Measurement of the correlation between the polar angles of leptons from top quark decays in the helicity basis at $s = 7$ TeV using the ATLAS detector

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A measurement of the correlations between the polar angles of leptons from pair-produced $t \bar{t}$ quarks was performed at the LHC by the ATLAS collaboration. The dataset corresponds to an integrated luminosity of 4.6 fb$^{-1}$ at a center-of-mass energy of $\sqrt{s} = 7$ TeV collected during 2011. Candidate events are selected in the dilepton topology with large missing transverse momentum and at least two jets. The angles $\theta_1$ and $\theta_2$ between the charged leptons and the direction of motion of the parent quarks in the $t \bar{t}$ rest frame are sensitive to the spin information, and the distribution of $\cos \theta_1 \cdot \cos \theta_2$ is sensitive to the spin correlation between the $t$ and $\bar{t}$ quarks. The distribution is unfolded to parton level and compared to the next-to-leading order prediction. A good agreement is observed.

I. INTRODUCTION

After the discovery of the top quark in 1995 at the Tevatron proton-antiproton collider [1,2], a new era of top quark precision measurements began in 2010 with the start of the Large Hadron Collider (LHC) at CERN at $\sqrt{s} = 7$ TeV. Due to its short lifetime the top quark decays before hadronization. This implies that top quarks can be studied as bare quarks and the spin information of the top quark can be deduced from the angular distributions of its decay products. At the LHC, $t \bar{t}$ production is dominated by gluon fusion with a smaller contribution from $q \bar{q}$ annihilation. However, many scenarios of physics beyond the Standard Model (SM) predict different spin correlations. For example, the measured spin correlation may differ from the SM if $t \bar{t}$ production from $q \bar{q}$ annihilation was enhanced by the top quark coupling to Higgs or extra gauge bosons [3–6], or if the top quark decayed into a scalar charged Higgs boson and a $b$-quark ($t \to H^+ b$) [7].

Both the CDF and D0 collaborations have performed measurements of the spin correlation [8–12] at the Tevatron where $t \bar{t}$ production via $q \bar{q}$ annihilation dominates. In addition, it has been measured at the LHC by both the ATLAS [13–15] and CMS experiments [16]. The different production mechanisms and center-of-mass energies make the measurements of the spin correlation at the two colliders complementary [17]. The results obtained from these analyses are all consistent with the SM prediction.

In this paper the decay $t \bar{t} \to W^+ W^- b \bar{b} \to \ell^+ \nu \ell^- \bar{\nu} b \bar{b}$ is used to measure the following differential distribution, which is related to the spin correlation of the $t \bar{t}$ system [18]:

$$\frac{1}{N} \frac{d^2N}{d \cos \theta_1 d \cos \theta_2} = \frac{1}{4} (1 + B_1 \cos \theta_1 + B_2 \cos \theta_2 - C_{\text{helicity}} \cos \theta_1 \cdot \cos \theta_2),$$

where $\theta_1$ ($\theta_2$) is the angle between the momentum direction of the charged lepton from the $t\bar{t}$ decay in the $t\bar{t}$ rest frame and the $t\bar{t}$ momentum direction in the $t\bar{t}$ center-of-mass frame. This is commonly referred to as the helicity basis. The helicity basis is not the only possibility and other bases are discussed in Ref. [17]. The top quark polarization parameters $B_1$ and $B_2$ are two orders of magnitude smaller than $C_{\text{helicity}}$ at next-to-leading-order (NLO) [19]. In Ref. [20] the polarization is measured to be $-0.035 \pm 0.040$ for the $CP$-conserving scenario, compatible with the measurement from CMS [16] and the SM expectation [21]. Consequently they are set to zero in this study. This analysis uses the measured distribution of $\cos \theta_1 \cdot \cos \theta_2$: it can be shown [18] that the mean of the distribution is proportional to the coefficient $C_{\text{helicity}}$ parametrizing the strength of the spin correlation.

Candidate events are selected with two isolated charged leptons and at least two jets in the final state, including a requirement to enhance the selection of jets originating from $b$-quarks. The $t$ and $\bar{t}$ are reconstructed using kinematic information from the event and invariant mass constraints. The distribution of $\cos \theta_1 \cdot \cos \theta_2$ at reconstruction level is obtained. Building upon previous studies the non-$t\bar{t}$ backgrounds are subtracted and the distribution is unfolded to parton level using an iterative
Bayesian technique. The parton-level distribution can be compared directly with the theoretical prediction without the need for templates derived from simulation.

II. ATLAS DETECTOR AND DATA SAMPLES

This analysis makes use of an integrated luminosity of 4.6 fb$^{-1}$ [22] of proton-proton collision data at a center-of-mass energy of 7 TeV, collected by the ATLAS detector at the LHC during 2011. The ATLAS detector [23,24] covers nearly the entire solid angle$^1$ around the collision point. It consists of an inner tracking detector (ID) covering $|\eta| < 2.5$, and comprising a silicon pixel detector, a silicon microstrip detector, and a transition radiation tracker. The ID is surrounded by a thin superconducting solenoid providing a 2 T magnetic field, followed by a liquid-argon calorimeter (LAr) with high granularity. An iron/scintillator tile calorimeter provides hadronic energy measurements in the central region ($|\eta| < 1.7$). The endcap and forward regions are instrumented with LAr calorimeters for electromagnetic (EM) and hadronic energy measurements up to $|\eta| = 4.9$. The calorimeter system is surrounded by a muon spectrometer (MS) with high-precision tracking chambers covering $|\eta| < 2.7$ and separate trigger chambers covering $|\eta| < 2.4$. The magnetic field is provided by a barrel and two endcap superconducting toroid magnets. A three-level trigger system is used to select events with high-$p_T$ leptons for this analysis [25].

Monte Carlo (MC) samples are produced for signal and background estimation. The SM $t\bar{t}$ signal events are modeled using the mc@nlo v4.01 generator [26]. Top quarks and the subsequent $W$ bosons are decayed conserving the spin correlation information. The decay products are interfaced with Herwig v6.520 [27], which hadronizes the $b$-quarks and $W$ boson decay products, and with Jimmy [28] to simulate multiparton interactions. The top quark mass is set to 172.5 GeV and the CT10 parton distribution [28] to simulate multiparton interactions. The top quark is reweighted to the number of interactions observed in the MRST LO PDF set [27]. They are all normalized to the theoretical predictions at NLO [43].

All MC samples use a GEANT4 based simulation [44,45] to model the ATLAS detector and the same reconstruction as used in data. During the 2011 data-taking period the average number of simultaneous $pp$ interactions per beam crossing (pileup) at the beginning of a fill of the LHC increased from 6 to 17. For each MC process, pileup is overlaid using simulated minimum-bias events from the pythia generator. The number of additional $pp$ interactions is reweighted to the number of interactions observed in data. Additional small corrections are made to the simulation to ensure that it describes the data well in terms of efficiencies and momentum or energy scales for the various objects used.

While all other backgrounds are based on MC simulation, the background arising from misidentified and nonprompt leptons (referred to as “fake leptons” in the figures and tables) is determined using a data-driven technique known as the matrix method [46].

III. EVENT SELECTION AND RECONSTRUCTION

A. Event selection

Candidate events are selected in the dilepton topology, referred to as the $e^+e^-$, $\mu^+\mu^-$, and $e^+\mu^-\bar{\nu}$ channels, according to the flavors of the two leptons. The full object and event selection are the same as described in Ref. [14], with the additional requirement that at least one $b$-jet is identified. The analysis requires events selected by an inclusive single-lepton($e$ or $\mu$) trigger [47]. The primary vertex with highest $p_T^2$ is taken as the primary vertex of the event if it has at least five associated tracks, with $p_T > 400$ MeV per track, consistent with the $x, y$ profile of the beam, and the other vertices are not considered.

Electron candidates are reconstructed using energy deposits in the EM calorimeter associated with reconstructed tracks in the ID [48]. Muon candidate reconstruction makes use of tracking in the MS and ID [49]. Both the electron and muon candidates have isolation criteria applied as in Ref. [14] and are matched to a triggered object. Jets are reconstructed with the anti-$k_t$ algorithm [50] with a radius parameter $R = 0.4$, starting

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$^1$ATLAS uses a right-handed coordinate system with its origin at the nominal interaction point (IP) in the center of the detector and the $z$-axis along the beam pipe. The $x$-axis points from the IP to the center of the LHC ring, and the $y$-axis points upward. Cylindrical coordinates $(r, \phi)$ are used in the transverse plane, $\phi$ being the azimuthal angle around the $z$-axis. The pseudorapidity is defined in terms of the polar angle $\theta$ as $\eta = -\ln \tan(\theta/2)$. Angular distance $\Delta R$ is defined as $\sqrt{\Delta \eta^2 + \Delta \phi^2}$ where $\Delta \phi$ and $\Delta \eta$ are the difference of azimuthal angle and pseudorapidity, respectively.
B. Topology reconstruction method

In order to reconstruct the distribution of $\cos \theta_1 \cdot \cos \theta_2$, the $t$ and $\bar{t}$ quarks must be reconstructed from their decay products. The momenta of the two neutrinos from the W boson decays in dilepton final states cannot be measured but can be inferred from the measured missing transverse momentum in the event. Since only the sum of the missing transverse momenta of the two neutrinos is measured as $E_{T}^{miss}$, the system is underconstrained. The kinematic information about the $b$-jets, leptons, and the missing transverse momentum are used in six independent equations describing the kinematic properties of $t$ and $\bar{t}$ decays. The equations describing the kinematic constraints are

$$
E_{x}^{miss}, \quad E_{y}^{miss}, \quad p_{x}, \quad p_{y}, \quad \mu_{x}, \quad \mu_{y}, \quad e_{x}, \quad e_{y}
$$

where $E_{x}^{miss}$ and $E_{y}^{miss}$ represent the missing momentum along the x- and y-axes, $p_{x}$ and $p_{y}$ are the four-momenta of the two charged leptons (two $b$-jets), and $\mu_{x}$ and $\mu_{y}$ are the masses of the W boson and top quark. The reconstruction algorithm requires the kinematic information for exactly two of the selected jets. Jets are ranked primarily by whether they are $b$-tagged or not, and then by descending $p_T$. The two highest-ranked jets are used in the reconstruction method.

Each selected event has two possible $b-\ell$ pairings. The pairing with the lower invariant mass is first considered for the $t\bar{t}$ reconstruction. This results in a higher probability to correctly identify the $b$-jets that originate from the respective top quarks than using the alternate pairing. When comparing the data and MC we find the fraction of events passing the reconstruction is consistent, indicating no systematic bias in the method. If no solution is found, the top quark mass is varied from the nominal value in steps of 1.5 GeV until a solution is found or the limits of 157.5 GeV and 187.5 GeV are reached. If it is still not possible to solve Eq. (2) then the alternative $b-\ell$ pairing is considered and the procedure is repeated. If more than one solution is found, the one with the minimum product of $p_T^2$ and $p_T^2$ is selected. About 70% of signal $t\bar{t}$ simulated events and 50% of background events are reconstructed.

The number of expected and observed events in each channel after selection and reconstruction is listed in Table I.

The distribution of reconstructed $\cos \theta_1 \cdot \cos \theta_2$ for the sum of the three dilepton channels, with the signal $t\bar{t}$ simulated sample from MC@NLO and backgrounds overlaid, is shown in Fig. 1. The backgrounds are highly
TABLE I. The number of expected and observed events after reconstruction for the $e^+e^-$, $\mu^+\mu^-$ and $e^+\mu^+$ channels. The combined statistical and systematic uncertainty due to the cross-section are included for samples derived from MC simulation. The $tt$ contribution in the table includes the signal events ($tt \rightarrow W^+W^-b\bar{b} \rightarrow \ell\ell c\bar{b}b\bar{b}$, with $\ell$ referring to $e/\mu$) and the $t$ events ($tt \rightarrow W^+W^-b\bar{b} \rightarrow \tau\ell\ell b\bar{b}$, with $\ell$ referring to $e, \mu$ or $\tau$ and $t$ decaying further to $e/\mu$).

<table>
<thead>
<tr>
<th>Source</th>
<th>$e^+e^-$ channel</th>
<th>$\mu^+\mu^-$ channel</th>
<th>$e^+\mu^+$ channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$tt$</td>
<td>352 ± 21</td>
<td>1016 ± 60</td>
<td>2950 ± 170</td>
</tr>
<tr>
<td>Single top (Wt-channel)</td>
<td>9.0 ± 0.9</td>
<td>25.4 ± 2.0</td>
<td>90 ± 7</td>
</tr>
<tr>
<td>Fake leptons</td>
<td>4.8 ± 2.2</td>
<td>8.0 ± 2.8</td>
<td>33 ± 6</td>
</tr>
<tr>
<td>$Z \rightarrow e^+e^-/\mu^+\mu^-$ + jets</td>
<td>0.6 ± 0.4</td>
<td>1.1 ± 0.6</td>
<td>--</td>
</tr>
<tr>
<td>$Z \rightarrow \tau^+\tau^-$ + jets</td>
<td>0.32 ± 0.29</td>
<td>1.5 ± 0.6</td>
<td>9.4 ± 1.4</td>
</tr>
<tr>
<td>Diboson</td>
<td>0.25 ± 0.08</td>
<td>1.4 ± 0.5</td>
<td>5.4 ± 1.3</td>
</tr>
<tr>
<td>Total expected</td>
<td>366 ± 21</td>
<td>1054 ± 60</td>
<td>3080 ± 180</td>
</tr>
<tr>
<td>Observed</td>
<td>383</td>
<td>1082</td>
<td>3132</td>
</tr>
</tbody>
</table>

W bosons decay to a $\tau$ that subsequently decays to an $e$ or $\mu$ and are taken from simulation and are also subtracted from data.

The number of bins used in the unfolding is chosen based on studies of the resolution of the $\cos \theta_1 \cdot \cos \theta_2$ reconstruction and taking into account the number of selected events in data, while minimizing the bin-to-bin correlations. Eight equally sized bins in $\cos \theta_1 \cdot \cos \theta_2$ are used.

A true physical observable in bin $C_i$ of distribution $n(C_i)$ is related to the reconstructed quantity in bin $E_j$ of a distribution $n(E_j)$ by the response matrix $P(E_j|C_i)$, which represents the event migration probability from bin $C_i$ to bin $E_j$:

$$n(E_j) = \sum_{i=1}^{n_C} P(E_j|C_i)n(C_i), \quad j = 1, \ldots, n_E,$$

where $e_i$ represents the measurement efficiency of events in bin $C_i$ and $n_C$ and $n_E$ represent the total number of bins in the true and reconstructed distributions respectively.

In this analysis an iterative Bayesian unfolding method is used [54,55], in which Bayes’ theorem is adopted to produce the following conditional probability:

$$P(C_i|E_j) = \frac{P(E_j|C_i)P_0(C_i)}{\sum_{j=1}^{n_C} P(E_j|C_i)P_0(C_i)},$$

where $P(C_i|E_j)$ represents the probability of having a true event in bin $C_i$, given a reconstructed event in bin $E_j$. And $P_0(C)$ is the normalized prior distribution of $n(C_i)$. Using $P(C_i|E_j)$, one can calculate $n(C_i)e_i$ using the following equation:

$$\hat{n}(C_i)e_i = \sum_{j=1}^{n_E} n(E_j)P(C_i|E_j).$$

The probability $P(C_i|E_j)$ depends on the prior distribution of $P_0(C)$, and $\hat{n}(C_i)e_i$ is a biased estimator if $P_0(C)$
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differs from data. To reduce the bias, an iterative procedure is introduced, replacing \( P_{0}(C) \) with the normalized \( \tilde{n}(C) \sigma_{i} \) in Eq. (4) to recalculate \( P(C|E_{i}) \) and then \( \tilde{n}(C) \sigma_{i} \) using Eq. (6). Finally, \( \tilde{n}(C) \) is obtained by scaling \( \tilde{n}(C) \sigma_{i} \) using \( 1/\epsilon_{i} \) from MC simulation.

Increasing the number of iterations reduces the bias of the estimator. However, fluctuations and correlations between bins of the estimator are increased. The iterative procedure is repeated until the unfolded distribution in the previous iteration within the statistical uncertainty. In this case, further iterations would not increase the sensitivity. Therefore the termination criterion is defined as

\[
\frac{\chi^{2}}{N_{\text{bins}}} \leq 1, \tag{7}
\]

where

\[
\chi^{2} = \sum_{j=1}^{N_{\text{bins}}} \sum_{i=1}^{N_{\text{bins}}} (n'_{i} - n_{i}) (\sigma_{i,j})^{-1} (n'_{j} - n_{j})^{T}, \tag{8}
\]

in which \( (n'_{i} - n_{i}) \) refers to the \( i \)th bin difference between the unfolded distribution \( n'_{i} \) in the current iteration and the unfolded distribution \( n_{i} \) in the previous iteration, \( \sigma_{i,j} \) is the covariance matrix of the unfolded distribution \( n'_{i} \), and \( N_{\text{bins}} \) is the number of bins in the unfolded distribution. The larger the difference between the prior distribution and the real distribution, the more iterations are required. Studies using MC simulation have shown that three iterations suffice to reduce the bias below the level of the statistical uncertainty.

**B. Method validation**

The unfolding method is validated and its uncertainty obtained using a MC sample containing the SM spin correlation. The simulated \( \cos \theta_{1} \cdot \cos \theta_{2} \) distribution after detector simulation, selection, and reconstruction is compared to that in collision data, and the ratio of the two is fitted by a smooth function that is used to weight the parton-level distribution. This is propagated through to the reconstructed distribution and results in a pseudomeasurement and a corresponding parton-level distribution. The unfolding method, using the nominal response matrix, is applied to the pseudomeasurement. The systematic uncertainty on the unfolding method is taken as the difference between the unfolded pseudomeasurement and the known parton-level distribution and is shown in Table II.

V. SYSTEMATIC AND STATISTICAL UNCERTAINTIES

**A. Systematic uncertainties**

Systematic uncertainties are evaluated by applying the unfolding procedure (using the nominal unfolding matrix) to pseudoexperiments created using MC samples modified to reflect the various systematic uncertainties. The systematic uncertainty of the unfolded distribution is then obtained by comparing the varied unfolded distributions to the nominal unfolded distribution. The following systematic uncertainty sources are considered in this analysis.

**1. MC generator modeling**

The uncertainty due to generator modeling is assessed using three different groups of samples. Powheg+pythia [56–59] is compared to MC@NLO+Herwig, where both the generator and parton showering are varied. Powheg+pythia is compared to Alpgen+Herwig and finally Powheg+pythia is compared to Powheg+herwig, where only the parton showering is different. The largest variation of the unfolded distributions found in these three comparisons is taken as the uncertainty.

<table>
<thead>
<tr>
<th>Bin range</th>
<th>-1: -0.75</th>
<th>-0.75: -0.5</th>
<th>-0.5: -0.25</th>
<th>-0.25:0</th>
<th>0:0.25</th>
<th>0.25:0.5</th>
<th>0.5:0.75</th>
<th>0.75:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator modeling</td>
<td>6.9</td>
<td>3.2</td>
<td>1.6</td>
<td>1.6</td>
<td>0.5</td>
<td>0.8</td>
<td>2.2</td>
<td>1.0</td>
</tr>
<tr>
<td>ISR/FSR</td>
<td>2.0</td>
<td>0.9</td>
<td>0.6</td>
<td>0.3</td>
<td>0.3</td>
<td>1.1</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>PDF</td>
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<td>0.1</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>UE/color reconnection</td>
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<td>1.1</td>
<td>1.0</td>
<td>0.7</td>
<td>0.1</td>
<td>0.5</td>
<td>1.1</td>
<td>1.3</td>
</tr>
<tr>
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<td>3.0</td>
<td>1.1</td>
<td>0.6</td>
<td>0.9</td>
<td>1.1</td>
<td>1.9</td>
<td>1.8</td>
</tr>
<tr>
<td>b-tagging SF</td>
<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$E_{T}^{\text{miss}}$</td>
<td>0.5</td>
<td>0.6</td>
<td>0.4</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Lepton reconstruction</td>
<td>1.5</td>
<td>0.6</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.5</td>
<td>0.6</td>
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<tr>
<td>Luminosity uncertainty</td>
<td>0.5</td>
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<td>0.1</td>
<td>0.0</td>
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<tr>
<td>Background uncertainty</td>
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<td>0.4</td>
<td>0.1</td>
<td>0.1</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Bayesian unfolding method</td>
<td>10.9</td>
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<td>1.4</td>
<td>1.0</td>
<td>2.6</td>
<td>0.6</td>
<td>7.8</td>
</tr>
<tr>
<td>Total</td>
<td>13.9</td>
<td>4.9</td>
<td>3.3</td>
<td>1.8</td>
<td>1.7</td>
<td>3.9</td>
<td>2.7</td>
<td>9.3</td>
</tr>
<tr>
<td>Top quark mass (±1 GeV)</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
<td>0.0</td>
<td>0.6</td>
</tr>
</tbody>
</table>
2. ISR/FSR

The uncertainty due to initial-state radiation/final-state radiation (ISR/FSR) is evaluated using Alpgen+pythia samples within which the parameters controlling ISR and FSR are varied within a range consistent with data. The average of the absolute values of the upwards and downwards variations of the unfolded distributions is taken as the systematic uncertainty.

3. PDF

The impact of the choice of PDF in simulation was studied by reweighting the MC samples to three PDF sets (CT10, MSTW2008 [60] NLO, and NNPDF20 [61]) and taking half of the maximum difference of the unfolded distributions using any two PDF sets.

4. Underlying event and color reconnection

To estimate the effect of the underlying event (UE), two samples simulated by Powheg+Pythia with the Perugia 11 and Perugia 11 mpiHI sets of tuned parameters are used [62]. The variation of the unfolded distributions between these two tunes is taken as the systematic uncertainty. The impact due to the modeling of color reconnection is studied by comparing two samples simulated with Powheg+Pythia. One has the nominal color reconnection model and the other has no color reconnection. The difference of the unfolded distributions between these two samples is taken as the systematic uncertainty.

5. Jet energy scale and jet reconstruction

The relative jet energy scale (JES) uncertainty varies from 1% to 3% depending on jet $p_T$ and $\eta$ [63]. The jet reconstruction efficiency for data and the MC simulation is evaluated exclusively from simulation. The change in the result due to the luminosity uncertainty is taken as systematic uncertainty.

6. b-tagging efficiency

Differences in the $b$-tagging efficiency as well as $c$-jet and light-jet mistag rates in data and simulation are parametrized using correction factors, which are functions of $p_T$ and $\eta$ [65]. The uncertainty on these correction factors is propagated to the unfolded distribution.

7. Modeling of $E_T^{\text{miss}}$

Uncertainties on the energy scale of jets and leptons are also propagated to the uncertainty on $E_T^{\text{miss}}$. Other contributions to this uncertainty originate from the energy scale and resolution of the soft calorimeter energy deposits that are not included in the reconstructed jets and leptons, and is propagated to the uncertainty of the unfolded distribution.

8. Lepton reconstruction

The modeling of the lepton momentum scale and resolution is studied using the reconstructed dilepton invariant mass distribution of $Z \rightarrow \ell^+\ell^-$ candidates and the simulation is adjusted accordingly. Any mismodeling of the electron and muon trigger, reconstruction, and selection efficiencies in the simulation is corrected using measurements of the efficiency in data. The systematic uncertainties on the correction factors applied are propagated to the unfolded distribution.

9. Luminosity uncertainty

The uncertainty on the measured integrated luminosity is 1.8% [22]. The effect of the luminosity uncertainty is evaluated by scaling the number of signal and background events by the luminosity uncertainty, for processes estimated exclusively from simulation. The change in the result due to the luminosity uncertainty is taken as systematic uncertainty.

10. Background uncertainties

The uncertainties due to the normalization of the non-prompt and fake lepton estimate, $Wt$ channel single top, $Z \pm$ jets, and diboson events are propagated to the uncertainty of the unfolded distribution.

11. Bayesian unfolding method

The residual bias in the unfolding method is taken as a systematic uncertainty as described in Sec. IV.

The evaluated systematic uncertainties are listed in Table II, for each bin of the $\cos \theta_1 \cdot \cos \theta_2$ distribution. The result of varying the top quark mass by $\pm 1$ GeV is shown in the last row of the table. The main sources of uncertainty are the unfolding method, followed by the uncertainties associated with jets. Some theoretical uncertainties (MC generator, top quark mass, UE/color connection) are estimated with uncorrelated MC samples, and

<table>
<thead>
<tr>
<th>Bin number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
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<td>1</td>
<td></td>
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</tr>
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<td>2</td>
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<td></td>
<td></td>
<td></td>
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<td>4</td>
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<td>0.071</td>
<td>0.44</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
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<td>5</td>
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<td>−0.18</td>
<td>−0.062</td>
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<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>−0.093</td>
<td>−0.14</td>
<td>−0.16</td>
<td>0.049</td>
<td>0.59</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>−0.068</td>
<td>−0.13</td>
<td>−0.18</td>
<td>−0.08</td>
<td>0.28</td>
<td>0.75</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>−0.035</td>
<td>−0.081</td>
<td>−0.13</td>
<td>−0.083</td>
<td>0.14</td>
<td>0.49</td>
<td>0.8</td>
<td>1</td>
</tr>
</tbody>
</table>
TABLE IV. The numerical summary of the unfolded $\cos \theta_1 \cdot \cos \theta_2$ distribution, with statistical and systematic uncertainties. The SM prediction is shown in the last column for comparison.

<table>
<thead>
<tr>
<th>Bin range</th>
<th>Unfolded data $1/\sigma d\sigma/d(\cos \theta_1 \cdot \cos \theta_2) \pm \text{stat} \pm \text{syst}$</th>
<th>MC@NLO prediction $1/\sigma d\sigma/d(\cos \theta_1 \cdot \cos \theta_2) \pm \text{stat}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.00: -0.75$</td>
<td>$0.0202 \pm 0.0020 \pm 0.0028$</td>
<td>$0.0215 \pm 0.0005$</td>
</tr>
<tr>
<td>$-0.75: -0.50$</td>
<td>$0.0696 \pm 0.0037 \pm 0.0034$</td>
<td>$0.0707 \pm 0.0008$</td>
</tr>
<tr>
<td>$-0.50: -0.25$</td>
<td>$0.1418 \pm 0.0045 \pm 0.0047$</td>
<td>$0.1384 \pm 0.0010$</td>
</tr>
<tr>
<td>$-0.25: 0$</td>
<td>$0.3106 \pm 0.0062 \pm 0.0057$</td>
<td>$0.3079 \pm 0.0014$</td>
</tr>
<tr>
<td>$0: 0.25$</td>
<td>$0.2882 \pm 0.0059 \pm 0.0048$</td>
<td>$0.2884 \pm 0.0013$</td>
</tr>
<tr>
<td>$0.25: 0.50$</td>
<td>$0.1078 \pm 0.0033 \pm 0.0042$</td>
<td>$0.1118 \pm 0.0009$</td>
</tr>
<tr>
<td>$0.50: 0.75$</td>
<td>$0.0489 \pm 0.0024 \pm 0.0013$</td>
<td>$0.0484 \pm 0.0006$</td>
</tr>
<tr>
<td>$0.75: 1.00$</td>
<td>$0.0129 \pm 0.0009 \pm 0.0012$</td>
<td>$0.0129 \pm 0.0003$</td>
</tr>
</tbody>
</table>

hence include a statistical uncertainty due to the MC sample size.

**B. Statistical uncertainties**

The uncorrelated bin-to-bin statistical uncertainties of the $\cos \theta_1 \cdot \cos \theta_2$ distribution at reconstruction level are propagated to the unfolded distribution, in which bin-to-bin correlations arise. The $8 \times 8$ correlation matrix is shown in Table III.

The statistical and systematic uncertainties for each bin of the unfolded $\cos \theta_1 \cdot \cos \theta_2$ distribution is summarized in Table IV.

**VI. RESULTS**

The unfolded distribution of $\cos \theta_1 \cdot \cos \theta_2$ is shown in Fig. 2 and presented in Table IV. The distribution is compared to the prediction from MC@NLO giving a $\chi^2/N_{\text{bin}} = 4.1/8$. Individual analyses for the $e^+e^-$, $\mu^+\mu^-$, and $e^+\mu^-$ channels are performed and the measurements are found to be consistent with the combined result.

Previous publications quote the results in terms of $A_{\text{helicity}} = \langle N_{\text{like}} - N_{\text{unlike}} \rangle / \langle N_{\text{like}} + N_{\text{unlike}} \rangle$ where $N_{\text{like}}$ ($N_{\text{unlike}}$) is the number of events where the top quark and top antiquark have parallel (antiparallel) spins with respect to the helicity basis. To compare with these quantitatively, the parameter $C_{\text{helicity}}$ in Eq. (1) is extracted from the unfolded distribution using $C_{\text{helicity}} = -9 \langle \cos \theta_1 \cdot \cos \theta_2 \rangle$ [18,66]. This is converted to $A_{\text{helicity}}$ using $C_{\text{helicity}} = -A_{\text{helicity}} \alpha_1 \alpha_2$, where $\alpha_1$ and $\alpha_2$ are the spin analyzing powers for the two charged leptons as in Ref. [14]. In dilepton final states the spin-analyzing power is effectively 100%; therefore $C = A$. This results in $A_{\text{helicity}} = 0.315 \pm 0.061 \text{(stat)} \pm 0.049 \text{(syst)}$, which agrees well with the NLO QCD prediction of $A_{\text{helicity}} = 0.31$ [67], the previous measurements using template fits to event properties without correcting for detector acceptance and efficiencies by ATLAS [13–15], and the unfolded parton level results reported by CMS [16].

**VII. CONCLUSION**

A differential cross-section measurement of the $\cos \theta_1 \cdot \cos \theta_2$ distribution is presented using 4.6 fb$^{-1}$ of proton-proton collision data collected at $\sqrt{s} = 7$ TeV by the ATLAS detector at the LHC during 2011. Events are selected in the dilepton topology with two jets. The background rejection is improved by the use of $b$-tagging. The distribution of $\cos \theta_1 \cdot \cos \theta_2$ is reconstructed using the kinematic information about the selected objects and unfolded to parton level using an iterative Bayesian unfolding algorithm. The unfolded distribution is in good agreement with the prediction from MC@NLO. The main sources of uncertainty are due to the unfolding method, theoretical modeling of the signal, and uncertainties related to the reconstruction of jets.
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