International risk cycles

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ABSTRACT

Recent work in international finance suggests that the forward premium puzzle can be accounted for if (1) aggregate uncertainty is time-varying, and (2) countries have heterogeneous exposures to a world aggregate shock. We embed these features in a standard two-country real business cycle framework, and calibrate the model to match the differences between low and high interest rates countries. Unlike traditional real business cycle models, our model generates volatile exchange rates, a large currency forward premium, "excess comovement" of asset prices relative to quantities, and an imperfect correlation between relative consumption growth and exchange rates. Our model implies, however, that high interest rate countries have smoother quantities, equity returns and interest rates than low interest rate countries, contrary to the data.

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1 Introduction

Dynamic stochastic general equilibrium models used in international macroeconomics have been reasonably successful at matching macroeconomic quantities, but are inconsistent with basic features of asset prices and exchange rates, such as their volatility, correlation across countries, and correlation with interest rates and other macroeconomic aggregates. Moreover, these models imply very small equity and currency risk premia, in contrast to the large average excess returns that are well documented in the empirical finance literature.

Recent work in international finance, notably Bekaert (1996), Bansal (1997), Backus et al. (2001), and Lustig et al. (forthcoming), suggests a possible reconciliation between asset prices (including real exchange rates) and quantities. The failure of the uncovered interest rate parity condition (UIP) and the high average return to a “carry trade” strategy imply two restrictions on any no-arbitrage model: (1) aggregate risk must be time-varying, and (2) countries must differ in their exposures to aggregate risk.

We embed these features in a standard two-country real business cycle (RBC) model in the tradition of Backus et al. (1992). Following Rietz (1988), Barro (2006), Gabaix (2008), and Gourio (2009), we model time-varying risk as a small, time-varying probability of a worldwide economic disaster, such as the Great Depression. Specifically, a disaster is modeled as a simultaneous large permanent decline in productivity and a capital destruction. One country is assumed to have a higher exposure to the world disaster—the productivity and capital shocks are larger if a disaster is realized. These assumptions imply that conditions (1) and (2) above are satisfied.

In the model, variations in disaster probability lead to changes in risk premia and asset prices and to variations in macroeconomic quantities. An increase in the probability of disasters leads to a decline of investment, because higher uncertainty makes it less attractive to hold risky capital. Output and employment also fall given the lower investment demand, leading to a recession. These business cycle dynamics occur with no change in total factor productivity (TFP), and even if no disaster is realized in sample. Moreover, stock prices and
interest rates fall, while spreads on risky securities increase. In our model, the two countries have different levels of riskiness, and the recession and the ensuing decline in stock prices are stronger in the more risky country—the country that would be hit hardest by a disaster realization.

Because of higher precautionary savings, the more risky country has a lower interest rate. When the disaster probability rises, the exchange rate of this country appreciates, because its marginal utility rises more. As a result, high interest rate currencies tend to depreciate during bad times, as they do in the data. This implies that the return on a carry trade strategy is risky, leading investors to require on average a positive excess return to compensate for this risk; hence, our model is consistent with the average return on the carry trade.

We evaluate our model by comparing its predictions to the data for a large set of moments. We start by documenting some new stylized facts about asset prices and business cycle dynamics for high and low interest rate countries. We show that equity returns, interest rates, and business cycles are more volatile in high interest rate countries, while average equity excess returns are lower.

We find that our simple framework helps to reconcile international RBC models with the data. The model implies that (1) real exchange rates are about twice more volatile than in the basic RBC model; (2) macroeconomic aggregates are more correlated across countries than in the RBC model; (3) asset returns are more correlated across countries than macroeconomic aggregates, with a correlation of about 0.9, (4) the correlation of relative consumption growth and the exchange rate is 0.6, versus 1.0 in the standard model (the Backus and Smith (1993) anomaly) and (5) the UIP condition is not satisfied and a carry trade strategy delivers a 2.5% annual average excess return, compared to essentially zero in the RBC model. As in Gourio (2009), our model is also consistent with business cycle and asset pricing facts within each country, such as the mean and volatility of equity and risk-free returns, the predictability of returns, and the correlation between asset prices and investment or output.
Our model, however, falters in a fundamental and important way: in our model low interest rate countries are the most risky, and hence have volatile quantities and returns. In the data, the low interest rate countries have lower volatilities. This tension appears difficult to resolve, and calls for future research.

One important element of our approach is recursive preferences (Epstein and Zin, 1989). We assume that risk aversion is higher than the inverse of the intertemporal elasticity of substitution (IES). This implies that agents prefer an early resolution for uncertainty. These preferences decouple consumption from marginal utility of wealth. As a result, the exchange rate is not perfectly correlated with the relative consumption growth, allowing us to make progress on the Backus and Smith (1993) anomaly. Moreover, recursive preferences imply that volatility shocks are priced: as a result, times of heightened uncertainty (when the probability of disaster is high) have a high marginal utility, even though consumption might initially rise. Intuitively, the marginal value of consumption now depends not only on current consumption, but also on future (continuation) utility. When risk increases, future utility is low, both because of a low mean and a high uncertainty. This low future utility increases the marginal utility of wealth immediately. Hence, assets that have low payoffs when disaster risk is high have positive excess returns.

Finally, we test the key mechanism of the model: an increase in disaster probability should negatively affect output and employment. Three clear results emerge. First, we find that equity implied volatility in the United States is highly correlated with the risk of large drops in U.S. equity prices, as measured in equity option markets over the last 15 years. This is consistent with our model, in which high probabilities of disaster are associated with high equity return volatilities. Thus, a change in volatility can be used as a proxy for a change in disaster probability. Building on this finding, we study the response of macroeconomic quantities and prices to volatility shocks. We measure volatility as the monthly standard deviation of realized daily equity returns. Second, we find that there is substantial co-movement of volatility worldwide. This is consistent with the model because we assume
that the disaster probability is common across countries. Third, aggregate volatility affects macroeconomic outcomes. We find that when global volatility (measured as average volatility of equity returns across G-7 countries) increases, industrial production falls and unemployment rises in all G7 countries. These results generalize Bloom’s (2009) empirical findings for the U.S. to a large set of Organisation for Economic Co-operation and Development (OECD) countries and different measures of volatilities.

The paper is organized as follows. The rest of the introduction reviews the related literature. Section 2 documents a set of stylized facts on macroeconomic quantities, exchange rates and assets prices, for low and high interest rate countries. Section 3 presents our model without international trade, parametrizes this model, and studies whether it can match the facts established in Section 2. Section 4 tests directly for the model key mechanism by studying empirically the impact of global uncertainty on macroeconomic variables. Section 5 concludes. A separate Online Appendix presents data sources, additional data statistics, our computational method, as well as some additional model results and robustness checks.

This paper contributes to the vast theoretical literature on international real business cycles. We do not attempt to summarize it here.\footnote{Some seminal contributions are Backus et al. (1992), Obstfeld and Rogoff (1995), Heathcote and Perri (2002), Chari et al. (2002), Devereux and Engel (2002), and Engel and West (2005). Some key recent contributions are Dotsey and Duarte (2008), Corsetti et al. (2008a), and Engel et al. (2007).} The paper is also closely related to the recent macroeconomics and finance literature on disaster risk.\footnote{See notably Rietz (1988), Barro (2006), Gabaix (2008), Gourio (2009), Liu et al. (2005), Wachter (2008), Barro et al. (2010). Jurek (2008), Backus et al. (forthcoming), and Farhi et al. (2009) confront the disaster risk model to option data.}

Closest to our work are recent studies by Farhi and Gabaix (2008) and Guo (2009), who are the first to show that a disaster risk model can reproduce the UIP puzzle. Our contribution is to extend their work to a standard production economy with Epstein and Zin (1989) preferences, using a wider set of data moments to evaluate the model, and to uncover a new tension between models of carry trades and macroeconomic volatilities. Our work is
also related to Gourinchas et al. (2010), who document that the U.S. provides insurance to the rest of the world, especially in times of global stress. They show that a simple disaster risk model accounts for the large collapse in U.S. net foreign assets.

While we model time-varying aggregate risk using a time-varying probability of worldwide disaster, similar results can be derived with stochastic volatility, as emphasized in the long-run risk model of Bansal and Yaron (2004). Bansal and Shaliastovich (2008), Colacito (2008), and Colacito and Croce (2011a) show that this class of models also offers a potential explanation to the UIP puzzle. For the case of a two-country endowment economy, Colacito and Croce (2011b) show that Epstein and Zin (1989) preferences and exposure to a small, but highly persistent, common source of risk can help explain some regularities of the behavior of exchange rates, replicating both the volatility of real exchange rates and the correlation of pricing kernels across countries. Our paper provides the production economy foundations for the common source of risk. Lustig, Roussanov, and Verdelhan (2010), by focusing on reduced form pricing kernels, show that heterogenous exposure to a world risk factor is needed in order to account for currency risk premia. Our paper provides an economic interpretation for the source of this heterogeneity, as the cross-sectional heterogeneity of the magnitude of capital destruction following a disaster.

This paper builds on the disaster risk literature but leaves open other interpretations. While the risk of an economic disaster is, in our model, a strictly rational expectation, it can also be interpreted as an irrational belief, which is potentially biased, or excessively volatile. We examine the macroeconomic effects of a time-varying belief for international economics. The time-varying disaster risk captures the idea that there are some asset price changes that are not obviously related to current or future TFP, i.e., “bubbles,” or “animal spirits,” and which in turn affect the macroeconomy.
2 Stylized facts

We begin with some stylized facts about business cycles, exchange rates, and asset prices in a novel form. While business cycles are often analyzed on a country-by-country basis, recent work in international finance—starting with Lustig and Verdelhan (2007)—shows that exchange rate puzzles and return differentials emerge most clearly when the researcher constructs artificial, composite “low interest rate” and “high interest rate” countries. We follow this approach: at each date \( t \), we sort countries into four groups according to their beginning-of-period nominal interest rate. We compute returns and growth rates of macroeconomic quantities, for each country in each group, from time \( t \) to time \( t + 4 \). We take, at each date, the average within each group of these returns or macroeconomic quantities and focus on the groups with the lowest and highest interest rates. This procedure produces time series for two artificial composite countries: a low interest country (i.e., group of the low countries) and a high interest rate country (i.e., group of high interest rate countries). Of course, the composition of countries in each group is time-varying, although some countries are almost always in one group. (For instance, Japan and Switzerland are often in the low interest rate group, and Australia and New Zealand are often in the high interest rate group.)

The stylized facts are reported in Panel I of Tables 1, 2, 3, and 4. We focus on OECD countries over the post-Bretton Woods sample and use quarterly series. The data sources are described in a separate Appendix. We report annual growth rates and, for each statistic, standard errors obtained by bootstrapping under the assumption that returns or growth rates are i.i.d. Each block in the bootstrap contains four observations and we repeat the estimation 1,000 times.

2.1 Business cycle moments

Table 1 provides the basic business cycle statistics for each composite group. For the low interest rate group, we obtain standard business cycle facts: investment is almost three times
more volatile than output, consumption is 20% less volatile than output, and employment volatility is about two-thirds that of output. For the high interest rate group, consumption is more volatile than output. Comparing the two groups, we see that all quantities are more volatile in the high interest rate group: the volatility of consumption growth, for example, ranges from 1.4% per year for the low interest rate group to 2.6% per year for the high interest rate group. Investment is more volatile, ranging from 4.4% to 7.7% per year. Consistent with the international macroeconomics literature, we also find that there is substantial synchronization of business cycles across countries, especially for investment and output: the correlation between output growth in the high and low interest rate groups is around 0.5.

[Table 1 about here.]

2.2 Equity risk premia and interest rates

It is well known that the average excess return on the U.S. stock market is large (the “equity premium puzzle”); however, this large average excess return is not unique to the U.S. Table 2 shows that the low interest rate group has large average excess returns (4.5% per year); the high interest rate group has a significantly lower average excess returns (1.1% per year). The two groups also differ by the volatilities of their interest rates and equity returns, which are clearly larger in the high interest rate group. Interest rates are highly persistent, and fairly smooth, in both groups. By construction, the high interest rate group has a higher average interest rate. Finally, there is a significant cross-country correlation of equity markets, and interest rates.

[Table 2 about here.]
2.3 Exchange rate and carry trades

We construct the real exchange rate between the low and high interest rate group, and compute exchange rate statistics in Table 3. Consistent with the literature, we find that this real exchange rate is volatile: the log change in the exchange rate has a standard deviation of about 8.6% per year, and exhibits little serial correlation at quarterly frequency (the overlapping observations—annual changes measured at a quarterly frequency—drive the high autocorrelation coefficient in the table). We also find that the log change in the exchange rate, while little skewed, is leptokurtic. In addition, the correlation of the change in the exchange rate with the relative stock market returns is low, the correlation with relative consumption growth is negative (the Kolman-Backus-Smith puzzle), and the correlations with market and interest rate differences across countries are small.

Table 4 focuses on the properties of the carry trade. According to the standard uncovered interest rate parity (UIP) condition, the change in the exchange rate should, on average, equal the corresponding interest rate differential, so that the average return to investing in either group of countries should be the same. A large body of empirical work, however, documents violations of UIP [see Hansen and Hodrick (1980) and Fama (1984), and surveys by Lewis (1995) and Engel (1996)]. The currency carry trade is the investment strategy to borrow in low interest rate currencies and invest in high interest rate currencies. This strategy delivers an excess return above 5.3% per year and a Sharpe ratio of 0.55 (higher than the Sharpe ratios on equity markets). Note that this excess return and Sharpe ratio do not take into account transaction costs. Net average excess returns would be lower. Lustig et al. (forthcoming) show that the average return on a carry trade strategy compensates investors for the exposure to world shocks. We thus report the correlation of the carry trade return with the world average consumption growth, investment growth, employment growth and output growth, and the world average stock return. These correlations, while small,
are positive, reaching 0.3 for consumption, investment, and output. Finally, in a standard regression of the change in the exchange rate on the interest rate differential, the slope coefficient is slightly negative, whereas it should equal one under UIP.

[Table 4 about here.]

After discussing the stylized facts of the low interest rate country and high interest rate country in the data, we now turn to our model.

3 The model

In this section, we study a two-country, one-good international real business cycle model. Following a recent literature (e.g., Brandt et al. (2006), Colacito and Croce (2011b), Bansal and Shaliastovich (2008), Alvarez et al. (2009), Verdelhan (2010), and Ang and Chen (2010)), we assume that asset markets are complete, but frictions in goods markets prevent any net trade of goods. As a result, we can solve for the allocation of each country separately. Under these assumptions, the exchange rate change is the ratio of the foreign and domestic stochastic discount factors.\footnote{This no-trade approach is a first step, as the ultimate goal is to build a model that generates the positive, but imperfect, risk-sharing that takes place in the data. However, because there is sometimes confusion regarding this approach, it is important to realize that it is internally consistent. From an individual point of view, agents are able to write state-contingent contracts that specify the delivery of goods in either country in any state of the world. However, in the aggregate there can be no net trade, hence the exchange rate has to adjust to make no trade an equilibrium. Hence, if our model generates the correct allocation of consumption and leisure across countries, and markets are complete, the exchange rate that we find must be the correct one. This is similar to the way the risk-free rate is determined in standard representative agent models: the risk-free asset is in zero net supply, so that no agent holds the risk-free asset in equilibrium, but agents are free to consider buying it, and the price of this asset is precisely such that net trade in this asset is nil.}

We make these assumptions for two reasons. First, it is difficult to solve a complete market model with two agents, production, and recursive preferences. The technical difficulty is that the planner problem is not recursive in the “natural” state variables, such as capital stocks and productivity. Continuation values must be added as state variables, leading to a curse of dimensionality. Second, the setup without net trade is a natural starting point.
because it is unlikely that the model with net trade can match the data if the model without net trade does not. The volatility of the exchange rate is likely to be lower in the model with trade. In Gourio et al. (2010), we relax this zero net trade assumption in the context of a small open economy model, and study the implications for capital flows.

### 3.1 Model setup

Our business cycle model differs from the standard RBC model in two ways: first, we introduce recursive preferences, and second, we introduce time-varying disaster risk. The model builds on the closed economy model developed in Gourio (2009).

#### 3.1.1 Domestic economy

In the home country, a representative consumer maximizes a recursive utility function,

\[
V_t = \left( ((1 - \beta)C_t^\upsilon (1 - N_t)^{1-\upsilon})^{1-\gamma} + \beta E_t \left( V_{t+1}^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1}{1-\gamma}}. \tag{1}
\]

Here \( \upsilon \) reflects the preference for consumption \( C_t \) as opposed to leisure \( 1 - N_t \), \( \gamma \) is the inverse of the intertemporal elasticity of substitution (IES) over the consumption-leisure bundle, and \( \theta \) measures risk aversion towards static gambles over the bundle. The risk aversion over consumption is \( \upsilon \theta \) (Swanson, 2009).

There is a representative firm that produces output using a standard Cobb-Douglas production function:

\[
Y_t = K_t^\alpha (z_t N_t)^{1-\alpha}, \tag{2}
\]

where \( K_t \) is the capital stock and \( z_t \) denotes the total factor productivity, to be described below. The firm accumulates capital subject to adjustment costs:

\[
K_{t+1} = \left( (1 - \delta)K_t + \phi \left( \frac{I_t}{K_t} \right) K_t \right) (1 - x_{t+1} b_k), \tag{3}
\]
where $\phi$ is an increasing and concave function, whose curvature captures adjustment costs. The dummy variable $x_{t+1}$ is 1 if a disaster hits at time $t + 1$ (with probability $p_t$) and is 0 otherwise (with probability $1 - p_t$). The parameter $b_k$ represents the capital destruction following a disaster.

The assumption that a disaster reduces the capital stock requires some discussion. While wars or natural disasters such as earthquakes or tsunamis physically destroy capital, economic depressions do not. Capital destruction can be interpreted more broadly to reflect the expropriation of capital holders (if the capital is taken away and then used very inefficiently) or the destruction of intangible capital (such as matches between firms, employees, and customers) Finally, one can imagine a situation where the demand for some goods falls sharply, rendering worthless the factories producing these goods. In terms of the economic mechanism, the assumption of capital destruction ensures that the return on capital is risky, i.e., the realized return on capital is low during disasters. From this standpoint, it is a fairly sensible assumption.\(^4\)

Since there is no net trade, the resource constraint is simply $C_t + I_t = Y_t$. TFP follows a unit root process, and is affected by standard “small normal shocks” $\varepsilon_{t+1}$, as well as disasters:

$$\log z_{t+1} = \log z_t + \mu + \sigma \varepsilon_{t+1} + x_{t+1} \log(1 - b_{tfp}), \quad (4)$$

where $\varepsilon_{t+1}$ is i.i.d., normally distributed with zero mean and unit variance $N(0, 1)$, $\mu$ is the drift of TFP, $\sigma$ is the standard deviation of Gaussian shocks, and $b_{tfp}$ is the reduction in TFP following a disaster. The probability of disaster $p_t$ follows an autoregressive process of order one (AR(1)) in log:

$$\log(p_{t+1}) = \rho \log(p_t) + \mu_p + \sigma_p \varepsilon_{p,t+1}, \quad (5)$$

\(^4\)One possibility is to make capital endogenously risky by assuming large adjustment costs. In this case a large negative shock to TFP reduces investment, leading the price of capital (“marginal Q”) to fall significantly.
where $\varepsilon_{p,t+1}$ is i.i.d. normally distributed with zero mean and unit variance.\(^5\)

### 3.1.2 Foreign economy

The foreign country has the same preferences and technology as the domestic economy. The disaster is perfectly correlated across the two countries: the same $x_{t+1}$ (indicator of disaster realization), $p_{t+1}$ (probability of disaster), and $\varepsilon_{p,t+1}$ (innovation to the log probability of disaster) apply to both. The domestic and the foreign economy differ only in their riskiness parameters, $b_k$ and $b_{tfp}$ and $b^*_k$ and $b^*_{tfp}$.\(^6\) We think of this simple assumption as capturing the fact that countries have different exposures to world risk, perhaps due to different industry compositions or different financial structures.

We allow the normally distributed shocks $\varepsilon_t$ and $\varepsilon^*_t$ to be contemporaneously correlated, consistent with our data. Compared to a standard international RBC model, the model adds a common source of shocks through the probability of disaster shock $\varepsilon_{p,t+1}$. Overall, the model has four shocks: the usual TFP shocks at home and abroad, $\varepsilon_{t+1}$ and $\varepsilon^*_{t+1}$, as well as the realization of disaster, $x_{t+1}$, and the shock to the probability of disaster, $\varepsilon_{p,t+1}$.

### 3.1.3 Asset prices and exchange rate

In this model, the stochastic discount factor in the home country is,

$$M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{\nu(1-\gamma)-1} \left( \frac{1 - N_{t+1}}{1 - N_t} \right)^{(1-\nu)(1-\gamma)} \left( \frac{V_{t+1}}{E_t \left(V_{t+1}^{1-\theta} \frac{1}{1-\theta} \right)^{\gamma-\theta}} \right)^{-1}$$

and similarly in the foreign economy. We define the real exchange rate $Q_t$ in units of domestic goods per foreign good, i.e., 1 foreign good = $Q_t$ home goods, so that a higher $Q_t$ reflects both a foreign appreciation and a home depreciation.

\(^5\)The probability of disaster needs to lie between 0 and 1. The AR(1) specification does not ensure that constraint. When we solve our model, however, we approximate the AR(1) process with a Markov chain, whose support does lie between 0 and 1.

\(^6\)An interesting extension of the model is to make the riskiness of each country, i.e. the parameters $b_k, b_{tfp}$, themselves stochastic. The identity of the more risky country would change over time, and shocks to $b_k, b_{tfp}$ would also create additional dynamics.
Under complete markets the exchange rate must satisfy,

\[ \frac{Q_{t+1}}{Q_t} = \frac{M_{t+1}^*}{M_{t+1}}. \] (7)

Last, and following Abel (1999) among others, we define an “equity” asset that pays out as dividend a levered claim on output \( (D_t = Y_t^{\lambda}) \) in the home country, and similarly in the foreign country. The parameter \( \lambda \) captures the financial and operating leverage of corporations. The model-based definition of equity is the asset that pays out the model dividends \( Y_t - w_t N_t - I_t \). As in most business cycle models, however, these dividends are significantly less volatile and procyclical than the dividends in the data. The model can then generate a significant equity premium, but not the high volatility of returns observed in the data. This motivates our introduction of leverage, which is standard in the asset pricing literature (e.g., Bansal and Yaron (2004) and Wachter (2008)).

### 3.2 Calibration

Parameters are listed in Table 5. The period is one quarter. A first group of parameters \((\alpha, \delta, v, \mu, \eta)\) follow the real business cycle literature (Cooley and Prescott, 1995). The functional form for the adjustment cost function follows Jermann (1998):

\[ \phi(x) = a_1 x^{1-\eta} + a_2, \]

where \(a_1\) and \(a_2\) are set such that the steady-state is independent of \(\eta\) and “marginal \(Q^*\) is one in the nonstochastic steady-state.

[Table 5 about here.]

A second group of calibration parameters pertains to the modeling of disasters and is more difficult to calibrate. Barro (2006) and Barro and Ursua (2008) document, using panel data on consumption and output for a sample that spans many countries and a century and a half, that large declines in economic activity are fairly frequent. We follow their work and assume that the probability of disaster is on average 1.7% per year, or 0.425% per quarter. Their work also suggests that the average (risk-adjusted) size of output disaster is 43%, and
similarly for consumption. Our model would require $b_k = b_{tfp} = 0.43$ to generate a decline of output or consumption equal to 43%. We set a significantly smaller size of disaster, equal to $b_k = b_{tfp} = 0.3$ in the low interest rate country (home country), to be conservative. We thus assume that disasters are 30% smaller than estimated. We set risk aversion $\theta$ equal to 8.5 over consumption-leisure, so as to match approximately the equity premium in the low interest rate group of countries in the data. This corresponds to a risk-aversion of about 2.5 over consumption alone. Next, we pick the disaster size for the high interest rate country (foreign country) to match the equity premium in the group of high interest rate countries, leading us to set $b_k^* = b_{tfp}^* = 0.24$. Finally, the persistence of the log probability of disaster is 0.92, and the unconditional standard deviation of the log probability is 2.05. These figures are picked to replicate approximately the volatility of equity returns in the group of low interest rate countries. It is noteworthy that our model requires that the probability of disaster be fairly volatile. Importantly, we do not target the second moments of quantities, the volatility of equity returns in the group of high interest rate countries, the level and volatility of interest rates, or any exchange rate or carry trade moments. These moments allow us to evaluate the fit of the model.

We consider an intertemporal elasticity of substitution of consumption (IES) equal to 2. Our value for the IES is larger than the standard estimates of Hall (1988). However, recent empirical evidence suggests that higher values are empirically plausible (see Bansal and Yaron (2004), Guvenen (2006), Mulligan (2004), and Vissing-Jørgensen (2002)). A high IES generates the sensible comparative statics that higher expected growth leads to higher asset prices, and higher uncertainty leads to lower asset prices. Finally, note that in the model, the correlation between consumption growth and the risk-free rate is low, and the standard Hall (1988) regressions are significantly biased towards zero.

The Solow residual is $z^{1-\alpha}$, so the downward shift in the production function during disasters is actually 20.9% in the home country and 16.6% in the foreign country. Reducing the disaster size further requires us to have somewhat higher risk aversion, but has otherwise little effect on the results.
Following Abel (1999) and Barro (2006), the leverage parameter $\lambda$ is set to 2. For simplicity, our calibration assumes that government bonds are risk-free, but this has little effect on our results, other than to change the mean return on government bonds. Last, we assume that the standard deviation of TFP shocks is $\sigma = 0.9\%$ (quarterly), and the cross-country correlation of TFP shocks is 0.3, consistent with our data. In the data, we find that the average volatility of productivity growth (where productivity is measured as $Y/N^{2/3}$) is 0.9% on average for the low interest rate countries and 1.3% on average for the high interest rate countries. In the interest of clarity, we assume that the countries have the same volatility of TFP.

In our benchmark model, the high interest rate country is the foreign country, which has the lowest fundamental risk (e.g., the lowest disaster risk, $b_k^* = b_{tfp}^* < b_k = b_{tfp}$). Precautionary savings are small, hence the interest rate is high. The home country in contrast has a low interest rate as the higher disaster risk implies high precautionary savings. On top of this benchmark calibration, we also present results from the model with no disasters (i.e., the basic RBC model), and the model where both countries are equally risky (i.e., $b_k = b_k^*$ and $b_{tfp} = b_{tfp}^*$). These two additional calibrations help illustrate the effects of (1) adding disasters with time-varying disaster probabilities to the standard RBC and (2) adding different exposure of countries during disaster to the model in (1).

### 3.3 Impulse response functions

We start by depicting the response of quantities and prices to each of the three shocks of our model to illustrate the mechanism. First, the standard (normally distributed, “small”) TFP shock has the usual effects of increasing output, investment, consumption, and employment, as shown in Figure 1. Moreover, equity prices and interest rates rise, while the exchange rate of the country with the positive TFP shock depreciates. This response of the exchange rate is consistent with the empirical literature (e.g., Acemoglu and Ventura (2002), Basu et al. (2006), and Pavlova and Rigobon (2007)). Standard models such as Heathcote and
Perri (2002) generate such a depreciation through a relative increase in the supply of the home good. In our model, the depreciation follows from the first-order condition for risk sharing: if consumption increases more in the home than in the foreign country, it must be that consumption is cheaper there than in the foreign country.

[Figure 1 about here.]

Second, Figure 2 depicts a disaster realization. By definition, the disaster leads to a permanent drop in productivity and an initial destruction of capital. Endogenously, consumption, investment, and output fall by the same amount, while employment and interest rates do not change. Equity prices fall significantly, which implies that stocks are risky since their payoffs are low when marginal utility is high. This is the main source of the equity premium in the model.

In the more risky country, all of the effects above are stronger. Because of the decreased supply of goods in that country, the exchange rate appreciates. Hence, the country that is most risky in terms of its stock market and quantities is the country whose currency is safest, i.e., appreciates during disasters. While this result may appear counterintuitive, it is actually consistent with experiences during several episodes, such as the recent nuclear facility accident in Japan. Within the first five days of the earthquake on March 11, 2011, the Japanese yen appreciated by about 6% against the U.S. dollar and roughly 4% against the euro. This exchange rate appreciation is essentially a direct implication of complete markets, since Equation (7) shows that the country with the largest increase in its marginal utility must have the exchange rate that appreciates the most.8

[Figure 2 about here.]

Finally, Figure 3 presents the impulse response functions to an increase to the probability of disaster. This is the most important shock in our model. (The effect of TFP shocks, 8In related research, we solve a small open economy model with incomplete markets and show that the exchange rate of the more risky country may depreciate. This is because the wealth effect, which is shut down by complete markets, can be quite important during disasters.)
which do not generate large risk premia, is well-known, while disaster themselves are rarely observed.) Within each country, an increase in disaster risk leads to a decline of investment, output, and employment, i.e., a recession. The shock to disaster probability affects investors’ expectations regarding future GDP; decreasing the mean and increasing risk. As shown in Gourio (2009), a shock to disaster risk is analytically equivalent (for quantities) to a preference shock: agents become more impatient and decide to invest less since the risk-adjusted return on capital is lower.

This result relies on the intertemporal elasticity of substitution (IES) being greater than unity, so that agents save less when the (risk-adjusted) return is lower. If the IES is smaller than unity, the wealth effect prevails, and agents save more when the return is lower. In this case, an increase in disaster risk leads to a boom. Finally, if the IES is exactly unity, there is no effect of the increase in risk on macroeconomic quantities. On the other hand, the IES has a small, limited effect on interest rates, stock prices, and exchange rates.

As a consequence, there is less incentive to produce and to work. Following an increase in disaster probability, consumption increases, and then later falls. These consumption dynamics break the perfect comovement that is predicted by the standard IRBC model and de-link consumption and asset prices. The two countries are affected by the same increase in disaster probability: the magnitude of the impact, however, is effectively larger in the country where disaster risk is larger, leading to larger recessions. Equity prices drop because of an increase in the discount rate, while the risk-free rate falls as demand for safe assets increases. Last, the exchange rate of the more risky country appreciates, because marginal utility increases more in this country.

3.4 Quantitative results

This section illustrates the effect of shocks to the world-wide disaster probability by comparing the implications for quantities, asset prices, and exchange rates of three models: (1)
a standard real business cycle model with only TFP shocks; (2) a model with time-varying risk of disaster, when both countries are equally risky (with a disaster size of 24%) and (3) our benchmark model with time-varying risk of disaster, when the home country is more risky (with a disaster size of 30%). The key result is that adding time-varying disaster risk makes our model closer to the data, if (and only if) the two countries have heterogeneous exposures to the disaster.

The separate online Appendix details our numerical solution algorithm. Given our interest in time-varying risk premia, we cannot log-linearize the model, thus we resort to a standard dynamic programming approach with discretization. All the moments are calculated by simulating the model in samples without disasters.\footnote{See the online Appendix for statistics in samples with disasters. Our results are largely unaffected, and the main effect is to increase the measured volatilities of all time series except employment and interest rates.}

### 3.4.1 Quantities

Panel I of Table 1 reports our data results, discussed in section 2. Panel II of Table 1 reports the quantity implications of the different models. The first row reports the results for the standard RBC model (i.e., no disaster risk). Since the two countries are perfectly symmetric, and there is no net trade, the quantity dynamics are exactly the same in the two countries. The quantity patterns within each country reflect the usual RBC results: consumption is smoother than GDP, investment is more volatile than GDP, and employment is volatile, but less than in the data. The cross-country correlation of consumption, employment, investment, and output are all equal to the assumed cross-correlation of TFP shocks, i.e., 0.3, since there is no endogenous interaction between the two countries and TFP shocks are the only source of fluctuations.

Turning to the model with both TFP shocks and disaster risk shocks (row 2), we see that when the two countries are equally risky, the additional shock raises the volatility of all series, but especially investment and employment. Investment and employment volatility are now closer to the data than what the standard RBC model predicts. The risk shock
is common across countries, hence it increases the cross-country correlation of quantities, especially investment and employment. Our model, however, still implies that the cross-country correlation of consumption should be greater than that of output, a puzzle noted at least since Backus et al. (1992). Finally, when one country is more risky (row 3), the same mechanism applies, to a larger extent for the more risky country. Our model replicates well the volatilities of consumption, investment, employment, and output for low interest rate countries.

However, the model underestimates substantially the volatilities of the high interest rate group of countries. In the model, the countries in this group are somewhat less volatile than in the low interest rate group, because they are less risky, whereas in the data they are about twice more volatile than the low interest rate countries. These shortcomings could be partly addressed by introducing more heterogeneity across countries, assuming for example that the less risky countries in terms of disasters experience more volatile TFP shocks. We discuss this extension below.

### 3.4.2 Asset prices

Panel I of Table 2 reports the mean, volatility, and cross-country correlation of the short-term interest rate and equity excess returns. Panel II reveals first that, in the absence of disaster risk, the model predicts tiny risk premia in the low and high interest rate country (since our model has low risk aversion), and equity returns are very smooth. Furthermore, the cross-country correlation of returns equals the correlation of TFP shocks. The model generates substantial volatility of returns as well as a large equity premium, and a stable risk-free rate, in our benchmark calibration with time-varying disaster risk. The annual average excess return is 5.64% in the low interest rate country and 2.74% in the high interest rate country.

In the data, high interest rate countries offer the lowest equity excess returns in local currencies. Our calibration slightly underestimates the volatility of equity returns in the
group of low interest rate countries, but we substantially under-predict (by a factor of about three) the volatility in the group of high interest rate countries. Finally, our model generates a range of interest rates that is too small: 1.85% to 2.4% in the model versus 0.8% to 5.2% in the data. Note that incorporating default on short-term loans would reduce the spread further in the model.

Returns are more correlated across countries in our model than in the standard RBC model since they are all driven by the fear of a worldwide disaster. Because risk has a powerful effect on returns, the correlation of returns is significantly higher than the correlation of “fundamentals,” such as output or investment. In this sense, there is “excess comovement” of stock markets. The benchmark model predicts a cross-correlation of returns that is even higher than in the data (0.89 for equity excess returns and 0.98 for risk-free rates), but this is not surprising since we abstract away from many idiosyncratic shocks or heterogeneities.

Finally, the model can also replicate the within-country correlation of quantities (such as output or investment), as well as asset prices or expected returns. These results are unreported since they are similar to Gourio (2009).

3.4.3 Exchange rates

Table 3 reports the corresponding moments for exchange rates. In our model with only TFP shocks, the exchange rate has a non-negligible volatility, since consumption growth rates are only weakly correlated. Adding disaster risk does not change this volatility if the home and the foreign country have the same exposure to disaster risk. This is because shocks that move countries’ marginal utilities by the same amount do not affect exchange rates. When the risk exposures of the two countries are different, the exchange rate volatility rises, because disaster risk now affects the countries’ marginal utilities differently, and the exchange rate volatility becomes close to the one in the data. Simulated exchange rates are quite volatile, with a standard deviation around 10.5% per year, versus about 8.6% in the data. Taking into account the uncertainty around these two estimates, the exchange rate volatility in the
model is not statistically different from that in the data.

In all models, the exchange rate is not far from a random walk, i.e., the autocorrelation of the change of the exchange rate is very small. This is line both with the literature (see notably Engel and West, 2005) and our data. Marginal utility is approximately a random walk in the model, as TFP shocks and disaster shocks generate permanent changes in consumption. In our benchmark model, there is some mean-reversion, because shocks to disaster risk are transitory and hence lead to a transitory appreciation of the real exchange rate of the more risky country. However, the persistence of the real exchange rate remains high and our benchmark model yields a leptokurtic distribution of changes in exchange rates, i.e., “fat tails,” even in samples without disasters. This is consistent with the data, and is an improvement over the RBC model with its approximately Gaussian distributions. The model does not, however, imply any significant skewness in the changes of the exchange rate, whereas Panel I suggests that there is some skewness in the data.

3.4.4 Backus-Smith Puzzle

Turning to the correlation of exchange rates with macroeconomic aggregates or asset prices, we see that in the basic RBC model the exchange rate of a country appreciates when its output or consumption goes down, or when its equity return goes down. In particular, as noted since Backus and Smith (1993) and Kollmann (1995), the correlation between changes in exchange rates and relative consumption growth rate equals one.\(^{10}\) This result holds exactly with power utility; while we have Epstein-Zin utility, the basic disaster risk model still generates a very strong correlation, over 0.99, when there are only TFP shocks. Our benchmark model, in contrast, leads to a weaker correlation of exchange rates with relative output growth and especially consumption growth (0.66). This is because shocks to

\(^{10}\)Backus and Smith (1993) note that in complete markets and with power utility, the change in the real exchange rate is equal to the relative consumption growth in two countries times the risk-aversion coefficient, thus implying a perfect correlation between the consumption growth and real exchange rate variations. Yet, in the data, Backus and Smith (1993) find that the actual correlation between exchange rate changes and consumption growth rates is low and often negative. Chari et al. (2002), Corsetti et al. (2008b), and Benigno and Thoenissen (2008) confirm their findings.
disaster probability generate a negative correlation between the country’s exchange rate and its consumption growth. Because agents have recursive preferences (a preference for early resolution of uncertainty), consumption and marginal utility are not proportional any more. Shocks to uncertainty drive them in opposite directions in our framework, provided that the IES is larger than unity. However, the benchmark model still implies a too strong correlation of exchange rates with macroeconomic variables and equity returns.

3.4.5 Carry trade returns and UIP

Table 6 reports the results for the implications of the riskiness of the exchange rate on carry trade returns and UIP. Consider a “carry trade” strategy: borrowing in a low interest rate country, and lending in a high interest rate country, taking on the exchange rate risk. This strategy generates a log excess return equal to:

$$rx_{t+1} = r_t^* - r_t + \Delta q_{t+1},$$

where $r$ and $r^*$ denote the domestic and foreign risk-free rates and $\Delta q$ the change in the log real exchange rate.

In the data, this strategy generates significant average excess returns, with a conditional volatility equal to that of the exchange rate. Table 4 shows that the annual average return on this strategy is 5.4%, with a volatility of 9.6%. Again, this average excess return does not include transaction costs. Lustig et al. (forthcoming), using forward contracts whose bid-ask spreads are easily available but for short samples, report an average net excess return of 3.1% (with a standard error of 0.5%) on samples of developed countries over the 1983–2009 period.

In our benchmark model, the high interest rate country is the foreign country, which has the lowest fundamental risk (e.g, the lowest $b_k = b_{tfp}$). Precautionary savings are small, hence the interest rate is high.\(^{11}\) If a disaster occurs, this country’s currency depreciates, since its

\[^{11}\text{In the model, interest rates also vary because of changes in TFP, hence the low interest rate country.}\]
marginal utility rises by a smaller amount. Hence, carry trades pay badly in bad times, and investing in high interest rate countries is risky, so that investors require compensation in the form of higher expected returns. Table 4 reports the mean and volatility of the carry trade excess return and the correlation of the carry trade return with the average (across our two groups of countries) of real consumption growth, real investment growth, real employment growth, real GDP growth, and real stock market returns.

In the standard RBC model, the carry trade strategy does not generate any excess return—the current interest rate is not correlated with the country’s riskiness. In our benchmark model, with heterogeneous exposures to disaster risk, the carry trade generates a significant excess return of 2.3%. The carry trade return is not fully captured by its correlation with consumption growth. Carry trades pay off badly both when disasters hit and when disaster risks rise; increases in disaster risk coincide with increases in consumption, thus the consumption CAPM mismeasures the risk of the carry trade. Note, however, that the market CAPM does not suffer from this measurement problem: the carry trade return is strongly correlated with the market return. We obtain similar results with world output, investment, and labor. In the data, as in the model, carry trade returns are positively correlated with the average growth rates of output, investment, and labor.

Finally, we note that our model reproduces the failure of traditional uncovered interest rate parity tests:

\[ \Delta q_{t+1} = \alpha_{UIP} + \beta_{UIP} (r_t - r_t^*) + \varepsilon_{t+1}. \]

The RBC model, where risk-neutrality holds almost perfectly, generates a slope coefficient very close to 1. Our benchmark model delivers a negative slope, lower than the one we obtain on portfolios, but consistent with a vast empirical literature on bilateral exchange rates. The failure of UIP is driven by variations in the probability of disaster. When \( p \) is high, the exchange rate risk premium is large. At the same time, the risk-free rate of the
more risky country is lower due to heightened precautionary savings. These two observations imply a negative UIP slope coefficient. Hence, our model reproduces the forward discount puzzle in an economy without frictions.\footnote{Additional frictions (like information heterogeneity or infrequent portfolio rebalancing) could strengthen deviations from UIP and add interesting dynamics to exchange rates (see Bacchetta and van Wincoop (2006, 2010) for the impact of such frictions on exchange rates).}

Finally, we report the correlation of carry trade excess returns with measures of aggregate volatility. In the data, Lustig et al. (forthcoming) show that high interest rate currencies tend to depreciate when aggregate equity volatility is high while low interest rate currencies tend to appreciate. They measure monthly volatility using the standard deviation of daily equity returns. Menkhoff et al. (forthcoming) obtain similar results with aggregate currency volatility. In the model, aggregate volatility is high when the disaster probability is high, given the time-series process for the disaster probability. The model also implies that carry trade returns are highly correlated to U.S. equity returns when volatility is high (a finding reported in Lustig and Verdelhan (2010)). Finally, Engel (2010) reports that while a positive interest rate differential leads to a positive excess return at short horizons, the effect switches signs at long horizons. Our benchmark model generates a positive excess return at short horizons that slowly mean-reverts at longer horizon towards zero, and hence does not fully capture this pattern (see the separate Appendix).

3.4.6 Fundamental risk and currency risk

A critical implication of our model is that countries that are more risky in terms of their domestic fundamentals such as consumption, output, or stock prices, have low interest rates and are less risky in terms of their exchange rates, because their currencies are countercyclical. This is a direct implication of Equation (7), which is itself a direct implication of complete markets. A key empirical question is whether this interpretation of low interest rate countries as more risky fits the data. On one hand, it is consistent with the fact that low interest rate countries deliver higher average equity excess returns. On the other hand,
it is inconsistent with the observed volatilities of macroeconomic quantities, equity returns and interest rates: low interest rate countries should have higher volatilities, but in the data they have lower volatilities. Empirically, this result appears fairly robust, and it is insensitive to the number of portfolios and the aggregation method inside each portfolio: we consider two, three, four, or five portfolios, using either simple means, GDP weights (measured at purchasing power parity), or medians. In all these cases, we never obtain a high interest rate portfolio that exhibits less volatile consumption growth than the low interest rate portfolio. Solving this puzzle is obviously an important question for future research.  

Estimating fundamental risk is difficult. It is not obvious that low interest rate countries such as Switzerland or Japan are more risky than high interest rate countries such as Australia or New Zealand, and fundamental risk might be time-varying. Our paper uncovers a potential tension between fundamental and currency risk.

3.4.7 Role of recursive utility

Most macroeconomic models use expected utility preferences, whereas we use recursive preferences as introduced by Epstein and Zin (1989), which allow for a separation between risk aversion and the inverse of the IES. Without recursive preferences, our model cannot generate the Backus-Smith anomaly. In the next paragraph we discuss how our results are

\[\text{If the IES equals unity, the model predicts no quantity response to a change in disaster probability, hence both countries have the same quantity volatilities. The model also predicts a positive carry trade return and volatile exchange rates. However, it is still the case that stock prices are more volatile in the low interest rate country, which is contrary to the data. The habit model of Verdelhan (2010) is here an exception: even if the consumption growth volatilities are the same in both countries, marginal utilities of wealth and interest rates differ across countries because of the different (time-varying) market prices of risk.}\]

\[\text{One way to illustrate the disconnect between fundamental risk and currency risk in our model is to use it to interpret the recent international financial crisis. There is evidence that investors around the world feared a Great Depression scenario in the Fall of 2008. Our model predicts that this increase in the probability of disaster leads to a worldwide recession and sharp declines in stock markets. The more risky countries endure larger recessions and see their currency appreciate. (In a fuller model with trade, capital flows out of the more risky countries.) The U.S. was perhaps, during this crisis, the more risky country, since the U.S. mortgage market was the source of the recession, and U.S. financial institutions were the ones most affected by the crisis. Consistent with our model, the U.S. exchange rate appreciated sharply during the crisis, and the U.S. current account increased. However, in contrast to our model, the recession was not larger in the U.S. than in other countries. This failure is perhaps due to the aggressive policy response in the U.S., which we do not model. If the U.S. is the less risky country, then the model cannot replicate the exchange rate and current account patterns during the crisis.}\]
affected if we change either the risk aversion coefficient, or the IES coefficient to return to the standard expected utility. This illustrates further the importance of recursive preferences for our approach.

In our model, risk aversion determines both the magnitude of risk premia and the size of the response of quantities to a disaster risk shock (since the importance of disaster risk depends on risk aversion). Hence, solving the model with a lower risk aversion ($\theta = 2$ so that $\theta = 1/\gamma$, and we are in the familiar case of expected utility) yields the same qualitative effects, but they are much reduced in size. For instance, the equity premium is 1.5% per year (5.6% in the benchmark). In contrast, changing the IES leads to a qualitative, rather than quantitative change in behavior of the model. If the IES is less than unity, the model implies that an increase in disaster risk leads to a boom of investment, employment, and output. However, the model implies volatile exchange rates, along with a failure of the UIP condition.

### 3.4.8 Robustness

The separate Appendix provides a detailed sensitivity analysis of the results. Introducing default risk has little effect on our results. The volatility puzzle that we emphasize is partially solved if one recognizes that high interest rate countries face higher TFP volatility; however, this does not explain the high volatility of their equity returns or interest rates. Last, a higher correlation of TFP across countries makes exchange rates less volatile.

### 4 Empirical Effect of Time-Varying Risk on Macroeconomic Quantities

We have shown how incorporating time-varying risk and the heterogeneous exposures to it helps our model generate moments closer to the data. In this section, we provide an empirical test of the main mechanism by studying the impact of time-varying risk on actual
macroeconomic quantities. Measuring time-varying disaster probabilities is challenging. We show that they can be approximated by measures of time-varying equity volatility: put prices on U.S. stock market indices are highly correlated with U.S. (implied and realized) equity volatility. The same is true in the model, where an increase in disaster probability leads to an increase in stock market volatility. For data availability reasons, we thus use shocks to equity return volatility as proxies for the shocks to the world disaster probability: realized stock market volatilities are available for many countries over long samples, allowing us to propose a measure of global volatility that starts in 1970. We then estimate impulse responses of macroeconomic quantities to a shock to global equity volatility using standard vector autoregressions (VAR).

4.1 Equity Volatility as Proxy for Disaster Probability

We find that high equity volatility is a reasonable proxy for disaster probability. We compare the implied volatilities from put options, risk reversals, and the VIX Index. The VIX Index uses a wide range of near-term and next-term in-the-money and out-of-the money S&P 500 call and put options, and then weights them to yield a constant, 30-day risk-neutral measure of the expected volatility of the S&P 500 Index. Risk-reversals are measured as the differences between the implied volatility of puts and calls with strike prices that are symmetric around the money (i.e., the current value of the S&P 500 index). A positive risk reversal means the implied volatility of puts is greater than the implied volatility of symmetric calls, which implies that more market participants are expecting a drop in the index than a rise, and vice versa if the risk reversal is negative.

Figure 4 presents the implied volatilities obtained from put options, risk reversals, and the VIX index. We focus on implied volatilities measured out-of-the-money, i.e., for strikes that differ from the value of the S&P 500 index. We measure the distance between the strike and the spot index in terms of options deltas; the delta of an option represents the sensitivity to changes in the underlying asset. The delta ($\delta$) of a put varies between 0 for extremely...
out-of-the-money options to −1 for extremely in-the-money-options. A 10-δ (25-δ) put is an option with a delta of 10% (25%). A 10-δ (25-δ) put option corresponds to a strike price that is on average 8.2% (3.6%) below the value of the spot S&P 500 index. A 10-δ put option with such a strike price thus offers a positive terminal payoff if the stock market decreases by more than 8.2% over the next 30 days. We focus on this range because of the liquidity of the option market.

The first panel of Figure 4 shows that the implied volatilities of out-of-the-money put options move in sync with the VIX Index. The correlation with the VIX is 0.99 for both 10-δ and 25-δ put options. This is not surprising since the VIX partially reflects these volatilities. It shows that most of the large variations in the VIX do not come from above-the-money calls (i.e., upside risk), but from downside risk.

The second panel of Figure 4 shows that risk reversals tend to be high when the VIX is high. As a result, high implied volatilities of puts and calls as measured by the VIX partly correspond to high expectations of large market decreases. The correlation between the VIX and the risk reversals is 0.88 (with a standard error of 0.03) at 10-δ and 0.85 (with a standard error of 0.04) at 25-δ. As a result, disentangling Gaussian risk from disaster risk is not an easy task; see Farhi et al. (2009) for a methodology using currency options. In this paper, we interpret shocks to equity volatility as shocks to disaster probabilities.

[Figure 4 about here.]

4.2 Comovement in realized and implied volatilities

We focus on a group of G7 countries for which monthly macroeconomic series are readily available over long periods of time: Canada, France, Germany, Italy, Japan, United Kingdom, and United States. The evidence presented above pertains to expected volatilities derived from equity options. These series are not available for many countries or over long samples. Fortunately though, implied volatilities and realized volatilities are highly correlated (despite the presence of a volatility risk premium). We thus resort to realized volatilities.
We import daily MSCI equity returns for the sample. We build monthly series of stock market volatility by recording standard deviations over calendar months. In the model, disasters are global; they are assumed to affect countries differently, but happen at the same time in all countries. How does this assumption compare to the data? Looking at realized equity return volatilities or option-implied volatilities, our assumption does not seem far-fetched. Option-implied or realized equity volatility series clearly contain a large common component across countries. The first principal component, which is close to the mean of all these series, accounts for more than 40% of total realized variance in a large set of 27 OECD countries over 40 years, close to 50% for the G7 countries over 40 years, and more than 90% of a set of nine option-implied volatilities over the last 15 years. As a result, we take, at each point in time, the mean of our different realized volatility series over the countries in sample: this mean volatility constitutes our measure of global disaster risk.

4.3 Impact of global volatility shocks on quantities

We also study the impact of global volatility shocks on monthly macroeconomic variables. Our methodology follows Bloom (2009) and focuses on simple VARs. Details about our VARs are in the separate online Appendix. Briefly, our VAR specification includes the market return, global volatility, and macroeconomic time series. The latter are the 12 month log differences of industrial production or unemployment rates. We use 12 lags in the VAR. The shock to volatility has no impact on the macroeconomic variable at time 0. This orthogonalization assumption has little impact on our results.

[Figure 5 about here.]

Figures 5 and 6 report the impulse response functions of industrial production and unemployment rates to a shock on aggregate volatility. In our core sample, unemployment rates increase and industrial production contracts when volatility increases. Bloom (2009) reports similar results for U.S. employment and industrial production. Note, however, that
we are using the average world volatility, and not the U.S. volatility as in Bloom (2009). Our results are in line with the model’s predictions: when the probability of a global disaster increases, a recession ensues with less production and employment. The magnitudes of the responses are economically and statistically significant. A one-standard deviation shock on aggregate volatility implies approximately a 0.8% decline in industrial production for the U.S. The 2007–2009 subprime mortgage crisis corresponds to a 9 standard deviation shock on aggregate equity volatility. In our VAR, such a shock would lead to a 7.2% decline in industrial production. As a comparison point, in the data, the 12-month growth rate of industrial production hit a bottom of −15% during the crisis. This strong result is not confined to the U.S. economy. All countries present a similar negative effect, and interestingly, even in this limited sample of OECD countries, there is some heterogeneity: for instance, Japan is significantly more affected by a volatility shock than the other countries.

We report in the separate Appendix the same impulse response functions country by country and with standard errors obtained by bootstrapping. All troughs are clearly negative and significantly different from zero for industrial production. The same result holds for unemployment, with clearly positive impulse response functions: peaks, however, are only significant in Canada, Japan, and the U.S. Overall, we find a clear impact of aggregate volatility shocks on macroeconomic aggregates.

5 Conclusion

This paper shows how the combination of time-varying risk and heterogeneous exposures to that risk make international business cycle models closer to the data. An increase in disaster risk leads to a worldwide recession, that is larger in the more risky countries, and to decline in stock prices, interest rates, and a negative return on the carry trade. We provide direct support for this mechanism by showing that aggregate volatility shocks have a significant
effect on macroeconomic aggregates. The main tension in our complete market model is that low interest rate countries are more risky in terms of fundamentals and hence more volatile, while in the data they are less so. This tension is an invitation to consider more elaborate models that incorporate default risk, monetary policy, and incomplete markets.
References


### Table 1: Business Cycle Statistics

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</tbody>
</table>

**Notes:** This table reports the standard deviations of log differences in consumption, investment, labor and output, along with the cross-country correlation of these variables. Panel I reports moments from the actual data. We use series from the OECD database, available on Datastream. Data are quarterly. The maximum sample period is 1970.I–2010.IV, but sample windows vary across countries. Additional information is available in Appendix A. We focus on high versus low interest rate countries. We sort countries on their interest rate levels and build four portfolios. We report statistics for countries in the first portfolio (low interest rate countries) for the domestic economy. For the foreign economy, we report statistics for countries in the last portfolio (high interest rate countries, denoted with a stat superscript $^\star$). Panel II is constructed by simulating the model, assuming no disasters are actually realized (see the Appendix D for simulations that include disaster realizations). We solve three variants of the model: (1) a standard real business cycle model with only TFP shocks; (2) a model with time-varying risk of disaster, when both countries are equally risky; (3) our benchmark model with time-varying risk of disaster, where the domestic country is more risky. We report moments obtained on one-year growth rates (measured at a quarterly frequency). Standard errors are obtained by block-bootstrapping (with blocks of four observations). For simulated data, we use a sample of the same size as our actual data by averaging across 3,000 simulations.
### Table 2: Financial Statistics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r_e)$</td>
<td>$E(r_f)$</td>
<td>$E(r_e^*)$</td>
<td>$Ac(r_f)$</td>
</tr>
<tr>
<td>4.45</td>
<td>0.81</td>
<td>1.06</td>
<td>5.17</td>
</tr>
<tr>
<td>[2.56]</td>
<td>[0.37]</td>
<td>[3.28]</td>
<td>[0.60]</td>
</tr>
<tr>
<td>$E(r_f^*)$</td>
<td>$Ac(r_f^*)$</td>
<td>$\sigma(r_e)$</td>
<td>$\sigma(r_f)$</td>
</tr>
<tr>
<td>0.99</td>
<td>0.94</td>
<td>19.81</td>
<td>2.50</td>
</tr>
<tr>
<td>[0.00]</td>
<td>[0.02]</td>
<td>[1.31]</td>
<td>[0.33]</td>
</tr>
<tr>
<td>$\sigma(r_e^*)$</td>
<td>$\sigma(r_f^*)$</td>
<td>$\sigma(r_e)$</td>
<td>$\sigma(r_f^*)$</td>
</tr>
<tr>
<td>4.13</td>
<td>0.77</td>
<td>25.23</td>
<td>4.13</td>
</tr>
<tr>
<td>[0.05]</td>
<td>[0.02]</td>
<td>[1.68]</td>
<td>[0.41]</td>
</tr>
<tr>
<td>$\sigma(r_e^<em>, r_f^</em>)$</td>
<td>$(r_e^<em>, r_f^</em>)$</td>
<td>(\sigma(r_e, r_f^*))</td>
<td>(\sigma(r_f, r_f^*))</td>
</tr>
<tr>
<td>0.73</td>
<td>0.73</td>
<td>19.81</td>
<td>2.50</td>
</tr>
<tr>
<td>[0.07]</td>
<td>[0.07]</td>
<td>[1.31]</td>
<td>[0.33]</td>
</tr>
</tbody>
</table>

**Panel I: Data**

<table>
<thead>
<tr>
<th>RBC</th>
<th>Equal Risk</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.17</td>
<td>3.26</td>
<td>0.17</td>
</tr>
<tr>
<td>[1.57]</td>
<td>[0.84]</td>
<td>[0.85]</td>
</tr>
<tr>
<td>(\sigma(r_e, r_f^*))</td>
<td>(\sigma(r_f, r_f^*))</td>
<td>(\sigma(r_e^<em>, r_f^</em>))</td>
</tr>
<tr>
<td>0.85</td>
<td>0.98</td>
<td>0.89</td>
</tr>
<tr>
<td>[0.07]</td>
<td>[0.03]</td>
<td>[0.07]</td>
</tr>
</tbody>
</table>

**Panel II: Model**

Notes: This table reports the averages and standard deviations of log equity excess returns and log short-term interest rates, along with the cross-country correlation of these variables and the first order autocorrelation of risk-free rates. Data are from the IMF, MSCI, and Datastream databases. See Table 1 for more details.
Table 3: Real Exchange Rates

<table>
<thead>
<tr>
<th></th>
<th>Moments of $\Delta q$</th>
<th>Corr. $\Delta q$ and $\Delta c - \Delta c^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta q)$</td>
<td>$\text{Ac}(\Delta q)$</td>
<td>$S(\Delta q)$</td>
</tr>
<tr>
<td>Panel I: Data</td>
<td>8.56</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>[0.83]</td>
<td>[0.05]</td>
</tr>
<tr>
<td>RBC</td>
<td>6.27</td>
<td>0.75</td>
</tr>
<tr>
<td>Equal Risk</td>
<td>6.24</td>
<td>0.75</td>
</tr>
<tr>
<td>Benchmark</td>
<td>10.71</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>[2.17]</td>
<td>[0.17]</td>
</tr>
</tbody>
</table>

Notes: This table reports the averages, standard deviations, annual autocorrelation, skewness and kurtosis of changes in log real exchange rates, along with the cross-country correlation of changes in log real exchange rates with cross-country differences in real equity returns, real risk-free rates, and real consumption growth. For actual data, we use series from the IMF, OECD, MSCI, Datastream databases. See Table 1 for more details.
Table 4: Carry Trade Excess Returns

<table>
<thead>
<tr>
<th>Moments of $rx$</th>
<th>Corr. $rx$ and $r_{t+1}^m$</th>
<th>UIP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r_{x,t+1})$</td>
<td>$\sigma(r_{x,t+1})$</td>
<td>$\Delta c$</td>
</tr>
<tr>
<td>5.31</td>
<td>9.59</td>
<td>0.29</td>
</tr>
<tr>
<td>[1.33]</td>
<td>[0.92]</td>
<td>[0.17]</td>
</tr>
<tr>
<td>Panel I: Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RBC</td>
<td>0.00</td>
<td>6.26</td>
</tr>
<tr>
<td>Equal Risk</td>
<td>0.00</td>
<td>6.23</td>
</tr>
<tr>
<td>Benchmark</td>
<td>2.36</td>
<td>10.87</td>
</tr>
<tr>
<td>[1.02]</td>
<td>[2.18]</td>
<td>[0.15]</td>
</tr>
<tr>
<td>Panel II: Model</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports the averages and standard deviations of carry trade excess returns, along with the cross-country correlation of these excess returns with the average (across portfolios) of macroeconomic and financial variables: real consumption growth, real investment growth, real employment growth, real GDP growth, real stock market returns, and the changes in stock market return volatilities. The last two columns report the UIP slope coefficient (obtained on quarterly series) and the associated $R^2$. See Table 1 for details. For the UIP regression we use 200 simulations of 50,000 periods to avoid a short sample bias. All other results do not depend on the length of the sample.
Table 5: Calibration Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Home / Foreign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share</td>
<td>$\alpha$</td>
<td>.34</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>.02</td>
</tr>
<tr>
<td>Share of consumption in utility</td>
<td>$\nu$</td>
<td>.30</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>.994</td>
</tr>
<tr>
<td>Adjustment cost curvature</td>
<td>$\eta$</td>
<td>.15</td>
</tr>
<tr>
<td>Trend growth of TFP</td>
<td>$\mu$</td>
<td>.0025</td>
</tr>
<tr>
<td>Standard deviation of ordinary TFP shock</td>
<td>$\sigma$</td>
<td>.009</td>
</tr>
<tr>
<td>Correlation of TFP shocks</td>
<td>$(\sigma, \sigma^*)$</td>
<td>0.3</td>
</tr>
<tr>
<td>IES</td>
<td>$1/\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>Risk aversion over consumption-leisure bundle</td>
<td>$\theta$</td>
<td>8.5</td>
</tr>
<tr>
<td>Drop in TFP in case of disaster</td>
<td>$b_{t_{fp}}/b_{t_{fp}}$</td>
<td>.3 / .24</td>
</tr>
<tr>
<td>Capital destruction in case of disaster</td>
<td>$b_k/b_k^*$</td>
<td>.3 / .24</td>
</tr>
<tr>
<td>Mean log disaster probability</td>
<td>$\mu_p$</td>
<td>-.0565</td>
</tr>
<tr>
<td>Persistence of log($p$)</td>
<td>$\rho_p$</td>
<td>.92</td>
</tr>
<tr>
<td>Unconditional std. dev. of log($p$)</td>
<td>$\sqrt{1-\rho_p}$</td>
<td>2.05</td>
</tr>
<tr>
<td>Leverage</td>
<td>$\lambda$</td>
<td>2</td>
</tr>
</tbody>
</table>

Notes: This table reports the parameters used to solve and simulate our model. The time period is one quarter. Home and foreign countries are identical unless otherwise indicated.
Figure 1: **Impulse Response Functions to a TFP shock**: This figure presents the impulse response functions of macroeconomic and financial variables to a standard TFP shock. Quantities are in percentage deviation from the balanced growth path (BGP). Asset returns are in percentage change per quarter. The disaster probability is in percentage, and the disaster realization is an indicator function.
Figure 2: Impulse Response Functions to a Disaster Realization: This figure presents the impulse response functions of macroeconomic and financial variables to a disaster realization. Quantities are in percentage deviation from the balanced growth path (BGP). Asset returns are in percentage change per quarter. The disaster probability is in percentage, and the disaster realization is an indicator function.
Figure 3: **Impulse Response Functions to a Disaster Probability Shock**: This figure presents the impulse response functions of macroeconomic and financial variables to an increase to the probability of disaster. Quantities are in percentage deviation from the balanced growth path (BGP). Asset returns are in percentage change per quarter. The disaster probability is in percentage, and the disaster realization is an indicator function.
Figure 4: Implied Volatilities from Put Options, Risk Reversals, and VIX Index: The first panel of this figure presents the implied volatilities of put options on the S&P 500 index, along with the VIX index. The VIX index measures the risk-neutral implied volatilities of a set of in-the-money and out-of-the-money puts and calls on the S&P 500. We use put options that are approximately 10-δ and 25-δ away from the money. The second panel of this figure reports 10-δ and 25-δ risk-reversals. Risk-reversals are measured as the differences between the implied volatility of puts and the implied volatility of calls with strike prices that are symmetric around the money. Implied volatilities are annualized and reported in percentages. For presentation purposes, risk-reversals are multiplied by 2. Data are from the Chicago Mercantile Exchange.
Figure 5: **Response of Industrial Production to a Shock on Average Realized Volatility**: This figure plots the impulse response functions of unemployment rates to a one-standard deviation shock on average volatility in the G7 countries: Canada, France, Germany, Italy, Japan, United Kingdom, and United States. Data are monthly. The sample is 1970.1 – 2009.12, except for the United Kingdom (1971.1 – 2009.12) and Canada (1995.1 – 2009.12). We do not have industrial production indices for Italy. Volatility measures correspond to the standard deviations of equity returns over calendar months. The average volatility is the mean of these different standard deviations over the same countries. VARs contain the following variables: market returns, volatility, and industrial production.
Figure 6: Response of Unemployment to a Shock on Average Realized Volatility: This figure plots the impulse response functions of unemployment rates to a one-standard deviation shock on average volatility in the following G7 countries: Canada, France, Germany, Italy, Japan, United Kingdom, and United States. Data are monthly. The sample is 1970.1 – 2009.12, except for France (1978.1 – 2009.12), Germany (1992.1 – 2009.12) and Italy (1980.1 – 2009.12). Volatility measures correspond to the standard deviations of equity returns over calendar months. The average volatility is the mean of these different standard deviations over the same G7 countries. VARs contain the following variables: market returns, volatility, and unemployment rates.