Inflationary paradigm after Planck 2013

The MIT Faculty has made this article openly available. Please share how this access benefits you. Your story matters.

| As Published | http://dx.doi.org/10.1016/j.physletb.2014.03.020 |
| Publisher | Elsevier |
| Version | Final published version |
| Accessed | Thu Feb 21 16:34:15 EST 2019 |
| Citable Link | http://hdl.handle.net/1721.1/105902 |
| Terms of Use | Creative Commons Attribution 3.0 Unported licence |
| Detailed Terms | http://creativecommons.org/licenses/by/3.0/ |
1. Introduction

Did our universe undergo a period of accelerated expansion in the early stage of its evolution? If so, does it play an important role in explaining observable features of our universe today?

We define the “inflationary paradigm” to mean that the answer to both of these questions is “yes” [1,2]. As we argue here, the inflationary paradigm draws upon well-motivated physical interactions and types of matter. The inflationary explanations for the homogeneity and the flatness of the universe can be understood in the context of classical general relativity, and even the origin of density fluctuations can be accurately described in the context of quantum field theory on a classical, curved spacetime [3], a theoretical framework that has been thoroughly studied for decades [4]. Moreover, reasoning about the behavior of fundamental scalar fields is on a stronger footing than ever, in the light of the recent observation of the Higgs boson at the LHC [5,6].

As is well known, inflation makes several generic predictions [7,8]. The observable universe today should be flat, i.e., $|\Omega_k| \ll 1$, where $\Omega_k \equiv 1 - \Omega$. There should exist primordial curvature perturbations whose power spectrum $P_R(k) \propto k^{n_s-1}$ has a slightly tilted spectral index, $|n_s - 1| \ll 1$, typically redshifted. Unless the inflaton potential or the initial conditions are fine-tuned, the primordial perturbations should be predominantly Gaussian [9]. Modes of a given (comoving) wavelength should “freeze out” upon first crossing the Hubble radius during inflation, remain (nearly) constant in amplitude while longer than the Hubble radius, and then resume oscillation upon reentering the Hubble radius. The temporal oscillations of modes with nearby wavelengths are therefore coherent [10], giving rise to a sharp pattern of peaks and troughs in the cosmic microwave background (CMB) power spectrum. These generic predictions are consequences of simple inflationary models, and depend only on the physics at the inflationary energy scale, i.e., the energy scale of the final stage of inflation, as observed in the CMB. We will refer to these as inflation-scale predictions. To date, every single one of these inflation-scale predictions has been confirmed to good precision [11], most recently with the Planck satellite [11].

Despite these successes, Ijjas, Steinhardt, and Loeb (ISL) [12] have recently argued that the inflationary paradigm is in trouble in the light of data from Planck. They agree that a class of inflationary models make predictions that agree with experiment, which is how theories are usually evaluated, but they bring up a different issue. They argue that if one starts at the Planck scale which is how theories are usually evaluated, but they bring up a different issue. They argue that if one starts at the Planck scale with reasonable assumptions about initial conditions, the successful inflationary models are “exponentially unlikely according to the inner logic of the inflationary paradigm itself.” In this paper we argue that this is not the case by addressing each of their specific points. We will argue that their negative conclusions rely on unfounded assumptions, and can be completely avoided under what we consider to be more reasonable assumptions about the physics between the inflationary scale and the Planck scale.
We also believe, as a matter of principle, that it is totally inappropriate to judge inflation on how well it fits with anybody's speculative ideas about Planck-scale physics—physics that is well beyond what is observationally tested. All theories of evolution begin with assumptions that are taken to be plausible, but which are usually not directly verifiable, and then the theories make predictions which can be tested against current observations. We do not reject Darwinian evolution because it does not explain the actual origin of life; we do not reject big-bang nucleosynthesis because it does not explain the homogeneous thermal equilibrium initial state that it requires; and we should similarly not even consider rejecting the inflationary paradigm because it is not yet part of a complete solution to the ultimate mystery of the origin of the universe. For us, the implications go the other way: the successes that inflation has had in explaining the observed features of the universe give us motivation to explore the speculative ideas about the implications of inflation for questions far beyond what we can observe.

If inflation occurred in the early universe, then the evidence of its own initial conditions would be effectively erased, as described by the cosmic no-hair conjecture [13]. Thus, the earliest moments of inflation, or anything that might have come before, are extremely difficult to probe observationally. Nonetheless, the inflationary framework does provide resources with which to address important open questions, such as the initial conditions at or near the Planck scale. Within that framework, important advances have been made in recent years on topics such as eternal inflation [14], the multiverse and various proposals to define probabilities [15–20], and the possible role of anthropic selection effects [21–23]. Most important, as we discuss below, the inflationary paradigm has expanded beyond what was once the dominant view, prevalent in the 1980s, which tended to focus on a single phase of "chaotic" inflation [24]. Given recent progress on both the observational and theoretical fronts, we believe that the inflationary paradigm is in far better shape than ever before.

The remainder of this paper is organized as follows. In Section 2, we discuss the implications of the Planck 2013 data. In Section 3 we discuss the initial conditions for inflation, in Section 4 we discuss the issue of predictions in the multiverse, in Section 5 we discuss what ISL call the inflationary "unlikelihood problem," and in Section 6 we discuss the possibility that the Higgs potential turns negative at large field values. We summarize in Section 7.

2. Planck 2013 data

ISL argue that the Planck 2013 data prefers single-field inflation over more complicated possibilities, and that a "plateau-like" potential looks better than other simple potentials such as power-law potentials. They argue that these facts lead to significant challenges to inflation.

The relevant observational constraints on the shape of the potential come from $r$, the ratio of the power spectra of tensor and scalar perturbations. For single-field models, $r$ is proportional to the slow-roll parameter $\epsilon \equiv -\dot{H}/H^2$ and hence to $(V_\phi/V)^2$. Thus small values of $r$ require modest slope of the inflationary potential, at least in the vicinity of $\phi_0 = \phi(t_1)$, where $t_1$ is the time during inflation when cosmologically relevant length scales first crossed outside the Hubble radius.

On their own, the Planck data constrain $r < 0.12$ at the pivot scale $k_p = 0.002 \text{ Mpc}^{-1}$ at 95% CL [11]. This bound represents an impressive improvement from the WMAP9 constraint ($r < 0.38$ [25]), although it is comparable to the constraints that arise from combining WMAP data with data from the South Pole Telescope (SPT) and measurements of the baryon acoustic oscillations (BAO): $r < 0.18$ for WMAP7+SPT and $r < 0.11$ for WMAP7+SPT+BAO [26]. The Planck constraint is little changed if one incorporates data from SPT, BAO, the Atacama Cosmology Telescope (ACT), and large-scale polarization data from WMAP9; these combinations yield $r < 0.11–0.13$ [11].

The constraint $r < 0.12$ is low enough that the simple, single-field model with $V = \lambda \phi^4$ falls outside the 95% CL contour if one makes the usual assumptions about reheating and the thermalization energy scale after inflation. Another simple model, with $V = \lambda m^4 \phi^2$, lies at the boundary of the 95% CL contour, although it moves more squarely into the allowed region if the pivot scale corresponds to $N_s = 63$ e-folds before the end of inflation [27] rather than $N_s \leq 60$.

Thus the latest data, while certainly impressive, hardly rule out simple models with polynomial potentials, although they do constrain parameter space at the 1σ–2σ level. Nonetheless, ISL raise the conceptual question of whether plateau-like potentials are evidence against the inflationary paradigm. The main point of this paper is to argue that even if the final stage of inflation, as observed in the CMB, is determined definitively to occur on a plateau-like potential, the inflationary paradigm is not in trouble at all. As we discuss in the next section, the preferred scenarios might simply depart from a view about the onset of inflation that was commonly held two to three decades ago.

3. Initial conditions

In this section we will assume, for the purpose of discussing ISL’s conclusions, that the observable phase of inflation—the phase which we believe produced the density perturbations that we now measure in the CMB—indeed occurred on a "plateau-like" potential. The constraints on $r$ discussed above require the height of the plateau $V_I \equiv V(\phi_I)$ to be no bigger than about $10^{-12} M_{Pl}^2$, where $M_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck scale. Because this energy density is so low, ISL argue that one needs very fine-tuned initial conditions at the Planck scale in order to have an approximately homogeneous region of Hubble size after the energy density falls to the needed value. In particular, they argue that one cannot use the simple chaotic picture $\frac{1}{2} \dot{\phi}^2 \sim -\frac{1}{2} |\nabla \phi|^2 \sim V(\phi) \sim M_{Pl}^4$, where $\frac{1}{2} \dot{\phi}^2 \sim V(\phi)$, to start the observable inflation, since the plateau potential energy density cannot be too high. With $\frac{1}{2} \dot{\phi}^2 \sim -\frac{1}{2} |\nabla \phi|^2 \gg V \sim 10^{-12} M_{Pl}^4$ at the Planck era, ISL argue that a Hubble-sized region of homogeneity at the onset of inflation would require a region of homogeneity at the Planck scale of at least 1000 Hubble lengths.

We do not agree with this estimate, which in our view is based on false assumptions. A very plausible way to cool from the Planck scale to energy densities of order $V_I$, while maintaining homogeneity, is to imagine starting from a region of negative spatial curvature, $k < 0$, so that it locally resembles an open Friedmann–Robertson–Walker universe. Note that $k = 0$ would be a very special case, and that regions with $k > 0$ would collapse before reaching $V_I$, unless they were very close to being flat. The curvature term in the Friedmann equation, like the gradient energy $\frac{1}{2} |\nabla \phi|^2$, scales as $1/a^2(t)$, where $a(t)$ is the scale factor. The scalar field kinetic energy $\frac{1}{2} \dot{\phi}^2$ scales as $1/a^6(t)$, so the $1/a^2(t)$ terms will

---

1 Here $V_\phi = \dot{\phi}^2/2\phi$, where $\phi$ is the (scalar) inflaton field and $V(\phi)$ is its potential. We use overdots to denote derivatives with respect to cosmic time, $t$. The Hubble parameter is defined as $H = \dot{a}/a$, where $a(t)$ is the scale factor of the Friedmann–Robertson–Walker line element.

2 We thank Alex Vilenkin for pointing this out. Alternatively, universes with non-trivial topology, such as a torus, can also cool from the Planck scale to low energies while maintaining homogeneity [28]. In this scenario it is even possible for initial inhomogeneities to be smoothed by "chaotic mixing" [29,28].
quickly dominate, leading to the behavior $a(t) \propto t$. If we make the plausible assumption that a region of homogeneity grows with the expansion of the universe, then both the size of the region and the Hubble length grow proportionally to $t$, and a Hubble-sized region at the onset of inflation requires no more than a Hubble-sized region at the Planck scale.

While the above argument seems reasonable, critics might argue that we are being overly optimistic, because perhaps the inhomogeneity originating from outside the region propagates inward, limited only by the speed of light. In that case, the physical radius $r(t)$ of the region of homogeneity at the onset of inflation can be related to the radius $r(t)_{\text{Planck}}$ of homogeneity at the Planck scale by

$$r(t) = a(t) \left[ \frac{r(t)_{\text{Planck}}}{a(t)} - \int_{t_{\text{Planck}}}^{t} \frac{dt}{a(t)} \right].$$

If we set $r(t)_{\text{Planck}} = H^{-1}(t)$ and $H^2(t_{\text{Planck}}) \equiv \frac{8\pi}{3} M_{\text{Pl}}^2$, and assume that $a(t) \propto t$, the above equation becomes

$$r(t_{\text{Planck}}) = \frac{1}{H(t_{\text{Planck}})} \left[ 1 + \ln \left( \frac{H(t_{\text{Planck}})}{H(t)} \right) \right] > 13.9 H^{-1}(t_{\text{Planck}}).$$

where we have used the Planck 2013 95% CL constraint [11] that $H(t_{\text{Planck}}) < 3.7 \times 10^{-3} M_{\text{Pl}} / \sqrt{8\pi}$. Thus, even in this worst case scenario, the factor of 1000 given by ISL is replaced by a factor of 13.9. Synthesizing the calculations described in this paragraph and the previous one, we conclude that a Hubble-sized homogeneous region at the onset of inflation requires only a region of homogeneity at the Planck scale of order 1–15 Hubble lengths.

Besides our disagreement about the required size of the region of homogeneity at the Planck scale, we more significantly disagree with the entire premise of the argument. ISL’s argument is predicated on the assumption that the final stage of inflation—whose last $N \sim 60$ e-folds correspond to the observable inflation—was the end of an uninterrupted phase of inflation that began at the Planck scale. That requirement is tantamount to assuming that the symmetry of the Coleman–De Luccia instanton would ensure that any particular state of inflation is selected by a Hubble-sized homogeneous region at the onset of inflation requires only a region of homogeneity at the Planck scale of order 1–15 Hubble lengths.

For example, the final stage of inflation could plausibly have begun by tunneling from some other metastable state. In that case, the inflation in the previous metastable state together with the symmetry of the Coleman–De Luccia instanton [32] would ensure spatial homogeneity (small $|\nabla \phi|^2$) prior to the last stage of inflation. (Since the bubble nucleation rate is exponentially suppressed, it is highly likely that the field before the tunneling event was in a metastable state, providing the right circumstances for Coleman–De Luccia tunneling.) Moreover, the evolution of the bubble universe after tunneling begins with $\phi = 0$ and strong Hubble damping of $\dot{\phi} (H \sim 1/t)$, so at least in simple models [33,34] $\dot{\phi}$ is never large enough to interfere with the onset of slow-roll inflation.

In this scenario, the universe would be homogeneous before the final stage of inflation. Yet the universe immediately after tunneling would be an empty, curvature-dominated (open) universe. To produce a matter-filled universe like the one in which we live, the tunneling would have to be followed by a period of slow-roll inflation. (Since the curvature term in the Friedmann equation falls off as $1/a^2$, after a time it can become dominated by vacuum energy, with $\rho_{\text{vac}} > \text{const}$. Neither matter ($\rho_{\text{mat}} \sim 1/a^4$) nor radiation ($\rho_{\text{rad}} \sim 1/a^6$) can overcome the curvature term, except through an intermediate stage of vacuum energy domination.) Followed by the standard reheating process, the inflation would lead to a hot big-bang universe. Moreover, if the duration of the slow-roll inflation were longer than $N \sim 60$ e-folds, then the flatness of the universe would be explained [1,2], and the origin of structures could proceed as envisioned in Ref. [35], and calculated in Ref. [36].

The point we wish to emphasize is that inflation with what we consider a realistic form of $V(\phi)$, containing many local minima and hence many metastable states, would generically lead to multiple phases of inflation. The observable properties of our universe today, as seen for example in the CMB, would be sensitive to the final phase of inflation, whereas details of the earlier processes would likely remain hidden from view, having been stretched far beyond the current horizon by the last $N \sim 60$ e-folds of inflation. Given the well-known attractor behavior of slow-roll inflation [6,37], quantitative predictions for observable quantities such as $\Omega_k$, $n_s$, $r$, and $\alpha = d\ln r / d\ln k$ are essentially independent of anything that preceded the final phase of inflation. Like any self-consistent effective field theory, inflation can be used to make specific predictions without knowing the exact behavior of the theory at arbitrarily high energies. In particular, the predictions do not require knowledge of the prior phases of inflation [8] or of Planck-scale physics [38].

ISL considered (in their Fig. 1d) the possibility that we discuss, with a tunneling episode prior to the slow-roll inflation, raising two criticisms with which we disagree. First, they argue that this approach involves adding “complicated features … for the purpose of turning an unlikely model into a likely one.” From our point of view, such “complicated features” are highly plausible in the context of the current understanding of particle theory. ISL further argue that the plateau shape of the low-energy part of the potential is not a consequence of inflation, but instead is chosen only to fit the Planck data, a situation which they describe as “trouble for the [inflationary] paradigm.” It is of course true that inflation does not determine the shape of the potential, and indeed most inflationary theorists, including us, would consider a $\frac{1}{2} m^2 \phi^2$ or a $\lambda \phi^4$ potential to be a priori quite plausible for the low-energy part of the potential. But this only means that (given current technological theory) the details of inflation will need to be determined by observation. Many of the features of the standard model of particle physics are also determined by observation; this situation might suggest that some deeper theory underlies the standard model, but we do not think that it spells trouble for the standard model paradigm.

So far our arguments have depended only on the recognition that $V(\phi)$ might plausibly be a complicated function, with many local minima, as suggested by current ideas in particle theory, such as string theory. But once we consider a potential energy function with more than one metastable local minimum—or any potential

---

3 The claim that this is the worst possible case can be investigated rigorously in the context of classical general relativity, and the relevant theorems are discussed in detail by Wald [30]. For Einstein’s equations in vacuum, Theorem 10.2.2 implies as a special case that if an initial spacelike slice contains a region $S$ described by a Robertson–Walker metric, then the future domain of dependence of $S$—the region calculated in Eqs. (1) and (2)—will be unperturbed, regardless of what is outside the region $S$. Conditions outside of $S$ cannot affect anything inside the future domain of dependence, and cannot affect where the boundary of that region occurs. On pp. 266 and 267, Wald discusses extensions of this theorem when matter is included, at least for simple forms of matter. While the theorem has not been proven for all forms of matter, we think it is safe to assume that any acceptable theory of matter would satisfy these basic causality properties.
energy function with a gentle plateau region—then eternal inflation seems unavoidable. ISL refer to this as the “multiverse problem.” While we do not consider it a “problem,” we agree that the multiverse is a very likely consequence. Regions filled with metastable “vacua” will generically inflate at a rate much faster than they decay, so the volume of inflating regions will grow exponentially as a function of the proper time, with no upper limit. The metastable vacua will decay by bubble nucleation, producing “pocket universes” at a rate that grows with the volume, and hence exponentially as a function of the proper time.

If this multiverse picture is combined with rather mild assumptions about anthropic selection effects, then it becomes very plausible that we live in a pocket universe which has undergone inflation, with no particular prejudice about whether the potential is plateau-like or not. As described above, the pocket universe after tunneling would be a homogeneous open universe, with the scalar field that tunneled starting with \( \phi = 0 \). The amount of slow-roll inflation that follows depends on the shape of the potential. Statistics alone would presumably favor small amounts of inflation if any, but Refs. [33] and [39] argue that simple assumptions about the probability distribution for slow-roll potentials imply that the probability density for having \( N \) e-folds of slow-roll inflation falls off for large \( N \) only as a power of \( N: P(N) \sim N^{-p} \), with \( p > 0 \) and \( p \sim O(1) \). Furthermore, Ref. [33] argues there is an anthropic minimum for the duration of the slow-roll inflation, \( N \geq 59.5 \), based on the requirement that galaxy formation is possible. We consider this a rather mild anthropic assumption, motivated by logic that parallels closely the logic of the anthropic bound on vacuum energy density [21]—both vacuum energy and curvature suppress the growth of structure.

Although we have invoked an anthropic constraint on \( N \), we emphasize that inflation is an essential part of the explanation. The anthropic constraint alone would give only \( \Omega_{k} \leq O(1) \). To explain the observed flatness of the universe, \( |\Omega_{k}| \lesssim 0.01 \) [11], we need inflation, with its exponential sensitivity of \( \Omega_{k} \) on \( N \), \( \Omega_{k} \propto e^{-\sigma} \). Only about 2.5 e-folds of inflation beyond the minimum are needed to explain the observed level of flatness. According to the estimates of Refs. [33] and [39], the relevant conditional probability is large: given that there is enough inflation to satisfy the anthropic cut \( (N \geq 59.5) \), the probability that there is enough inflation to explain the observed level of flatness \( (N \geq 62) \) is very nearly one.

To summarize, the possibility that the final stage of inflation was preceded by a bubble nucleation event is at least one way that fine-tuning issues can be avoided. The prior inflation in a metastable state can occur without any significant fine-tuning of initial conditions—all that is necessary is that the inflaton field roll down to a local minimum with positive energy density. The tunneling must be followed by a period of slow-roll inflation, but with a complicated \( V(\phi) \), as we would expect, it is very plausible that this occurs somewhere in field space. Anthropic selection effects can then make it plausible that we live in a pocket universe that evolved in this way. Moreover, one need not know how our observable universe came to undergo its final phase of inflation in order to make specific, quantitative predictions for observable quantities today.

4. The multiverse and predictability

ISL refer to a “multiverse–unpredictability” problem, and in the discussion they raise two issues. First, they argue that the plateau potentials favored by Planck will lead to eternal inflation, and hence the measure problem [40]. We agree that if the observable inflation occurred on a plateau-like potential, eternal inflation seems very likely. It can occur either while the scalar field is at or near the top of the plateau, or in a metastable state that preceded the final stage of inflation. We also agree that this leads to the measure problem: in an infinite multiverse, we do not know how to define probabilities. Since anything that can happen will happen an infinite number of times, the distinction between common events and extremely rare events requires a comparison of infinities, and that requires some method of regularization. We do not yet know what is the correct method of regularization, or even what physical principles might determine the correct answer. While we agree that this question is unanswered, we feel that acceptable measures (i.e., regularization prescriptions) have been proposed (e.g. [16–20]), and that the mere fact that we have not solved this problem is no reason to believe that nature would avoid eternal inflation. Nature does not care whether we understand it or not. However, since the measure problem is not fully solved, ISL are certainly justified in using their intuition to decide that eternal inflation seems unlikely to them. To us, the measure problem is simply an important problem that remains to be solved.

One reason for believing that the measure problem must be solved anyway is that the circumstances that lead to it are hard to avoid. The cyclic model, for example, seems to also have a measure problem. In Ref. [41], Johnson and Lehners study cyclic models that include a dark-energy-dominated phase, concluding that there is a measure problem, with probabilities that depend on a cut-off procedure for which there is no a priori way to determine. They go on to claim that it is easier to find an acceptable measure in the context of cyclic cosmology, but the existence of the measure problem is not avoided. While Johnson and Lehners confined their remarks to a subclass of cyclic models, we believe that the measure problem exists in all cyclic models. The measure problem pertains to all probabilities, not just the probability of finding oneself in a particular phase. For any model in which anything that can happen will happen an infinite number of times, the measure problem applies.

The second issue that ISL include in the multiverse–unpredictability discussion is a claim that if there is a multiverse, then we should observe a large number of many-\( \sigma \) deviations from predictions of our theories. That is, they argue that inflation fails because it describes the data too well. We emphasize that the existence of a measure problem does not mean that probability theory fails. The different measures that have been proposed, and presumably the correct measure that we seek, obey all the standard properties of probability theory. (Anything that can happen will happen, but not with equal probability.) The predicted probabilities will depend on the measure, but we should expect that they will not differ radically from naive expectations, just as physicists in the 1920’s could have expected that the emerging theory of quantum mechanics was not going to predict that cars should start tunneling out of their garages. That is, any acceptable extension of the laws of physics must be consistent with the older theories in the regime where the older theories have been tested successfully. The measures discussed in Refs. [16–20] all fit this criterion. Since all the basic axioms of probability theory are intact, an event with a probability of 1/10 would be expected to occur about once in every 10 trials, as usual.

5. Inflationary “unlikeliness problem”

ISL admit that inflationary plateau-like models obviously pass the test of giving predictions that agree with observation, thereby satisfying the criterion that is generally used to define the success of a theory. They argue, however, that this is not enough. In what they call the inflationary “unlikeliness problem,” they contend that inflation occurring on a plateau is exponentially unlikely compared to inflation in a power-law potential. As a simple example, they consider the plateau potential
\[ V(\phi) = \lambda (\phi^2 - \phi_0^2)^2. \] (3)

which has a plateau for \( |\phi| \ll \phi_0 \), but behaves like a power-law potential for \( |\phi| \gg \phi_0 \). They argue that there is a much larger range \( \Delta \phi \) of scalar field values available in the power-law region, and a much larger maximum for the number \( N_{\text{max}} \) of e-folds of inflation, implying that inflation on the power-law part of the potential is exponentially more likely than inflation on the plateau.

In making this claim, ISL seem to have put themselves in the peculiar position of arguing, on the one hand, that eternal inflation leads to infinities, “potentially rendering inflationary theory totally unpredictable,” while at the same time arguing that they can tell us what inflation predicts, and that it is unambiguously at odds with the plateau behavior that the Planck observations favor.

At the level of inflation-scale physics, inflation on the power-law and the plateau parts of the potential of Eq. (3) are two distinct models, each perfectly consistent. In comparing the likelihood of the two we need to consider Planck-scale physics, asking which inflation-scale scenario is more likely to develop from an assumed description at the Planck scale. Since we and ISL agree that these models lead to eternal inflation, it is in this context that we will discuss their argument. We believe that it is possible to make plausible predictions about Planck-scale issues in the context of eternal inflation, but they must be made carefully, choosing a probability measure which is at least free of known problems. Unlike ISL, we would view the success or failure of such predictions not as a test of the inflationary paradigm, but rather as part of our exploration of the measure problem. So far predictions based on multiverse calculations have been pretty much limited to gross dynamical properties of the universe, such as the cosmological constant [16, 42] or \( \Omega_m \) [43]. Detailed particle physics issues, like the relative likelihood of finding a scalar field in one range of values vs. another, depend sensitively on the underlying particle dynamics, and do not appear to be even approachable at the present time. Thus, if we assume that the final slow-roll inflation occurred in the potential of Eq. (3), in our view there is no way of knowing whether we should expect it to have occurred on the plateau or on the power-law part of the potential.

Nevertheless, ISL’s argument for an “inflationary unlikeliness problem” sounds reasonable on a first reading, so we would like to look at it more carefully. They argue that inflation on the power-law side of the potential is more likely because it allows a much larger range \( \Delta \phi \) of the scalar field, and a much larger maximum number \( N_{\text{max}} \) of e-folds of inflation.

We consider first the claim that the larger range \( \Delta \phi \) of scalar field values implies that inflation on the power-law side is more likely because it allows a much larger range \( \Delta \phi \) of the scalar field, and a much larger maximum number \( N_{\text{max}} \) of e-folds of inflation.

We consider first the claim that the larger range \( \Delta \phi \) of scalar field values implies that inflation on the power-law side is more likely. If the physical system consisted of a single scalar field \( \phi \) which started with random initial conditions at the Planck scale, then ISL’s argument would be persuasive. But the situation is not so clear if we consider the multiverse, or even if we just consider a more complicated system of scalar fields. For example, if the plateau inflation was preceded by a tunneling event, then \( \Delta \phi \) and \( N_{\text{max}} \) for the power-law side would have to be compared not with the corresponding values for the plateau, but instead with the corresponding values for the metastable state from which the tunneling occurred. In this case ISL’s argument for the larger amount of inflation on the power-law part of the potential completely disappears.

Furthermore, it is easy to construct models in which the plateau center, \( \phi = 0 \), is an enhanced symmetry point, which can make it a likely endpoint of either tunneling or the stochastic evolution of scalar fields, even if the range of \( \phi \) on the plateau is very small. For example, with multiple scalar fields in the theory, it is quite plausible that there are metastable vacuum states for which the inflaton field \( \phi = 0 \), with nonzero values for some or all of the other fields. The dominant decay of such states could very plausibly maintain \( \phi \approx 0 \). Similarly, if the potential energy function includes a term \( \lambda \phi^2 \psi^2 \), where \( \psi \) is another scalar field which has a large value at early times, then \( \phi \) can plausibly settle into its minimum energy state, \( \phi \approx 0 \), before \( \psi \) becomes small.

Turning now to the claim that the probability of inflation on the power-law side is exponentially enhanced by the larger value of \( N_{\text{max}} \), we first point out that this argument also disappears if we assume that the final stage of inflation was preceded by a tunneling event. But even if that is not the case, the issue of whether a large \( N_{\text{max}} \) leads to a large probability is precisely the kind of question that plays a major role in the discussion of measures, and hence must be handled with care.

The simplest measure, known as proper-time cutoff measure [17, 44], selects a finite sample spacetime volume of the multiverse by considering only events that occur before a final cutoff hypersurface that is chosen as a hypersurface of constant proper time. The relative likelihood of events of different types is determined by counting the numbers of events in this sample spacetime volume, and then taking the limit as the final proper-time hypersurface is taken to infinity.

While the proper-time cutoff measure seems intuitive, it has been found to lead to a gross inconsistency with experience, often called the “youngness problem" [8, 45–47]. The problem is driven by the huge disparity in time scales: the scale factors of the most rapidly inflating metastable false vacua are expected to have time constants of perhaps \( \tau_{\text{min}} \sim 10^{-37} \) s, while the time scales relevant to the questions we ask might range from seconds to gigayears. The growth of the sample spacetime volume is dominated by the most rapidly inflating vacua, so it is expected to grow as a function of \( \tau_{\text{cutoff}} \) with a time constant close to \( \tau_{\text{min}} \). Since the growth is exponential, most of the spacetime volume will lie within a few time constants of the final hypersurface. Thus, most of the pocket universes that form in the sample spacetime volume nucleate within a few time constants of the final hypersurface, and pocket universes that are older by some time interval \( \Delta \tau \) are suppressed in probability by the smaller volume available at these earlier times, proportional to \( e^{-3\Delta \tau/\tau_{\text{min}}} \). Proper-time cutoff measure implies, therefore, that the statistical distribution of pocket universes is strongly biased toward very young universes. Pocket universes as old as \( \Delta \tau = 14 \) billion years, for example, are suppressed by a factor such as \( e^{-3\Delta \tau/\tau_{\text{min}}} \sim 10^{-10^{35}} \). Tegmark [8] connects this strongly biased probability distribution to observation by estimating the probability that we find ourselves in a pocket universe old enough for the CMB temperature to be less than 3 K, finding that \( P(T_{\text{CMB}} < 3 \text{ K}) \sim 10^{-10^{36}} \). Thus, the proper-time cutoff measure is emphatically ruled out by observation.

ISL recognize the failure of the proper-time cutoff measure, which they describe as “weighting by volume,” and yet their ar-

---

4 It is conceivable that string theory might not even allow field values much larger than \( M_P \), in which case the large range of \( \Delta \phi \) on the power-law side would be an illusion.

5 Proper time is of course not a globally defined quantity in general relativity, so the meaning of a proper-time cutoff needs to be described more carefully. One begins by choosing an arbitrary initial spacelike hypersurface of finite extent. One then constructs a congruence of geodesics that begin on this hypersurface and normal to it, extending toward the future. The sample spacetime region is then chosen to be the region swept out by these geodesics, each followed for a proper time \( \tau = \tau_{\text{cutoff}} \). It is expected that as \( \tau_{\text{cutoff}} \to \infty \), the resulting probabilities become independent of the choice of the initial hypersurface.

6 Tegmark’s result is more extreme than the probability of \( 10^{-10^{37}} \) that we quote based on age, a discrepancy that appears to be due to unimportant approximations. The bottom line however is clear: in proper-time cutoff measure, it is absurdly improbable for us to find ourselves in a universe as old as what we see.
argument about a purported “inflationary unlikeliness problem” critically depends upon that flawed measure. Measures that are currently considered acceptable, such as those in Refs. [16–20], all avoid the youngness problem in one way or another, and they also satisfy several other consistency checks (see, for example, Ref. [48]). All lead to the conclusion that the probability of finding oneself in a particular type of pocket universe is not enhanced by the amount of slow-roll inflation that occurs in that type of pocket universe.

Of the various known ways to avoid the youngness problem, perhaps the simplest to describe is the scale-factor time cutoff measure, which differs from the proper-time cutoff measure only by the choice of the time variable. Instead of using proper time to define the final cutoff hypersurface, one uses scale factor time \( t_{sf} \), which in the context of a Friedmann–Robertson–Walker universe is just equal to the logarithm of the scale factor. In an arbitrary spacetime, scale factor time can be defined as \( 1/3 \) times the logarithm of the volume expansion factor. The volume of any comoving region then increases as \( e^{3t_{sf}} \), so there is no youngness problem, since there is no large discrepancy in time scales, and regions with large values of \( H \) are no longer given an enhanced weight. Note that this measure is just as much volume-weighted as the proper-time cutoff measure—it is just the time coordinate that is different. Once the cutoff is chosen, all volumes under the cutoff are counted equally. Note also that we did not use the Planck data to influence our choice of measure; the absence of an enhancement from slow-roll inflation was implied by other—rather basic—considerations about probabilities, independent of any CMB data.

Without the strong exponential preference, the relative probabilities of the two starting points for the last stage of inflation—plateau-like or outer wall—become the issue of complicated dynamics in the multiverse, and we are unable to compute which will dominate with our current knowledge and technology.

To summarize, we have argued that there is no reason to conclude that inflation predicts that plateau inflation is unlikely. We have indicated that the larger range \( \Delta \Phi \) of scalar field values allowed for power-law inflation is irrelevant if the final stage of plateau inflation is preceded by a tunneling event, and that in multifield models the plateau can be favored as an enhanced symmetry point. We have also pointed out that in proposals for a probability measure that are currently considered acceptable, the larger amount of inflation that might be expected for power-law potential inflation does not lead to an enhanced probability.

6. Inflation and the LHC

ISL close their paper with one final argument, which in this case is based on the LHC, rather than Planck 2013. They argue that perhaps the absence of evidence for physics beyond the standard model should be extrapolated to a claim that there is no new physics up to the Planck scale. In that case, given the measurements of the top quark and Higgs masses, the Higgs field potential energy function is predicted to reach a maximum, and then to decrease to a large negative vacuum energy density. While the barrier is high enough to give a lifetime for our vacuum that is large compared to the age since the big bang, ISL argue that it is highly unlikely for the Higgs field to end up in the tiny pocket around the correct electroweak breaking minimum, since there is a vastly larger region of field space in which it would roll toward Planck values.

As ISL noted themselves, the problem may evaporate if there is new physics below the scale at which the Higgs potential energy turns around. For example, if there is supersymmetry around a TeV scale or at a scale not too far from a TeV, e.g., a few orders of magnitude above a TeV as in the scenario in Ref. [49], then the problem is avoided. Alternatively, supersymmetry may exist at an intermediate scale [50], which may also prevent the Higgs potential from turning around. (This is the case if the supersymmetric threshold is slightly below the scale at which the standard model Higgs quartic coupling would vanish.) Nonminimal couplings of the Higgs field, either to gravity [51] or to the inflaton [51,52], can also obviate the problem. It was also pointed out in Ref. [53] that the turnaround of the Higgs potential can be avoided by the Peccei–Quinn mechanism, which with other assumptions can even lead to a phenomenologically successful relationship between the Higgs mass and the dark matter density.

Nevertheless, the suggestions of the previous paragraph are speculative, and it is possible that the Higgs potential actually does become negative above an intermediate scale, e.g., around \( 10^{11} \) GeV, and that there is no new physics up to the Planck scale, as ISL suggest. We would argue that, even in this case, there is no problem for inflation in the context of an eternally inflating multiverse. The key issue is that there is no plausible way that regions in which the Higgs field has run off to Planckian values could support life. The large negative vacuum energy density is enough to ensure that these regions would collapse to a crunch on time scales far shorter than a second, leaving only those (initially very rare) regions where the Higgs field has rolled toward small values. It has always been assumed that the multiverse includes a large number of types of pocket universes that do not support life, so the possibility described by ISL merely adds one to that number. For the multiverse framework to be consistent, it is only necessary that the probability that intelligent observers find themselves in a pocket universe like ours is not unreasonably small.

For the Higgs field to remain in the region within the potential maxima during inflation, there are constraints on various inflationary parameters, derived in references cited by ISL [51,54], that must be obeyed. In particular there are constraints on the energy scale of inflation, on the amplitude of tensor fluctuations, and on the amplitude of density perturbations for the case of power-law potentials, but none of these pose trouble for inflationary models.

While we see no reason to be concerned with the case described by ISL—the case in which the standard model holds exactly up to the Planck scale, with a Higgs potential that turns negative—in the context of the multiverse there are other interesting possibilities. One could imagine, for example, a vacuum in the landscape for which the physics is given by the standard model, except for an offset in the vacuum energy density which makes the value very near zero when the Higgs field is at the Planck scale. Such a universe would still be inhospitable to life: with the values of gauge and Yukawa couplings taken from the standard model, a Planck-scale Higgs expectation value would make all the standard model particles so heavy that they would presumably not even be created during reheating, leaving a universe populated only by photons (and possibly neutrinos).

One might argue that if the Yukawa couplings vary, becoming vanishingly small in such a way that the masses of the quarks and leptons are fixed at their standard model values, then the resulting universe might not be very much different from ours [55]. In this case, the probability of finding ourselves in such a universe is limited by the probability of obtaining such tiny couplings in

---

Footnote: In more detail, one begins by choosing an arbitrary initial spacelike hypersurface of finite extent, as in proper-time cutoff measure, and again one constructs a congruence of geodesics that begin normal to this surface. An interval of scale factor during reheating, leaving a universe populated only by photons

\[ \frac{dt}{Hsf} = \frac{dH}{H} \]

7 This interval of scale factor time along a worldline is given by \( dt_{sf} = HdH \), where \( dt \) is the proper time interval and \( H \) is the local expansion rate of the geodesic congruence. Equivalently, one could imagine filling the initial hypersurface with a uniform density of dust particles that subsequently follow the geodesics of the congruence. Scale factor time is then \( -1/3 \) times the logarithm of the density of dust.
the landscape, which is likely to be small. In addition, there may be anthropic reasons associated with the absence of weak interactions that prevent life in such a universe, despite the analysis of Ref. [55]. In any event, while one may continue to speculate about conceivable vacua, the exercise would only pose trouble for the inflationary paradigm if someone identified a class of vacua in the landscape which could be shown to strongly dominate over our vacuum in probability. This has not happened.

7. Conclusions

Inflationary cosmology rests on very firm foundations. Rather than relying on untested (though certainly interesting) speculations about additional spatial dimensions or repeated collisions of hypothetical branes, inflation builds upon decades of in-depth study of quantum field theory in curved spacetime. Like many other successful modern physical theories, inflation may be understood as an effective field theory, capable of making specific, quantitative predictions for observables in various energy regimes of interest, even in the absence of complete knowledge of physics at arbitrarily high energy scales. Many of those quantitative predictions have been subjected to empirical tests across a wide range of experiments, including most recently with the Planck satellite. Every single test to date has shown remarkably close agreement with inflationary predictions.

We agree with Lijias, Steinhardt, and Loeb [12] that important questions remain. A well-tested theory of physics at the Planck scale remains elusive, as does a full understanding of the primordial singularity and of the conditions that preceded the final phase of inflation within our observable universe. Likewise, although significant progress has been made in recent years, a persuasive theory of probabilities in the multiverse has not yet been found. We strongly disagree with ISL, however, that these remaining challenges represent any sort of shortcoming of inflationary cosmology. Quite the opposite: the inflationary paradigm, with its many successes, provides a framework within which such additional questions may be pursued.

In assessing the criticisms of inflation by ISL, we have identified several assumptions in their arguments that we consider problematic. Most stem from an outdated view in which a single phase of inflation is assumed (or required) to persist from the Planck scale to the observable scale. None of the quantitative predictions from inflationary cosmology for various observables require such an assumption, nor does such an assumption seem at all realistic in the light of recent developments in high-energy theory.

Recent experimental evidence, including the impressive measurements with the Planck satellite of the CMB temperature perturbation spectrum and the strong indication from the LHC that fundamental scalar fields such as the Higgs boson really exist, put inflationary cosmology on a stronger footing than ever. Inflation provides a self-consistent framework with which we may explain several empirical features of our observed universe to very good precision, while continuing to pursue long-standing questions about the dynamics and evolution of our universe at energy scales that have, to date, eluded direct observation.

Acknowledgements

We would like to thank Bruce Bassett, Andrei Linde, and Alex Vilenkin for helpful conversations. Since our writing of this article Linde has posted a manuscript based on his 2013 Les Houches lectures [56], which also contains counterarguments to ISL in the appendix; see also his 2013 lecture at the KITP [57]. This work was supported in part by the U.S. Department of Energy (DOE) under cooperative research agreement DE-FG02-05ER41360. The work of Y.N. was also supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC02-05CH11231, the National Science Foundation under grant PHY-1214644, and the Simons Foundation grant 230224.

References


A. Linde, D. Linde, A. Mezhlumian, From the big bang theory to the theory of quantum cosmology, Phys. Rev. D 49 (1994) 1783, arXiv:gr-qc/9306035. This paper is mainly concerned with the properties of the proper time cutoff measure, which is no longer considered acceptable, but it also introduces scale factor cutoff measure as an alternative.


See also, T. Banks, TCP, quantum gravity, the cosmological constant and all that…, Nucl. Phys. B 249 (1985) 332;


A.H. Guth, Inflation and eternal inflation, in Ref. [40].


B. Freivogel, Making predictions in the multiverse, in Ref. [40].


