phi^0 \) is upper semi-continuous. Last define \( \Phi : P \times B^A \rightarrow P \times B^A \) by,

\[
\Phi(p, x) = (\Phi^0 \times \Phi^f \times \Phi^b \times \Phi^h)
\]

Since this too is upper semi-continuous, by Kakutani's fixed point theorem, \( \Phi \) has a fixed point.
Appendix B

Proofs for Chapter Four

Analysis of the trading game of section 3.3.

- (U,U) Since both parties are uninformed, they will in expectation exactly split gains from trade. Trade relevant payoffs are (unconditional on realization of information):

\[ \pi_b = \frac{1}{2}(T - k_2\hat{\phi}) \]
\[ \pi_s = \frac{1}{2}(T - k_2\hat{\phi}) \]

Gains from trade are fully realized, and the monitoring cost is only paid in the second period conditional on the report of the entrepreneur.

- (I,I):

\[ \pi_b = \frac{T}{2} - k_1 \]
\[ \pi_s = \frac{T}{2} - k_1 \]

Information acquisition at date 1 renders monitoring at date 2 unnecessary. However both parties pay the acquisition cost. There is excessive monitoring in this case.
\( \cdot (i, U): \) Buyer informed, seller uninformed. \(^1\)

\[
\pi_b(q) = E[(g(x, R) - q)1_{g(x, R) > q}] - k_1 \\
\pi_s(q) = E[(q - \Gamma g(x, R))1_{g(x, R) > q}]
\]

There are two cases, differentiated by which party initiates the offer.

- B has bargaining power: \( q_b \) chosen s.t. \( \pi_s(q_b) = 0 \).
- S has bargaining power: \( q_s \) chosen s.t. \( \pi'_s(q_s) = 0 \). \(^2\)

Let \( T(q) = \pi_s(q) + \pi_b(q) = (1 - \Gamma)E[g(x, R)1_{g(x, R) > q}] \). Then, \( T(q) \) is maximized when \( g(x, R) \) is always greater than \( q \). However, this will generally not be true. Consider the second case. Since S does not know \( x \), \( q_s \) cannot be conditioned on \( x \). Notice that \( \pi'_s(0) = 1 + \Gamma f(0)E[g(x, R)] > 0 \). Thus, \( q_s > 0 \), which implies that \( g(x, R) < q_s \) for some \( x \). Under adverse selection, not all efficient trades are realized.

\( \cdot (U, I): \) Buyer uninformed, Seller informed.

\[
\pi_b(p) = E[(g(x, R) - p)1_{g(x, R) < p}] \\
\pi_s(p) = E[(p - \Gamma g(x, R))1_{g(x, R) < p}]
\]

Again, there are two cases,

- S has bargaining power: \( p_s \) chosen s.t. \( \pi_b(p_s) = 0 \).
- B has bargaining power: \( p_b \) chosen s.t. \( \pi'_b(p_b) = 0 \).

In this case, \( T(p) = (1 - \Gamma)E[g(x, R)1_{g(x, R) < p}] \). Which is maximized when \( p \) is always greater than \( \Gamma g(x, R) \). Consider the first case this time. \( T(p) \) is maximized when \( p \) is equal to \( R \). \( \pi'_b(R) = E[\min(x, R) - R] = -E[R - x] < 0 \). Therefore, \( p \) will be set less than \( R \), or gains from trade will not be fully realized.

---

\(^1\) A fixed price is an optimal price discrimination scheme in this case because of the linearity in buyer and seller valuations.

\(^2\) The solution to the F.O.C. gives the optimal scheme as long as \( f(x) \) satisfies the MHRC.
While it is easy to see that \((U, U)\) is most efficient, it is less clear to identify whether or not it is an equilibrium. \((U, U)\) is an equilibrium as long as neither B nor S have a private incentive to become informed. This will generally depend on parameter values.

**Security design in section 4.1**

We restrict ourselves to securities of the form

\[ g(x, R, \alpha) = \alpha \min(x, R) \]

Now, consider the choice of \(q_s\). \(q_s\) solves \(\pi_s'(q_s) = 0\), where,

\[ \pi_s(q_s) = E[q_s 1_{\min(x, R) > q_s}] \]

Clearly, we must have that \(R > q_s / \alpha\). Then,

\[ \pi_s(q_s) = (1 - F(q_s / \alpha))q_s \]

The FOC for this (which is necessary and sufficient given MHRC) is,

\[ 1 - F(q_s / \alpha) = q_s / \alpha f(q_s / \alpha) \]

Let, \(x^*\) solve \(1 - F(x) = xf(x)\). Then, \(q_s = x^* \alpha\). Returning to the expression for the buyer's profit,

\[ \pi_b(q_s) = \alpha \int_{x^*}^{\hat{x}} (\min(x, R) - x^*) f(x) dx \]

which we wish to maximize subject to the constraint that \(E_x[\alpha \min(x, R)] = C\). It is relatively easy to show that this program yields a corner solution of \(R = \hat{X}\).