Financial integration, entrepreneurial risk and global dynamics


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Abstract

How does financial integration impact capital accumulation, current-account dynamics, and cross-country inequality? This paper investigates this question within a two-country, general-equilibrium, incomplete-markets model that focuses on the importance of idiosyncratic entrepreneurial risk—a risk that introduces, not only a precautionary motive for saving, but also a wedge between the interest rate and the marginal product of capital. Our contribution is then to show that this friction provides a simple explanation for the emergence of global imbalances, a simple resolution to the empirical puzzle that capital often fails to flow from the rich or slow-growing countries to the poor or fast-growing ones, and a distinct set of policy lessons regarding the intertemporal costs and benefits of capital-account liberalization.

JEL codes: E13, F15, F41.

Keywords: Financial integration, capital-account liberalization, incomplete markets, idiosyncratic risk, entrepreneurship, current-account deficits, global imbalances.

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1 Introduction

The last two or three decades have been characterized by significant liberalization of international capital flows. This, in turn, appears to have facilitated the rise of significant global imbalances—a large foreign debt on the side of the United States along with vast currency reserves and big positive holdings of US Treasury bills on the side of emerging countries such as China. Furthermore, whereas the standard neoclassical paradigm predicts that capital should be flowing from the rich to the poor, or from the least-growing to the fastest-growing countries, the empirical evidence often suggests the opposite direction of capital flows (Gourinchas and Jeanne, 2006).

These observations, and more generally the themes of financial integration and global imbalances, have motivated a large body of research.\(^1\) In this paper, we contribute to this growing literature by studying the global macroeconomic effects of financial integration in the presence of a certain market friction—uninsurable idiosyncratic entrepreneurial risk.

Our focus on this friction is motivated, not only by the fact that entrepreneurship is of obvious empirical relevance, but also by the observation that this friction can play a crucial role in capital accumulation and productivity growth. Indeed, this friction introduces both a precautionary motive for saving, as entrepreneurs seek to self-insure against the uninsurable risk in their income, and a wedge between the interest rate and the marginal product of capital, as entrepreneurs require a (private) risk premium in compensation for the risk they face in their entrepreneurial activity. Furthermore, this wedge is likely to vary across countries, with, say, entrepreneurs in China presumably enjoying less risk sharing and hence facing a higher wedge than those in the United States. Our contribution is to show how cross-country differences in this wedge may help explain a number of stylized facts—such as the persistence of cross-country inequality, the emergence of global imbalances, and the failure of capital to flow from the rich or slow-growing countries to the poor or fast-growing ones—while also providing a distinct set of policy lessons regarding the dynamic effects of capital-account liberalization.\(^2\)

**Preview of model.** We conduct our theoretical exercise within a tractable, general-equilibrium, incomplete-markets model. There are two economies (countries), each of which is populated by a continuum of households (families). Each family includes a worker and an entrepreneur. The worker supplies his labor in the domestic labor market; the entrepreneur runs a private business

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\(^2\) Borrowing constraints, although not explicitly considered here, are complementary sources of a wedge between the “external” and the “internal” return to capital. This offers a useful re-interpretation of our contribution. As it will become clear, our key results hinge on the properties that the aforementioned wedge is positive and decreasing with wealth—properties that may hold whether the wedge originates in idiosyncratic risk or borrowing constraints.
that operates a constant-returns-to-scale technology, employs labor from the domestic labor market, and uses the capital stock owned by her family. All households can freely trade a safe asset, but can diversify only a fraction of the idiosyncratic shocks hitting their private firms. The two countries differ in the magnitude of the uninsurable risk—with the “North” enjoying better risk-sharing possibilities and hence less risk than the “South”—but are otherwise identical.

Within this model, we define “financial autarchy” as the regime in which the market for the safe asset clears on a country-wide level, and “financial integration” as the regime in which this market clears on a world-wide level. We then study the steady states that obtain under these two regimes, as well as the entire transitional dynamics of the global economy between the two steady states.

**Preview of results.** Under financial autarchy, the South features a lower interest rate. This is due to the stronger demand for precautionary saving implied by the larger amount of undiversifiable idiosyncratic risk (or, in an extension, due to the lower supply of the safe asset). Despite its lower interest rate, however, the South may also feature a lower capital stock and a lower level of income than the North. This is because the South faces a higher wedge between the marginal product of capital and the interest rate. It follows that, prior to financial integration, the South identifies the poor, capital-scarce country, whereas the North identifies the rich, capital-abundant country.

Because the South has a lower autarchic interest rate than the North, financial integration triggers the North to run large current-account deficits and, symmetrically, the South to accumulate a large positive foreign asset position. Intuitively, this is because the North has a comparative advantage in supplying the safe asset: the North “exports” this asset by running current-account deficits. What is more, as financial integration causes interest rates to rise in the South, the opportunity cost of capital goes up and the capital stock goes down, thereby depressing domestic wages and output. Conversely, the North experiences a boom.

If the North is interpreted as the United States, and the South as China or other emerging economies, these result helps explain the significant “global imbalances” that the world economy has experienced in recent history. Furthermore, they help explain why financial globalization may initially exacerbate cross-country inequality, and why capital may often fail to flow from the rich, capital-abundant countries to the poor, capital-scarce ones.

Interestingly, though, the long-run effects of financial integration can be quite different. Because financial integration permits the South to save abroad at higher returns than otherwise, the South is able to accumulate more and more wealth over time. As this happens, the willingness to take risk increases, the wedge between the interest rate and the marginal product of capital falls, and the capital stock increases. As a result, in the new steady state the South may well end up with higher levels of capital, wages, output and consumption than in its autarchic steady state. Our model therefore predicts that financial integration may help poor countries in the long run, even as it hurts them in the short run—and may reduce cross-country inequality in the long run, even as it increases it in the short run.
Furthermore, because of the aforementioned wealth accumulation and the consequent increase in risk taking, the transition in the South may feature a reallocation of saving from safe but low-return investment opportunities to risky but high-return ones. As a result, the South experiences an acceleration in its TFP growth, while the converse is true for the North. Along with the property that the South runs current-account surpluses, while the North runs current-account deficits, this result means that capital flows from the faster growing countries to the slower growing ones—a prediction that is the opposite of the one made by the standard neoclassical paradigm and that helps resolve the empirical puzzle documented by Gourinchas and Jeanne (2008).

Combined, our results provide, not only a possible explanation to certain stylized facts, but also a distinct policy lesson: the benefits of capital-account liberalization for less developed economies may be higher in the long run than in the short run. As already noted, the key intuition is that financial integration helps agents in the South accumulate more wealth over time, which in turn permits them to mitigate the friction they face in their entrepreneurial activities. We reinforce this intuition by studying the welfare effects of financial integration in our model.

Upon financial integration, the South’s poor tend to loose for two complementary reasons: the increase in interest rates means an increase in the cost of borrowing; and the initial outflow of capital means a reduction in their wages. In contrast, the middle class and the rich gain because of the higher returns to their saving and of the lower labor costs in their private businesses. But as time passes and capital eventually reaches higher levels than under autarchy, the resulting increase in wages alleviates the burden of all poor agents and even reverses the fortunes of some of them, so that they too gain in the long run. Once again, this highlights the distinct short-run and long-run effects that our analysis brings to light.3

Relevant literature. Our paper belongs to a large, and growing, literature that uses Bewley-type models to study various macroeconomic implications of incomplete markets. Key references include Aiyagari (1994), Huggett (1997), Krusell and Smith (1998), and Rios-Rull (1995); see Heathcote, Storesletten, and Violante (2008) and Krusell and Smith (2006) for eclectic reviews. The bulk of this literature focuses on idiosyncratic endowment or labor-income risk. Important exceptions are Angeletos and Calvet (2000, 2006) and Angeletos (2007), which are among the first papers to emphasize the distinct implications of idiosyncratic investment risk for aggregate saving within the context of the neoclassical growth model.4 Our paper starts by extending Angeletos (2007) to a two-country open-economy setting. Our contribution is then to study how cross-country differences in the level of idiosyncratic investment risk impact global macroeconomic dynamics. In independent parallel work, Corneli (2010) undertakes a similar exercise and obtains closely related results.

3Complementary in this regard is Aoki, Benigno and Kiyotaki (2009), which also stresses the importance of studying the intertemporal costs and benefits of financial integration, albeit in a different model than ours.
4Other papers that touch the same theme, but focus on different questions, include Angeletos and Panousi (2009), Basin, Benhabib and Zhu (2009), Cagetti and De Nardi (2006), Goldberg (2010), Quadrini (2000), Covas (2006), Mall (2009), Meh and Quadrini (2006), Kitao (2007), and Panousi (2010).
Closely related in this regard is Mendoza, Quadrini, and Rios-Rull (2008). Like our paper, this work studies how cross-country differences in domestic risk sharing can help explain significant and persistent global imbalances. See also Willen (2004) for an earlier take on the same key insight. However, unlike our paper, this work rules out endogenous capital accumulation and/or idiosyncratic investment risk.\(^5\) It is precisely the combination of these two features that distinguishes our theoretical exercise and that explains the novelty of our results.

Also closely related are Buera and Shin (2010) and Sandri (2010). Buera and Shin’s model shares the two key features of our model, namely capital accumulation and entrepreneurial risk, but adds a number of other ingredients, such as borrowing constraints, occupational choice, and cross-sectional distortions in the allocation of capital. By assuming that capital-account liberalization comes in tandem with a structural reform that removes these distortions, they obtain an acceleration in TFP growth. At the same time, a surge in current-account surpluses occurs for reasons similar to ours. Their paper and ours are thus highly complementary.\(^6\) Sandri, on the other hand, considers a one-country model that also features entrepreneurial risk, but focuses on a different policy exercise. In particular, he studies a reform that permits some agents to switch from “farmers” to “entrepreneurs”. Because entrepreneurial activity is assumed to face more risk than farming, this means an increase in the level of idiosyncratic risk and hence a surge in precautionary saving, which in turn helps generate current-account surpluses. A similar mechanism operates in Carroll and Jeane (2009), except that there the driving force is an increase in idiosyncratic labor-income risk.

Our paper also shares with Caballero, Farhi, and Gourinchas (2008) the idea that global imbalances are explained, in a certain sense, by a shortage of assets in the South. But whereas that paper assumes that the South has a lower capacity in supplying any asset, we only assume that the North has a comparative advantage in supplying the relatively safer assets. This in turn can be the case, not because of different technologies, but simply because the North has a weaker demand for precautionary saving. Furthermore, that paper rules out capital accumulation, thus also ruling out the distinct dynamic effects that are at the core of our contribution.

**Layout.** The rest of the paper is organized as follows. Section 2 introduces the model and Section 3 characterizes the general equilibrium. Section 4 studies the autarchic and integrated steady states, section 5 the transitional dynamics between the two, and section 6 the welfare implications. Section 7 considers a useful extension. Section 8 concludes. The proofs are delegated to the Appendix.

\(^5\)Mendoza et al. (2008) allow for a certain type of investment risk, but rule out capital accumulation: the investment opportunity in that paper is an exogenous “Lucas tree”. Mendoza et al. (2009), on the other hand, allow for capital accumulation, but rule out idiosyncratic investment risk. Finally, Willen (2004) studies an endowment economy, thus ruling out both capital accumulation and idiosyncratic investment risk.

\(^6\)The comparative advantage of their paper is that it contains a richer quantitative exercise, while that of our analysis rests on its increased tractability and the consequent clarity of the theoretical insights.
2. The model

Our model is a two-country variant of the closed-economy model of Angeletos (2007). There are two countries, indexed by \( j \in \{1, 2\} \), and a single good, which can be used for either consumption or investment purposes. Each country is populated by a continuum of infinitely-lived households, indexed by \( i \) and distributed uniformly over \([0, 1]\). Each household includes a worker and a producer ("entrepreneur"). The worker supplies his labor inelastically to the domestic labor market. The entrepreneur runs a privately-held firm ("family business"). Each household can freely save or borrow in the riskless bond—up to a natural borrowing constraint—and can accumulate physical capital within its own family business. Firms are hit by idiosyncratic shocks, which the households can only partially diversify. Finally, to maintain tractability, we abstract from any aggregate uncertainty. We also let the time be continuous, indexed by \( t \in [0, \infty) \).

Preferences take an Epstein-Zin specification, which permits us to distinguish intertemporal substitution from risk aversion. Fix a household \( i \) in county \( j \). Her preferences are defined as the limit, for \( \Delta t \to 0 \), of the solution to the following recursive specification:

\[
U_{ijt} = \left( 1 - e^{-\beta \Delta t} \right) c_{ijt}^{1-\gamma / \theta} + e^{-\beta \Delta t} \left( E_t \left[ U_{ijt+\Delta t}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}} ,
\]

where \( \beta > 0 \) is the discount rate, \( \gamma > 0 \) is the coefficient of relative risk aversion, and \( \theta > 0 \) is the elasticity of intertemporal substitution.\(^7\)

The financial wealth of this household, denoted by \( x_{ijt} \), is the sum of its holdings in private capital, \( k_{ijt} \), and in the riskless bond, \( b_{ijt} \):

\[
x_{ijt} = k_{ijt} + b_{ijt} .
\]

(2)

The evolution of \( x_{ijt} \) is given by the following budget constraint:

\[
dx_{ijt} = d\pi_{ijt} + [R_t b_{ijt} + \omega_{jt} - c_{ijt}] dt + dT_{ijt} .
\]

(3)

Here, \( d\pi_{ijt} \) is the household’s capital income (i.e., the profits from the private firm it owns), \( R_{jt} \) is the interest rate on the riskless bond, \( \omega_{jt} \) is the wage rate, \( c_{ijt} \) is the household’s consumption, and \( dT_{ijt} \) is a transfer that captures risk-sharing opportunities (to be defined later on).

Whereas the sequences of the wage and the interest rate are deterministic (due to the absence of aggregate risk), firm profits, and hence household capital income, are subject to undiversified

\(^7\)Standard expected utility is nested for \( \theta = 1/\gamma \); in this case, \( U_{ijt} = E_t \int_0^\infty e^{-\beta s} U(c_{ijt}) ds \), where \( U(c) = \frac{c^{1-1/\theta} - 1}{1-1/\theta} \). We allow for \( \theta \neq 1/\gamma \) so as to facilitate a more precise understanding of the underlying forces in our environment and a better calibration. However, none of our results rest on letting \( \theta \neq 1/\gamma \).
idiosyncratic risk:
\[
d\pi_{ijt} = [F(k_{ijt}, n_{ijt}) - \omega_{jt}n_{ijt} - \delta k_{ijt}]dt + \sigma_j k_{ijt} d\tilde{z}_{ijt}.
\] (4)

Here, \(n_{ijt}\) is the amount of labor the firm hires in the competitive labor market, \(\delta\) is the mean depreciation rate, and \(F\) is a constant-returns-to-scale neoclassical production function. For simplicity, we assume a Cobb-Douglas specification: \(F(k, n) = k^\alpha n^{1-\alpha}\), with \(\alpha \in (0, 1)\).

Idiosyncratic risk is introduced through \(d\tilde{z}_{ijt}\), a standard Wiener process that is i.i.d. across agents and time. Literally taken, \(d\tilde{z}_{ijt}\) represents a stochastic depreciation, or productivity, shock. However, we wish to interpret this shock more broadly as encompassing various sources of idiosyncratic risk in the entrepreneurial activity and, more generally, in the returns to private investment. The scalar \(\sigma_j\) then parameterizes the level of this risk in country \(j\).

Since this risk is purely idiosyncratic, agents would be able to obtain full insurance against it if financial markets were complete. A number of reasons—moral hazard, adverse selection, costly state verification, inefficient legal and enforcement systems, or mere lack of sophistication—may explain why this does not happen in the real world. In this paper, as in most other papers in the Bewley tradition, we abstract from the deeper micro-foundations of incomplete markets. Instead, we exogenously impose that the available risk-sharing possibilities are limited, and more severely so in the South. We capture this by assuming that:
\[
dT_{ijt} = -\lambda_j \sigma_j k_{ijt} d\tilde{z}_{ijt},
\] (5)
for some \(\lambda_j \in (0, 1)\). This assumption can also be justified by introducing an exogenous asset structure that permits agents to diversify only certain components of their idiosyncratic risk, or by letting them sell equity on only a fraction of their profits. Either way, the scalar \(\lambda_j\) measures the fraction of idiosyncratic risk that agents are able to diversify in country \(j\); this is what defines the level of financial development in our model.

Combining conditions (3)-(5), we get that the household budget reduces to:
\[
dx_{ijt} = d\tilde{\pi}_{it} + [R_t b_{ijt} + \omega_{jt} - c_{ijt}]dt,
\] (6)
where
\[
d\tilde{\pi}_{ijt} \equiv d\pi_{ijt} + dT_{ijt} = [F(k_{ijt}, n_{ijt}) - \omega_{jt}n_{ijt} - \delta k_{ijt}]dt + (1 - \lambda_j)\sigma_j k_{ijt} d\tilde{z}_{ijt}.
\]

It is then evident that the quantity \(\tilde{\sigma}_j \equiv (1 - \lambda_j)\sigma_j\) measures the amount of undiversifiable idiosyncratic risk in country \(j\). We henceforth impose \(\tilde{\sigma}_2 < \tilde{\sigma}_1\), which permit us to identify country 1 as the country with a lower level of uninsurable entrepreneurial risk—and, in this particular sense, as the country with the more advanced financial markets. We accordingly refer to country 1 as the “North” or the “developed” economy, and to country 2 as the “South” or the “developing” economy.
At this point, we would like to invite the reader to maintain a flexible interpretation of the assumption that $\tilde{\sigma}_2 > \tilde{\sigma}_1$. For example, entrepreneurial risk may be higher in developing economies because, in comparison to developed economies such as the United States, developing economies such as China, India, and Mexico appear to face more severe agency and/or enforcement problems. Government corruption and weak property rights also contribute to higher levels of idiosyncratic entrepreneurial risk in developing economies: some times the entrepreneur is the fortunate recipient of preferential treatment by corrupt politicians and bureaucrats, some times he is the unfortunate victim. Finally, as tax and regulatory policies tend to be more volatile in these economies, the idiosyncratic incidence of these policies appears to be more volatile as well, contributing to additional risk in entrepreneurial activity.

3 Equilibrium

Let $Y_{jt}, C_{jt}, N_{jt}, K_{jt},$ and $B_{jt}$ denote the aggregate levels of output, consumption, employment, capital, and bond holdings in country $j$ at date $t$ (that is, the cross-sectional averages of $y_{ijt}, c_{ijt}$ and so on). We consider two policy regimes. In the first, countries are in financial autarchy: the riskless bond cannot move across borders. In the second, they are financially integrated: countries can borrow and lend to one another. We define the corresponding equilibrium concepts as follows.

Definition 1. An autarchic equilibrium consists of a deterministic sequence of country-specific interest rates, wages, and macroeconomic quantities, \{R_{jt}, \omega_{jt}, Y_{jt}, C_{jt}, N_{jt}, K_{jt}\}_{t \in [0, \infty)} for $j \in \{1, 2\}$, and a collection of individual contingent plans, \{c_{ijt}, n_{ijt}, k_{ijt}, b_{ijt}\}_{t \in [0, \infty)} for $i \in [0, 1], j \in \{1, 2\}$, such that the following are true: (i) individual plans are optimal given the sequences of prices; (ii) macroeconomic quantities are obtained by aggregating individual plans; (iii) labor and bond markets clear at the country level, namely $N_{jt} = 1$ and $B_{jt} = 0$ for all $j, t$.

Definition 2. An integrated equilibrium consists of a deterministic sequence of world-wide interest rates, \{R_{t}\}_{t \in [0, \infty)}), a deterministic sequence of country-specific wages and macroeconomic quantities, \{\omega_{jt}, Y_{jt}, C_{jt}, N_{jt}, K_{jt}\}_{t \in [0, \infty)} for $j \in \{1, 2\}$, and a collection of individual contingent plans, \{(c_{ijt}, n_{ijt}, k_{ijt}, b_{ijt})_{t \in [0, \infty)}| i \in [0, 1], j \in \{1, 2\}\}, such that the following are true: (i) individual plans are optimal given the sequences of prices; (ii) macroeconomic quantities are obtained by aggregating individual plans; (iii) labor markets clear at the country level, namely $N_{jt} = 1$ for all $j, t$; (iv) the bond market clears at the world level, namely $B_{1t} + B_{2t} = 0$ for all $t$.

In the remaining of this section, we first characterize the individual household’s problem for a given sequence of wages and interest rates. We then proceed to characterize the general equilibrium under both regimes.
3.1 Individual behavior

Since employment is chosen after the capital stock has been installed and the idiosyncratic shock has been observed, optimal employment maximizes profits state-by-state. Furthermore, by constant returns to scale, optimal employment and profits are linear in own capital. We therefore have that:

\[ n_{ijt} = \bar{n}_{jt}k_{ijt} \quad \text{and} \quad d\pi_{ijt} = \bar{r}_{jt}k_{ijt}dt + \sigma_jk_{ijt}dz_{ijt}, \]

where \( \bar{n}_{jt} = \arg \max_n [F(1, n) - \omega_{jt}n] \) and \( \bar{r}_{jt} = \bar{r}(\omega_{jt}) \equiv \max_n [F(1, n) - \omega_{jt}n] - \delta \). As in Angeletos (2007), the key finding here is that households face linear, albeit risky, returns to their capital. This linearity, together with the homotheticity of preferences, ensures that the household’s consumption-saving problem reduces to a tractable homothetic optimization problem, much like in Samuelson’s and Merton’s classic portfolio analysis. It then follows that the optimal policy rules are linear in wealth, as shown in the next lemma.

**Lemma 1.** Let \( \{\omega_{jt}, R_{jt}\}_{t \in [0, \infty)} \) be equilibrium price sequences (with \( R_{1t} = R_{2t} = R_t \) if the world is integrated) and let \( h_{jt} \equiv \int_t^\infty e^{-\rho_s}R_s\omega_{js}ds \) denote the present value of labor income (a.k.a. human capital). Then, optimal consumption, investment and bond holdings are given by

\[ c_{ijt} = m_{jt}(x_{ijt} + h_{jt}), \quad k_{ijt} = \phi_{jt}(x_{ijt} + h_{jt}), \quad \text{and} \quad b_{ijt} = (1 - \phi_{jt})(x_{ijt} + h_{jt}) - h_{jt}, \]

where \( m_{jt} \) denotes the marginal propensity to consume and \( \phi_{jt} \) the marginal propensity to invest in private capital. The marginal propensity to consumer solves the following recursion:

\[ \frac{\dot{m}_{jt}}{m_t} = m_t + (\theta - 1)\hat{\rho}_{jt} - \theta \beta, \]

where \( \hat{\rho}_{jt} \equiv \rho_{jt} - \frac{1}{2}\gamma\phi_{jt}^2\hat{\sigma}_j^2 \) denotes the risk-adjusted return to saving and \( \rho_{jt} \equiv \phi_t\bar{r}_{jt} + (1 - \phi_{jt})R_t \) the mean return to saving. Finally, the marginal propensity to invest is given by

\[ \phi_{jt} = \frac{\bar{r}_{jt} - R_{jt}}{\gamma\hat{\sigma}_j^2}. \]

Condition (8) establishes the linearity of optimal consumption, capital and bond holdings in wealth. Condition (10) identifies the propensity to invest in the risky asset as an increasing function of the risk premium, \( \mu_t \equiv \bar{r}_t - R_t \), and a decreasing function of the amount of uninsurable risk, \( \hat{\sigma}_j = (1 - \lambda_j)\sigma \). Finally, condition (9) is essentially the Euler condition: it describes the growth rate of the marginal propensity to consume as a function of the anticipated path of risk-adjusted returns to saving.\(^8\)

\(^8\)Note that higher risk-adjusted returns reduce the propensity to consume (i.e., increase the propensity to save) if and only if the elasticity of intertemporal substitution \( \theta \) exceeds one; this is due to the familiar tension between the income and substitution effects implied by an increase in the rate of return.
3.2 General equilibrium

Let \( f(K) \equiv F(K,1) = K^\alpha \). From Proposition 1, we have that the equilibrium values of the propensity to invest and the risk-adjusted return to saving are given by \( \phi_{jt} = \phi(K_{jt}, R_t, \tilde{\sigma}_j) \) and \( \bar{\rho}_{jt} = \bar{\rho}(K_{jt}, R_t, \tilde{\sigma}_j) \), where

\[
\phi(K, R, \tilde{\sigma}) \equiv \frac{f'(K) - \delta - R}{\gamma \tilde{\sigma}^2} \quad \text{and} \quad \bar{\rho}(K, R, \tilde{\sigma}) \equiv R + \frac{(f'(K) - \delta - R)^2}{2\gamma \tilde{\sigma}^2}.
\]

Furthermore, the equilibrium wage satisfies \( \omega_{jt} = f(K_{jt}) - f'(K_{jt})K_{jt} = (1-\alpha)f(K_{jt}) \). Using these facts, aggregating the policy rules of the agents, and imposing market clearing for the risk-free bond, we arrive at the following tractable characterization of the general equilibrium of the economy.

**Proposition 1.** In either the autarchic or the integrated equilibrium, the aggregate dynamics of country \( j \) satisfy the following ODE system

\[
C_{jt} + \dot{K}_{jt} + \dot{B}_{jt} = f(K_{jt}) - \delta K_{jt} + R_{jt} B_{jt} \quad (11)
\]

\[
\frac{\dot{C}_{jt}}{C_{jt}} = \theta (\bar{\rho}_{jt} - \beta) + \frac{1}{2\gamma \tilde{\sigma}_j^2} \phi_{jt}^2 \quad (12)
\]

\[
\dot{H}_{jt} = R_{jt} H_{jt} - (1-\alpha)f(K_{jt}) \quad (13)
\]

\[
B_{jt} = (1 - \phi_{jt})(K_{jt} + B_{jt}) - \phi_{jt} H_{jt} \quad , \quad (14)
\]

where \( \phi_{jt} = \phi(K_{jt}, R_{jt}, \tilde{\sigma}_j) \) and \( \bar{\rho}_{jt} = \bar{\rho}(K_{jt} R_{jt}, \tilde{\sigma}_j) \). The autarchic equilibrium is then obtained by letting \( R_{1t} \neq R_{2t} \) and requiring that, for each \( j \), \( R_{jt} \) adjusts so that

\[
B_{jt} = 0 \quad . \quad (15)
\]

In contrast, the integrated equilibrium is obtained by imposing \( R_{1t} = R_{2t} = R_t \) and requiring that \( R_t \) adjusts so that

\[
B_{1t} + B_{2t} = 0 \quad . \quad (16)
\]

Conditions (11) and (12) give, respectively, the resource constraint and the aggregate Euler condition. Condition (13) gives the law of motion for human capital, whereas condition (14) gives the equilibrium level of aggregate holdings of the riskless bond or, equivalently, the net foreign asset position of the country. Conditions (15) and (16) then complete the characterization of the equilibrium: under financial autarchy, the domestic interest rate of each country must be such that the net foreign asset position of that country is zero; under financial integration, the world-wide interest rate must be such that the asset positions of the two countries balance one another.

At this point, it is important to recognize how idiosyncratic risk impacts the general-equilibrium system. When \( \tilde{\sigma}_j = 0 \), arbitrage imposes that \( R_t = f'(K_{jt}) - \delta = \bar{\rho}_{jt} \), and the Euler condition
reduces to its familiar complete-markets version, \( \frac{C_{jt}}{C_{jt-1}} = \theta (R_t - \beta) \). When instead \( \tilde{\sigma}_j > 0 \), there are two important changes. First, the precautionary motive for saving introduces a positive drift in consumption growth, represented by the term \( \frac{1}{2} \gamma \tilde{\sigma}_j^2 \phi_j^2 \) in the Euler condition (12). This is the key force in Bewley-type models such as Aiyagari (1994) and Mendoza et al. (2008). Second, the fact that investment is subject to undiversifiable idiosyncratic risk introduces a wedge between the risk-free rate and the marginal product of capital, so that \( R_{jt} < \rho_{jt} < f'(K_{jt}) - \delta \). This wedge plays a crucial role in the results of our paper and distinguishes it from the aforementioned work.

4 Steady state

In this section we first explain how long-run wealth accumulation impacts the wedge between the interest rate and the marginal product of capital, and thereby the steady-state level of capital for given interest rate. This identifies the key mechanism behind the long-run effects in our framework. We then complete the characterization of the autarchic and integrated steady states by studying the determination of the interest rate.

4.1 Long-run wealth accumulation and the wedge on investment

In steady state, whether under autarchy or under integration, the growth rate of aggregate consumption in each country must be zero. The Euler condition (12) then reduces to the following:

\[
\hat{\rho}_j = \beta - \frac{1}{2} \gamma \tilde{\sigma}_j^2 \phi_j^2 .
\] (17)

This condition simply requires that the risk-adjusted return to saving in country \( j \) be lower than the discount rate as much as it takes for the associated negative intertemporal substitution effect to just offset the positive precautionary motive. Using the facts that \( \hat{\rho}_j = R_j + \frac{1}{2 \gamma \tilde{\sigma}_j^2} \mu_j \) and \( \phi_j = \frac{1}{\gamma \tilde{\sigma}_j^2} \mu_j \), where \( \mu_j = f'(K_j) - \delta - R_j \) is the risk premium, we can restate condition (17) as follows:

\[
f'(K_j) - \delta = R_j + \sqrt{\frac{2 \theta \gamma \tilde{\sigma}_j^2 (\beta - R_j)}{\theta + 1}} .
\] (18)

We infer that this condition pins down the combinations of the domestic capital stock and the interest rate that are consistent with stationarity of aggregate consumption—equivalently, with stationarity of aggregate wealth—in country \( j \).

If there were no uninsurable idiosyncratic risk (\( \tilde{\sigma} = 0 \)), condition (18) would have reduced to the familiar condition \( f'(K) - \delta = R \); that is, the marginal product of capital would have been equated to the interest rate. Furthermore, this would have implied that the capital stock is a decreasing function of the interest rate. Now, instead, we have that the marginal product of capital exceeds the interest rate: \( f'(K) - \delta > R \). This is because agents require a positive risk premium in order
to be willing to hold their risky entrepreneurial capital. In addition, the steady-state value of this premium, which is given by the square-root term in (18), is decreasing in the interest rate. This is because a higher interest rate permits the domestic agents to accumulate more wealth in the long run, which in turn increases their willingness to take risk and thereby reduces the wedge between the interest rate the marginal product of capital.

Indeed, for any given initial level of aggregate wealth, a higher interest rate necessarily increases the mean return to saving and therefore also increases the level of aggregate wealth in subsequent periods. It follows that the long-run level of aggregate wealth also increases. The accumulation of more wealth, in turn, increases agents’ willingness to take risk—due to diminishing absolute risk aversion—and thereby reduces the premium they require in order to hold any given amount of capital. Hence, the overall impact of the interest rate on capital accumulation is now ambiguous: a higher interest rate may actually induce more investment in the long run, due to the wealth effect on risk taking. This wealth and risk-taking effect plays a central role in the results of our paper; we will revisit it shortly.

Going back to the determination of the steady state, we now note that, because the interest rate and the wage are constant in steady state, the present value of labor income is also constant. In particular, it is given by

\[ H_j = (1 - \alpha) f(K_j)/R_j. \]

Using this into condition (14), we infer that aggregate bond holdings—equivalently, the net foreign asset position—of country \( j \) satisfy the following condition:

\[ B_j = \frac{1 - \phi_j K_j - (1 - \alpha) f(K_j)}{\phi_j R_j}. \]  

(19)

Combining this result with the one in condition (18), we reach the following lemma.

**Lemma 2.** (i) There exist continuous functions \( K, B : (0, \beta) \times \mathbb{R}_+ \rightarrow \mathbb{R} \) such that, under either autarchy or integration, the steady-state levels of aggregate capital and bond holdings satisfy

\[ K_j = K(R_j, \tilde{\sigma}_j) \quad \text{and} \quad B_j = B(R_j, \tilde{\sigma}_j)K_j \]  

(20)

These functions are defined by

\[ K(R, \tilde{\sigma}) \equiv (f')^{-1} \left( R + \mu(R, \tilde{\sigma}) + \delta \right) \quad \text{and} \quad B(R, \tilde{\sigma}) \equiv \frac{1 - \phi(R, \tilde{\sigma})}{\phi(R, \tilde{\sigma})} - \frac{(1 - \alpha)f(K(R, \tilde{\sigma}))}{R K(R, \tilde{\sigma})}, \]

where \( \mu(R, \tilde{\sigma}) \equiv \sqrt{\frac{2\theta\gamma \tilde{\sigma}^2}{1 + \theta}} (\beta - R) \) and \( \phi(R, \tilde{\sigma}) \equiv \frac{1}{\gamma \sqrt{\theta}} \mu(R, \tilde{\sigma}). \)

(ii) \( \frac{\partial K(R, \tilde{\sigma})}{\partial R} > 0 \) if and only if \( \phi(R, \tilde{\sigma}) < \frac{\theta}{1 + \theta} \), which in turn is true if and only if \( R > \hat{R}(\tilde{\sigma}) \), where \( \hat{R}(\tilde{\sigma}) \equiv \beta - \frac{\theta}{1 + \theta} \frac{\gamma \tilde{\sigma}^2}{2} < \bar{R} \).

(iii) \( \frac{\partial K(R, \tilde{\sigma})}{\partial \tilde{\sigma}} < 0 \) necessarily.

(iv) \( \frac{\partial B(R, \tilde{\sigma})}{\partial R} > 0 \) necessarily.

(v) \( \frac{\partial B(R, \tilde{\sigma})}{\partial \tilde{\sigma}} > 0 \) if and only if \( R > \bar{R}, \) where \( 0 < \bar{R} \equiv \beta \frac{2\theta(1 - \alpha)}{\alpha + (2 - \alpha)\theta} < \bar{R} \).
Part (i) follows from conditions (18) and (19). The functions $K$ and $B$ give, respectively, the domestic capital stock and the net foreign-asset position that are consistent with stationarity of aggregate wealth when the interest rate is $R$ and the level of risk is $\sigma$. These functions will turn out to be particularly helpful in the characterization of the steady states.

Parts (ii) through (iv) then provide us with the comparative statics of these functions with respect to the interest rate and the level of risk. Part (ii), in particular, establishes that the steady-state capital stock is a U-shaped function of the interest rate. What lies behind this U-shaped relation is our wealth-and-risk-taking effect: for sufficiently high $R$, this effect dominates the familiar opportunity-cost effect, guaranteeing that a higher interest rate increases the capital stock in the steady state. This result plays a crucial role in our subsequent analysis. Part (iv), then, complements this result by showing that, as the interest rate increases, the propensity to save in the bond also increases: as the risk-free rate increases, saving in the riskless asset (bond) increases relative to aggregate saving in the risky asset (capital).

Finally, parts (iii) and (v) establish that, for any given interest rate, an increase in the level of risk necessarily reduces the steady-state capital stock, while it increases the propensity to save in the bond as long as the interest-rate is not too low. These properties capture, respectively, the risk-aversion and precautionary-saving effects of higher idiosyncratic risk.

Combined, these results facilitate the characterization of the autarchic and integrated steady states. To sharpen this characterization, we now introduce the following assumption, which we will invoke for a subset of our results.

**Assumption 1.** Suppose that either of the following conditions holds:

$$\tilde{\sigma}_j > \sqrt{\frac{2\alpha \beta (1 + \theta)}{\theta \gamma (\alpha + \theta (2 - \alpha))}} \quad \text{or} \quad \frac{\alpha - s_{j}^{aut}}{1 - s_{j}^{aut}} < \frac{\theta}{1 + \theta},$$

where $s_{j}^{aut} \equiv \delta K_{j}^{aut} / f(K_{j}^{aut})$ is the autarchic steady-state saving rate of country $j$.

This assumption requires either (i) that the uninsurable idiosyncratic risk exceeds some minimal level, or (ii) that the elasticity of intertemporal substitution, $\theta$, is sufficiently high relative to saving rates. It can be shown that the former property implies the latter (see Appendix). The advantage of the former property is that it is stated in terms of purely exogenous parameters, thus guaranteeing the existence of economies for which the assumption holds. The advantage of the latter property is that it can easily be mapped to data.

In particular, consider the following back-of-the envelope exercise. Using US data, we can set $\alpha \approx .36$ and $s^{aut} \approx .23$. It then follows that Assumption 1 is satisfied for the United States if $\theta > .2$. For countries with higher saving rates, this condition might be satisfied for even lower values of $\theta$. Since most recent estimates of $\theta$ are almost always above .5, and often above 1, we conclude that Assumption 1 is a very plausible benchmark.
In any event, the role of this assumption is to guarantee that the autarchic steady states lie in the increasing portion of the function $K$. In words, this means that, in the neighborhood of the autarchic steady state, the wealth-and-risk-taking effect of a higher interest rate dominates the standard opportunity-cost effect.

4.2 Autarchy

We are now ready to provide our first main result, which concerns the characterization of the autarchic steady state.

**Proposition 2.** There always exists an autarchic steady state, it is unique, and it features the following properties:

(i) The autarchic interest rates are given by $R_{aut}^j$, where $R_{aut}^j$ solves $B(R_{aut}^j, \tilde{\sigma}_j) = 0$, and satisfy $\bar{R}_2 < R_{aut}^j < R_{aut}^1 < \tilde{R}$, where $\bar{R}$ is the complete-markets interest rate, $\bar{R} = \beta$.

(ii) The autarchic capital stocks are given by $K_{aut}^j = K(R_{aut}^j, \tilde{\sigma}_j)$. Furthermore, under Assumption 1, $0 < K_{aut}^2 < K_{aut}^1 < \bar{K}$, where $\bar{K}$ is the complete-markets capital stock, defined by $f'(\bar{K}) = \beta + \delta$.

(iii) The autarchic consumption levels are given by $C_{aut}^j = f(K_{aut}^j) - \delta K_{aut}^j$. Furthermore, under Assumption 1, $0 < C_{aut}^2 < C_{aut}^1 < \bar{C}$, where $\bar{C}$ is the complete-markets consumption level, defined by $\bar{C} = f(\bar{K}) - \delta \bar{K}$.

The existence and the uniqueness of the autarchic steady state follow from the continuity and monotonicity of the function $B$ with respect to $R$ (which we established in Lemma 2), along with appropriate limit properties (which we establish in the Appendix).

Part (i) characterizes the steady-state levels of the interest rate: it establishes that the interest rate is lower than the discount rate in both countries, and more so in the South than in the North. The first property, namely that the autarchic interest rates are lower than the discount rate, reflects the presence of a precautionary motive for saving. As noted earlier, this is similar to Aiyagari (1994) and Mendoza et al. (2008). The second property, that the interest rate in the South is lower than the one in the North, is then a consequence of the fact that the precautionary motive is stronger in the South, due to the higher level of idiosyncratic risk. Formally, this is captured by the monotonicity of the function $B$ with respect to $\sigma$: the higher the level of undiversifiable idiosyncratic risk, the higher the steady-state demand for the risk-free asset for any given $R$; but since the net supply of
this asset is zero when the economy is in autarchy, it must be that the autarchic interest rate is lower the higher is $\sigma$.

This result is also illustrated in Figure 1. The interest rate is on the horizontal axis. The solid line is the curve $B$ for the North; the dashed line is the curve $B$ for the South. These curves can be interpreted as the aggregate demand for the safe asset in each country (normalized, though, by the corresponding capital stocks). Both curves are increasing in $R$, but the one for the South lies above the one for the North, reflecting the stronger precautionary motive in the South. The autarchic steady-state interest rates are given by the intersections of the two curves with the horizontal zero line. Clearly, the South has a lower autarchic interest rate, $R_{2}^{\text{aut}} < R_{1}^{\text{aut}}$.

Part (ii) characterizes the steady-state levels of the capital stock: it establishes, under Assumption 1, that the capital stock is lower than its complete-markets counterpart in both countries, and more so in the South than in the North. The first property, namely that the autarchic capital stocks are lower than their complete-markets counterparts, revisits the key result in Angeletos (2006). As mentioned in the introduction, this is a core prediction that differentiates our framework from prior work, including Aiyagari (1994), Krusell and Smith (1998), Mendoza et al. (2008, 2009), and most other Bewley-type models where incomplete risk sharing is typically associated with higher capital accumulation. Furthermore, this prediction is obviously more consistent with the data than the alternative featured in the aforementioned class of models: our framework predicts that the least financially developed countries are the poorest ones, not the richest ones.

The key for this difference is the type of risk featured in those models versus the type of risk in our model. In those models, agents face only idiosyncratic labor-income risk. This risk introduces a precautionary motive for saving, which reduces the interest rate, but does not break the equality between the interest rate and the marginal product of capital. In contrast, our model features entrepreneurial, or capital-income, risk. This risk introduces not only a precautionary motive, but also a positive wedge between the interest rate and the marginal product of capital; this wedge is the risk premium on private investment. It follows that, while incomplete risk-sharing necessarily encourages more capital accumulation in Bewley models by reducing the interest rate, it can discourage capital accumulation in our model by introducing the risk-premium wedge. The conditions in Assumption 1 then suffice for this wedge to dominate the reduction in the interest rate, thus guaranteeing that the capital stock is lower than under complete markets. Finally, the result that the autarchic capital stock is lower in the South than in the North reflects the fact that the wedge is higher in the South. Formally, this last result follows combining the facts that $\sigma$ is higher in the South, that $R$ is lower in the South, that the function $K$ is necessarily decreasing in $\sigma$, and that, under Assumption 1, this function is also increasing in $R$ for all $R \geq R_{j}^{\text{aut}}$.

Finally, part (iii) characterizes the steady-state level of consumption: it establishes, under Assumption 1, that the aggregate level of consumption is lower than its complete-markets counterpart in both countries, and more so in the South than in the North.
Combined, the above results show that, under autarchy, the South—the economy with more severe financial frictions—features a lower risk-free rate, a higher marginal product of capital, and lower levels of aggregate capital, wealth and consumption.

### 4.3 Financial integration

We now proceed to our second main result, the characterization of the integrated steady state.

**Proposition 3.** An integrated steady state exists, and it necessarily features the following properties:

(i) The interest rate is given by $R^{\text{int}}$, where $R^{\text{int}}$ solves $\sum_{j \in \{1,2\}} B(R^{\text{int}}, \tilde{\sigma}_j)K(R^{\text{int}}, \tilde{\sigma}_j) = 0$, and satisfies

$$R_2^{\text{aut}} < R^{\text{int}} < R_1^{\text{aut}} < \beta.$$

(ii) The foreign asset positions are given by $B_j^{\text{int}} = B(R^{\text{int}}, \tilde{\sigma}_j)K_j^{\text{int}}$ and satisfy

$$B_1^{\text{int}} < 0 < B_2^{\text{int}}.$$

(iii) The capital stocks are given by $K_j^{\text{int}} = K(R^{\text{int}}, \tilde{\sigma}_j)$. Furthermore, under Assumption 1,

$$K_2^{\text{aut}} < K_2^{\text{int}} < K_1^{\text{int}} < K_1^{\text{aut}}.$$

(iv) The consumption levels are given by $C_j^{\text{int}} = f(K_j^{\text{int}}) - \delta K_j^{\text{int}} + R^{\text{int}}B_j^{\text{int}}$. Furthermore, under Assumption 1,

$$C_2^{\text{aut}} < C_2^{\text{int}} < C_1^{\text{int}} < C_1^{\text{aut}}.$$

Part (i) establishes that the interest rate in the integrated steady state falls between the two autarchic values, while part (ii) states that in the integrated steady state the South is a net creditor, while the North is a net debtor. As we will see in the next section, this steady-state position is attained after a long transition throughout which the North runs persistent current-account deficits (and, symmetrically, the South runs persistent current-account surpluses).

These two results contain the explanation that our model offers for global imbalances:

**Corollary 1.** Along the transition from the autarchic to the integrated steady state, the North must accumulate a negative foreign asset position, that is, it must run a series of current-account deficits.

Intuitively, this is because the North has a comparative advantage in supplying the riskless asset. More precisely, the autarchic price of the riskless asset is lower (i.e., the autarchic interest rate is higher) in the North than in the South because of the weaker precautionary motive in the North. Extrapolating from standard trade theory, one would thus expect that the North will become a net supplier of the riskless asset once the two countries are allowed to trade. Of course, this intuition could have been misleading both because we are talking here about capital flaws, not goods trade,
and because of the rich dynamics that are involved in our environment. Nevertheless, our results show that this basic intuition is largely correct.

Parts (ii) and (iii) then study the long-run implications of financial integration for economic activity and world-wide inequality. In particular, part (ii) establishes that the South has a higher capital stock in the integrated steady state than in the autarchic one. Formally, this is a direct implication of our earlier result in Lemma 2 that the function $K$ is increasing in $R$. More intuitively, this is because of the dynamics of wealth accumulation that we highlighted earlier: agents in the South enjoy a higher capital stock in the integrated steady state because a prolonged access to higher safe returns permits them to accumulate more wealth, and therefore to take more risk. The converse is true for the North.

Part (iv) spells out the implications for aggregate consumption. The South enjoys a higher level of consumption in the integrated steady state than under autarchy, both because it has accumulated more capital domestically and because it has accumulated a positive position against the North. Once again, the converse is true for the North.

Clearly, similar properties as those for capital and consumption hold if we look at GDP, wages, and labor productivity. This gives the key prediction of our model regarding the long-run impact of financial integration on cross-country inequality:

**Corollary 2.** *In the long run, financial integration reduces cross-country inequality.*

As we will see next, however, the short-run effects are quite different.

## 5 Transitional dynamics and numerical example

In this section we examine in more detail the dynamic responses of the two countries to the integration of their financial markets, starting from an initial position that coincides with the autarchic steady states. For this purpose, we henceforth have to abandon generality and focus on a particular numerical exercise. While we base this numerical exercise on a somewhat plausible calibration of the model, we invite the reader not to focus on the precise numbers: the simplicity of our model and data limitations preclude a rich, serious quantitative assessment. That being said, the numerical exercise indicate that the effects can be of non-trivial magnitude. Furthermore, the qualitative patterns we identify with this particular numerical exercise are extremely robust: as one should anticipate from our earlier theoretical results, they obtain for a wide range of parameters that we have experimented with as long as Assumption 1 is maintained.

### 5.1 Parameterization

The two economies are parameterized by $(\alpha, \beta, \gamma, \delta, \theta, \tilde{\sigma}_1, \tilde{\sigma}_2)$, where $\alpha$ is the income share of capital, $\beta$ is the discount rate, $\gamma$ is the coefficient of relative risk aversion, $\delta$ is the depreciation rate,
\( \theta \) is the elasticity of intertemporal substitution, and \( \tilde{\sigma}_j \) is the undiversifiable risk in country \( j \).

The time period is interpreted as one year. All the preference and technology parameters are set in a manner that is broadly consistent with the macro and macro-finance literatures. In particular, the discount rate is \( \beta = 0.05 \). The elasticity of intertemporal substitution is \( \theta = 1 \), a value broadly consistent with recent micro and macro estimates,\(^9\) while the coefficient of relative risk aversion is chosen to be \( \gamma = 8 \), a value commonly used in the macro-finance literature to help generate plausible risk premia. Finally, the depreciation rate is \( \delta = 0.10 \) and the share of capital in production is \( \alpha = 0.40 \).

This leaves us with \( \tilde{\sigma}_j \) or, equivalently, with \( \sigma_j \) and \( \lambda_j \). We first focus on \( \sigma_j \), which we interpret as the idiosyncratic volatility of the rate of return that an individual entrepreneur faces in his investment, regardless of whether this risk is insurable or not. This interpretation is analogous to the notion of idiosyncratic volatility for stock market returns, except that here we are primarily interested in privately-held businesses.\(^10\)

Unfortunately, there is no direct measure \( \sigma \) in the US economy because of the unavailability of data about entrepreneurial returns. However, there are various indications that idiosyncratic investment risks in the United States are significant. For instance, the probability that a privately held firm survives five years after entry is less than 40%. Furthermore, even conditional on survival, the risks faced by entrepreneurs and private investors appear to be very large: as Moskowitz and Vissing-Jörgensen (2002) document, not only is there a dramatic cross-sectional variation in the returns to private equity, but also the volatility of the book value of an index of private firms is twice as large as that of the index of public firms. Since this index already diversifies the firm-specific risk in private equity, and since the volatility of the entire pool of public firms is about 15% per annum, this gives another indication of the significant risks faced by entrepreneurs. Finally, if one takes the idiosyncratic volatility of public firms as a proxy for that of private firms, this would suggest a value of \( \sigma \) over 50% for the United States. This is actually the value preferred by Moskowitz and Vissing-Jörgensen (2002), Bitler et al. (2005) and Roussanov (2009).

Another indirect estimate for the private-sector volatility in the US could be motivated by the work of Davis et al. (2006), who use the Longitudinal Business Database (LBD). They find that in 2001 the ratio of private to public volatility of employment growth rates was in the range of 1.43-1.75. Given that the average annual standard deviation of returns for public firms over 1990-1997 was 0.11 according to Campbell et al. (2001), and that there is, at least in the context of the present model, a close relationship between the volatility of profits and the volatility of labor demand, a choice of \( \sigma \) near 0.20 for the US economy could be justified from this perspective. Finally, if one also takes into account that, in the data, sales and profits are more volatile than employment (at

\(^9\)See Angeletos (2007) for a more thorough discussion of the relevance of this parameter within the type of model we have employed here, and also for references on the empirical estimates of this parameter.

\(^10\)Note, though, that idiosyncratic risk may affect the investment decisions of public traded firms as well. See Papanikolaou and Panousi (2010) for evidence.
least for public firms), this would suggest even higher values for $\sigma$.

Combining the above observations, and interpreting the North in our model as the United States, we conclude that a value for $\sigma_1$ near 0.5 is a plausible benchmark. However, as already mentioned, the entrepreneur could actually be able to diversify away a fraction $\lambda$ of that risk, so that the volatility of the remaining undiversifiable risk is in fact lower than $\sigma$. Furthermore, our model assumes that all capital is held in private businesses, whereas in reality an important fraction is held in publicly-traded companies. For these reasons, we next proceed to discuss the value of $\lambda$ and/or $\tilde{\sigma}$.

Although we have not explicitly modeled the distinction between private and public equity, the following conceptual exercise provides a possible mapping between our model and the data. Suppose that each household in our model is able to split its family business in two accounting identities. The one, which takes a fraction $\lambda$ of the business's output and profits, "goes public": it is sold in the market for its expected value, so that the household diversifies the risk in that component. The other, which takes the residual $(1-\lambda)$ of the business's output and profits, "stays private": the household has to bear the risk in that component. This interpretation then suggests that $\lambda$ can be matched to the ratio of public firm profits over total profits in the United States, where total profits are the sum of privately held firm profits plus corporate profits. In the National Income and Product Accounts (NIPAs), the ratio of proprietors' profits over total profits (proprietors plus corporate) is 47% on average over the period 1981-2006. This gives a value for $\lambda_1$ around 0.5 or 0.6, which is consistent with other estimates of the size of public equity relative to total capital.

Finally, a direct calibration of the uninsurable risk $\tilde{\sigma}_1$ can be obtained as follows. In our model, the idiosyncratic volatility of individual consumption growth is proportional to $\tilde{\sigma}_1$. We could then ask what is the value of $\tilde{\sigma}_1$ that makes our model's prediction about idiosyncratic consumption volatility match the one found in the data. Using studies that estimate this idiosyncratic variance of consumption growth in US data, such as Ait-Sahalia et al. (2001) and Malloy et al. (2006), we then infer that the appropriate value for $\tilde{\sigma}_1$ is close to 0.2. On the basis of this observation and all the preceding discussion, we pick $\sigma_1 = 0.5$, $\lambda_1 = 0.6$ and $\tilde{\sigma}_1 \equiv (1-\lambda_1)\sigma_1 = 0.2$ as our favorable parameterization for the North.

Turning to the South, we note that data on entrepreneurial activity and idiosyncratic investment risk are even more scarce in developing countries than in the United States. Nevertheless, there are multiple indications that idiosyncratic risk is higher in developing countries. For lack of a better alternative, we assume that the overall amount of risk $\sigma_2$ in the South is the same as the one assumed for the North. We then set $\lambda_2$ at the conservative value of 0.2. This is at the upper range of available estimates of the ratio of public equity to total capital in less advanced economies such as

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11 Incidentally, note that this distinction is unclear in the data too, since ownership of many public companies is often concentrated in the hands of few key investors.

12 See Moskowitz and Vissing-Jørgensen (2002) for a more extensive discussion of the relative size of private and public equity in the United States.
China, India, Brazil and Mexico (e.g., Demirguc and Levine, 1996; La Porta, 1997). This approach gives $\sigma_2 = 0.5$, $\lambda_2 = 0.2$ and $\tilde{\sigma}_2 \equiv (1 - \lambda_1)\sigma_1 = 0.4$ as our favorable parameterization for the South.

In any event, what matters for the qualitative properties of all the results we document below is merely the fact that $\tilde{\sigma}_2$ is higher than $\tilde{\sigma}_1$, not the precise numbers we have picked. With this qualification in mind, we summarize the baseline parameterization of our model in Table 1 and proceed to document the dynamic effects of financial integration.

### 5.2 Dynamic responses

We are now ready to conduct the experiment of interest, namely a reform that lets the two economies integrate their financial markets (i.e., to trade the riskless asset). This reform is assumed to be unexpected and irreversible. Before this reform, the the two economies are assumed to rest at their respective autarchic steady states. The objective is then to study the dynamics responses of these economies to this reform.

Tracking the transitional dynamics of incomplete-market models is often a daunting exercise. This is not the case here, thanks to the low dimensionality of the general-equilibrium system of our model. In particular, note from Lemma 1 that, when $\theta = 1$, the marginal propensity to consume out of total wealth reduces to $m_{jt} = \beta$ for all $j, t$. It then follows from Proposition 1 that the transitional dynamics of the world economy can be reduced to a simple system of four first-order ODE’s in $(X_{jt}, H_{jt})_{j \in \{1,2\}}$, where $X_{jt} \equiv K_{jt} + B_{jt}$. Our numerical algorithm then works as follows. First, we solve for both the autarchic and the integrated steady-state aggregates. Next, we numerically solve the aforementioned ODE system using the autarchic steady-state values of capital, $X_{j0} \equiv K_{j aut}$, as initial conditions and the integrated steady-state values of human wealth, $H_{j int}$, as terminal conditions.

The dynamic path of the South is illustrated in Figure 2, and that of the North in Figure 3. Time in years is on the horizontal axis, and levels of several macroeconomic variables are on the vertical axis. The dotted lines indicate the levels of the variables at the autarchic steady state. The dashed lines indicate the levels of the variables at the integrated steady state. The solid lines show the dynamic response of the variables.

Figure 2 shows that, immediately upon integration, the capital stock in the South falls below its autarchic steady-state level. But after this initial fall, the capital stock starts recovering. In fact, it is back to the autarchy level in about thirty years and it keeps increasing after that, eventually
converging to the new, higher, integrated steady state. In other words, the South faces a bleak picture in the short run, with a significant outflow of capital immediately after integration, but this picture is reversed in the long run, as capital starts flying back into the country, eventually reaching a higher level than under autarchy. In particular, the capital stock in the South falls by almost 4% immediately after integration, compared to its autarchic steady state. But, at the long-run integrated steady state, the capital stock in the South has increased almost 9% above its autarchic level. The same qualitative picture is true for the other aggregate variables, such as aggregate output, consumption, and the wage. For example, aggregate output in the South falls by almost 2% in the short run, and it increases by almost 3% in the long run, compared to its autarchic value.

Figure 3 demonstrates the exact opposite picture for the North. Immediately upon integration, the North experiences an inflow of capital, and capital remains above its autarchic level for about fifty years. However, in the long run, capital settles at an integrated level lower than the autarchic one. The same is true for the other aggregate variables. The interest rate jumps down from the autarchic steady state upon integration, and it settles at an even lower level in the long run. Finally, in the long run the North ends up borrowing from the South. In other words, the North experiences an initial period of prosperity, but in the long run this picture is reversed. For example, capital in the North increases by about 2.5% upon integration, but it falls by about 5% in the long-run steady state, compared to its autarchy level. And aggregate output in the North increases by 1% upon integration, but it falls by 2% in the long run, compared to autarchy.

The intuition behind these results is as follows. While in autarchy, the South faces higher levels of idiosyncratic risk and therefore features a higher demand for precautionary saving than the North. This stronger precautionary motive keeps the domestic (risk-free) interest rate suppressed in the South relative to the North. Upon integration, however, the precautionary saving of the South is partly absorbed by the North, implying that the domestic interest rate has to increase in the South (and decrease in the North). This in turn has very different implications for the macroeconomic outcomes of the South depending on whether we look at the short run or the long run. In the short run, the increase in interest rates means an increase in the opportunity cost of capital, causing a reduction in the capital stock of the South. In the long run, however, this increase in interest rates permits the residents of the South to accumulate more wealth. As they do so, they become willing to undertake more investment risk, which explains why the capital stock recovers over time. The fact that the capital stock eventually increases beyond its autarchic value then follows from Proposition 3.

Finally, note that, along the transition to the new steady state, the South runs significant current-account surpluses, so that it keeps increasing its financial position abroad. Conversely, the North runs significant current-account deficits, eventually reaching a dramatic level of foreign debt, equal to about 3.5 times its GDP. Clearly, this is the manifestation of the precautionary saving of the South rushing for safety in the North.
Our findings thus provide a novel perspective on the ongoing debate on the costs and benefits of capital-market liberalization. In particular, while many fear that such a reform may cause an outflow of capital, and while this fear seems to be validated by the recent emergence of global imbalances, here we find that this effect may be reversed in the long run thanks to the endogenous accumulation of capital.

**Corollary 3.** Financial integration can trigger an outflow of capital from the poor country in the short run, thereby exacerbating cross-country inequality. These effects, however, are reversed in the long run.

### 6 Welfare implications

In this section we examine the welfare effects of integration within each country. In so doing, we are interested to distinguish how these effects may vary between the poor and the rich, as well as across different generations. This motivates us to consider two exercises. The first studies welfare at the moment the reform takes place, taking into account the entire transitional dynamics that will follow; the second compares welfare across the two steady states.

More precisely, the first exercise seeks to answer the following question. Suppose that a country rests in its autarchic steady state and the current generation contemplates the option to undertake a reform that would let it integrate with the other country. Pick a particular level of wealth. What is the minimal compensation an agent with that particular level of wealth would be willing to accept in return for the failure of the reform to take place? The second exercise, on the other hand, seeks to answer the following question. Suppose that future generations are offered with the option to be born in the autarchic steady state versus be born in the integrated steady state. Fix a particular ranking in the wealth distribution (say, the 7-th percentile). What is the minimal compensation an agent with that particular ranking would have to receive under the autarchic steady state in order to be as happy as an agent with the same ranking under the integrated steady state? In short, the first exercise studies how financial integration impacts the welfare of the poor and the rich in the current generation, while the second exercise studies how it impacts the welfare of the poor and the rich in generations in the distant future.

At this point, we note that our baseline model features an explosive level of wealth inequality within each country. This is because individual dynamics follow a random walk in steady state. To fix this issue, we can modify the model to let some agents die with a constant Poison rate $\nu > 0$ and get replaced with other agents who “inherit” the average level of wealth; see Panousi (2010) for further details on this approach. We can then adjust the subjective discount rate so that the effective discount rate, which is now $\beta + \nu$, remains the same as in our baseline model. This guarantees that the aggregate dynamics of the modified model remain exactly the same as those of our baseline model, while at the same time the modified model admits a unique, well-defined steady-state wealth distribution. For our numerical exercise, we set $\nu = 1/150$; this is motivated by the fact that the average mortality rate is about $1/75$ per year and the fact that agents are imperfectly altruistic towards future generations.
Table 2: Welfare Effects. This table summarizes the welfare effects of financial integration across different quantiles of the wealth distribution. Q1 is the first quantile, Q2 is the second quantile, and so on. The numbers in the cells of the table report the within-quantile averages of the short-run and long-run compensating differentials. The latter are measured as percent of permanent income. The “short run” refers to welfare at the moment integration takes place, while the “long run” compares welfare across the autarchic and integrated steady states. (See the main text for detailed definitions.)

Formally, fix a country $j$ and let $V_{\infty,\text{aut}}^j(x)$, $V_{0,\text{int}}^j(x)$ and $V_{\infty,\text{int}}^j(x)$ denote the value functions at, respectively, the autarchic steady state, the time the reform initiates, and the integrated steady state. The first welfare exercise is to compute, for each level of financial wealth $x$, a compensating differential $\tau^j(x)$ such that $V_{\infty,\text{aut}}^j(x + \tau^j(x)) = V_{0,\text{int}}^j(x)$. The second exercise is to compute a compensating differential $\tau'_{\infty}^j(x)$ such that $V_{\infty,\text{aut}}^j(x + \tau'_{\infty}^j(x)) = V_{\infty,\text{int}}^j(g^j(x))$, where $g^j(x)$ is the level of wealth that corresponds to the same relative wealth position under the integrated steady state as the one obtained with wealth $x$ under the autarchic steady state. For either of these two exercises, we then express the corresponding compensating differential as a fraction of the agent’s permanent income. The resulting number represents a welfare gain if it is positive, and as a welfare loss if it is negative. Finally, to fix language, and notwithstanding the fact that both exercises concern life-time utility, we refer to the effects that are computed with the first exercise as the “short-run” welfare effects, and to the ones that obtain from the second exercise as the “long-run” welfare effects.

These welfare gains and losses are then illustrated in Table 2 and in Figure 4, for each of the two countries and for different levels of wealth. Table 2 summarizes the welfare gains and losses across the four different quantiles of the autarchic steady-state wealth distribution. Figure 4 gives a similar but finer picture, by illustrating the welfare effects across all percentiles of the wealth distribution. The solid line in this figure represents the “short-run” welfare effects (that is, those obtained by the first of the aforementioned welfare exercises), while the dashed line represents the “long-run” welfare effects (that is, those obtained by the second exercise).

We first consider the South, which is in panel (a) of Figure 4. On impact (solid line), financial integration benefits the rich at the expense of the poor: the poor of the current generation suffer losses, whereas the rich enjoy gains. These effects, however, are reversed in the long run (dashed line): the poor of future generations are better off living under integration than under autarchy.

\[14\] That is, a number equal to, say, 5% means that the agent must receive either a lump sum equal to 5% of his effective wealth or, equivalently, a perpetuity with annual dividend equal to 5% of his permanent income.
while the converse is true for the rich. For example, as shown in Table 2, agents at the bottom 25% of the wealth distribution suffer an average loss equal to $-8.5\%$ of their permanent income on impact, but enjoy an average gain of $+3.8\%$ in the long run. The corresponding numbers for the top 25% of the wealth distribution are $+0.9\%$ and $-0.8\%$.

The intuition behind these results is as follows. In the short run, financial integration causes the South’s wages to fall and its interest rates to rise, as we have seen in Figure 2. Both these forces tend to reduce the present discounted value of wages, that is, the human wealth of the households. In turn, this hurts all agents, but more so the poorer ones, since a larger fraction of poor agents’ effective wealth comes from labor income. At the same time, the reduction in wages means that private business now have to face low labor costs, a force that increases the average return on private investment. Along with the fact that the interest rate has also increased, this means that the overall return to saving has increased. This effect tends to benefit the rich, who have large amount of financial wealth relatively to human wealth. In our example, this positive effect is strong enough to offset the negative effect of the reduced human wealth for richer agents, and it explains why richer agents gain whereas poorer agents lose from integration at impact.

In the long run, on the other hand, wages eventually settle at a higher level than under autarchy. This tends to increase human wealth. The increase in interest rates contributes in the opposite direction, but does not offset the positive effect of higher wages. The long-run increase in human wealth then benefits both the poor and the rich. Along with the fact that the wealth distribution shifts to the right, this explains why the poor and the middle class of future generations are most likely to benefit from integration. The rich, however, may end up losing because the new steady state is associated with higher labor costs and lower mean returns to entrepreneurship.

We next consider the North, which is illustrated in panel (b) of Figure 4. In the short run (solid line), the poor and the middle class gain, while the very rich loose. Once again, these effects are reversed in the long run (dashed line): the poor loose and the rich gain. For example, as shown in Table 2, the bottom 25% make gain of $+3.3\%$ in the short run and a loss of $-11.6\%$ in the long run, while the top 25% make a loss of $-0.7\%$ in the short run and a gain of $+6.8\%$ in the long run. The intuition for these results is analogous to that for the South. The North’s poor gain immediately upon integration because of the increase in human wealth, while the rich loose because of the lower return to their bond holdings and the higher labor costs in their private businesses. But as time passes and capital starts going down, the consequent reduction in wages hurts the poor, while it benefits the rich, and welfare effects are reversed.

In Table 3, we study the sensitivity of the aforementioned findings to three variant parameterizations of the model. For simplicity, we focus on long-run welfare effect (comparisons across steady states). The first variant raises the level of uninsurable risk in the South, from $\tilde{\sigma}_1 = 0.4$ to $\tilde{\sigma}_1 = 0.6$. The second variant raises the income share of capital in both countries to $\alpha = 0.7$; this is meant to capture the broader definition of capital one may wish to use for long-run considerations. The third
Table 3: Sensitivity analysis. This table revisits the long-run welfare effects of financial integration for three alternative parameterizations and a variant policy reform.

variant combines the aforementioned two variants. In all cases, the poor continue to make gains in the South and to suffer loses in the North. However, the poor’s gains in the South now tend to be much bigger, while the poor’s losses in the North are not much different. Furthermore, the long-run benefits of integration are now more widespread in the South, with all quantiles actually gaining in the last two variant. Notwithstanding the limitations of our quantitative exercises, these findings suggest that the long-run welfare gains of capital-account liberalization are likely to be highest for economies where idiosyncratic risk impacts a broad range of entrepreneurial, investment, and human-capital choices.

Finally, in the last column of Table 3, we return to the baseline parameterization but consider an alternative policy exercise: we now assume that financial integration permits the South to obtain access, not only to the higher safe returns of the North, but also to improved risk-sharing possibilities of the latter. That is, we let financial integration be associated with an increase of $\lambda_2$ from 0.2 to 0.6, and hence with a reduction in $\sigma_2$ from 0.4 to 0.2. The implied welfare gains are then much bigger than those of our baseline policy exercise, and also more widespread in the population. For example, the bottom 25% gain $+40.2\%$ instead of $+3.8\%$, and the top 25% gain $13.9\%$ instead of losing $-0.8\%$. This finding underscores that the benefits of capital-account liberalization for developing economies are likely to be maximal if the reform helps these countries alleviate their own agency, enforcement and institutional problems by gaining access to the more efficient financial institutions of developed economies.

The numerical findings we have reported in this section are, of course, only illustrative. A serious quantitative exercise would require a richer model, one that would allow for more sources of heterogeneity (e.g., different levels of entrepreneurial ability), for diminishing returns in entrepreneurial investment, and for endogenous occupational and educational choices. Nevertheless, the qualitative properties we have uncovered are likely to be robust and highlight the distinct short- and long-run effects that are at the focus of our analysis.
7 TFP growth and shortage of assets

In this section we discuss an extension of our model that helps accommodate the idea that developing countries suffer from a shortage of assets (Caballero, Farhi and Gourinchas, 2008), uncovers the possible implications of our analysis for TFP growth, and helps resolve the puzzle that capital often flows from fast-growing to slow-growing countries (Gourinchas and Jeanne, 2008).

This extension introduces a “safe sector”. The technology in this sector has a lower mean return than entrepreneurial activity, but entails no risk. One can think of this as, say, “farming”, or as some form of storage technology. The broader idea here is that entrepreneurs face a trade-off between risk and return as they choose among an array of investment opportunities—a trade-off that is known to play a crucial role in aggregate TFP and growth dynamics (e.g., Acemoglu and Zilibotti, 1997).

The production function in the “safe sector” is assumed to take the form $g_j(M_{jt}) = A_j M_{jt}^\alpha$, where $M_{jt}$ is the corresponding level of capital and $A_j$ is a productivity parameter that determines the size of the safe sector relative to that of the risky, entrepreneurial sector. Clearly, the equilibrium must now satisfy $R_{jt} = g_j'(M_{jt})$ for each country $j$ and all periods $t$: the marginal product of capital in the safe sector is equated to the interest rate. This pins down the capital stock of the safe sector—which can be interpreted as the supply of safe assets—as an increasing function of the interest rate. The rest of the equilibrium characterization then proceeds in similar lines as in our benchmark model, and is omitted here because of space limitations.

Consider now the following exercise. Restrict $\tilde{\sigma}_1 = \tilde{\sigma}_2$ but let $0 < A_2 < A_1$. Letting $A_1 > A_2$ captures the idea that the North may have a technological or institutional superiority in supplying the safe asset; restricting $\tilde{\sigma}_1 = \tilde{\sigma}_2$ seeks to isolate this possibility from the possibility of differential levels of uninsurable entrepreneurial risk, the implications of which we have already studied. It is then possible to check that all our findings continue to hold as before. In particular, the South is poorer than the North under both autarchy and integration; the North runs persistent current-account deficits upon integration; capital initially flies out of the South and into the North in the short run; and finally this effect is reversed in the long run.

This extension thus offers a direct re-interpretation of the preceding analysis: our results originate interchangeably in the relatively higher level of uninsurable risk faced by entrepreneurs in the South and/or in the relative superiority of the North in supplying the global economy with safe stores of value. In turn, this builds a bridge between our paper and Caballero, Farhi and Gourinchas (2008). Like this earlier work, our analysis indicates that global imbalances may originate from a shortage of assets in emerging countries. But unlike this earlier work, our analysis requires only a shortage of the relatively safe assets, not of all assets. Indeed, emerging economies appear to be producing a lot of assets in reality. Yet, most of these assets are risky and their residents seem to be searching abroad for safer assets such as US Treasury bills. It is thus the shortage of

\[15\] That the safe sector does not employ labor is for simplicity.
such “quality” assets, and not of all assets, that explain why financial capital may be flowing from emerging countries to the United States and other advanced economies.

Finally, our analysis has distinct implication for aggregate TFP and growth dynamics. To see this, note that along the transition from the autarchic to the integrated steady state, agents in the South become increasingly willing to take risk. Like in our baseline mode, this is because the increase in interest rates induces agents in the South to accumulate more wealth. But now that the agents face a choice between the safe sector and the risky, entrepreneurial sector, this increase in the willingness to take risk also means a reallocation of resources from the safe sector to the more risky, but also more efficient, entrepreneurial sector. As this happens, the South enjoys an increase in TFP. Conversely, because the North de-cumulates wealth and reallocates capital away from its entrepreneurial sector, it experiences a drop in its TFP. This is illustrated in Figure 5, which shows the dynamics of TFP in the two countries for a numerical version of the extended model.16

Along with our model’s prediction regarding current-account dynamics, this provides a simple resolution to the empirical puzzle documented by Gourinchas and Jeanne (2008). This work showed that, in the data, capital often appears to flow from countries that experience higher productivity growth to those that experience lower productivity growth. While this fact is inconsistent with the standard neoclassical growth paradigm, it is easily accommodated in our model.17

8 Conclusion

This paper studies the global macroeconomic implications of financial integration within a tractable incomplete-markets model that features uninsurable idiosyncratic entrepreneurial risk—a friction that introduces, not only a precautionary motive for saving, but also a wedge between the interest rate and the marginal product of capital.

Because of this wedge, a financially underdeveloped economy (“South” or “China”) can feature both a lower interest rate and a lower capital stock under autarchy than a more advanced economy (“North” or “US”). As the two economies open up their capital accounts, interest rates rise in the South and fall in the North; the North starts running large current-account deficits; and the South suffers an outflow of capital. Over time, however, integration permits the South to accumulate more wealth, in part by saving in the North. As this happens, the bite of the aforementioned friction diminishes. Eventually, this helps boost capital accumulation and growth, thereby reducing cross-

16 The numerical exercise here assumes \( \delta_1 = \delta_2 = .50 \), \( \alpha = 0.7 \), and \( (A_1, A_2) \) chosen so that in autarchy the capital in the “safe sector” accounts for 50% of total capital in the North and for 20% of total capital in the South. Also, note that TFP growth is negative in the North and positive in the South, but both countries could feature positive TFP growth if we had allowed for an exogenous constant drift in technology. The robust prediction is that integration speeds up TFP growth in the South while it slows it down in the North.

17 In fact, if we focus on labor productivity (output per worker) rather than TFP, this statement holds true even for our baseline model: along the transition from the autarchic to the integrated steady states, the South experiences higher growth in physical capital and labor productivity than the North, and yet it is the North that is borrowing from the South. The extension of this section helps reinforce this point by establishing a similar property for TFP.
country inequality in the long run. Combined, these results provide a simple explanation for the emergence of global imbalances, a simple resolution to the empirical puzzle that capital often fails to flow from the rich or slow-growing to the poor or fast-growing countries, and a distinct set of policy lessons regarding the intertemporal costs and benefits of capital-account liberalization.

Underlying these findings are two key properties. First, a positive wedge between the marginal product of capital and the risk-free rate. Second, the tendency of this wedge to diminish as wealth increases. In our model, the first property is due to uninsurable idiosyncratic investment risk; the second property then follows from diminishing absolute risk aversion. Interestingly, these properties may naturally emerge also in models with borrowing constraints. These models feature a positive wedge between the marginal product of capital (“internal returns”) and the interest rate faced by savers (“external returns”), either because constraints bind now or because they are expected to bind in the future. What is more, this wedge typically falls with wealth, as more wealth helps overcome current and future borrowing constraints. We thus conjecture that similar results would obtain in a variant of our model that would introduce realistic borrowing constraints in addition to, or in place of, the entrepreneurial risk that we have focused on in this paper.
9 Appendix

Proof of Lemma 1 (individual policy rules). This result is essentially a variant of the Merton-Samuelson optimal portfolio problem; see the proof of Proposition 1 in Angeletos and Panousi (2009).

Proof of Proposition 1 (equilibrium dynamics). For simplicity, we drop the index \( j \). Since aggregate labor demand is \( \int n_t^i = \bar{n}(\omega_t)K_t \) and aggregate labor supply is 1, the labor market clears if and only if \( \bar{n}(\omega_t)K_t = 1 \). It follows that the equilibrium wage satisfies \( \omega_t = F_L(K_t, 1) \) and, similarly, the equilibrium mean return to capital satisfies \( \bar{\mu}_t = F_K(K_t, 1) - \delta \). The bond market, on the other hand, clears if and only if \( B_t = 0 \). We define total effective wealth for an agent \( i \) as the sum of his financial wealth, which in turn is the sum of his capital holdings and bond holdings, plus human wealth (or human capital), i.e. \( w_t^i \equiv k_t^i + b_t^i + h_t = x_t^i + h_t \). Then, in general equilibrium of the autarchic economy, \( W_t = K_t + H_t \), which, combined with the aggregation of bond holdings from (8), gives (14). Aggregating over the definition of human capital in Lemma 1, we get

\[
H_t = h_t = \int_t^\infty e^{-\int_t^s R_t^j \omega_s ds} ds. 
\]

Expressing this in recursive form gives condition (13). Aggregating the household budget, which can be written as \( dw_t^i = [\bar{\mu}_t k_t^i + R_t (b_t^i + h_t) - c_t^i] dt + \sigma k_t^i dz_t^i \), using the aggregated policy functions from (8), using (9) and (13), and the fact, in equilibrium, \( \bar{\mu}_t K_t + \omega_t = F(K_t, 1) - \delta K_t \), we get the resource constraint (11). Finally, using \( C_t = m_t W_t \), and therefore \( \dot{C}_t/C_t = \dot{m}_t/m_t + \dot{W}_t/W_t \), together with (10) and the definition of \( \dot{\rho}_t \), gives the aggregate Euler condition (12).

Proof of Lemma 2. (i) The form of the function \( K \) is evident from condition (18), while the form of the function \( B \) follows from condition (19).

(ii) Using our Cobb-Douglas assumption for the production function and equation (18), we get that \( K(R) = \left[ \frac{\mu(R)+\delta+R}{\alpha} \right]^{\frac{1}{\alpha-1}} \). It follows that \( K_R \) has the same sign as \( \frac{1}{\alpha-1}(\mu_R+1) \). Since from (18) \( \mu(R) = (\frac{2\theta^2}{1+\theta^2})(\beta - R)^{1/2} \), we get that \( \mu_R = (\frac{1}{2}\frac{\theta^2}{1+\theta^2})^{1/2}(\beta - R)^{-1/2} \). Using this, we have that \( K_R > 0 \iff R > \beta - \frac{1}{2}\frac{\theta^2}{\bar{\gamma} + 1} \equiv \bar{R}(\bar{\gamma}) < \beta \equiv R \).

In addition, since \( \dot{W}_t = \dot{\rho}_t W_t - C_t = (\rho_t - m_t) W_t \), where \( \rho_t \equiv \phi_t \bar{\mu} + (1 - \phi_t) R_t \), wealth stationarity requires \( \dot{\rho} = m \). Combining this with the Euler equation in steady state, we get

\[
\frac{\theta + 1}{2} \phi(f'(K) - \delta - R) - \theta(\beta - R) = 0. 
\]

From this, and for steady-state capital to be lower than under complete markets, that is, for \( f'(K) -
\( \delta > \beta \), it has to be the case that

\[
\frac{\theta + 1}{2} \phi(\beta - R) - \theta(\beta - R) < 0,
\]

which, since \( \beta - R > 0 \), gives \( \theta > \phi/(2 - \phi) \) or \( \phi < \theta/(\theta + 1) \).

(iii) Since \( K(R) = \left[ \frac{\mu(R) + \delta + R}{\alpha} \right]^{1/2} \), it follows that \( K_\delta \) has the same sign as \(-\mu_\delta\). Since \( \mu(R) = (2\theta_2^2/\gamma_1) (\beta - R)^{1/2} \), we get that \( \mu_\delta = \theta_2^2/\gamma_1 (2\theta^2/\gamma_1)^{-1/2}(\beta - R) \). Using this, we have that \( K_\delta < 0 \).

(iv) From (19) we have that

\[
B(R) = -(1 - \alpha) \frac{K(R)^\alpha}{R} + \frac{1 - \phi(R)}{\phi(R)} K(R). \tag{21}
\]

Consider the limits of \( B \) as \( R \to 0^+ \) and \( R \to \beta^- \). Note that \( \mu(0) = (2\theta_2^2/\gamma_1) \) is finite and hence both \( \phi(0) \) and \( K(0) \) are finite. It follows that

\[
\lim_{R \to 0^+} B(R) = -(1 - \alpha) K(0)^\alpha \lim_{R \to 0^+} \frac{1}{R} + \left( \frac{1}{\phi(0)} + 1 \right) K(0) = -\infty.
\]

Furthermore, \( \mu(\beta) = 0 \), implying \( \phi(\beta) = 0 \) and \( K(\beta) = (f')^{-1}(\beta) \) is finite. It follows that

\[
\lim_{R \to \beta^-} B(R) = -(1 - \alpha) K(\beta)^\alpha \frac{1}{\beta} + \lim_{R \to \beta^-} \left( \frac{1}{\phi(R)} + 1 \right) K(\beta) = +\infty.
\]

Next, note that, from (21),

\[
\frac{\partial B}{\partial R} = -(1 - \alpha) \frac{K(R)^\alpha}{R^2} \left[ \alpha R K'(R) - 1 - \frac{\phi'(R)}{\phi(R)^2} K(R) + \frac{1}{\phi(R)} K'(R) \right].
\]

Now note that, since \( K(R) = \left[ \frac{\mu(R) + \delta + R}{\alpha} \right]^{1/2} \) and \( \phi(R) = \sqrt{\frac{2\theta}{\gamma^2 (1 + \theta)} (\beta - R)} \), we have

\[
K^{\alpha - 1} = \frac{f'(K)}{\alpha}, \quad \frac{K'}{K} = \frac{1}{\alpha - 1} \frac{\mu' + 1}{f'(K)}, \quad \text{and} \quad \frac{\phi'}{\phi^2} = \frac{\gamma^2 \mu'}{\mu^2},
\]

where we suppress the dependence of \( K, \mu, \) and \( \phi \) on \( R \) for notational simplicity. It follows that

\[
\frac{\partial B}{\partial R} = -\frac{1 - \alpha}{\alpha} \frac{R \mu' + R - f'(K)}{R^2} - \frac{\gamma^2 \mu'}{\mu^2}.
\]

Since \( \mu'(R) < 0 \) and \( R < f'(K(R)) \) for all \( R \in (0, \beta) \), we have that \( \partial B/\partial R > 0 \) for all \( R \in (0, \beta) \).

(v) Using the formulas for \( \mu(R) \) and \( \phi(R) \) from above, we get

\[
\frac{\partial B}{\partial \sigma} = \frac{\partial}{\partial \sigma} \left( \frac{B}{K} \right) = \frac{\partial}{\partial \sigma} \left( \phi^{-1} - 1 - (1 - \alpha) K^{\alpha - 1} R^{-2} \right) = -\phi^{-2} \phi_\sigma - \frac{1 - \alpha}{\alpha} R^{-1} \mu_\sigma,
\]

30
where \( \phi_\tilde{\sigma} = -\frac{1}{\sigma^2} (\frac{2\theta(\beta-R)}{\gamma(1+\gamma)})^{1/2} \) and \( \mu_\tilde{\sigma} = (\frac{2\theta(\beta-R)}{\gamma(1+\gamma)})^{1/2} \). Substituting this into \( \frac{\partial B}{\partial \tilde{\sigma}} > 0 \) yields

\[
R > \frac{2\theta\beta(1-\alpha)}{\alpha + \theta(2-\alpha)} \equiv \bar{R} < \beta .
\]

**Proof that the first part of Assumption 1 implies its second part.** Using the definitions of \( \hat{R} \) and \( \bar{R} \), we get

\[
\hat{R} < R \iff \tilde{\sigma} < \frac{2\alpha\beta(1+\theta)}{\theta\gamma(\alpha + \theta(2-\alpha))} .
\]

In this region of interest rates, \( K_R > 0 \), and therefore \( \phi < \theta/(1+\theta) \). Next, let \( f(K) = K^\alpha \), \( \hat{f}(K) = \delta K \), and \( s = \delta K / \hat{f} \). From (19) in autarchic bond market clearing, we have that

\[
\frac{1 - \phi}{\phi} = \frac{H}{K} = \frac{\omega}{RK} = \frac{f(K) - f'(K)K}{RK} > \frac{f/K - f'}{f'},
\]

and therefore

\[
\phi < \frac{\hat{f}'K/\hat{f} - \delta K/\hat{f}}{1 - \delta K/\hat{f}} = \frac{\alpha - s}{1 - s} .
\]

For \( \tilde{\sigma} \) very small, \( \phi \simeq \frac{\alpha - s}{1 - s} \), which implies that \( K_R > 0 \iff \frac{\alpha - s}{1 - s} < \frac{\theta}{1+\theta} \).

**Proof of Proposition 2.** (i) This part follows from the proof of Lemma 2, part (iii). The limits of \( B(R) \), together with the continuity of \( B(R) \) in \( R \), establish the existence of an \( R \) that solves \( B(R) = 0 \). This is in fact the unique steady-state \( R \), since \( B_R > 0 \) always.

(ii) The equation \( B(R^\text{aut}_1, \tilde{\sigma}_j) = 0 \) is simply bond market clearing for each country. Under Assumption 1, we are in the region where \( B_\tilde{\sigma} > 0 \). From (1) we have that \( B = B/K \equiv D \). Using a proof similar to that in Proposition 1(iv), we get that \( D_R < 0 \). Hence, \( B_R < 0 \). We also have that \( B_{\tilde{\sigma}} = B_R R_{\tilde{\sigma}} > 0 \), with \( B_R < 0 \). Therefore, it has to be that \( R_{\tilde{\sigma}} < 0 \) in autarchy. In other words, \( R^\text{aut}_1 > R^\text{aut}_2 \).

(iii) Under Assumption 1, we are in the region where \( K_R > 0 \). Hence, the fact that \( R^\text{aut}_1 > R^\text{aut}_2 \) implies that \( K^\text{aut}_1 > K^\text{aut}_2 > 0 \). Since consumption is increasing in capital, we also have that \( C^\text{aut}_1 > C^\text{aut}_2 \).

**Proof of Proposition 3.** (i) Consider the function \( WB(R) \) defined by

\[
WB(R) \equiv B(R, \tilde{\sigma}_1)K(R, \tilde{\sigma}_1) + B(R, \tilde{\sigma}_2)K(R, \tilde{\sigma}_2) .
\]

An integrated steady state is given by any solution to \( WB(R) = 0 \). Note that the function \( K \) is always positively valued, while the function \( B \) can take both signs and is increasing in \( R \) and \( \tilde{\sigma} \).
Furthermore, recall that $R_{2}^{out} < R_{1}^{out}$. Whenever $R \leq R_{2}^{out} (< R_{1}^{out})$, by the monotonicity of $B$ in $R$ we have that $B(R, \tilde{\sigma}_{2}) \leq B(R_{2}^{out}, \tilde{\sigma}_{2}) = 0$ and $B(R, \tilde{\sigma}_{1}) < B(R_{2}^{out}, \tilde{\sigma}_{2}) = 0$; it follows that $WB(R) < 0$. Similarly, whenever $R \geq R_{1}^{out}$, we have that $WB(R) > 0$. Along with the fact that the function $WB(R)$ is continuous in $R$, this implies that a solution $R^{int}$ to $WB(R) = 0$ always exists and it necessarily satisfies $R_{2}^{out} < R^{int} < R_{1}^{out}$.

(ii) Since $K_{\tilde{\sigma}} < 0$, it follows that $K_{1}^{int} > K_{2}^{int}$. Since Assumption 1 ensures that $K_{R} > 0$, and using (i), we get the desired result.

(iii) Under Assumption 1, we are in the area where $B_{\tilde{\sigma}} > 0$, which implies that $B_{1}^{int} < B_{2}^{int}$, and since the world bond market has to clear, this means that $B_{1}^{int} < 0 < B_{2}^{int}$.

(iv) This part follows directly from parts (ii) and (iii).
References


Figure 1: Steady state. This figure illustrates the determination of the autarchic and integrated steady states. The interest rate is on the horizontal axis, the net foreign asset position of a country is on the vertical axis. The solid line is the function $B(R)$ for the North. The dashed line is the function $B(R)$ for the South. The intersection of these curves with the zero line gives the autarchic interest rates, where $R_{2}^{\text{aut}} < R_{1}^{\text{aut}}$. The integrated interest rate, $R^{\text{int}}$, falls in between the two autarchic values.
Figure 2: South’s dynamic adjustment to financial integration. This figure illustrates the transition of the South from its autarchic steady state to the integrated one. Time in years is on the horizontal axis. Integration occurs at time zero. The dotted line indicates the value of the variables in the autarchic steady state. The dashed line indicates the value of the variables in the integrated steady state. The solid line indicates the dynamic path of the variables. Capital, output, consumption, and the wage are normalized by the corresponding autarchy values of the North. The net foreign asset position is given as a fraction of contemporaneous GDP.
Figure 3: North’s dynamic adjustment to financial integration. This figure illustrates the transition of the North from its autarchic steady state to the integrated one. Time in years is on the horizontal axis. Integration occurs at time zero. The dotted line indicates the value of the variables in the autarchic steady state. The dashed line indicates the value of the variables in the integrated steady state. The solid line indicates the dynamic path of the variables. Capital, output, consumption, and the wage are normalized by their corresponding autarchy values. The net foreign asset position is given as a fraction of contemporaneous GDP.
Figure 4: Welfare effects. This figure illustrates the welfare effects of financial integration across different wealth levels. The horizontal axis measures wealth in terms of percentiles in the autarchic steady-state distributions. The vertical axis measures the welfare gains (positive numbers) or losses (negative numbers), evaluated as percent of individual permanent income. The solid line represents the welfare effects for the current generation (the “short-run” effects that obtain by evaluating welfare at the time of reform); the dashed line represents the welfare effects for future generations (the “long-run” effects that obtain from comparing welfare across the two steady states).
Figure 5: TFP dynamics. This figure illustrates the dynamic adjustment of TFP to financial integration, within the context of the extended model. Time in years is on the horizontal axis. Integration occurs at time zero. The TFP of each country, normalized by its corresponding value at the integrated steady state, is on the vertical axis.