The Contribution of Striations to the Eddy Energy Budget and Mixing: Diagnostic Frameworks and Results in a Quasigeostrophic Barotropic System with Mean Flow

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ABSTRACT

Low-frequency oceanic motions have banded structures termed “striations.” Since these striations embedded in large-scale gyre flows can have large amplitudes, the authors investigated the effect of mean flow on their directions as well as their contribution to energetics and mixing using a β-plane, barotropic, quasigeostrophic ocean model. In spite of the model simplicity, striations are always found to exist regardless of the imposed barotropic mean flow. However, their properties are sensitive to the mean flow. Rhines jets move with the mean flow and are not necessarily striations. If the meridional component of the mean flow is large, Rhines jets become high-frequency motions; low-frequency striations still exist, but they are nonzonal, have small magnitudes, and contribute little to energetics and mixing. Otherwise, striations are zonal, dominated by Rhines jets, and contribute significantly to energetics and mixing. This study extends the theory of β-plane, barotropic turbulence, driven by white noise forcing at small scales, to include the effect of a constant mean flow. Theories developed in this study, based upon the Galilean invariance property, illustrate that the barotropic mean flow has no effect on total mixing rates, but does affect the energy cascades in the frequency domain. Diagnostic frameworks developed here can be useful to quantify the striations’ contribution to energetics and mixing in the ocean and more realistic models. A novel diagnostic formula is applied to estimating eddy diffusivities.

1. Introduction

Geostrophic eddies, which are ubiquitous in the global ocean, dominate the oceanic kinetic energy reservoir (Ferrari and Wunsch 2009). As a key component of the oceanic circulation, they stir and mix tracers (e.g., heat and potential vorticity) and therefore greatly influence the oceanic circulation and climate variability. Given their importance, much effort has been made recently toward estimating and interpreting eddy energy budgets (e.g., Ferrari and Wunsch 2009; Xu et al. 2011; Chen et al. 2014a) and eddy mixing rates (e.g., Gille et al. 2007; Griesel et al. 2010; Rypina et al. 2012; Klocker et al. 2012a,b; Abernathey et al. 2013; Chen et al. 2014b; Chen et al. 2015b) in the global ocean.

The temporal average (i.e., low-frequency component) of eddy motions has banded structures in most of the ocean that are often termed “striations” (e.g., Cox 1987; Galperin et al. 2004; Maximenko et al. 2005; Nakano and Hasumi 2005; Richards et al. 2006; Maximenko et al. 2008; Chen 2013). Spectral analysis of the output from an eddying global model suggests that these striations account for more than 10% of the zonal velocity variability in the upper 1000 m of the central North Pacific and east North Pacific regions (Chen et al. 2015a). This significant percentage suggests that striations are an important component of the oceanic eddy field and that their contribution to eddy energetics and tracer mixing is probably nonnegligible. Yet, while their origin has received much attention, little effort has been made toward quantifying these contributions (e.g., Maximenko et al. 2008; Schlax and Chelton 2008; Afanasyev et al. 2012; Chen et al. 2015a).

In addition, striations are also embedded in large-scale oceanic background flows, such as subtropical and
subpolar gyres. How these background flows influence the mixing properties and energetics of striations is not known. Exploring these questions will help us further understand the consequences of eddies in the ocean and their impact on the climate system. The focus of this study is to assess the contribution of striations to eddy energy budgets and mixing in a barotropic quasigeostrophic (QG) ocean model on a β plane with a background mean flow imposed.

Use of the barotropic QG model, which is one of the simplest systems that produces banded structures, allows us to focus on both the elementary dynamics and consequences of striations. In this model, Rhines jets can arise as a result of the dramatic slowing of the inverse cascade due to the β effect (e.g., Rhines 1975; Sukoriansky et al. 2007). Recent studies have demonstrated that this model, though highly idealized, is a useful tool for the theoretical exploration of banded structures and many related frontiers (e.g., Danilov and Gurarie 2004; Danilov and Gryanik 2004; Farrell and Ioannou 2007; Srinivasan and Young 2012). Indeed, this study shows that striations in this simple system have rich behaviors and that their effects on mixing and energy budgets are sensitive to the imposed barotropic mean flow. The simplicity of the model allows ease of interpretation of these behaviors. We also develop theories relevant to the ocean about the effect of barotropic mean flows on eddy mixing and energy cascades in the frequency domain.

This study presents a computational/conceptual framework to diagnose the contributions that striations have on mixing and energetics. The computational efficiency and ease of interpretation of the barotropic model make it a good tool to test the validity of the computational/conceptual framework. It is important to reveal the eddy dynamics and thereby allow for the improvement of eddy parameterization schemes. As summarized by Abernathey et al. (2013), many methods have been used to diagnose eddy mixing rates, such as flux-gradient diffusivities, Nakamura effective diffusivities, and Lagrangian diffusivities. These methods essentially estimate mixing rate contributions from all of the eddies. However, partitioning of the contributions to mixing from striations and other eddies has rarely been explored. We present a new diagnostic framework for this partitioning, based on special tracers with eddy velocity as their initial condition. Thompson and Richards (2011) presents a diagnostic framework to assess the energetics of jets in the Southern Ocean by assuming there is a spectral gap between low-frequency and high-frequency eddies. This study presents a framework to diagnose the energy budget of (time dependent) striations without the need for the spectral gap assumption. Our diagnostic methods can be easily extended to investigating realistic striations.

In summary, this paper 1) develops a diagnostic framework to evaluate the contributions of striations to mixing and the energy budget and 2) examines the elementary dynamics of striations and the effect of mean flow on them using a barotropic QG model on a β plane. Section 2 describes the model and experimental setup. Section 3 characterizes and interprets striations in the model from a spectral perspective. Sections 4 and 5 present the key results: the effect of mean flow on the contribution of striations to mixing and energy budgets. Corresponding diagnostic frameworks are described in detail in these two sections. Section 6 provides the summary and discusses applications of our work to more realistic fluid systems.

2. Numerical model and experimental setup

We use the barotropic QG equation

\[
\left( \frac{\partial}{\partial t} + \mathbf{U} \frac{\partial}{\partial x} + \mathbf{V} \frac{\partial}{\partial y} \right) \mathbf{\hat{q}} + J(\psi, \mathbf{\hat{q}} + \beta y) = \mathcal{F}(x, y, t) - r \mathbf{\hat{q}},
\]

where \((\mathbf{U}, \mathbf{V})\) is the imposed mean flow, \(\psi\) denotes the eddy streamfunction, \(\mathbf{\hat{q}}\) is the relative eddy vorticity \(\nabla^2 \psi\), \(J\) is the Jacobian operator, \(\beta\) is the meridional gradient of the Coriolis parameter, \(\mathcal{F}(x, y, t)\) is the external forcing, and \(r\) is the friction coefficient. The equation is solved using the pseudospectral method in a doubly periodic domain (Arbic 2000). The domain is discretized at 128 × 128 grid points and its size is 3000 km × 3000 km.

As described in section 1, one of our main goals is to investigate how the background flow \((\mathbf{U}, \mathbf{V})\) influences the striation properties. Though the background flow varies spatially in the ocean, here we only consider the case when \((\mathbf{U}, \mathbf{V})\) are constants. The reasons are as follows. First, in some oceanic regions (e.g., the subtropical gyre away from topography and the oceanic boundary), the background flow varies slowly in space. Thus, it can be assumed to be constant, if we examine striations in a small oceanic patch. Second, this choice reduces the model [Eq. (1)] to the simplest system and thus allows us to focus on the basic physics about the mean flow effect on striations. Third, the constant mean flow assumption has been widely used in previous studies about striations, eddies, energetics, and mixing, leading to results/conclusions relevant to the ocean, at least to some extent (e.g., Arbic 2000; Killworth 1997; Flierl and Pedlosky 2007; Smith 2007; Thompson 2010;
Our study includes three main experiments: Exp1 has no mean flow imposed and represents the original case presented in Rhines (1975). Exp2 has a southward mean flow imposed, and Exp3 has eastward mean flow imposed. The choice of southward and eastward mean flow in Exp2 and Exp3 is based on the fact that both quasi-zonal and quasi-meridional background flow are prevalent in the ocean (e.g., in the subtropical gyre). The magnitude of time–mean barotropic velocity in the eddying ocean model, the ECCO2 state estimate, varies spatially, ranging from $O(0.01)$ m s$^{-1}$ in quiescent regions to $O(0.1)$ m s$^{-1}$ in coastal regions and in the Antarctic Circumpolar Current. Within the relevant parameter range, we arbitrarily choose the magnitude of the imposed mean flow in Exp2 and Exp3 to be $0.03$ m s$^{-1}$. We also carried out sensitivity experiments with a range of mean flow magnitudes, to test our physical interpretations.

The external forcing $F(x, y, t)$ is white noise in time and has a narrow banded wavenumber spectrum. The external forcing at time step $n\Delta t$ is

$$F_n = \frac{\hat{A} F^{-1}\{\exp[-0.01(\{K^2 - K_E^2\})^2 + i\theta_n(k, l)]\}}{\max(F^{-1}\{\exp[-0.01(\{K^2 - K_E^2\})^2 + i\theta_n(k, l)]\}),}$$

(2)

where $\hat{A}$ is the forcing amplitude, $F^{-1}$ is the inverse Fourier transform operator, $K = ki + lj$ is the wavenumber vector, $K_E = 7 \times 10^{-3}$ cycle km$^{-1}$, and $\theta_n(k, l)$ are uniformly distributed random numbers between 0 and $2\pi$. In the wind-driven barotropic model, $F(x, y, t)$ is essentially equal to $\text{curl}[(\tau/\rho_0)H]$, where $\tau$ is the wind stress vector (Pedlosky 1987). Typical values of the ocean depth $H$, density $\rho_0$, and the instantaneous curl($\tau$) are $1000$ m, $1000$ kg m$^{-3}$, and $2 \times 10^{-8}$ Pa m$^{-1}$, respectively. Therefore, we choose $\hat{A}$ to be $2 \times 10^{-12}$ s$^{-2}$. The friction coefficient $r$ is $2.9 \times 10^{-10}$ s$^{-1}$, leading to nonlinear motions with eddy velocity magnitudes of $0.03$ m s$^{-1}$. The value $0.03$ m s$^{-1}$ falls within the range of the zonally averaged magnitude of the barotropic eddy velocity in the ECCO2 state estimate ($0.05$ m s$^{-1}$). We choose $\beta$ to be $2 \times 10^{-11}$ m$^{-1}$ s$^{-1}$, which is representative of midlatitude values. All of the experiments start from rest, and our analysis uses the 200-yr output from the statistical equilibrium state.

3. Rhines jets versus striations

a. Descriptions of Rhines jets

In a barotropic QG model on a $\beta$ plane with no mean flow imposed, elongated structures exist in the instantaneous eddy snapshots (Rhines 1975). We define such banded structures in the eddy snapshots as “Rhines jets.” We chose not to use the terminology “zonal jets,” because zonal jets in literature can refer to either bands in the flow snapshot (e.g., Panetta 1993) or bands in the temporally averaged flow field (e.g., Maximenko et al. 2005; Richards et al. 2010). Rhines jets show up in the instantaneous eddy snapshots in all of our experiments and they are all elongated in roughly the zonal direction (Fig. 1). However, the Rhines jets in Exp2 are advected southward by the imposed southward mean flow (Fig. 2).

The averaged width of a single jet, which is either eastward or westward, can be determined by counting the number of jets in the spatial domain (Thompson 2010). We examined the zonal averages of zonal eddy velocities and found that the number of jets is 12 in all of our experiments. Therefore, the width of a single jet is $250$ km, which is roughly consistent with the Rhines scale ($230$ km). In this study, our definition of the Rhines scale, $l_{\text{Rhines}}$, follows Thompson (2010):

$$l_{\text{Rhines}} = 2\pi \sqrt{V_e/\beta},$$

(3)

where $V_e$ is the temporal and spatial average of eddy velocity magnitudes. In our experiments, the dominant meridional wavenumber of Rhines jets, $2\pi/(2l_{\text{Rhines}})$, is also consistent with the meridional wavenumber centroid of the eddy kinetic energy (EKE) spectra.

Rhines jets contain 86% of total EKE in our experiments. This percentage estimation is based on the approach from Chen et al. (2015a), which is briefly described next. The normalized wavenumber spectrum of EKE, $S_{\text{EKE}}^N$, is defined as

$$S_{\text{EKE}}^N = \frac{(k^2 + l^2)S_k(k, l)}{[2(k^2 + l^2)S_k(k, l)]_{\text{max}}.}$$

(4)

Here $k$ and $l$ are zonal and meridional wavenumbers, $S_k(k, l)$ is the wavenumber spectrum of eddy streamfunction $\psi$, and $[\ ]_{\text{max}}$ denotes the maximum value over the entire available wavenumber space $(k, l)$. We identify an optimum ellipse, which is the smallest one for which all the wavenumbers with $S_{\text{EKE}}^N$ larger than 0.2 are inside. As shown in Fig. 3, the optimum ellipse is very narrow and has its major axis aligning with $k = 0$, which is consistent with the fact that zonal elongated structures dominate the flow field. The percentage of EKE associated with Rhines jets is

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1. ECCO2 refers to “Estimating the Circulation and Climate of the Ocean, phase II: High resolution global-ocean and sea-ice data synthesis” (Menemenlis et al. 2008; Chen 2013).
percent \[ R \] denotes the wavenumber space inside the optimum ellipse and \[ \mathcal{V} \] denotes the entire available wavenumber space. More details of this approach are available in Chen et al. (2015a).

b. Effect of mean flow on Rhines jets

The formation mechanism of Rhines jets has not yet been fully resolved. Rhines (1975) examined the barotropic decaying turbulence and proposed that Rhines jets can be formed from the arrest of inverse cascade on a \( \beta \) plane. Sukoriansky et al. (2007) found that, in the forced barotropic flow on a \( \beta \) plane, the inverse cascade cannot be arrested by \( \beta \), but can be ended by large-scale dissipation. Other proposed mechanisms include, but are not limited to, the mixing of potential vorticity by eddies (Dritschel and McIntyre 2008), zonostrophic instability (Srinivasan and Young 2012), and the shearing of the eddies by the local shear (Bakas and Ioannou 2013). Our discussion next focuses on the effect of mean flow on jet characteristics, rather than mechanisms for the jet origin.

The insensitivity of the existence, width, and zonal orientation of the Rhines jets to the imposed mean flow is illustrated in FIG. 2. The figure uses the zonally averaged eddy streamfunction as an approximate indicator of Rhines jets. Rhines jets are advected southward by the imposed southward mean flow in Exp2. Meridional drift of the jets does not appear over the 500-day time scale in Exp1 and Exp3 because of the absence of a meridional component of the imposed mean flow.
(Fig. 1) is due to the Galilean invariance of the barotropic model. Ignoring forcing and friction in Eq. (1), we obtain

$$\frac{\partial}{\partial t} U + \frac{\partial}{\partial x} U \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \frac{\partial}{\partial y} - \frac{\partial}{\partial y} \frac{\partial}{\partial x} + \beta \frac{\partial}{\partial x} = 0.$$  \( \tag{6} \)

Equation (6) reduces to the original Rhines jet model from Rhines (1975), in a moving coordinate system

$$x' = x - Ut, \quad y' = y - Vt \quad \text{and} \quad t' = t.$$  

In that system, freely evolving eddy characteristics would be independent of the imposed mean flow: the dominant eddies become "zonal" when the energy cascades to low "wavenumbers" and low "frequencies" and Rhines jets are generated and align with the \( x' \) (i.e., zonal) direction (note: here the wavenumber and frequency are defined in the moving coordinates).

Even though the imposed white noise forcing is not Galilean invariant, the above mechanism is still a good approximation, since the forcing is of small spatial scale and does not end up affecting the basic statistics of larger-scale eddies. Basic eddy statistics differ little in our experiments, indicating Galilean invariance holds reasonably here. The difference of the domain-averaged EKE among the three experiments is only 0.3% of the EKE magnitude. The difference of the domain-averaged vorticity magnitude among the three experiments is only 0.5% of the vorticity magnitude. The meridional wavenumber centroid of the EKE wavenumber spectra ranges from 0.0020 to 0.0021 cycle km\(^{-1}\) in our experiments, and the corresponding zonal wavenumber centroid ranges from \( 3.0 \times 10^{-4} \) to \( 3.3 \times 10^{-4} \) cycle km\(^{-1}\).

c. Effect of mean flow on frequency–wavenumber spectra

To prepare for the discussion of the dynamics and consequences of the striations, we first examine the spectral characteristics of eddies. Define \( S_A(k, l, \omega) \) as the frequency–wavenumber spectra for any arbitrary variable \( A(x, y, t) \),

$$S_A(k, l, \omega) = \langle |\hat{A}(k, l, \omega)|^2 \rangle.$$  \( \tag{7} \)

Here \( \langle \cdot \rangle \) denotes the ensemble average and \( \hat{\cdot} \) denotes the Fourier transform over the three-dimensional space \( x, y, t \). Define \( \phi_A(x, y, t) \) as the solution in Exp1, where no mean flow is imposed. Because of the Galilean invariance, the function

$$\phi_B(x, y, t) = \phi_A(x - Ut, y - Vt, t)$$  \( \tag{8} \)

describes eddy statistics for cases with an imposed mean flow (e.g., Exp2 and Exp3). Equation (8) leads to

$$S_{\phi_B}(k, l, \omega) = \langle |\hat{\phi_B}(k, l, \omega)|^2 \rangle = \langle |\hat{\phi_A}(k, l, \omega - kU - lV)|^2 \rangle = S_{\phi_A}(k, l, \omega - kU - lV),$$  \( \tag{9} \)

which is confirmed by the comparison of spectra from Exp1, Exp2, and Exp3 (not shown). Therefore, the spectra in a case with mean flow are the frequency-shifted versions of those in the case with no mean flow. This implies that the wavenumber spectra of eddies do not depend on the imposed spatially uniform mean flow, as
d. Effect of mean flow on striation amplitudes

As summarized in section 1, striations in literature often refer to banded structures in the temporal average of oceanic fields. Note that the temporal average retains only low-frequency motions. Following Chen et al. (2015a), we define striations as banded structures in low-frequency eddies to be consistent with previous studies. One natural question is how to determine the separation frequency between high- and low-frequency motions ($\Omega_S$). To quantify the amplitude of striations in the North Pacific, Chen et al. (2015a) proposed an “optimum ellipse” approach to separate low-frequency eddies from high-frequency eddies. First, we define the normalized wavenumber spectrum of the eddy streamfunction $\psi$ at frequency $\omega$ as

$$S_N^\psi(k, l, \omega) = \frac{S_\psi(k, l, \omega)}{[S_\psi(k, l, \omega)]_{\text{max}}},$$  

(11)

where $S_\psi(k, l, \omega)$ is the frequency–wavenumber spectrum of $\psi$ and $[S_\psi(k, l, \omega)]_{\text{max}}$ is the maximum value of $S_\psi(k, l, \omega)$ at frequency $\omega$. Second, for each frequency, we can determine an optimum ellipse, which denotes the smallest ellipse containing all the wavenumbers with $S_\psi(k, l, \omega)$ larger than 0.2. Finally, we diagnose the ratio between the major and minor axes of this ellipse. If the ratio between the major and minor axes is larger than three, the ellipse is considered narrow, the wavenumber spectrum at this frequency is very anisotropic, and banded structures dominate. We define $\Omega_S$ as the highest frequency where all the optimum ellipses at frequencies lower than $\Omega_S$ are narrow. Based on this approach, $\Omega_S$ is 0.14, 0.4, and 0.14 cycle yr$^{-1}$ in Exp1, Exp2, and Exp3, respectively.

Striations exist in all three of our experiments, and they are not necessarily dominated by Rhines jets (Fig. 4). In Exp2, where the imposed mean flow is in the meridional direction, the striations are tilted south-westward with amplitudes that are much weaker than that of Rhines jets (Fig. 1). In Exp1 and Exp3, the direction, width, and magnitude of the striations are roughly the same as Rhines jets (Fig. 1).

Rhines jets contain most of the eddy energy in this turbulent system (section 3a). Whether Rhines jets can be classified as striations depends upon the Rhines jet frequency, $\omega_{\text{Rhines}}$. The spatial structure of Rhines jets, and thus their dominant wavenumbers, are insensitive to the imposed mean flow (sections 3a and 3b). In our experiments, the dominant zonal wavenumber of Rhines jet is approximately zero and the dominant meridional wavenumber of Rhines jets is $2\pi/(2l_{\text{Rhines}})$, where $l_{\text{Rhines}}$ is the Rhines scale for a single eastward or westward jet, defined in Eq. (3). In the case where no mean flow was imposed (Exp1), $\omega_{\text{Rhines}}$ is roughly zero because of the inverse cascade (Rhines 1975). Based on Eq. (9) for cases with mean flow (e.g., Exp2 and Exp3), the Rhines frequency is

$$\omega_{\text{Rhines}} = \mathbf{U} \ast 0 + \nabla \ast 2\pi/(2l_{\text{Rhines}}) = \nabla \pi/l_{\text{Rhines}}. \quad (12)$$

If the separation frequency between striations and high-frequency eddies ($\Omega_S$) satisfies
Rhines jets are low-frequency motions and striations will be dominated by them and will have large amplitudes. For example, in Exp3, $V_S$ is zero so that $v_{Rhines}$ is zero and striations have large amplitudes. On the other hand, if Eq. (13) does not hold (e.g., Exp2), striations are not composed of Rhines jets and they have lower amplitudes and are less energetic.

Figure 5 further illustrates the dependence of striation amplitudes on $\Omega_S/\omega_{Rhines}$. The percentage of total EKE that is contained in striations is

$$\text{percent}_S = \frac{\int_{-\Omega_S}^{\Omega_S} \int_{-\infty}^{\infty} S_{EKE}(k, l, \omega) dk dl d\omega}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{EKE}(k, l, \omega) dk dl d\omega},$$

(14)

where $S_{EKE}(k, l, \omega)$ is the EKE spectra, that is,

$$S_{EKE}(k, l, \omega) = (k^2 + l^2)S_\theta(k, l, \omega).$$

(15)

As shown in Fig. 5, as $\Omega_S$ increases or as $V_S$ decreases, $\Omega_S/\omega_{Rhines}$ increases and a sharp increase of percent$_S$ occurs at

$$\Omega_S > \omega_{Rhines} = \frac{\nabla \pi l_{Rhines}}{\sqrt{\beta}},$$

(13)

$$\omega_{Rhines} = \frac{\nabla \pi l_{Rhines}}{\sqrt{\beta}} = \Omega_S,$$

(16)
Here $\epsilon$ is 0.2 cycle yr$^{-1}$ and $\Omega_{\text{Rossby}}$ is the Rossby wave frequency. These eddies can be viewed as a set of weakly interacting Rossby waves.

Striations are dominated by a group of quasi-stationary Rossby waves. Because most of the energy lies near the Rossby wave dispersion relation, and striations are defined as low-frequency motions, wave-numbers with large amplitudes in the striation spectra lie near the zero Rossby wave frequency curve, defined as (Fig. 7)

$$
\Omega_{\text{Rossby}} = \frac{U}{k} + \frac{V}{l} - \frac{\beta}{k^2 + l^2} = 0.
$$

In the case with $V \neq 0$ (Exp2), the zero frequency Rossby wave curve is shifted from $k = 0$ and striations are nonzonal. Conversely, in the case with $V = 0$ (Exp1 and Exp3), the zero frequency Rossby wave curve occurs at $k = 0$ and striations are zonal.

### 4. Effect of mean flow on energy cascades and striations’ energy budget

#### a. Diagnostic approach

The domain-averaged kinetic energy density from low-frequency motions is

$$
\langle KE_{\Omega} \rangle = \langle \frac{1}{2} \langle \nabla \psi_{\Omega} \cdot \nabla \psi_{\Omega} \rangle \rangle_{\Omega},
$$

where $\langle \cdot \rangle$ is the spatial average and $\langle \cdot \rangle_{\Omega}$ denotes the part of the motion at frequencies lower than $\Omega$. For example, $\psi_{\Omega}$ is defined as

$$
\psi_{\Omega}(x, y, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi(k, l, \omega) e^{i(kx + ly - \omega t)} H(\omega, \Omega) \, dk \, dl \, d\omega,
$$

where $H(\omega, \Omega)$ is an arbitrary low-pass filter operator, such as the boxcar filter:

$$
H(\omega, \Omega) = \frac{\sin(\pi \omega / \Omega)}{\pi \omega / \Omega}.
$$

The $\langle KE_{\Omega} \rangle$ budget equation in our doubly periodic barotropic model is

$$
\frac{\partial}{\partial t} \langle KE_{\Omega} \rangle = \underbrace{\langle \psi_{\Omega} [J(\psi, \nabla^2 \psi)]_{\Omega} \rangle}_{\Pi_{\Omega}} + \underbrace{\langle -\psi_{\Omega} [F(x, y, t)]_{\Omega} \rangle}_{F_{\Omega}} + \underbrace{\langle -r KE_{\Omega} \rangle}_{D_{\Omega}},
$$

where $\Pi_{\Omega}$ denotes the energy flux to the $KE_{\Omega}$ reservoir due to eddy–eddy interaction, $F_{\Omega}$ denotes the energy input into the $KE_{\Omega}$ reservoir from external forcing, and $D_{\Omega}$ denotes the dissipation of $KE_{\Omega}$ due to friction. The term on the left-hand side represents the temporal rate of change of $KE_{\Omega}$. The mean flow $(U, V)$ does not explicitly enter into Eq. (21) because of the doubly periodic boundary condition. The derivation of Eq. (21) is provided in appendix A.

The low-frequency motions are dominated by banded structures (Fig. 4). Assuming all the low-frequency motions
are striations, and replacing $\Omega$ in Eq. (21) with $\Omega_S$, we obtain the energy budget equation for the striations:

$$\frac{\partial}{\partial t} \langle \Pi_{\Omega_S} \rangle = \langle \mathcal{F}_{\Omega_S} \rangle + \langle D_{\Omega_S} \rangle. \tag{22}$$

Note that any type of low-pass filter $H(\omega, \Omega)$ can be used when diagnosing Eqs. (21) and (22).

### b. Comparison with previous diagnostic approaches

Our diagnostic framework is distinct from others presented previously. For example, by assuming a spectral gap exists between low-frequency and high-frequency motions, Thompson and Richards (2011) developed an approximate energy budget equation for studying the low-frequency jets in the Southern Ocean. However, a spectral gap assumption is not used in our diagnostic framework for the energetics for the striations [i.e., Eq. (22)].

Nonlinear energy cascades in the frequency domain have recently been discussed in a two-layer model context (Arbic et al. 2012). However, their energy cascade [Eq. (21) in Arbic et al. (2012)] does not include the local time rate of change term ($\frac{\partial}{\partial t} \langle \Pi_{\Omega} \rangle$, which is needed to study the temporal (e.g., seasonal and interannual) variability of striations. The one-layer version of their Eq. (21) can be shown to be essentially the time average, over the entire time series, of our Eq. (21) with a rectangular filter. Averaging Eq. (21) over the entire time series leads to

$$\frac{\partial}{\partial t} \langle \Pi_{\Omega} \rangle = \langle \mathcal{F}_{\Omega} \rangle + \langle D_{\Omega} \rangle = 0. \tag{23}$$

Appendix A illustrates that the time mean of the nonlinear energy fluxes ($\langle \Pi_{\Omega} \rangle$), which represent energy cascades in the frequency domain, can be expressed as an integral over the frequency and wavenumber domain:

$$\langle \Pi_{\Omega}(x, y, t) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{\psi}(k, l, \omega) \tilde{H}(\omega, \Omega) \tilde{J}(-k, -l, -\omega) dk \, dl \, d\omega, \tag{24}$$

where $\tilde{J}$ is the Fourier transform of $J(\psi, \nabla^2 \psi)$. The barotropic equivalent of the nonlinear energy fluxes in Arbic et al. (2012) is

$$\langle \Pi_{\Omega}(x, y, t) \rangle_{\text{Arbic}} = \int_{|\omega|<\Omega} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{\psi}(k, l, \omega) \tilde{J}(-k, -l, -\omega) dk \, dl \, d\omega. \tag{25}$$

Therefore, $\langle \Pi_{\Omega}(x, y, t) \rangle$ in their paper matches ours when one uses a rectangular filter, while ours is more general in that it allows for the use of any type of filter $H$.

### c. Results

#### 1) Effect of mean flow on energy cascades in the frequency domain

Although the energy cascade in the wavenumber domain is not sensitive to the mean flow in the barotropic model, the energy cascade in the frequency domain is [section 5.8.2 in Chen (2013)]. Appendix B
Fig. 8. The normalized integral energy budget in the frequency domain [Eq. (23)] in each experiment. Red, green, and black curves denote \((\Pi_{\Omega}(x,y,t))\), \((\mathcal{F}_{\Omega}(x,y,t))\), and \((\mathcal{D}_{\Omega}(x,y,t))\), respectively. Black boxes are used to highlight the magnitude of each term at the frequency \(\Omega_s\). The shaded area in light red is within the frequency range of striations \([0, \Omega_s]\). Blue dashed lines mark \(\Omega = \Omega_s\), and \(\Omega_s\) is the same as that in Fig. 4. In (b), the magenta dashed line denotes \(\Omega = \omega_{\text{Rhines}}\) in Exp2 and the red dashed line denotes \(\Omega = \Omega_M\), where \((\Pi_{\Omega})\) reaches the maximum. The low-pass filter \(H(\omega, \Omega)\) we chose in this diagnosis is the Butterworth filter.

illustrates that, using the Galilean invariance and the convolution theorem, nonlinear energy fluxes in the case with any arbitrary mean flow imposed, \([\Pi_{\Omega}(x,y,t)]\), can be calculated from the eddy streamfunction for the case with no mean flow imposed \([\phi_A(x,y,t)]\). 

\[
\langle \Pi_{\Omega}(x,y,t) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_A(k,l,\omega) \hat{\phi}_A(-k,-l,-\omega) H^2(\omega + k\mathbf{U} + l\mathbf{V}, \Omega) \, dk \, dl \, d\omega,
\]

where \(J_A = J(\phi_A, \nabla^2 \phi_A)\). Obviously, because the low-pass filter \(H\) depends on the mean flow, so does \(\langle \Pi_{\Omega}(x,y,t) \rangle\).

The sensitivity of \((\Pi_{\Omega})\) to the imposed mean flow, predicted by Eq. (26), can be interpreted as follows. In the \(\beta\)-plane barotropic QG system with no mean flow imposed, energy cascades to low frequencies and low wavenumbers; Rhines jets are low-frequency motions that extract energy from high-frequency motions (Rhines 1975). Using the Galilean invariance of the barotropic system, we know that Rhines jets extract energy from other eddy motions regardless of the imposed mean flow. However, because the frequency of Rhines jets [Eq. (12)] is sensitive to the imposed meridional flow, the structure of \((\Pi_{\Omega})\) is also sensitive to the imposed meridional flow, as confirmed through numerical analysis and presented in Fig. 8.

In Fig. 8, a positive (negative) slope of \((\Pi_{\Omega})\) at the frequency \(\Omega\) means that eddies at that frequency gain (lose) energy through eddy–eddy interaction. In Exp1 and Exp3, the slope of \((\Pi_{\Omega})\) is negative at most frequencies higher than \(\Omega_S\), which denotes the separation frequency between striations and high-frequency eddies. Therefore, energy primarily flows to motions with frequencies lower than \(\Omega_S\). However, in Exp2 over the frequency range \([\Omega_S, \Omega_M]\), \((\Pi_{\Omega})\) decreases from its maximum value at \(\Omega_M\) to roughly zero at \(\Omega_S\) (Fig. 8).

Therefore, in this case energy primarily flows to the frequency range \([\Omega_S, \Omega_M]\). Note that the Rhines jet frequency in Exp1 and Exp3 is roughly zero, but it lies within \([\Omega_S, \Omega_M]\) in Exp2. Consequently, the energy always moves into the frequency range where Rhines jets reside.

2) ENERGY BUDGET OF STRIATIONS

The energy budget of the striations [Eq. (22)] is also found to be sensitive to the imposed mean flow. The time average of the terms in Eq. (22) is highlighted by the black boxes drawn in Fig. 8. In Exp2, where the Rhines jets are not classified as striations, external forcing and eddy–eddy interaction are equally important energy sources for the striations. However, the amount of energy extracted by the striations from other eddies is small. In Exp1 and Exp3, where the striations are dominated by Rhines jets, the dominant energy source for the striations is eddy–eddy interaction and the amount of energy extracted by striations from other eddies is large.

5. Effect of mean flow on total mixing and striations’ contribution

We introduce a diagnostic framework to assess the role of striations in mixing and then apply this framework to the specific example of barotropic striations.
a. Diagnostic approach

1) MIXING BY ALL THE EDDIES

Mixing rates of passive tracers induced by eddies are often quantified using the eddy mixing tensor \( D_{ij} \), (Plumb and Mahlman 1987; Bachman 2012; Bachman and Fox-Kemper 2013; Abernathey et al. 2013), which is defined as

\[
\mathbf{T}_i = \mathbf{u}'' b'' = -D_{ij} \frac{\partial b(x, t)}{\partial x_j}, \tag{27}
\]

where \( \mathbf{T}_i \) denotes the ensemble average of the eddy tracer flux, \( \mathbf{u}'' b'' \), with the overbar denoting an ensemble average (the mean) and the prime denoting a deviation from the mean (eddies). For example, \( \bar{u}_i \) is the mean velocity and \( u'_i \) is the eddy velocity. Here \( b \) denotes the concentration of a passive tracer, which satisfies

\[
\frac{\partial b}{\partial t} + \mathbf{u} \cdot \nabla b - \kappa \nabla^2 b = 0, \tag{28}
\]

where \( \mathbf{u} \) denotes the velocity vector, which is the sum of the mean velocity \( \overline{\mathbf{u}} \), the low-frequency eddy velocity \( \overline{\mathbf{u}}' \), and the high-frequency eddy velocity \( \mathbf{u}' \). Here \( \kappa \) can either represent molecular diffusivity or artificial numerical diffusivity to ensure computational stability (Shuckburgh and Haynes 2003). The symmetric part of \( D_{ij} \) is eddy diffusivity and the antisymmetric part provides information about the transport due to Stokes drift (Plumb and Mahlman 1987).

Equation (27) is an underdetermined system for one tracer, and therefore, previous studies solve for \( D_{ij} \) by using the pseudoinversion method, which involves initializing multiple tracers with different initial tracer gradients and then calculating \( D_{ij} \) through the least squares approach (Plumb and Mahlman 1987; Bachman 2012; Abernathey et al. 2013). However, we chose to use a novel approach from G. Flierl (2014, unpublished manuscript). This approach is an extension of the work by Kraichnan (1987), and it has been successfully applied to mixing estimations in both an idealized barotropic model (Chen 2013) and a realistic regional ocean model (Woods 2013). He found that, if both \( \mathbf{V} \mathbf{F} \) and \( \mathbf{T}_i \) vary slowly in space, \( D_{ij} \) is

\[
D_{ij} = \int_{t_0}^{t} \overline{u'_i(x, t) C_j(x, t)} dt. \tag{29}
\]

The term \( C_j(x, t) \) denotes the concentration of a “special tracer” at time \( t \). The special tracer is stirred by the total flow field and has the same value as the eddy velocity \( u'_i \) at time \( t = t_0 \). In other words, \( C_j(x, t) \) essentially records the initial velocity and it can be solved for numerically by integrating

\[
\left( \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla - \kappa \nabla^2 \right) C_j(x, t) = 0, \tag{30}
\]

with the initial condition

\[
C_j(x, t_0) = u'_j(x, t_0). \tag{31}
\]

The detailed derivation of this mixing formula [Eqs. (29)–(31)] can be found in section 5.8.3 of Chen (2013), and it is very long. Here are some key steps. First, we obtain the equation for eddy tracer concentration \( b' \) from the equation for the total tracer concentration \( b' \) [Eq. (28)]; second, we rewrite \( b' \) based on the Green’s function \( G \), satisfying

\[
\left( \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla - \kappa \nabla^2 \right) G(x, t | x', t') = \delta(x - x') \delta(t - t'). \tag{32}
\]

We can then rewrite the eddy tracer flux \( \mathbf{T}_i \), the eddy mixing tensor \( D_{ij} \), and the special tracer \( C_j \) based on the Green’s function \( G \) as well. The mixing formula can then be obtained by comparing \( D_{ij} \) and \( C_j \).

Compared to the pseudoinversion method, the new mixing formula [Eqs. (29)–(31)] has several advantages. First, the pseudoinversion approach can be computationally expensive; for example, six tracers are used in Abernathey et al. (2013) to infer the eddy mixing tensor in an idealized channel model. However, to diagnose \( D_{ij} \) using the new formula, we only need \( N \) special tracers in an \( N \)-dimensional flow. Second, \( D_{ij} \) computed from the pseudoinversion approach can be sensitive to the choice of the initial tracer concentration pattern, and improper choices lead to unusable solutions (Bachman 2012). However, the initial concentration of the special tracer field in Eq. (29) is just the eddy velocity. Computation of \( D_{ij} \) in Eq. (29) involves an ensemble average, and it is insensitive to the initial concentration of the special tracer in each realization.

2) MIXING BY LOW-FREQUENCY EDDIES

The eddy tracer flux \( \mathbf{T}_i \) includes contributions from both low- and high-frequency eddies,

\[
\mathbf{T}_i = \mathbf{u}'' b'' = \overline{u'_s b''} + \overline{u'_h b''}, \tag{33}
\]

where \( u_{s,i} \) and \( u_{h,i} \) denote the low-frequency and high-frequency eddy velocity, respectively. Thus, \( \overline{u'_s b''} \) and \( \overline{u'_h b''} \) represent the contribution of low-frequency and high-frequency eddies to eddy tracer flux, respectively. As in section 4, we assume all the low-frequency motions are striations, and thus \( D_{s,ij} \) is approximately the eddy mixing tensor induced by striations. Similar to Eq. (27), we can parameterize the two parts as follows:
Ds,ij denotes the eddy mixing tensor induced by low-frequency eddies and Dh,ij is that induced by high-frequency eddies. Following the approach in section 5.8.3 from Chen (2013), replacing u0,ib0 there with u0,s,ib0, we obtain Ds,ij:

\[ D_{s,ij} = \int_{t_0}^{t} u'_{s,j}(x,t) \overline{C}_j(x,t) \, dt, \]  

(34)

Similarly, if we replace u'ib' in section 5.8.3 from Chen (2013) with u'h,jb', we obtain Dh,ij:

\[ D_{h,ij} = \int_{t_0}^{t} u'_{h,i}(x,t) \overline{C}_j(x,t) \, dt, \]  

(36)

where \( \overline{C}_j \) is the special tracer defined by Eqs. (30) and (31). Note that

\[ D_{s,ij} + D_{h,ij} = D_{ij}. \]  

(37)

Analogous to \( D_{ij} \), the symmetric parts of \( D_{s,ij} \) and \( D_{h,ij} \) represent the contribution to mixing, and their antisymmetric parts provide information about the Stokes drift.

**b. Numerical results**

1) A DIAGNOSTIC EXAMPLE: EDDY MIXING TENSOR IN EXP1

To illustrate the methodology, we compute an estimate of the mixing tensor \( D_{ij} \) for Exp1. The first step is to estimate the special tracer \( C_j(x,t) \) defined by Eqs. (30) and (31). Choose an arbitrary \( t_0 \) as an initial time and specify the eddy velocity at that time as the initial value of the special tracer. Then employ the total velocity field to advect the special tracer forward to time \( t \). Figure 9 shows representative snapshots of the special tracer distribution at \( t_0 \) and two subsequent times. Considering
that $D_{ij}$ from Eq. (29) is an integral from time $t_0$ to time $t$, we define $t - t_0$ as the “integration time.” As the integration time increases, both the magnitude and spatial gradient of $C_j(x, t)$ decrease because of mixing and transport. The second step, after obtaining $C_j(x, t)$ above, is to diagnose $D_{ij}$ using Eq. (29). This two-step procedure is repeated multiple times to obtain more ensembles in order to obtain more reliable estimated values of $D_{ij}$ and its uncertainty (Fig. 10). In addition to the smoothing of the special tracer, as the integration time increases, the spatial pattern of the special tracer gradually decorrelates from its initial pattern (Fig. 9). Thus, $D_{ij}$ gradually levels off and we use $D_{ij}$ in this equilibrated time range, indicated by the red lines, for our final estimate (Fig. 10). The mixing tensor for striations ($D_{sij}$) can be estimated in a similar way.

2) EXPERIMENTAL RESULTS

Figure 11 show the domain-averaged mixing tensor for total eddies ($\langle D_{ij} \rangle$) and that for striations ($\langle D_{sij} \rangle$) in the three experiments. For a two-dimensional flow the tensor is of rank 2. The imposed mean flow does not affect the domain-averaged mixing in the barotropic system. The magnitude of $\langle D_{ij} \rangle$ is roughly the same in the three experiments (Fig. 11) because of Galilean invariance as both tracers and eddies are advected by the mean flow (appendix C).

Mixing is anisotropic and primarily in the zonal direction regardless of the direction of the imposed mean flow. In all three experiments, the zonal eddy diffusivity $\langle D_{11} \rangle$ has the largest magnitude among the four components of $\langle D_{ij} \rangle$ (Fig. 11). This result can be understood by realizing that for the coordinates moving with the imposed mean flow, the eddy velocity is dominated by that of the Rhines jets, which are roughly zonal. Therefore, water parcels move in a quasi-zonal direction, and as a result mixing is predominantly zonal. Rhines jets are always the dominant contributor to zonal eddy diffusivity $\langle D_{11} \rangle$. Consequently, striations contribute significantly to zonal mixing only when striations are dominated by Rhines jets (Fig. 11). In Exp1 and Exp3, where the meridional component of the imposed mean flow is zero, striations are predominantly Rhines jets and contribute much more to zonal mixing than the high-frequency eddies. In Exp2, where the meridional component of the imposed mean flow is significant, striations are not composed of Rhines jets and they contribute much less to zonal mixing than the high-frequency eddies.

Figures 9–11 show results for the intrinsic diffusivity $\kappa$ in Eqs. (28) and (30) set to 3200 m$^2$ s$^{-1}$. Other values of $\kappa$ were tried, and it was found that as $\kappa$ decreases, $D_{ij}$ levels off more slowly as a function of $t - t_0$ and its magnitude increases. However, the effect of mean flow
on $D_{ij}$ and on the role of striations in the total eddy mixing was found to be insensitive to $\kappa$.

6. Summary and discussion

In the low friction, $\beta$-plane, barotropic, QG system driven by small-scale narrow-banded white noise forcing, most of the eddy energy in frequency and wavenumber space is concentrated on the surface of the barotropic Rossby wave dispersion relation. Therefore, striations can be viewed as quasi-stationary Rossby waves. In the case with an imposed meridional mean flow, Rhines jets move meridionally and have the frequency $\nu_{\text{Rhines}}$. If $\nu_{\text{Rhines}}$ is larger than the separation frequency $\nu_S$, Rhines jets are not part of low-frequency motions. As a result, striations are tilted, have weak amplitudes, contribute little to tracer mixing, and extract little energy from the high-frequency eddies. When there is no mean flow imposed or when $\nu_S$ is small, the Rhines jet frequency is smaller than $\nu_S$. In this case, striations are zonal, dominated by Rhines jets, contribute significantly to tracer mixing, and extract significant energy from the high-frequency eddies.

Our experiments summarized above are forced by white noise, which has zero decorrelation time scale. We also examined the case when the forcing has the same narrow-banded wavenumber spectra but has a decorrelation time scale of a few days, as described in Lilly (1969), Williams (1978), Maltrud and Vallis (1991), and Chen et al. (2015a). Our choice is motivated by the fact that the decorrelation time scale of observed winds is on the order of a few days (Schlax et al. 2001; Gille 2005; Monahan 2012). We found that in this case, the striation properties summarized above still hold. On the other hand, Srinivasan and Young (2014) found that Galilean invariance breaks down if the forcing has finite decorrelation time scale. Thus, striation properties in the case when the forcing decorrelation time scale is very long could be different from those in the white noise forcing case. Detailed exploration is left for future work.

Besides forcing, many more regimes about striations in the background flow are left for future work. For example, we only consider the case with constant mean flow, which is a good approximation in some ocean regions (e.g., subtropical gyre), but not others (e.g., near topography). The large horizontal shear of the background mean flow may lead to barotropic instability and thus eddy–mean flow interaction. Cases with sheared mean flow, eddy–mean flow interaction, strong friction, topography, or multiple vertical models are to be explored. Coastal geometry and eastern boundary current can also greatly influence the generation of striations (e.g., Davis et al. 2014). Based on the Galilean invariance in the barotropic QG model, we developed some simple theories for the energy (appendix B), mixing (appendix C), and spectral (section 3c) characteristics for the striations. As a result of these, there are several conclusions that can be drawn.

First, energy cascades in the wavenumber domain are not sensitive to the imposed mean flow (Chen 2013), but energy cascades in the frequency domain are (appendix B). Second, total eddy mixing is insensitive to the barotropic mean flow (appendix C). Previous theories suggest that the mixing in the cross-mean flow direction can be suppressed by the wave propagation relative to the mean flow (e.g., Green 1970; Ferrari and Nikurashin 2010; Klocker et al. 2012a; Chen et al. 2014b). In other
words, the difference between the mean flow magnitude and the wave phase speed along the mean flow direction contribute to the mixing suppression (Chen et al. 2014b). However, the barotropic mean flow does not affect the magnitude of the mismatch between mean flow magnitude and wave speed, as waves are Doppler shifted by the barotropic mean flow. Therefore, we can conclude from these previous theories that the barotropic mean flow does not affect the mixing rates in the cross-mean flow direction, which is consistent with the findings in this study. Third, we found that at a given wavenumber vector the barotropic mean flow can shift the spectra in the frequency domain (section 3c). For example, in an ocean region, if the barotropic mean flow magnitude is 0.01 m s\(^{-1}\) and the dominant eddy length scale is 100 km, eddy motions with intrinsic frequency of zero will have the frequency of \(1 \times 10^{-7}\) cycle s\(^{-1}\), corresponding to a period of four months, because of the barotropic mean flow effect. In summary, in this simple barotropic model, the barotropic mean flow does affect energetics and mixing of low-frequency structures, the energy cascade in the frequency domain, and frequency–wavenumber spectra; however, the barotropic mean flow does not influence the total mixing rates by all the eddies.

The Galilean invariance also holds in the N-layer QG model (W. R. Young 2013, personal communication). The N-layer QG model is shown to be a reasonable model for realistic oceanic flows in many regions, such as the Southern Ocean and midlatitude oceanic interior (e.g., Arbic and Flierl 2004; Thompson 2010; Venaille et al. 2011). In consequence, the effect of barotropic (not baroclinic) mean flow on the energy, mixing, and spectral characteristics for the striations in the N-layer QG model or possibly in the ocean can also be described by these simple theories summarized in the last paragraph (i.e., appendices B and C and section 3c).

Our Exp2 is potentially applicable to the Southern Ocean from a kinematical perspective. In contrast to most oceanic regions, banded structures are visible in the flow snapshot in the Southern Ocean, and these structures are termed “Southern Ocean jets” (Thompson 2010). These jets are greatly influenced by topography (e.g., Thompson 2008; Ivenchenko et al. 2008), and topography with the scale comparable to the jet width can cause the meridional drifts of jets in the QG two-layer model (Thompson 2010). These meridional drifts have also been identified in some regions of the Southern Ocean in realistic eddying models (Thompson and Richards 2011). Our Exp2 is analogous to these Southern Ocean regions in two kinematic aspects: banded structures are visible in the instantaneous images of the flow field and they have meridional drifts. Results from Exp2 suggest that bands in low-frequency motions in the Southern Ocean might be different from the jets visible in the snapshots.

One can use our diagnostic framework and their potential descendants to investigate the energy and mixing consequences of realistic striations. This may be an important result considering that the amplitude of realistic striations is noticeable (Chen et al. 2015a). Following the techniques in appendix A, the energy budget equation for striations can be easily generalized to a form consistent with the governing equations of realistic models. The framework in section 5 is directly applicable to quantifying the contribution of striations to mixing in selected regions from observations and models of varying complexity, provided that the local spatial homogeneity assumption is applicable. The separation frequency between realistic striations and high-frequency eddies can be determined using the method from Chen et al. (2015a). These efforts can help reveal the role of striations in the spatial distribution of tracers (e.g., temperature, salinity) and thus the mean state of the climate and its variability.

We employed a new formula to estimate eddy mixing tensor [Eq. (29)]. This approach requires fewer tracers to estimate the mixing tensor than the pseudoinverse approach (e.g., Bachman 2012; Abernathey et al. 2013). Note that this new formula is based on the assumption that both the mean tracer gradient and the mean eddy tracer fluxes vary slowly in space. This assumption is also employed often in previous mixing studies (e.g., Ferrari and Nikurashin 2010; Klocker et al. 2012a). Comparison and reconciliation with other diffusivity diagnostic approaches are to be explored. Further application of this formula to oceanic and atmospheric mixing studies is to be carried out.

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APPENDIX A

Energy Budget for Low-Frequency Motions

The evolution equation of \(\langle KE_{12}^{l} \rangle\) is obtained by applying the temporal low-pass filter \(\hat{\omega}_{12}^{l}\) to our barotropic
OG model [Eq. (1)], then multiplying it by $-\psi_{\Omega}^f$ and integrating over the doubly periodic domain. Using integration by parts, one finds

$$\frac{\partial}{\partial t} \langle KE_{\Omega}^f \rangle = \langle \psi_{\Omega}^f [J(\psi, \nabla^2 \psi)]_{\Omega} \rangle_{\Omega} - \langle \psi_{\Omega}^f \rangle_{\Omega} \frac{\partial}{\partial t} \langle F(x,y,t) \rangle_{\Omega}$$

$$+ (\psi_{\Omega}^f)_{\Omega} = 0,$$

(A1)

where $\langle \cdot \rangle$ denotes the domain average. One key to obtaining Eq. (A1) is noting that the low-pass temporal filter operator $\hat{\cdot}$ and temporal deviation operator $\partial/\partial t$ can commute

$$\frac{\partial}{\partial t} \langle \psi_{\Omega}^f \rangle_{\Omega} = \frac{\partial}{\partial t} \langle \nabla^2 \psi_{\Omega} \rangle_{\Omega}.$$

(A2)

The temporal and domain-averaged nonlinear energy fluxes $\Pi_\Omega$ can be written in the frequency–wavenumber space. Using the convolution theorem, we express the Fourier transform of $\Pi_\Omega$ in terms of the Fourier transform of $\psi_{\Omega}^f$ and $[J(\psi, \nabla^2 \psi)]_{\Omega}^f$.

$$\Pi_{\Omega}(k,l,\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{\psi}(k',l',\omega') H(\omega',\Omega) \hat{J}(k-k',l-l',\omega-\omega') H(\omega-\omega',\Omega) dk' dl' d\omega',$$

(A3)

where $J$ denotes $J(\psi, \nabla^2 \psi)$ and $H$ denotes the low-pass filter, as that in Eqs. (19) and (20). Using the definition of Fourier transform,

$$\langle \Pi_{\Omega}(x,y,t) \rangle = \Pi_{\Omega}(k,l,\omega) \big|_{k=0,l=0,\omega=0},$$

(A4)

where $\cdot$ here denotes time mean. Noting that $H(-\omega',\Omega) = H(\omega',\Omega)$, Eqs. (A3) and (A4) lead to

$$\langle \Pi_{\Omega}(x,y,t) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{\psi}(k',l',\omega') [H(\omega',\Omega)]^2 \hat{J}(-k',-l',-\omega') dk' dl' d\omega'.$$

(A5)

**APPENDIX B**

**Effects of Mean Flow on Energy Cascades**

The eddy streamfunction in the experiment with mean flow $\psi(x,y,t)$ has the same statistical characteristics as $\phi_B(x,y,t)$, where

$$\phi_B(x,y,t) = \phi_A(x - \bar{U}t, y - \nabla t, t)$$

(B1)

with $\phi_A(x, y, t)$ denoting the eddy streamfunction in the experiment with no mean flow (section 3c). Thus, the nonlinear energy fluxes in the experiments with mean flow can be represented as a function of $\phi_B(x,y,t)$:

$$\Pi_{\Omega}(x,y,t) = \langle \psi(x,y,t) \rangle_{\Omega}^f [J(x,y,t)]_{\Omega}^f = \langle \phi_B(x,y,t) \rangle_{\Omega}^f [J_B(x,y,t)]_{\Omega}^f,$$

(B2)

where $\omega_0 = \omega - k'\bar{U} - l'\nabla$. Therefore, nonlinear energy fluxes in the case with mean flow [i.e., Eq. (A5)] can be obtained from the eddy field in the case without mean flow (i.e., $\phi_A$).
APPENDIX C

Effects of Mean Flow on Total Mixing

Here we discuss, in our barotropic model, why the imposed mean flow does not influence the spatially averaged total mixing rate. First, consider the eddy mixing case as 

\[ D_{Ai,j} = \int_0^t \langle u'_{Ai}(x,t) C_{Ai}(x,t) \rangle \, dt. \]  

(C1)

Here the special tracer \( C_{Ai}(x,t) \) satisfies

\[ \left( \frac{\partial}{\partial t} + u_A \cdot \nabla - \kappa \nabla^2 \right) C_{Ai}(x,t) = 0 \]  

with the initial condition

\[ C_{Ai}(x,t)|_{t=0} = u'_{Ai}(x,t)|_{t=0}. \]  

(C3)

Next, consider the case with mean flow \( \bar{u} \) imposed. We define the eddy velocity and the mixing tensor in this case as \( u'_B \) and \( D_{Bij} \). Again, we know from Eq. (29) that

\[ D_{Bij} = \int_0^t \langle u'_{Bj}(x,t) C_{Bj}(x,t) \rangle \, dt. \]  

(C4)

Here the special tracer \( C_{Bj}(x,t) \) satisfies

\[ \left( \frac{\partial}{\partial t} + u \cdot \nabla + u'_B \cdot \nabla - \kappa \nabla^2 \right) C_{Bj}(x,t) = 0 \]  

with the initial condition

\[ C_{Bj}(x,t)|_{t=0} = u'_{Bj}(x,t)|_{t=0}. \]  

(C6)

We know from the Galilean invariance that, from a statistical point of view,

\[ u'_B(x,t) = u'_A(x',t) = u'_A(x - \bar{u},t). \]  

(C7)

The term \( (x', t) \) denotes the moving coordinates, where \( x' = x - \bar{u}t \) and \( t' = t \). Using Eq. (C7), we can write Eqs. (C5) and (C6) in the moving coordinates,

\[ \left( \frac{\partial}{\partial t'} + u'_A(x',t') \cdot \nabla - \kappa \nabla^2 \right) C_{Bj} = 0 \]  

with the initial condition

\[ C_{Bj}|_{t=0} = u'_{Bj}(x,t)|_{t=0} = u'_{Bj}(x',t')|_{t=0}. \]  

(C9)

Comparing Eqs. (C8) and (C9) with Eqs. (C2) and (C3) leads to

\[ C_{Bj}(x,t) = C_{Ai}(x',t') = C_{Ai}(x - \bar{u},t). \]  

(C10)

Equations (C7) and (C10) lead to

\[ u'_B(x,t)C_{Bj}(x,t) = u'_A(x - \bar{u},t)C_{Ai}(x). \]  

(C11)

Therefore, at each time step in the doubly periodic domain, \( u'_B(x,t)C_{Bj}(x,t) \) has the same spatial pattern as \( u'_A(x - \bar{u},t)C_{Ai}(x) \). Consequently, the spatially averaged mixing tensor is independent of the imposed mean flow, that is, \( \langle D_{Ai,j} \rangle = \langle D_{Bij} \rangle \).

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