Forces and Gauge Groups
Beyond the Standard Model

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The discovery of the Higgs boson in 2012 completed the particle content of the Standard Model, but brought into sharp relief two outstanding problems: why is the Higgs so light, and what is the identity of 80% of the matter content of the universe? Neither appears to have an answer within the Standard Model. This thesis attempts to address these problems with the introduction of new forces and gauge groups. I investigate a model where dark matter interacts through a new massive $U(1)$ gauge boson which kinetically mixes with the photon, and show how this model can be tested at neutrino experiments. Supersymmetry may explain the smallness of the Higgs mass compared to the Planck scale, but reconciling the measured value of 126 GeV with the absence of superpartners at colliders is difficult. By gauging various global symmetries of the Standard Model, I show that a variant of Higgsed gauge mediation called auxiliary gauge mediation can provide acceptable supersymmetric spectra. Finally, the astrophysical dark sector may be complicated, with many kinds of allowed interactions, and I describe techniques to diagnose the presence of dark matter at direct-detection experiments independent of its velocity distribution.
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This thesis is based on both published and unpublished work: principally Ref. [184] in collaboration with Jesse Thaler, Gordan Krnjaic, and Matthew Toups; Ref. [185], in collaboration with Jesse Thaler and Matthew McCullough; and Ref. [135], in collaboration with Patrick Fox and Matthew McCullough. It also draws from Refs. [188] and [189] with Jesse Thaler, as well as unpublished work with Dan Roberts, Jesse Thaler, Adam Anderson, Patrick Fox, and Matthew McCullough.
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Right (b): Electron energy spectra due to various DM signal points and principal beam-on backgrounds (unstacked histograms) assuming the on-axis DAEδALUS/LENA configuration. The color shaded region under each signal curve represents the signal window that maximizes $S/\delta B$ for each parameter point. The $\nu_\mu$ CCQE distribution shows the residual background after a 70% reduction from vetoing Michel electrons; the remaining muons are mis-identified as electrons in LENA, and their kinetic energy spectrum is shown. The $\nu_e$ CCQE distribution was only simulated above 100 MeV where it begins to dominate. The $e^2$ values for each signal point are chosen to match the minimum value for which the DAEδALUS/LENA setup has the 3$\sigma$ sensitivity displayed in Fig. 2-3.
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Comparison of LSND sensitivities as computed using methods in the existing literature [57, 97] (magenta curve) and those obtained using the full three-body matrix element that includes DM production via an off-shell $A'$. 

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4-1 A schematic representation of all halo possibilities for $\tilde{g}(v_{\text{min}})$. If an experiment observes a number of events consistent with DM scattering, in this case three events of energy $E_i$, then hypothetical values of $\tilde{g}(\tilde{v}_{i-1} < v_{\text{min}} \leq \tilde{v}_i) = \tilde{g}_i$ may be chosen where the positions of the steps $\tilde{v}_i$ are given by $v_{\text{min}}(E_i)$ in the case of perfect energy resolution, and are allowed to float as free parameters if the energy resolution is non-zero. The solid blue curve will always minimize the extended log-likelihood, both in the case of perfect energy resolution and also with resolution effects included as demonstrated in App. C. Conversely the dashed red curve corresponds to the worst possible fit out of all halos, which is infinitely bad if the velocity integral between $v_{\text{low}}$ and $v_1$ is taken to infinity. Here, $v_{\text{low}}$ ($v_{\text{high}}$) is the velocity that corresponds to the low (high) energy threshold of the experiment. To determine the range of halos implied by the DM candidate events the parameters $\tilde{g}_i$ and $\tilde{v}_i$ may be varied, consistently choosing the solid blue curve in the likelihood, in order to determine the best-fit values and confidence intervals for $\tilde{g}_i$. 

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B-1 Generation of $B_\mu$ at two loops from gauginos and messengers. The diagram for $A$-terms is analogous, except with the Higgsino mass $\mu_H$ replaced by a scalar vertex. The two-loop calculation performed here includes all orders in $F/M^2$, however the perturbative mass insertions for the messengers have been depicted here to demonstrate the chirality flips required for the generation of the lowest-order term. The red arrows show the momentum routing.
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3.3 Parameters for the minimal auxiliary gauge mediation model with a single $U(1)_X$ gauge symmetry with lepton, quark, and Higgs charges $q_L = 1$ and $q_Q = q_H = 1/3$. 

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Chapter 1

Introduction

The Standard Model (SM) is perhaps the most spectacularly successful physical theory to date. Based on the gauge group $SU(3) \times SU(2) \times U(1)$ and describing three generations of quarks and leptons, its predictions have been verified to extraordinary accuracy both through precision low-energy experiments and at high-energy colliders [237]. With the discovery of the Higgs boson in July 2012 at a mass of 126 GeV [73, 1], the experimental support for the major theoretical components of the Standard Model is now complete. However, there is convincing evidence that the SM cannot be the end of the story: observational evidence from without, in the form of dark matter (DM), and theoretical evidence from within, in the form of a Higgs much lighter than dimensional analysis would suggest. In particular, explaining the size of the Higgs mass motivates the introduction of supersymmetry (SUSY), which faces its own observational challenges.

In this thesis, I will propose several ideas addressing pieces of the puzzles of dark matter and supersymmetry, which deal with forces and gauge groups beyond the SM. A dark photon is a promising candidate for mediating DM interactions, representing the addition of a new $U(1)$ gauge group. I will show that MeV-scale dark photons and dark matter are discoverable at low-energy neutrino experiments, and I will propose novel experimental setups and analysis techniques to conduct these searches. Concerning supersymmetry, I will demonstrate that auxiliary gauge mediation, which involves gauging an $SU(3) \times U(1) \times U(1)$ global symmetry of the SM,
provides a concrete realization of a “mini-split” SUSY spectrum. Such a spectrum could reconcile the observed Higgs mass with the non-observation of superpartners thus far at the LHC. Finally, I will discuss more model-independent tests for dark matter. The as-yet-unknown interactions of DM may cause it to have nontrivial structure in velocity space, and I will show that “halo-independent” methods for direct detection allow one to make strong statements about whether or not DM has been seen at an experiment, independent of its velocity distribution in our galaxy. I now review the key features and motivations for each of these ideas.

1.1 Dark matter and dark photons

Precision cosmological measurements from the Planck satellite [10] tell us that the SM, with all its rich phenomenology, accounts for only 4.9% of the total energy budget of the universe. A much larger component, weighing in at 26.8%, is cold, non-baryonic, and by all appearances does not couple to either the strong or electromagnetic forces. For lack of a better name, this material is called dark matter (DM), and its properties and interactions lie outside the purview of the SM. The existence of DM has been unambiguously established by multiple astrophysical measurements. Galactic rotation curves which flatten out at large radius imply the existence of galactic DM halos (see e.g. [258]), N-body simulations such as Via Lactea [202] show that DM with only gravitational interactions forms structures and substructures, and observations of the Bullet Cluster [218] can even constrain the DM self-interaction cross section, $\sigma/m < 1.3$ barn/GeV. Despite this body of evidence, dark matter particles have never been unambiguously detected at terrestrial accelerators, nor have their decay or annihilation products been unambiguously identified. The nature of dark matter remains one of the key observational puzzles for physics beyond the SM.

While the effects of dark matter on the SM can be encoded in higher-dimensional operators in an effective field theory (EFT) framework, arguably the best prospects
for discovery come from "portal" operators:

\[ \mathcal{O}_H = |H|^2|\phi|^2 \]  
\[ \mathcal{O}_N = e^{ab}L_a H^1_b N \]  
\[ \mathcal{O}_{A'} = F'_{\mu\nu} B^{\mu\nu} \]

These are the only renormalizable, Lorentz-invariant, gauge-invariant operators which
couple SM fields (the Higgs doublet \( H \), the hypercharge field strength \( B^{\mu\nu} \), and
left-handed lepton doublets \( L \)) to new dark-sector fields (a complex scalar \( \phi \), a new U(1)
field strength \( F'_{\mu\nu} \), and a fermion \( N \)). The Higgs and neutrino portals may be difficult
to observe because the dark-sector fields \( \phi \) and \( N \) may be singlets under all gauge
groups in both the SM and the dark sector.\(^1\) However, there are myriad observational
prospects for the dark photon portal.

The dark photon is a U(1) vector boson \( A' \), a spin-1 particle with a mass of
\( \mathcal{O}(1 \text{ MeV} - 10 \text{ GeV}) \). The dark photon portal describes kinetic mixing of the U(1)
field strength with hypercharge, or with electromagnetism after electroweak sym-
metry breaking. Crucially, this gives the \( A' \) a coupling to any electrically-charged
SM particles. The kinetically-mixed dark photon was first suggested in Ref. [171]
(see also Refs. [236, 145]). The \( A' \) can acquire mass either through a St"uckelberg
field or a dark Higgs, the latter possibility giving rise to interesting phenomeno-
logical scenarios.\(^2\) The focus on the MeV mass scale is motivated by the vast lit-
erature which invokes light, weakly-coupled particles to resolve anomalies in direct
and indirect detection experiments; build models that relate dark and baryonic en-
ergy densities; resolve puzzles in simulations of cosmological structure formation;
introduce new relativistic degrees of freedom during big bang nucleosynthesis; and
resolve the proton charge radius and other low-energy standard model anomalies
[66, 67, 127, 175, 242, 242, 187, 174, 121, 126, 39, 235, 240, 25, 82, 230, 217, 72,
53, 256, 118, 191, 113, 30, 229, 84, 26, 157, 173, 92, 69, 192, 131, 250, 201, 180, 98].

\(^1\)However, a promising indirect avenue is through measurements of the Higgs invisible decay
width [241, 243].

\(^2\)Indeed, the \( \phi \) of the Higgs portal could be the dark Higgs.
In particular, a dark photon has been an attractive candidate for reconciling the longstanding discrepancy between the theoretical prediction and experimental measurement of the anomalous magnetic moment of the muon, \((g - 2)_\mu\) [60].

In the minimal scenario, DM is a single species \(\chi\), which carries unit charge under the new \(U(1)_D\) (\(D\) for “dark”). The Lagrangian for this model is

\[
\mathcal{L} \supset \frac{\epsilon_Y}{2} F_{\mu\nu}^D B^{\mu\nu} + \frac{m_{A'}^2}{2} A'_\mu A'^\mu + \bar{\chi}(i\gamma^\nu - m_\chi)\chi.
\]  

(1.4)

While DM can be either a scalar or a Dirac fermion, I will focus on fermionic DM for concreteness. Here, \(D_\mu \equiv \partial_\mu + ig_D A'_\mu\), where \(g_D\) is the dark coupling constant. After electroweak symmetry breaking, one can diagonalize the kinetic terms and show that the \(A'\) inherits a universal coupling to electromagnetic currents with strength \(\epsilon e\), where \(\epsilon \equiv \epsilon_Y \cos \theta_W\). This model has four free parameters,

\[
\{m_{A'}, \epsilon, m_\chi, \alpha_D\},
\]  

(1.5)

namely the \(A'\) mass \(m_{A'}\), the kinetic mixing parameter \(\epsilon\), the DM mass \(m_\chi\), and the dark fine-structure constant \(\alpha_D \equiv g_D^2/4\pi\). To facilitate the comparison between \(U(1)_D\) and \(U(1)_{EM}\), I will often refer to \(\alpha' \equiv \epsilon^2 \alpha_{EM}\), the effective \(A'\) coupling to charged matter. Most studies explore only the \(\{m_{A'}, \epsilon\}\) portion of dark photon parameter space, but in Chapter 2, I will show that \(m_\chi\) is an essential third dimension that introduces qualitatively different kinematic effects.

Due to its universal coupling to electromagnetism, the \(A'\) can replace a photon in any kinematically-allowed process, with an accompanying factor of \(\epsilon\), such that the event rate for any tree-level process coupling the visible sector to the dark sector is proportional to \(\epsilon^2\). Such an \(A'\) could mediate annihilation of MeV-scale dark matter into electrons via \(\chi \chi \rightarrow A' \rightarrow e^+e^-\) [66, 242] which could explain the excess [260] of 511 keV photons from the galactic center [67, 175] and a 3\(\sigma\) anomaly [7, 109] in the \(\pi^0 \rightarrow e^+e^-\) decay rate [187]. Alternatively, dark matter could be at the TeV scale, with the 511 keV excess explained with excited dark matter states [127, 39, 229]. In such models, pair annihilation \(\chi \chi \rightarrow A'A'\) followed by the decay \(A' \rightarrow e^+e^-\)
[242, 39] could also explain the high-energy positron excess in the PAMELA [13] and FERMI [5] data. In any of these models, indirect constraints from electron and muon anomalous magnetic moments [119, 240] force the $A'$ to have extremely weak coupling to matter, $\alpha' \approx 10^{-6} \times \alpha_{\text{EM}}$, which could explain why the $A'$ has evaded detection thus far.

While the dark photon model is a viable, renormalizable theory of DM in its own right, it is also useful to regard this scenario as a simplified model for an entire class of theories in which sub-GeV particles mediate interactions between dark and visible matter. That said, it has been observed that certain realizations of light DM ($\lesssim$ GeV) face strong constraints from out-of-equilibrium annihilation to charged leptons during cosmic microwave background (CMB) freeze-out [251, 146, 81, 147, 176]. However, these bounds are model dependent and can be evaded if DM is asymmetric, scatters inelastically with the visible sector [182], has a velocity-suppressed annihilation cross section [211], or if the annihilating particles are a subdominant fraction of the DM abundance, none of which affect the projections for the fixed-target searches I consider in this thesis.\(^3\) One can therefore consider the kinetically-mixed dark photon as a simplified model of a portal to the dark sector for which the experimental constraints and future projections can be adapted to study a plethora of other, more elaborate scenarios.

Several experiments have been designed to search for the distinctive signatures of a weakly-coupled $A'$. One possible decay mode is $A' \rightarrow e^+e^-$ (the visible mode), inviting a search for low-mass resonances in the $e^+e^-$ invariant mass spectrum [245, 114, 64]. There are currently several experiments being developed to look specifically for the visible decay mode: APEX[115, 8] and HPS [172] at the CEBAF facility at JLab, HIPS [31] at DESY, A1 [225] at the MAMI facility in Mainz, and DarkLight\(^4\) [142, 128] at the JLab FEL.

\(^3\)A thorough analysis of model-dependent cosmological constraints is beyond the scope of this work, but see Ref. [181] for a more in-depth discussion of these issues. I simply note here that in the region of parameter space I will consider in this thesis, $\alpha_D$ is typically large enough to make the relic density of $\chi$ a subdominant fraction of the observed total DM abundance.

\(^4\)“Detecting A Resonance Kinematically with eLectrons Incident on a Gaseous Hydrogen Target.” I am indebted to Jesse Thaler for coming up with this spectacular acronym.
Alternatively, the $A'$ could decay primarily into invisible final states such as neutrinos or dark matter (the *invisible mode*), in which case one must either perform a missing energy/momentum or missing invariant mass search, or alternatively search for rescattering of the “invisible” states at a detector downstream. The missing energy search was initially proposed in Ref. [169], and subsequent proposals include Refs. [265, 266, 183, 231]. There has recently been a resurgence of studies reanalyzing beam-dump and neutrino experiments for hints of missing energy from an $A'$ [99, 57]. Proposals to search for DM with rescattering include Refs. [58, 181, 96, 97, 182].

With an eye towards the connection to dark matter, this thesis will focus primarily on the region of parameter space $\alpha_D \gg e^2 \alpha_{EM}$ where the $A'$ primarily decays into DM when kinematically allowed, rather than into visible-sector particles. In Chapter 2, based on work in Ref. [184], I will show that neutrino experiments can also produce dark photons through proton-nuclear scattering, with sufficient luminosity such that the subsequent decay products can be seen downstream through rescattering at a detector optimized for neutrino detection. Similar work was also performed in Ref. [189], where I showed that DarkLight could perform a missing invariant mass search for the $A'$, provided that photon detectors were added to the DarkLight design to veto on two-photon bremsstrahlung. Results for the reach in $\alpha'$ (taken from Ref. [189]) are shown in Fig. 1-1 as a function of photon efficiency.

### 1.2 The Higgs boson and mini-split supersymmetry

The Higgs itself may bring more questions than answers. As the only fundamental scalar field in the Standard Model Lagrangian, its mass is unprotected by any symmetry, and loop effects should generically push its mass to the highest possible scale in the effective theory; if the Standard Model is truly the fundamental theory of all non-gravitational interactions, the Higgs mass should be at the Planck scale, $10^{19}$ GeV. The fact that the measured Higgs mass is 16 orders of magnitude smaller
Figure 1-1: **Left:** DarkLight reach for invisibly-decaying $A'$ ($A' \rightarrow \chi \chi$) for various photon efficiencies. The gray shaded area indicates constraints from anomalous magnetic moment measurements, with the green region indicating the “welcome” region where an $A'$ could explain the $(g - 2)\mu$ discrepancy. The fluctuations in the reach for high photon efficiency are an artifact of the difficulty of achieving high enough Monte Carlo statistics for events where both photons miss the detector. $\Delta_{\text{cut}}$ is the maximum value of $E_{\text{miss}}/m_{\text{miss}} - 1$, to improve the invariant mass resolution; more details are given in Ref. [189]. **Right:** DarkLight reach for visibly-decaying $A'$ ($A' \rightarrow e^+e^-$), adapted from Ref. [142]. The visible search reach is shown for comparison and includes additional constraints from beam dump experiments.
begs for an absurdly fine-tuned cancellation or some new theoretical framework to protect its mass.

Supersymmetry is one of the best-motivated proposals for addressing the problem of the light Higgs, as well as many other theoretical questions such as gauge coupling unification.\(^5\) It accomplishes these feats by introducing “superpartners,” fermionic partners for every SM boson and vice versa. This ensures the approximate cancellation of loop diagrams contributing to the Higgs mass, and almost miraculously, changes the running of the SM gauge couplings with energy such that they appear to unify below the Planck scale. Indeed, models of SUSY usually furnish a dark matter candidate as well (the lightest supersymmetric particle, or LSP), offering the tantalizing promise of addressing two of the most pressing issues facing the Standard Model in one fell swoop.

The fact remains, though, that the experiments responsible for confirming the Standard Model have seen no evidence for the extra particles or interactions predicted by SUSY. This implies that supersymmetry must be spontaneously broken, lifting the masses of superpartners such that they have avoided detection. A challenge for any SUSY model is to provide an explanation for the superpartner mass spectrum. SUSY-breaking can be parameterized with certain “soft” terms in the Lagrangian, so called because they avoid reintroducing the quadratic divergence which plagues the Higgs mass in the Standard Model. Given the values of all the soft terms, the spectrum may be calculated at any desired renormalization group (RG) scale using software packages such as Refs. [22] and [106].

The discovery of a 126 GeV Higgs boson [1, 73] places considerable restrictions on SUSY model building. At tree-level, the Higgs mass is constrained to satisfy \(m_h \leq m_Z\) [220]. Loop effects can raise the Higgs mass, with the main correction driven by the top squark \(\tilde{t}\) (the SUSY partner of the top quark, also called the “stop”),

\[
\delta m_h^2 \sim m_{\tilde{t}}^2 \ln(m_{\tilde{t}}^2/m_t^2).
\]  

\(^5\)For reviews, see Refs. [220, 262].
However, as $m_h$ increases, the top squark is required to be many orders of magnitude heavier than the top quark. But these large loop corrections are precisely those that SUSY was designed to cancel, threatening to destroy the very motivation for the introduction of SUSY in the first place. Nonetheless, one can take the observed Higgs mass as empirical evidence for the approximate mass of scalar superpartners. On the other hand, the gauginos (the fermionic partners of the SM gauge bosons) are expected to be light in order to preserve gauge coupling unification [100, 101]. This sort of spectrum, with light fermions and heavy scalars, was initially proposed as “split supersymmetry” [261, 151, 38]. Before the measurement of the Higgs mass, there was no upper limit on the scalar mass scale, but a Higgs at 126 GeV forces the third generation squarks to lie between 1 and $10^6$ TeV, depending on the parameter $\tan \beta$ which controls the ratio of up-type to down-type Higgs vevs [41, 44]. One is thus led to a “mini-split” spectrum, where the scalar superpartners are heavier than the gauginos, but not arbitrarily so.\(^6\)

Many of the direct experimental bounds on SUSY particles are easily evaded by squarks as heavy as a PeV, so the fact that superpartners have not yet been observed at the LHC may be seen as a feature of mini-split SUSY, not a bug. Interestingly, the strongest constraint on mini-split models may come from the theoretical challenge of obtaining the correct SM vacuum structure.\(^7\) The masses of the various superpartners feed into the RG equations for the SUSY mass spectrum evaluated at low energies, and in particular, the light gluino does not protect top squarks from running tachyonic under RG flow. This can often lead to unacceptable color- and charge-breaking vacua [178, 44]. This problem is exacerbated by two-loop RG effects if the first- and second-generation squarks are split from the third [42, 16]. Finally, any complete model of mini-split must also generate appropriate Higgs sector soft terms $m_{H_u}^2$, $m_{H_d}^2$, and most acutely $B_\mu$, the non-holomorphic Higgs mass term. Of course, mini-split models always include some degree of fine-tuning of parameters to get the correct vacuum, but even to begin fine-tuning, the Higgs soft terms must be at least “in the ballpark,”

\(^6\)For other models realizing a similar spectrum, see Refs. [163, 36, 55, 177, 158, 224, 110, 246].

\(^7\)Of course, there are also constraints if one chooses to require a suitable dark matter candidate with the correct relic density.
which in this context means a value of $\sqrt{B_\mu}$ close to the scalar mass scale. Thus, mini-split model building is not as simple as “heavy sfermions, light gauginos,” since one must also ensure the consistency of the Higgs sector.

A promising general framework for generating a mini-split spectrum is gauge mediation [102, 233, 24, 103, 105, 104]. There, SUSY-breaking is communicated to the Standard Model through loops of “messenger” fields which carry SM gauge charges.\(^8\) One can try to arrange for gauginos to acquire masses at a higher loop level than squarks, achieving the desired separation for a mini-split spectrum. However, in the standard gauge mediation paradigm, squarks and gauginos get mass at parametrically the same order. In Chapter 3, I construct a model of mini-split through gauge mediation where the messengers are charged under gauged anomaly-free global symmetries of the SM (which we refer to collectively as the “auxiliary group”), rather than the SM gauge groups. This structure has the advantages of not requiring any extra fields with SM charges, while at the same time naturally providing a U(1)\(_H\) symmetry acting on the Higgs fields which generates the correct Higgs sector SUSY-breaking terms. This model can therefore reproduce the correct SM vacuum without any fine-tuning in model space.

\subsection*{1.3 Halo-independent methods for dark matter}

Despite the large body of evidence in favor of dark matter, many crucial properties of DM remain unknown. Its mass could lie anywhere from sub-eV to $10^{13}$ GeV, with many well-motivated candidates located at all mass scales. Its non-gravitational interactions with visible matter could be elastic or inelastic, proceed through a light or heavy mediator, or might be nonexistent. In analogy to the complexity of the Standard Model in the visible sector, there could be an entire dark sector with multiple states, gauge groups, self-interactions, and decays. Finally, we have no direct measurements of the local DM velocity distribution in our own galactic halo. A common

\footnote{This additional sector is necessary to evade the supertrace sum rule, which would force the superpartner mass spectrum to bracket the SM spectrum from above and below, rather than raising all superpartner masses as is empirically required.}
assumption is a Maxwellian distribution, but N-body simulations suggest deviations from this distribution [203], which can have a strong impact on direct detection experiments.

Given our uncertainties about the properties of dark matter, it is advantageous to develop experiments and analysis techniques which make as few assumptions as possible about these properties. Specifying to direct detection experiments, which search for DM scattering off nuclei, the differential event rate for spin-independent scattering as a function of nuclear recoil energy is

$$\frac{dR}{dE_R} = \frac{N_A \rho_X \sigma_n m_n}{2m_X \mu_{nX}^2} C_T^2(A, Z) \int dE_R' G(E_R, E_R') \epsilon(E_R') F^2(E_R') \int_{v_{\text{min}}(E_R')}^{\infty} \frac{f(v + v_E)}{v} d^3v. \quad (1.7)$$

This expression contains input from the DM model (density $\rho_X$, nuclear cross section $\sigma_n$, masses $m_X$ and $\mu_{nX}$), the detector properties (target-dependent coherent scattering enhancement $C_T^2(A, Z)$, detector resolution function $G(E_R, E_R')$, and detector efficiency $\epsilon(E_R')$), nuclear physics (nuclear form factor $F^2(E_R')$), and the halo model (DM velocity distribution $f(v + v_E)$, where $v_E$ is the velocity of the Earth). In addition, the lower limit $v_{\text{min}}(E_R')$ of the halo integral, which is the minimum dark matter velocity required to provoke a nuclear recoil $E_R'$, depends on the kinematics of the DM model. The traditional method for analyzing direct detection experiments is to choose a dark matter model and a halo model (for example, a Maxwellian velocity distribution), and present exclusion limits or preferred regions in $m_X - \sigma_n$ space. However, an alternate, “halo-independent” analysis [137, 136] is possible: rather than choosing a halo model, one can simply change variables and present exclusion limits or preferred regions in $v_{\text{min}} - g(v_{\text{min}})$ space, where

$$g(v_{\text{min}}) = \int_{v_{\text{min}}}^{\infty} \frac{f(v + v_E)}{v} d^3v \quad (1.8)$$

is the halo integral written as a function of its lower limit. This requires no assumptions about the DM halo, and makes it easy to compare multiple experiments, because two experiments with different nuclear targets may have overlapping ranges of $v_{\text{min}}$. 

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even if they have non-overlapping ranges of $E_R$.

The present null results from direct detection experiments such as XENON100 [35] and LUX [20] suggest that an emerging dark matter signal will likely consist of only a handful of events, and thus it is advantageous to retain as much information about each event as possible. Direct detection experiments have extremely low backgrounds and excellent energy resolution, so one should avoid binning the data and instead use unbinned analysis techniques. In Chapter 4, I will extend the methods of [137] to unbinned data, and prove that the method still works even in the presence of finite energy resolution. In forthcoming work, I show that halo-independent analyses can be carried out without a fiducial choice of DM mass, allowing strong conclusions to be drawn from experimental data in a halo- and mass-independent way. I also generalize the unbinned halo-independent techniques to inelastic kinematics, relaxing the requirement of a monotonic $v_{\text{min}}(E_R)$ function.

1.4 Other research directions

The body of research I performed while at MIT includes other directions not directly related to the theme of forces and gauge groups beyond the SM. Some of this work is briefly outlined here.

1.4.1 Locality from the IR perspective

Locality is a fundamental guiding principle in the construction of quantum field theories to describe physical systems. It appears in many different guises, from the causal structure of Lorentz-invariant theories to the analyticity of the $S$-matrix. Theories with compact dimensions offer an interesting context in which to think about locality, since for a low-energy observer, locality in the compact dimensions is qualitatively different from locality in the noncompact ones. From an ultraviolet (UV) or top-down perspective, various mechanisms exist to ensure compact locality. In the usual picture of dimensional reduction, locality in the UV is assumed, and interactions in the compact dimensions remain local after geometric compactification. In models of dimen-
sional deconstruction [37], a UV-complete four-dimensional gauge theory condenses at low energies to yield a theory with a compact fifth dimension, and five-dimensional locality is ensured by the irrelevance of nonlocal operators before condensation. A deeper mechanism exists in the AdS/CFT correspondence [215, 160, 264], where bulk locality emerges from the large-$N$ limit of the boundary CFT [167, 168, 130, 254, 129].

From an infrared (IR) or bottom-up perspective, however, compact locality is baffling. In the far IR, a compact dimension can be described by a tower of Kaluza-Klein (KK) modes, and locality simply enforces certain constraints on the spectrum and interactions of these modes. If there are spin-1 degrees of freedom, there is a cutoff scale $\Lambda$ where longitudinal scattering of the massive spin-1 KK modes becomes strongly coupled. From an IR point of view, there is no apparent reason to exclude additional nonlocal interactions, and one might even expect nonlocal terms could render the theory better behaved in the IR. Indeed, in the local case, it is precisely the interactions among different KK levels which partially unitarize KK scattering, pushing $\Lambda$ above the naive expectation from considering the KK modes as independent massive vectors. It is therefore plausible that including nonlocal interactions with the correct sign could yield a similar interference effect, possibly driving the cutoff scale $\Lambda$ higher than in the local case.

In Ref. [188], I presented a system where precisely the opposite is true: insisting on the highest possible cutoff scale $\Lambda$ implies locality in the compact dimension. We studied the case of a deconstructed five-dimensional SU(2) Yang-Mills theory in a flat geometry, described by a “theory space” cyclic moose diagram as in Fig. 1-2 (left), perturbed by nonlocal interactions (right). This four-dimensional theory has an intrinsic cutoff scale $\Lambda$, and maximizing $\Lambda$ is correlated with locality in theory space. This gives a purely low-energy perspective on why compact locality is special, in the sense that local theories are the most weakly coupled in the IR. Strictly speaking, our analysis only holds for small nonlocal perturbations, and we cannot exclude the possibility that large nonlocal terms could lead to a larger value of $\Lambda$. While unitarity violation in higher-dimensional gauge theories has been investigated before [80, 79, 94], to our knowledge the only studies of extra-dimensional nonlocality had been in a
Figure 1-2: **Left:** Local cyclic $N$-site SU($n$) moose diagram, known also as “theory space,” corresponding to the latticization of compactified five-dimensional Yang-Mills. The link fields $\Sigma$ transform as bifundamentals of the gauge groups, represented by shaded circles at either end of the link. When each link field acquires a vacuum expectation value, the moose describes $N - 1$ interacting massive SU($n$) gauge bosons, one massless SU($n$) gauge boson, and one Goldstone winding mode. **Right:** A graphical representation of nonlocal terms, showing an $N = 6$ cyclic moose with nonlocal terms of nonlocal length scale $N_{\text{hop}} = 1$ (red long-dashed lines) and $N_{\text{hop}} = 2$ (blue short-dashed lines).

gravitational context [248],\(^9\) and our work was the first to investigate nonlocality in pure gauge theory.\(^{10}\)

### 1.4.2 Inflation and supergravity

Cosmological evidence indicates that the universe is very nearly spatially flat, and homogeneous at the level of $10^{-5}$ on large scales [10]. Both of these facts can be explained if the early universe underwent inflation, a period of approximately exponential expansion [161, 212, 21]. The rapid expansion would drive the curvature to zero, and the homogeneity would be explained by the fact that the observable universe today arose from an extremely small patch before inflation: regions which are out of causal contact today were originally in causal contact, allowing them to equilibrate.

\(^9\)When discretizing gravity, nonlocal interactions are necessary to have a local continuum limit [248], which is not the case for gauge theories.

\(^{10}\)Subsequent work was performed in Ref. [249].
The observed fluctuations in the CMB could then be traced to quantum fluctuations during inflation [232, 78, 162, 166, 253, 52], establishing a deep connection between the physics of the very small (quantum field theory) and the physics of the very large (temperature correlations across the universe).

To make the concept of inflation concrete, let the metric of spacetime take the Friedmann-Robertson-Walker (FRW) form

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2). \quad (1.9)$$

Accelerated expansion occurs if $\ddot{a} > 0$, and if $\ddot{a}$ is small, expansion is approximately exponential with time constant $H = \dot{a}/a$ (the Hubble parameter). Einstein's equations for the metric (1.9) allow for accelerated expansion if the Einstein tensor is sourced by a stress-energy tensor with negative pressure. This is not nearly as exotic as it sounds. The stress-energy tensor of an ordinary real scalar field $\varphi$ (the inflaton) can correspond to negative pressure:

$$p_{\varphi} = \frac{1}{2} \dot{\varphi}^2 - V(\varphi). \quad (1.10)$$

This shows that $p_{\varphi} < 0$ if the dynamics of $\varphi$ is dominated by its potential $V(\varphi)$ rather than its kinetic energy $\frac{1}{2} \dot{\varphi}^2$. To quantify this argument, we can introduce the slow-roll parameter

$$\varepsilon = -\frac{\dot{H}}{H^2}. \quad (1.11)$$

Einstein's equations may be used to relate this combination of Hubble parameters to the stress-energy tensor for $\varphi$, and it can be shown that $\varepsilon < 1$ implies $p_{\varphi} < 0$. If $\varepsilon \ll 1$, the expansion is approximately exponential.

Unlike the topics discussed in the body of this thesis, which can be understood with a fixed background metric and non-dynamical gravity, inflation necessarily involves a dynamical metric. Thus, the supersymmetric version of inflation must be understood in the context of supergravity.\footnote{For a brief review, see Ref. [262] and references therein. For a much more detailed review, see Ref. [141].} Supergavity promotes Lorentz symmetry to a local
symmetry and introduces a corresponding gauge field, the gravitino $\psi_{\mu}$, a spin-3/2 fermion carrying both Lorentz and spinor indices. The gravitino can be understood as the supersymmetric partner of the graviton, and its existence is a generic consequence of a supersymmetric theory of gravity.

In fact, the gravitino is necessary even in the context of spontaneously-broken SUSY without gravity. The fermionic analogue of Goldstone’s theorem states that spontaneously-broken global SUSY implies a massless fermion, known as a goldstino, with couplings to matter determined by the SUSY-breaking structure. As no such fermion has been observed, it must be that the goldstino gets “eaten” by the gauge field of local SUSY [91, 90], in precisely the same way that the problem of massless goldstones in the SM is resolved by the Higgs mechanism for spontaneously-broken local gauge symmetries. Thus, for spontaneously-broken SUSY to be consistent with observations, the goldstino must become the longitudinal spin-1/2 components of the gravitino.

In Ref. [186], I argue that an EFT of inflation can be constructed containing only the inflaton and the gravitino. Both SUSY and supergravity typically augment the particle spectrum with a plethora of new states, but I show that most of these can be consistently decoupled, leaving only the irreducible ingredients common to any model of supersymmetric inflation. Because the gravitino contains the goldstino, its equations of motion contain information about spontaneous SUSY breaking. In the minimal supersymmetric EFT of inflation, the inflaton stress-energy tensor breaks SUSY due to both its large potential energy and the explicit time-dependence inherent in $\dot{H}$, and these effects are communicated to the gravitino through an unusual Lorentz-violating dispersion relation for its spin-1/2 components. Finally, the minimal EFT contains the leading interactions of the inflaton and goldstino/gravitino, allowing one to make predictions for inflationary observables which could in principle diagnose the presence of SUSY during inflation.
1.5 A perspective on beyond-the-Standard-Model physics

At the present time, the positive data we have for physics beyond the Standard Model (including the discovery of the Higgs boson, and the gravitational evidence for dark matter) do not point in an obvious direction to look for new physics. This is in distinct contrast to the situation with previous discoveries, where (for example) Gell-Mann’s Eightfold Way [148] predicted the $\Omega^{-}$, the Glashow-Iliopoulos-Maiani mechanism [152] predicted the charm quark, and the need for a CP-violating phase led Kobayashi and Maskawa to predict a third generation of quarks [198]. In each of these cases, the properties of the hypothetical new particles, including their masses, charges, and spins, were sharply predicted. The problem of the Higgs boson mass offered the prospect of a similar resolution, predicting weak-scale particles with specific couplings to the Higgs to cancel the quadratic divergences. Unfortunately, the most straightforward realizations of this scenario have been unambiguously ruled out by experiments. Even more frustratingly, the basic properties of dark matter – its mass, spin, and gauge charges – are completely unknown.

Given this situation, one may choose various paths. The path of model-building aims to connect concrete models with experimental data as directly as possible. The path of model-independence aims to derive general properties and consistency conditions which can be useful no matter what the correct theory turns out to be. In the first part of this thesis (Chapters 2 and 3), I will focus on concrete models of both dark matter and supersymmetry. In Chapter 4, on the other hand, I will discuss a model-independent approach to dark matter direct detection, where the “model” from which we are striving for independence describes dark matter self-interactions, which can modify the velocity distribution of the dark matter halo in our galaxy. I strongly believe that a combination of these approaches will allow maximal use of the limited hints for new physics that we have, without violating Sherlock Holmes’s maxim, “It is a capital mistake to theorize before one has data.”

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12I am indebted to Wally Melnitchouk for making me aware of this excellent quotation.
Chapter 2

DAEδALUS and Dark Matter Detection

Most realistic DM scenarios predict some kind of non-gravitational interactions between DM and ordinary matter. One ubiquitous prediction is that DM should have non-zero scattering cross sections off nuclei, which is the mechanism by which direct detection experiments search for DM in the galactic halo [154, 209]. DM can also be produced in laboratory experiments, either at high energies at machines like the Large Hadron Collider [45], or at low energies through bremsstrahlung or rare hadron decays (see Ref. [112] for a review). This low energy mode has been exploited to use fixed-target neutrino experiments such as LSND [46] and MiniBooNE [19] as production and detection experiments for sub-GeV DM [57, 97, 96], and it has been recently proposed to use the main injector beam at Fermilab paired with the NOνA detector [50] to search for GeV-scale DM [108].1 A similar logic applies to electron beam fixed-target experiments [181, 182, 99].

In this chapter, we propose to use the DAEδALUS neutrino experiment [11] in close proximity to a large-volume neutrino detector such as the proposed LENA detector [268].2 DAEδALUS uses cyclotrons (peak power 8 MW, average power 1–2

1As of this writing, MiniBooNE is currently analyzing data taken in off-target mode for a dark sector search. The expanded off-shell reach we discuss in this chapter could have important consequences for this search.

2The study in Ref. [180] also considers an underground accelerator paired with a large neutrino
MW) to produce a high-intensity 800 MeV proton beam incident on a graphite and copper target (1 m of graphite liner inside a 3.75 m copper beam stop), creating a decay-at-rest neutrino source from stopped charged pions. Proton-carbon scattering is also a rich source of neutral pions, and in scenarios involving a light weakly-coupled dark sector, rare $\pi^0$ decays to an on-shell dark mediator $A'$ can produce pairs of DM particles $\chi\overline{\chi}$ when $2m_\chi < m_{\pi^0}$. These DM particles can then be detected through neutral-current-like scattering in detectors designed to observe neutrinos, as illustrated in Figs. 2-1 and 2-2. A similar setup was the basis for existing LSND bounds on light DM [57, 97], but we find that for light $\chi$, DAE\delta\text{ALUS} can improve the reach of LSND by an order of magnitude in the visible-dark sector coupling $c^2$ after only one year of running. This DM search is therefore an important physics opportunity for the initial single-cyclotron phase of DAE\delta\text{ALUS}.

We also find that both DAE\delta\text{ALUS} and LSND are sensitive to DM production through off-shell mediators in two distinct regimes, a fact that has been overlooked in the literature. Surprisingly, in the lower regime ($m_{A'} < 2m_\chi$), sensitivity to an off-shell $A'$ can be superior compared to a heavier on-shell $A'$. In the upper regime ($m_{A'} > m_{\pi^0}$), existing LSND limits are considerably stronger than previously reported, and the DAE\delta\text{ALUS} sensitivity limits can extend up to $m_{A'} \simeq 800$ MeV rather than cutting off at $m_{A'} \simeq m_{\pi^0}$. Indeed, the observation that DM produced from meson decays can probe $A'$ masses much heavier than the meson mass expands the sensitivity of the entire experimental program to discover DM in proton-beam fixed-target searches. As Figs. 2-3 and 2-4 illustrate, the combination of updated LSND bounds and projected DAE\delta\text{ALUS} sensitivity covers a broad range of DM and mediator masses, and is even competitive with searches for visibly-decaying mediators in certain regions of parameter space.

The search strategies for MeV-scale DM at both DAE\delta\text{ALUS} and LSND are very similar, so it is worth pointing out the potential advantages of DAE\delta\text{ALUS} compared to LSND:

- **Higher energy range.** The LSND $\nu_e - e^-$ elastic scattering measurement detector to search for light scalars of relevance to the proton radius puzzle.
[49], which has been used to set limits on light DM, focused on the recoil electron energy range $E_e \in [18, 52]$ MeV, where a Čerenkov detector can use directionality to discriminate against decay-at-rest neutrino backgrounds. This strategy is optimal for a heavier DM search ($m_X > 40$ MeV) where the kinetic energy available for scattering is smaller. Here, we propose a search with DAEδALUS/LENA in the higher energy range $E_e \in [106, 400]$ MeV, well above the thresholds from decay-at-rest backgrounds, which is optimal for lighter DM ($m_X \lesssim 20$ MeV). The specialized target at DAEδALUS, designed to reduce the decay-in-flight component of the neutrino beam, makes such a high-energy search possible by reducing decay-in-flight backgrounds.

- **Higher luminosity.** A single DAEδALUS cyclotron with a 25% duty cycle and peak power 8 MW can deliver $4.9 \times 10^{23}$ protons on target per year, producing $7.5 \times 10^{22} \pi^0$ per year, compared to $10^{22} \pi^0$ over the life of the LSND experiment.

- **Larger acceptance.** At LSND, the source was placed a distance of 30 m from the neutrino detector, whereas the DAEδALUS source can be placed as close as 20 m to the detector, increasing the angular acceptance for DM scattering. In addition, the detector length of LSND was 8.3 m, whereas DAEδALUS can be paired with a large neutrino detector like LENA in a geometry where the average path length through the detector is closer to 21 m, and the maximum path length is over 100 m.

Because we consider a dedicated DM search with DAEδALUS, we will optimize our cuts for each point in the dark sector parameter space. We will show that under conservative assumptions, a light DM search at DAEδALUS/LENA is systematics dominated. In particular, the improvements compared to LSND come almost exclusively from the optimized cuts rather than the higher luminosity and larger acceptance, though that conclusion could change with relatively modest improvements to the systematic uncertainties of neutrino-nucleon scattering cross sections.

3In principle, LSND could have done such a high-energy search as well. It may be possible to derive stronger limits than those from the LSND electron scattering measurement by using LSND's measurement of $\nu_e C \rightarrow e^- X$ at 60–200 MeV [47].
Figure 2-1: **Left (a):** schematic diagram of DM production in proton-carbon collisions, through on- or off-shell dark photons $A'$ from exotic $\pi^0$ decays. **Right (b):** DM scattering at a detector through the same dark photon $A'$. We focus on electron scattering in this chapter, but the detector target may be protons or nuclei in alternative experimental setups.

The full DAEδALUS program [86] includes multiple cyclotron-based neutrino sources placed at three different distances from a single detector such as LENA. Because the earliest phase of DAEδALUS involves just a single “near” cyclotron-based neutrino source, we focus on pairing this neutrino source with a neutrino detector to perform a dedicated DM search. For studies of other physics opportunities with a near cyclotron, see Refs. [27, 15, 29].

To directly compare to previous studies, we will focus on vector portal models of the dark sector as described in the Introduction, with special attention to $m_\chi$, an essential third dimension of parameter space that introduces qualitatively different phenomenology. We focus primarily on the region of parameter space $\alpha_D \gg e^2 \alpha_{EM}$, though we do look at a wider range of $\alpha_D$ values in Fig. 2-4.5

As shown in Fig. 2-1, DM can be produced and detected via

$$\pi^0 \rightarrow \gamma A'_{(*)} \rightarrow \gamma \chi \chi, \tag{2.1}$$

$$\chi e^- \rightarrow \chi e^-, \tag{2.2}$$

---

*4One could also pair DAEδALUS with the proposed JUNO [210], Hyper-K [6], or water-based liquid scintillator [23] detectors. While it may be possible to use an existing neutrino detector such as NOνA, beam-off backgrounds for an above-ground detector appear prohibitive.

*5Changing $\alpha_D$ results in a simple linear scaling of the sensitivity when the DM is produced via an on-shell $A'$, and a quadratic scaling when the DM is produced via an off-shell $A'$. We discuss scaling with $\alpha_D$ in Sec. 2.5.
where the $A'$ can either be on- or off-shell in the production process, and the scattering process proceeds through a $t$-channel $A'$. The main detection backgrounds come from neutrinos, either elastic scattering off electrons or charged-current quasi-elastic (CCQE) scattering off nucleons, but because the spectra of neutrinos produced from decays at rest have sharp kinematic cutoffs, much of the neutrino background can be mitigated by a simple cut on the electron recoil energy in the detector.

The rest of this chapter is organized as follows. In Sec. 2.1, we describe the mechanism of DM production at the DAE6ALUS source, for both on- and off-shell mediators. We describe the mechanism and signals of DM scattering at the LENA detector in Sec. 2.2, and we survey the backgrounds to such a search in Secs. 2.3 and 2.4. In Sec. 2.5, we discuss the sensitivity of DAE6ALUS/LENA to DM production in various regions of parameter space, and compare with re-evaluated bounds from LSND and limits from searches for $A' \rightarrow e^+e^-$. We conclude in Sec. 2.6. Details of the various production and scattering calculations can be found in Appendix A. The work described in this chapter was undertaken in collaboration with Jesse Thaler.

---

6Since $\chi$ and $\bar{\chi}$ are indistinguishable in the detector, we only write $\chi$ for simplicity.
Figure 2-3: Summary of DAEδALUS/LENA 3σ sensitivity to the kinetic mixing parameter $\epsilon^2$ assuming the on-axis configuration (see Fig. 2-2c) and a full year of run-time with $7.5 \times 10^{22} \pi^0$ produced. **Left column:** DAEδALUS sensitivity as a function of $m_{A'}$ for various DM masses. **Right column:** DAEδALUS sensitivity as a function of $m_X$ for various $A'$ masses. The thick green band is the region where $A'$ could resolve the long-standing $(g - 2)_\mu$ anomaly to within $\pm 2\sigma$ [240]; see Sec. 2.5 for information about the other projected sensitivities and constraints. Where applicable, the dashed vertical black line marks the transition between the on- and off-shell $A'$ regimes for $\pi^0 \rightarrow \gamma A'^{(\pm)} \rightarrow \gamma \chi \bar{\chi}$. In the lower off-shell regime we emphasize that the LSND and DAEδALUS limits assume the existence of the off-shell process $A'^{(\pm)} \rightarrow \chi \bar{\chi}$.
Figure 2-4: Parameter space for the dark photon mass $m_{A'}$ and dark coupling $\alpha_D$, taking $\epsilon$ to be the smallest value which resolves the $(g - 2)_\mu$ anomaly for $m_\chi = 1$ MeV. The DAEå·ALUS/LENA curve shows $3\sigma$ sensitivity. The solid black curve is the boundary where $\text{Br}(A' \rightarrow e^+e^-) = \text{Br}(A' \rightarrow \bar{\chi}\chi) = 50\%$. Note that for $\text{Br}(A' \rightarrow e^+e^-) \sim 100\%$ (just below the black curve) recent results from NA48/2 [85] have ruled out the remaining parameter space for a visibly decaying $A'$ that explains the discrepancy.
Gordan Krnjaic, and Matthew Toups, with special thanks to Adam Anderson, Brian Batell, Janet Conrad, Rouven Essig, Joseph Formaggio, Eder Izaguirre, Patrick de Nieverville, Maxim Pospelov, Philip Schuster, Joshua Spitz, and Natalia Toro for many helpful conversations. It is largely based on [184].

2.1 Dark matter production at DAEδALUS

As mentioned in Sec. 1.1, production of dark photons $A'$ can be achieved by replacing a photon with an $A'$ in any kinematically-allowed process. At the 800 MeV proton kinetic energies of the DAEδALUS beam, photons come primarily from $\pi^0$ decays, where the pions are produced mostly from $\Delta$ resonances:

$$\Delta^+ \rightarrow p + \pi^0, \quad \Delta^0 \rightarrow n + \pi^0.$$  

(2.3)

$A'$s can also be produced directly from radiative $\Delta$ decays, $\Delta \rightarrow N + A'$, where $N$ is a proton or neutron. The branching ratio for $\Delta \rightarrow N + \gamma$ is approximately 0.5%, and so is subdominant to $A'$ production from pion decays, except in the range $m_{\pi^0} < m_{A'} < m_{\Delta} - m_N$ where the $A'$ is on-shell from $\Delta$ decay but off-shell from $\pi^0$ decay. Lacking a reliable way to simulate $\Delta$ production and decay, we neglect this signal mode in our analysis, though we estimate that it may improve signal yield by as much as a factor of 2 over the range $m_{A'} \in [135, 292]$ MeV.\textsuperscript{7} Other sources of photons are expected to be negligible for our sensitivity estimates: $\rho$ and $\eta$ mesons are kinematically inaccessible, and bremsstrahlung photons produced in the hadronic shower are suppressed by $\alpha_{EM}$, $m_p$, and phase space factors, making them subdominant to photons from $\Delta$ decays. Consequently, we will focus on DM production through $\pi^0 \rightarrow \gamma A'(\ast) \rightarrow \gamma \chi \overline{\chi}$, where the $A'$ can be either on- or off-shell depending on the masses of the DM and the $A'$.

We simulated DM production by obtaining a list of $\pi^0$ events from GEANT 4.9.3 [18] with a simplified model of the DAEδALUS target geometry, and generated the

\textsuperscript{7}We thank Rouven Essig for pointing out the importance of on-shell $A'$ production from $\Delta$ decays.
DM kinematics by decaying the pions as predicted by the dark photon model; details are given below and in App. A.1.\footnote{We used the “QGSP.BIC” physics list in GEANT4.} Previous studies [57, 97] have assumed that the \( \pi^0 \) energy spectrum from proton-carbon collisions is similar to the \( \pi^+ \) spectrum, and used fits to \( \pi^+ \) data [70] to model the \( \pi^0 \) production. We find reasonable agreement with this assumption based on the GEANT simulation, though the spectra of \( \pi^+ \) versus \( \pi^0 \) differ considerably at high energies. Similarly, in previous studies, the total \( \pi^+ \) production rate was estimated by working backwards from the observed neutrino flux within the detector acceptance, and assuming that all neutrinos came from \( \pi^+ \) decays at rest; the \( \pi^0 \) total rate was assumed to be equal to the \( \pi^+ \) rate up to a factor of 2 uncertainty [97]. In our approach, the same GEANT simulation can simulate both \( \pi^0 \) and \( \pi^+ \) production, allowing an estimate of the \( \pi^0 \) rate which does not rely on such assumptions about the \( \pi^+ \) rate.

If \( 2m_\chi < m_{A'} < m_{\pi^0} \), the \( A' \) can be produced on-shell and decay to DM. The narrow width approximation [247] can be used to obtain a simple expression for the branching ratio,

\[
\text{Br}(\pi^0 \to \gamma \chi \bar{\chi}) = \text{Br}(\pi^0 \to \gamma \gamma) \times 2 \epsilon^2 \left( 1 - \frac{m_{A'}^2}{m_{\pi^0}^2} \right)^3 \times \text{Br}(A' \to \chi \bar{\chi}) \quad \text{(on-shell).} \tag{2.4}
\]

In the region of parameter space where \( \alpha_D \gg \epsilon^2 \alpha_{\text{EM}} \), \( \text{Br}(A' \to \chi \bar{\chi}) \approx 1 \). Then \( \text{Br}(\pi^0 \to \gamma \chi \bar{\chi}) \) is independent of \( m_\chi \) and \( \alpha_D \) and depends only on the \( A' \) mass and the kinetic mixing parameter \( \epsilon \). Since the kinematics of two-body decays are fixed by energy-momentum conservation, the double-differential angular and energy distribution \( d^2N_\chi/(d\Omega dE_\chi) \) (summed over the DM polarizations and the unobserved photon polarizations) of the DM is also independent of \( m_\chi \), and is inherited directly from the analogous distribution of the \( A' \)'s, which is, in turn, inherited from the parent pions. However, we caution that the narrow-width approximation breaks down if \( m_{A'} \) is sufficiently close to \( m_{\pi^0} \) from below [62, 194, 257]. In particular, there is no sharp kinematic threshold at \( m_{\pi^0} \).

If \( m_{A'} < 2m_\chi \) or \( m_{A'}^2 \gtrsim m_{\pi^0}^2 - 2\Gamma_{A'} m_{A'} \), the narrow-width approximation is not
applicable, and DM is produced through a three-body decay.\(^9\) Details of our treatment of the narrow width approximation are given in Appendix A, Secs. A.1.4 and A.1.5. The expression for the branching ratio involves a phase-space integral which cannot be computed analytically,

\[
\text{Br}(\pi^0 \to \gamma\chi\bar{\chi}) = \frac{1}{\Gamma_{\pi^0}} \times \frac{e^2 \alpha_D}{2m_{\pi^0}} \int d\Phi_{\pi^0 \to \gamma A'} d\Phi_{A' \to \gamma \chi \bar{\chi}} \frac{ds}{2\pi} \langle |\hat{A}_{\pi^0 \to \gamma \chi \bar{\chi}}|^2 \rangle \quad \text{(off-shell),} \tag{2.5}
\]

where \(s\) is the mass-squared of the virtual \(A'\), \(\Gamma_{\pi^0} = 7.74\) eV is the total \(\pi^0\) width, and \(\hat{A}_{\pi^0 \to \gamma \chi \bar{\chi}}\) is the three-body decay amplitude normalized to \(\epsilon = \alpha_D = 1\). This normalization was chosen to make the dependence of the branching ratio on \(\epsilon\) and \(\alpha_D\) explicit. In contrast to the on-shell case, the branching ratio now depends on both the dark fine structure constant \(\alpha_D\) and the DM mass \(m_\chi\). Full expressions for the three-body amplitudes for fermionic and scalar \(\chi\), as well as the \(A'\) width, are given in App. A.1. The double-differential distribution \(d^2N_\chi/(d\Omega dE_\chi)\) can be obtained in a straightforward manner from Eq. (2.5) by only performing the first phase space integral, which gives the distribution in the \(\pi^0\) rest frame, then boosting according to the \(\pi^0\) lab-frame distribution.

Putting these pieces together, the total number of DM particles produced at DAE\(\delta\)ALUS is

\[
N_\chi = 2N_{\pi^0} \text{Br}(\pi^0 \to \gamma\chi\bar{\chi}), \tag{2.6}
\]

where our GEANT simulation yields \(N_{\pi^0} = 7.5 \times 10^{22} \pi^0/\text{yr}\), and \(\text{Br}(\pi^0 \to \gamma\chi\bar{\chi})\) is given by Eq. (2.4) for on-shell production and Eq. (2.5) for off-shell production. The maximum energy of DM produced at DAE\(\delta\)ALUS as a function of its mass \(m_\chi\) is

\[
E_{\chi}^{\text{max}} = \frac{1}{2} \gamma_{\text{max}} m_{\pi^0} \left( 1 + \beta_{\text{max}} \sqrt{1 - \frac{4m_\chi^2}{m_{\pi^0}^2}} \right), \tag{2.7}
\]

\(^9\)This illustrates a subtlety of the narrow-width approximation. Although the \(A'\) can go on-shell for \(m_{A'} < m_{\pi^0}\), the phase space suppression means that the phase space integral in Eq. (2.5) is actually dominated by the off-shell region of the amplitude, giving a smooth behavior through the \(\pi^0\) threshold. The effect of near-degeneracies on the efficacy of the narrow width approximation in resonant three-body decays has been previously noted in Ref. [257], where it is shown that phase-space factors distort the shape of the Breit-Wigner and lead to errors parametrically greater than \(\Gamma/M\).
Figure 2-5: Sensitivity contours at DAEδALUS/LENA showing the effect of changing experimental geometries. All curves assume a 3σ signal-to-background sensitivity, see Secs. 2.4 and 2.3. Existing limits from the multi-year data set at LSND [97] are shown for comparison. The signal contours are computed by integrating the electron recoil profile over the interval that maximizes $S/\delta B$ for each value of $m_A$.

where $(\gamma_{\text{max}}, \beta_{\text{max}}) \approx (5, 0.98)$ are the maximum boost and velocity respectively for $\pi^0$'s produced at DAEδALUS.

## 2.2 Dark matter scattering at LENA

The LENA detector [268] is a proposed cylindrical scintillator detector with a target volume of radius 13 m and height 100 m; we assume the target volume is filled with linear-alkyl-benzene ($C_{18}H_{30}$), giving a fiducial mass of 45.8 kiloton, though other choices of scintillator are under consideration. Dark sector particles produced at the DAEδALUS target can travel unimpeded through the surrounding material to scatter in the LENA detector. For low mass mediators, the dominant channel is coherent scattering off detector nuclei, which enjoys an $A^2$ enhancement since small momentum transfers are unable to resolve nuclear substructure. However, this channel suffers from a severe form-factor suppression for momentum transfers in excess of our electron recoil cuts which are necessary to discriminate the signal from the beam-on neutrino.
backgrounds. DM particles can also scatter off atomic electrons in the detector, and it is this \( \chi e^- \rightarrow \chi e^- \) channel which we will focus on, though the discussion below can be adapted to a generic detector target.\(^{10}\)

The total scattering yield for the electron channel is

\[
N_{\text{sig}} = n_e \int_{E_{\text{low}}(m_\chi)}^{E_{\text{high}}(m_\chi)} dE_e \int_{E_{\chi}^{\text{min}}(E_e)}^{E_{\chi}^{\text{max}}(E_e)} dE_\chi \int_{\text{LENA}} d\Omega \ell(\Omega) \frac{d^2 N_\chi}{d\Omega dE_\chi dE_e},
\]

(2.8)

where \( n_e = 3.0 \times 10^{23}/\text{cm}^3 \) is the number density of target electrons, \( \ell(\Omega) \) is the DM path length through LENA, \( d\sigma/dE_e \) is the recoil electron energy distribution, and the angular integral is taken over the region covered by the LENA detector for the chosen geometry. \( E_{\text{low}}(m_\chi) \) and \( E_{\text{high}}(m_\chi) \) are electron recoil energy cuts which are chosen for each \( m_\chi \) to optimize signal-to-background sensitivity for that mass point; we discuss these cuts further in Sec. 2.4. In principle, we should also include a factor accounting for any muon veto dead time or reconstruction efficiencies, but we neglect these here. The minimum incoming energy for \( \chi \) to induce an electron recoil of energy \( E_e \) is

\[
E_{\chi}^{\text{min}}(E_e) = \frac{T_e}{2} \left[ 1 + \sqrt{\left(1 + \frac{2m_e}{T_e}\right) \left(1 + \frac{2m_\chi^2}{m_e T_e}\right)} \right], \quad T_e \equiv E_e - m_e,
\]

(2.9)

where \( m_e \) is the electron mass and \( T_e \) is the electron kinetic energy. Another useful expression is the maximum possible recoil electron energy for a given DM mass,

\[
E_{e}^{\text{max}}(m_\chi) = m_e + \frac{2(E^{\text{max}}_\chi)^2 - 2m_\chi^2}{2E^{\text{max}}_\chi m_e + m_\chi^2 + m_e^2},
\]

(2.10)

where \( E^{\text{max}}_\chi \) is given in Eq. (2.7). In App. A.2, we present the details of our numerical signal rate computation, including cross sections for scalar and fermion DM particles scattering off a generic target.

\(^{10}\)If there are mass splittings in the dark sector and the \( A' \) coupling is off-diagonal between mass eigenstates, scattering inside the detector will be inelastic and may feature striking de-excitation signals that are not easily mimicked by neutrino or cosmic backgrounds [182]. Although this scenario is beyond the scope of this work, we note that the experimental setups discussed in this work should have promising discovery potential for these signals as well, and in App. A.2 we derive cross sections appropriate to this more general case.
In terms of geometry, we consider three possible locations for DAE6ALUS relative to LENA, shown in Fig. 2-2:

- **midpoint**—pointed horizontally at the vertical midpoint of the detector, 16 m away from the cylindrical face;
- **oblique**—pointed horizontally near the upper corner of the detector, at a lateral distance 16 m and height 5 m;
- **on-axis**—pointed downwards into the endcap of the detector, 16 m above the top face.

The LENA design is self-shielding and includes a 2 m buffer and 2 m muon veto between the outer face and the target volume, so the effective source-detector distance in all three cases is at least 20 m. The signal yield for a 1 MeV DM particle for the three proposed geometries is shown in Fig. 2-5. The choice of geometry only affects the sensitivity in $\epsilon^2$ by a factor of order 10%. The midpoint and on-axis geometries are essentially identical, and provide superior sensitivity compared to the oblique geometry for the entire range of $A'$ masses; the effective detector length and solid angle acceptance are larger for these geometries, and because the signal and background angular distributions are so similar after energy cuts are imposed (see Fig. 2-6a and the discussion below), no additional signal/background separation is achieved in the oblique configuration. For simplicity, we will focus on the on-axis configuration because it preserves cylindrical symmetry.

In terms of electron energy cuts, we consider three benchmark cuts on $E_e$ based on avoiding various beam-on background thresholds:

- $E_{e}^{\text{low}} = 106$ MeV—Above the low-energy muon capture and stopped pion and muon backgrounds;
- $E_{e}^{\text{low}} = 147$ MeV—Above the energy threshold for muon production from beam-on sources;
- $E_{e}^{\text{low}} = 250$ MeV—Above the dominant decay-in-flight neutrino-electron scattering background.
### Table 2.1: One-year rates for all beam-off backgrounds resulting in an outgoing lepton $\ell = e, \mu$ with kinetic energy $T_\ell > 106$ MeV in the final state. “Elastic” refers to elastic neutrino-electron scattering, and “CCQE” refers to charged-current quasi-elastic neutrino-nucleon scattering. A cut $\cos \theta_\ell > 0.9$ has been imposed on all outgoing charged leptons.

<table>
<thead>
<tr>
<th>Source</th>
<th>Flavor</th>
<th>Reaction</th>
<th>106–147 MeV</th>
<th>147–250 MeV</th>
<th>250–400 MeV</th>
<th>Tag</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atm.</td>
<td>$\nu_\mu$</td>
<td>elastic</td>
<td>$&lt; 1$</td>
<td>$&lt; 1$</td>
<td>$&lt; 1$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CCQE</td>
<td>6</td>
<td>13</td>
<td>12</td>
<td>Michel</td>
</tr>
<tr>
<td></td>
<td>$\nu_e$</td>
<td>elastic</td>
<td>$&lt; 1$</td>
<td>$&lt; 1$</td>
<td>$&lt; 1$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CCQE</td>
<td>3</td>
<td>9</td>
<td>9</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$\bar{\nu}_\mu$</td>
<td>elastic</td>
<td>$&lt; 1$</td>
<td>$&lt; 1$</td>
<td>$&lt; 1$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CCQE</td>
<td>2</td>
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<td>elastic</td>
<td>$&lt; 1$</td>
<td>$&lt; 1$</td>
<td>$&lt; 1$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CCQE</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>neutron</td>
</tr>
</tbody>
</table>

Roughly speaking, the 106 MeV cut is optimal for heavy DM, the 147 MeV cut is optimal for medium-mass DM, and the 250 MeV cut is optimal for light DM. This can be seen from Eq. (2.10): for example, $m_\chi = 42$ MeV implies $E_\ell^\text{max} = 146$ MeV, so the lowest of the energy thresholds (with all its additional backgrounds) is necessary to retain any signal acceptance at all. We give more details justifying these cuts in Sec. 2.4 below, and discuss how to optimize them based on the various background spectra.

### 2.3 Beam-off backgrounds

The signal process $\chi e^- \rightarrow \chi e^-$ faces backgrounds from any process which results in an energetic lepton in the final state. There are two main sources of backgrounds, beam-off and beam-on. The principal advantage of using an underground detector such as LENA is the reduction in beam-off backgrounds from sources other than neutrinos. The target depth of LENA is approximately 4000 m.w.e. with a cosmic muon flux of $\sim 1 \times 10^{-4} \text{m}^{-2}\text{s}^{-1}$. Therefore, external backgrounds related to untagged cosmic muons interacting in the rock surrounding the detector are expected to be negligible in our energy range of interest, $E > 106$ MeV. Consequently, we focus only on backgrounds involving neutrinos. Elastic neutrino-electron scattering from
atmospheric neutrinos of any flavor,
\[ \nu e^- \rightarrow \nu e^-, \quad (2.11) \]
poses an irreducible beam-off background since it has the same final state as the signal process. However, there is an additional type of background from charged-current quasi-elastic (CCQE) scattering of neutrinos,
\[ \nu \ell n \rightarrow \ell^- p, \quad \bar{\nu} \ell p \rightarrow \ell^+ n. \quad (2.12) \]
Despite the fact that this event has a completely different final state from the signal process (with for example hadronic activity in addition to the lepton), for \( \nu_e \) this process is an irreducible background at LENA because the energy from the vertex activity cannot be separated from the energy of the produced electron.\(^{11}\) For all other neutrino flavors, this process is at least partially reducible, by detecting the Michel electron from the muon decay for \( \ell = \mu^\pm \), and by tagging the neutron for \( \ell = e^+ \) when the CCQE reaction takes place on hydrogen. However, since the duty cycle of the DAEnALUS cyclotron is only 25\%, all of these backgrounds can be measured directly during beam-off time and then scaled to the beam-on time with a systematic uncertainty of \( \sqrt{3B}/3 \). This is combined in quadrature with the statistical uncertainty \( \sqrt{B} \) on the background during beam-on time, giving a total background uncertainty which scales as \( \delta B = \sqrt{4B}/3 \).

The spectrum of atmospheric neutrinos extends to very high energies, so to reduce the rate of high-energy neutrino scattering feeding down into lower electron recoil energies, we will impose a maximum recoil energy \( E_{\ell}^{\text{max}} \) for the recoil electron depending on the DM mass (see below). Furthermore, the resultant lepton is produced nearly isotropically, while high-energy electrons from DM scattering are principally scattered in the direction of the initial proton beam, as shown in Fig. 2-6a. By requiring the outgoing lepton to be within 25° of the beamline (\( \cos \theta_{\ell} > 0.9 \)) and exploiting

\(^{11}\)In principle, events with delayed vertex activity such as \( \nu_e ^{12}\text{C} \rightarrow e^- ^{12}\text{N}_{\text{gs}}, ^{12}\text{N}_{\text{gs}} \rightarrow ^{12}\text{C} \beta^+ \) can be tagged, but we do not consider event-by-event rejection of this class of events here.
Table 2.2: One-year rates for all beam-on backgrounds resulting in an outgoing lepton \( \ell = e, \mu \) with kinetic energy \( T_\ell > 106 \) MeV in the final state. “Elastic” refers to elastic neutrino-electron scattering, and “CCQE” refers to charged-current quasi-elastic neutrino-nucleon scattering. A cut \( \cos \theta_\ell > 0.9 \) has been imposed on all outgoing charged leptons. Bolded entries are dominant backgrounds in their respective energy ranges. We expect backgrounds from \( \mu^+ \) decay-in-flight (DIF) to be subdominant; see text for details.

The directional detection capabilities of LENA, we can further reduce the beam-off background while keeping \( \approx 99\% \) of the signal over most of the kinematically-allowed parameter space.\( ^{12} \) The rates for these processes in three benchmark energy ranges of interest are given in Table 2.1; more details of our beam-off estimates are given in App. A.3.1. We note that with these cuts, all the beam-off backgrounds are subdominant to the beam-on backgrounds, which we discuss below.

### 2.4 Beam-on backgrounds

We now consider the possible beam-on backgrounds. By imposing kinematic cuts which select for neutrino energies \( E_\nu > 52.8 \) MeV, we eliminate the large decay-at-rest neutrino background from

\[
\pi^+ \rightarrow \mu^+ \nu_\mu, \quad \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu.
\]  

\( ^{12} \)LENA is able to resolve paths of outgoing electrons with energies above 250 MeV and muons with kinetic energies above 100 MeV to an accuracy of a few degrees [268]. Extending this cut for electrons down to energies of 106 MeV is perhaps optimistic at LENA, but may be possible with a future detector paired with the DAE\$ALUS source.
Figure 2-6: Left (a): Angular distributions for DM production and beam-on neutrinos produced at the DAEδALUS source. The neutrino distribution is roughly isotropic while the signal is strongly peaked in the forward direction ($\cos \theta \approx 1$). The slight excess of neutrino production in backward direction is an artifact of the simplified target geometry used in the simulation; see text for details. Above 106 MeV both the DM and neutrino distributions are strongly peaked in the forward direction; the relative normalizations of the curves with and without the cut show the reduction in signal and background due to this cut alone, though the actual signal is also determined by the geometric acceptance of LENA. For different DM masses, the normalization of the DM distribution changes, but not its shape. Although LENA cannot resolve electron-recoil angles for which $\cos \theta > 0.9$, imposing a stronger angular cut of $\cos \theta > 0.95$ would preserve an order-one fraction of signal events and dramatically reduce both beam-off and beam-on backgrounds discussed in Secs. 2.3 and 2.4. To be conservative, we assume $\cos \theta > 0.9$ for all of our sensitivity projections, but this is a potential avenue for improving new-physics searches in the electron scattering channel. Right (b): Electron energy spectra due to various DM signal points and principal beam-on backgrounds (unstacked histograms) assuming the on-axis DAEδALUS/LENA configuration. The color shaded region under each signal curve represents the signal window that maximizes $S/\delta B$ for each parameter point. The $\nu_\mu$ CCQE distribution shows the residual background after a 70% reduction from vetoing Michel electrons; the remaining muons are mis-identified as electrons in LENA, and their kinetic energy spectrum is shown. The $\nu_e$ CCQE distribution was only simulated above 100 MeV where it begins to dominate. The $\epsilon^2$ values for each signal point are chosen to match the minimum value for which the DAEδALUS/LENA setup has the $3\sigma$ sensitivity displayed in Fig. 2-3.
A further cut at $E_\nu > 70$ MeV eliminates the neutrino background from helicity-suppressed $\pi^+$ decays-at-rest,

$$\pi^+ \rightarrow e^+ \nu_e,$$

which could pose a significant background because of the large number of stopped pions at DAEδALUS. Finally, a cut at $E_\nu > m_\mu \approx 106$ MeV mitigates the neutrino background from muon capture,

$$\mu^- + _{Z}^A N \rightarrow \nu_\mu + _{Z-1}^A N',$$

where $N$ is a nucleus in the DAEδALUS target, either carbon or copper. The rate of muon capture is not well-modeled by our GEANT simulation since the true DAEδALUS target contains copper, and the cross section for $\mu^-$ capture on copper is much higher than on graphite. However, the neutrinos produced from muon capture have a sharp kinematic endpoint at or below the muon mass, and suffer an acceptance penalty because they are produced isotropically, so we expect this background to be negligible above 106 MeV.

The remaining beam-on sources of neutrinos above 106 MeV are all decays-in-flight,

$$\pi^+ \rightarrow \mu^+ \nu_\mu, \quad (2.16)$$
$$\pi^+ \rightarrow e^+ \nu_e, \quad (2.17)$$
$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu, \quad (2.18)$$
$$\pi^- \rightarrow e^- \bar{\nu}_e, \quad (2.19)$$
$$\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e. \quad (2.20)$$

Note the inclusion of the helicity-suppressed pion decay modes to electrons and positrons, which will in fact pose the main backgrounds above 250 MeV. To estimate the beam-on backgrounds, we used the same GEANT simulation which generated our signal events to generate the parent pions and muons, and GENIE [32] to simulate the CCQE processes; details are given in App. A.3.2. The simplified DAEδALUS target
geometry used in this simulation consisted of a single block of graphite with a flat face, whereas the full DAEδALUS design consists of a graphite and copper target with a re-entrant hole. Since the stopping power for copper is greater than for graphite, we expect the decay-in-flight background from this simulation to be an upper limit on the true decay-in-flight background from the DAEδALUS neutrino source. Furthermore, we expect our simulation to over-estimate the number of backscattered pions, since in the full DAEδALUS target design, some pions will stop in target material surrounding the re-entrant hole. That said, since we focus on energies above decay-at-rest neutrino spectrum, these backscattered pions do not pose a background in this analysis.

A few words are in order regarding our treatment of the muon decay-in-flight backgrounds. For the LSND experiment, the $\nu_e$ background from $\mu^+$ decays was of the same order of magnitude as that from $\pi^+$ decays, in the electron recoil range 60–200 MeV [47]. However, at DAEδALUS, we expect the $\nu_e$ background from $\pi^+$ decay to be dominant for a number of reasons. First, a significant number of the decay-in-flight $\mu^+$ at LSND were due to isotope stringers placed in the LAMPF beam upstream of the LSND target, whereas the DAEδALUS target will be optimized to suppress decay-in-flight backgrounds. Second, the spectrum of decay-in-flight $\mu^+$ at DAEδALUS is much softer than the $\pi^+$ spectrum due to the longer muon lifetime and correspondingly larger energy loss in the DAEδALUS target. Third, the daughter neutrinos are less energetic: 52.4 MeV in the muon rest frame, as compared to 70 MeV in the pion rest frame. Therefore we expect this background to be subdominant to the $\pi^+$ decay-in-flight $\nu_e$ CCQE background for energies above 250 MeV, and subdominant to the $\pi^+$ decay-in-flight $\nu_\mu$-electron elastic scattering background between 106 and 250 MeV. We attempted to directly simulate this background with GEANT, but statistics proved prohibitive; we leave a full simulation of this background to more detailed studies.

Exactly as with beam-off backgrounds, beam-on backgrounds consist of both $\nu - e^-$ elastic scattering and CCQE events. Elastic events tend to have the outgoing electron scattered at small angles with respect to the initial neutrino direction when $T_e >
106 MeV, while CCQE events tend to have the lepton (electron or muon) produced more isotropically. As shown in Fig. 2-6a, the DM distribution is strongly peaked in the forward direction, such that much of the signal at large recoil energies will have electrons nearly parallel to the beamline.\textsuperscript{13} Thus beam-on CCQE background events can be mitigated with the same cut on the outgoing charged lepton angle $\theta_l < 25^\circ$ as was used for beam-off events. The uncertainty for beam-on backgrounds is dominated by the systematic uncertainty in the neutrino flux. For each flavor of neutrino, a charged-current (CC) channel is available to measure the flux:

\begin{align}
\nu_\mu \, ^{12}\text{C} \rightarrow \mu^- \, X & \quad \text{(tagged muon),} \\
\nu_e \, ^{12}\text{C} \rightarrow e^- \, ^{12}\text{N}_{\text{gs}} & \quad \text{(tagged } ^{12}\text{N}_{\text{gs}} \text{ beta decay),} \\
\bar{\nu}_\mu \, p \rightarrow \mu^+ \, n & \quad \text{(tagged muon and neutron),} \\
\bar{\nu}_e \, p \rightarrow e^+ \, n & \quad \text{(tagged neutron).} 
\end{align}

There has been a considerable experimental effort to measure these CC cross sections \cite{133}, and recently it was proposed to measure the inclusive CC reaction in Eq. (2.21) with a mono-energetic 236 MeV $\nu_\mu$ beam from kaon decays \cite{252}. When presenting the reach of DAE\textdelta ALUS/LENA, we will assume a 20\% uncertainty in all of these cross sections, translating to an approximate 20\% uncertainty in all beam-on background rates, $\delta B = 0.2B$.\textsuperscript{14}

The elastic and CCQE rates for all beam-on backgrounds above 106 MeV with the angular cut imposed are summarized in Table 2.2 for the three benchmark energy ranges. The main irreducible background in the recoil energy range 106–147 MeV is $\nu_\mu - e^-$ elastic scattering. The main reducible background is $\nu_\mu$ CCQE, which

\textsuperscript{13}The fact that the beam-on neutrino angular distribution appears to rise in the backwards direction is an artifact of our simplified GEANT simulation; without a re-entrant hole, we have a large number of backscattered pions.

\textsuperscript{14}The high statistics of the JPARC-MLF experiment \cite{165}, which should see nearly 200,000 CCQE events at 236 MeV, would give a much better than 20\% uncertainty on the differential energy spectrum. However, there would still be considerable uncertainty on the overall normalization, since theoretical predictions for the inclusive CC cross section can differ up to 25\% (see Ref. [252] for a discussion). That said, the exclusive channel in Eq. (2.22), which accounts for about 1\% of the $\nu_e$ CCQE cross section, has a smaller $\simeq 10\%$ uncertainty and may be useful for determination of the absolute flux to 10\%. We thank Joshua Spitz for bringing this point to our attention.
produces an outgoing muon; 70% of the time this muon can be identified through its Michel electron decay product [268], which as described above also provides the channel with which to calibrate the $\nu_\mu$ flux. Above 147 MeV, muons can no longer be produced in CCQE events from beam-on neutrino sources, leaving the $\nu_\mu - e^-$ elastic background as the dominant irreducible background in the recoil energy range 147–250 MeV, with a significant contribution from $\nu_e$ CCQE. Above 250 MeV, the rate due to beam-on $\nu_\mu - e^-$ elastic scattering is less than 1 event per year. Here, the dominant background is $\nu_e$ CCQE. Amusingly, the source of these electron neutrinos is the helicity-suppressed decay $\pi^+ \rightarrow e^+\nu_e$, which despite its branching ratio of $1.23 \times 10^{-4}$, has a very broad $\nu_e$ energy spectrum and a large CCQE cross section. The corresponding decay $\pi^- \rightarrow e^-\bar{\nu}_e$ leads to a subdominant reducible background with a taggable neutron.

The optimal recoil cuts as a function of $m_\chi$ and $m_{A'}$ can now be determined based on the various background thresholds. For light $\chi$, Fig. 2-6b shows that the DM recoil spectrum is relatively flat and extends to high energies, so the optimal $E_{e}^{\text{low}}$ is around 210 MeV where the only significant background is $\nu_e$ CCQE. As $m_\chi$ increases, the DM distribution begins to fall more steeply with energy, such that for $m_\chi \approx 20$ MeV the signal and $\nu_\mu$ elastic background fall at approximately the same rate. Thus, one needs to apply a lower energy cut to retain a sufficient yield of signal events; this is true for both on- and off-shell DM production. Below 250 MeV the only new background is $\nu_\mu$ elastic scattering, so to keep the maximum number of signal events, the optimal $E_{e}^{\text{low}}$ should be close to 147 MeV. For heavier DM, $m_\chi \gtrsim 30$ MeV, the 147 MeV cut is too severe because the DM is not produced with enough kinetic energy to provoke recoils above 147 MeV at an appreciable rate. As described above, to avoid the numerous low-energy backgrounds, the lowest realistic energy cut is $E_{e}^{\text{low}} = 106$ MeV. We determined $E_{e}^{\text{high}}$ as a function of $m_\chi$ and $m_{A'}$ by optimizing signal-to-background sensitivity $S/\delta B$ using $\delta B = \sqrt{4B/3}$ (systematic and statistical errors combined) for beam-off and $\delta B = 0.2B$ (systematic only) for beam-on; the result for $m_{A'} = 50$ MeV is shown in Fig. 2-7. Due to the broad neutrino background spectra, the optimal signal window is as narrow as possible for all DM masses. However, the energy resolution
Figure 2-7: Optimal electron recoil cuts $E_{e}^{\text{low}}$ (green curve) and $E_{e}^{\text{high}}$ (red curve), which optimize the signal-to-background sensitivity $S/\delta B$ as a function of $m_{\chi}$ for fixed $m_{A'} = 50$ MeV, assuming a minimum signal window width of 50 MeV. The shaded region between the red and green curves defines the optimal signal window for each mass point. Also shown is the maximum electron recoil energy $E_{e}^{\text{max}}$ (black, dotted curve) for each $m_{\chi}$ assuming an initial proton energy of 800 MeV (see Eq. 2.10). The blue dashed lines at $E_{e} = 106, 147,$ and $250$ MeV respectively denote the electron energies beyond which beam-on backgrounds from $\mu^{-}$ capture, $\nu_{\mu}$ CCQE (from $\pi^{+}$ DIF), and $\nu_{e}$ elastic scattering (from $\pi^{+}$ DIF) become irrelevant; these lines can be regarded as a heuristic estimate of $E_{e}^{\text{low}}(m_{\chi})$. Above 250 MeV, the only significant beam-on background is from the $\nu_{e}$ CCQE process (see Table 2.2).

at LENA is on the order of a few percent in the energy range we consider [268]. To be conservative, we use signal windows of 50 MeV or greater in electron recoil energy.

2.5 Sensitivity

The main results of this chapter are shown in Fig. 2-3, which give the 3$\sigma$ sensitivity of the DAE$\delta$ALUS/LENA setup to the dark photon/DM parameter space. We also show updated results for the LSND exclusions, which extend the analysis of Ref. [97] into both off-shell $A'$ regimes. Our LSND exclusions are based on rescaling our GEANT simulation for the DM signal rates in DAE$\delta$ALUS/LENA to match the
collision rate and target geometry of LSND. We make no attempt to simulate the backgrounds at LSND, but instead assume that the 55-event upper limit quoted in Ref. [49] accounts for background subtraction. Our signal yields are expected to be very similar to the analysis in Ref. [97], because the $\pi^0$ spectrum depends very little on the target geometry; we verified that in the on-shell $A'$ regime, we obtain nearly identical results to Ref. [97]. A key feature to note is the dark gray bands in Figs. 2-3c and 2-3e, which indicate the region of parameter space where LSND can place bounds on visible $A' \rightarrow e^+e^-$ decays by searching for DM produced in $\pi^0 \rightarrow \gamma A'^* \rightarrow \gamma \chi \bar{\chi}$ via an off-shell $A'$. The extended exclusion limits from LSND compared to the previously-reported limits are demonstrated in Fig. 2-8 for $m_\chi = 20$ MeV; we discuss the reason for this extended coverage in more detail below.

Also plotted in Fig. 2-3 are constraints and projected sensitivities for a variety of dark photon searches; for a comprehensive review of this parameter space see Ref. [112] and citations therein. The constraints are from E137 [65, 56], Orsay [93], muon $g-2$ [240, 111], electron $g-2$ [149, 164], E141 [64], E787 [12], E949 [43], the BaBar visible search for $A' \rightarrow e^+e^-$ [208] denoted “BaBar V” in Fig. 2-3, the BaBar invisible search for monophoton and missing energy [48] denoted “BaBar I” in Fig. 2-3, and recent results from NA48/2 [85]. Other visible constraints from A1 [225], and the APEX test run [8] are shown in Fig. 2-4; recent constraints from PHENIX [9] are subdominant to NA48/2 in this region of parameter space. The projected sensitivities involve a combination of visible $A' \rightarrow e^+e^-$ and invisible $A' \rightarrow \chi \bar{\chi}$ searches: BDX [58], APEX [115, 8], HPS [227], MESA and MAMI [61], VEPP-3 [266], and DarkLight [142, 51, 189]. The thick green band is the parameter space for which $A'$ resolves the long-standing $(g-2)_\mu$ anomaly [240].

The plots in the left column of Fig. 2-3 show the DAEεALUS/LENA sensitivity in $\epsilon^2$ for fixed $(\alpha_D, m_\chi)$ as a function of $m_{A'}$, where for each point $(m_{A'}, m_\chi)$ the signal window is chosen to optimize the sensitivity, as in Fig. 2-7. For light $\chi$ ($m_\chi = 1$ MeV in Fig. 2-3a), the sensitivity curve is essentially parallel to that of LSND, but better by an order of magnitude due to the optimized cuts. The projected sensitivity of the BDX experiment is shown in dashed green for comparison. For this DM mass, the $A'$
Figure 2-8: Comparison of LSND sensitivities as computed using methods in the existing literature [57, 97] (magenta curve) and those obtained using the full three-body matrix element that includes DM production via an off-shell $A'$. 

is produced on-shell for $m_{A'} < m_{\pi^0}$, and off-shell when $m_{A'} > m_{\pi^0}$. However, there is no sharp kinematic threshold at $m_{A'} = m_{\pi^0}$, and both DAEδALUS/LENA and LSND still have sensitivity in the upper off-shell regime; this observation was neglected in previous studies, due to an improper application of the narrow-width approximation. 

Going to heavier DM, $m_\chi = 20$ MeV in Fig. 2-3c, we can probe the on-shell region $2m_\chi < m_{A'} < m_{\pi^0}$, as well as the two off-shell regions $m_{A'} < 2m_\chi$ and $m_{A'} > m_{\pi^0}$. The large mass of DM compared to the $A'$ results in two key differences compared to the light DM case. First, there is a true kinematic threshold for on-shell production of the $A'$ at $m_{A'} = 40$ MeV. Just above threshold, the phase space suppression of DM particles produced nearly at rest in the on-shell $A'$ rest frame competes with the matrix element suppression of DM produced through an off-shell $A'$, and so the cut on electron recoil energy tends to shift the point of maximum sensitivity in $\epsilon^2$ to larger $A'$ masses. This results in a dip at $m_{A'} \gtrsim 40$ MeV rather than a sharp drop exactly at threshold. Second, in the lower off-shell regime $m_{A'} < 40$ MeV,
both DAE6ALUS and LSND are still sensitive to DM production and scattering, and in fact the sensitivity to very light off-shell \( A' \)'s is superior to the on-shell sensitivity. This surprising observation has also been neglected in previous studies, and is possible because the virtuality of the \( A' \) does not generate all that much of a suppression in the decay \( \pi^0 \to \gamma A'^* \to \gamma \chi \chi \). Indeed, phase space constraints at high \( m_{A'} \) can be more restrictive than matrix element suppression at low \( m_{A'} \), such that there is a region of parameter space at very low \( m_{A'} \) where the off-shell reach of both experiments in \( \epsilon^2 \) is stronger than the on-shell reach.

Furthermore, because the \( A' \) couples to electrons by assumption, if \( A' \) decays to DM are kinematically forbidden, then the decay channel \( A' \to e^+e^- \) must be open. This leads to the key feature mentioned above that the sensitivity of DAE6ALUS/LENA and LSND in the lower off-shell \( A' \) regime can overlap with visible \( A' \to e^+e^- \) searches. Indeed, for \( m_\chi = 20 \text{ MeV} \), the reach of LSND and DAE6ALUS/LENA is comparable to experiments like E141 [64] and HPS [227]. Of course, the visible limits are independent of \( m_\chi \) whereas the LSND and DAE6ALUS/LENA limits require a dark sector state of the appropriate mass. Still, this emphasizes the importance of studying the full \( \{m_{A'}, \epsilon, m_\chi, \alpha_D\} \) parameter space. Note that as \( \alpha_D \) increases, the LSND and DAE6ALUS curves on these plots shift downward. DM production is independent of \( \alpha_D \) in the on-shell regime but proportional to \( \alpha_D \) in the off-shell regime, while DM scattering is proportional to \( \alpha_D \) for any \( m_\chi \) and \( m_{A'} \) (see App. A.1 and App. A.2). Thus, the scaling of the sensitivity with \( \alpha_D \) is quadratic in the off-shell regime and linear in the on-shell regime. In contrast, the visible searches remain unaffected as \( \alpha_D \) is changed since the on-shell \( A' \to e^+e^- \) process is independent of the dark coupling \( \alpha_D \).

Going to even heavier DM, \( m_\chi = 40 \text{ MeV} \) in Fig. 2-3e, we see that constraints from LSND data already cover the entire region which would be probed by DAE6ALUS in one year of running. This is due to the fact that LSND is a Čerenkov detector and can use directionality to discriminate against neutrino backgrounds at lower energies than LENA. For the DAE6ALUS/LENA setup, the minimum recoil cut of 106 MeV which is necessary to mitigate the backgrounds also cuts out the majority of the signal,
since the heavy DM is produced with relatively low kinetic energy. This also results in an even greater degradation of sensitivity near the on-shell threshold at $m_{A'} = 2m_\chi$ compared to LSND. Thus we see that experiments like LSND, which have sensitivity to low electron recoil energies, are optimal for larger $m_\chi$.

The plots in the right column of Fig. 2-3 show the sensitivity in $\epsilon^2$ for fixed $(m_{A'}, \alpha_D)$ as a function of $m_\chi$, where again the electron recoil cuts are chosen for each $m_\chi$ to optimize the sensitivity as in Fig. 2-7. The DAEδALUS/LENA reach improves on LSND by an order of magnitude for light $\chi$, but the improvement weakens for heavier $\chi$ for the same reasons discussed above: the LSND recoil cuts favor heavier DM because it is produced with less kinetic energy. The constraints from visible searches now appear as horizontal lines in the off-shell regime because they depend only on $m_{A'}$ and not on $m_\chi$.

Finally, Fig. 2-4 shows a different slice through parameter space. Here, we fix $m_\chi$ and show the sensitivity to $\alpha_D$ as a function of $m_{A'}$, where for each $A'$ mass, $\epsilon$ assumes the lowest value consistent with the $(g - 2)_\mu$ preferred band (as shown in green in Fig. 2-3a). We see that DAEδALUS/LENA can improve considerably on LSND bounds over the entire kinematically-accessible parameter space of the dark photon model, and nearly covers all of the remaining parameter space that resolves the $(g - 2)_\mu$ anomaly. The prospect of reconciling this anomaly with a dark photon is usually discussed for an $A'$ which decays purely to $e^+e^-$ or purely to dark-sector states (see Ref. [207] for a discussion of current constraints), but presenting the parameter space in this fashion shows that DAEδALUS/LENA is sensitive to dark photons that decay predominantly to visible states, and that visible decay experiments already cover some regions in which the $A'$ decays invisibly.\textsuperscript{15} Note that after including recent results from NA48/2 [85], the $(g - 2)_\mu$ window for a visibly decaying $A'$ is now fully closed (see also Fig. 2-3e).

\textsuperscript{15}We thank Natalia Toro for pointing out the sensitivity of visible searches in this region of parameter space.
2.6 Conclusion

A rich dark sector remains a well-motivated possibility, and light DM coupled to a kinetically-mixed dark photon provides excellent opportunities for discovery. In this chapter we have shown that intensity frontier experiments like DAE\delta ALUS, in conjunction with a large underground neutrino detector such as LENA, will have unprecedented sensitivity to light (sub-50 MeV) DM, light (sub-400 MeV) dark photons, and other light, weakly coupled particles. Previous analyses have emphasized the $m_{A'} > 2m_X$ region of parameter space where the $A'$ decays almost exclusively to the dark sector via $A' \rightarrow \chi \bar{\chi}$. This focus was motivated by the typical size of $\epsilon$, which ensures that if light dark-sector states exist, then $\text{Br}(A' \rightarrow \chi \bar{\chi}) \approx 1$. Here, we have shown that existing LSND data places strong constraints on two additional regions: the $m_{A'} < 2m_X$ regime, where on-shell $A'$'s decay via the visible channel $A' \rightarrow e^+e^-$ but DM can be produced via an off-shell $A'$, and the $m_{A'} > m_{\pi^0} > 2m_X$ regime, which does not actually contain a kinematic threshold forbidding DM production. Because DM can be produced through both on- and off-shell dark photons, the full four-dimensional parameter space $\{m_{A'}, \epsilon, m_X, \alpha_D\}$ contains interesting regimes which are not captured in the usual $\{m_{A'}, \epsilon\}$ plots. DAE\delta ALUS is uniquely sensitive to this larger parameter space, even up to $A'$ masses of 500 MeV. We also encourage the current search at MiniBooNE to explore this expanded parameter space.

In addition to the potential advantages of higher luminosity and larger acceptance compared to previous experiments, a light DM search at DAE\delta ALUS/LENA would not require a separate running mode, such as the off-target mode used for MiniBooNE. While the sensitivity is best in the on-axis configuration, the reach is relatively insensitive to the detector geometry, and so a DM search could run simultaneously with a decay-at-rest neutrino experiment, provided analysis cuts are performed offline after data-taking. In fact, pairing DAE\delta ALUS with a large-volume underground Čerenkov detector like the proposed Hyper-K, with sensitivity to both low and high electron recoil energies and good electron-muon separation to reduce CCQE backgrounds, could cover a broad region of the full four-dimensional parameter space of the dark pho-
ton model. The fact that both neutrino and DM experiments share essentially the
same signals and backgrounds, though often well-separated kinematically, is an ad-
vantageous feature of such a setup, and suggests exciting opportunities for symbiosis
between beyond-the-standard-model and neutrino physics in the coming years.
Chapter 3

Auxiliary Gauge Mediation: A New Route to Mini-Split Supersymmetry

As mentioned in Sec. 1.2, gauge mediation is an attractive framework for preserving desired features of SUSY such as gauge coupling unification and suppression of flavor-violating observables. However, gauge mediation with SM gauge groups generically leads to sfermion and gaugino masses which are parametrically of the same order, making it difficult to realize the mini-split SUSY spectrum suggested by the measured value of the Higgs mass. That said, in any incarnation of gauge mediation, one is already committed to introducing scales intermediate between the weak scale and the Planck scale (at minimum, the messenger scale), so it is attractive to entertain the possibility of new gauge groups which are spontaneously broken at high scales.

In this chapter, we present a new approach for mini-split model building, which we dub auxiliary gauge mediation. We consider gauging $G_{aux}$, the auxiliary group containing all anomaly-free continuous symmetries of the SM in the limit of vanishing
Yukawas, consistent with grand unified theories (GUTs). As we will show,

\[ G_{\text{aux}} \equiv SU(3)_F \times U(1)_{B-L} \times U(1)_H, \]  

(3.1)

which contains an SU(3)_F flavor symmetry that rotates the three generations, the well-known U(1)_{B-L} symmetry, and most importantly a U(1)_H symmetry acting on the Higgs doublets. Gauge mediation via this spontaneously-broken U(1)_H generates precisely the Higgs sector soft terms one needs for consistent mini-split model building. Furthermore, auxiliary gauge mediation ensures that gaugino masses stay two loop factors smaller than scalar masses, automatically realizing the mini-split spectrum.

Auxiliary gauge mediation is a special case of Higgsed gauge mediation [155], and we review how to obtain the spectrum at lowest order in the SUSY-breaking parameter F using the techniques of Refs. [88, 87]. We also present, for the first time in the literature, a Feynman diagrammatic calculation of the two-loop contribution to A- and B-terms to all orders in F in Higgsed gauge mediation, which also sheds light on the two-loop result in standard gauge mediation [244]. Contrary to a common misconception, we find two-loop contributions to A- and B-terms which are non-zero at the messenger scale, in addition to the well-known contributions proportional to \( \log(M/\mu) \) which vanish when the RG scale \( \mu \) equals the messenger scale \( M \). Our result is consistent with the known results from analytic continuation into super-space [150, 40], where logarithmically-enhanced two-loop A- and B-terms arise from one-loop RG evolution. The two-loop contributions we find are not logarithmically-enhanced and therefore a small effect in standard gauge mediation. They are important, however, to include when studying mini-split models where visible-sector gaugino-loop contributions to \( B_\mu \) are suppressed.

1By “anomaly-free” we mean that \( G_{\text{aux}} \) has no mixed anomalies with SM gauge groups. \( G_{\text{aux}} \) may have its own internal anomalies whose cancellation requires the addition of new matter, but these new fields have no SM gauge charges.

2A similar U(1)_H was discussed in Ref. [197] in the context of non-supersymmetric two-Higgs-doublet models.

3The bar on \( \mu \) emphasizes that throughout this paper, we work in the dimensional reduction scheme DR. This is particularly relevant for the discussion in Sec. 3.2, where we want to track finite two-loop contributions. In an earlier calculation [244], these contributions were absorbed into a redefinition of the messenger scale.
For mini-split model building, auxiliary gauge mediation exhibits a number of interesting features. For concreteness, we will keep our discussion within the context of the minimal supersymmetric standard model (MSSM [101]), though auxiliary gauge mediation could be adapted to non-minimal scenarios as well.4

- While only SU(3)$_F$ contributes to the gluino soft mass, all three factors in $G_{aux}$ contribute to the wino and bino soft masses. This allows the gaugino spectrum to be significantly altered relative to more conventional scenarios. In particular, using the $U(1)_H$ factor, the wino or bino could be close in mass to (or possibly heavier than) the gluino.

- The spontaneous breaking of SU(3)$_F$ allows splittings between the third-generation squarks and those of the first two generations. This can significantly enhance the branching ratio of gluino decays into third-generation quarks, leading to “flavored” mini-split LHC signatures.

- Because of the $U(1)_{B-L}$ factor, auxiliary gauge mediation can accommodate scenarios with sleptonst significantly heavier than squarks.

- As is typical in gauge mediation, the gravitino is the LSP, but generic low-scale models have gravitinos which are too light to be dark matter. Auxiliary mediation using all three factors of $G_{aux}$ can provide a low-scale mini-split spectrum with super-WIMP [123, 124] gravitino dark matter, thanks to a bino NLSP of the correct mass.

- Economical models of mini-split can be constructed based on the single gauge symmetry $U(1)_{B-L+kH}$, where $k$ encodes the freedom to choose a variety of Higgs charges. These “minimal mini-split” models generate novel, testable gaugino spectra, as well as the necessary Higgs sector soft terms.

The structure of this paper is as follows. In Sec. 3.1, we review the mechanism of Higgsed gauge mediation for a general gauge group $G$, giving expressions at lowest

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4In the context of the next-to-minimal supersymmetric standard model (NMSSM), it would be interesting to augment $G_{aux}$ with additional U(1) symmetries acting on the singlet superfield.
non-trivial order for all the soft terms. We take a short detour in Sec. 3.2 and App. B, calculating the $A$- and $B$-terms for the case of standard gauge mediation and demonstrating the presence of non-zero contributions at the messenger scale. Sec. 3.3 motivates and defines the auxiliary group $G_{\text{aux}}$ and contains the main technical results of our paper. We provide example spectra and consider associated phenomenology in Sec. 3.4, including scenarios with and without flavor structure. We describe a minimal $U(1)_{B-L+kH}$ benchmark model in Sec. 3.5, and conclude in Sec. 3.6. The work described in this chapter was undertaken in collaboration with Jesse Thaler and Matthew McCullough, with special thanks to Nathaniel Craig for collaborating during the early stages of this work; Ben Allanach, Matthew Dolan, and Andrew Larkoski for helpful conversations; and Ian Low and the participants of TASI 2013 for stimulating discussion. It is largely based on [185].

3.1 Review of Higgsed gauge mediation

Before studying auxiliary gauge mediation in particular, we first review the broad features of Higgsed gauge mediation. The reader familiar with this material and the notation in Ref. [88] can safely skip to Sec. 3.2.

3.1.1 Soft masses from the effective Kähler potential

In Higgsed gauge mediation [155], SM soft masses arise from messengers charged under a spontaneously broken gauge symmetry. For simplicity, consider an Abelian gauge group $U(1)'$ and a single vector-like messenger $\Phi, \Phi^c$ with charge $q_\Phi$. As in minimal gauge-mediated scenarios, these messengers are coupled to the SUSY-breaking spurion $\langle X \rangle = M + \theta^2 F$ in the superpotential

$$W \supset X \Phi \Phi^c.$$  \hspace{1cm} (3.2)

The generalization to non-Abelian gauge groups and multiple messengers is straightforward.
Because $U(1)'$ is spontaneously broken at a high scale, the calculation of soft-masses is considerably more complicated than for standard gauge mediation, and the elegant technique of analytically-continuing RG thresholds [150, 40] cannot be directly employed due to the multiple mass thresholds. As shown in Ref. [88] and later applied in Ref. [87], the full soft spectrum can be obtained by employing the two-loop effective Kähler potential and analytically continuing both the messenger mass and the vector superfield mass,

$$|M_\phi|^2 \rightarrow X^\dagger X, \quad M_V^2 \rightarrow M_V^2 + 2g'^2 q^2 q^\dagger q,$$

where $q$ are visible-sector fields with charge $q_q$ under the $U(1)'$.

Using the two-loop effective Kähler potential result from Ref. [234] and the two-loop sunrise-diagram integral evaluated in Ref. [132], we have

$$K_{2L} \supset \frac{q^2 g^2}{(4\pi)^4} |M_\phi|^2 \left[ 2\Delta \log(\Delta) \left( \log \left( \frac{|M_\phi|^2}{\mu^2} \right) - 2 \right) + (\Delta + 2) \log \left( \frac{|M_\phi|^2}{\mu^2} \right) \left( \log \left( \frac{|M_\phi|^2}{\mu^2} \right) - 4 \right) + \Omega(\Delta) \right], \quad \Delta = \frac{M_V^2}{|M_\phi|^2},$$

where $\mu$ is the $\overline{\text{DR}}$ renormalization scale, and we can express the function $\Omega(\Delta)$ using dilogarithms as

$$\Omega(\Delta) = \sqrt{\Delta(\Delta - 4)} \left( 2\zeta(2) + \log^2(\alpha) + 4\text{Li}_2(-\alpha) \right), \quad \alpha = \left( \sqrt{\frac{\Delta}{4}} + \sqrt{\frac{\Delta}{4} - 1} \right)^{-2}.$$

Applying the shift in Eq. (3.3) and expanding Eq. (3.4) to first order in $|q|^2$ and lowest non-trivial order in $F/M^2$, we are left with a two-loop Kähler potential for the visible-sector fields

$$K_{2L} \supset -\frac{q^2 g_0^2}{(2\pi)^2} \alpha^2 \left( h(\delta) \left( \frac{F}{M} \frac{\theta^2 + F^2}{M^2} + f(\delta) \left| \frac{F}{M} \right|^2 \frac{\theta^2 \theta^2}{M} \right) \right) |q|^2, \quad \delta = \left| \frac{M_V}{M} \right|^2,$$

where the factors $h(\delta)$ and $f(\delta)$ track the difference between Higgsed gauge mediation
and standard gauge mediation,\(^5\) and are given explicitly by

\[
h(\delta) = 2\frac{(\delta - 4)\delta \log(\delta) - \Omega(\delta)}{\delta(\delta - 4)^2},
\]

\[
f(\delta) = 2\frac{\delta(\delta - 4)((\delta - 4) + (\delta + 2)\log(\delta)) - 2(\delta - 1)\Omega(\delta)}{\delta(\delta - 4)^3}.
\]

From Eq. (3.6), we will derive two-loop scalar mass-squared, two-loop A- and B-terms, and three-loop gaugino masses in the subsections below.

As expected, the SUSY breaking contributions vanish as \(\delta \to \infty\) since the gauge superfield becomes infinitely massive and no longer mediates SUSY breaking. This can be seen from the limiting behavior

\[
\lim_{\delta \to \infty} h(\delta) = \frac{2\log \delta}{\delta}, \quad \lim_{\delta \to \infty} f(\delta) = \frac{2(\log \delta - 1)}{\delta}.
\]

The unbroken limit \(\delta \to 0\) corresponds to standard gauge mediation,

\[
\lim_{\delta \to 0} h(\delta) = (1 - \log \delta), \quad \lim_{\delta \to 0} f(\delta) = 1.
\]

Note the large logarithm in \(h(\delta)\), corresponding to the \(\theta^2\) components in Eq. (3.6), which arises from the running of the gauge coupling between the messenger scale \(M\) and the vector mass scale \(M_V\). We will return to this function in some detail in Sec. 3.2.

### 3.1.2 Two-loop scalar masses

When the mediating gauge group is Abelian, we can read off the scalar soft mass-squared directly from Eq. (3.6):

\[
\tilde{m}_q^2 = q_f^2 q_f^2 \left( \frac{\alpha^2}{2\pi} \right)^2 \left| \frac{F}{M} \right|^2 f(\delta), \quad \delta \equiv \left( \frac{M_V}{M} \right)^2.
\]

\(^5\)For a generalization of the function \(h(\delta)\) to all orders in \(F/M^2\) see App. B, and for a similar generalization of \(f(\delta)\) see Ref. [155].
where $M_V$ is the mass of the U(1)$'$ gauge superfield, $\alpha = g^2/4\pi$ is the corresponding fine-structure constant, and $q$ and $\Phi$ have respective charges $q_q$ and $q_\Phi$. It is straightforward to generalize to the non-Abelian case \[88\],

$$
(\tilde{m}_q^2)_{ij} = C(\Phi) \frac{\alpha^2}{(2\pi)^2} \left| \frac{F}{M} \right|^2 \sum_a f(\delta^a) (T^a q_i T^a q_j), \quad \delta^a \equiv \frac{M^a}{M},
$$

(3.12)

where $M^a$ is the mass of the gauge superfield corresponding to the generator $T^a$, $\{ij\}$ indicates that these indices have been symmetrized and $C(\Phi)$ is the Dynkin index of the messenger superfield representation. Generalizing to multiple gauge groups and multiple messengers is more complicated if the gauge groups mix (see Ref. \[88\]). We will consider scenarios where mixing is not present for simplicity of presentation, in which case we need only include a sum over various messenger/gauge group contributions.

The formulae in Eqs. (3.11) and (3.12) are the values of the soft masses at the effective messenger scale, which is the lower of the scales $M$ or $M_V$. Specifically, if the gauge symmetry is spontaneously broken far below the messenger scale $M$, the effective messenger scale is $M_V$ rather than $M$ since the “running” from the scale $M$ down to $M_V$ has already been accommodated by the effective Kähler potential.\[6\]

Hence, the proper definition of the effective messenger scale $M_{\text{eff}} = \min\{M, M_V\}$ is important when RG-evolving the soft terms from high scales down to the weak scale through their interactions with the visible sector.

### 3.1.3 Two-loop A-terms and B-terms

To find the two-loop $A$- and $B$-terms, it is easiest to holomorphically rescale each visible-sector superfield to eliminate terms linear in $\theta^2$ in Eq. (3.6):

$$
q \rightarrow \left(1 + q_q^2 q_\Phi^2 \frac{\alpha^2}{(2\pi)^2} h(\delta) \frac{F}{M} \theta^2\right) q.
$$

(3.13)

\[6\] Strictly speaking, the effective Kähler potential does not include resummation of logarithms, but this prescription for the effective messenger scale is needed to avoid double-counting of the momentum scales between $M$ and $M_V$.  

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or in the non-Abelian case

\[ \mathbf{q}_i \rightarrow \left( \delta_{ij} + C(\Phi) \frac{\alpha^2}{(2\pi)^2} \sum_a h(\delta^a) (T^a_q T^a_q)_{\{ij\}} \frac{F}{M \delta^2} \right) \mathbf{q}_j. \] (3.14)

This rescaling does not affect the value of the soft masses at two-loop order since the resulting corrections appear formally at four loops. With this holomorphic rescaling, the SUSY breaking terms are pulled into the superpotential, leading to SUSY-breaking holomorphic terms in the scalar potential.

Adapting the notation of Ref. [150], we can write the soft scalar potential as

\[ V_{\text{soft}} \supset \sum_{ij} A_{ij} \mathbf{q}_i \frac{\partial W}{\partial \mathbf{q}_j} \bigg|_{q^2 \to 0}. \] (3.15)

In the Abelian case we have

\[ A_{ij} = A_i \delta_{ij}, \quad A_i = q^2 q^2 \frac{\alpha^2}{(2\pi)^2} h(\delta) \frac{F}{M}, \] (3.16)

and in the non-Abelian case

\[ A_{ij} = C(\Phi) \frac{\alpha^2}{(2\pi)^2} \frac{F}{M} \sum_a h(\delta^a) (T^a_q T^a_q)_{\{ij\}}. \] (3.17)

Again, these soft terms should be considered to appear at the effective messenger scale \( M_{\text{eff}} = \min\{M_V, M\} \). In Sec. 3.2, we will discuss how to interpret the \( M_V \to 0 \) limit.

### 3.1.4 Three-loop gaugino masses

If the messengers \( \Phi, \Phi^c \) are uncharged under SM gauge groups, then visible-sector gaugino masses first arise at three-loop order. Though this might seem computationally daunting, one can again use the power of holomorphy and analytic continuation to extract this three-loop effect from Eq. (3.6). The field rescaling in Eqs. (3.13) and
(3.14) is anomalous [83, 199], leading to a shift of the gauge kinetic function
\[
\int d^2 \theta \ f \ W_\alpha W^\alpha \to \int d^2 \theta \left( f - \sum_{q_r} \frac{C_G(q_r)}{8\pi^2} \log Z_{q_r}(\mu) \right) W_\alpha W^\alpha. \tag{3.18}
\]

Since this rescaling contains SUSY-breaking components, it leads to Majorana gaugino masses.\(^7\)

If the visible-sector chiral superfields \(q_r\) are charged under an Abelian mediating gauge group, then the gaugino mass for a visible-sector gauge group \(G\) is
\[
\tilde{M}_{\lambda_G} = \alpha_G^2 \frac{\alpha^2}{(2\pi)^2} h(\delta) \frac{F}{M} \sum_{q_r} q_r^2 C_G(q_r), \tag{3.19}
\]
where the sum is over all rescaled fields. For a non-Abelian mediating gauge group \(G'\),
\[
\tilde{M}_{\lambda_{G'}} = C(\Phi) \frac{\alpha_G^2}{(2\pi)^2} \frac{\alpha^2}{M} \sum_{q_r} C_G(q_r) C_{G'}(q_r) \sum_a h(\delta^a). \tag{3.20}
\]
Here the sum over the generators appearing in Eq. (3.14) simplifies using \(\text{Tr}(T^a T^b) = C_{G'} \delta^{ab}\), hence the appearance of the Dynkin index of \(q_r\) with respect to the mediating group \(G'\). This simplification still holds even after an orthogonal rotation of the generators \(T^a\) to the mass eigenstate basis, since the Dynkin index is just the magnitude of \(T^a\) with respect to the trace norm.

### 3.2 A-terms and B-terms in standard gauge mediation

Before applying the above expressions to the case of auxiliary gauge mediation, it is worthwhile to pause and consider the \(\delta \to 0\) limit in more detail, since this should yield the familiar results of standard gauge mediation where the mediating gauge

\(^7\)For a discussion of how this effect can be seen from the point of view of the real gauge coupling superfield, see Refs. [150, 40, 87].
group $G \equiv G_{\text{SM}}$ is unbroken.\(^8\) Because $f(\delta \to 0) = 1$, the two-loop scalar soft-masses in Sec. 3.1.2 clearly match those for standard gauge mediation. At first glance, the $A$- and $B$-term results in Sec. 3.1.3 also appear to match the standard gauge-mediated results if we reinterpret the vector mass $M_V$ as the RG scale $\mu$ and take $h(\delta) \simeq -\log \delta \simeq \log(M^2/\mu^2)$. Indeed, this logarithmic factor is a well-known one-loop effect of RG evolution driven by the gaugino masses.

Upon closer inspection, however, there appears to be a mismatch between the standard lore about $A$- and $B$-terms in gauge mediation and our expressions. Applying the general results found in Sec. 3.1 to standard gauge mediation, the SM gauge groups are unbroken above the weak scale so the low energy cutoff in the path integral is the SM gaugino mass rather than the gauge superfield mass. Thus, in the $\delta \to 0$ limit in Eq. (3.10), we should really make the replacement

$$h(\delta) \to 1 + \log \left( \frac{M^2}{\mu^2} \right), \quad (3.21)$$

where $M$ is the messenger mass and $\mu$ is the RG scale which should be ultimately set to the gaugino mass (which by design is close to the weak scale). From the results in Sec. 3.1.3, we therefore find $A, B_\mu \propto (1 + \log(M^2/\mu^2))$. Naively, this seems to be at odds with previous results based on analytic continuation with one-loop threshold RG matching, where $A, B_\mu \propto \log(M^2/\mu^2)$ vanishes at the messenger scale [150, 40]. In a common misconception, it is often assumed that $A$- and $B$-terms always vanish at the messenger scale in gauge mediation, although this statement is in fact only true at one-loop.\(^9\)

There are two different ways to see why this standard lore is not quite correct. First, we can revisit the arguments in Ref. [150] on analytic continuation to show why threshold matching and one-loop RG running does not yield the complete answer at

\(^8\)Of course, the three-loop gaugino masses in Sec. 3.1.2 are subdominant in the standard gauge mediation case where gaugino masses first arise at one-loop order, whereas the three-loop gaugino mass is the desired leading effect in auxiliary gauge mediation to get light gauginos in mini-split SUSY.

\(^9\)We are not sure where this misconception comes from, since Refs. [150, 40] only make this statement for the matched one-loop calculation and not as a claim for the full two-loop result, and a two-loop finite contribution had been calculated previously with Feynman diagrams in Ref. [244].
two-loop order. The wavefunction renormalization of a visible-sector superfield $Q$ is in general a function of the ultraviolet (UV) gauge coupling $\alpha_{\text{UV}}$ defined at the cutoff scale $\Lambda$, and the logarithms $L_X = \log(\mu^2/|X|^2)$ and $L_{\text{UV}} = \log(\mu^2/\Lambda^2)$, which can be written generally as

$$\log(Z_Q) = \sum_{\ell} \alpha_{\text{UV}}^{\ell-1} P_{\ell}(\alpha_{\text{UV}} L_X, \alpha_{\text{UV}} L_{\text{UV}}), \quad (3.22)$$

where $\ell$ is the loop order. The soft-masses are calculated from

$$\tilde{m}_Q^2 = -\left. \frac{\partial^2 \log(Z_Q)}{\partial \log(X) \partial \log(X^\dagger)} \right|_{\frac{F}{M}}^2 \quad (3.23)$$

$$\propto \sum_{\ell} \alpha^{\ell+1}(\tilde{\mu}) P''_{\ell}(\alpha(\tilde{\mu}) L_X), \quad (3.24)$$

where in the second line the loop function $P_{\ell}$ has been differentiated twice. Thus the $\alpha^2(\tilde{\mu})$ soft-masses can be evaluated simply with the one-loop running $P_1$, which is the beauty of the argument presented in Ref. [150]. However, if we consider the value of $A_Q$ (see Eq. (3.15)) that enters into $A$- and $B$-terms, we have

$$A_Q = -\left. \frac{\partial \log(Z_Q)}{\partial \log(X)} \right|_{\frac{F}{M}}^2 \quad (3.25)$$

$$\propto \sum_{\ell} \alpha^{\ell}(\tilde{\mu}) P'_{\ell}(\alpha(\tilde{\mu}) L_X), \quad (3.26)$$

where now the loop function has only been differentiated once. Thus, the full $\alpha^2(\tilde{\mu})$ $A$- and $B$-terms require the full two-loop result; one-loop running and matching cannot capture all of the contributions. Thus, the general arguments of Ref. [150] already accommodate a discrepancy between the full two-loop result obtained here and the result obtained from one-loop RG threshold matching.

Second, we can perform a brute force calculation in component fields to show that Eq. (3.21) is the proper replacement. In App. B, we perform a full two-loop calculation of $A$- and $B$-terms to all orders in $F/M^2$. For a broken mediating gauge
group in App. B.1, this yields an effective \( \tilde{h}(F/M^2, \delta) \), with the expansion

\[
\tilde{h}(F/M^2, \delta) = h(\delta) + \mathcal{O}\left(\frac{F}{M^2}\right), \tag{3.27}
\]

in agreement with the answer obtained using our analytic continuation method. For an unbroken mediating gauge group in App. B.2, the two-loop diagram contains an IR divergence. In this case, if we regulate this divergence with dimensional reduction (DR) (following e.g Eq. (2.21) of Ref. [219]), we find that \( A, B_\mu \propto (1 + \log(M^2/\mu^2)) \), which is precisely the form arising from the analytic continuation method used here.\(^{10}\) This justifies the replacement of \( M_V \to \mu \) in the case of an unbroken gauge group, and demonstrates that \( M_V \) can be identified with with the \( \overline{\text{DR}} \) RG scale \( \mu \), making a direct connection (and highlighting the discrepancy) with results based solely on threshold matching.\(^{11}\)

Practically speaking, the difference between the full two-loop answer \( A, B_\mu \propto (1 + \log(M^2/\mu^2)) \) and the lore \( A, B_\mu \propto \log(M^2/\mu^2) \) has been relatively unimportant up until now since the logarithmic term typically dominates.\(^{12}\) In mini-split models, though, the finite piece is more relevant, since visible-sector gaugino masses can be very small and the precise values of Higgs sector parameters such as \( B_\mu \) are important.

### 3.3 Auxiliary gauge mediation

In the framework of auxiliary gauge mediation, SM Yukawa couplings are generated via spontaneous breaking of the auxiliary group

\[
G_{\text{aux}} \equiv \text{SU}(3)_F \times U(1)_{B-L} \times U(1)_H \tag{3.28}
\]

\(^{10}\)Ref. [244] also finds a finite piece, though it is a factor of two larger than what we find here. See App. B.2 for a more detailed discussion.

\(^{11}\)This result also has implications for the three-loop gaugino mass contributions, since they arise from precisely the same \( \theta^2 \) terms in the scalar wavefunction renormalization that generate the \( A \)- and \( B \)-terms.

\(^{12}\)Getting the precise value of \( A \) terms is important when appealing to naturalness considerations, though, since non-zero \( A \) terms at the messenger scale help push down the stop masses required for a Higgs at 126 GeV by increasing stop mixing.
Figure 3-1: General structure of auxiliary gauge mediation, where hidden sector SUSY breaking is communicated to the MSSM via messengers charged only under $G_{\text{aux}} \equiv \text{SU}(3)_F \times \text{U}(1)_{B-L} \times \text{U}(1)_H$ (and not under $G_{\text{SM}} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$).

at a high scale, which we shall refer to as the "auxiliary scale". Above the auxiliary scale, it is consistent for the full gauge group of the MSSM to be

$$G_{\text{total}} = G_{\text{SM}} \times G_{\text{aux}}, \quad G_{\text{SM}} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y.$$ (3.29)

This auxiliary gauge symmetry $G_{\text{aux}}$ can then play a role in mediating SUSY breaking to the MSSM fields as shown in Fig. 3-1, leading to new connections between MSSM soft terms and flavor structures. Gauge mediation by the SU(3)$_F$ flavor group was previously considered in Refs. [88, 87], where its role was to augment the contribution from standard $G_{\text{SM}}$ gauge mediation. Here, we will take auxiliary $G_{\text{aux}}$ gauge mediation as the sole mediation mechanism, leading to a novel and economical realization of the mini-split SUSY scenario with a (predictive) hierarchy between sfermions and gauginos.

### 3.3.1 Motivating the auxiliary group

Before calculating the soft spectrum, we want to justify the choice of $G_{\text{aux}}$ in Eq. (3.28). This can be achieved by switching off the SM Yukawa couplings and considering all possible gauge symmetries consistent with anomaly cancellation. A powerful simplifying criteria is to require that $G_{\text{aux}}$ has no mixed SM gauge anomalies, such that no new SM charged matter is need to cancel anomalies. This has the appealing feature of not spoiling gauge-coupling unification, though one could of course consider more
general gauge groups with exotic matter.

With this criteria imposed, we are left with a small set of possibilities. In the flavor sector one could have an SU(3)\(_F\) gauge symmetry (with all quark and lepton multiplets transforming in the fundamental) or an SO(3)\(_L\) \(\times\) SO(3)\(_R\) gauge symmetry (with the electroweak doublets \(Q\) and \(L\) transforming separately from the electroweak singlets \(U^c, D^c,\) and \(E^c\)). An SO(3)\(_L\) \(\times\) SO(3)\(_R\) gauge symmetry is likely inconsistent with the simplest GUT models, since left-handed and right-handed fields often live in the same GUT multiplets. For this reason we opt for the SU(3)\(_F\) gauge symmetry in defining \(G_{aux}\).\(^{13}\)

Gauge mediation by additional U(1) gauge groups has been considered before [190, 75, 76, 116, 206, 205]; all of these models require extra matter with SM gauge charges for anomaly cancellation. An obvious anomaly-free gauge symmetry is U(1)\(_{B-L}\), which has has received considerable attention [14, 107, 196]. This, and the SU(3)\(_F\) flavor symmetry, can both be used to generate scalar soft-masses for all of the matter fields. However gauge mediation by SU(3)\(_F\) \(\times\) U(1)\(_{B-L}\) alone leads to issues in the Higgs sector since the Higgs multiplets are uncharged under both mediating groups and, at two loops, Higgs soft-masses squared and the \(B_\mu\) term are both vanishing at the messenger scale. This can be remedied by mixing U(1)\(_{B-L}\) with U(1)\(_Y\) [226, 44], though this option is not in the spirit of this work, where we wish to separate \(G_{SM}\) from \(G_{aux}\).\(^{14}\)

The crucial ingredient for auxiliary gauge mediation is a U(1)\(_H\) gauge symmetry, under which \(H_u\) and \(H_d\) have equal and opposite charges and all other fields are neutral.\(^{15}\) This possibility was missed in the first treatment of flavor mediation [87], though in that context it was relatively unimportant since standard \(G_{SM}\) gauge mediation was employed to realize a natural SUSY spectrum. Here, U(1)\(_H\) is crucial

\(^{13}\)One could also choose to gauge just an SU(2) or U(1) subgroup of the flavor SU(3)\(_F\), acting e.g. on the first two generations. Given that a larger gauge symmetry is possible and there is no obvious reason why only some subgroup would be gauged, we will always gauge the full SU(3)\(_F\).

\(^{14}\)For this case of mixing U(1)\(_{B-L}\) with U(1)\(_Y\), avoiding issues such as tachyonic stops requires the tuning of tree-level \(D\)-term contributions against two-loop soft masses as well as very particular values of the mixing angle.

\(^{15}\)Additional anomaly-free U(1) symmetries acting on Higgs doublets are discussed in Ref. [197], but these only apply to the Type I two-Higgs-doublet models, not Type II relevant for SUSY.
Table 3.1: Representations under $G_{\text{total}} \equiv G_{\text{SM}} \times G_{\text{aux}}$ of the MSSM superfields and additional superfields required for anomaly cancellation and the generation of Yukawa couplings. The notation $C(\Phi)$ means that the messenger $\Phi$ lives in a representation with Dynkin index $C(\Phi)$. Also shown are the coupling constants $\alpha_i = g_i^2/4\pi$ for the various groups.

<table>
<thead>
<tr>
<th>Field</th>
<th>$SU(3)_C$</th>
<th>$SU(2)_L$</th>
<th>$U(1)_Y$</th>
<th>$SU(3)_F$</th>
<th>$U(1)_{B-L}$</th>
<th>$U(1)_H$</th>
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<tr>
<td>$Q$</td>
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<td>2</td>
<td>1/6</td>
<td>3</td>
<td>1/3</td>
<td>—</td>
</tr>
<tr>
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<td>—</td>
<td>-2/3</td>
<td>3</td>
<td>-1/3</td>
<td>—</td>
</tr>
<tr>
<td>$D^c$</td>
<td>3</td>
<td>—</td>
<td>1/3</td>
<td>3</td>
<td>-1/3</td>
<td>—</td>
</tr>
<tr>
<td>$L$</td>
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<td>2</td>
<td>-1/2</td>
<td>3</td>
<td>-1</td>
<td>—</td>
</tr>
<tr>
<td>$E^c$</td>
<td>—</td>
<td>—</td>
<td>1</td>
<td>3</td>
<td>1</td>
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<tr>
<td>$H_u$</td>
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<td>1/2</td>
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<tr>
<td>$H_d$</td>
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<td>$N^c_{B-L}$</td>
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<td>—</td>
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<td>—</td>
<td>$\pm 2$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$S^\pm_H$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>$\pm 1$</td>
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</tr>
</tbody>
</table>

Thus, we arrive at the most general auxiliary group consistent with the requirements of anomaly cancellation and gauge coupling unification: $G_{\text{aux}} \equiv SU(3)_F \times U(1)_{B-L} \times U(1)_H$. In fact, we may obtain acceptable phenomenology by mediating with $U(1)_H$ and just one of the other two factors, but in the interest of completeness we will retain this full gauge symmetry in the soft-mass expressions in Sec. 3.3.3. The representations of the MSSM fields under these gauge symmetries are detailed in Table 3.1. While we have ensured the absence of mixed SM-auxiliary anomalies, additional fields with no SM gauge charges are of course needed to cancel anomalies within $G_{\text{aux}}$ itself. An example of a fully anomaly-free spectrum is given in Table 3.1, motivated by the states needed below to break $G_{\text{aux}}$ and generate Yukawa couplings.

for successful electroweak symmetry breaking since $U(1)_H$ leads to Higgs soft-masses and also a $B_u$ term at two loops.
3.3.2 Flavor boson mass spectrum

In order to calculate soft terms, we need to know some details about the breaking of \( G_{\text{aux}} \) at the auxiliary scale. While a complete model of Yukawa coupling generation is beyond the scope of this work, we do need to choose a specific field content and vacuum expectation value (vev) structure to know the auxiliary gauge boson mass spectrum. Following Ref. [87] and summarized in Table 3.1, we assume that the only sources of \( SU(3)_F \) breaking are fields \( S_u \) and \( S_d \) (both transforming as a \( \bar{6} \) under \( SU(3)_F \)), which get vevs along a \( D \)-flat direction as to not break SUSY. The fields \( S_{B-L}^\pm \) (\( S_H^\pm \)) are responsible for breaking \( U(1)_{B-L} \) \( (U(1)_H) \). The additional right-handed neutrino fields \( N_e^c \) and \( N_{B-L}^c \) ensure that all \( SU(3)_F \) and \( U(1)_{B-L} \) anomalies cancel, respectively.\(^{16}\)

There are a number of different options for how to generate the SM Yukawa couplings. For pedagogical purposes, we will choose a structure that allows us to clearly delineate the role played by the different gauge groups in \( G_{\text{aux}} \) in generating the soft mass spectrum. In the quark sector, we assume that the following dimension six operators arise after integrating out heavy vector-like fields:

\[
W \supset \frac{1}{\Lambda_u^2} S_H^- S_u H_u Q U^c + \frac{1}{\Lambda_d^2} S_H^+ S_d H_d Q D^c.
\]  

(3.30)

Here, the up-type Yukawa matrix comes from \( \langle S_H^- S_u \rangle / \Lambda_u^2 \) and the down-type Yukawa matrix comes from \( \langle S_H^+ S_d \rangle / \Lambda_d^2 \). Instead of Eq. (3.30), we could have considered a more economical model where the \( S_u \) and \( S_d \) fields are charged under both \( SU(3)_F \) and \( U(1)_H \), allowing the Yukawa couplings to arise from dimension five operators.\(^{17}\)

Note that \( S_{B-L}^\pm \) need not play a role in generating the Yukawa couplings, though, due to the charges chosen, it can be used to generate right-handed neutrino masses. If

\(^{16}\)Assigning charges \( \pm 2 \) to \( S_{B-L}^\pm \) allows \( N_{B-L}^c \) to get a Majorana mass when \( S_{B-L}^- \) gets a vev. However, a complete model of flavor needs additional field content beyond those in Table 3.1, including a \( \bar{6} \) to give a Majorana mass to \( N_e^c \) and a \( \bar{6} \) to generate the lepton Yukawas. See Ref. [87] for further discussion.

\(^{17}\)In this case, the \( S_{u,d} \) vevs lead to mixing between the \( SU(3)_F \) and \( U(1)_H \) generators, giving the breaking pattern \( SU(3)_F \times U(1)_H \rightarrow SU(2)' \times U(1)' \rightarrow U(1)'' \rightarrow 0 \). The resulting soft mass spectrum contains mixed contributions proportional to \( \alpha_H \alpha_F \), which is interesting but inconvenient for pedagogical purposes.
we only gauge a subset of $G_{aux}$, then we can set the corresponding field in Eq. (3.30) to a constant value.\footnote{For example, if $U(1)_H$ is gauged but $SU(3)_F$ is not, then we can use the simpler superpotential

$$W = \frac{\lambda_u}{\Lambda_u} S_H^{-1} H_u Q U^c + \frac{\lambda_d}{\Lambda_d} S_H^{+} H_d Q D^c,$$  

(3.31)

where $\lambda_u$ and $\lambda_d$ are proportional to the SM Yukawa matrices, avoiding the need to dynamically generate the hierarchical $S_{u,d}$ vevs.}

Given the superpotential in Eq. (3.30), the pattern of $SU(3)_F$ gauge boson masses is determined by the measured flavor parameters. We will make the simplifying assumption that $\langle S_u \rangle \gg \langle S_d \rangle$, such that the flavor boson mass-spectrum is dominated by the up-quark Yukawa. After performing a global $SU(3)_F$ rotation we can diagonalize the flavor breaking matrices and denote

$$\langle S_u \rangle = \begin{pmatrix} v_{u1} & 0 & 0 \\ 0 & v_{u2} & 0 \\ 0 & 0 & v_{u3} \end{pmatrix}, \quad \langle S_d \rangle = V_{CKM} \begin{pmatrix} v_{d1} & 0 & 0 \\ 0 & v_{d2} & 0 \\ 0 & 0 & v_{u3} \end{pmatrix} V_{CKM}^T. \quad (3.32)$$

This leads to the hierarchical flavor breaking pattern $SU(3)_F \rightarrow SU(2)_F \rightarrow 0$ where the flavor boson masses are

$$M^2_{V[\sim SU(3)_F/SU(2)_F]} = 4\pi \alpha_F \left\{ \frac{8}{3} v_{u3}^2, (v_{u3} + v_{u2})^2, v_{u3}^2, v_{u3}^2, (v_{u3} - v_{u2})^2 \right\}, \quad (3.33)$$

$$M^2_{V[\sim SU(2)_F]} = 4\pi \alpha_F \left\{ 2 v_{u2}^2, v_{u2}^2, v_{u2}^2 \right\}. \quad (3.34)$$

Explicitly inputting both the up-quark and down-quark Yukawa couplings, taking $\Lambda_u = \Lambda_d$ for simplicity ($\alpha = 1$ in the language of Ref. [87]), and denoting $v_{u3} \equiv v_F$, we have the flavor boson mass spectrum

$$M^2_{V[\sim SU(3)_F/SU(2)_F]} \approx 4\pi \alpha_F v_F^2 \{ 2.67, 1.02, 1.00, 1.00, 0.99 \}, \quad (3.35)$$

$$M^2_{V[\sim SU(2)_F]} \approx 4\pi \alpha_F v_F^2 \{ 11.0, 5.60, 5.55 \} \times 10^{-5}, \quad (3.36)$$

clearly demonstrating the hierarchical symmetry breaking pattern for $SU(3)_F$.

For the $U(1)_{B-L}$ and $U(1)_H$ gauge bosons, their masses are determined by the
vevs $\langle S_{B-L}^\pm \rangle = v_{B-L}^\pm$ and $\langle S_H^\pm \rangle = v_H^\pm$:

\[
M_V^2[U(1)_{B-L}] = 32\pi\alpha_{B-L} \left( (v_{B-L}^+)^2 + (v_{B-L}^-)^2 \right), \\
M_V^2[U(1)_H] = 8\pi\alpha_H \left( (v_H^+)^2 + (v_H^-)^2 \right).
\]

With the chosen field content, we can freely adjust the masses of the SU(3)$_F$, U(1)$_{B-L}$, and U(1)$_H$ gauge bosons.

### 3.3.3 Soft terms in auxiliary gauge mediation

Once we choose $G_{aux}$ representations for the messenger fields $\Phi$, the soft terms in auxiliary gauge mediation follow directly from the general formulas in Sec. 3.1. The Dynkin index of $\Phi$ under SU(3)$_F$ is $C(\Phi)$, and $\Phi$ has charge $p_\Phi$ ($q_\Phi$) under U(1)$_{B-L}$ (U(1)$_H$). We denote

\[
\delta_i \equiv \left( \frac{M_{V_i}}{M} \right)^2,
\]

where $M_{V_i}$ is the mass of the appropriate gauge superfield (SU(3)$_F$, U(1)$_H$, or U(1)$_{B-L}$), and the generators $T^a$ always correspond to the SU(3)$_F$ generators in the gauge boson mass eigenstate basis. The soft terms are then given at the effective messenger scale (see Sec. 3.1.2), and must be RG evolved down to the weak scale.

Using the results of Sec. 3.1.2, the Higgs soft masses are given by

\[
\tilde{m}_{H_u,H_d}^2 = q_\Phi \left( \frac{\alpha_H^2}{2\pi} \right)^2 \frac{F^2}{M} f(\delta_H).
\]

The squark and slepton soft masses are given by

\[
(m_q^2)_{ij} = C(\Phi) \left( \frac{\alpha_F^2}{2\pi} \right)^2 \frac{F^2}{M} \left| \sum_a f(\delta_F^a) (T^a_q T^a_q)_{\{ij\}} + \eta p_\Phi^2 \alpha_{B-L}^2 \right| \left( \frac{F}{M} \right)^2 f(\delta_{B-L}) \delta_{ij},
\]

where $\eta = 1$ for sleptons and $1/9$ for squarks, and $\{ij\}$ indicates that these indices have been symmetrized. As noted in Ref. [87], the assumption that the up-quark Yukawa dominates implies that the off-diagonal terms in the squark and slepton mass matrices in the gauge interaction basis are extremely small, so as to be irrelevant for
flavor constraints.

Next, applying the results from Sec. 3.1.3 for the MSSM $B_\mu$ term:

$$B_\mu = 2\mu_H q_\Phi^2 \frac{\alpha_H^2}{(2\pi)^2} \frac{F}{M} h(\delta_H), \quad (3.42)$$

where the $\mu_H$ is the Higgsino mass. We can similarly calculate the $A$-terms. The holomorphic $h_{uL\tilde{L}R}$ coupling is

$$A_{h_{uL\tilde{L}R}} = \frac{\lambda_t}{(2\pi)^2} \left( 2C(\Phi)\alpha_F^2 \sum_a h(\delta_F^a) (T_q^a T_q^a)_{33} + \frac{2}{g} p_\Phi^2 \alpha_{B-L}^2 h(\delta_{B-L}) + q_\Phi^2 \alpha_H^2 h(\delta_H) \right) \left( \frac{F}{M} \right). \quad (3.43)$$

Even though the messengers are charged under all factors of $G_{aux}$, there are no crossterms containing e.g. $\alpha_H \alpha_F$. This can be seen directly from the field rescalings, Eqs. (3.13) and (3.14), which give rise to the $A$-terms.

Finally, we have the gaugino masses at three loops from Sec. 3.1.4. Summing over all visible-sector fields in Eqs. (3.19) and (3.20), we have the gluino, wino, and bino masses

$$\tilde{M}_g = \frac{\alpha_s}{4\pi^3} \frac{F}{M} \left( \frac{1}{2} C(\Phi)\alpha_F^2 \sum_a h(\delta_F^a) + \frac{1}{3} p_\Phi^2 \alpha_{B-L}^2 h(\delta_{B-L}) \right), \quad (3.44)$$

$$\tilde{M}_W = \frac{\alpha_W}{4\pi^3} \frac{F}{M} \left( \frac{1}{2} C(\Phi)\alpha_F^2 \sum_a h(\delta_F^a) + \frac{1}{2} q_\Phi^2 \alpha_H^2 h(\delta_H) + 4 p_\Phi^2 \alpha_{B-L}^2 h(\delta_{B-L}) \right), \quad (3.45)$$

$$\tilde{M}_B = \frac{\alpha_Y}{4\pi^3} \frac{F}{M} \left( \frac{5}{6} C(\Phi)\alpha_F^2 \sum_a h(\delta_F^a) + \frac{1}{2} q_\Phi^2 \alpha_H^2 h(\delta_H) + \frac{23}{9} p_\Phi^2 \alpha_{B-L}^2 h(\delta_{B-L}) \right), \quad (3.46)$$

where the prefactors from the SU(3)$_F$ contribution come from the fact that all quark superfields are flavor fundamentals and have Dynkin index 1/2. Note that the gluino mass does not depend on $\alpha_H$ at this order, and we may exploit this freedom to obtain non-standard gaugino spectra.\textsuperscript{19}

\textsuperscript{19}Due to matter charged under both gauge groups, hypercharge may mix kinetically with $U(1)_H$ and/or $U(1)_{B-L}$, and gaugino mass-mixing may also occur. However, one can show that even if this mixing is present the bino mass is still given by Eq. (3.46).
The various soft terms at the messenger scale in auxiliary gauge mediation, in particular the gaugino masses, are considerably different from those in standard gauge mediation. In auxiliary gauge mediation, the gaugino masses \( \tilde{M} \) are suppressed by two loops compared to the scalar masses \( \tilde{m} \), as opposed to standard gauge mediation where gauginos obtain mass at one loop and \( \tilde{M} \sim \tilde{m} \). For \( \alpha_H = \alpha_B-L = 0 \) we have the familiar GUT-motivated gaugino masses hierarchy at the messenger scale, \( \tilde{M}_g : \tilde{M}_W : \tilde{M}_B = \alpha_S : \alpha_W : \alpha_1 \), where \( \alpha_1 = \frac{5}{3} \alpha_Y \) is the GUT-normalized hypercharge coupling. However, by turning on \( \alpha_H \) and \( \alpha_B-L \), we can change the hierarchy among the gaugino masses at the messenger scale and the wino or bino may end up closer in mass to (or even heavier than) the gluino.

### 3.3.4 Renormalization group evolution

The above soft terms are the values at the effective messenger scale \( \min\{M_V, M\} \), which then must be RG evolved to the weak scale to determine the resulting phenomenology. The RG behavior of the soft terms has important implications for the mini-split spectrum, particularly for the Higgs and third-generation squarks, which we will focus on here. In the benchmark studies below, we perform the RG evolution of all soft parameters numerically.

In the MSSM, the RG equations for the third-generation squark masses and up-type Higgs masses contain the following terms [42, 16]:

- a one-loop term proportional to squared gaugino masses \( \tilde{M}_A^2 \);
- a two-loop term proportional to the first- and second- generation scalar masses-squared;
- a one-loop hypercharge \( D \)-term \( \alpha_Y Y_i \text{Tr}(Y \tilde{m}^2) \); and
- a one-loop term proportional to

\[
X_t = |\lambda_t|^2(\tilde{m}_{Hu}^2 + \tilde{m}_{t_R}^2 + \tilde{m}_{t_L}^2) + |A_{H_u t_L t_R}|^2. \tag{3.47}
\]
In auxiliary gauge mediation, the gaugino squared masses $M_A^2$ appear formally at six loops and are therefore negligible in the RG evolution. As has been pointed out previously in Refs. [178, 44], this absence of the gaugino contribution to the sfermion beta functions can allow the stops to run tachyonic at the weak scale. The two-loop term only contributes above the scale $\mu \approx m_{1,2}$, but if $m_{1,2} \gg m_{3,i}$, this term can also push the stops tachyonic [42, 16].

Therefore, it is non-trivial to have a mini-split spectrum with the desired vacuum structure after RG evolution of the soft parameters. In the case of auxiliary gauge mediation, the leading RG equation for the third-generation scalar soft masses and up-type Higgs in auxiliary gauge mediation is

$$\frac{d\tilde{m}^2_{Hu}}{d \log \mu} = \frac{3}{8\pi^2} X_t, \quad \frac{d\tilde{m}^2_{iR}}{d \log \mu} = \frac{2}{8\pi^2} X_t, \quad \frac{d\tilde{m}^2_{iL}}{d \log \mu} = \frac{1}{8\pi^2} X_t. \quad (3.48)$$

Compared to the full RG equation, we have kept only the $X_t$ term since the hypercharge $D$-term vanishes at the messenger scale, and as long as $m_{1,2} \approx m_t$, the two-loop term can also be neglected. Ignoring also the running of $\lambda_t$ and $A_{H_u\tilde{t}_L\tilde{t}_R}$, we can find an analytic solution to the RG equation in Eq. (3.48):

$$\tilde{m}^2_{Hu}(\mu) = \tilde{m}^2_{Hu}(M) - \frac{3\lambda^2_t}{8\pi^2} \left( \frac{|A_{H_u\tilde{t}_L\tilde{t}_R}|^2}{\lambda^2_t} + \tilde{m}^2_{Hu}(M) + 2\tilde{m}^2_t(M) \right) \log \frac{M}{\mu}, \quad (3.49)$$

$$\tilde{m}^2_{iR}(\mu) = \tilde{m}^2_{iR}(M) - \frac{2\lambda^2_t}{8\pi^2} \left( \frac{|A_{H_u\tilde{t}_L\tilde{t}_R}|^2}{\lambda^2_t} + \tilde{m}^2_{Hu}(M) + 2\tilde{m}^2_t(M) \right) \log \frac{M}{\mu}, \quad (3.50)$$

$$\tilde{m}^2_{iL}(\mu) = \tilde{m}^2_{iL}(M) - \frac{\lambda^2_t}{8\pi^2} \left( \frac{|A_{H_u\tilde{t}_L\tilde{t}_R}|^2}{\lambda^2_t} + \tilde{m}^2_{Hu}(M) + 2\tilde{m}^2_t(M) \right) \log \frac{M}{\mu}. \quad (3.51)$$

Here $\tilde{m}^2_t(M) \equiv \tilde{m}^2_{iL}(M) = \tilde{m}^2_{iR}(M)$ since both stops have the same soft mass at the messenger scale. We see that by adjusting $\tilde{m}^2_{Hu}$ to be small enough compared to $\tilde{m}^2_t$ at the messenger scale, we can always arrange for $\tilde{m}^2_{Hu}$ to run tachyonic and trigger electroweak symmetry breaking while $\tilde{m}^2_{iL,R}$ remains positive. Since the stop soft masses are controlled by the SU(3)$_F \times$ U(1)$_{B-L}$ groups while the Higgs masses is

---

$^{20}$The two-loop term only contributes when the first and second generation are moderately split from the third.
Table 3.2: Parameters for five auxiliary gauge mediation benchmark points: “Low Scale” with a low messenger mass, “High Scale” with a large messenger mass, “Flavored” with non-negligible splittings between the third-generation and first-two-generation scalars, “B − L” which employs only the \(U(1)_{B-L}\) gauge groups, and a “superWIMP” scenario which can accommodate gravitino dark matter. In SOFTSUSY, \(\tan\beta\) is an input which sets the Higgsino mass \(\mu_H\) after solving for electroweak breaking conditions. The Higgs mass is 126 GeV for each benchmark, consistent with LHC results. Except for \(\tan\beta\), all of these values are specified at the effective messenger scale \(M_{\text{eff}}\) described in Sec. 3.1.2 and set the UV boundary condition for RG evolution to the weak scale. For benchmarks where each factor of \(G_{\text{aux}}\) has its own \(\delta\), each soft term should really be run down from its corresponding effective messenger scale. However, since none of our benchmarks feature vastly different values of \(\delta\), the error incurred by taking a single messenger scale for all soft terms (here taken to be the minimum of the various effective messenger scales) is small and does not significantly change the phenomenology.

controlled by \(U(1)_H\), there is ample parameter space where this occurs.\(^{21}\)

### 3.4 Benchmark scenarios

As proof of principle that auxiliary gauge mediation can generate a realistic mini-split spectrum, we present five benchmark points which result in a Higgs mass of approximately 126 GeV. The messenger scale parameters for these benchmarks are given in Table 3.2. The RG evolution to the weak scale is performed using SOFTSUSY 3.3.8.

\(^{21}\)If we had \(S_u\) and \(S_d\) fields charged both under \(SU(3)_F\) and \(U(1)_H\) as in footnote 17, then there would be mixed contributions proportional to \(\alpha_F\alpha_H\). In that case, one may have to rely more on the \(U(1)_{B-L}\) contribution to the stop masses to find viable parameter space.
Figure 3-2: Weak scale spectra for the five benchmark points specified in Table 3.2 and described in the text. Each benchmark is split into four columns depicting (from left to right) Higgs sector scalars, inos, squarks, and sleptons. In the third and fourth columns, third generation scalars are shown in dotted lines and first two generations in solid lines.

[22], modified to allow the auxiliary gauge mediation boundary conditions at the messenger scale, and the resulting spectrum is shown in Fig. 3-2. Phenomenological discussions of the benchmarks appear in the subsequent subsections.

In all of the benchmarks, the overall scale of the spectrum is set by requiring the gluino masses to be above 1.5 TeV, to ensure consistency with current collider bounds for scenarios where the lightest SUSY particle (LSP) is a gravitino [?, 74, 2]. For the auxiliary gauge couplings to remain perturbative, this requires \( F/M > 100 \) TeV. This in turn places the sfermion mass scale at about \( \tilde{m}^2 > (10^4 \text{ GeV})^2 \), which is precisely the required scale for a 126 GeV Higgs [44]. The Higgs soft masses are independent from the squark and slepton masses, since they depend only on \( \alpha_H \) and not \( \alpha_F \) or \( \alpha_{B-L} \), but to ensure the vacuum does not break color we must have \( \tilde{m}_{H}^2 \lesssim \tilde{m}_3^2 \) (see

22It may well be the case that the operating accuracy of SOFTSUSY is less than the fine-tuning required to achieve the electroweak symmetry breaking conditions and that additional uncertainty arises through the hierarchical RG thresholds. However, we expect that the true physical spectrum is likely to be close enough to the spectrum given by SOFTSUSY for the practical purpose of demonstrating the features of this setup.
Sec. 3.3.4 and Sec. 3.5). The gravitino mass $m_{3/2}$ should be taken as a lower bound, since its mass could be lifted with multiple SUSY breaking [77] or gravitino decoupling [214, 89].

As previously mentioned in the introduction, in any mini-split model there are two different types of tunings which one must be aware of. The first tuning, which is widely appreciated, is the tuning of the Higgs sector parameters necessary to obtain a hierarchy between the electroweak symmetry breaking scale and the scalar soft masses. In the case of auxiliary gauge mediation, the Higgsino mass $\mu_H$ is a free parameter which can be tuned for this purpose.

The second tuning, not often discussed, is when one has to tune model parameters to precise values in order for the model to be viable. This is the case, for example, if typical model parameters lead to color-breaking vacua or if the model generically leads to inappropriate values for $B_\mu$. Our models avoid this second type of tuning, with only the first type of tuning which is irreducible in mini-split models. Indeed, in the benchmarks discussed here, only one parameter needs to take finely adjusted values, and the mini-split spectrum, including an acceptable Higgs sector, can be accommodated within much of the parameter space of the model.

### 3.4.1 Two SU(3)$_F \times$ U(1)$_H$ models

Our first two benchmarks utilize just the SU(3)$_F \times$ U(1)$_H$ subgroup of $G_{aux}$ to mediate SUSY breaking. Here, squarks and sleptons of a given generation receive identical soft masses from the SU(3)$_F$ mediation. The gluino obtains mass at three loops from diagrams involving just the SU(3)$_F$ gauge group, whereas the wino and bino feel two loop contributions from both gauge groups. Thus the ratio of gaugino masses is different from those found in other scenarios such as anomaly or gauge mediation. In particular, it is possible for the mass of the bino and wino to be raised closer to the gluino than in other models.

We consider two benchmark scenarios: “Low Scale” with a relatively low messenger masses, and “High Scale” with a higher messenger mass scale. We take $\delta_F \lesssim 1$ such that the generation-dependent splitting is small, and all the squark and slep-
ton generations obtain similar soft masses at the messenger scale. These scenarios economically realize the "mini-split" spectrum. There is some small splitting of generations, particularly due to the running of the stop mass, however the scalars all have mass beyond the LHC reach of $\tilde{m} \gtrsim 10$ TeV. The Higgsinos are also reasonably heavy, requiring smaller values of $\tan \beta \sim 5$. Both of these scenarios would lead to generic mini-split LHC phenomenology, with gluinos decaying through off-shell squarks in a decay chain which terminates with an invisible gravitino. Displaced vertices could potentially arise from bino decays.

A feature of this scenario compared to other mini-split models is that by including the $U(1)_H$ symmetry, the appropriate Higgs sector soft parameters, including $B_\mu$, can be generated without requiring additional couplings between the Higgs and SUSY-breaking sectors.

### 3.4.2 A flavored SU(3)$_F \times U(1)_H$ model

Taking the same SU(3)$_F \times U(1)_H$ subgroup, we can realize a "Flavored" benchmark point by taking $\delta_F \gtrsim 1$. In this case, flavor mediation generates greater masses for the first and second generation scalars, with third generation scalar masses somewhat suppressed, as described in Ref. [87]. This can make for novel mini-split spectra with some smoking gun phenomenological features. For the "Flavored" benchmark point we choose a large value of $\delta_F$ such that the third-generation squark mass is suppressed by a factor $\sim 6$ relative to the first-two-generation squarks. Since the gluino decays proceed via off-shell squarks this would lead to extremely top- and bottom-rich gluino decays, with third-generation decays a factor $6^4$ more frequent than decays involving the first-two-generation squarks. Top- and bottom-tagging would then enhance the LHC sensitivity to such flavored mini-split scenarios. Another notable feature of this scenario is that, since the SU(3)$_F$ gauge symmetry treats sleptons and squarks equally (a feature demanded by anomaly-cancellation) any flavored spectrum automatically keeps the sbottoms and staus light, alongside the stop.

This flavored benchmark also features reasonably light higgsinos, with $m_{\tilde{H}} \sim 750$ GeV and a larger value of $\tan \beta \sim 20$. Such light Higgsinos are possible as $m_{\tilde{H}_u}^2$ can be
tuned small if the amount of running is tuned. Then to obtain electroweak symmetry breaking a smaller $|\mu_H|^2$ can be tuned against $m_{H_u}^2$, leading to Higgsinos significantly lighter than the squarks and sleptons, although this is not specific to the auxiliary gauge mediation scenario.

### 3.4.3 A $U(1)_{B-L} \times U(1)_H$ model

Another interesting scenario to consider is whenever the mediation is entirely flavorless, such that gauge mediation only occurs via the $U(1)_{B-L} \times U(1)_H$ subgroup. Mediation via a $U(1)_{B-L}$ symmetry was previously considered in Ref. [44] for generating a mini-split spectrum. However, in order to generate Higgs soft parameters this gauge symmetry had to be significantly mixed with $U(1)_Y$, with the mixing parameter taking a specific value to avoid color-breaking vacua. These issues are circumvented here simply by employing the $U(1)_H$ symmetry, which can generate Higgs sector soft masses and the $B_\mu$ term at the appropriate scale.

The “$B-L$” benchmark has some very interesting features, which can be traced back to the fact that squarks carry $U(1)_{B-L}$ charge which is three times smaller than sleptons. The first obvious feature is that sleptons tend to have masses a factor $\sim 3$ larger than squarks. This would also further suppress leptonic high intensity probes. This is in sharp contrast to the situation in standard gauge mediation, where the squarks are several times heavier than the sleptons, as well as in the hypercharge-mixed mini-split model of Ref. [44].

A less immediate consequence follows from the fact that gluino soft masses are mediated via loops involving squarks, whereas the winos and bino also obtain contributions from loops of sleptons. Due to the larger slepton $U(1)_{B-L}$ charge, the bino and wino masses can be raised significantly, close to, or above the gluino mass. This is demonstrated in Fig. 3-2 where the wino is much heavier than the gluino, and the bino and gluino are almost degenerate. Such gaugino mass patterns are rather unique and do not arise in ordinary gauge-mediated realizations of mini-split. In Sec. 3.5, we show how the same gross features can arise in a more economical model with a single mediating $U(1)$ gauge group.
3.4.4 SuperWIMPs from $SU(3)_F \times U(1)_{B-L} \times U(1)_H$

Our final benchmark employs all three factors of $G_{aux}$, and was chosen to realize the superWIMP scenario [123, 124] discussed in Ref. [125]. The "SuperWIMP" benchmark has a gravitino mass of $1.9$ GeV and a bino mass of $1.6$ TeV. In gauge mediation with only a single SUSY-breaking sector, the gravitino is almost always the LSP, but once the the gravitino is heavy enough to be a viable cold dark matter candidate, gravity-mediated contributions to SUSY breaking can pollute the flavor-blind gauge-mediated soft terms and cause flavor problems. One solution is to have the current relic abundance of gravitino dark matter be produced non-thermally, through the decay of a long-lived WIMP after freeze-out. In gauge mediation, the bino typically plays the role of the WIMP and a light gravitino can be a superWIMP. Indeed a gravitino LSP and bino NLSP of the appropriate masses can also satisfy conditions on the bino lifetime from big bang nucleosynthesis and ensure that small-scale structure formation is not disrupted by free-streaming gravitinos. A full analysis of these cosmological constraints is beyond the scope of this chapter, but we note that the preferred parameter space (gravitino at $1 - 10$ GeV, bino at $1 - 5$ TeV) given in Ref. [125] is easily accommodated in our model.

3.5 A minimal mini-split model

The examples of Sec. 3.4 demonstrate a wide variety of possibilities for mini-split model building with auxiliary gauge mediation. Motivated by minimality, it is interesting to consider the smallest gauge symmetry required to generate a mini-split spectrum with the correct SM vacuum. In this case the auxiliary gauge group is some subgroup of the full available symmetry which, requiring appropriate Higgs sector soft terms and masses for colored superpartners, is

$$U(1)_{X=B-L+kH} \subset U(1)_{B-L} \times U(1)_H.$$  \quad (3.52)
Table 3.3: Parameters for the minimal auxiliary gauge mediation model with a single $U(1)_X$ gauge symmetry with lepton, quark, and Higgs charges $q_l = 1$ and $q_q = q_H = 1/3$.

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Minimal Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{\text{eff}}$ [GeV]</td>
<td>$10^{10}$</td>
</tr>
<tr>
<td>$F/M$ [GeV]</td>
<td>$7 \times 10^5$</td>
</tr>
<tr>
<td>$q_\Phi \alpha_X$</td>
<td>3.0</td>
</tr>
<tr>
<td>$\delta_X$</td>
<td>0.04</td>
</tr>
<tr>
<td>$\tan \beta$</td>
<td>3.045</td>
</tr>
<tr>
<td>$\mu_H$ [TeV]</td>
<td>51.5</td>
</tr>
<tr>
<td>$\sqrt{B}_\mu$ [TeV]</td>
<td>88.3</td>
</tr>
<tr>
<td>$m_{3/2}$ [GeV]</td>
<td>$5.3 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Figure 3-3: Particle spectra for the minimal $U(1)_X$ auxiliary gauge mediation model. Conventions follow Fig. 3-2. Due to the $B-L$ nature of the squark and slepton charges the sleptons are a factor $\sim 3$ more massive than squarks. The wino is the heaviest of the gauginos due to the large three-loop contributions involving sleptons. The gluino and bino happen to be close in mass for this benchmark.

Here $k$ denotes the freedom to choose the normalization of the Higgs charges relative to $B-L$ charges. The parameter $k$ is not entirely free as there are constraints on the charge of Higgs fields from RG evolution. From Eqs. (3.49)–(3.51) it is clear that to have radiative EW symmetry breaking and a color-preserving vacuum one requires $2\tilde{m}_{H_u}^2 \lesssim 3\tilde{m}_t^2$ at the messenger scale (assuming small $A$-terms and only considering one-loop running). For the $U(1)_X$ symmetry considered above, choosing the overall normalization by setting the usual baryon charge $q_l = 1$ constrains $q_q = 1/3$ constrains $q_H^2 \lesssim 1/6$. As long as this criterion is satisfied, there is no barrier to constructing a minimal model of auxiliary gauge mediation based on this single $U(1)_X$ gauge symmetry, with the understanding that the MSSM Yukawa couplings are generated as in Eq. (3.31) and a separate spurion may be responsible for the generation of Majorana neutrino masses.

As an example minimal scenario, consider $U(1)_X$ where the lepton charge is $q_l = 1$ and the Higgs and quark charges are $q_H = q_q = 1/3$ (i.e. $k = 1/3$). We show a "Minimal" benchmark parameter choice in Table 3.3 and the corresponding particle
spectrum in Fig. 3-3.\textsuperscript{23} As expected, the sleptons are heavier than the squarks by a factor $\sim 3$, and due to large three-loop contributions from sleptons the wino and bino masses have increased relative to the gluino, leading to a non-standard gaugino spectrum.

A full study of this minimal auxiliary gauge mediation scenario is beyond the scope of this work. However, this benchmark demonstrates that the full mini-split spectrum, with the necessary Higgs sector soft parameters and scalars two loop factors heavier than gauginos, can all be generated from a single U(1) gauge symmetry.

\section*{3.6 Conclusion}

Naturalness has long been a guiding principle for constructing models of weak scale SUSY, but the observed Higgs boson at 126 GeV raises the possibility that some tuning of parameters might be necessary for successful electroweak symmetry breaking. In this light, mini-split SUSY is an attractive scenario, and we have shown that a spectrum of heavy sfermions with light gauginos automatically arises in gauge mediation by the auxiliary group $G_{\text{aux}} = \text{SU}(3)_F \times \text{U}(1)_{B-L} \times \text{U}(1)_H$. The key ingredient is the U(1)$_H$ symmetry acting on the Higgs doublets, which generates the appropriate Higgs sector soft parameters (including $B_{\mu}$) such that only a single parameter needs to be tuned to have a viable spectrum.

The phenomenology of auxiliary gauge mediation shares many of the same features as generic mini-split models, with a few unique features. The U(1)$_H$ factor raises the masses of the bino and wino compared to standard scenarios, leading to lighter gluinos within phenomenological reach. If SU(3)$_F$ is present with $\delta^a \gtrsim 1$, then the third-generation sfermions are lighter than those of the first two generations, leading to gluino decays with top- and bottom-rich cascade decays. Mediation with the U(1)$_{B-L}$ factor gives much larger masses to sleptons than squarks, and auxiliary gauge mediation with the full auxiliary group can give rise to superWIMP gravitino

\textsuperscript{23}Again, due to the inherent uncertainties introduced with such large fine-tuning, this spectrum should be taken as demonstrative of the overall qualitative features.
dark matter. Finally, we have shown that auxiliary gauge mediation with a single abelian group $U(1)_{B-L+kH}$ can reproduce the gross features of a mini-split spectrum with the correct Higgs mass.

In our analysis, we have treated the breaking of $G_{\text{aux}}$ and the mediation of SUSY breaking as independent modules, but it is attractive to consider the possibility that auxiliary gauge breaking and SUSY breaking might be more intimately related, since both can occur at intermediate scales. Indeed, models with dynamical SUSY breaking often include spontaneously broken gauge symmetries [14, 179], some of which could be potentially be identified with $G_{\text{aux}}$. Given the model building challenge of generating the hierarchical SU(3)$_F$ flavor breaking, it is encouraging that auxiliary gauge mediation with just $U(1)_{B-L}\times U(1)_H$ (or $U(1)_{B-L+kH}$) is sufficient to generate a mini-split spectrum. On the other hand, tying SU(3)$_F$ breaking to SUSY breaking may give new insights into SM flavor. More generally, auxiliary gauge mediation is a reminder that there can be rich dynamics in the “desert” between the weak scale and Planck scale, and these dynamics may leave their imprint in novel SUSY spectra.
Chapter 4

Unbinned Methods for Halo-Independent Direct Detection

Convincing evidence for the particle nature of dark matter can come from three general areas: collider production, indirect detection (observation of the decay or annihilation products of DM), and direct detection. The latter relies on interactions of dark matter in our galactic halo with terrestrial experiments, through scattering of DM off nuclei at underground detectors. A few experiments have observed some potential signals of DM scattering, such as the long-standing DAMA annual modulation [63], the CoGeNT excess and modulation [3, 4], CRESST-II excess [34], and most recently the CDMS-Si excess [17]. However, null results from other experiments have put DM interpretations under increasing tension, with the recent results from the LUX experiment excluding the simplest possibility, spin-independent elastically scattering DM where the DM couples equally to protons and neutrons [20, 159, 95, 134].

In this chapter, we propose a new analysis technique for DM direct detection experiments, which is both independent of the unknown DM velocity distribution in our halo, and especially well suited to comparing null results at one experiment.

Further analysis of the LUX results reveal that DM interpretations of the CDMS-Si excess with unequal DM couplings to protons and neutrons [122, 159, 95, 134] now face increased tension with the LUX results. Models with exothermic scattering [156, 139, 134, 140] are now also in considerable tension with the LUX results, however there is no tension between LUX and an interpretation of the CDMS-Si excess in terms of a DM sub-component such as exothermic double-disk dark matter [223].
with a small emerging excess at another. A lesson learned from studying past DM hints is that the interplay between signals and constraints at different detectors may depend heavily on the local velocity distribution of DM, making this unknown a particularly troubling (or in some cases useful) nuisance parameter [117, 216, 221, 238]. To mitigate this uncertainty, halo-independent methods which allow the comparison of scattering rates at different detectors irrespective of the DM velocity distribution were developed [137, 136] and subsequently extended to treat detectors with multiple target nuclei [138], detector energy resolution effects [153], annual modulation signals [170], and inelastic DM scattering [68].

While the halo-independent methods are very effective in interpreting null results from DM searches in order to place unambiguous limits on the allowed scattering rates at other detectors, the interpretation of an emerging DM signal using current halo-independent methods is open to some ambiguities. Current methods require that candidate DM scattering events be grouped into bins of recoil energy. The total rate in each bin is then mapped into a halo-independent rate to be compared with the limits from other detectors. For many applications this method is appropriate (see Ref. [120] for example, which emphasizes the computational efficiency of this method), but for an emerging DM signal it is not ideal for the following reasons:

- State-of-the-art detectors achieve expected backgrounds which are very low, typically expecting $\mathcal{O}(\lesssim 1)$ background events in the DM acceptance region. As each new experimental run often leads to less than an order of magnitude increase in sensitivity, an emerging DM signal will likely come in the form of a small number of events. Many more events may follow with further experimental runs, but it is unlikely that the discovery of DM will begin with a large number of events. The binning of a small number of events is undesirable, since it is ambiguous and introduces sensitivity to the choice of bins. Hence, methods which rely on binning will not be optimal in the early stages of DM discovery.

- Current and future DM direct detection technology typically achieves excellent

\footnote{For a review of halo-independent and related approaches see [239].}
energy resolution. As the uncertainty in the energy of each candidate DM scattering event is likely to be small, bins wider than the energy resolution can only lead to the loss of important information about each event, effectively reducing the interpreted resolution and efficacy of the detector. Ideally, as much information as possible about each event should be retained in any comparison between candidate DM events and constraints from other detectors. For an emerging discovery, halo-independent methods which do not rely on binning are desirable.

The ability to compare this signal to limits from other detectors, independent of the DM velocity distribution, will be critical in assessing the validity of a signal. In addition, if a true signal of DM scattering begins to emerge, the initial stages of discovery will likely begin with a small statistical excess within a particular detector, where analysis techniques based on binning events will introduce unwanted ambiguities. In this chapter, we propose a new halo-independent method for analyzing candidate DM events which, by the above arguments, would be useful in the early stages of a DM discovery, and beyond. This builds on previous methods and relies on well-known properties of the integral over the velocity distribution of DM. The method allows for candidate DM events to be interpreted as best-fit points, with associated confidence intervals, for the DM velocity integral. These best-fit points and confidence intervals are shown to hold over all possible DM halos, and are in this sense halo-independent. Once determined, the implied values of the DM velocity integral can then be compared to limits from other detectors, allowing a halo-independent comparison between candidate DM signals and null DM experiments, free from the need to bin events and the ambiguities this introduces.

The remainder of this chapter is organized as follows. In Sec. 4.1 we review the standard halo-independent methods, including the calculation of constraints from null experiments in Sec. 4.1.1. We introduce the new method for an unbinned halo-independent interpretation of candidate DM events in Sec. 4.2; we discuss comparisons between positive signals and null results in Sec. 4.2.2, and point out a simple scaling with the DM mass in Sec. 4.2.3. The reader only interested in a short explanation
of how to apply the methods can proceed directly to Sec. 4.2.4 where all necessary
calculation steps for setting limits and for interpreting signals are briefly set out. In
Sec. 4.3 we apply the new unbinned halo-independent methods to the three anomalous
events observed in the CDMS-Si detector and compared to the current constraints
from XENON10 and LUX. We conclude in Sec. 4.4 with suggestions for areas of future
development. App. C contains a proof that our method works equally well for both
the idealized case of perfect energy resolution and the more realistic case of finite
experimental energy resolution. The work described in this chapter was undertaken
in collaboration with Patrick Fox and Matthew McCullough, with special thanks
to Prateek Agrawal, Kyle Cranmer, Brian Feldstein, Felix Kahlhoefer, Joe Lykken,
Christopher McCabe, Jesse Thaler, David J. E. Marsh, Grace Haaf, Joshua Batson,
and Tiankai Liu for helpful conversations. It is largely based on [135].

\section{4.1 Halo-independent analysis methods}

The differential event rate\footnote{Throughout this chapter we consider only spin-independent coupling of DM to nuclei, the generalization of these techniques to the spin-dependent case is straightforward.} at a direct detection experiment is

\[
\frac{dR}{dE_R} = \frac{N_A \rho \sigma n m_n}{2 m_X \mu_{nX}^2} C_T(A, Z) \int dE'_R G(E_R, E'_R) \epsilon(E'_R) F^2(E'_R) g(v_{\text{min}}(E'_R)) ,
\]

where \(m_X\) is the DM mass, \(m_n\) the nucleon mass, \(\mu_{nX}\) the nucleon-DM reduced mass,
\(\sigma_n\) the DM-nucleon scattering cross-section, \(\rho\) the local density, \(N_A\) is Avogadro’s
number, \(F(E_R)\) is the nuclear form factor which accounts for loss of coherence as the
DM resolves sub-nuclear distance scales, \(C_T(A, Z) = (f_p/f_n Z + (A - Z))\) is the usual
coherent DM-nucleus coupling factor, \(\epsilon(E_R)\) is the detector efficiency, and \(G(E_R, E'_R)\)
is the detector resolution function. The velocity integral is

\[
g(v_{\text{min}}) = \int_{v_{\text{min}}}^{\infty} \frac{f(v + \nu_E)}{v} d^3v ,
\]
where \( f(v) \) is the DM velocity distribution, and \( v_E \) is the Earth's velocity, both in the galactic frame. We ignore the small time-dependence introduced by the Earth's motion around the Sun. For elastically scattering DM the minimum DM velocity required to produce a nuclear recoil energy \( E_R \) is

\[
  v_{\text{min}}(E_R) = \sqrt{\frac{m_N E_R}{2 \mu_{N_X}^2}},
\]

where \( \mu_{N_X} \) is the nucleus-DM reduced mass. As is now standard, the constant factors which are common to all DM detectors are absorbed into a rescaled velocity integral

\[
  \tilde{g}(v_{\text{min}}) = \rho_X \sigma_n \frac{g(v_{\text{min}})}{m_X}. \tag{4.4}
\]

An observation critical to the halo-independent methods, first noted in \cite{137, 136}, is that because the velocity integrand is positive definite, \( \tilde{g}(v_{\text{min}}) \) is a monotonically decreasing function of \( v_{\text{min}} \) for any DM halo. This observation becomes very powerful in developing halo-independent methods for the comparison of multiple experiments, as now described.

### 4.1.1 Constraining \( \tilde{g}(v_{\text{min}}) - \) null results

Before considering the possibility of positive DM search results it is worthwhile to first consider the case of null experiments which can be used to constrain the velocity integral \( \tilde{g}(v_{\text{min}}) \). We follow the discussion of \cite{137}. Once a specific value of the DM mass \( m_X \) is chosen it is possible to place limits on the velocity integral \( \tilde{g}(v_{\text{min}}) \). If, at some reference minimum velocity \( v_{\text{ref}} \), the velocity integral is non-zero \( \tilde{g}(v_{\text{ref}}) \neq 0 \) then, since the velocity integral is monotonically decreasing, the unique form for the velocity integral which minimizes the total number of events for a given \( \tilde{g}(v_{\text{ref}}) \neq 0 \) is

\[
  \tilde{g}(v_{\text{min}}) = \tilde{g}(v_{\text{ref}}) \Theta(v_{\text{ref}} - v_{\text{min}}). \tag{4.5}
\]

Thus, for a given choice of DM mass, it is possible to constrain the largest value of \( \tilde{g}(v_{\text{ref}}) \) allowed by a given null experiment by constraining the velocity integral.
Eq. (4.5) with standard methods. As this choice minimizes the total number of events for a given \( \tilde{g}(v_{\text{ref}}) \), limits calculated in this way represent the most conservative limits possible over all halos. In other words, if a certain value of \( \tilde{g}(v_{\text{ref}}) \) constrained in this way is excluded it is excluded for all possible halos. However, if it is not excluded by this approach it may still be excluded for many reasonable halos, e.g. the standard halo model (SHM), but just not for the distribution of Eq. (4.5) which corresponds to a DM stream at speed \( v_{\text{ref}} \). This process is repeated for different values of \( v_{\text{ref}} \) to build up a continuous exclusion contour in \( \tilde{g}(v_{\text{ref}}) \) over all \( v_{\text{ref}} \).

### 4.2 Discovering \( \tilde{g}(v_{\text{min}}) \) – positive results

The most sensitive, and arguably least ambiguous DM detectors, strive to keep backgrounds low enough that \( \lesssim \mathcal{O}(1) \) background events are expected in a given run. They also typically have excellent energy resolution, such that \( \Delta E_R / E_R \ll 1 \). These factors combined suggest that the initial emergence of a DM discovery will likely be in the form of a relatively small number, \( N_0 \), of events observed at discrete energies \( E_i \). To establish the consistency of such a scenario it will be important to then compare this potential DM discovery with limits from other experiments, ideally in a context free of uncertainties in the DM halo. Clearly an optimal route is to compare constraints on \( \tilde{g}(v_{\text{min}}) \) from the null experiments (described in Sec. 4.1.1) with the non-zero values of \( \tilde{g}(v_{\text{min}}) \) hinted at by the emerging DM signal.

For positive signals, all current methods require the ad-hoc choice of a set of energy bins and then the calculation of upper and lower limits on the signal within these bins using the observed events and estimated backgrounds. These energy bins and preferred rates in each bin are then converted into \( v_{\text{min}} \)-space bins and preferred values of \( \tilde{g}(v_{\text{min}}) \) in each bin, subject to the constraint that the velocity integral is monotonically decreasing. The problems with such a method are immediately apparent. For emerging hints the number of events in the energy range of the detector will be small and binning a small number of events is a statistically questionable exercise from the outset, open to ambiguities and introducing issues with bin choice.
Also, if a detector has good energy resolution, then valuable information is lost by binning the data in bins much larger than the experimental resolution, reducing the efficacy of any interpretation of the DM hint. Most crucially, binning data in bins of width much greater than the experimental resolution may lead to misinterpretation of the halo-independent constraints on this DM hint. Conversely, choosing bins of width much smaller than the energy resolution would, in the limit of a small number of events, smear single events across bins.

Ideally, it would be possible to map an emerging DM hint to $\tilde{g}(v_{\text{min}})-v_{\text{min}}$ space in a way which preserves as much information as possible. In the case of detectors with excellent energy resolution this is of the utmost importance. But even for detectors with poor energy resolution there is information in the positions of the events and maintaining that information means employing methods which avoid binning the data.

4.2.1 The method

A method commonly used in fitting a model with free parameters to unbinned data is the extended maximum likelihood method [54] which is desirable over the standard likelihood method as the normalization of a given rate is taken into account. When applied to a DM direct detection experiment which has observed $N_O$ events, in the energy range $[E_{\text{min}}, E_{\text{max}}]$, the extended likelihood is

$$\mathcal{L} = \frac{e^{-N_E}}{N_O!} \prod_{i=1}^{N_O} \left. \frac{dR_T}{dE_R} \right|_{E_R=E_i},$$

(4.6)

where $dR_T/dE_R$ contains signal and background components and

$$N_E = \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{dR_T}{dE_R} dE_R,$$

(4.7)

is the total number of events expected for a given set of parameters. We may compare different parameter choices by considering the log-likelihood, $L = -2 \log(\mathcal{L})$ which is minimized for a good fit and grows with decreasing quality of fit. Discarding constants
irrelevant to the fitting procedure we have

\[ L/2 = N_E - \sum_{i=1}^{N_0} \log \left| \frac{dR_T}{dE_R} \right|_{E_R=E_i}. \] (4.8)

Using the DM rate, in terms of \( \tilde{g}(v_{\text{min}}) \), as presented in Eq. (4.1), and including a background component

\[ \frac{dR_T}{dE_R} = \frac{dR_{BG}}{dE_R} + \frac{dR_{DM}}{dE_R} \] (4.9)

\[ = \frac{dR_{BG}}{dE_R} + \frac{N_A m_n}{2 \mu_{nx}^2} C_f^2(A, Z) \int dE_R' G(E_R, E_R') \delta(E_R') \tilde{g}(v_{\text{min}}(E_R')) \] (4.10)

where the first term accounts for the (small) estimated backgrounds and the last term the DM signal. There now appears to be a barrier to calculating \( L \) since there are an infinite set of possible DM halos to consider as one must also make a choice of the form of \( \tilde{g}(v_{\text{min}}(E_R)) \) not only at each event, but over the whole range of measurable energies since the total number of events is calculated as the integral over this energy range.

For simplicity let us first consider the case with perfect energy resolution \( G(E_R, E_R') = \delta(E_R - E_R') \). A given set of events corresponds to a set of \( N_O \) hypothetical values of \( \tilde{g}_i \equiv \tilde{g}(v_{\text{min}}(E_i)) \) as well as the form of \( \tilde{g}(v_{\text{min}}(E_R)) \) interpolating between the \( \tilde{g}_i \). However, Eq. (4.8) penalizes against the total number of events predicted, since \( L \) increases as \( N_E \) increases. Thus, since \( \tilde{g}(v_{\text{min}}(E_R)) \) is monotonically decreasing, the best fit out of all possible DM halos is the one which minimizes the total number of events predicted in any interval \( E_{i-1} < E_R < E_i \) between events. This is accomplished by choosing a constant value \( \tilde{g}(v_{\text{min}}(E_{i-1} < E_R \leq E_i)) = \tilde{g}_i \)

which is illustrated in Fig. 4-1.

This form of \( \tilde{g}(v_{\text{min}}) \) is quite robust. Indeed, in App. C we prove using variational techniques that the best-fit \( \tilde{g}(v_{\text{min}}) \) is still a sum of \( N_O \) step functions even in the case of a very general resolution function; the only difference is that the positions \( \tilde{v}_i \) of the steps may now shift to the right of their position in the scenario with perfect

\(^4\)We define \( E_0 \) to be the lower threshold of the experiment, \( E_{\text{min}} \).
Figure 4-1: A schematic representation of all halo possibilities for $\bar{g}(v_{min})$. If an experiment observes a number of events consistent with DM scattering, in this case three events of energy $E_i$, then hypothetical values of $\bar{g}(\vec{v}_{i-1} < v_{min} \leq \vec{v}_i) = g_i$ may be chosen where the positions of the steps $\vec{v}_i$ are given by $v_{min}(E_i)$ in the case of perfect energy resolution, and are allowed to float as free parameters if the energy resolution is non-zero. The solid blue curve will always minimize the extended log-likelihood, both in the case of perfect energy resolution and also with resolution effects included as demonstrated in App. C. Conversely the dashed red curve corresponds to the worst possible fit out of all halos, which is infinitely bad if the velocity integral between $v_{low}$ and $v_1$ is taken to infinity. Here, $v_{low}$ ($v_{high}$) is the velocity that corresponds to the low (high) energy threshold of the experiment. To determine the range of halos implied by the DM candidate events the parameters $g_i$ and $\vec{v}_i$ may be varied, consistently choosing the solid blue curve in the likelihood, in order to determine the best-fit values and confidence intervals for $g_i$.

energy resolution, $\vec{v}_i \geq v_{min}(E_i)$. Thus, in all cases of interest, the form of the velocity integral which minimizes the extended likelihood for $N_O$ observed events is a sum of at most $N_O$ step functions\textsuperscript{5}, whose $2N_O$ free parameters (heights and positions) may be determined numerically in a straightforward manner, or analytically in the case of perfect energy resolution.

To calculate the log-likelihood it helps to define

$$
\mu_i = \left. \frac{dR_{BG}}{dE_R} \right|_{E_i},
$$

(4.11)

\textsuperscript{5}Two step functions of the same height are equivalent to one step function, so in practice there may be fewer than $N_O$ steps.
the differential background rate evaluated at the energy of each event \( E_i \), and

\[
\tilde{\mu}_i = \left. \frac{dR_{DM}}{dE_R} \right|_{E_i} = \frac{N_A m_n}{2 \mu_{nx}^2} C_T^2(A, Z) \sum_{j=1}^{N_O} \tilde{g}_j \int_{E_{j-1}}^{E_j} dE'_R G(E_i, E'_R) \epsilon(E'_R) F^2(E'_R),
\]

(4.12)

which is the differential scattering rate at each event \( E_i \). Here \( \tilde{E}_i \) are the positions of the steps in the halo velocity integral \( \tilde{g} \) (written as a function of recoil energy \( E_R \)) satisfying \( \tilde{E}_i = E_i \) in the case of perfect energy resolution. Another useful quantity is

\[
\tilde{N}_T = \frac{N_A m_n}{2 \mu_{nx}^2} C_T^2(A, Z) \sum_{j=1}^{N_O} \tilde{g}_j \int_{E_{j-1}}^{E_j} dE'_R \epsilon(E'_R) F^2(E'_R),
\]

(4.13)

which is simply the total number of DM events expected. In terms of these quantities (which depend on the \( \tilde{g}_i \) and the \( \tilde{E}_j \)) the extended log-likelihood now decomposes as,

\[
L = \sum_{i=1}^{N_O} L_i = 2 \left( \tilde{N}_T + N_{BG} - \sum_{i=1}^{N_O} (\log(\tilde{\mu}_i + \mu_i)) \right)
\]

(4.14)

\[
\rightarrow 2 \left( \tilde{N}_T - \sum_{i=1}^{N_O} (\log(\tilde{\mu}_i + \mu_i)) \right),
\]

(4.15)

where in going to the last line irrelevant constants have again been discarded. In this way the construction of the likelihood function for \( N_O \) events simply requires the straightforward calculation of the quantities defined in Eq. (4.11), Eq. (4.12), and Eq. (4.13).

Eq. (4.15) contains all of the information required to find the best-fit values and confidence intervals for the DM halo integral. To find the best-fit values \( \tilde{g}_{i,\min} \) and the best-fit positions of the steps \( \tilde{E}_{i,\min} \), the likelihood may be numerically minimized to find \( L_{\min} \), subject to the monotonicity constraint which must be imposed for any DM interpretation i.e. \( \tilde{g}_{i,\min} \geq \tilde{g}_{i+1,\min} \). The confidence intervals in each \( \tilde{g}_i \), denoted \( \Delta \tilde{g}_i^{\pm} \), may be found by determining the extremum values satisfying \( L(\tilde{g}_i \pm \Delta \tilde{g}_i^{\pm}) = L_{\min} + \Delta L \), for some \( \Delta L \) which is determined from the statistical confidence desired. The other values of \( \tilde{g}_{j \neq i} \) and the positions of the steps \( \tilde{E}_j \) should also be allowed to

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\(^6\)The resolution function has already been integrated over in this expression.
vary when determining the extremum values. It should be noted that in determining the confidence intervals, the monotonicity constraint must still be imposed, thus in determining $\Delta \bar{g}_i^\pm$, the other $\bar{g}_{i' \neq i}$ may not always take their best-fit values.

To determine the allowed region at a given statistical confidence level, one would typically use the $\Delta L$ corresponding to the $\chi^2$ value for the number of parameters in the fit. However, this approach breaks down when the number of events is small. Furthermore, since parameter points which extremize $\Delta L$ typically live on the boundary of the parameter space, where the monotonicity constraint is saturated (possibly multiple times), the constraint reduces the number of effective parameters. Thus the determination of $\Delta L$ is best done through Monte Carlo simulation. Taking the underlying probability distribution function to be given by the best fit values $(E_{\text{min}}, \bar{g}_{\text{min}})_i$, combined with the background model, one generates a large set of fake data. Each iteration contains the same number of events as was observed in the experiment. For every pseudo-experiment the likelihood is extremized as before and the best fit values for that run are recorded. For a large enough set of pseudo-experiments, the mean best fit values $(\bar{E}, \bar{g})_i$ should lie close to the original best fit found for the actual data. Together the mean $\bar{\mu}$ and the covariance matrix $\sigma$ of the best-fit parameters define the surface of a hyper-ellipsoid of radius $\sqrt{\Delta L}$ in parameter space,

$$(\bar{x} - \bar{\mu})\sigma^{-1}(\bar{x} - \bar{\mu}) = \Delta L. \quad (4.16)$$

The $\Delta L$ corresponding to, for example, 90% C.L. is determined by the radius of the hyper-ellipsoid that contains 90% of the pseudo-experiments. The region of parameter space that contains the best-fit parameters for the actual data at 90% confidence is within this $\Delta L$ of the actual best fit $L_{\text{min}}$.

As an infinite number of possible halos have been discarded, one may wonder whether this method actually captures the full ranges for $\bar{g}_i$ at the desired confidence level. For a given $\bar{g}_i$, a non-minimal halo not saturating the monotonicity constraint, i.e. one for which $\bar{g} (v_{\text{min}}(E_{i-1} < E_R < E_i)) > \bar{g}_i$, would only increase the value of $\bar{N}_T$ and therefore the log-likelihood, meaning that a smaller range of $\bar{g}_i$ would be allowed.
with respect to the global minimum of the likelihood. Thus, rather than testing all possible halos to determine the best-fit values of, and allowed range of, the $\tilde{g}_i$ one can instead make the the minimal (saturating) choice, $\tilde{g}(v_{\text{min}}(E_{i-1} < E_R < E_i)) = \tilde{g}_i$. The best-fit points found this way will be the best fit out of all possible halos and the confidence intervals $\Delta \tilde{g}_i^\pm$ necessarily encompass the maximally allowed ranges. This means that the envelope of allowed $\tilde{g}_i$ captures all halos for which the extended likelihood is within $\Delta L$ of the minimum.

4.2.2 Comparing with null results

Although a DM hint may suggest non-zero values of $\tilde{g}_i$ for each anomalous event, it is desirable to compare these values in a halo-independent way with constraints from detectors which do not observe a signal. As described in [137], and Sec. 4.1.1, the most conservative limit on the velocity integral at a specific value of $v_{\text{min}} = v_{\text{ref}}$, denoted $\tilde{g}(v_{\text{ref}})$, may be determined by considering limits on the function $\tilde{g}(v_{\text{min}}) = \tilde{g}(v_{\text{ref}})\Theta(v_{\text{ref}} - v_{\text{min}})$.

Calculating limits on $\tilde{g}(v_{\text{min}})$ in this way, and the best-fit values and confidence intervals for $\tilde{g}_i$ suggested by a DM hint using the method above, leads to plots such as Fig. 4-2, showing experimental limits and the best-fit values and confidence envelope for the velocity integral. It should be emphasized that the envelope of $\tilde{g}(v_{\text{min}})$ does not imply that any curve passing through the envelope will have a log-likelihood value of $L \leq L_{\text{min}} + \Delta L$, but it does imply that there exists a curve which passes through any single point in the envelope within a confidence interval satisfying $L \leq L_{\text{min}} + \Delta L$. Furthermore, no curve with $L \leq L_{\text{min}} + \Delta L$ lies outside the envelope.

The most important information in any such plot is the interplay between the limits curve and the preferred envelope in the velocity integral. Consider a point on the lowest boundary of the envelope in $\tilde{g}(v_{\text{min}})$ at a point $v'_{\text{min}}$, denoted $\tilde{g}_-(v'_{\text{min}})$. The halo which leads to this value of $\tilde{g}(v'_{\text{min}})$ at $v'_{\text{min}}$, but would predict the smallest possible number of events in any detector, corresponds to $\tilde{g}(v_{\text{min}}) = \tilde{g}_-(v'_{\text{min}})\Theta(v'_{\text{min}} - v_{\text{min}})$. However, it is precisely this halo shape which has been constrained by the null experiment. Hence, if a single point along the lowest boundary of the preferred
envelope for $\tilde{g}(v_{\text{min}})$ is excluded by a null experiment then there is no halo within $\Delta L$ of the minimum of the likelihood for which the hint could be consistent with the null experiment. In other words, it is excluded by the null experiment independent of any uncertainties in the DM halo.

4.2.3 Varying $m_\chi$

The halo-independent methods are clearly of great value in comparing experimental results whilst avoiding the significant uncertainties in the velocity distribution of the DM. One perceived weakness of this approach is that it appears the calculations must be performed under the hypothesis of a single DM mass, $m_\chi$, and to consider a different DM mass $m'_\chi$ the entire calculation must be repeated again, leading to a proliferation of plots when presenting the results. However, assuming the detector is built from a single material, once limits and best-fit velocity integrals have been calculated for a single DM mass $m_\chi$, it is simple to map them to the analogous quantities for a different mass $m'_\chi$.

Let us first consider the energy of a scattering event. The minimum DM velocity required is given by Eq. (4.3) which, for a specific scattering energy, immediately gives the relationship between $v_{\text{min}}(E_R)$ for a DM mass $m_\chi$ and $v'_{\text{min}}(E_R)$ for a DM mass $m'_\chi$,

$$v'_{\text{min}}(E_R) = \frac{\mu_{N_X}}{\mu_{N_X'}} v_{\text{min}}(E_R),$$

(4.17)

mapping a point on the $v_{\text{min}}$ axes for $m_\chi$ to a point on the $v'_{\text{min}}$ axes for $m'_\chi$ while preserving the ordering of the scattering events. It should be noted that this mapping is nucleus-dependent, and shifts limits and hints from different detectors by differing amounts. Furthermore, as halo-independent limits and best-fit points are calculated assuming a flat velocity integral between any neighboring events, the total number of events predicted between any two events only changes by a global normalization factor. This normalization can be found from Eqs. (4.1) and (4.4), where it is clear that under a change in the DM mass, $m_\chi \rightarrow m'_\chi$, the required normalization of $\tilde{g}$,
whether as a best-fit point, or a point on an exclusion curve, will be shifted to

\[ \tilde{g}' = \frac{\mu_{nx}'^2}{\mu_{nx}^2} \tilde{g} \]  

(4.18)

These two transformations, Eq. (4.17) and Eq. (4.18), define a unique mapping between any point on the \( \tilde{g} - u_{\min} \) plane for a DM mass \( m_X \), to a new point on the \( \tilde{g}' - u'_{\min} \) plane for a DM mass \( m'_X \).

This has important implications for the presentation of DM direct detection results: if new DM limits, or hints, are presented in plot form in the halo-independent framework for a specific DM mass, then a single plot alone contains all of the information required for all DM masses. Thus, if an experimental collaboration released such a plot it would be possible to study halo-independent limits for any DM mass. Even if many details of the experimental analysis are not publicly available, this would enable the robust application of the DM results to different halo-independent scenarios by external groups.

As the shift in the normalization affects all \( \tilde{g} \) equally, the same minimum value of the log-likelihood Eq. (4.15) will be found for any DM mass, and the halo-independent method contains no information on the preference of data from a single experiment for a specific DM mass. A preferred mass may only be determined by appealing to a specific halo, or through requirements on the upper limits on \( u_{\min} \) due to the galactic escape velocity, or by combining data from multiple experiments.

4.2.4 Summary

It is useful at this point to summarize the steps required to perform an unbinned halo-independent analysis with real data. To calculate exclusion contours in \( \tilde{g}(u_{\min}) \) from a null experiment, it is only necessary to calculate limits in the usual way, with the exception that at a point \( u_{\text{ref}} \) the usual velocity integral is replaced with \( \tilde{g}(u_{\min}) = \tilde{g}(u_{\text{ref}})\Theta(u_{\text{ref}} - u_{\min}) \) and an upper bound is calculated for the constant \( \tilde{g}(u_{\text{ref}}) \). This process, which was described in [137] is repeated for different values of \( u_{\text{ref}} \) to build up an exclusion contour.
In the case of an experiment with good energy resolution which observes an excess of events over an expected $O(\lesssim 1)$ background events, it is only necessary to assume the velocity integral $\tilde{g}(v_{\text{min}})$ takes the form of at most $N_O$ step functions with undetermined heights and positions. It is then necessary to calculate $\mu_i$, $\tilde{\mu}_i$, for each event and the total number of events predicted $\tilde{N}_T$, where these quantities are defined in Eq. (4.11), Eq. (4.12), and Eq. (4.13). With these quantities in hand one simply varies the heights and positions of the steps to find the minimum of the sum $L/2 = \tilde{N}_T - \sum_{i=1}^{N_O} \log (\tilde{\mu}_i + \mu_i)$ with the additional constraints that $\tilde{g}_{i,\text{min}} \geq \tilde{g}_{i+1,\text{min}}$. The uncertainty on these determinations, at a given confidence level, is given by finding the variations, $\pm \Delta \tilde{g}_{v_{\text{min}}}$, which saturate $L = L_{\text{min}} + \Delta L$ (also allowing the positions of the steps to vary) to construct an envelope of preferred values for $\tilde{g}(v_{\text{min}})$. The ranges $\pm \Delta \tilde{g}_i$ encapsulate the full envelope of possibilities of all DM distributions which are monotonic and satisfy $L = L_{\text{min}} + \Delta L$. The determination of the relevant $\Delta L$ is best done by carrying out pseudo-experiments, as described in Section 4.2.

Once these limits and hints have been calculated and compared for a particular DM mass they have effectively been compared for all masses, assuming the DM scatters elastically and the detector consists of a single target.

### 4.3 Application to real data:

**CDMS-Si versus XENON and LUX**

The new halo-independent method is now employed to investigate the consistency between the $\sim 3\sigma$ excess of events observed by the CDMS-Si collaboration [17] and the most constraining null results from the xenon-based detectors, which are currently the XENON10 and LUX experiments. This not only illustrates the utility of the method for detectors with good energy resolution and a small number of observed events, but also represents the first halo-independent unbinned comparison between the CDMS-Si excess and the recent LUX results.

The S2-only XENON10 analysis [33] is used, with the ionization yield $Q_y$ also
Figure 4-2: Halo-independent interpretation of the CDMS-Si events versus constraints from XENON10 and LUX assuming elastic, spin-independent scattering with equal couplings to protons and neutrons (left panel) and with couplings tuned to maximally suppress the sensitivity of xenon experiments (right panel). The preferred envelope and constraints are both calculated at 90%. The best-fit halo is inconsistent with the LUX results and only a small section of the lower boundary of the preferred halo envelope for CDMS-Si is compatible with the null LUX results, meaning that only a small range of DM halos are compatible with the LUX results for which the extended likelihood is within $\Delta L$ of the best-fit halo. If the DM-nucleon couplings are tuned to maximally suppress scattering on xenon, the best-fit DM interpretation is still inconsistent with the LUX results, however the range of viable halos is increased. The curve for the SHM is also shown, giving a good fit to the CDMS-Si data as well as a curve for the best-fit halo which minimizes the extended likelihood.

The LUX collaboration have recently announced results from the first run [20]. The estimated LUX background distributions are not yet publicly available, making a profile likelihood ratio (PLR) test statistic analysis impossible. In [144] it was shown that for light DM the vast majority of nuclear recoil events would actually lie below the mean of the AmBe and Cf-252 nuclear recoil calibration band. The reason for this is that for a given low $S_2$ signal the $S_1$ signal is likely to have appeared above threshold due to a Poisson fluctuation. As there are no events in the region expected for light DM scattering (or equivalently low energy events) the DM event detection efficiency provided in [20] can be used to calculate the total number of expected events.

taken from [33]. We take the detector resolution function $G(E_R, E'_R)$ to be a Gaussian with energy-dependent width $\Delta E_R = E_R/\sqrt{E_R Q_y(E_R)}$. The acceptance is 95%, and the exposure is 15 kg days. Yellin’s ‘Pmax’ method [269] is used to set limits.
for a light DM candidate and then a Poisson upper limit can be set for zero observed events. We find excellent agreement with the estimated limits from [95] and good agreement with the official LUX results for the light DM region.

For CDMS-Si three events were found in 140.2 kg days of data [17]. We take the detector resolution function $G(E_R, E'_R)$ to be a Gaussian and assume a conservative detector resolution of 0.5 keV. The acceptance is taken from [17]. The background contributions are taken from [222] with normalization such that surface events, neutrons, and $^{206}$Pb, give 0.41, 0.13, and 0.08 events respectively. The best-fit points and confidence regions are calculated following the method described in Sec. 4.2, the confidence intervals are calculated for a variation $\Delta L = 9.2$, where $L$ is the total log-likelihood. This value of log-likelihood corresponds to a chi-squared distribution for six degrees of freedom and one constraint, thus five free parameters altogether where there are six degrees of freedom from the heights of each step and the step positions and there is one constraint due to the monotonicity constraint. As parameter points which extremize $\Delta L$ typically live on the boundary of the parameter space where the constraint is saturated the constraint effectively reduces the number of effective parameters. Thus we choose $\Delta L = 9.2$ as this corresponds to the $\chi^2$ value for five parameters and a confidence interval of 90%. We were led to this choice numerically by generating large sets of fake data from a given underlying three-step-function distribution. For each set of fake data we then perform the usual procedure of allowing a step for each event, and then varying the heights and positions of the steps to find the best-fit halo for those events. We then compare the best-fit value of the log-likelihood for these generated events to the best-fit value for the true underlying halo and find that 90% of the results lie within a distribution which we find to be very well approximated by a $\chi^2$.\footnote{We thank Brian Feldstein and Felix Kahlhoefer for conversations regarding the choice of log-likelihood.}

In Fig. 4-2 we show the halo-independent constraints on an elastically scattering spin-independent DM scattering interpretation of the CDMS-Si events. There is some tension between the CDMS-Si excess and the LUX results independent of the DM halo.
if the DM couples equally to protons and neutrons. The lower energy events may still be consistent with LUX, however with a reduced number of events the significance of the excess is reduced. Even when couplings are tuned to maximally suppress scattering on xenon [122], the best-fit elastically scattering DM interpretation of the highest energy event is in tension with the LUX results. The best-fit halo interestingly takes the form of two step functions. Although there are three events, the Gaussian smearing leads to a best-fit halo which only has two steps, whereas in the case of perfect energy resolution there would be three steps.

Thus, independent of uncertainties in the DM halo, and free from uncertainties introduced by binning the three anomalous CDMS-Si events, a DM interpretation of this excess faces some tension with the LUX results. This tension is reduced if the DM-proton and neutron couplings are tuned to maximally suppress scattering on xenon, however even when exploiting this freedom there is still tension with the LUX results. A $\tilde{g}(v_{\text{min}})$ curve is also shown for the SHM to demonstrate that the CDMS-Si events give a good fit to the SHM. We also note that the CDMS data alone prefer a DM contribution over a background-only description, $\tilde{g} = 0$. This is not surprising, since by allowing for general speed distributions, the lowest energy excess event in any data can always be fit by the DM hypothesis, and thus the overall fit can be improved.

Unlike the relative quality of the fits from the background-only hypothesis compared to signal plus background, the absolute quality of the fit cannot be determined by the methods employed here. Approaches which determine the goodness of fit but do not requiring binning have been developed (for a review see [263]), but their application to small data sets is not well understood. To determine the behavior of these techniques for the small number of events in direct detection experiments would require extensive modelling in Monte Carlo, which is beyond the scope of this work.

In Fig. 4-3 we show the result of using the mapping, Eq. (4.17) and Eq. (4.18), from the points in Fig. 4-2 for $m_X = 9$ GeV to curves for other masses, demonstrating that an exclusion curve, or best-fit points, for a single DM mass contains all of the information necessary to translate the curve of best-fit points in a halo-independent way for different masses. This confirms that the presentation of new experimental results
in a halo-independent manner for a single DM is a very efficient way to communicate the halo-independent information.

### 4.4 Conclusion

The DM direct detection field continues to evolve rapidly. The richness and effectiveness with which this dark frontier is explored relies on multiple experiments and detection strategies being employed. If an experiment begins to observe events consistent with DM scattering it will be crucial to confirm or refute this possibility with a separate independent experiment which uses different techniques and a different target nucleus. Previously developed halo-independent methods significantly reduce the systematic errors in such a comparison by eliminating the uncertainties due to the unknown DM velocity distribution. In this work these methods have been extended to enable a halo-independent analysis of candidate DM events without having to resort to event binning, which is inappropriate for a small number events and for detectors with good energy resolution, as would be expected in the circumstances of an emerging DM discovery. This method was developed for the simplest scenario of elastically scattering DM, however it would be interesting to extend it to include non-minimal
scenarios such as inelastic or exothermic DM, or non-isotropic scattering.

The method we have described uses the standard approach of minimizing the extended likelihood, which has the advantage of being a well known technique in the field and thus straightforward for experimental collaborations to implement. Furthermore, it has the feature that results from multiple experiments can be straightforwardly added to the likelihood to carry out a combined analysis, although we have not studied such combinations here. This is true for both excesses and limits. It would be interesting to see if other statistical techniques, which do not require binning, give similar results. In addition to being straightforward for the experimental collaborations to implement, and reducing one of the systematic uncertainties that plague the interpretation of their results, we reemphasize that this does not come at the expense of complicating the presentation of their results. For DM scattering elastically in a detector with a single target, the results need only be presented for a single DM mass as this contains all necessary information; the extension to other masses is straightforward to calculate from the results for a single mass. In addition this method provides a halo-independent analogue of the usual comparison between limits and preferred regions.

Finally, as a test example we applied our technique to the recent results from CDMS and LUX. In accordance with expectations an unbinned halo-independent analysis of the three anomalous CDMS-Si events shows that for elastic, spin-independent scattering the CMDS-Si events are in tension with the null results from the LUX detector. If a DM interpretation of the CDMS-Si excess is to be found with no tension from the LUX results, this analysis suggests it will require non-standard DM scenarios.
In this thesis I have described several approaches towards resolving the outstanding (and related) problems of the identity of dark matter and the mass of the Higgs boson in light of supersymmetry. The unifying theme behind these approaches has been the introduction of new forces and gauge groups beyond the Standard Model, either in concrete models, or in a model-independent fashion which allows for more complicated possibilities beyond the minimal scenario. In Chapter 2, I described a search for dark matter using neutrino experiments, with interactions mediated by the dark photon $A'$ of a new U(1) gauge group. With the minimal visibly-decaying dark photon recently ruled out as an explanation for the muon $g - 2$, an invisibly-decaying dark photon is still a viable explanation in some regions of parameter space, and the search for dark photons at DAEδALUS can help close this remaining window. In Chapter 3, I constructed a concrete model of mini-split supersymmetry using auxiliary gauge mediation, reconciling the observed Higgs mass of 126 GeV with the non-observation of superpartners thus far at the LHC while ensuring that RG evolution of soft terms does not spoil the SM vacuum structure. In particular, one benchmark spectrum provided a viable gravitino dark matter candidate through the super-WIMP paradigm. Finally, in Chapter 4, I investigated the extent to which one could attribute a direct-detection signal to dark matter, given a signal with very few events and assuming nothing about the velocity distribution of the dark matter halo in our galaxy. The methods I described can be put to immediate use by
experimentalists in drawing conclusions about dark matter, independent of the dark matter self-interactions which may generically give rise to non-Maxwellian velocity distributions.

Here, I briefly outline some future directions for research, building on the body of work in this thesis.

5.1 Halo-independent generalizations

The principal observation of halo-independent methods, that the velocity integral \( g(v_{\text{min}}) \) is monotonically decreasing, is extremely robust and lends itself to a number of generalizations.

5.1.1 Analyses without mass assumptions

As described in Chapter 4, the limits and preferred value regions in the \( \tilde{y} - v_{\text{min}} \) plane exhibit a simple scaling with the DM mass \( m_X \). For each choice of \( m_X \), this scaling is linear, so there exists a change of variables which compresses the halo-independent analysis onto a single plot, containing information about all DM masses \[98\]. This facilitates the comparison between different experiments without a fiducial choice of DM mass, allows one to draw conclusions about the relative consistency of two experiments which are valid for all DM masses, and may even allow an analytic determination of the best-fit DM mass from the joint likelihood function of several experiments.\[1\]

5.1.2 Beyond elastic scattering

Alternatively, one may generalize away from elastic scattering kinematics. If the DM spectrum consists of two or more closely-spaced states, there may be up- or down-scattering at direct detection experiments, known as “inelastic” [255] or “exothermic” [156] DM respectively. In that case, the function \( v_{\text{min}}(E_R) \) is not monotonic, instead

\[\text{1 I am indebted to Matthew McCullough for this observation.}\]
taking the form
\[ v_{\text{min}}(E_R) = \sqrt{\frac{1}{2m_N E_R}} \left| \frac{m_N E_R}{\mu_N} + \delta \right|, \tag{5.1} \]
where positive (negative) \( \delta \) corresponds to inelastic (exothermic) scattering with mass difference \( |\delta/2| \) between the initial and final DM states. However, the monotonicity of \( g(v_{\text{min}}) \) as a function of \( v_{\text{min}} \), combined with knowledge of the functional form of \( v_{\text{min}}(E_R) \), allows a straightforward generalization of the unbinned halo-independent method even for finite energy resolution. The same techniques may be used to derive halo-independent bounds in the case of a multi-level DM system, independent of the relative cross-sections for scattering into the various DM states. This is yet another step towards model-independence, allowing for the possibility of more complicated interactions between the dark sector and the Standard Model.

5.1.3 Cross section bounds and escape velocities

Since the monotonicity of \( g(v_{\text{min}}) \) has proven so useful in deriving halo-independent conclusions, one might wonder whether it is also possible to exploit the normalization of the velocity distribution,\(^2\)
\[ \int_0^\infty f(v) d^3v = 1. \tag{5.2} \]
In fact, this condition, combined with the positivity of \( f(v) \), gives rise to the simple inequality
\[ g(v_{\text{min}}) \leq \frac{1}{v_{\text{min}}} \tag{5.3} \]
(first noted in Ref. [195] as a constraint on parameterizations of pseudodata) which can be used to set a lower bound on the normalized DM-nucleon cross section \( \sigma_n \) for a given DM number density. In addition, while the halo-independent methods thus far described have made no assumptions whatsoever about the DM halo, real astrophysical halos have escape velocities. Given the reasonable assumption of a maximum velocity \( v_{\text{esc}} \) beyond which the DM velocity distribution has no support,

\(^2\)I am indebted to David Pinner for suggesting this strategy.
one may also derive an upper bound on $\sigma_a$ from the inequality

$$g(v_{\text{min}} = 0) \geq \frac{1}{v_{\text{esc}}}.$$  \hfill (5.4)

In fact, $v_{\text{min}} = 0$ is accessible for finite $E_R$ in the case of exothermic DM, so the generalized methods for non-monotonic $v_{\text{min}}$ are extremely useful here.

### 5.2 More dark photon phenomenology

With the closing of the visible $(g - 2)_\mu$ window, a dark photon coupled to a MeV dark sector remains an interesting phenomenological possibility. Two research directions seem especially promising.

#### 5.2.1 Asymmetric MeV DM and the INTEGRAL excess

MeV dark matter was proposed several years ago as a possible explanation for the excess of 511 keV photons observed from the galactic center by INTEGRAL [260], but obtaining the correct relic density compared to the annihilation rate required today proved difficult [67]. Instead, one could postulate that DM and anti-DM are distinct, with the present dark matter density dominated by DM with a small "asymmetric" anti-DM component. The difference between the primordial annihilation rate (which sets the DM relic density) and the present-day annihilation rate (which controls the flux of 511 keV photons) can be explained by the present-day ratio of anti-DM to DM, which is exponentially sensitive to the annihilation cross section. Preliminary calculations indicate that a dark photon of mass 5 MeV and fermionic DM of mass 1 MeV can evade all current constraints, while explaining the INTEGRAL excess and furnishing a technically-natural model of the dark sector.\(^3\)

\(^3\)Unfortunately, it seems impossible to reconcile the INTEGRAL result with $(g - 2)_\mu$ in this scenario.
5.2.2 Dedicated proton beam searches

In Chapter 2, I showed that the DAEδALUS/LENA experiment could double as a dark matter experiment with essentially no modifications to its functionality as a neutrino experiment. However, one can imagine a dedicated proton beam search for dark photons and dark matter, which may have better sensitivity. The limiting factor for DAEδALUS/LENA was the large CCQE background coming from rare charged pion decays, so an ideal setup would involve sweeping the charged pions out of the way of the detector, leaving the $\pi^0$ to decay in flight to DM. In that case, it would no longer be necessary to stop the charged pions since the boosted decay products would be collimated with the charged pions and hence out of the way of the detector. It would be challenging to realize this setup without also bending the beam of charged protons, but a thin foil target immediately followed by magnetic focusing horns and a beam dump for the remnants of the proton beam may suffice.

5.3 Signatures of mini-split SUSY

The existence of a concrete model for mini-split SUSY, in the form of auxiliary gauge mediation, makes it possible to examine the phenomenology of various mini-split spectra without worrying that they are secretly inconsistent at a high scale. Most obviously, the prediction of PeV-scale scalar superpartners (which could be lighter by an order of magnitude or more for large $\tan\beta$) provides a strong motivation for searching for mini-split at a future 100 TeV collider. Such a collider would likely have enough luminosity to also copiously produce TeV-scale gluinos, and observations of an extremely large branching ratio to 3rd generation quarks would be a smoking-gun signature for a flavored mini-split spectrum as described in Chapter 3. The connection between mini-split SUSY and dark matter is also interesting. Auxiliary gauge mediation unambiguously predicts that dark matter should consist of gravitinos, which are likely unobservable at any conceivable terrestrial direct-detection experiment. However, auxiliary gauge mediation is certainly not the only model of mini-split SUSY, and a multi-sector model of auxiliary gauge mediation and gravity or anomaly me-
diation (whose contributions are generically expected to be present in any realistic model of SUSY-breaking) may furnish a detectable dark matter candidate. Finally, given that the dark photon explanation for \((g - 2)_\mu\) is becoming more and more constrained, one might try to explain this anomaly with supersymmetric particles. This is hardly a new idea [228], but some of its recent incarnations [200, 71, 259] invoke non-universal gaugino masses, which the three factors of the auxiliary group conveniently furnish in auxiliary gauge mediation.

5.4 Summary and outlook

The coming decade promises to be an exciting one for fundamental physics. The LHC is restarting collisions at 13 TeV in mid-2015, dark matter direct-detection experiments are growing in size and sensitivity, and numerous space- and ground-based experiments are poised to look for astrophysical signatures of dark matter. While I am optimistic that the identity of dark matter and the role of naturalness in protecting the Higgs mass may be resolved in my research lifetime, in the interim it is crucial to make maximum use of every positive data point, and keep an open mind for less-traditional solutions. The extensions to the Standard Model I have proposed in this thesis are a promising start, and the model-independent techniques I have described allow for more exotic possibilities. With luck, evidence for dark matter and supersymmetry will point us towards a new iteration of the Standard Model, and the search will continue.
Appendix A

Production and Scattering of DM at DAEδALUS

A.1 Dark matter production rates

For calculating the DM production rates and kinematics at DAEδALUS in Sec. 2.1, we need the three-body matrix element for $\pi^0 \rightarrow \gamma A'(\ast) \rightarrow \gamma \chi \bar{\chi}$, summed over photon polarizations and DM spins if necessary. The calculations below are sufficiently general to be used for either an on-shell or off-shell $A'$, so we will keep the width $\Gamma_{A'}$ in the $A'$ propagator. We will give expressions both for complex scalar DM and Dirac fermion DM, though we only show plots for fermionic DM in the text.

A.1.1 Dark photon width

For the parameter space $m_{A'} > 2m_e$ and assuming that $\chi$ is the only dark-sector particle coupled to $U(1)_D$, the $A'$ width is

$$\Gamma_{A',\text{tot}} = \begin{cases} 
\Gamma_{A'\rightarrow \chi \bar{\chi}} + \Gamma_{A'\rightarrow e^+e^-} & (m_{A'} > 2m_\chi), \\
\Gamma_{A'\rightarrow e^+e^-} & (m_{A'} < 2m_\chi).
\end{cases} \quad (A.1)$$
The two-body widths are given by

$$\Gamma_{A' \rightarrow \chi \bar{\chi}} = \frac{|p|}{8\pi m_{A'}} \langle |A|^2 \rangle,$$  

(A.2)

with $|p| = \sqrt{m_{A'}^2/4 - m_{\chi}^2}$, and $m_{\chi} = m_{\chi}$ or $m_e$ as appropriate. The spin-averaged squared amplitudes for $A'$ decay to DM and leptons are

$$\langle |A_{A' \rightarrow \chi \bar{\chi}}|^2 \rangle = \frac{g_D^2}{3} \left\{ \begin{array}{ll} m_{A'}^2 - 4m_{\chi}^2 & \text{(scalar)},

m_{A'}^2 + 8m_{\chi}^2 & \text{(fermion)},
\end{array} \right.$$  

(A.3)

$$\langle |A_{A' \rightarrow e^+e^-}|^2 \rangle = \frac{4}{3} \epsilon^2 e^2 (2m_e^2 + m_{A'}^2),$$  

(A.4)

where $g_D$ is the U(1)$_D$ gauge coupling. The total $A'$ width is therefore

$$\Gamma_{A'} = \frac{1}{6m_{A'}^2} \times \left\{ \begin{array}{ll} \alpha_D(m_{A'}^2 - 4m_e^2)\sqrt{m_{A'}^2/4 - m_{\chi}^2} + 4\epsilon^2\alpha_{EM}(2m_e^2 + m_{A'}^2)\sqrt{m_{A'}^2/4 - m_e^2} & \text{(scalar)},

4\alpha_D(m_{A'}^2 + 2m_e^2)\sqrt{m_{A'}^2/4 - m_{\chi}^2} + 4\epsilon^2\alpha_{EM}(2m_e^2 + m_{A'}^2)\sqrt{m_{A'}^2/4 - m_e^2} & \text{(fermion)},

4\epsilon^2\alpha_{EM}(2m_e^2 + m_{A'}^2)\sqrt{m_{A'}^2/4 - m_e^2} & \text{(off-shell)},
\end{array} \right.$$  

(A.5)

where $\alpha_D \equiv g_D^2/4\pi$ and $\alpha_{EM} \equiv e^2/4\pi$ are the U(1)$_D$ and electromagnetic fine structure constants, respectively. The last expression is valid when $m_{A'} < 2m_{\chi}$ such that on-shell decays $A' \rightarrow \chi \bar{\chi}$ are kinematically forbidden.

### A.1.2 Scalar DM production

The matrix element for DM production can be obtained by replacing a photon leg with an $A'$ leg in the $\pi^0 \rightarrow \gamma\gamma$ effective vertex mediated by the chiral anomaly, with the $A' \rightarrow \chi \bar{\chi}$ part of the diagram determined by the U(1)$_D$ coupling to $\chi$. For the case of scalar DM, the matrix element is

$$A_{\pi^0 \rightarrow \gamma\chi\bar{\chi}} = \epsilon g_D \frac{e^2}{4\pi^2} \frac{1}{f_{\pi}} \epsilon_{\gamma}^{(\gamma)} \epsilon_{\mu\alpha\beta} p_{\alpha} q_{\beta} \frac{i(g_{\mu\nu} - q_{\mu} q_{\nu} / m_{A'}^2)}{s - m_{A'}^2 + i m_{A'} \Gamma_{A'} (k_2^\nu - k_1^\nu)},$$  

(A.6)

where $p$ is the photon momentum, $k_1$ and $k_2$ are the DM momenta, $q = k_1 + k_2$ is the virtual $A'$ momentum, $s = q^2$, $\epsilon_{\gamma}^{(\gamma)}$ is the polarization vector of the outgoing
photon, and \( f_\pi \) is the pion decay constant. Squaring and summing over the two photon polarizations gives

\[
\langle |A_{\pi^0\rightarrow\gamma\chi}\chi|^2 \rangle = \frac{-e^2 g_D^2 \alpha_{EM}^2}{\pi^2 f_\pi^2} g_{\lambda\mu\alpha\beta} \epsilon^{\mu\nu\sigma\delta} \rho \rho_\alpha \rho_\beta \rho_\sigma \rho_\delta \left( \frac{g_{\mu\nu} - q_\mu q_\nu / m_{A'}^2 \right) \left( g_{\sigma\epsilon} - q_\sigma q_\epsilon / m_{A'}^2 \right) \left( k_2^\sigma - k_1^\nu \right) \left( k_2^\epsilon - k_1^\nu \right). \tag{A.7}
\]

There are six contractions of the \( \epsilon \) tensors; two of them vanish identically because they result in a prefactor of \( p^2 = 0 \), and the remaining four can be simplified using

\[
q \cdot (k_2 - k_1) = (k_2 + k_1) \cdot (k_2 - k_1) = k_2^2 - k_1^2 = 0. \tag{A.8}
\]

This last identity ensures that all terms resulting from the \( q_\mu q_\nu / m_{A'}^2 \) part of the \( A' \) propagator vanish, which must happen because the \( A' \) couples to the conserved electromagnetic current. We can also simplify some of the dot products using

\[
p \cdot q = \frac{m_\pi^2 - s}{2}, \quad k_1 \cdot k_2 = \frac{s}{2} - m_\chi^2, \tag{A.9}
\]

which leads to the final result

\[
\langle |A_{\pi^0\rightarrow\gamma\chi}\chi|^2 \rangle = \frac{e^2 \alpha_{EM}^2 \alpha_D}{\pi f_\pi^2 [(s - m_{A'}^2)^2 + m_{A'}^2 \Gamma_{A'}^2]} \left[ (s - 4m_\chi^2) (m_{\pi^0}^2 - s)^2 - 4s (p \cdot k_1 - p \cdot k_2)^2 \right]. \tag{A.9}
\]

If \( m_{A'} < 2m_\chi \), the \( A' \) is off-shell, and the \( A' \) width (which is proportional to \( \epsilon^2 \)) can be neglected in the denominator; see Eq. (A.5).

### A.1.3 Fermionic DM production

The matrix element for fermionic DM is identical to the scalar case apart from the external spinors which replace the momentum factor \( k_2^\nu - k_1^\nu \). The matrix element is

\[
A_{\pi^0\rightarrow\gamma\chi} = \epsilon g_D \frac{e^2}{4\pi^2} \frac{1}{f_\pi} \epsilon^{(\gamma)} \epsilon^{(\nu)} \epsilon^{(\rho)} p_\alpha q_\beta \left( \frac{g_{\mu\nu} - q_\mu q_\nu / m_{A'}^2 \right) \left( g_{\sigma\epsilon} - q_\sigma q_\epsilon / m_{A'}^2 \right) \left( \bar{\nu}(k_2) \gamma^\nu u(k_1) \right). \tag{A.10}
\]

The additional spin sum is straightforward:

\[
\langle |A_{\pi^0\rightarrow\gamma\chi}\chi|^2 \rangle = \frac{-4e^2 g_D^2 \alpha_{EM}^2}{\pi^2 f_\pi^2} g_{\lambda\mu\alpha\beta} \epsilon^{\mu\nu\sigma\delta} \rho \rho_\alpha \rho_\beta \rho_\sigma \rho_\delta \left( \frac{g_{\mu\nu} - q_\mu q_\nu / m_{A'}^2 \right) \left( g_{\sigma\epsilon} - q_\sigma q_\epsilon / m_{A'}^2 \right) \left( k_2^\sigma - k_1^\nu \right) \left( k_2^\epsilon - k_1^\nu \right) \left( k_2^\nu - k_1^\sigma \right) \left( k_2^\epsilon - k_1^\nu \right) - g^{\mu\nu}(k_1 \cdot k_2 + m_\chi^2). \tag{A.11}
\]
The same two contractions as in the scalar case vanish from \( p^2 = 0 \), and indeed, the longitudinal part of the propagator still vanishes when contracted into the last term above. Simplifying this expression using the dot product identities above gives

\[
\langle |A_{\pi^0 \to \gamma \chi\bar{\chi}}|^2 \rangle = \frac{4\epsilon^2 \alpha_{\text{EM}}^2 \alpha_D}{\pi f_\pi^2 (s - m_{\pi^0}^2)^2 + m_{\pi^0}^2 \Gamma_{\pi^0}^2} \left[ (s + 2m_{\chi}^2) (m_{\pi^0}^2 - s)^2 - 8s(p \cdot k_1)(p \cdot k_2) \right].
\]

(A.12)

Again, if \( m_{A'} < 2m_{\chi} \), the \( A' \) width can be neglected.

### A.1.4 On-shell regime

If the pole of the \( A' \) propagator is well within the physical kinematical region \( s \in [4m_{\chi}^2, m_{\pi^0}^2] \), we can use the narrow-width approximation [247],

\[
\frac{1}{(s - m_{A'}^2)^2 + m_{A'}^2 \Gamma_{A'}^2} \to \frac{\pi}{m_{A'} \Gamma_{A'}} \delta(s - m_{A'}^2).
\]

(Make this substitution in the appropriate matrix elements and integrating over the phase space gives Eq. (2.4) in the text. In particular, when \( \alpha_D \gg \epsilon^2 \alpha_{\text{EM}} \), \( \Gamma_{A'} \propto \alpha_D \) (see Eq. (A.5)), so the factors of \( \alpha_D \) cancel and \( \Gamma_{\pi^0 \to \gamma \chi\bar{\chi}} \) is independent of \( \alpha_D \). However, if \( \alpha_D \ll \epsilon^2 \alpha_{\text{EM}} \) (as in a portion of parameter space that we consider in Fig. 2-4), then \( \Gamma_{A'} \propto \epsilon^2 \) since the visible width dominates; in that case the factors of \( \epsilon^2 \) cancel and \( \Gamma_{\pi^0 \to \gamma \chi\bar{\chi}} \) is proportional to \( \alpha_D \) but independent of \( \epsilon \).

As a check of the narrow-width approximation, we find the expected result

\[
\Gamma_{\pi^0 \to \gamma \chi\bar{\chi}} = \Gamma_{\pi^0 \to \gamma A'} \times \text{Br}(A' \to \chi\bar{\chi}) \text{ (on-shell),}
\]

valid for both fermionic and scalar DM.

### A.1.5 Pion threshold regime

If the pole of the \( A' \) propagator is sufficiently close to \( m_{\pi^0}^2 \), the narrow width approximation breaks down because the Breit-Wigner is no longer completely contained in the physical kinematical region \( s \in [4m_{\chi}^2, m_{\pi^0}^2] \). In that case, we must integrate the

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appropriate full three-body matrix element over phase space as in Eq. (2.5) to obtain the branching ratio \( \text{Br}(\pi^0 \to \gamma\chi\bar{\chi}) \). Now, however, the width must be included in the denominator because it is not parametrically small with our choice of parameters; it is proportional to \( \alpha_D \) rather than \( \epsilon^2 \). In practice, the three-body matrix element must be used for \( |m_{A'}^2 - m_{\pi^0}^2| \lesssim 10\Gamma_A m_{A'} \); for \( \alpha_D = 0.1 \) and \( m_\chi = 1 \) MeV, this translates to \( 120 \) MeV \( \lesssim m_{A'} \lesssim 140 \) MeV.

In the limit of large \( m_{A'} \) and small \( m_\chi \), the decay width for \( \pi^0 \to \gamma\chi\bar{\chi} \) can be written as

\[
\Gamma_{\pi^0 \to \gamma\chi\bar{\chi}} = \frac{m_{\pi^0}^4}{120} \left( \frac{\epsilon g_D}{m_{A'}^2} \right)^2 \Gamma_{\pi^0 \to \gamma\gamma}.
\]  

(A.15)

Thus we can view \( \epsilon g_D/m_{A'}^2 \) as a "Fermi constant" for the dark sector arising from integrating out the \( A' \), analogous to integrating out the \( W \) boson in the weak sector. This gives the scaling of the limits in the curves in Figs. 2-3 and 2-8 for \( m_{A'} \gg m_{\pi^0} \).

### A.2 Dark matter scattering rates

For calculating the scattering of DM at LENA in Sec. 2.2, we need to calculate the \( \chi e^- \to \chi e^- \) differential cross section \( d\sigma/dE_e \). While we have in mind elastic scattering off electrons, we will present formulas that are sufficiently general to apply to any point-like (fermionic) target \( T \), and any inelastic splittings between DM masses which could lead to alternative signals. We let the incoming (outgoing) DM have four-momentum \( p_1 \) (\( k_1 \)) and mass \( m_1 \) (\( m_2 \)). We assume the target \( T \) is initially at rest in the lab frame, with mass \( m_T \) and initial (final) four-momentum \( p_2 \) (\( k_2 \)). The case of \( \chi e^- \to \chi e^- \) in the text is obtained with \( m_1 = m_2 \equiv m_\chi \) and \( T = e^- \).

#### A.2.1 DM scattering amplitudes

For scalar DM and a fermionic target \( T \) (i.e. electron or nucleon), the amplitude for scattering via a \( t \)-channel kinetically mixed photon is

\[
\mathcal{A} = \frac{\epsilon \epsilon g_D}{(t - m_{A'}^2)} \bar{u}(k_2)(\gamma_i + k'_1)u(p_2) \quad \text{(scalar).}
\]

(A.16)
Unlike in the production case, here we can always ignore the $A'$ width. Squaring and averaging (summing) over the initial (final) state target spins gives

$$
|\langle A |^2 = \frac{32\pi^2\epsilon^2\alpha_{\text{EM}}\alpha_D}{(t-m_{A'}^2)^2}
\left[
(k_2 \cdot p_1)(p_2 \cdot p_1) + (k_2 \cdot p_1)(p_2 \cdot p_1) - (k_2 \cdot p_2)(p_1 \cdot p_1)
\right.
\left. + (k_2 \cdot p_1)(p_2 \cdot k_1) + (k_2 \cdot k_1)(p_2 \cdot p_1) - (k_2 \cdot p_2)(p_1 \cdot k_1) + (k_2 \cdot k_1)(p_2 \cdot k_1)
\right.
\left. + (k_2 \cdot k_1)(p_2 \cdot k_1) - (k_2 \cdot p_2)(k_2 \cdot p_1) + (k_2 \cdot p_1)(p_2 \cdot k_1)
\right.
\left. - (k_2 \cdot p_2)(k_1 \cdot p_1) + m_T^2\left[m_1^2 + m_2^2 + 2p_1 \cdot k_1\right]\right]
$$

(scalar), \hspace{1cm} (A.17)

where $t \equiv (k_1 - p_1)^2 = (k_2 - p_2)^2 = 2m_T^2 - 2m_TE_{k_2}$ and $E_{k_2} = k_2^0$ in the lab frame. All quantities can now be written in terms of the incoming $\chi_1$ energy $E_{p_1}$ and the target recoil energy $E_{k_2}$ in the lab frame.

For fermionic DM, the analogous matrix element is

$$
\mathcal{A} = \frac{\epsilon g_D}{(t-m_{A'}^2)}[\bar{u}(k_2)\gamma_\mu u(p_2)][\bar{u}(k_1)\gamma^\mu u(p_1)] \hspace{1cm} (\text{fermion}). \hspace{1cm} (A.18)
$$

Squaring and averaging/summing over the spin states gives

$$
|\langle \mathcal{A} |^2 = \frac{128\pi^2\epsilon^2\alpha_{\text{EM}}\alpha_D}{(t-m_{A'}^2)^2}
\left[
(k_1 \cdot k_2)(p_1 \cdot p_2) + (k_2 \cdot p_1)(p_2 \cdot k_1) - m_1 m_2 (k_2 \cdot p_2)
\right.
\left. - m_T^2(p_1 \cdot k_1) + 2m_1 m_2 m_T^2\right] \hspace{1cm} (\text{fermion}). \hspace{1cm} (A.19)
$$

### A.2.2 Differential distributions

From the amplitudes above, we can obtain the differential cross section. Letting * denote quantities in the center-of-mass (CM) frame, the angular distribution is

$$
\frac{d\sigma}{d\Omega^*} = \frac{1}{2\pi} \frac{d\sigma}{d\cos\theta^*} = \frac{|\langle \mathcal{A} |^2 | \tilde{k}^* |}{64\pi^2 s |\tilde{p}|}, \hspace{1cm} (A.20)
$$

where the initial and final state three-momenta in the CM frame are

$$
|\tilde{p}|^2 = \frac{(s - m_T^2 - m_1^2)^2 - 4m_T^2 m_1^2}{4s}, \hspace{1cm} |\tilde{k}|^2 = \frac{(s - m_T^2 - m_2^2)^2 - 4m_T^2 m_2^2}{4s}. \hspace{1cm} (A.21)
$$
To go to the lab frame (without *s), we can use the relations

\[ s = (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2m_T E_{p_1}, \]  
\[ k_1 \cdot p_1 = -\frac{1}{2} (2m_T^2 - m_1^2 - m_2^2 - 2m_T E_{k_1}) = E_{p_1}^* E_{k_1}^* - |\vec{p}^*||\vec{k}^*| \cos \theta^*, \]

(A.22)

where the incoming DM energy in the lab frame \( E_{p_1} \) is known. This allows us to obtain simple expressions for the flux factor \( |\vec{k}^*|/|\vec{p}^*| \) and the scattering angle \( \cos \theta^* \), giving

\[ d \cos \theta^* = \frac{m_T}{|\vec{p}^*||\vec{k}^*|} dE_T, \]  
(A.23)

where \( E_T \equiv E_{k_2} \) is the energy of the recoiling target. The recoil energy distribution is

\[ \frac{d\sigma}{dE_T} = \frac{m_T (|A|^2)}{32\pi s |\vec{p}^*|^2}, \]  
(A.24)

which contains the particle physics information about \( d\sigma/dE_e \) needed to evaluate the signal yield in Eq. (2.8). In particular, the cross section is proportional to \( e^2\alpha_{EM}\alpha_D \).

### A.2.3 Numerical signal rate

Specializing to the case of elastic electron scattering \( T = e^-, m_1 = m_2 = m_{\chi} \), we can obtain the DM signal yield in Eq. (2.8) given a total production rate of \( N_{\pi^0} \) neutral pions by

\[ N_{\text{sig}} = 2N_{\pi^0} \text{Br}(\pi^0 \rightarrow \chi\chi) n_e \sum_{i=1}^{N_{\chi}} \int_{E_{e,i}^{\text{low}}}^{E_{e,i}^{\text{high}}} \frac{d\sigma}{dE_e} (E_e, E_{\chi}^i) \Theta [E_{\chi}^i - E_{\chi}^{\text{min}}(E_e)]. \]  
(A.25)

Here \( n_e \) is the target electron density, and we have used our GEANT simulation to generate a population of \( N_{\chi} \) DM four-vectors \( \{E_{\chi}^i, \vec{p}_{\chi}^i\} \). The sum is over all \( N_{\chi} \) events passing geometric cuts; the path length through the detector for event \( i \) is \( \ell(\vec{p}_{\chi}^i) \), and the total geometric acceptance is \( N_{\chi}^C / N_{\chi} \). To induce an electron recoil of magnitude \( E_e \), the DM energy must be above the \( E_{\chi}^{\text{min}}(E_e) \) threshold defined in Eq. (2.9).

For a LENA-like cylindrical detector of radius \( R \) and height \( h \) as discussed in Sec. 2.2, we can compute the path length through the detector for a DM particle or
neutrino. For each geometry, we take the z axis to point in the beam direction. For the midpoint scenario depicted in Fig. 2-2a, we define the y axis to be parallel to the cylindrical detector axis. The path length is

$$
\ell(p_{\chi}) = \begin{cases} 
S \sec \theta_y & (\chi \text{ exits through side}), \\
(h/2 - L \tan \theta_z) \csc \theta_y & (\chi \text{ exits through top/bottom}), 
\end{cases}
$$

(A.27)

where \( \tan \theta_{x,y} = |p_{x,y}|/p_z \) and

$$
S = \frac{D(D + 2R)}{L} - L, \quad L = (R + D) \cos \theta_x - \sqrt{(R + D)^2 \cos^2 \theta_x - D(D + 2R)}. 
$$

(A.28)

Here \( L \) is the horizontal distance (parallel to the ground) \( \chi \) travels prior to reaching the detector, and \( D \) is the horizontal distance between the DAE\( \delta \)ALUS source and the detector.

For the oblique scenario in Fig. 2-2b, the path lengths are

$$
\ell(p_{\chi}) = \begin{cases} 
[(h + D \cos \theta_0) \tan(\theta_0 - \theta_d) - L] \csc(\theta_0 - \theta_d) & (\chi \text{ enters top/exits side}), \\
[(D \cos \theta_0 + h) - L \cot(\theta_0 - \theta_d)] \sec(\theta_0 - \theta_d) & (\chi \text{ enters side/exits bottom}), \\
S \csc(\theta_0 - \theta_d) & (\chi \text{ enters side/exits bottom}), 
\end{cases}
$$

(A.29)

where \( \tan \theta_0 = 2R/h \) and and \( \theta_d \) is the angle with respect to the beam in the plane spanned by the beam-line and the detector’s cylindrical axis.

Finally, for the on-axis scenario in Fig. 2-2c, the cylindrical detector axis is aligned with the z axis (i.e. the beam direction). The path length is

$$
\ell(p_{\chi}) = \begin{cases} 
h \sec \theta_x & (\chi \text{ exits through bottom}), \\
(R - D \tan \theta_x) \csc \theta_x & (\chi \text{ exits through side}), 
\end{cases}
$$

(A.30)

where \( \tan \theta_x = \sqrt{p_x^2 + p_y^2}/p_z \) is the DM angle with respect to the z axis. Here \( D \) is the (vertical) distance between the DAE\( \delta \)ALUS source and the detector.
A.3 Neutrino backgrounds

A.3.1 Beam-off backgrounds

The irreducible background due to neutrino-electron scattering from atmospheric neutrinos is estimated from the calculated spectra of Gaisser et. al. [143] with a latitude-dependent scaling factor applied to translate the flux from Kamioka to Pyhasalmi as in Ref. [267]. To determine this rate, we convolved the neutrino flux with the elastic scattering cross section. The resulting event rates were less than 1 event per year for each neutrino flavor in each energy range 106–147 MeV, 147–250 MeV, and 250–400 MeV. The CCQE scattering of atmospheric electron and muon neutrinos and antineutrinos poses an additional beam-off background. For this channel, we generated 1 million sample events on C_{18}H_{30} using GENIE 2.8.0 [32], with atmospheric flux spectra from Ref. [59] as input. The event sample was re-weighted to match the expected number of $\nu - e$ events calculated above. After a cut requiring the outgoing lepton $\ell = e, \mu$ to be within 25° of the beam direction, $\cos \theta_{\ell} < 0.9$ (which we take to reduce the nearly-isotropic CCQE backgrounds by a factor of 20), the raw rates for these processes are given in Table 2.1. We then assumed a 70% reduction in the $\nu_\mu$ and $\bar{\nu}_\mu$ CCQE background rate by rejecting events followed by a Michel electron candidate, as described in Ref. [268]. Furthermore, roughly 25% of the CCQE events for $\bar{\nu}_\mu$ and $\bar{\nu}_e$ are on hydrogen, and produce a neutron that can be tagged to reject the event; we assumed a 80% neutron tagging efficiency. After these reductions, the dominant process in each energy range is $\nu_e$ CCQE. Using the 75% beam-off time of DAE\textsuperscript{3}ALUS to measure this background gives a statistical uncertainty of $\sqrt{B}$ and a systematic uncertainty of $\sqrt{B/3}$, for a total uncertainty of $(\delta B)^2 = 4B/3$. We have checked that additional backgrounds such as excited resonances, coherent scattering, and deep inelastic scattering also have negligible rates compared to CCQE; in addition, these backgrounds are reducible if one can identify vertex activity or pions in the final state.
A.3.2 Beam-on backgrounds

There are two main types of beam-on backgrounds, neutral-current elastic muon neutrino-electron scattering and CCQE neutrino-nucleon scattering. For neutrino energies $E_\nu \ll m_Z$, the differential cross section for elastic muon neutrino-electron scattering ($\nu_\mu e^- \rightarrow \nu_\mu e^-$) is

$$\frac{d\sigma_\nu}{dE_e} = \frac{G_F^2 s}{4\pi E_\nu} \left[ g_L^2 + g_R^2 \left( 1 - \frac{E_e}{E_\nu} \right)^2 \right],$$

(A.31)

where $G_F$ is the Fermi constant, $s = m_e^2 + 2m_eE_\nu$, and $g_{L,R} = g_\nu \pm g_A$, where $g_\nu = -\frac{1}{2} + 2\sin^2 \theta_W$, $g_A = -\frac{1}{2}$. For antineutrino scattering ($\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$), $g_L$ and $g_R$ are interchanged.

As outlined in Sec. 2.4, we used population of decay-in-flight pion events generated in GEANT to simulate our neutrino background events. Given a total flux $N_{\pi^+}$ of decay-in-flight $\pi^+$, each of which produces one $\nu_\mu$, the total $\nu_\mu$ background count is

$$N_{\text{bg}} = N_{\pi^+} n_e \frac{1}{N_\nu} \sum_{i=1}^{N_\nu} \ell(\vec{p}_\nu^i) \int_{E_e^\text{low}}^{E_e^\text{high}} dE_e \frac{d\sigma_\nu}{dE_e}(E_e, E_\nu^i) \Theta \left[ E_\nu^i - E_{\nu}^{\text{min}}(E_e) \right],$$

(A.32)

where as above, $n_e$ is the detector electron density, $N_\nu$ is the number of sample neutrino events generated, $N_\nu^c$ is the number of neutrino events passing geometric cuts, $\ell(\vec{p}_\nu^i)$ is the path length through the detector for a muon-neutrino with three-momentum $\vec{p}_\nu$, and

$$E_{\nu}^{\text{min}}(E_e) = \frac{T_e}{2} \left( 1 + \sqrt{1 + \frac{2m_e}{T_e}} \right), \quad T_e \equiv E_e - m_e,$$

(A.33)

is the minimum neutrino energy to trigger an electron recoil of energy $E_e$. For neutrinos produced from incident $\pi^-$ or $\mu^+$, we replace $N_{\pi^+}$ by the flux of the particle in question. We have checked that the neutrino events produced by GEANT, and neutrinos obtained from manually decaying a sample of energetic pion events from GEANT, give the same results.

For the CCQE events, we used the same GENIE simulation [32] as for beam-off
backgrounds, with an input neutrino flux spectrum generated from our GEANT simulation. We manually decayed the GEANT sample of decay-in-flight pions to obtain the spectrum for the relevant neutrino flavors, input the neutrino spectrum into GENIE with the angle-dependent path length appropriate for the geometry in question, and re-weighted the event sample to match our elastic scattering simulations. The resulting raw rates are given in Table 2.2; the same Michel electron and neutron tagging reductions apply for the background rates. We cross-checked the GENIE results by implementing the Llewellyn Smith CCQE parameterization [213] in our own simulation. We find excellent agreement with GENIE, which is somewhat surprising as it implies that Pauli blocking is not significant in this energy range. We leave a detailed study of the kinematics of the CCQE background to future work. We attempted to directly simulate the background from $\mu^+$ decays with GEANT, but limited statistics proved prohibitive; as explained in the text, we expect this background to be subdominant to the other processes we have considered.
Appendix B

All-Orders Result for A-terms and B-terms in Auxiliary Gauge Mediation

In this appendix, we present the first two-loop calculation of the soft SUSY-breaking A- and B-terms in gauge mediation, to all orders in $F/M$, by a component Feynman diagram calculation. This calculation is simplified as only a single diagram contributes, shown in Fig. B-1.

B.1 Result in Higgsed gauge mediation

We start with the case of a broken gauge group, where the diagram in Fig. B-1 is finite. For the $B_\mu$ term, the result is

$$B_\mu = 16\mu g_H^4 q_\phi^2 M F I(M_V, M, F),$$

(B.1)

where the familiar two loop integral is

$$I(M_V, M, F) = \int \frac{d^4 p d^4 q}{(2\pi)^8} \frac{1}{(p^2 - M_V^2)^2} \frac{1}{q^2 - M^2} \frac{1}{((q + p)^2 - (M^2 + F))((q + p)^2 - (M^2 - F))}.$$  

(B.2)
Figure B-1: Generation of \( B_\mu \) at two loops from gauginos and messengers. The diagram for \( A \)-terms is analogous, except with the Higgsino mass \( \mu_H \) replaced by a scalar vertex. The two-loop calculation performed here includes all orders in \( F/M^2 \), however the perturbative mass insertions for the messengers have been depicted here to demonstrate the chirality flips required for the generation of the lowest-order term. The red arrows show the momentum routing.

Here, \( M_V \) is the gaugino mass, \( M \) is the fermionic messenger mass, and \( M^2 \pm F \) are the scalar messenger masses-squared. After summing over the two scalar messenger mass eigenstates, the upper messenger loop gives the last factor in the loop integral of Eq. (B.2). This finite integral can be evaluated by the usual method of Feynman parameters, giving

\[
B_\mu = 2\mu_H q_0^2 \frac{\alpha_H^2}{(2\pi)^2} \frac{F}{M} \tilde{h}(\kappa, \delta),
\]  

(B.3)

where \( \kappa = F/M^2 \), \( \delta = M_V/M \), and

\[
\tilde{h}(\kappa, \delta) = \int_0^1 dw \int_0^1 dx \int_0^{1-x} dy \frac{2(1-w)}{w(1+(x-y)\kappa) - (1-w)((x+y)^2 - (x+y))\delta}. 
\]  

(B.4)

Making the change of variables \( u = x + y \), \( v = x - y \), two of the Feynman integrals can be evaluated analytically, giving

\[
\tilde{h}(\kappa, \delta) = \frac{1}{\kappa} \int_0^1 du \left\{ \text{Li}_2 \left( 1 + \frac{1 - \kappa u}{u(u-1)\delta} \right) - \text{Li}_2 \left( 1 + \frac{1 + \kappa u}{u(u-1)\delta} \right) 
\right. \\
+ \frac{\kappa \delta u^2(u-1) \log \left( \frac{1 - \kappa^2 u^2}{u^2(1-u^2)\delta^2} \right) - 2(\delta(u-u^2) + \kappa^2 u^2 - 1) \tanh^{-1}(\kappa u)}{u^2 \kappa^2 - (1-(u-u^2)\delta)^2} \right\}. 
\]  

(B.5)
For $\kappa = 0$, one can perform the $u$ integral analytically to show that $h(0, \delta)$ matches precisely with $h(\delta)$ given in Eq. (3.7). The $A$-terms lead to the same loop integrals and functional form for $h(\kappa, \delta)$.

### B.2 Results in standard gauge mediation

To make contact with results from standard gauge mediation, the $A$- and $B$-terms must be determined for an unbroken mediating gauge group. In this case, the internal gauginos become massless in Fig. B-1, leading to an IR divergence which, although vanishing in physical observables, must be regulated to enable comparison with expressions for $A$-terms and $B$-terms in the literature.\footnote{It should be noted that the gauginos obtain mass at one-loop. However, inserting this one-loop mass to regulate the two-loop diagram in Fig. B-1 formally leads to a three-loop result, and is thus not included in the leading result, though they were included in the calculation of Ref. [244].} Formulae in the gauge mediation literature are often quoted using dimensional reduction with the minimal subtraction scheme, i.e. $\overline{\text{DR}}$. Hence it makes sense to regulate the divergence in a way which makes contact with the $\overline{\text{DR}}$ RG scale $\overline{\mu}$, allowing a comparison with the standard results for $A$- and $B$-terms in gauge mediation.

We regulate this IR divergence following the prescription used in e.g Eq. (2.21) of Ref. [219].\footnote{The specific integral regulated in this manner in Ref. [219] is the same as each of the contributing integrals of Fig. B-1 which are summed to give Eq. (B.2). Hence the structure of the IR divergence is identical and we can employ the same prescription here.} The regulated integral is evaluated as

\[
I(0, M, F) = \lim_{M_V \to 0} \left[ I(M_V, M, F) + G(M, F) \log \left( \frac{M_V^2}{\overline{\mu}^2} \right) \right], \tag{B.6}
\]

where $\overline{\mu}$ is the $\overline{\text{DR}}$ RG scale and $G(M, F)$ is the finite one-loop subintegral involving only messenger fields. This cancels the logarithmic divergence in $M_V$ and, practically speaking, amounts to making the replacement $M_V \to \overline{\mu}$ in $I(M_V, M, F)$ and taking the limit $\overline{\mu} \to 0$. We obtain the final result

\[
B_\mu = 2\mu_H q_\phi^2 \frac{\alpha_H^2}{(2\pi)^2} \frac{F}{M} h_{\overline{\text{DR}}}, \tag{B.7}
\]
where

\[ h_{\text{DR}} = 1 + \log\left(\frac{M^2}{\mu^2}\right), \]  

and similarly for A-terms as they arise from the same diagram. Thus we find that in standard gauge mediation the A- and B-terms do not vanish at the messenger scale when the IR-divergent contributions are regulated with DR. Note that in Ref. [244], the finite piece (which is regulator dependent) was absorbed into a redefinition of the messenger threshold, \( M \rightarrow eM \). However, if one uses DR then the messenger threshold really is \( M \) and the finite piece is genuine. Furthermore we can make a direct connection with the analytic continuation methods developed in Refs. [88, 87] for an unbroken mediating gauge group. This once again shows the consistency between the analytic continuation methods of Refs. [88, 87] and brute force Feynman diagram calculations, in this case for unbroken mediating gauge groups.
Appendix C

Optimal Halos and Finite Energy Resolution

For a DM direct detection experiment with finite energy resolution \( G(E_R, E'_R) \), one may worry that due to smearing effects, the halo integral which minimizes the log-likelihood is no longer a sum of step functions, but perhaps a more complicated function whose many free parameters preclude a simple numerical minimization of the kind described in Sec. 4.2.4. Here we present a proof to the contrary – for any physically reasonable resolution function, the only effects of smearing are to shift slightly the positions of the steps of \( \tilde{g}(E_R) \) away from the measured energies \( E_i \), and possibly to merge some of the steps. In particular, the optimal halo integral is still a sum of at most \( N_o \) step functions.

Although we have in mind Gaussian smearing, this analysis holds for any reasonable form of the resolution function. We define a physically reasonable resolution function \( G(E_R, E'_R) \) to have the following properties:

(i) \( \int G(E_R, E'_R)dE'_R = 1 \) for any \( E_R \).

(ii) As a function of \( E'_R \) for fixed \( E_R \), \( G(E_R, E'_R) \) has a single local maximum at \( E'_R = E_R \) and no other local extrema.

(iii) For \( E_R \neq E'_R \), either \( G(E_R, E'_R) = 0 \) or \( \partial G(E_R, E'_R)/\partial E'_R \neq 0 \).
Property (i) simply states that the resolution function is normalized and doesn't change the total number of events. Property (ii) states that $G$ has a single peak where the detected energy equals the true energy, and no other structure. Property (iii) is a technical assumption which will be used in the arguments below, and states that if $G$ is flat on some interval, it must vanish. A normalized Gaussian resolution function $G(E_R, E'_R) \propto e^{-(E_R-E'_R)^2/2\sigma^2}$ clearly satisfies all three properties, as does a delta function $G(E_R, E'_R) = \delta(E_R - E'_R)$. Certain models for energy resolution may violate property (iii), for example a "top hat" shape where $G(E_R, E'_R)$ is constant in some interval about $E_R$ and zero everywhere else, but one may always assume some infinitesimal deviations from flatness which otherwise has no measurable effect.

To simplify the notation, we write the differential scattering rate (4.1) as

$$\frac{dR}{dE_R} = \int dE'_R G(E_R, E'_R)K(E'_R)\tilde{g}(E'_R),$$  \hspace{1cm} (C.1)

where we have absorbed the form factor, efficiency, and all prefactors into $K(E'_R)$. For reasonable choices of the form factor and efficiency functions, $K(E'_R) > 0$ for all $E'_R$ within the experimental sensitivity, and in addition $dK/dE'_R$ is small. We have also written $\tilde{g}(E'_R)$ as a function of $E'_R$ directly rather than $\nu_{\text{min}}$. Note however that since $\nu_{\text{min}}$ is a monotonic function of $E'_R$, $\tilde{g}(E'_R)$ is also monotonic function of $E'_R$.

Consider now the expression for the log-likelihood (4.8), written in the suggestive form

$$L[\tilde{g}] = \int dE'_R K(E'_R)\tilde{g}(E'_R) - \sum_{i=1}^{N_O} \log \left( \mu_i + \int dE'_R G(E_i, E'_R)K(E'_R)\tilde{g}(E'_R) \right). \hspace{1cm} (C.2)$$

Property (i) above ensures the resolution function $G$ does not appear in the first integral.

We can now view the log-likelihood minimization as a variational problem: minimize the functional $L[\tilde{g}]$ with respect to the function $\tilde{g}(E'_R)$, subject to the monotonic-

\footnote{A Gaussian with an energy-dependent width $\sigma(E_R)$ also satisfies these properties as long as the form of $\sigma(E_R)$ is physically reasonable. For example $\sigma(E_R) \sim 1/\sqrt{E_R}$ in the XENON experiment, and $G(E_R, E'_R)$ satisfies property (ii) as long as the region of $E_R$ close to zero is avoided.}
ity constraint \( \frac{d\tilde{g}}{dE'} \leq 0 \). The subject of variational problems with inequality constraints may be somewhat unfamiliar to physicists, but is well-known in economics and related fields; the solution is given by the Karush-Kuhn-Tucker conditions [193, 204], which generalize the concept of Lagrange multipliers. In a similar fashion to imposing an equality constraint with a Lagrange multiplier, we can impose the inequality constraint by introducing an auxiliary function \( q(E'_R) \) and modifying the log-likelihood, \( L[\tilde{g}] \rightarrow L[\tilde{g}] + \int dE'_R \frac{d\tilde{g}}{dE'_R} q(E'_R) \). The solution that minimizes the log-likelihood while satisfying the monotonicity constraint will satisfy

\[
\frac{\delta L}{\delta \tilde{g}} - \frac{dq}{dE'_R} = 0, \tag{C.3}
\]

\[
\frac{d\tilde{g}}{dE'_R} \leq 0, \tag{C.4}
\]

\[
q(E'_R) \geq 0, \tag{C.5}
\]

\[
\int dE'_R \frac{d\tilde{g}}{dE'_R} q(E'_R) = 0. \tag{C.6}
\]

Eq. (C.3) is the familiar equation resulting from varying the modified functional with respect to \( \tilde{g} \), and Eq. (C.4) is the desired monotonicity constraint. Eq. (C.6) is a complementarity condition which ensures that the shift in \( L \) vanishes on the solution, just as the extra Lagrange multiplier term vanishes on the solution in the case of equality constraints. When combined with Eqs. (C.4) and (C.5), Eq. (C.6) enforces that at every point \( E'_R \), either \( d\tilde{g}/dE'_R = 0 \) (saturating the inequality constraint), or \( q(E'_R) = 0 \).

Suppose that the solution \( \tilde{g}(E'_R) \) has nonzero derivative at some point \( E_0 \). Then by Eq. (C.6), \( q(E_0) = 0 \). Moreover, we must have \( dq/dE'_R = 0 \) at \( E_0 \) since if not, we would violate the positivity condition (C.5) at \( E_0 + \epsilon \) for arbitrary \( \epsilon > 0 \). Thus, Eq. (C.3) becomes \( \delta L/\delta \tilde{g} = 0 \), or taking the functional derivative explicitly,

\[
\sum_{i=1}^{N_O} \frac{G(E_i, E_0)}{\gamma_i} = 1, \tag{C.7}
\]

\(^2\)We have divided out by \( K(E'_R) \) which is legitimate so long as we are considering \( (a, b) \in [E_{\text{min}}, E_{\text{max}}] \).
where

\[ \gamma_i = \mu_i + \int dE''_R G(E_i, E''_R)K(E''_R)\tilde{g}(E''_R), \quad (C.8) \]

is the total differential event rate at \( E_i \). By property (ii), as a function of \( E_0 \), the left-hand side of (C.7) is a sum of \( N_O \) peaked functions, weighted by various factors \( \gamma_i \). By property (iii), \( G \) has no flat regions unless it vanishes, so a sum of functions of this form will cross a horizontal line at most \( 2N_O \) times; thus, only isolated points \( E_0 \) are solutions. This proves that \( \tilde{g}(E'_R) \) must be flat except at isolated points \( E_0 = \tilde{E}_j \); in other words, it is a sum of step functions.

To determine the number and position of the points \( \tilde{E}_j \), we can read Eq. (C.3) as a differential equation for \( q \):

\[ \frac{dq}{dE'_R} = K(E'_R)\left(1 - \sum_{i=1}^{N_O} \frac{G(E_i, E'_R)}{\gamma_i}\right). \quad (C.9) \]

The solution to this equation depends on the \( \gamma_i \), which in turn depend on the full solution function \( \tilde{g}(E'_R) \), so we cannot integrate this equation directly. In fact this turns out not to be necessary.

By the complementarity condition, we must have \( q(\tilde{E}_j) = 0 \), but to preserve the positivity condition (C.5), we must also have

\[ \left. \frac{dq}{dE'_R} \right|_{\tilde{E}_j} = 0, \quad \left. \frac{d^2q}{dE'_R^2} \right|_{\tilde{E}_j} \geq 0 \quad (C.10) \]

at the roots \( \tilde{E}_j \) of \( q \). Taking the derivative of Eq. (C.9), and using the assumption \( dK/dE'_R \approx 0 \), the condition on the second derivative becomes

\[ -\sum_{i=1}^{N_O} \frac{1}{\gamma_i} \left. \frac{\partial G(E_i, E'_R)}{\partial E'_R} \right|_{E'_R=\tilde{E}_j} \geq 0. \quad (C.11) \]

By property (ii), \( G(E_i, E'_R) \) will be peaked at \( E'_R = E_i \), so for \( E'_R \) close but not equal to \( E_i \), the \( i \)th term in the sum should dominate. The derivative of a peaked function is negative to the right of the peak, so to satisfy the inequality, we must have \( \tilde{E}_j > E_i \). In other words, the positions of the steps shift to the right slightly.
The positions $\tilde{E}_j$ are given by solving $dq/dE'_R|_{\tilde{E}_j} = 0$, which we already used as Eq. (C.7) to derive the shape of $\tilde{g}$. There are at most $2N_O$ solutions, but only $N_O$ of these solutions will satisfy the convexity condition (C.11) and could qualify as roots of $q$. For sufficiently large $\gamma_i$, the peak of $G(E_i, E'_R)$ may dip below the line of height 1, so there may be fewer than $N_O$ solutions; the same is true if two peaks are close enough to one another to merge together. Furthermore, it may be the case that both conditions (C.10) are satisfied at $\tilde{E}_j$, but $q(\tilde{E}_j) \neq 0$, in which case there would be no step at $\tilde{E}_j$. We conclude that in the case of physically reasonable $G(E_R, E'_R)$, the optimal halo integral $\tilde{g}(E_R)$ is given by a sum of at most $N_O$ step functions.

Rather than integrate Eq. (C.9), one may simply use this knowledge to perform at most a $2N_O$-parameter numerical minimization of the log-likelihood subject to the monotonicity constraint on $\tilde{g}$. 
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