Integrated Airline Scheduling:
Models and Algorithms
by
Fang Lu
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Author ............................................ Department of Civil and Environmental Engineering
May 16, 1997

Certified by .................................................. Cynthia Barnhart
Associate Professor of Civil and Environmental Engineering
Thesis Supervisor

Accepted by ............................................ Joseph Sussman
Chairman, Department Committee on Graduate Studies

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Abstract

Schedule planning is concerned with generating a schedule (a feasible plan of what cities
to fly to and at what times) that has the most revenue potential, and resolving a host of
related issues involving aircraft assignments and crew scheduling. Current practice in the
airline industry is to divide the overall problem into a set of smaller subproblems and
then to solve these subproblems sequentially. We develop an approximate integrated
model and a new solution approach to measure the potential benefits that may be accrued
by solving the entire schedule planning problem simultaneously. We show that solutions
generated by current approaches can be far from optimal, resulting in significant
monetary losses to the airline industry.

Thesis Supervisor: Cynthia Barnhart
Title: Associate Professor of Civil and Environmental Engineering
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Chapter One

Introduction

1.1 Motivation

The industry of air transport in recent decades can be characterized by high growth rate and marginal profitability. Very few industries have enjoyed such growth for such a long period of time [Hanlon, 1996]. World passenger traffic has increased more than ten-fold over the past three decades (Figure 1-1), and most forecasts predict that the average growth rates will be between five and seven percent in the next twenty years. To accommodate this growth in demand, the airline industry has expanded rapidly (Figure 1-1). Even so, load factors (which are revenue passenger miles divided by available seat miles\(^1\)), and thus a measure of the percentage of available seats that are filled with paying passengers) for most airlines are still as high as 70% (Figure 1-2).

---

\(^1\) A revenue passenger mile (RPM) is one paying passenger flown one mile. It is the principle measure of airline passenger traffic. An available seat mile (ASM) is one available seat flown one mile. It is the principle measure of airline capacity. For example, an airliner with 200 passenger seats and 100 passengers
Despite the high rate of growth in the airline industry, profitability during the past decades has been marginal. Figure 1-3 shows that the constant dollar yield (which is a measure of airline revenue derived by dividing passenger revenue by passenger miles) has decreased over recent years. Figure 1-4 shows that the airline industry actually enjoyed only three brief periods of reasonable profitability, during the years of 1976-78, 1986-89, and the past three years. Through the years, airlines have earned a net profit of between one and two percent compared to an average of five percent for U.S. industries as a whole [ATA, 1995].

![Graph of World Passenger Traffic]

**Figure 1-1:** World passenger traffic (Source: Air Transport Association)

---

on board, flown a distance of 100 miles, represents 20,000 available seat miles (ASMs) and 10,000 revenue passenger miles (RPMs) [ATA, 1995].
Figure 1-2: U.S. airlines load factors (Source: Air Transport Association)

Figure 1-3: U.S. scheduled airlines yield (Constant Dollar 1982) (Source: Air Transport Association)
The factors that contribute to the low profit in airline industry are the high cost structure for airlines and fierce competition.

**High Cost Structure**

The airline industry is a capital intensive business. Airlines need an enormous range of expensive equipment and facilities, such as aircraft and maintenance hangers, to maintain the business. A huge amount of money needs to continually finance all equipment.
maintenance and replacement. Most equipment are purchased or leased through loans or the issuance of stock. High debt burden is typical for an airline.

Another characteristic of the airline cost structure is its labor intensiveness. More than one third of the revenue generated by the airlines goes to payment for its workforce, which includes pilots, flight attendants, management employees, and so on. According to reports filed with the Department of Transportation in 1993, airline costs broke down as Figure 1-5 [ATA, 1995].

Figure 1-5: Breakdown of the airline cost structure [ATA, 1995]
Flying operations cost is essentially any cost associated with the operation of the aircraft, such as fuel cost and pilot salaries. Among the flying operations cost, fuel is the largest component. It makes up about 15% of total operating expenses. Despite the fact that the airline industry has increased fuel efficiency nearly 50% over the past two decades, major U.S. airlines still spend more than $10 billion a year on fuel. Next to the fuel cost, crew (pilots) cost is the second highest component of flight operating cost. Figure 1-6 compares labor cost with the total cost per available seat miles (ASM). It shows that labor cost is nearly one third of the flight operating cost. The average annual pilot salary at major U.S. airlines is more than $123,000 per pilot [Peltz, et al., 1997]. A 15% salary increase would translate into hundreds of millions of dollars in cost increments for a major airline.

**Figure 1-6:** Labor cost for major U.S. airlines [ATW, 3/1997]
Fierce Competition

The primary product of an airline is the flight schedule. The nature of this product is quite homogenous among airlines. From a passenger’s point of view, one airline’s seat is very similar to another and aircraft do not differ significantly. Therefore, if the schedule times for the flights from different airlines are similar, a traveler does not have a preference, aside from possibly choosing the lower fare.

The homogenous nature of the airline product pushes airlines into marketing their products in a very costly way. The efforts to differentiate their product from that of their competitors include being the first to introduce new aircraft types, increasing frequency of service, increasing the quality of in-flight catering and advertising [Doganis, 1991].

The consequences of these efforts are two-fold. First, these approaches increase the cost substantially. With higher frequency service or new aircraft types, the airline needs to finance more aircraft, which leads to huge interest payments where needed to finance the debt burdens from purchasing or leasing more aircraft. In addition, advertising and better in-flight service drives up the expense.

However, better service does not necessarily bring in a higher yield. Across the world as a whole, breakdown of leisure/business travel is approximately 80/20, and leisure travel has been growing more rapidly than business travel [Hanlon, 1996]. The demand for leisure travel is price elastic, while the demand for business travel is not. With the declining percentage of business travel, the aggregate demand for air travel is price elastic; that is, most travelers are very price sensitive. Thus, airlines have little room for fare increases even with better service.
The intense competition and high cost structure of the airline industry have pushed airlines to minimize costs and maximize revenues. This objective can be achieved by applying operations research in planning. Operations research professionals have been working on the development of methods for obtaining optimal planning since the 1950s. Decades of application proved that operations research can improve aircraft and crew utilization effectively, and generate substantial savings and profits for airlines. Even so, there is still a lot of room for further improvement.

1.2 Planning in the Airline Industry

Airline planning includes five major components: fleet planning, routes planning, schedule planning, pricing, and revenue management. Fleet planning decides whether to purchase or lease new or used aircraft and what type of aircraft should be bought or leased. Routes planning decides where to fly the aircraft. Scheduling planning is designing system-wide flight patterns that provide optimum public service, in both quantity and quality, consistent with the financial health of the carrier. [Wells, 1994]. The goal of pricing is to determine the fare levels subject to passenger “willingness to pay”, level of service, and competitive response. The purpose of revenue management is to maximize the revenue subject to the seat capacity and the fare types.

Among airline planning methods, schedule planning is one of the most vital decision making tools in airline planning. Schedule planning is concerned with generating a schedule (a feasible plan of what cities to fly to and at what times) that has the most revenue potential, and resolving a host of related issues involving aircraft
assignments and crew scheduling. [Barnhart, and Talluri]. Figure 1-7 shows a widely used schedule planning paradigm.

![Airline scheduling paradigm diagram]

**Figure 1-7:** Airline scheduling paradigm

**Flight Schedule**

A flight schedule is generated based on traffic forecasts for the month, tactical and strategic initiatives, and seasonal demand variations. Various internal and external
factors, such as flight operations, facility constraints, marketing, and potential competitors' reactions, are involved in making a flight schedule. The estimation of the costs that would be incurred and the revenues that would accrue from flying a proposed schedule are tabulated during this period. Figure 1-8 depicts the schedule development process and a number of factors considered [Wells, 1994]. Due to the complicated factors involved, constructing the flight schedule is very difficult and requires much time and labor.

With a given flight schedule, a significant portion of costs and revenues are fixed. The task of scheduling planning is the optimization of the flight schedule, i.e., to find the most efficient and effective deployment of an airline's resources [Etschmaier, et al. 1985].

**Aircraft Scheduling**

Once the flight schedule is determined, the aircraft scheduling problem can be solved. The aircraft scheduling problem is composed of fleet assignment, through flight assignment, and aircraft maintenance routing. The fleet assignment problem is to determine which type of aircraft should fly each flight segment. The goal of fleet assignment is to minimize the total cost of assigning aircraft to flights, satisfying a number of requirements. The fleet assignment problem will be detailed in Chapter Three.
Figure 1-8: Conceptual framework for the schedule-development process [Wells, 1994]
Following the fleet assignment, a set of “through flights” is selected. A set of through flights is a pair of flights that are flown by the same plane, and have the same flight number. Through flights may result in additional revenue, which is called through revenue, because passengers are willing to pay a premium to stay on the same aircraft rather than make a connection at an airport [Talluri, et al. 1995]. The maintenance routing problem then follows fleet assignment and through flight selection. The challenge of the maintenance routing problem is to determine the actual routings (called rotations) of each aircraft so that each aircraft has adequate opportunities to undergo maintenance required by the FAA [Talluri, et al 1995]. Currently, the aircraft scheduling problem is solved sequentially, with fleet assignment solved first. Given the fleet assignment solution, the through assignment problem is solved second. Finally, given fleet assignment and through assignment solutions, the maintenance routing problem is solved.

**Crew Planning**

Given a set of scheduled flights flown by a particular fleet or aircraft type, and the through assignments and aircraft routings, crew planning can be performed. The first component of crew planning is to solve the crew pairing problem, which involves the construction of a minimum cost set of pairings (a pairing is typically a three to five work day schedule for U.S. domestic operations and a one to two weeks schedule for international operations) that partition the flights, for each crew-compatible fleet\(^2\). Each

\(^2\) Pilots are qualified for particular cockpits. The fleet types can be partitioned into crew compatible fleet groups. Two fleet types are crew-compatible if pilots qualified for one are qualified for the other. An
pairing must satisfy a host of rules and regulations set forth by the FAA and/or collective bargaining agreements. Once crew pairings are generated, the bidline or rostering problem is solved. Bidlines are constructed in such a way as to provide adequate staffing for all scheduled flights. Each bidline or roster is composed of a sequence of pairings and represents typically one month’s work for one or more crew members. Like pairings, bidlines and rosters must adhere to FAA and contractual rules. If bidlines are constructed, crews bid for bidlines and lines are awarded based on seniority. With rostering, work schedules are constructed for individual crew members specifically, and the award process does not require bidding.

1.3 Focus of the Thesis

Typically, airline planning is achieved by sequentially solving a series of subproblems. This approach has been adopted because of the limitations imposed by computer hardware and solution algorithms to solve the overall airline planning problem directly. However, this sequential approach might not yield feasible solutions, and even if feasible solutions are achieved, these solutions might be far from the global optimum. In contrast to the sequential approach, simultaneous decision making will produce more economical plans that may be closer to global optimality. In addition, a simultaneous solution process will also reduce incompatibilities between decisions. This should lead to fewer difficulties in making the schedule operational, thus allowing a reduction in overall delays.

example of a crew compatible fleet group is the 757, 767 and 767 stretch. They all have the same cockpit but are considered different fleet types because of seating capacity. For purposes of crew modeling they are in the same group [Clarke, et al. 1996].
schedule generation time and improved productivity of the schedule developers [Shenoi, 1996].

**Challenges of Integrated Modeling**

Although it is natural to think of making schedule decisions simultaneously, an integrated model is difficult to solve. Even sequentially, the crew pairing problem and the aircraft scheduling problem are each hard problem to solve. A huge number of variables and constraints are involved in the crew pairing and aircraft scheduling problems. For example, a conventional crew pairing problem considering only the flights assigned to a crew compatible fleet group, can contain billions of variables. Thus, it is impossible to solve the crew pairing problem using a workstation class computer by explicitly considering all the variables. The column generation method has to be used. In an integrated model, since the decision of assigning fleets to flights is made simultaneously with the decision of assigning crew pairings to flights, a number of crew pairing problems will have to be solved simultaneously. This would result in an exponential increase in the number of pairing variables. Given the limitation of computer hardware and the complexity of the model, the integrated problem is, practically speaking, intractable.

The focus of this thesis then is on the development of an advanced solution approach, which involves an approximate, rather than an exact, integrated model that will improve the sequential solution process for aircraft scheduling and crew planning problems. Our approximate integrated model also provides a lower bound on the best
possible solution for the combined aircraft and crew scheduling problems. By comparing the solutions of our process and traditional processes, we will provide a measure of how much the solutions of the current sequential models can be improved.

Approximate Models

Since it is impractical at this point to solve the integrated aircraft and crew scheduling problems, our advanced solution approach continues to solve the aircraft scheduling and crew scheduling problems sequentially. However, in solving the fleet assignment problem, we approximate the impacts on crew scheduling of various fleetings and include these effects in the fleet assignment model to create an approximate integrated model. Specifically, this model integrates the basic fleet assignment model and an approximate crew scheduling model (Figure 1-9).

![Diagram showing the flow of operations from Fleet Assignment through Aircraft Scheduling and Maintenance Routing to Crew Planning, connected to the Approximate Integrated Model (Basic Fleet Assignment + Approximate Crew Pairing Model), leading to Crew Pairing.]

**Figure 1-9:** Our advanced solution approach for solving the airline scheduling problem
Quantification of Benefits of Integrated Models and Solution Approaches

We are interested in comparing the solution value (or lower bounds on those values) of our approach and those of currently used sequential models. If our approach achieves a considerably improved solution compared to the sequential approach, then we demonstrate that the sequential approach can produce solutions that are far from optimality. In this case, expending effort to develop more accurate and fully integrated models and solution approaches is warranted since significant cost benefits might be produced. However, if the two solutions are close in value, then we can compare a lower bound on the optimal integrated solution with the sequential solution. This lower bound is a by-product of our solution approach since an optimal IP or LP solution to our approximate integrated model is a lower bound on the costs of the optimal fleet and crew solutions. Thus if the optimal solution value for the approximate integrated fleet and crew IP (or LP) is close to the sequential solution value, then we cannot obtain a significantly improved solution by optimizing the entire aircraft and crew scheduling processes. Instead, the sequential approach achieves near-optimal global solutions. In this case, development and solutions of huge integrated models are not necessary; sequential solutions are sufficient.
1.4 Thesis Outline

In Chapter Two and Chapter Three respectively, we review some definitions in fleet assignment and crew scheduling, and introduce some fundamental models and solution approaches for them. In Chapter Four, we present our integrated approximate airline scheduling model, and detail our advanced solution approach. A case study and examination of the potential benefits of our integrated model and solution approaches are also presented in Chapter Four. Finally, contributions of the thesis, conclusions and future research directions are detailed in Chapter Five.
2.1 Introduction

Given a flight schedule, a set of aircraft, and the costs of flying each flight segment with each fleet type, the fleet assignment problem is to determine which fleet should fly each flight segment. This chapter reviews the fleet assignment problem, provides some basic definitions, describes the time line flight network, and presents a basic fleet assignment model and an enhanced fleet assignment model.

2.2 Definitions

This section presents the terms associated with fleet assignment that will be encountered in later sections [Hane, et al. 1995].

1. **Flight Segment**: a flight segment is a non-stop flight between two cities. It is also called a *flight leg*. 

25
2. **Fleet:** a fleet is a set of aircraft with similar equipment parameters, such as the number of seats, cockpit size, aircraft type, and the operating cost, etc.

3. **Turn Time:** the turn time is the time needed for deplaning and enplaning passengers, baggage handling, and. Turn time varies with aircraft type and usually ranges from 30 minutes to 60 minutes.

4. **Maintenance Time:** the maintenance time is the time period scheduled for an aircraft to be maintained. Maintenance time varies with the aircraft type and the type of maintenance the aircraft receives.

5. **Ready Time:** ready time is the arrival time of the flight plus the turn time for the aircraft flying the flight.

6. **Maintenance Ready Time:** the maintenance ready time is the arrival time of the flight plus the maintenance time for the aircraft flying this flight.

### 2.3 Time-Line Flight Network

A flight network is at the core of the fleet assignment problem. Each fleet type has its separate flight network to maintain the balance for each fleet. The time-line network is the type of flight network that is used throughout the models presented in this thesis. The time-line network consists of a set of nodes associated with each station and arcs that represent flight legs, aircraft on the ground, and overnighting aircraft [Talluri, 1996].

1. **Node/Event:** each node/event is an arrival or a departure of a flight. The location and time of a node corresponding to a departure is the origin and departure time of the flight; and the location and time of a node corresponding to
an arrival is the destination and ready time of the flight. All nodes are sorted chronologically at each station for each fleet’s flight network.

2. **Arc**: each flight network has four types of arcs — *flight arcs, maintenance arcs, ground arcs*, and *wrap-around arcs*.

   - A *flight arc* and its start and end nodes in a fleet network denote a flight in the schedule.

   - A *maintenance arc* represents a maintenance opportunity in a maintenance station. The location and time of the start node of a maintenance arc is the origin and departure time of a flight respectively; and the location and time of the end node of a maintenance arc is the destination and maintenance ready time of the corresponding flight respectively.

   - A *ground arc* connects the successive nodes in one station in each fleet’s flight network.

   - A *wrap-around arc* is a special type of ground arc. It connects the last departure/arrival event and the first departure/arrival event at one station. These arcs represent aircraft overnighting on the last day of the schedule. Through wrap-around arcs, we can model the fleet assignment problem as a circulation problem.

3. **Count time**: count time is a time that is arbitrarily chosen in the flight network. It is used to restrict the number of the aircraft used in the fleet assignment solution. The total number of aircraft of any fleet type assigned to arcs crossing the count time defines the number of assigned aircraft of that fleet type.
Let us consider a flight network with three cities (e.g., A, B, and C), and four flights (e.g., F1, F2, F3, and F4). City C is a maintenance station. The turn-time for the fleet is 30 minutes and the maintenance time for this fleet is 4 hours. Table 2-1 lists the relevant data for each flight, and Figure 2-1 is the resulting time-line flight network.

<table>
<thead>
<tr>
<th>Flight Number</th>
<th>Origin</th>
<th>Destination</th>
<th>Departure</th>
<th>Arrival</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>B</td>
<td>6:30</td>
<td>8:30</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>C</td>
<td>9:30</td>
<td>11:00</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>B</td>
<td>16:00</td>
<td>17:00</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>A</td>
<td>18:00</td>
<td>20:00</td>
</tr>
</tbody>
</table>

*Table 2-1:* Flight leg information
Figure 2-1: An example time-line network

F: flight arc
F_m: maintenance arc
G: ground arc
W: wrap-around arc
2.4 Models

The fleet assignment problem has been well studied for a number of years [Abara 1989, Daskin and Panayotopoulos 1989, Berge and Hopperstad 1993, Dillon et al. 1993, Subramanian et al. 1994, Hane et al. 1994, Gu et al. 1994, Desaulniers et al. 1994b, 1995b, Clarke et al. 1996]. In this chapter, we review the models that will be used in the integrated airline scheduling model.

The objective of fleet assignment models varies with the models' purposes. The primary objective is cost minimization, i.e., minimizing the total cost of assigning the fleets to the flights. For fleet planning purposes, the objective function can be changed to minimize the total number of aircraft used to fly the schedule; and for route planning purposes, the objective can be to maximize the potential profit, i.e., revenue minus operating cost, in assigning fleets to flights. Cost minimization will be the only type of objective function to be discussed in this thesis.

The decision variables for the fleet assignment model are those defining which fleet is assigned to a flight. In the formulation of the fleet assignment model that corresponds to cost minimization, the coefficient of the decision variable $e_{ik}$ is the cost of assigning fleet type $k$ to flight $i$. This cost is calculated as follows:

$$e_{ik} = \text{(operating cost of fleet } k \text{ for flight } i + \text{ opportunity cost of spilling passengers if fleet } k \text{ is assigned to flight } i)$$
The operating cost includes fuel cost, maintenance cost, and so on. It depends primarily on the size and efficiency of the aircraft and the distance of the flight leg. The spill cost captures the relationship between passenger demand and seat capacity. Spill cost is incurred when passenger demand is greater than the number of aircraft seats. In the airline industry, passenger demand varies dramatically from day to day. Forecasting passenger demand is a hard task for an airline planner. Airlines cannot guarantee sufficient capacity for all potential passengers, because excess capacity can cost a great deal of money. When seat capacity is less than demand, passengers will be spilled. If the spilled passengers cannot be recaptured\(^3\) by the same airline, they are lost to the airline, and it incurs damage to its reputation, and loses the potential revenue. Given the projected demand \(D_i\) for flight \(i\), the recapture rate \(b_{ij}\) (i.e., the percentage of the passengers who want to take flight \(i\) but take flight \(j\) instead), the fares \(Fare_i\) and \(Fare_j\) for flight \(i\) and \(j\) respectively, and the seat capacity \(CAP_k\) for fleet \(k\), the spill cost \(SC_{ki}\) of assigning fleet \(k\) to flight \(i\) is as follows:

\[
SC_{ki} = \begin{cases} 
0, & \text{if } D_i \leq CAP_k \\
(Fare_i - \sum_{j \neq i, j \in F} Fare_j b_{ij})(D_i - CAP_k), & \text{if } D_i \geq CAP_k 
\end{cases}
\]

\(^3\) Recapture is the fact that the airline retakes the spilled passengers to another flight operated by the same airline.
2.4.1 The Basic Fleet Assignment Model

The fleet assignment model considered by Hane, et al. 1995 is presented below. It minimizes the total cost of assigning fleets to flights.

\[ \text{Min } \sum_{k \in K} \sum_{i \in F} e_{ik} z_{ik} \]  

s.t.

\[ \sum_{k \in K} \sum_{i \in F} \delta_{pik} z_{ik} = 1 \quad \forall p \in F \]  

\[ \sum_{i \in F} b_{3nk} z_{ik} + \sum_{i \in F} b_{4nhk} w_{hk} = 0 \quad \forall n \in M_k, \forall k \in K \]  

\[ \sum_{i \in F} d_{3nk} z_{ik} + \sum_{i \in F} d_{4nhk} w_{hk} \leq N_k \quad \forall k \in K \]  

\[ z_{ik} \in \{0,1\} \quad \forall p \in F, \forall k \in K \]  

\[ w_{hk} \geq 0 \quad \forall h \in H_k, \forall k \in K \]  

where:

- \( F \) = set of flights
- \( K \) = set of fleets
- \( N_k \) = number of aircraft in fleet \( k \)
- \( M_k \) = set of all nodes for fleet \( k \)
- \( H_k \) = set of all ground arcs in fleet \( k \)
- \( e_{ik} \) = cost of flight \( i \) flown by fleet \( k \)
- \( z_{ik} \) = 1, if flight arc \( i \) is used by fleet \( k \); 0 otherwise
\[ \delta_{pqk} = 1, \text{if } p = q; 0, \text{otherwise} \]

\[ w_{nk} = \text{number of aircraft on ground arc } h \text{ in fleet } k \]

\[ b3_{nk} = 1, \text{if flight arc } i \text{ in fleet } k \text{ begins at node } n; 0 \text{ otherwise} \]
\[ = -1, \text{if flight arc } i \text{ in fleet } k \text{ ends at node } n; 0 \text{ otherwise} \]

\[ b4_{nhk} = 1, \text{if ground arc } i \text{ in fleet } k \text{ begins at node } n; 0 \text{ otherwise} \]
\[ = -1, \text{if ground arc } i \text{ in fleet } k \text{ ends at node } n; 0 \text{ otherwise} \]

\[ d3_{ik} = 1, \text{if flight arc } i \text{ in fleet } k \text{ crosses the count time}; 0 \text{ otherwise} \]

\[ d4_{hk} = 1, \text{if ground arc } i \text{ in fleet } k \text{ crosses the count time}; 0 \text{ otherwise} \]

Constraints (2.2) are the cover constraints. They force each flight leg to be flown by exactly one fleet. Constraints (2.3) are the balance constraints. They force the aircraft to circulate through the network of flights. If the balance constraints cannot be satisfied in the network, aircraft need to be ferried, that is, repositioned, to a destination without carrying any passenger—a very expensive option. Flight balance is achieved by ensuring flow conservation at each node of each fleet’s flight network. The last set of constraints (2.4) are the fleet size constraints, which force the number of aircraft of each fleet used in the model to be less than the available number. Constraints (2.5) and (2.6) force the integrality of the flight arcs and ground arcs.

In the basic fleet assignment model, maintenance, noise limitations, gate availability and crew constraints and cost are not considered. Therefore, the basic fleet assignment model is an approximation of the true fleet assignment problem.
2.4.2 An Enhanced Fleet Assignment Model

Since the basic fleet assignment model does not take into consideration maintenance of the aircraft, its solution might not be maintenance feasible, i.e., some aircraft might not have the opportunity to be maintained within the time that the FAA requires. By forcing a minimum flow on the maintenance arcs, the enhanced fleet assignment model incorporates certain FAA maintenance requirements [Barnhart, and Shenoi 1996].

Following is the enhanced fleet assignment model presented by Barnhart and Shenoi 1996.

\[
\text{Min} \sum_{k \in K} \sum_{i \in F} e_{ik} z_{ik} + \sum_{k \in K} \sum_{i \in F} e_{ik} z_{ik}^m \tag{2.7}
\]

s.t.

\[
\sum_{k \in K} \sum_{i \in F} \delta_{pik} z_{ik} + \sum_{k \in K} \sum_{i \in F} \delta_{pik}^m z_{ik}^m = 1 \quad \forall p \in F \tag{2.8}
\]

\[
\sum_{i \in F} b_3 z_{nrik} + \sum_{i \in F} b_3^m z_{nrik}^m + \sum_{h \in H_k} b_4 w_{hk} = 0 \quad \forall n \in M_k, \forall k \in K \tag{2.9}
\]

\[
\sum_{i \in F} d_3 z_{nrik} + \sum_{i \in F} d_3^m z_{nrik}^m + \sum_{h \in H_k} d_4 w_{hk} \leq N_k \quad \forall k \in K \tag{2.10}
\]

\[
\sum_{i \in F} z_{ik}^m \geq N_t \quad \forall t, \forall k \in K \tag{2.11}
\]

\[
\sum_{i \in F} z_{ik}^m \geq \left[ \frac{7N_k}{4} \right] \quad \forall k \in K \tag{2.12}
\]

\[
z_{ik}, z_{ik}^m \in \{0,1\} \quad \forall p \in F, \forall k \in K \tag{2.13}
\]

\[
w_{hk} \geq 0 \quad \forall h \in H_k, \forall k \in K \tag{2.14}
\]

where
$z_{ik}^m = \begin{cases} 1, & \text{if maintenance flight arc } i \text{ is used by fleet } k; \\ 0, & \text{otherwise} \end{cases}$

$\delta_{pk}^m = \begin{cases} 1, & \text{if } p = q, \text{ and maintenance arc } p \text{ exists for fleet } k; \\ 0, & \text{otherwise} \end{cases}$

$b_{nk}^m = \begin{cases} 1, & \text{if maintenance arc } i \text{ in fleet } k \text{ begins at node } n; \\ -1, & \text{if maintenance arc } i \text{ in fleet } k \text{ ends at node } n; \\ 0, & \text{otherwise} \end{cases}$

$d_{nk}^m = \begin{cases} 1, & \text{if maintenance arc } i \text{ in fleet } k \text{ crosses the count time}; \\ 0, & \text{otherwise} \end{cases}$

$R_t^Q = \text{set of flights that begin on any of } Q \text{ consecutive days starting on day } t$

Compared to the basic fleet assignment model, the enhanced version includes maintenance arcs in the flight network, maintenance cost in the objective function, and two more sets of constraints (2.11) and (2.12). Constraints (2.11) and (2.12) force a minimum flow on the maintenance arcs to provide each fleet with a sufficient number of maintenance opportunities.

Although maintenance is considered, the enhanced model does not exactly formulate the maintenance constraints. Three constraints provide an adequate number of maintenance opportunities, but do not guarantee that the intervals between maintenance are sufficiently short or that the maintenance visits are appropriately distributed among aircraft, i.e., some aircraft may be maintained frequently, while others not. An aircraft routing model must be solved, given the fleet assignment solution, to determine if maintenance-feasible aircraft routings can be constructed. This sequential approach of fleet assignment and aircraft routing does not guarantee an optimal solution, or even a feasible one. To achieve a global optimal solution, an integrated aircraft scheduling
model must be solved, as presented by Barnhart et al. 1997. Their approach is to assign fleets to flights and maintenance-feasible routings to aircraft simultaneously.
Chapter Three

Crew Scheduling:
Models & Solution Approaches

3.1 Introduction

This chapter introduces the crew scheduling problem and some basic definitions related to it. Work rules and cost structures for crew scheduling will be illustrated. Finally, two formulations for the crew scheduling problem will be presented; one is exact, and the other is approximate.

3.2 Overview of the Crew Scheduling Problem

The objective of crew scheduling is to find a minimum cost assignment of flight crews to work schedules, such that a given flight schedule is partitioned among the work schedules and all work rule restrictions are satisfied. The given flight schedule to be considered includes all flights that have been assigned to a single type of aircraft. The crews to be
assigned are all qualified to fly this particular aircraft type and can thus be treated identically.

3.2.1 Definitions

This section presents the terms associated with crew scheduling that will be encountered in later sections [Minoux, 1984; Rannou, 1986; Ball, and Roberts 1985; Barnhart, et al. 1993].

1. **Crew Base**: the city where the crew is domiciled, also called the *crew domicile*.

2. **Layover / Sit Time**: the time that a crew waits on the ground between flights at a city other than the crew base. If the length of the layover period is larger than a certain allowable time, then this layover period is called a crew *rest period/overnight*.

3. **Duty Period**: a sequence of flight legs that is separated from other flight legs by a rest period.

4. **Brief Time**: the time taken to brief the crew before the first flight in a duty period.

5. **Debrief Time**: the time taken by the crew for debriefing after the last flight in a duty period.

6. **Time-Away-From-Base (TAFB)**: the total duration of a pairing, that is, the difference between the arrival time of the last flight leg and the departure time of the first flight leg in the pairing, plus the brief time for the first flight leg and debrief time for the last flight leg.
7. **Crew Pairings:** a sequence of flight segments that begins at a crew base station, and returns to the same crew base station, within the maximum allowable time-away-from-base. A pairing is a set of duty periods, separated by *rest periods/overnights* that satisfy all work rules.

8. **Connect Time:** the time interval between two successive flight segments belonging to the same duty period.

9. **Flying Time:** the total duration of a schedule spent in flying. It does not include the brief and debrief times.

10. **Deadhead:** a flight leg that has one or more crews flying as passengers. Deadheading is used to transport a crew from one station to a target station, in order to utilize the crew (by assigning them to fly flights departing the target station) more effectively. Since deadheading crews occupy revenue seats on the aircraft, the practice of deadheading results in unproductive flying time.

    Duty periods typically last several hours; and pairings last for a few days. All pairings and duty periods must conform to the limitations imposed by the FAA regulations and by the carrier’s specific rules.

### 3.2.2 Work Rule Restrictions

The limitations established by the FAA are designed to ensure that crew members can fulfill their duties without significant risk of degradation of performance due to fatigue. Such rules include a minimum of eight hours rest time between duty periods, and not
more than eight hours’ flying time per duty period for domestic operations; more than eight hours’ flying time may be scheduled for international operations.

In addition to the FAA regulations, the agreement between labor unions and airlines also impose certain rules on crew scheduling, such as the limit on the maximum elapsed time for a crew pairing. Examples of typical rules from these labor agreements follow [Gershkoff, 1989].

- The agreements may guarantee a minimum flying time per duty period or a guaranteed average flying time per day, typically four to five hours. Such guarantees are negotiated to discourage management from assigning crews to very short duty periods.

- The flying time in a duty period will meet some minimum percentage of the total time on duty, say 50 percent. If the crew does not fly at least this much, they are paid as though they had. This discourages the scheduling of long sit periods between flights.

- The total flying time in the entire trip will exceed some percentage of the total hours the crew is away from its home base; this discourages excessively long layovers.

- Deadheading crews are to be paid for 50 percent or more of the flying time of the leg, even though there is no productive work in a deadhead flight.

Other restrictions might be prescribed, such as maximum and minimum sit times between flight segments within duty periods, and a maximum number of duty periods to compose a legal pairing.
3.2.3 Cost

Specific components of cost depend on the airline's crew salary structure and work rules; however, crew salaries and hotel expenses are common to most airlines. Crew salaries are the dominant cost components in crew scheduling. Hotel expenses are incurred when crews stay overnight at stations other than their home bases. The airline must provide a hotel room for each member of the crew. The large airlines usually pay several million dollars each month for hotel rooms.

In most North American airlines, crews' pay is based on the *credit hours* associated with a pairing. Credit hours are determined by the maximum of: 1) flying time, 2) a guaranteed minimum of credited flying time, and 3) the total elapsed time or time away from base of the pairing. Since a variety of penalties are applied, the credit hours can be much larger than the total flying time associated with a pairing [Ball, and Roberts 1985]. *Pay-and-credit* measures the productivity of the crews. Pay-and-credit refers to the difference between the number of hours for which a crew member is paid and the number of hours the crew actually flies. *Pay-and-credit*, usually expressed as a percentage of total flying time, represents the unproductive crew time and should be minimized.

The various components of the credit hours lead to the nonlinear formula for the crew cost. In addition, flight legs may not be priced individually because the effect of the same flight leg on the cost of a pairing will depend upon the characteristics of the particular pairing and, further, flight cost will vary with its position in the pairing [Crainic, and Rousseau 1987]. Therefore, the crew cost of a pairing cannot be
determined until the pairing has been fully generated. This creates particular difficulties in determining the crew cost [Crainic, and Rousseau, 1987].

In the long-haul crew scheduling problem, time-away-from-base is the dominant factor in determining overall cost. Therefore, we use an approximate pairing cost structure, that is, we assume that the cost of a pairing is its time-away-from-base cost, rather than the maximum of duties, the TAFB, and the minimum guaranteed costs. Thus, crew cost is a linear function of time, and the objective is to minimize the overall time-away-from-base. Approximating costs in this manner provides only a lower bound estimate on the true pairing costs but achieves a substantial reduction in problem complexity.

3.2.4 Network Representation

The time-line network of duties [Barnhart, et al. 1993] can be used for crew pairing problems, especially those involving non hub-and-spoke operations. This network is quite similar to that of the fleet assignment except for the following:

1. A network of duties is created for each crew base, and

2. The cost assigned on each arc is the elapsed time of that arc.

3.3 Models

The crew pairing problem has been extensively studied in the past decades (i.e., Thiriez 1969, Arabeyre et al. 1969, Rubin 1973, Baker et al. 1979, Shepardson and Marsten

3.3.1 Set Partitioning and Covering Models

The crew pairing problem (CPP) is typically formulated as a set partitioning problem, where each row corresponds to a scheduled flight and each column corresponds to a feasible pairing. Since the pairings have implicitly captured the constraints on the maximum time-away-from-base, minimum rest, maximum duty time, and so on, the only constraint explicitly enforced is flight coverage, that is, each flight must be included in exactly one pairing. The objective is to select a subset of pairings with minimum cost.
The CPP formulation is as follows.

\[(CPP) \quad \text{Min} \sum_{j=1}^{n} c_j x_j \quad (3.1)\]

s.t.

\[\sum_{j=1}^{n} a_{ij} x_j = 1 \quad i = 1,\ldots,m \quad (3.2)\]

\[x_j \in \{0,1\} \quad j = 1,\ldots,n \quad (3.3)\]

where

- \(m\) = number of operational flights
- \(n\) = number of crew pairings
- \(a_{ij} = 1\), if operational flight \(i\) is covered by the pairing \(j\); and 0, otherwise
- \(c_j\) = time-away-from-base cost of crew pairing \(j\)
- \(x_j = 1\) if a crew is assigned to pairing \(j\); and 0, otherwise

When deadheading is allowed, the flight coverage constraints (3.2) can be relaxed to allow a flight leg to be covered by more than one pairing. The revised formulation is a set covering problem. When TAFB is the determining cost component, and revenue-generating passengers are not displaced by deadheading crews, the cost of a crew flying equals the cost of a crew deadheading on the same flight. In this case, the set covering pairing formulation is:
(CPP - Revised) \[ \text{Min} \sum_{j=1}^{n} c_j x_j \]

s.t. \[ \sum_{j=1}^{n} a_{ij} x_j \geq 1 \quad i = 1, \ldots, m \] \[ x_j \in \{0, 1\} \quad j = 1, \ldots, n \]

In the case of multiple crew bases, there may be some restrictions on the available crew credit hours. However, when this constraint is added to the problem, it is no longer a "pure" set partitioning problem or set covering problem. In our discussion, we assume this constraint can always be satisfied, and thus it is ignored.

Since the set partitioning model requires that deadhead flights be specified a priori for each pairing, compared to set covering models, there is a significant increase in the number of decision variables and solution time required. Computational experiments show that the set covering formulation is much easier to solve, and so, we use it as our CPP model.

As we saw above, the columns of CPP are all the feasible pairings that can be formed. For an airline crew scheduling problem, billions of possible pairings are common. Therefore, if we attempt to enumerate all possible pairings, the problem can be intractable and unsolvable. Thus, column generation is needed to solve this problem. The basic idea of column generation is to generate a subset of the columns containing the optimal solution, and solve only restricted problems containing only the selected columns to obtain the optimal solution. Column generation has been well studied for decades [Bradley, et al. 1977, Barnhart, et al. 1993]. The most recent study on column generation
by Barnhart, et al. 1995 (Branch-and-Price), reviews the literature and details a branch-
and-bound solution approach for obtaining optimal solutions for large scale integer
programs.

3.3.2 A Multicommodity Flow Model

It is time-consuming to solve CPP, because it involves the column generation method, in
which many LPs are solved before an optimal or a near-optimal solution is obtained. Barnhart, et al. (1993) developed an approximate integer multicommodity flow model by
replacing crew pairing variables with duty variables. The duty period based formulation
(DPP) for a specific fleet type is:

\[(\text{DPP}) \quad \text{Min} \sum \sum C_j^r X_j^r + \sum \sum D_H^r Y_H^r \quad (3.7)\]

\[\text{s.t.} \]

\[\sum \sum A_{ij} X_j^r \geq 1 \quad \forall i \in F \quad (3.8)\]

\[\sum J B_{Lj} X_j^r + \sum H B_{Lh} Y_H^r = 0 \quad \forall L \in M; \forall R \quad (3.9)\]

\[X_j^r \in \{0,1\} \quad \forall J; \forall R \quad (3.10)\]

\[Y_H^r \in Z^+ \quad \forall H; \forall R \quad (3.11)\]

where:
\( J = \text{Set of duties} \)

\( F = \text{Set of operational flights} \)

\( H = \text{Set of ground arcs} \)

\( M = \text{Set of network nodes} \)

\( R = \text{Set of crew types (i.e., Set of crew bases)} \)

\( X^R_J = 1, \text{if duty } J \text{ is flown by crew type } R; \ 0, \text{otherwise} \)

\( Y^R_H = \text{number of type } R \text{ crews using ground arc } H \)

\( C^R_J = \text{elapsed time cost of duty } J \text{ flown by crew type } R \)

\( D^R_H = \text{elapsed time cost of ground arc } H \text{ when used by crew type } R \)

\( A_{ij} = 1, \text{if duty } J \text{ covers operational flight } i; \ 0, \text{otherwise} \)

\( B_{ij} = 1, \text{if duty } J \text{ enters node } L; \ -1 \text{if it leaves node } L; \ 0, \text{otherwise} \)

\( B_{jL} = 1, \text{if ground arc } H \text{ enters node } L; \ -1 \text{if it leaves node } L; \ 0, \text{otherwise} \)

The objective function (3.7) minimizes total time-away-from-base crew costs. Bundle constraints (3.8) guarantee that each operational flight is covered at least once. Constraints (3.9) ensure the conservation of flow for each crew type. Constraints (3.10) ensure that the number of crews assigned to any duty is 0 or 1. Constraints (3.11) guarantee that the number of crews on the ground at each point in time remains integral. The last constraints can be relaxed to non-negative constraints only because integrality of the duty variables ensures integrality of ground variables.

The duty period model (DPP) is an approximate model of the crew pairing problem, and its optimal objective value is a lower bound on the optimal value of CPP. DPP undervalues the true crew cost of any schedule, since 1) it sets pairing cost equal to the time-away-from-base cost of the pairing, and 2) DPP is a relaxation of CPP, i.e.,
constraints on the maximum time-away-from-base and the maximum number of duties within a pairing are eliminated in DPP. The result is that any CPP solution is feasible to DPP, however, a solution of DPP might not be feasible to CPP.

For long-haul crew scheduling, DPP has a significant advantage over CPP. In the long-haul crew scheduling problem, the number of duty periods is often two to three times the number of flights. Therefore, DPP can have a significantly reduced number of columns compared to CPP, and is much easier and faster to solve.

3.4 Deadheading in Crew Scheduling

The flights making up a duty period can be classified into two types: operational and deadhead. Operational flights are the flights scheduled during the planning period that must be assigned a crew. Deadhead flights are either operational flights with two or more assigned crews or flights operated by another airline with one or more crews assigned as passengers. Unlike domestic U.S. operations in which deadheading may be limited, deadheading is an essential component in long-haul operations where relatively few flights may be scheduled in and out of a particular location. The reason is that the long-haul crew pairing problems are characterized by sparsity of flights and extended periods of inactivity for crews at some stations. To eliminate these extended rest periods and reduce overall costs, it is advantageous in some cases to deadhead crews, and reposition them for better utilization. [Barnhart, et al. 1995]

The deadhead selection approach presented by Barnhart, et al 1995, iterates between solving the LP relaxation of the crew pairing problem, and selecting additional
deadheads. Their application to the long-haul crew pairing problem of a U.S. carrier had achieved reductions of over 27% in pay-and-credit. Figure 3-1 depicts this deadhead selection procedure. Since this approach involves iteratively solving the LP relaxation of CPP, it is very computationally expensive to select deadheads. Barnhart, et al. 1996, replace CPP with DPP allowing more deadheads to be selected, while dramatically reducing solution time, and improving solution quality [Barnhart, et al. 1996].

![Diagram of Crew Pairing Solver and Deadheads Selector](image)

**Figure 3-1:** Deadhead Selection Procedure [Barnhart, et al. 1995]
Chapter Four

The Integrated Airline Scheduling Problem

4.1 Introduction

This chapter presents current advances in integration of airline scheduling and an advanced sequential approach to generate a feasible fleeting and crew solution to the airline scheduling problem by solving an integrated approximate airline scheduling model, followed by solving several crew pairing problems. Finally, a case study is presented to evaluate this advanced sequential approach.

4.2 Background
It is well known that fleet assignment has a huge impact on the revenue and cost of airlines, while crew costs are the second largest component in the operating costs. In the conventional sequential approach, fleet assignment is solved first, followed by the solution of the crew pairing problem for each fleet type. Figure 5-1 depicts this solution approach. We let $Z_{FAM}^*$ denote the optimal solution value of the fleet assignment model, $Z_{CPP}^k$ denote the optimal solution value of CPP for fleet type $k$, $k = 1, \ldots, K$, and $\sum_{k=1}^{K} Z_{CPP}^k$ denote the total crew cost. Since the basic fleet assignment model does not reflect the effects of fleeting decisions on crew costs, the crew costs resulting from a sequential solution approach might be much higher than crew costs in a global optimal airline scheduling solution. In addition, this sequential solution process might not yield a globally feasible solution to the airline scheduling problem, and modifying it to achieve feasibility can result in a significant degradation of the solution value.
Although fleet assignment and crew scheduling should be considered jointly in decision making process, the combined mathematical model is unsolvable in practical terms. To our knowledge, there has been no work that models and solves both scheduling problems simultaneously. Clarke, et al. 1996 incorporate some aspects of crew scheduling into the fleet assignment model while retaining solvability. They attempt to provide better crew connections to balance the costs between lonely overnights and fleeting; however, they do not explicitly consider crew costs in their model.

It is straightforward to include some crew constraints in fleet assignment. For example, a lower bound on the number of departures of a fleet from crew bases for that fleet and an upper bound on the number of flying hours for the fleet can be incorporated. To achieve the best balance in fleet and crew costs, however, an exact, integrated fleet assignment and crew pairing model and algorithm is necessary.

**Past Achievements on the Integrated Approach**

Prior to Barnhart, et al. (1996), the decisions about fleet assignment, through assignment, and maintenance routing were made sequentially. Barnhart, et al. (1996) developed an integrated model for aircraft scheduling. They integrated fleet assignment, through flight assignment and maintenance routing into one aircraft scheduling model. As for the crew planning problem, Barnhart, et al. (1995) have developed the integrated crew pairing model (CPP), integrating crew pairing and deadhead selection in the long-haul crew scheduling problem, and thus achieving millions in savings for the airline.
4.3 An Advanced Solution Approach

For computational reasons, it is not practical to solve exactly a model that combines the fleet assignment and crew pairing problems. We develop an advanced sequential solution approach in which an integrated approximate airline scheduling model is solved, then given its fleeting decisions, we solve one crew pairing problem for each fleet type. Figure 5-2 depicts our advanced solution approach.

The integrated approximate model combines the basic fleet assignment model and the DPP crew scheduling model, and produces a feasible solution to the fleet assignment and crew pairing problems. DPP is used in the integrated approximate model because: 1) for long-haul problems, DPP, a multi-commodity flow formulation, has significantly reduced size compared to CPP, and 2) DPP is a very good approximation to CPP.

Given the fleet assignments of the integrated model, CPP is solved for each fleet type $k$. The fleet assignment solution obtained from the integrated approximate model and the CPP solutions for each fleet $k = 1, 2, \ldots , K$ yield feasible fleeting and crew solutions. We compare these solutions to those achieved by the conventional approach of solving the basic fleet assignment model followed by solving one CPP for each fleet. We also can compare a lower bound on the optimal integrated solution with the sequential solution value. The lower bound is produced by solving our approximate integrated fleet assignment and crew model.

We let $Z_I$ denote the optimal solution to the integrated approximate model, $Z_{FAM-I}$ denote the total fleeting cost in the solution to the integrated model, $Z^k_{CPP-I}$ denote the optimal solution to CPP for fleet $k$, and $Z_{DPP-I}$ denote the total crew cost in the solution
of the integrated model. Then we make the following observations about bounds: 1) \( Z_I \) is a lower bound on the optimal solution to the integrated fleet assignment and CPP models since DPP provides a lower bound on the optimal solution value of CPP; 2) \( Z_{FAM-I} \geq Z_{FAM}^* \) since crew constraints are considered in the integrated model, but are omitted from the basic fleet assignment model; and 3) \( Z_{DPP-I} = \sum_{k=1}^{K} Z_{CPP-I}^k \leq \sum_{k=1}^{K} Z_{CPP}^k \) because DPP is a relaxation of CPP and the integrated model will produce fleeting solutions that allow lower cost crew pairing solutions.

\[
Z_I = Z_{FAM-I} + Z_{DPP-I}
\]

**Figure 4-2:** The advanced sequential solution approach for the airline scheduling problem
4.4 The Integrated Approximate Airline Scheduling Model

The integrated approximate model used in our advanced sequential approach combines the basic fleet assignment model and the DPP crew scheduling model. The formulation of the integrated model is shown below. This model is comprised of various subproblems: one fleet assignment subproblem and a number of crew scheduling subproblems. As compared with the fleet assignment and duty period models, all the variables are defined the same except the DPP variables all have an added subscript $k$, corresponding to fleet $k$.

\[
\begin{split}
\text{Min} & \quad \sum_{k} \sum_{i} e_{ik} z_{ik} + \sum_{k} \sum_{R} \sum_{J} C_{j}^{Rk} X_{j}^{Rk} + \sum_{k} \sum_{R} \sum_{H} D_{H}^{Rk} Y_{H}^{Rk} \\
\text{s.t.} & \quad \sum_{k} \sum_{i} \delta_{pik} z_{ik} = 1 \quad \forall \ p \\
& \quad \sum_{i} b_{3nk} z_{ik} + \sum_{h} b_{4nhk} w_{hk} = 0 \quad \forall \ n, \forall \ k \\
& \quad \sum_{i} d_{3nk} z_{ik} + \sum_{h} d_{4nhk} w_{hk} \leq N_{k} \quad \forall \ k \\
& \quad \sum_{i} (-\delta_{pik}) z_{ik} + \sum_{R} \sum_{J} A_{ij}^{k} X_{j}^{Rk} \geq 0 \quad \forall \ i, \forall \ k \\
& \quad \sum_{J} B_{Lj}^{k} X_{j}^{Rk} + \sum_{H} B_{LH}^{k} Y_{H}^{Rk} = 0 \quad \forall \ L, \forall \ R, \forall \ k \\
& \quad z_{ik} \in \{0,1\} \quad \forall \ i, \forall \ k \\
& \quad w_{hk} \geq 0 \quad \forall \ h, \forall \ k \\
& \quad X_{j}^{Rk} \in \{0,1\} \quad \forall \ j, \forall \ R, \forall \ k
\end{split}
\]
\[ Y_{H}^{Rk} \geq 0 \quad \forall H, \forall R, \forall k \quad (4.10) \]

where,

\[ X_{J}^{Rk} = 1, \text{if duty } J \text{ flown by crew type } R \text{ and fleet type } k; 0, \text{otherwise} \]

\[ Y_{H}^{Rk} = \text{number of type } R \text{ crews who fly type } k \text{ fleet using ground arc } H \]

\[ C_{J}^{Rk} = \text{elapsed time cost of duty } J \text{ flown by crew type } R \text{ and fleet type } k \]

\[ D_{H}^{Rk} = \text{elapsed time cost of ground arc } H \text{ when used by crew type } R \text{ and fleet type } k \]

\[ A_{iJ}^{k} = 1, \text{if duty } J \text{ flown by fleet } k \text{ covers operational flight } i; 0, \text{otherwise} \]

\[ B_{LJ}^{k} = 1, \text{if duty } J \text{ flown by fleet } k \text{ enters node } L; -1, \text{if it leaves node } L; 0, \text{otherwise} \]

\[ B_{LH}^{k} = 1, \text{if ground arc } H \text{ using by fleet } k \text{ enters node } L; -1, \text{if it leaves node } L; 0, \text{otherwise} \]

The objective function (4.1) minimizes the sum of the total fleet assignment costs and total TAFB for all fleet types. Constraints (4.2)-(4.4) are the fleet assignment constraints. Flight cover constraints (4.2) ensure that all the flights in the schedule are assigned to exactly one fleet. Balance constraints (4.3), one set for each fleet type, ensure that balance is maintained for each fleet at each station. Constraints (4.4) ensure that the number of aircraft for each fleet type does not exceed the number available in that fleet.

Constraints (4.6) are balance constraints for crews. One set of balance constraints for each fleet for each crew base, ensures that balance is maintained for each crew at each station for each crew.

Constraints (4.5) bind the fleet assignment subproblem and crew scheduling subproblems together. These bundle constraints guarantee that if a flight is assigned to a fleet, it will be flown by a crew that is eligible to fly that fleet. Constraints (4.7) and (4.8), (4.9) and (4.10) ensure integrality of the decision variables [Shenoi, 1996].
Model Depiction

Figure 4-3 depicts an example of the constraint matrix for the integrated model. In this example, there are two fleet groups (Fleet type I and II). The crews assigned to the first group have three crew bases (CR1, CR2, and CR3), while the crews in the second group have two bases (CR2, CR4). Both crews have a common crew base CR2. Constraints (A), (B), and (C) correspond to constraints (4.2), (4.3), and (4.4) in the fleet assignment model respectively. Constraints (D) and (E) are the bundle constraints corresponding to constraints (4.5). Constraints (F) and (G) are crew balance constraints corresponding to constraints (4.6).
**Figure 4-3:** Integrated approximate airline scheduling model’s constraint matrix
4.5 Case Study

We conduct our experiments using the data provided by a long-haul U.S. airline. Table 4-1 shows the data set for two fleet types. Fleet type I has one crew base, while fleet type II has three crew bases. The crew base of fleet I is one of the crew bases of fleet II. 964 operational flights are in the data set. These flights need to be covered by exactly one fleet and at least one crew. 452 deadhead flights are considered in the data set. These flights will not be covered by any fleet, and need not be covered by crews. A total of 1527 duty periods are constructed from the operational flights and deadhead flights. These duties include the operational flights, deadheads, and duty periods composed of two or more operational flights and deadheads.

<table>
<thead>
<tr>
<th>Fleet Type</th>
<th>I</th>
<th>II</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crew Bases</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Number of aircraft</td>
<td>17</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>Seat Capacity</td>
<td>418</td>
<td>298</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4-1: Characteristics of the data set

Basic Fleet Assignment Solution: Baseline for Comparison

Given the fleet operating cost for the flights, the projected demand, and the fares of the flights, the total cost of assigning each fleet to each flight is computed. We then solve the basic fleet assignment model (2.1-2.6). The result is shown in Table 4-2.
<table>
<thead>
<tr>
<th>Fleet Type</th>
<th>I</th>
<th>II</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Flights</td>
<td>795</td>
<td>169</td>
<td>964</td>
</tr>
<tr>
<td>Fleeting Cost</td>
<td>135,434,286</td>
<td>20,032,096</td>
<td>155,466,382</td>
</tr>
</tbody>
</table>

**Table 4-2**: Basic fleet assignment solution: baseline

**DPP and CPP Solution: Baseline for Comparison**

To provide a baseline against which we can compare the result of the integrated model, we solve the LP and IP of the DPP and CPP models twice, once for flights assigned to fleet I in the solution of the basic fleet assignment model, and once for the flight assigned to fleet II. The results are shown in Table 4-3. It is observed that the optimality gap between IP and LP solutions is very small; although the gap between the solutions of DPP-IP and CPP-IP is large for fleet I, the solution of DPP-IP is almost equal to that of CPP-IP for fleet II and all following crew solutions (Table 4-4 and Table 4-5). We conclude that DPP is a good approximation of CPP, and so, our integrated approximate model should provide a tight lower bound to the optimal combined fleet assignment and CPP solution value.
<table>
<thead>
<tr>
<th>Fleet Type</th>
<th>I</th>
<th>II</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crew Cost</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DPP-LP</td>
<td>119,476,800</td>
<td>22,987,371</td>
<td>142,464,171</td>
</tr>
<tr>
<td>DPP-IP</td>
<td>119,476,800</td>
<td>22,987,371</td>
<td>142,464,171</td>
</tr>
<tr>
<td>Optimality Gap</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Crew Cost</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPP-LP</td>
<td>126,398,571</td>
<td>22,987,371</td>
<td>149,385,942</td>
</tr>
<tr>
<td>CPP-IP</td>
<td>126,648,943</td>
<td>22,987,371</td>
<td>149,636,314</td>
</tr>
<tr>
<td>Optimality Gap</td>
<td>0.9%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Approximation Gap</td>
<td>5.46%</td>
<td>0%</td>
<td>4.62%</td>
</tr>
</tbody>
</table>

**Table 4-3: Crew scheduling solutions: baselines**

**Solution of DPP and CPP for the Mixed Data, Ignoring Fleet Type**

To determine the maximum cost reduction in crew costs achievable by an integrated airline scheduling model, we mix the flights for fleet I and II, ignoring the fleet type, and find the least cost crew pairing solution. Table 4-4 compares the result from mixed data with the results from the separated data of the basic fleet assignment solution. It is observed that crew costs are reduced by more than 20%.

---

4 Optimality Gap = \( \frac{\text{IP} - \text{LP}}{\text{IP}} \times 100\% \)

5 Approximation Gap = \( \frac{(\text{CPP - IP}) - (\text{DPP - IP})}{\text{CPP - IP}} \times 100\% \)
<table>
<thead>
<tr>
<th></th>
<th>Baseline (A)</th>
<th>Mixed Data (B)</th>
<th>Cost Reduction (B-A)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DPP</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DPP-LP</td>
<td>142,464,171</td>
<td>113,240,743</td>
<td>20.51%</td>
</tr>
<tr>
<td>DPP-IP</td>
<td>142,464,171</td>
<td>113,240,743</td>
<td>20.51%</td>
</tr>
<tr>
<td><strong>CPP</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPP-LP</td>
<td>149,385,942</td>
<td>113,240,743</td>
<td>24.18%</td>
</tr>
<tr>
<td>CPP-IP</td>
<td>149,636,314</td>
<td>113,240,743</td>
<td>24.33%</td>
</tr>
<tr>
<td>Approximation Gap</td>
<td>5.46%</td>
<td>0%</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 4-4:** Effects of ignoring fleet type in computing minimum possible crew cost

**Solution of the Advanced Sequential Approach**

We solve the integrated approximate airline scheduling model (4.1-4.10), which combines the basic fleet assignment model (a close approximation to the fleet assignment problem), and the DPP (a close approximation to the crew pairing problem). The result provides a lower bound on the global optimal fleeting and crew solutions, and a feasible fleeting solution for the approximate airline scheduling problem.

From the solution of the integrated model, we extract the flights that are flown by fleet I and II respectively. 752 flights are assigned to fleet I, while 212 flights are assigned to fleet II. We use the extracted flights for each fleet as the input of CPP, and solve CPP for each fleet type. We compare this solution with the baseline solutions we generated using a conventional sequential approach. Table 4-5 shows that we are able to reduce total fleeting and crew costs by more than 3% — a savings that could translate to
millions of dollars annually for large U.S. airlines. In addition to these savings, we observe:

1. The gaps between the solutions of CPP and DPP are typically very small and there is little difference between the combined FAM and DPP solution value and the combined FAM and CPP solution value. Therefore, the DPP model is a very good approximation of CPP.

2. Compared to the baselines, the fleet assignment solution in our approach is slightly increased (by about 1.03%), but crew costs decrease dramatically (by about 7.55%). This reduction in crew costs offsets the increase in fleeting costs to produce an overall cost reduction of 3.21%. This provides evidence that fleeting decisions should not be made in a vacuum since their resulting effects on crew costs can be significant.

3. The gap between the optimal solution value for the approximate integrated fleet and crew IP (or LP) and the optimal solution value for the conventional sequential approach is about 3.50% (Table 4-6). Since an optimal IP or LP solution to the approximate integrated model is a lower bound on the costs of the optimal fleet and crew solutions, integrating fleet assignment and crew scheduling could have a potential savings of 3.50% (possibly many millions of dollars annually to large U.S. airlines) compared to current fleeting and crew solutions.

4. In Table 4-4, we report that the minimum achievable crew cost is more than 20% lower than the baseline. In Table 4-5, however, we show that the actual reduction in crew costs is only 7.55%. This discrepancy results because the 20% reduction is possible only when fleet balance does not have to be enforced and the number
of aircraft used of any type is allowed to exceed the number available. When these constraints are enforced, however, the potential improvement in crew costs is reduced significantly, as indicated by the DPP solution values in our approximate integrated model.

<table>
<thead>
<tr>
<th></th>
<th>Lower Bound of Fleet and Crew Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential (A)</td>
<td></td>
</tr>
<tr>
<td>LP</td>
<td>304,852,324</td>
</tr>
<tr>
<td>IP</td>
<td>305,102,696</td>
</tr>
<tr>
<td>Integrated (B)</td>
<td></td>
</tr>
<tr>
<td>LP</td>
<td>294,416,424</td>
</tr>
<tr>
<td>IP</td>
<td>294,416,424</td>
</tr>
<tr>
<td>Optimality Gap (B-A)</td>
<td></td>
</tr>
<tr>
<td>LP</td>
<td>(3.50%)</td>
</tr>
<tr>
<td>IP</td>
<td>(3.51%)</td>
</tr>
</tbody>
</table>

**Table 4-6: Lower bound comparison**
<table>
<thead>
<tr>
<th></th>
<th>Fleet I + II (Sequential) (A)</th>
<th>Mixed Data (Minimum Crew Cost) (B)</th>
<th>Fleet I + II (Advanced) (C)</th>
<th>Decrement (C-A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPP Solution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPP-LP</td>
<td>149,385,942</td>
<td>113,240,743</td>
<td>138,320,742</td>
<td>7.56%</td>
</tr>
<tr>
<td>CPP-IP</td>
<td>149,636,314</td>
<td>113,240,743</td>
<td>138,320,742</td>
<td>7.55%</td>
</tr>
<tr>
<td>DPP Solution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DPP-LP</td>
<td>142,464,171</td>
<td>113,240,743</td>
<td>137,402,314</td>
<td>3.51%</td>
</tr>
<tr>
<td>DPP-IP</td>
<td>142,464,171</td>
<td>113,240,743</td>
<td>137,402,314</td>
<td>3.51%</td>
</tr>
<tr>
<td>FAM Solution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>155,466,382</td>
<td>(FAM infeasible)</td>
<td>157,014,114</td>
<td>(1.03%)</td>
</tr>
<tr>
<td>Total (FAM+CPP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LP</td>
<td>304,852,324</td>
<td>--</td>
<td>295,334,856</td>
<td>3.12%</td>
</tr>
<tr>
<td>IP</td>
<td>305,102,696</td>
<td>--</td>
<td>295,334,856</td>
<td>3.21%</td>
</tr>
<tr>
<td>Total (FAM+DPP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LP</td>
<td>297,930,553</td>
<td>--</td>
<td>294,416,424</td>
<td>1.17%</td>
</tr>
<tr>
<td>IP</td>
<td>297,930,553</td>
<td>--</td>
<td>294,416,424</td>
<td>1.17%</td>
</tr>
</tbody>
</table>

**Table 4-5:** Result of the advanced sequential airline scheduling approach
Chapter Five

Summary and
Future Research Directions

5.1 Summary

This thesis concentrates on quantifying the difference in solution quality between simultaneous and sequential solution methods for airline scheduling.

Airline scheduling typically includes fleet assignment, through flight assignment, aircraft maintenance routing, and crew pairing optimization. These models are solved sequentially due to the limitations of computer technology. However, simultaneous solution approaches are preferred because they do not create incompatibilities like those generated in sequential approaches and they generate lower cost solutions.

The advanced solution approach for the approximate airline scheduling problem presented in the thesis solves an integrated approximate fleet assignment and crew pairing model and then, given the fleet assignments, solves the crew pairing problems. The integrated model combines the basic fleet assignment model with duty period crew scheduling models. Since the basic fleet assignment optimal solution has cost that is a
good estimate of the true optimal fleet assignment cost, and since the duty period crew pairing optimal solution cost is a good estimate of the true optimal crew pairing cost, our integrated model provides a good estimate of the optimal cost of an airline scheduling solution. In addition, since the integrated model explicitly considers the fleet cost and constraints, the solution of the integrated model is feasible to fleet assignment. However, the DPP solution might not satisfy some work rules, and so, CPP is solved to generate feasible crew solutions. We show that the potential improvement of our solution approach compared to conventional solution approaches could be more than 3% — savings of millions of dollars annually for a major U.S. airline.

Our integrated approximate model may not be solvable for hub-and-spoke operations since the number of duties in DPP will grow exponentially. Further, the DPP model might not be a good approximation of CPP for domestic crew scheduling problems since maximum allowable time-away-from-base is quite short, from two to five days. In this case, time-away-from-base might not be the dominant cost of crew costs, and the DPP solution might provide a poor estimate of optimal crew costs.

5.2 Further Work

In this thesis, we conduct our computational experiments using one set of data. Additional computational experimentation should be performed to evaluate further the potential benefits of this integrated approach.
The integrated model presented in this thesis does not consider aircraft maintenance requirements. One possible extension of our model is to combine the enhanced fleet assignment model (2.7-2.14) and the duty period crew scheduling model, i.e.:

\[
\text{Min } \sum_{k} \sum_{i} e_{ik} z_{ik} + \sum_{k} \sum_{i} e_{ik} z_{mk} + \sum_{k} \sum_{j} \sum_{R} c_{jk}^{Rk} x_{jk}^{Rk} + \sum_{k} \sum_{R} \sum_{H} d_{H}^{Rk} y_{H}^{Rk} \tag{5.1}
\]

s.t.

\[
\sum_{k} \sum_{i} \delta_{pik} z_{ik} + \sum_{k} \sum_{i} \delta_{pik}^{m} z_{mk} = 1 \quad \forall p \tag{5.2}
\]

\[
\sum_{i} b_{3n_k} z_{ik} + \sum_{i} b_{3n_k}^{m} z_{mk} + \sum_{h} b_{4n_h} w_{hk} = 0 \quad \forall n, \forall k \tag{5.3}
\]

\[
\sum_{i} d_{3n_k} z_{ik} + \sum_{i} d_{3n_k}^{m} z_{mk} + \sum_{h} d_{4n_h} w_{hk} \leq N_k \quad \forall k \tag{5.4}
\]

\[
\sum_{r \in R_k} z_{rk}^{m} \geq N_k \quad \forall t, \forall k \tag{5.5}
\]

\[
\sum_{r} z_{rk}^{m} \geq \left\lceil \frac{7N_k}{4} \right\rceil \quad \forall k \tag{5.6}
\]

\[
\sum_{i} (-\delta_{pik}) z_{ik} + \sum_{i} (-\delta_{pik})^{m} z_{mk} + \sum_{j} A_{jk}^{i} x_{jk}^{Rk} \geq 0 \quad \forall i, \forall k \tag{5.7}
\]

\[
\sum_{j} B_{Lj}^{i} x_{jk}^{Rk} + \sum_{H} B_{LH}^{k} y_{H}^{Rk} = 0 \quad \forall L, \forall R, \forall k \tag{5.8}
\]

\[\begin{align*}
z_{ik}, z_{mk} & \in \{0,1\} \quad \forall i, \forall k \tag{5.9} \\
w_{hk} & \geq 0 \quad \forall h, \forall k \tag{5.10} \\
x_{jk}^{Rk} & \in \{0,1\} \quad \forall J, \forall R, \forall k \tag{5.11} \\
y_{H}^{Rk} & \geq 0 \quad \forall H, \forall R, \forall k \tag{5.12}
\end{align*}\]
This model will provide a tighter lower bound on the integrated scheduling solution because certain maintenance requirements are incorporated into this enhanced integrated model.
References


TITLE VARIES: □ in degree book, last word is "Algorithm" (no "s")

NAME VARIES:

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SUPERVISORS:

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YEAR: 1997 DEGREE: M.S.

NAME: LU Fang