Model-independent confirmation of the $Z(4430)^-$ state

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I. INTRODUCTION

Almost all known mesons and baryons can be described in the quark model with combinations of two or three quarks, although the existence of higher multiplicity configurations, as well as additional gluonic components, is, in principle, not excluded [1]. For many years, significant effort has been devoted to the search for such exotic configurations. In the baryon sector, resonances with a five-quark content have been searched for extensively [2]. Recently, LHCb has observed a resonance in the $J/\psi p$ channel, compatible with being a pentaquark-charmonium state [6]. In the meson sector, several charmoniumlike states, that could be interpreted as four-quark states [7,8], have been reported by a number of experiments, but not all of them have been confirmed.

The existence of the $Z(4430)^-$ hadron, originally observed by the Belle Collaboration [9–11] in the decay $B^0 \rightarrow \psi(2S)K^-\pi^-$ with $Z(4430)^- \rightarrow \psi(2S)\pi^-$, was confirmed by the LHCb Collaboration [12] (the inclusion of charge-conjugate processes is implied). This state, having a minimum quark content of $c\bar{c}ud$, is the strongest candidate for a four-quark meson [13–24]. Through a multidimensional amplitude fit, LHCb confirmed the existence of the $Z(4430)^-$ resonance with a significance of 13.9$\sigma$, and its mass and width were measured to be $M_{Z^\pm} = 4475 \pm 7^{+15}_{-28}$ MeV/$c^2$ and $\Gamma_{Z^\pm} = 172 \pm 13^{+30}_{-19}$ MeV/$c^2$. Spin parity of $J^P = 1^+$ was favored over the other assignments by more than 17.8$\sigma$, and through the study of the variation of the phase of the $Z(4430)^-$ with mass, LHCb demonstrated its resonant character.

The BABAR Collaboration [25] searched for the $Z(4430)^-$ state in a data sample statistically comparable to Belle’s. They used a model-independent approach to test whether an interpretation of the experimental data is possible in terms of the known resonances in the $K\pi$ system. The $K\pi$ mass and angular distributions were determined from data and used to predict the observed $\psi(2S)\pi$ mass spectrum. It was found that the observed $\psi(2S)\pi$ mass spectrum was compatible with being described by reflections of the $K\pi$ system. Therefore, no clear evidence for a $Z(4430)^-$ was established, although BABAR’s analysis did not exclude the observation by Belle.

The present article describes the details of an LHCb analysis that was briefly reported in Ref. [12]. Adopting a model-independent approach, along the lines of BABAR’s strategy, the structures observed in the $\psi(2S)\pi$ mass spectrum are predicted in terms of reflections of the $K\pi$ system mass and angular composition, without introducing any modelling of the resonance line shapes and their interference patterns. The compatibility of these predictions with data is quantified.

II. LHCb DETECTOR

The LHCb detector [26,27] is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, designed for the study of particles containing $b$ or $c$ quarks. The detector includes a high-precision tracking system consisting of a silicon-strip vertex detector surrounding the $pp$ interaction region [28], a large-area silicon-strip detector located upstream of a dipole magnet with a bending power of about 4 Tm, and three stations of silicon-strip detectors and straw drift tubes [29] placed downstream of the magnet. The tracking system provides a measurement of momentum, $p$, of charged particles with a relative uncertainty that varies from 0.5% at low momentum to 1.0% at 200 GeV/$c$. The minimum distance of a track to a primary vertex, the impact parameter, is measured with a resolution of $(15 + 29/p_T)$ μm, where $p_T$ is the component of the momentum transverse to the beam, in GeV/$c$. Different types of charged hadrons are distinguished using...
information from two ring-imaging Cherenkov detectors [30]. Photons, electrons, and hadrons are identified by a calorimeter system consisting of scintillating-pad and preshower detectors, an electromagnetic calorimeter, and a hadronic calorimeter. Muons are identified by a system composed of alternating layers of iron and multihit proportional chambers [31]. The online event selection is performed by a trigger [32], which consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction.

III. DATA SAMPLES AND CANDIDATE SELECTION

The results presented in this paper are based on data from pp collisions collected by the LHCb experiment, corresponding to integrated luminosities of 1 and 2 fb⁻¹ at center-of-mass energies of 7 TeV in 2011 and 8 TeV in 2012, respectively. In the simulation, pp collisions are generated using PYTHIA [33] with a specific LHCb configuration [34]. The decays of hadronic particles are described by EvtGen [35], in which final-state radiation is generated using PHOTOS [36]. The interaction of the generated particles with the detector, and its response, are implemented using the GEANT4 toolkit [37] as described in Ref. [38]. Samples of simulated events, generated with both 2011 and 2012 conditions, are produced for the decay \( B^0 \rightarrow \psi(2S)K\pi \) with a uniform three-body phase-space distribution and the \( \psi(2S) \) decaying into two muons. These simulated events are used to tune the event selection and for efficiency and resolution studies.

The selection is similar to that used in Ref. [12] and consists of a cut-based preselection followed by a multivariate analysis. Track-fit quality and particle identification requirements are applied to all charged tracks. The \( B^0 \) candidate reconstruction starts by requiring two well-identified muons, with opposite charges, having \( p_T > 2 \text{ GeV/c} \) and forming a good quality vertex. The dimuon invariant mass has to lie in the window 3630–3734 MeV/c², around the \( \psi(2S) \) mass. To obtain a \( B^0 \) candidate, each dimuon pair is required to form a good vertex with a kaon and a pion candidate, with opposite charges. Pions and kaons are required to be inconsistent with coming from any primary vertex (PV) and to have transverse momenta greater than 200 MeV/c.

The \( B^0 \) candidate has to have \( p_T > 2 \text{ GeV/c} \), a reconstructed decay time exceeding 0.25 ps, and an invariant mass in the window 5200–5380 MeV/c² around the nominal \( B^0 \) mass. Contributions from \( \phi \rightarrow K^+K^- \) decay, where one of the kaons is misidentified as a pion, are removed by vetoing the region 1010–1030 MeV/c² of the dihadron invariant mass calculated assuming that the \( \pi^- \) candidate has the \( K^- \) mass.

To reduce the combinatorial background, a requirement is imposed on the output of a multivariate discriminator based on the likelihood ratio [39]. The four variables used as input are the smaller \( \chi^2_{IP} \) of the kaon and the pion, where \( \chi^2_{IP} \) is the difference in the PV fit \( \chi^2 \) with and without the track under consideration; the \( \mu^+\mu^-K\pi \) vertex-fit quality; the \( B^0 \) candidate impact parameter significance with respect to the PV; and the cosine of the largest opening angle between the \( \psi(2S) \) and each of the charged hadrons in the plane transverse to the beam. After the multivariate selection, the \( B^0 \) candidate invariant-mass distribution appears as shown in Fig. 1 with a fitted curve superimposed. The fit model consists of a Hypatia distribution [40] to describe the signal and an exponential function to describe the background.

Table I provides the fit results and the signal and background yields in the signal region. The width of the distribution, \( \sigma_{B^0} \), is defined as half the symmetric interval around \( M_{B^0} \) containing 68.7% of the total signal. The signal region is defined by the \( \pm 2\sigma_{B^0} \) interval around \( M_{B^0} \).

Sideband subtraction is used to remove the background which is dominated by combinations of \( \psi(2S) \) mesons from b-hadron decays with random kaons and pions. Sidebands are identified by the intervals \( [M_{B^0} - 80, M_{B^0} - 7\sigma_{B^0}] \) MeV/c² and \( [M_{B^0} + 7\sigma_{B^0}, M_{B^0} + 80] \) MeV/c². A weight, \( W_{\text{signal}} \), is attributed to each candidate: unit

<table>
<thead>
<tr>
<th>Variable</th>
<th>Fit results</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{B^0} )</td>
<td>5280.83 ± 0.04 MeV/c²</td>
</tr>
<tr>
<td>( \sigma_{B^0} )</td>
<td>5.77 ± 0.05 MeV/c²</td>
</tr>
<tr>
<td>Signal yield</td>
<td>23,801 ± 158</td>
</tr>
<tr>
<td>Background yield</td>
<td>757 ± 14</td>
</tr>
</tbody>
</table>

FIG. 1 (color online). Spectrum of the \( \psi(2S)K\pi \) system invariant mass. Black dots are the data, the continuous (blue) line represents the fit result, and the dashed (red) line represents the background component.
weight is assigned to candidates in the signal region; the ratio of the background yield in the signal region and in the sidebands, with a negative sign, is the weight assigned to sideband candidates; and zero weight is assigned to candidates in the remaining regions.

IV. EFFICIENCY AND RESOLUTION

The reconstruction and selection efficiency has been evaluated using simulated samples. The efficiency is calculated as a function of four variables: the $K\pi$ system invariant mass, $m_{K\pi}$; the cosine of the $K^{*0}$ helicity angle, $\cos\theta_{K^{*0}}$; the cosine of the $\psi(2S)$ helicity angle, $\cos\theta_{\psi(2S)}$; and the angle between the $K\pi$ and the $\mu^+\mu^-$ planes calculated in the $B^0$ rest frame, $\Delta\phi_{K\pi,\mu\mu}$ (this variable is called $\phi$ in Ref. [12]). The helicity angle of the $K^{*0}$ [$\psi(2S)$] is defined as the angle between the $K^+$ ($\mu^+$) direction and the $B^0$ direction in the $K^{*0}$ [$\psi(2S)$] rest frame. This four-dimensional (4D) space is subdivided in 24, 25, 5, and 4 bins of the respective variables. The value of the efficiency, at each point of the 4D space, is evaluated as a multilinear interpolation of the values at the 16 bins’ centers surrounding it. To the points falling in a border 4D bin, where interpolation is not possible, the value of the efficiency at the bin center is assigned.

To visualize the behavior, 2D efficiency plots are shown, as functions of all the possible variable pairs, in Fig. 2.

Table II lists the resolutions (average uncertainty) of the reconstructed event variables as evaluated on simulated events. They are found to be very small compared to the width of any possible structure searched for in this analysis; therefore, no resolution corrections are applied. In addition, the smooth behavior of the $m_{\psi(2S)\pi}$ resolution, shown in Fig. 3, demonstrates that structures in the $m_{\psi(2S)\pi}$ spectrum could not be caused by resolution effects.

V. $K^+$ RESONANCES

A number of $K^{*0}$ resonances with masses up to slightly above the kinematic limit of 1593 MeV/$c^2$ can decay to the $K\pi$ final state and contribute to the $B^0 \rightarrow \psi(2S)K^+\pi^-$ decay. Table III lists these $K^{*0}$ states as well as resonances just above the kinematic limit.

### Table II. Experimental resolution of kinematical quantities, as estimated from Monte Carlo simulations.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{K\pi}$</td>
<td>1.5 MeV/$c^2$</td>
</tr>
<tr>
<td>$m_{\psi(2S)\pi}$</td>
<td>1.8 MeV/$c^2$</td>
</tr>
<tr>
<td>$\cos\theta_{K^{*0}}$</td>
<td>0.004</td>
</tr>
<tr>
<td>$\cos\theta_{\psi(2S)}$</td>
<td>0.005</td>
</tr>
<tr>
<td>$\Delta\phi_{K\pi,\mu\mu}$</td>
<td>0.3°</td>
</tr>
</tbody>
</table>
TABLE III. Mass, width, spin, and parity of resonances known to decay to the $K\pi$ final state [5]. The list is limited to masses up to just above the maximum invariant mass for the $K\pi$ system which, in the decay $B^0 \to \psi(2S)K^+\pi^-$, is 1593 MeV/c$^2$.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Mass (MeV/c$^2$)</th>
<th>Width (MeV/c$^2$)</th>
<th>Spin</th>
<th>Parity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^*(800)^0$</td>
<td>682 ± 29</td>
<td>547 ± 24</td>
<td>0$^+$</td>
<td></td>
</tr>
<tr>
<td>$K^*(892)^0$</td>
<td>895 ± 0.19</td>
<td>47.4 ± 0.6</td>
<td>1$^-$</td>
<td></td>
</tr>
<tr>
<td>$K^*(1410)^0$</td>
<td>1414 ± 15</td>
<td>232 ± 21</td>
<td>1$^-$</td>
<td></td>
</tr>
<tr>
<td>$K_0^*(1430)^0$</td>
<td>1425 ± 50</td>
<td>270 ± 80</td>
<td>0$^+$</td>
<td></td>
</tr>
<tr>
<td>$K_2^*(1430)^0$</td>
<td>1432 ± 1.3</td>
<td>109 ± 5</td>
<td>2$^+$</td>
<td></td>
</tr>
<tr>
<td>$K^*(1680)^0$</td>
<td>1717 ± 27</td>
<td>322 ± 110</td>
<td>1$^-$</td>
<td></td>
</tr>
<tr>
<td>$K_4^*(1780)^0$</td>
<td>1776 ± 7</td>
<td>159 ± 21</td>
<td>3$^-$</td>
<td></td>
</tr>
</tbody>
</table>

The $m_{K\pi}$ spectrum of candidate events, shown in the left plot of Fig. 4, is dominated by the $K^*(892)^0$ meson. A structure in the $K^*(1410)^0$, $K_0^*(1430)^0$, and $K_2^*(1430)^0$ mass regions is also clearly visible. In addition, a non-resonant component is evident. A contribution from the low-mass tail of excited states above the kinematic limit is expected, in particular from the spin-1 $K^*(1680)^0$ and the spin-3 $K_4^*(1780)^0$ due to their large widths. The right plot of Fig. 4 shows the $\cos \theta_{K^\pi}$ distribution which highlights the rich angular structure of the $K\pi$ system. The resonant structures of the $K\pi$ system can be also seen in the 2D distributions shown in Fig. 5. The plot on the right illustrates how the structures present in the $K\pi$ system considerably influence the $\psi(2S)\pi$ system.

VI. EXTRACTION OF THE MOMENTS OF THE $K\pi$ SYSTEM

Background-subtracted and efficiency-corrected data are subdivided in $m_{K\pi}$ bins of width 30 MeV/c$^2$, which is suitable for observing the $K\pi$ resonance structures. For each $m_{K\pi}$ bin, the $\cos \theta_{K^\pi}$ distribution can be expressed as an expansion in terms of Legendre polynomials. The coefficients of this expansion contain all of the information on the angular structure of the system and characterize the spin of the contributing resonances. The angular distribution, after integration over the $\psi(2S)$ decay angles, can be written as

$$
\frac{dN}{d\cos \theta_{K^\pi}} = \sum_{j=0}^{l_{\text{max}}} \langle P_j \rangle P_j(\cos \theta_{K^\pi}),
$$

where $l_{\text{max}}$ depends on the maximum orbital angular momentum necessary to describe the $K\pi$ system, $P_j(\cos \theta_{K^\pi}) = \sqrt{2j+1}Y_j^0(\cos \theta_{K^\pi})$ are Legendre polynomials, and $Y_j^0$ are spherical harmonic functions. The coefficients $\langle P_j \rangle$ in Eq. (1) are called unnormalized moments (moments, in the following) and can be calculated as integrals of the product of the corresponding Legendre polynomial and the $\cos \theta_{K^\pi}$ distribution. Resonances of the $K\pi$ system with spin $s$ can contribute to the moments up to $\langle P_{s+s} \rangle$. Interference between resonances with spin $s_1$ and $s_2$ can contribute to moments up to $\langle P_{s_1+s_2} \rangle$.

For large samples, the moments are determined from the data as

$$
\langle P_j \rangle = \sum_{i=1}^{N_{\text{reco}}} \frac{W_i}{c^2} P_j(\cos \theta_{K^\pi}),
$$

where $N_{\text{reco}}$ is the number of reconstructed and selected candidates in the $m_{K\pi}$ bin. The superscript $i$ labels the

![Figure 3](https://example.com/figure3.png)  
**FIG. 3** (color online). The $\psi(2S)\pi$ invariant mass resolution as determined from simulated data (red dots). The continuous line is a spline-based interpolation.

![Figure 4](https://example.com/figure4.png)  
**FIG. 4**. Distributions of (left) $m_{K\pi}$ and (right) $\cos \theta_{K^\pi}$, after background subtraction and efficiency correction.
candidate, $W_{\text{signal}}$ is the weight which implements the sideband background subtraction, and $\epsilon_i = \epsilon(m_{K\pi})$, $\cos\theta_{K\pi}$, $\cos\theta_{\psi(2S)}$, $\Delta\Phi$, is the efficiency correction, obtained as described in Sec. III.

The dependence of the first six moments on $m_{K\pi}$ is shown in Fig. 6. Together with moment $(P_{4i}^0)$, represented in the left plot of Fig. 4, moments $(P_{3i}^0)$ and $(P_{6i}^0)$ show the S, P, and D wave amplitudes in the mass regions of the $K^*(892)^0$, $K^*(1410)^0$, $K^*_0(1430)$, and $K^{*+}_2(1430)$ resonances. The behavior of the moment $(P_{6i}^0)$, generated by an F wave, shows that any contribution from $K^{*+}_2(1780)^0$ is small. A resonant $\psi(2S)\pi$ state would, in general, contribute to all $K\pi$ moments.

A detailed discussion of these moments, together with the expressions relating moments to the amplitudes, can be found in Ref. [25] and references therein.

VII. ANALYSIS OF THE $m_{\psi(2S)\pi}$ SPECTRUM

The reflection of the mass and angular structure of the $K\pi$ system into the $\psi(2S)\pi$ invariant mass spectrum is investigated to establish whether it is sufficient to explain the data distribution. This is achieved by comparing the experimental $m_{\psi(2S)\pi}$ spectrum to that of a simulated sample which accounts for the measured mass spectrum and the angular distribution of the $K\pi$ system by means of appropriate weights. The comparison is performed in three
configurations of the $K\pi$ spin contributions. The simplest configuration corresponds to including the contributions of S, P, and D waves, which account for all resonances with mass below the kinematic limit and the $K^*(1680)^0$ meson, just above it (see Table III). In the second configuration, the $K^*(1780)^0$ meson is also allowed to contribute. This represents a rather unlikely assumption since it implies a sizeable presence of spin-3 resonances at low $m_{K\pi}$. This configuration can be considered as an extreme case that provides a valuable test for the robustness of the method. In the third configuration, a more realistic choice is made by limiting the maximum spin as a function of $m_{K\pi}$.

For each of the three configurations, 50 million simulated events are generated according to the $B^0 \to \psi(2S)K^+\pi^-$ phase-space decay. The simulation does not include detector effects because it will be compared to efficiency-corrected data. The simulated $m_{K\pi}$ distribution is forced to reproduce the $K\pi$ spectrum in the data (left plot of Fig. 4) by attributing to each event a weight proportional to the ratio between the real and simulated $m_{K\pi}$ spectra in the appropriate bin. Finally, the angular structure of the $K\pi$ system accounts for most of the features seen in the $m_{K\pi}$ spectrum except for the peak around 4430 MeV/c$^2$. The dashed (yellow) band in the figure is derived from the $\pm 1\sigma$ values of the normalized moments. The borders of the band are calculated by attributing to each simulated event the weight in Eq. (3) assuming the values of $+\sigma$ or $-\sigma$, simultaneously for all the contributing normalized moments. Due to the negative contributions of the moments, the borders may cross the central continuous (blue) line. The band should not be considered as an additional weight to each event computed as

$$w_i = 1 + \sum_{j=1}^{l_{\text{max}}} \langle P^N_j \rangle P_j (\cos \theta_{K\pi}^j), \tag{3}$$

where $\langle P^N_j \rangle = 2 \langle P^\psi_j \rangle / N_{\text{corr}}$ are the normalized moments, derived from the moments $\langle P^\psi_j \rangle$ of Eq. (2), and $N_{\text{corr}}$ is the background-subtracted and efficiency-corrected yield of the $m_{K\pi}$ bin where the event lies. The behavior of the first six normalized moments is shown in Fig. 7. The value and the uncertainty of these moments, at a given $m_{K\pi}$ value, are estimated by linearly interpolating adjacent points and their $\pm 1\sigma$ values, respectively, as shown by the shaded (yellow) bands in the figures.

The experimental distribution of the $\psi(2S)\pi$ system invariant mass, $m_{\psi(2S)\pi}$, is shown by the black points in the left plot of Fig. 8. The dotted (black) line represents the pure phase-space simulation; the dash-dotted (red) line shows the effect of the $m_{K\pi}$ modulation; while in the continuous (blue) line, the angular structure of the $K\pi$ system has been taken into account by allowing S, P, and D waves to contribute, which corresponds to setting $l_{\text{max}} = 4$ in Eq. (3). The effect of the angular structure of the $K\pi$ system system accounts for most of the features seen in the $m_{\psi(2S)\pi}$ spectrum except for the peak around 4430 MeV/c$^2$. The dashed (yellow) band in the figure is derived from the $\pm 1\sigma$ values of the normalized moments. The borders of the band are calculated by attributing to each simulated event the weight in Eq. (3) assuming the values of $+1\sigma$ or $-1\sigma$, simultaneously for all the contributing normalized moments. Due to the negative contributions of the moments, the borders may cross the central continuous (blue) line. The band should not be considered as an additional weight to each event computed as

\[w_i = 1 + \sum_{j=1}^{l_{\text{max}}} \langle P^N_j \rangle P_j (\cos \theta_{K\pi}^j), \tag{3}\]

where $\langle P^N_j \rangle = 2 \langle P^\psi_j \rangle / N_{\text{corr}}$ are the normalized moments, derived from the moments $\langle P^\psi_j \rangle$ of Eq. (2), and $N_{\text{corr}}$ is the background-subtracted and efficiency-corrected yield of the $m_{K\pi}$ bin where the event lies. The behavior of the first six normalized moments is shown in Fig. 7. The value and the uncertainty of these moments, at a given $m_{K\pi}$ value, are estimated by linearly interpolating adjacent points and their $\pm 1\sigma$ values, respectively, as shown by the shaded (yellow) bands in the figures.
The disagreement is more evident when looking at the same spectra in different intervals of $m_{\psi(2S)\pi}$, as shown in Fig. 10. Here, the candidates are subdivided according to the $m_{K\pi}$ intervals defined in Eq. (4). The last interval is further split into $1000 \text{ MeV}/c^2 < m_{K\pi} < 1390 \text{ MeV}/c^2$ and $m_{K\pi} > 1390 \text{ MeV}/c^2$. Except for the mass region around 4430 MeV/$c^2$, all slices exhibit good agreement between the data and the simulation. The peaking structure is particularly evident in the region $1000 \text{ MeV}/c^2 < m_{K\pi} < 1390 \text{ MeV}/c^2$, between the $K^*(892)^0$ and the resonances above 1400 MeV/$c^2$.

Figure 9 demonstrates that with this better-motivated $l_{\text{max}}$ assignment, the simulation cannot reproduce adequately the $m_{\psi(2S)\pi}$ distribution.

The disagreement is more evident when looking at the same spectra in different intervals of $m_{K\pi}$, as shown in Fig. 8. Here, the candidates are subdivided according to the $m_{K\pi}$ intervals defined in Eq. (4). The last interval is further split into $1000 \text{ MeV}/c^2 < m_{K\pi} < 1390 \text{ MeV}/c^2$ and $m_{K\pi} > 1390 \text{ MeV}/c^2$. Except for the mass region around 4430 MeV/$c^2$, all slices exhibit good agreement between the data and the simulation. The peaking structure is particularly evident in the region $1000 \text{ MeV}/c^2 < m_{K\pi} < 1390 \text{ MeV}/c^2$, between the $K^*(892)^0$ and the resonances above 1400 MeV/$c^2$.
VIII. STATISTICAL SIGNIFICANCE OF THE RESULT

The fact that the \( m_{\psi(2S)\pi} \) spectrum cannot be explained as a reflection of the angular structure of the \( K\pi \) system has been illustrated qualitatively. In this section, the disagreement is quantified via a hypothesis-testing procedure using a likelihood-ratio estimator. The compatibility between the expected \( m_{\psi(2S)\pi} \) distribution, accounting for the reflections of the \( K\pi \) angular structure, and that observed experimentally is tested for the three \( l_{\text{max}} \) assignments described in the previous section, with three sets of about 1000 pseudoexperiments each. For each pseudoexperiment, data and simulated samples involved in the analysis chain are reproduced as pseudosamples, generated at the same experimental situations.

The Monte Carlo method described in Sec. VII is used for each pseudoexperiment to produce an \( m_{\psi(2S)\pi} \) probability density function, \( F_{l_{\text{max}}} \), for each of the three \( l_{\text{max}} \) configurations. To test for the presence of possible contributions from the \( \psi(2S)\pi \) dynamics, which are expected to be present in moments of all orders, a fourth configuration is introduced by setting \( l_{\text{max}} \) to the unphysically large value of 30. By including moments up to \( l_{\text{max}} = 30 \), most of the features of the \( m_{\psi(2S)\pi} \) spectrum in data are well described, as can be seen in Fig. 11. The logarithm of the likelihood ratio is used to define the test statistic from two distributions, in the same 4D space, obtained from the simulation with full detector effects, before and after the application of the analysis chain. The sum of the signal and background samples is then subject to background subtraction and efficiency correction, exactly as for the real data, and moments are calculated. In the pseudoexperiments, events are simulated with equal amounts of each \( \psi(2S) \) polarization state. Effects related to \( \psi(2S) \) polarization are only included in the pseudosample via their correlation with the \( K\pi \) mass and \( \cos \theta_{K\pi} \) distributions, which are derived from the data. It has been checked that this does not significantly influence the results although the validity of such approximate treatment of \( \psi(2S) \) polarization is, in general, analysis dependent and not necessarily appropriate in other experimental situations.

The two uncertainties should not be combined when comparing data and Monte Carlo predictions. See the text for further details.
where the sum runs over the events in the pseudo- or real experiments.

An exotic state in the $\psi(2S)\pi$ system would give contributions to all $K\pi$ Legendre polynomial moments, whereas the conventional $K\pi$ resonances contribute only to moments corresponding to their spin and their interferences. If, for instance, the $B^0 \to \psi(2S)K^+\pi^-$ decay proceeds through S, P, and D $K\pi$ resonances, then only moments with $l_{\text{max}} \leq 4$ would exhibit significant activity. Therefore, activity in moments of order $l_{\text{max}} > 4$ would suggest the presence of other resonant states contributing to the decay. Lower-order $K\pi$ Legendre polynomial moments, determined from data and used to build the prediction, although strongly dominated by the conventional $K\pi$ resonances, could also contain a contribution from the exotic state. As a consequence, a relatively small $\psi(2S)\pi$ resonant contribution could be accommodated by the Monte Carlo prediction. Conversely, a significant disagreement would imply that the $\psi(2S)\pi$ invariant mass spectrum cannot be explained as a reflection of the activity of known resonances in the $K\pi$ system and would therefore constitute strong evidence for the presence of exotic states in the decay $B^0 \to \psi(2S)K^+\pi^-$. The $\Delta\text{NLL}_{l_{\text{max}}}$ distributions of the pseudoexperiments are shown in Fig. 12 (points with error bars) for each of the three $l_{\text{max}}$ settings. They are consistent with Gaussian distributions. The statistical significance, $S$, to rule out the different hypotheses is the distance, in units of standard deviations, at the mean value of the $\Delta\text{NLL}_{l_{\text{max}}}$ (dashed red arrow in Fig. 12) and the observed value of the real experiment (continuous black arrow in Fig. 12). This ranges from 8 to 15 standard deviations, as listed in Table IV.

The table also gives the statistical significance obtained by restricting the analysis to the region $1000 \text{ MeV}/c^2 < m_{K\pi} < 1390 \text{ MeV}/c^2$, where the presence of the structure around the $Z(4430)^-$ mass is most evident, as shown in Fig. 13. Thus, the hypothesis that the data can be explained solely in terms of plausible $K\pi$ degrees of freedom can be

### Table IV

<table>
<thead>
<tr>
<th>$l_{\text{max}}$</th>
<th>$S_{\psi(2S)\pi}$</th>
<th>$S_{\psi(2S)\pi}^{K\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>13.3σ</td>
<td>18.2σ</td>
</tr>
<tr>
<td>6</td>
<td>8.0σ</td>
<td>14.1σ</td>
</tr>
<tr>
<td>$l_{\text{max}}(m_{K\pi})$</td>
<td>15.2σ</td>
<td>17.3σ</td>
</tr>
</tbody>
</table>
ruled out without making any assumption on the exact shapes of the $K\pi$ resonances present and their interference patterns.

IX. SUMMARY AND CONCLUSIONS

A satisfactory description of the $\psi(2S)\pi$ mass spectrum in the decay $B^0 \to \psi(2S)K^+\pi^-$ cannot be obtained solely from the reflections of the angular structure of the $K\pi$ system. In particular, a clear peaking structure in the $4430\text{ MeV}/c^2$ mass region remains unexplained. Through a hypothesis-testing procedure based on the likelihood-ratio estimator, compatibility between the data and predictions taking into account the reflections of $K\pi$ states up to spin 3, is excluded with a significance exceeding $8\sigma$. The most plausible configuration, which allows $K\pi$ states with spin values depending on the $K\pi$ mass, is excluded with a significance of more than $15\sigma$.

This work represents an alternative and model-independent confirmation of the existence of a $\psi(2S)\pi$ resonance in the same mass region in which previous model-dependent amplitude analyses have found signals [9,11,12].

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