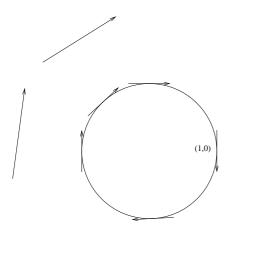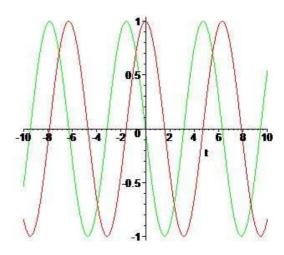## Solutions to Recitation 21 Problems

1. At (1,0), it's a vertical arrow downward; at (0,1), it's a horizontal arrow to the right; at (-1, 0), it's a vertical arrow upward; at (0,-1), it's a horizontal arrow to the left. And the arrow length gets longer as we go outward from the origin. So we can see that a curve that connects these arrows must be a circle going in clockwise direction. So the curve that passes through (1,0) is the circle $x^2 + y^2 = 1$.

Differentiating $\dot{x} = y$, we get $\ddot{x} = \dot{y} = -x$, so the equation is $\ddot{x} + x = 0$. The general solution is $C_1 \cos(t) + C_2 \sin(t)$. Since the initial value of x is 1 and of $\dot{x} = y = 0$, we see that $x = \cos(t)$. So $y = \dot{x} = -\sin(t)$. The graph of $\mathbf{u}(t)$ looks exactly as drawn in the sketch on the left. The red graph is $x(t)$ and green one is $y(t)$.

2. Let $y = \dot{x}$. Then $\dot{y} = \ddot{x} = -\frac{1}{2}\dot{x} - \frac{17}{16}x = -\frac{1}{2}y - \frac{17}{16}x$. So the system is

$$\begin{cases} \dot{x} = y \\ \dot{y} = -\frac{17}{16}x - \frac{1}{2}y \end{cases}.$$

Note that we have successfully described $\dot{\mathbf{u}}$ in terms of $\mathbf{u}$. The roots of the characteristic equation of the 2nd order equation is $-\frac{1}{4} \pm i$, so the general solutions are
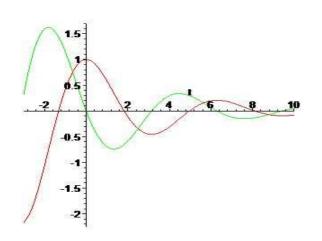
$$C_1 e^{-t/4} \cos(t) + C_2 e^{-t/4} \sin(t)$$

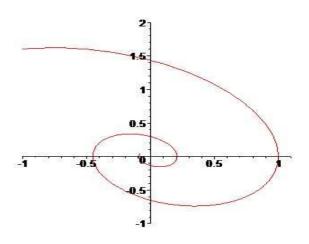Because $x(0) = 1$, $C_1 = 1$. Taking derivative, $-\frac{1}{4} + C_2 = 0$, so $C_2 = \frac{1}{4}$. So we must have

$$x = e^{-t/4}\cos(t) + \frac{1}{4}e^{-t/4}\sin(t)$$

$$y = \dot{x} = -\frac{17}{16}e^{-t/4}\sin(t)$$

Sketch of $x(t)$ is in red and of $y(t)$ is in green below. The resulting sketch of the trajectory $\mathbf{u}(t)$ is on the right. The graph of $\mathbf{u}(t)$ is a spiral inward as $t \to \infty$ and it is on the positive x-axis every time t is an integer multiple of 2π. The velocity is $\dot{x} = y$, so at each point of the trajectory, the velocity is represented by the y-coordinate.

3.

$$(x + 2y), \begin{pmatrix} x & y \\ 2x & 2y \end{pmatrix}, \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}, \begin{pmatrix} ax + by & au + bv \\ cx + dy & cu + dv \end{pmatrix}$$