Systemic risk and the refinancing ratchet effect

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Systemic Risk and the Refinancing Ratchet Effect

Amir E. Khandani†, Andrew W. Lo‡, and Robert C. Merton§

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Abstract

We show that the confluence of rising home prices, declining interest rates, and near-frictionless refinancing opportunities can lead to substantial risk in the financial system. The interaction between these three factors causes an unintentional synchronization of homeowner leverage. Coupled with the indivisibility and sole ownership of residential real estate—which prevents homeowners from deleveraging when property values decline—this synchronization conspires to create a “ratchet” effect in homeowners’ leverage. To assess the magnitude of potential risk through this mechanism, we simulate the U.S. housing market with and without equity extractions and estimate the losses absorbed by mortgage lenders by valuing the embedded put-option in non-recourse mortgages. In our simulation, this mechanism alone can generate losses of $1.7 trillion from June 2006 to December 2008, compared with simulated losses of $330 billion in the absence of equity extractions. Irrespective of its role in the Financial Crisis of 2007–2009, the refinancing ratchet effect is a new and more complex form of systemic risk in the residential mortgage system that does not rely on any of the dysfunctional behaviors on which most studies of the crisis are based.

Keywords: Systemic Risk; Financial Crisis; Household Finance; Real Estate; Subprime

JEL Classification: G01, G12, G13, G18, G21, E17, E27, E37, E47, R21, R38

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1 Introduction

Home mortgage loans—one of the most widely used financial products by U.S. consumers—are collateralized mainly by the value of the underlying real estate. This feature makes the market value of the collateral very important in measuring the risk of a mortgage. To reduce the risk of default, mortgage lenders usually ask for a down payment of 10% to 20% of the value of the home from the borrower, creating an “equity buffer” that absorbs the first losses from home price declines. Any event or action that reduces the value of this buffer, e.g., an equity extraction or a drop in home values, increases the risk of the lending institution.

A number of secular trends over the last two decades, including the increased efficiency of the refinancing process and the growth of the refinancing business, have made it much easier for homeowners to refinance their mortgages to take advantage of declining interest rates, increasing housing prices, or both. Consequences of these trends have been documented by Greenspan and Kennedy (2008, p. 120), who observe, “since the mid-1980s, mortgage debt has grown more rapidly than home values, resulting in a decline in housing wealth as a share of the value of homes.” They attribute most of this effect to discretionary equity extractions via home sales, “cash-out” refinancing (where the homeowner receives cash after the refinancing), and home-equity loans.

In this paper, we focus on a previously unstudied dimension of risk in the mortgage market: the interplay among the growth of the refinancing business, the decline in interest rates, and the appreciation of property values. Each of these three trends is systemically neutral or positive when considered in isolation, but when they occur simultaneously, the results can be explosive. We argue that during periods of rising house prices, falling interest rates, and increasingly competitive and efficient refinancing markets, cash-out refinancing is

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1 Although most models of household finance assume that residential mortgages are non-recourse loans, the legal procedure for foreclosure and obtaining a deficiency judgment is complex, varying greatly from state to state. In fact, Ghent and Kudlyak (2009, Table 1) observe that home mortgages are explicitly non-recourse in only 11 states. Not surprisingly, some of those states are experiencing severe foreclosure problems in the current crisis such as Arizona and California. However, in certain populous states with recourse, such as Florida and Texas, generous homestead-exemption laws can make it virtually impossible for lenders to collect on deficiency judgments because borrowers can easily shield their assets. Ghent and Kudlyak (2009) study the effect of lender recourse on mortgage defaults across the U.S. and conclude that recourse does decrease the probability of default for homeowners who have negative equity.

2 See, for example, Danis and Pennington-Cross (2005); Downing, Stanton, and Wallace (2005); Gerardi, Shapiro, and Willen (2007); Doms, Furlong, and Krainer (2007); Bajari, Chu, and Park (2008); Bhardwaj and Sengupta (2008a); and Gerardi et al. (2008).
like a ratchet. It incrementally increases homeowner leverage as real-estate values appreciate without the ability to symmetrically decrease leverage by increments as real-estate values decline. This self-synchronizing “ratchet effect” can create significant systemic risk in an otherwise geographically and temporally diverse pool of mortgages.

The potential magnitude of the risk created due to the refinancing ratchet effect is most clearly illustrated though a hypothetical scenario in which all homeowners decide to keep their leverage at a level generally associated with extreme prudence and good lending practices, for example, a loan-to-value (LTV) of 80% for a conventional fully-amortizing 30-year fixed-rate mortgage. Suppose the refinancing market is so competitive, i.e., refinancing costs are so low and capital is so plentiful, that homeowners are able to extract any equity above the minimum each month. In such an extreme case, cash-out refinancing has the same effect as if all mortgages were re-originated at the peak of the housing market. When home prices fall, as they must eventually, the ratchet “locks” because homeowners cannot easily unwind their real-estate positions and incrementally deleverage due to indivisibility and illiquidity. The unintentional synchronization of leverage during the market’s rise naturally leads to an apparent shift in regime during the market’s decline, in which historically uncorrelated defaults now become highly correlated.\(^3\)

Indivisibility and sole ownership of residential real-estate are two special characteristics of this asset class that make addressing this issue particularly challenging. The impact of indivisibility can be crystallized by comparing an investment in residential real estate with a leveraged investment in a typical exchange-traded instrument such as shares of common stock. While the latter is subject to both an initial margin requirement as well as a maintenance margin requirement, home mortgages only have a homeowner equity requirement that plays a role similar to that of an initial margin. It is hard to imagine that homeowners would willingly finance large capital purchases using short-term debt like margin accounts, and long-term debt may have become the standard method for financing home purchases precisely because of the indivisible nature of the collateral. Furthermore, the owner is typically the sole equityholder in an owner-occupied residential property which makes it difficult to bring incrementally additional capital in to reduce risk by issuing new equity.

These two special features of residential real estate create an important asymmetry in

\(^3\)If mortgages were recourse loans and borrowers had uncorrelated sources of income, the aggregate risk of the mortgage market would be lower. However, as discussed in footnote 1, recourse does not exist in all states; hence this diversification channel is not always available.
the housing market that does not exist in most financial markets. While a leveraged investor may decide not to incrementally deleverage as prices decline due to optimistic expectations of a price reversal, the indivisibility and sole ownership of owner-occupied homes makes incremental deleveraging impossible, even for those who want to reduce their exposure to real estate. Therefore, the only option available to homeowners in a declining market is to sell their homes, recognize their capital losses, and move into less expensive properties that satisfy their desired LTV ratio. The enormous costs—both financial and psychological—of such a transaction make it a highly impractical and implausible response to addressing the issue raised in this paper.

We propose to gauge the magnitude of the potential risk caused by the refinancing ratchet effect by creating a numerical simulation of the U.S. housing and mortgage markets. By calibrating our simulation to the existing stock of real estate, and by specifying reasonable behavioral rules for the typical homeowner’s equity extraction decision—which satisfy common standards of prudence and good lending practices in the U.S.—our simulation can match some of the major trends in this market over the past decade such as the rapid rise in the amount of mortgages outstanding and the massive equity extractions from U.S. residential mortgages during this period. We are able to show that refinancing-facilitated home-equity extractions alone can create significant risk in the residential mortgage system.

Using a standard derivatives-pricing model, we construct an estimate of losses absorbed by mortgage lenders—banks, asset management firms, and government-sponsored enterprises (GSEs)—from the decline in real-estate prices and compare these estimates with the scenario of no equity extractions over the same period. Our simulation yields an approximate loss of $1.7 trillion from the housing-market decline since June 2006 compared to a loss of $330 billion if no equity had been extracted from U.S. residential real estate during the boom.

While we have attempted to construct as realistic a simulation as possible, we acknowledge at the outset that our approach is intended to capture “reduced-form” relations and is not based on a general equilibrium model of households and mortgage lenders. Instead of relying on a stylized general equilibrium model, we adopt a simple refinancing rule that seems to capture plausible behavior among U.S. homeowners over the recent past. Also, we do not model the supply of refinancing and the behavior of lenders, but rather assume that households can refinance as much as they wish at prevailing historical interest rates. While the plentiful supply of credit had been close to reality during the decade leading up to
the peak of the housing market in June 2006, our motivation for this assumption is to isolate the impact of the refinancing ratchet effect. Lending behavior undoubtedly contributed to the magnitude of the Financial Crisis of 2007–2009, as did many other factors (see Lo, 2012, for a review of the burgeoning crisis literature). An empirically accurate stochastic dynamic general equilibrium model of the housing and mortgage markets that endogenizes these factors is a much more challenging undertaking and beyond the scope of this paper.

Our objective is not to explain the crisis, but rather to show that even in the absence of any dysfunctional behavior such as excessive risk-taking, fraud, regulatory forbearance, political intervention, and predatory borrowing and lending, large system-wide shocks can occur in the housing and mortgage markets. The refinancing ratchet effect is a considerably more subtle and complex form of systemic risk, arising from the confluence of three familiar and individually welfare-improving economic trends. The simplicity of our simulation approach makes the refinancing ratchet effect more transparent, and the potential magnitude of its impact suggests that further attention is warranted.

We begin in Section 2 with a brief review of the literature, and Section 3 provides some basic facts about the U.S. mortgage system. We outline the design of our simulation and describe the results of the calibration exercise in Section 4. We use these results in conjunction with a simple option-pricing model in Section 5 to estimate the impact of mortgage refinancing on the aggregate risk of the U.S. mortgage market as home prices declined from 2006 to 2008. We provide some qualifications for and extensions of our results in Section 6, and conclusions in Section 7.

2 Literature Review

Given the magnitude of the subprime mortgage crisis, a number of recent papers have attempted to trace its root causes. Dell’Ariccia, Igan, and Laeven (2008), Demyanyk and Van Hemert (2008), Bhardwaj and Sengupta (2008b), Keys et al. (2008), and Mian and Su (2008) are only a few of the examples in this vast and growing literature. While the issues discussed in these papers, such as lax lending standards and the impact of institutional changes like securitization or the expansion of the subprime market, were certainly of primary importance in causing the recent crisis, none of these papers have focused on the unique interplay between refinancing and systemic risk in the residential mortgage system that we examine in this paper.
The uncertain durations and credit risk of mortgages—due to prepayment or default by the borrower—make their risks different from other fixed-income products. The approach to modeling these risks can be divided into two categories: structural and reduced-form. Structural models focus on the underlying dynamics of the collateral value and the interest rates to arrive at a model of consumer behavior, while reduced-form models take a more statistical approach. Kau, Keenan, Muller, and Epperson (1992, 1995) and Kau and Keenan (1995) provide some early examples of the structural approach while Schwartz and Torous (1989), Deng, Quigley, and Van Order (2000), and Deng and Quigley (2002) take the reduced-form approach in their studies. LaCour-Little (2008) provides a recent review of this literature.

The earliest structural models adopted simplifying assumptions that yielded elegant closed-form solutions at the expense of certain stylized facts of the U.S. mortgage market that could not be captured by those assumptions. For example, consider the decision to default on a mortgage. The value of the underlying real estate is obviously the most important factor in driving this decision. However, while negative equity may be a necessary condition to trigger default, it is apparently not sufficient (Foote, Gerardi, and Willen, 2008), perhaps due to concerns such as moving costs, the desire to preserve reputational capital, default penalties, or even sentimental attachment to the home. Similarly, homeowners seeking to refinance into a lower interest-rate mortgage when rates decline may be constrained by their financial circumstances or insufficient amounts of equity in their homes.

Such complexities make complete modeling of risk in the residential mortgage system challenging. To avoid the possibility that our main message could be lost while dealing with these complexities, we have adopted a simple yet realistic behavioral rule that can be easily understood and allows us to focus on the main subject of our paper. Our approach for evaluating risk and pricing mortgage guarantees uses option-pricing technology that makes the analytical aspects of our approach closer to the structural models.

We argue that the increasing familiarity of borrowers with the refinancing process; the invention of new mortgage products; and the corresponding institutional, social, and political changes over the last decade contributed to an environment fertile for the type of risk that is the focus of this paper. Other researchers have studied and commented on this topic.

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4Stanton (1995) and Downing, Stanton, and Wallace (2005) incorporate some of these effects into their models of mortgage termination.

5Archer, Ling, and McGill (1997) and Peristiani et al. (1997) consider the impact of household financial conditions such as income, credit history, and the amount of homeowner’s equity on the ability to refinance.
as well. For example, by comparing the refinancing decision of homeowners in the 1980s relative to the 1990s, Bennett, Peach, and Peristiani (2001) find evidence that over time, a combination of technological, regulatory, and structural changes has reduced the net benefit needed to trigger a refinancing decision. They conjecture that homeowners' familiarity with the refinancing process and their increased financial sophistication are possible drivers behind this phenomenon.⁶

Two examples of new mortgage products that enabled easier refinancing are the “sub-prime” and “Alt-A” mortgages. As Mayer and Pence (2008, p. 1) observe, “these new products not only allowed new buyers to access credit, but also made it easier for homeowners to refinance loans and withdraw cash from houses that had appreciated in value.” They point out that “subprime mortgages are used a bit more for refinancing than home purchase” and “almost all subprime refinances are cash-out refinances” (Mayer and Pence, 2008, p. 10). Moreover, some of the more exotic products like non- or negative-amortization mortgages are contractual equivalents to dynamic strategies involving frequent cash-out refinancing to maintain a desired leverage ratio. These product innovations may have facilitated large-scale equity extractions by making refinancing significantly easier, cheaper, and virtually automatic.⁷

The behavioral and social aspects of the decision to default on a residential mortgage is considered by Guiso, Sapienza, and Zingales (2009) using surveys of American households in late 2008 and early 2009. They find that those who consider it immoral to default are 77% less likely to declare their intention to do so. They also find that households who have been exposed to defaults are more willing to default strategically, i.e., to default even though they can afford their mortgage payments. For example, holding social stigma constant, individuals who know someone who defaulted strategically are 82% more likely to declare their intention to do so. And as defaults become more common within a given social network, the social stigma of default is likely to decline, lowering the threshold for new defaults to occur.

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⁶ Specifically, they compare the refinancing behavior during two major refinancing cycles: 1986–1987 and 1992–1993. They find that measurable transactions costs, such as points and fees, are quite important in the refinancing decision, and these costs have declined over time due to competition and growth in the refinancing market. However, even after controlling for these costs and other factors that are known to impact the refinancing decision, the estimated refinancing probability is still considerably higher in the later part of their sample (9% vs. 14%; see Bennett, Peach, and Peristiani, 2001, pp. 970–971). Motivated by this analysis, we will propose refinancing rules with a structural break in the year 1988 (see Appendix A.3).

⁷ Many of these innovations may also have important tax or transaction-cost benefits to the borrower; hence they may have been demand-driven rather than the result of overly aggressive mortgage lenders. In fact, these products may be essential for achieving optimal risk-sharing, Piskorsi and Tchistyj (2006).
Perhaps a similar set of forces was at play during the most recent cash-out refinancing boom. Institutional changes, heightened competition, and technological advances made it easier and cheaper for consumers to engage in mortgage refinancing, and increased awareness of and familiarity with the refinancing process made it more popular. Even though many homeowners were undoubtedly aware of the potential dangers of equity extractions, the fact that many of their neighbors or co-workers were extracting equity from their homes made it more socially acceptable to do so at the height of the housing boom.

3 Basic Facts About the U.S. Mortgage Market

We begin with some basic facts about the overall size and trends of the U.S. mortgage market that are most relevant for our study. Figure 1 shows the time series of conventional 30-year fixed-rate mortgage rates, and purchase and refinancing mortgage-origination volumes in the U.S. from the first quarter of 1991 (1991Q1) to the fourth quarter of 2008 (2008Q4). The data collected by the Mortgage Bankers Association breaks down origination volume into two components: origination of loans intended for new purchase, and those intended for refinancing purposes. Refinancing volume can be further broken down by loan type based on the data collected by the U.S. Federal Housing Finance Agency (FHFA), formerly the Office of Federal Housing Enterprise Oversight (OFHEO). In particular, FHFA data, available at http://www.fhfa.gov/Default.aspx?Page=87 classifies loans into the following three categories: Purchase, Cash-Out Refinancing, and Rate/Term Refinancing.

Several prominent themes emerge from Figure 1. While purchase volume is highly seasonal, the increase and subsequent decline closely matches the trend in overall real-estate prices. There is also a clear relationship between decline in mortgage rates and rate-refinancing volume. For example, the decline in interest rates in the early 1990s is followed by a period of high rate-refinancing activity from 1992 to 1993. The next period of increased rate-refinancing occurs in 1998, again coinciding with a drop in mortgage rates. However, the most active period of rate refinancing takes place in 2001Q4 through 2003Q3, where the average volume is $342 billion per quarter, far exceeding the peak of each of the previous two refinancing booms. There is also indirect evidence that mortgage-lending competition in-

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creased during this period—according to Freddie Mac’s surveys (see www.freddiemac.com), the average number of points associated with conventional 30-year fixed-rate mortgages declined from 1.8 in December 1997 to 1.0 in December 1998 to 0.6 in December 2002, and to 0.7 by June 2009.

Home-equity extraction is a process in which a homeowner converts a portion of the equity in the home into cash by retiring the existing loan and taking out a new and larger loan. Such loans are categorized as “cash-out” refinancing in the FHFA data set. It is not surprising that equity extraction is more common in a rising real-estate market because during such periods, homeowners’ equity increases dollar-for-dollar with home prices, giving homeowners more equity to extract. Figure 1 documents a seemingly permanent increase in cash-out refinance volume in the second half of the sample. The first peak in cash-out refinancing occurs in 1998Q4, when volume surpasses $100 billion for the first time. Although the volume in the following 9 quarters (1999Q1 to 2001Q1) was less than $100 billion per quarter, the average value of cash-out refinancing per quarter was $204 billion in the subsequent 30 quarters (2001Q2 to 2008Q3), far exceeding the average value in the preceding 41 quarters from 1991Q1 to 2001Q1. Not surprisingly, as home prices fell from 2006 to 2008, cash-out refinancing volume rapidly subsided, declining to only $84 billion in the last quarter of 2008.

Figure 2 shows the relation between gross equity extraction and aggregate U.S. home prices during the period from 1991Q1 to 2008Q4. The increase and subsequent decline in the gross equity extraction closely mirrors the pattern of aggregate U.S. residential real-estate prices. According to this estimate, U.S. homeowners extracted an average of $160 billion in each of the 32 quarters between 1999Q3 to 2007Q2, far outstripping the $87 billion extracted during the previous peak in 1998Q4.

Other things being equal, equity extraction leads to a larger mortgage on a given home, implying a link between the amount of equity extracted and the volume of mortgages outstanding that is confirmed in Figure 3. This figure shows that outstanding mortgages grew from $2,648 billion in 1991Q1 to a peak of $11,142 billion in 2008Q1. During this period, homeowners extracted $6,720 billion in equity. These figures suggest that equity extractions represent a non-trivial portion of outstanding mortgages, and the risk transferred from home-

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9The estimates of gross equity extractions are from Greenspan and Kennedy (2005). We are grateful to Jim Kennedy for providing us with updated estimates. See Appendix A.1 for details.
Figure 1: 30-year fixed-rate mortgage rates, and purchase, cash-out refinancing, and rate-refinancing origination volume from 1991Q1 to 2008Q4.

Figure 2: The appreciation in and subsequent decline of U.S. residential real-estate values, as measured by the S&P/Case-Shiller Composite-10 Home Price Index, and the corresponding growth of equity extractions from 1991Q1 to 2008Q4. The equity extraction data is based on Greenspan and Kennedy (2005).
owners to the financial sector due to these extractions may have had a significant impact on the overall risk exposure of this sector to real-estate prices. The objective of the simulation in this paper is to quantify this effect.

4 Simulating the U.S. Mortgage Market

In this section we provide the details of our simulation approach and report the results of our calibration exercise for different simulation scenarios.

4.1 Simulation Assumptions

In designing our simulation, we must balance the desire for realism against the availability of data and the tractability of the computations required. To that end, we make the following assumptions:

(A1) Each house is purchased at an initial LTV ratio drawn from a distribution that is fixed through time and does not have any geographical dependency.
(A2) All homes are purchased with conventional fixed-rate mortgages that are non-recourse loans with initial maturity drawn from a distribution that is fixed through time and does not have geographical dependency.

(A3) The market value of homes follows a geometric random walk with a mean given by a “Home Price Index” and volatility given by “Home Price Volatility”. We incorporate geographical heterogeneity into the Home Price Index but assume that home price Volatility is constant through time and across all regions.

(A4) We allow homeowners the possibility of engaging in “Cash-Out Refinancing” or “Rate Refinancing” in each month.

(A5) For Cash-Out Refinancing, we assume that the $i^{th}$ homeowner’s decision is random with probability $REFI_{i,t}$ which is a function only of the current equity in the home and the prevailing mortgage rate. In particular, we assume that the refinancing decision does not depend on factors such as the price and age of the home, or the time elapsed since any previous refinancing. We also assume that the homeowner will refinance into a new loan with the initial LTV ratio and maturity specified in Assumptions (A1) and (A2).

(A6) The owners will engage in a Rate Refinancing as soon as mortgage rates have fallen by more than the “Rate Refinance Threshold” (RRT) from the existing mortgage rate. The new mortgage is assumed to have the same maturity as the existing mortgage, and the principal of the new mortgage is equal to the remaining value of the existing mortgage. Therefore, the homeowner will save in monthly payments due to the lower mortgage rate, but no equity is extracted.

(A7) Homeowners’ refinancing decisions are random and independent of each other, apart from the dependence explicitly parameterized in the refinancing rule.

(A8) Once fully paid for, a home will not re-enter the housing market.

(A9) We do not incorporate taxes or transaction costs explicitly into our simulations.

Given the central role that these assumptions play in our simulations and their interpretation, a few words about their motivation are in order.
Assumptions (A1) and (A2) determine the initial leverage and type of mortgages we assume for new homeowners. Here we have assumed the initial LTV distribution did not change throughout time and all mortgages were standard and fully-amortizing mortgages. Of course, in the years leading up to the peak of the housing market in 2006, considerably more aggressive and exotic loans were made, including the now-infamous NINJA (“no income, no job or assets”) mortgages and many others with embedded options. Assumptions (A1) and (A2) are motivated by our desire for simplicity; we also wish to err on the side of caution with respect to default-related loss implications wherever possible.

Assuming that mortgages are non-recourse loans greatly simplifies our simulations because we do not need to model the dynamics of other sources of collateral. However, by assuming that lenders have no recourse to any other sources of collateral, our simulation may yield over-estimates of potential losses, and it may also oversimplify the behavior of borrowers (see Ghent and Kudlyak, 2009). To take on the more complex challenge of matching the mix of recourse and non-recourse loans in the mortgage system in our simulations, we would require information about the types of recourse that are permitted and the practicalities of enforcing deficiency judgments in each of the fifty states, as well as cross-sectional and time-series properties of homeowner income levels, assets, and liabilities. While this task is beyond the scope of our current study, it is not insurmountable given sufficient time, resources, and access to financial data at the household level.

Assumption (A3) allows us to calibrate the price dynamics of our simulated housing stock, and the dynamics we have assumed are consistent with the standard weighted-repeat-sales index construction methodology (see, for example, Calhoun, 1996). We introduce geographical heterogeneity in the mean home price appreciation rate but assume that volatility is constant. Once again, these assumptions are meant to err on the conservative side. For example, it is likely that home price volatility spiked in regions with sharp price declines, or after national price levels dropped; we ignore such empirical regularities in our simulations.

Assumptions (A4)–(A6) are simple behavioral rules meant to encapsulate the economic deliberations of homeowners as they decide whether or not to refinance. Accordingly, implicit in these rules are many factors that we do not model explicitly, e.g., transactions costs, opportunity costs, homeowner characteristics such as income and risk preferences, macroeconomic conditions, and social norms. While it may be possible to derive similar rules from first principles (e.g., Stanton, 1995), the computational challenges may outweigh the benefits,
especially from the perspective of producing estimates of potential losses for the aggregate housing sector.

Assumptions (A5) and (A6) outline the two polar opposites of our simulated refinancing activities—(A5) describes the situation where owners decide to increase their mortgage debt to extract equity from their homes while (A6) describes the situation where owners do not change the size of their mortgage debt but refinance to take advantage of declining interest rates. Clearly a number of intermediate cases can be considered, but we focus only on these two extremes to delineate the boundaries that separate them.

Implicit in (A4)–(A6) is the assumption that the supply of credit to households is infinitely elastic at prevailing market rates, and it is motivated by our interest in measuring the impact of household refinancing behavior in and of itself. The complexities of consumer credit markets warrant a separate simulation study focusing on just those issues.

Assumption (A7) requires some clarification because the refinancing rules in (A5) and (A6) imply that refinancing decisions are not independent across households. Assumption (A7) simply states that there are no other sources of dependence (e.g., peer pressure and social norms arising from the refinancing activity of neighbors). The only channel through which refinancing decisions are correlated across households in our simulation is through interest rates and home prices via the behavioral rules in (A5) and (A6). Remarkably, this single source of commonality is sufficient to generate an enormous amount of synchronized losses when home prices decline. Finally, assumption (A8) is motivated primarily by the desire for simplicity, and can easily be amended to allow fully paid houses to re-enter the real-estate market.

Ignoring issues such as relocation or renting vs. owning is not likely to affect our estimates of aggregate risk and losses. For example, consider the case of an individual who decides to rent after selling for $200,000 a home that was recently purchased for $100,000 with a down payment of $15,000. Assuming a 0% interest rate for simplicity, this fortunate individual has taken $115,000 of equity out of the housing market. However, the new buyer of this home will likely borrow all but 10% to 20% of the purchase price. For the purpose of measuring aggregate risk, this transaction is virtually identical to a cash-out refinancing by the original homeowner.

Assumption (A9) is a standard simplification but is not equivalent to the usual “perfect markets” assumption where taxes and transactions costs are assumed to be zero. In
fact, assuming away market frictions may seem particularly incongruous in the context of a simulation of refinancing activity, which some consider to be driven largely by transactions costs. Assumption (A9) does not assert that these frictions do not exist, but merely that we do not model their impact on behavior explicitly. Instead, our behavioral rules for the homeowner’s refinancing decision implicitly incorporate these costs into our simulation in a “reduced-form” manner.

With these assumptions in place, we can now turn to the specific inputs of the simulation.

4.2 Simulation Input Data

Our simulations depend on a number of input parameters and time series, which this section describes. To conserve space, we have summarized the data used to calibrate our simulations in Table 1 and left the more detailed information to Appendix A.2. For three of our parameters, “Rates Refinance Threshold,” “LTV Refinance Threshold,” and “Prepayment Probability” (see Table 1 for definitions), we were unable to find appropriate data to calibrate their behavior over time. In lieu of formal calibration, we set these parameters to plausible values. We can report, however, that we have conducted a series of sensitivity analyses and our results are not sensitive to the levels of these parameters.

4.3 Calibration Reference Series

Our goal is to calibrate the parameters of our refinancing model so that the simulation results come as close as possible to matching the following two historical time series:

1. **Outstanding Mortgage Volume.** We use the value of residential mortgage liabilities as reported in the Federal Reserve *Flow of Funds Accounts*. This data is available at a quarterly frequency from 1951Q4 to 2008Q4, and annually from 1945 to 1951.

2. **Equity Extractions.** We use the series produced by Greenspan and Kennedy (2005), which is available at a quarterly frequency from 1968Q1 to 2008Q4. Although their approach decomposes the “Total Gross Equity Extractions” series into three components (home sales, home equity loans net of unscheduled payments, and cash-out refinancing), we use their aggregated series in our calibration process. This is motivated by the fact that home sales and cash-out refinancing have a similar impact on the aggregate risk of the housing market (see Appendix A.1 for further details).
<table>
<thead>
<tr>
<th>Data Item</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Price Appreciation</td>
<td>Prior to 1987, we assume that all homes grow at a rate specified by a single nationwide Home Price Index constructed using data from Robert Shiller prior to 1975Q1 and data from FHFA for 1975Q4 to 1986Q4. After 1987, we use the 10 individual series from the S&amp;P/Case-Shiller Composite-10 Home Price Index to introduce geographical heterogeneity in our simulations.</td>
</tr>
<tr>
<td>New Homes Entering the</td>
<td>The number of new homes entering the mortgage system is calculated using data from the U.S. Census Bureau after 1963. We make some adjustments to take into account homes built by owner or by contractors. For years prior to 1963, we use a statistical approach outlined in the Appendix to backfill the data. Post-1987, we use the weights as given by the S&amp;P/Case-Shiller Composite-10 Home Price Index to calculate the number of new homes in each of the 10 regions.</td>
</tr>
<tr>
<td>Mortgage System</td>
<td></td>
</tr>
<tr>
<td>New Home Purchase Price</td>
<td>We construct a time series of the average price of new homes using data from the U.S. Census Bureau since 1963. We backfill the data using our Home Price Appreciation index to January 1919. Finally, we use data from the 2007 American Housing Survey to create a distribution with 15 different price levels around the average series.</td>
</tr>
<tr>
<td>Initial Loan-to-Value Ratio</td>
<td>We assume that the initial loan-to-value ratios are uniformly distributed between 75% and 95%.</td>
</tr>
<tr>
<td>Initial Mortgage Maturity</td>
<td>Based on the data from the 2007 American Housing Survey, 80% of mortgages are assumed to be 30-year fixed-rate and the remaining 20% are 15-year fixed-rate, all with standard amortization.</td>
</tr>
<tr>
<td>Long-Term Risk-Free Rate</td>
<td>After February 1977, we use the yield on constant maturity 30-year treasury bonds. For earlier periods, we use the annual long-term interest rate data collected by Robert Shiller and interpolate it to arrive at monthly series.</td>
</tr>
<tr>
<td>Mortgage Rates</td>
<td>For 30-year mortgages, we use the data available from Freddie Mac for periods after April 1971. For earlier periods, we add 150 basis points (bps) to our Long-Term Risk-Free Rate and use the resulting time series. 150 bps is selected based on the average difference between these series in the post-1971 period. For 15-year mortgages, we use the data available from Freddie Mac since September 1991. For earlier periods, we subtract 46 bps from the 30-year mortgages to arrive at the 15-year series. 46 bps is the average difference between two series in the post-1991 period.</td>
</tr>
</tbody>
</table>

Continued on the next page
<table>
<thead>
<tr>
<th>Data Item</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Price Volatility</td>
<td>Determines the volatility of individual home values around the mean given by the Home Price Index. The observed volatility of national home price indexes masks much of the idiosyncratic volatility of individual home prices. Fortunately, an estimate for the volatility of individual homes is produced in the process of estimating the repeated-sales index. Based on the data available from FHFA, we use 8% annual volatility in our study. Since this is central to all of our derivatives-related calculations, we also report results assuming for 6% and 10% annualized volatility.</td>
</tr>
<tr>
<td>Rent Yield</td>
<td>Determines the service flow, akin to dividend payouts from common stock, from home ownership. This parameter affects derivatives-related calculations. Motivated by Himmelberg, Mayer, and Sinai (2005), we use 4% annual yield in the base case, but also report results assuming values of 3% or 5% rent yield.</td>
</tr>
<tr>
<td>Rates Refinance Threshold</td>
<td>Determines the minimum drop in mortgage rates needed to trigger a rates refinancing. We set this threshold to 2% in all simulations, but the simulation results are not very sensitive to this value.</td>
</tr>
<tr>
<td>LTV Refinance Threshold</td>
<td>Determines the maximum level of LTV, beyond which homeowners cannot engage in cash-out refinancing. Given our assumption that the initial LTV is distributed between 75% to 95%, we set this parameter to 75%.</td>
</tr>
<tr>
<td>Prepayment Probability</td>
<td>Determines the propensity of individuals to pre-pay their mortgage. We use a value of 1 bps per month, but the simulation results are not very sensitive to this value.</td>
</tr>
</tbody>
</table>

Table 1: (From previous page) Time series data used to calibrate a simulation of the U.S. housing and mortgage markets (see Appendix A.2 for details).

---

4.4 Parameterization of the Refinancing Probability

In our simulations, we follow the evolution of the value of each home as well as its outstanding mortgage from when it first enters the mortgage system until the mortgage is fully paid and, according to assumption (A8), it exits the mortgage system. The simulation takes in a number of input series and parameters as given in Table 1.

To ensure that our simulation is computationally feasible given the computing power available to us, we simulate 1000 paths for homes that enter the mortgage system in a given year. Therefore, for each complete run of our simulations, we simulate approximately 1.1 million individual simulation paths. For each simulation path, we keep track of home value, interest rate, mortgage outstanding, and several option-based risk measures. The information is then aggregated to arrive at system-wide time series of interest such as total mortgage debt outstanding, total equity extracted, and our various option-based risk metrics.

The path each home follows is driven by factors such as the evolution of mean home prices, mortgage rates, and the realization of idiosyncratic home price movement around the mean home price, as well as the owners’ refinancing decisions as described in Assumptions (A5) and (A6). The main driver of our simulation results is $\text{REFI}_{i,t}$ (see Assumption (A5)), which determines the probability that homeowner $i$ may participate in a cash-out refinancing in month $t$. We assume that $\text{REFI}_{i,t}$ has the following functional form:

$$\text{REFI}_{i,t} = (LTV_{i,t} < 75\%) \times \text{[Base Refinancing Rate + (MR}_t < \text{MR}_{i,0}) \times \text{Refinancing Intensity}(t)]_i.$$ (1)

Our motivation for selecting this particular characterization requires some discussion. As noted earlier, our object in this paper is to construct a simple yet realistic simulation that matches the size and growth of the U.S. residential mortgage system and to use that to study the potential risk caused by the refinancing ratchet effect alone. This objective reduces the burden on us to have a rule in place that is plausible for the behavior of owners. The particular characterization proposed here is simple, yet it has a number of intuitively plausible properties. First, its assumed form ensures that owners do not participate in a cash-out refinancing unless they have at least 25% equity in their home. Given the assumed distribution for the initial LTV, this constraint ensures that individuals participate in cash-out refinancing only if their home equity is at least 25%.
refinancing only after they have had time to build some equity above their initial equity level. For those owners who satisfy this LTV constraint, the refinancing intensity has two parts. One part, given by the “Base Refinancing Rate,” is constant through time and independent of the rate on the outstanding mortgage relative to the prevailing market rate. The second component, given by “Refinancing Intensity(t),” is active when the rate on the current mortgage, \( MR_{i,0} \), is above the prevailing rate of \( MR_t \). This second component can be a function of time. In fact, in some of our simulations we assume that refinancing intensity increases through time perhaps due to technological change, consumer familiarity, or other such factors. Ultimately, the ability of this model to capture the main drivers of refinancing trends will be judged by the success in reproducing the calibration time series. We turn to this issue in the next section.

4.5 Calibration Results

The calibration of our simulation consists of finding a base refinancing rate and a refinancing intensity function that can produce the Outstanding Mortgage Volume and Equity Extractions time series of Section 4.3. Since the time series used in these calibrations are non-stationary, traditional measures, such as correlation and \( R^2 \), may be misleading indicators of goodness-of-fit. A simpler alternative is to compute the mean of the quarterly absolute deviations between the simulated and actual series as a percentage of the actual quarterly values:

\[
\text{Mean Absolute Deviation} \equiv \frac{1}{T} \sum_{k=1}^{T} \frac{|\text{Simulated}_k - \text{Actual}_k|}{\text{Actual}_k} .
\]  

(2)

We begin with a base refinancing rate of 0.1% per month and assume that the refinancing intensity is constant over the entire sample period. By varying the level of the refinancing intensity function, we try to achieve a low level for the Mean Absolute Deviation (MAD) for both our calibration series. Table 2 contains the MAD results of this calibration exercise for three time periods: 1980–2008, 1990–2008, and 2000–2008. The results suggest that a refinancing intensity level of 4.00% achieves the lowest level of MAD across both calibration reference series during the 2000–2008 period, which is the most relevant period for our purposes.

Figure 4 depicts the entire time series produced by our simulations for the combination
<table>
<thead>
<tr>
<th>Uniform</th>
<th>MAD of Mortgages Outstanding (%)</th>
<th>MAD of Cumulative Equity Extractions (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80-08  90-08  00-08</td>
<td>80-08  90-08  00-08</td>
</tr>
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<td>2.00%</td>
<td>24.39  23.78  21.82</td>
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<tr>
<td>2.25%</td>
<td>22.78  21.50  18.84</td>
<td>31.00  28.99  22.64</td>
</tr>
<tr>
<td>2.50%</td>
<td>21.07  19.15  15.88</td>
<td>28.26  25.49  18.42</td>
</tr>
<tr>
<td>2.75%</td>
<td>19.78  17.26  13.39</td>
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<tr>
<td>3.00%</td>
<td>18.46  15.44  11.06</td>
<td>24.07  20.07  11.69</td>
</tr>
<tr>
<td>3.25%</td>
<td>17.54  14.09  9.33</td>
<td>22.71  18.23  9.47</td>
</tr>
<tr>
<td>3.50%</td>
<td>16.46  12.58  7.42</td>
<td>21.37  16.48  7.75</td>
</tr>
<tr>
<td>3.75%</td>
<td>15.72  11.55  6.20</td>
<td>20.52  15.48  7.13</td>
</tr>
<tr>
<td>4.00%</td>
<td>15.12  10.72  5.51</td>
<td>19.89  14.76  7.22</td>
</tr>
<tr>
<td>4.50%</td>
<td>14.32  9.74   5.32</td>
<td>19.13  14.15  8.76</td>
</tr>
</tbody>
</table>

Table 2: Summary of Mean Absolute Deviations (MAD), defined in (2), between the results produced by our simulation approach and the calibration reference series for Total Mortgages Outstanding and Cumulative Equity Extractions. Cash-out refinancing takes place according to the probabilistic rule given in (1) where the Base Refinancing Rate is 0.1% per month and the refinancing intensity is constant and given by the first element of each row. The time series of Total Mortgages Outstanding is compared with the Total Mortgage Liability series from Flow of Funds Accounts; the time series of Cumulative Equity Extractions is compared with the series produced by Greenspan and Kennedy (2005) (see Section 4.3). The MAD is reported for three different time periods to prove additional details about the success of the calibration procedure.
of input parameters selected. As a basis for comparison, we have shown the Total Mortgages Outstanding series for the case of no-cash-out refinancing in Figure 4(a). While our simulations capture the massive growth in the amount of mortgages outstanding and cumulative equity extractions after 2000 very well, they fall behind the calibration series between the mid-1980s and the late 1990s. However, since our main focus in this paper is to evaluate risk in the mortgage system in the years leading up to the Financial Crisis of 2007–2009, the lack of fit in earlier periods is not as problematic.

The refinancing intensity levels of 4.00% per month may seem excessively high, but note this level is only relevant for homes that meet both the LTV and the mortgage rate conditions in (1). To develop further intuition for this aspect of our simulation, we computed the fraction of homes in our simulation that meet both conditions in (1). Figure 5(a) shows the time series of the percentage of homes that meet the LTV condition of (1) and Figure 5(b) provides the corresponding time series for the percentage of homes that meet the Mortgage Rate condition. Figure 5(c) contains the time series for the percentage of homes that meet both conditions; such homes are candidates for potential cash-out refinancing. It can be seen that there are only a few periods of time, for example the early and late 1990s and the period between 2001 and 2005, during which a large fraction of homes meet both constraints and for which the assumed 4.00% refinancing intensity represents the actual likelihood of cash-out refinancing.

Based on the of goodness-of-fit metrics reported in Table 2 and the full time series shown in Figure 4 we feel comfortable that our simulation under refinancing rule (1), where the refinancing intensity is constant through time at the level of 4.00%, is properly calibrated to assess the impact of refinancing on the systemic risk of the U.S. residential mortgage market. We adopt this specification in our analysis of such risks in Section 5. While this rule implies a uniform probability of cash-out refinancing, we have studied two alternative rules in which the refinancing intensity curve is either linearly increasing in time or where the refinancing intensity undergoes a discrete break in 1988 (as motivated by Bennett, Peach, and Peristiani, 2001). The results based on these refinancing rules are provided in Appendix A.3.

5 Options-Based Risk Analysis

Armed with a properly calibrated simulation of the U.S. residential mortgage market, we now turn to assessing the systemic risk posed by the refinancing ratchet effect. Given our
Figure 4: A comparison of Total Mortgages Outstanding and Cumulative Equity Extractions produced by our simulation and Total Mortgage Liability series from *Flow of Funds Accounts* data and the Cumulative Equity Extractions series of Greenspan and Kennedy (2005). Cash-out refinancing takes place according to the probabilistic rule given in (1), where the Base Refinancing Rate is 0.1% per month and the refinancing intensity is 4.00%.
Figure 5: Simulated time series of the percentage of homes meeting the (a) LTV condition in (1); (b) the MR condition in (1); (c) both conditions. The results are based on the optimized parameters (see Table 2) with the base refinancing rate set to 0.1% per month and the refinancing intensity equal to 4.00%/month.
assumption that all mortgages in our simulations are non-recourse loans—collateralized only by the value of the underlying real estate—the homeowner has a guarantee or put option that allows him to put or “sell” the home to the lender at the remaining value of the loan if the value of the home declines below the outstanding mortgage. Such guarantees can be evaluated using derivatives pricing theory as described in Merton (1977) and Merton and Bodie (1992), and can be applied to quantify macro-level risks as described in Gray, Merton, and Bodie (2006, 2007a, 2007b, 2008) and Gray and Malone (2008).

As mortgages are placed in various structured products like collateralized mortgage obligations and then sold and re-sold to banks, asset management firms, or GSEs (see Figure 6), the ultimate entities exposed to these guarantees may be masked. However, it is clear that all mortgage lenders must, in the aggregate, be holding the guarantees provided to all homeowners. To the extent that some owners may be liable for the deficiency in their collateral value through recourse, those owners share some of the burden of the loss caused by a decline in home prices. See footnote 1 and Ghent and Kudlyak (2009) for further discussion. Therefore, we can circumvent the complexities of these intermediate transactions—those in the dotted box of Figure 6—and use the aggregate value of the guarantees and their various risk metrics to evaluate the overall risk in the mortgage system.

Figure 6: Interlinked balance sheet of entities backed by the underlying real estate, based on Gray, Merton, and Bodie (2008). Intermediate risk redistributions (through, for example, CDOs) will be ignored in our simulations.
5.1 The Aggregate Value of Mortgage Put Options

We measure the value of the guarantee embedded in each non-recourse mortgage as the value of a put option written on the underlying real estate. Since Merton’s (1977) analysis of deposit insurance, the use of derivatives pricing models to value guarantees has become standard. Such an approach is forward-looking by construction, providing a consistent framework for estimating potential losses based on current market conditions—in particular, the price and volatility of the guaranteed asset—rather than historical experience. Of course, derivatives pricing models do require additional assumptions, e.g., complete markets and a specific stochastic process (one that is consistent with completeness such as geometric Brownian motion). We adopt a discrete-time version of these assumptions in (A10):

\[(A10)\text{ Housing-price dynamics can be approximated by a discrete-time geometric random walk represented by a recombining binomial tree, and markets are dynamically complete so options on property values can be priced by no-arbitrage arguments alone.}\]

Whether home prices follow random walks is debatable,\(^{11}\) and a number of studies have documented departures from geometric Brownian motion in several financial assets (see Lo and MacKinlay, 1999, and the many references they provide to this burgeoning literature). However, for constructing an initial benchmark for valuing the embedded option in non-recourse mortgages, Assumption (A10) is a natural starting point from which more sophisticated models can be built.\(^{12}\)

\(^{11}\) Certainly aggregate home price series are not consistent with geometric Brownian motions—they are far too “smooth.” However, much of this smoothness is due to the artificial averaging implicit in all real-estate index-construction methods. For our purposes, what matters more is the price process for individual homes since there is an embedded put option within each mortgage. Specifically, Calhoun (1996) describes the FHFA index construction method as extracting the component \(\{\beta_t\}\) from the following time series model:

\[
\log P_{it} = \beta_t + H_{it} + N_{it}.
\]

While it is true that \(\beta_t\) is not well-approximated by a Brownian motion because it is the systematic component of individual home prices, for our purposes, the volatility of the idiosyncratic term \(H_{it}\) is the key input into determining the value of the embedded put. Therefore, the Black-Scholes model may be a reasonable approximation for this purpose, and we use the estimated volatility of \(H_{it}\) reported by FHFA in our simulations. To the extent that \(\beta_t\) induces a smoother time-varying expected return, this can be addressed by the mean-reverting diffusion processes in Lo and Wang (1995).

\(^{12}\) Mean reversion can easily be accommodated as in Lo and Wang (1995), and the implications for option-pricing analysis are particularly straightforward (only the drift is affected by mean reversion, implying that the option-pricing formula is unchanged but the estimated volatility must be adjusted for serial correlation). Another extension is to consider price dynamics that reflect the U.S. real-estate “bubble.” However, developing a precise definition of a bubble is not a simple task. For example, while some studies concluded that real estate prices were too high in 2004–2006 (Shiller, 2006), other studies came to the opposite conclusion.
Under (A10), we model the guarantee in non-recourse mortgages as a “Bermuda” put option—an option that can be exercised at certain dates in the future, but only on those fixed dates—and we set these exercise dates to be once a month, just prior to each mortgage payment date. The exercise price is the amount of the outstanding loan, which declines over time due to the monthly mortgage payments. Before we can implement this option-pricing model, we must determine the volatility of the underlying asset on which the option is written, as well as any “dividend yield” that may affect the value of that asset. We set these two parameters to the values reported in Table 1.

Figure 7 shows the simulated time series for the aggregate value of put options for the cases of no-cash-out vs. cash-out refinancing using our calibrated uniform refinancing rule, and Table 3 contains the numerical values for each quarter between 2005Q1 and 2008Q4. During normal times, homeowners’ equity absorbs the first losses from a decline in residential real-estate prices (see Figure 6). However, the process of equity extraction causes the size of this buffer to decrease, resulting in a larger portion of the losses transferred to the equity-holders and debt-holders of various mortgage-lending entities (through the complex risk redistribution methods shown in the dotted section of Figure 6). Our simulations show that with the downturn in residential real estate in 2007 and 2008, the value of the guarantees extended to homeowners by mortgage lenders increased substantially.

(McCarthy and Peach, 2004, and Himmelberg, Mayer and Sinai, 2005). Even ex post, estimating the appropriate “price correction” is not obvious, as Wheaton and Nechayev (2007) illustrate. This lack of consensus underscores the empirical challenges in identifying stable relations between prices and the most obvious fundamentals (in particular, see Gallin, 2004, 2006). But to the extent that a “bubble” refers, instead, to an impending tail event, this case can easily be accommodated by assuming a jump component in the stochastic process of the Home Price Index, and then using Merton’s (1976) jump-diffusion option-pricing model to price the guarantee. Although this is a simple extension, it is likely to lead to larger loss estimates than Assumption (A10) because of the additional tail risk component; hence we adopt the simpler assumption in the spirit of conservatism.

We use the Cox-Ross-Rubinstein (see Cox and Rubinstein (1985)) binomial tree algorithm to price these options, and implement it in Matlab (version 7.2) using the Financial Derivatives Toolbox (Version 4.0) and the functions: crrtimespec, crrsens, crrtree, instoptstock, intenvset, and stockspec. See http://www.mathworks.com for documentation and additional details.

Note that negative correlation between volatility and prices has been documented in several asset classes (see, for example, Bekaert and Wu, 2000, for the equities case). To the extent that this negative correlation holds in real-estate markets as well, our volatility parameter—which is an approximate long-term average—is likely to under-estimate the realized volatility during a market downturn, which, in turn, will under-estimate aggregate losses.
Figure 7: Simulated time series of the aggregate value of total guarantees extended to homeowners by mortgage lenders for cash-out and no-cash-out refinancing scenarios. For the cash-out refinancing case, refinancing takes place according to probabilistic rule (1) in which the base refinancing rate is 0.1% per month and the Refinancing Intensity is constant over time and equal to 4.00%.
5.2 U.S. Mortgage System Delta, Gamma, and Vega

The overall level of risk in the mortgage system can be calculated based on the sensitivities of the value of mortgages’ embedded put options to changes in the level or volatility of home prices. While the exact value of these risk metrics depends on the particular model used for option pricing, the nature of risk in the mortgage system transcends the specifics of the models. For example, regardless of the exact model, the value of the guarantees increases as the value of the collateral declines. Using the language of option pricing, the delta of the mortgage guarantees is positive. Furthermore, the rate of the increase is itself increasing, or in the option language, the gamma is positive. To emphasize the full effect of non-linearity, we have used our simulations to produce the value of the put guarantees for the stock of homes that existed as of June 2006. We then subjected all home prices to a certain percentage drop or increase in home values and recalculated the total put values. This analysis, as shown in Figure 8, emphasizes that the non-linearity is an important aspect of this risk. An estimate of losses based on the linear approximation would have substantially under-estimated the true losses under price declines in the range similar to those that we observed in the last few years. Furthermore, this figure shows that a sudden shock to the volatility level, as is usually the case during periods of prices decline, causes the entire curve to shift higher.

Figure 8: Value of aggregated put options based on the calibrated simulations for June 2006 subject to different levels of drop or increase in home values.

Figure 9 and Table 3 report these metrics for the simulation based on the uniform re-
financing rule (we provide these metrics for alternative refinancing rules in Appendix A.3). As reported in Table 3, in the first quarter of 2005 we estimate that the aggregate value of all embedded put options would increase by $18.17 billion for each 1% drop in home prices. By the last quarter of 2008, this sensitivity almost doubled to $38.13 billion for each 1% drop in home values. This large increase is due to the large gamma of these embedded options, as reported in Table 3. For the same simulation, the estimated gamma was $573.79 million per 1% drop in home values in the first quarter of 2005, which increased to $801.13 million for each 1% drop in home values by the last quarter of 2008. The size and increase in the gammas of these options indicate substantial non-linearity in the risk of the mortgage system that may need to be accounted for in systemic risk measurement and analysis. Table 3 contains delta and gamma estimates for simulations without cash-out refinancing as well.

Another aspect of risk in the mortgage system can be measured by estimating the sensitivity of the value of embedded options to an increase in home price volatility, also known as the option’s “vega.” Based on our calculations, we estimate that the total value of the embedded put options in non-recourse mortgages would increase by approximately $70 to $80 billion for each 1% increase in home price volatility in the years leading up to the crisis. While Figure 9 shows a large increase in vega over time, during the recent crisis this measure of risk first increased and then declined. Portfolios of options can exhibit such counter-intuitive behavior because of the non-linearity of these metrics. For example, in the Black-Scholes model, the vega of options way in- or out-of-the-money is low, but is quite high for options near the money. These non-linearities can give rise to the effects documented here for the U.S. mortgage system.

Figure 9 shows that all three risk metrics are increasing over time, even in the case of no-cash-out refinancing. This trend is simply due to the fact that the total number of homes (along with population) is increasing over time. To normalize this effect, and to develop a better sense for the relative increase as well as the timing of the increase in these metrics, we have plotted the ratios of the cash-out vs. no-cash-out delta, gamma, and vega measures for the heterogeneous inputs case in Figure 10. Comparing the pattern observed with the drivers of the refinancing intensity shown in Figure 5, we can see the ratios of these risk measures increase when both drivers of refinancing activity are present. For example, the ratios remained relatively stable from the 1960s to the early 1980s. During this period, although most homeowners met the LTV condition of our refinancing driver (see Figure
5(a)), the high level of interest rates implied that only a small portion of the population was in a favorable state to refinance (Figure 5(b)). The first shift in the three ratios occurred in the mid-1980s as interest rates fell while home prices remained high, causing large increases in overall refinancing activity. Refinancing activity slowed down in the early 1990s due to the drop in home prices during this period (Figure 5(a)). The next major shift occurred in the early 2000s due to the combination of interest-rate declines and home price appreciation. In fact, the three ratios began declining in 2006 as home prices declined, indicating that the risk metrics increased at a higher rate for simulations with no-cash-out refinancing relative to those with cash-out refinancing.

In summary, we see that the refinancing ratchet effect causes the greatest increases in the three relative risk measures during periods when home prices are stable or increasing, and interest rates are declining.
Figure 9: Simulated time series of the sensitivities of the aggregate value of total guarantees extended to homeowners by mortgage lenders for cash-out and no-cash-out refinancing scenarios. Figure (a) plots the sensitivity to a 1% drop in home prices, Figure (b) plots the rate of change of (a) with respect to home prices, and Figure (c) plots the sensitivity to a 1% increase in home price volatility. For the cash-out refinancing case, refinancing takes place according to probabilistic rule (1) in which the base refinancing rate is 0.1% per month and the refinancing intensity is constant over time and equal to 4.00%.
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<th>Cash-Out</th>
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<td>Put Value</td>
<td>Delta</td>
<td>Gamma</td>
<td>Vega</td>
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<td>Delta</td>
<td>Gamma</td>
<td>Vega</td>
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<td>19.72</td>
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<td>38.13</td>
<td>801.13</td>
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Table 3: Simulated time series of the aggregate value and sensitivities of total guarantees extended to homeowners by mortgage lenders for cash-out and no-cash-out refinancing scenarios for each quarter from 2005Q1 to 2008Q4 (put values are in $billions, deltas are in $billions per 1% decline in home prices, gammas are in $millions per 1% decline in home prices, and vegas are in $billions per 1% increase in home price volatility). For the cash-out refinancing case, refinancing takes place according to probabilistic rule (1) in which the base refinancing rate is 0.1% per month and the refinancing intensity is constant over time and equal to 4.00%. 
5.3 Sensitivity to Model Parameters and the Refinancing Rule

The accuracy of the simulation results of Sections 5.1 and 5.2 depends, of course, on the values of the various parameters of our simulations, as well as the assumed form of the behavioral refinancing intensity function. We have conducted a series of sensitivity analyses that we will summarize in this section.

We first consider the estimates for the aggregate value of the put options in non-recourse mortgage loans under the two alternative refinancing rules. As reported in Table 4, the results are remarkably stable across different refinancing rules. This stability is likely due to the fact that each of these rules is calibrated to match the two reference series in Section 4.3 (See Appendix A.3).

To gauge the sensitivity of our simulations to the rent yield and home price volatility assumptions, we performed additional simulations for rent yields of 3% and 5%, and home price volatilities of 6% and 10%. To conserve space, we have reported only the resulting
estimates for the total put value under these parameter values in Table 5. These results show that in the fourth quarter of 2008, the simulated loss estimates range from a low of $1,241 billion (3% rent yield, 6% volatility) to a high of $2,256 billion (5% rent yield, 10% volatility). As volatility increases, or as the rent yield increases, ceteris paribus, the embedded guarantee becomes more valuable. While rent yields may be relatively stable over time, it can be argued that home price volatility is more variable. In particular, as the national home price index reached its peak in June 2006 and began to decline, home price volatility is likely to have increased significantly beyond historical levels, which implies that our estimates for the aggregate put value may under-estimate actual losses. Tables A.6–A.8 in Appendix A.4 provide similar sensitivity analyses for the estimated delta, gamma, and vega measures for the aggregate put.
<table>
<thead>
<tr>
<th></th>
<th>Uniform (Base Case)</th>
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<th>Uniform with Break</th>
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<td>Put Value</td>
<td>Delta</td>
<td>Gamma</td>
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<td>Dec-08</td>
<td>1727.20</td>
<td>38.14</td>
<td>801.13</td>
</tr>
</tbody>
</table>

Table 4: Simulated time series of the aggregate value and sensitivities of total guarantees extended to homeowners by mortgage lenders for cash-out and no-cash-out refinancing scenarios for each quarter from 2005Q1 to 2008Q4 (put values are in billions, deltas are in billions per 1% decline in home prices, gammas are in millions per 1% decline in home prices, and vegas are in billions per 1% increase in home price volatility) for three refinancing rules: Uniform (4.00%), Linear (4.50%), Uniform with Break (3.75% before 1988; 4.25% from 1988 onward). See Tables 2, A.4 and A.5 for calibration details for these rules.
<table>
<thead>
<tr>
<th></th>
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<td>2014.40</td>
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<td>1727.20</td>
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Table 5: Simulated total value of guarantees extended to homeowners by mortgage lenders (in $billions) for various values of home price volatility (Vol) and rent yield (RY) under the Uniform (4%) refinancing rule from 2005Q1 to 2008Q4.
6 Discussion

In this section, we present a number of qualifications, extensions, and implications of our simulation of the U.S. residential housing market. In Section 6.1, we contrast the heuristic nature of our simulations to traditional general equilibrium analysis. We acknowledge in Section 6.2 that we have not modeled the behavior of lenders in our simulations. We distinguish between market risk and systemic risk in Section 6.3. In Section 6.4 we observe that the welfare implications of the recent financial crisis, and the events leading up to it, are not yet fully understood.

6.1 Heuristic vs. General Equilibrium Analysis

Our simulations are based on a number of simplifying assumptions. While we have attempted to err on the side of lower implied losses whenever possible, some assumptions may have the opposite effect, e.g., assuming that all mortgages are non-recourse loans. Incorporating more realistic features of the housing market, such as adjustable-rate and negative-amortization mortgages with teaser rates, NINJA loans, and regional differences in the U.S. residential real-estate market, bankruptcy laws, and homeowner asset and income dynamics, may increase the accuracy of the simulation.

However, our analysis is not designed to capture feedback effects among all endogenous variables such as home prices, interest rates, household income, and borrowing and lending behavior. Therefore, standard comparative-statics questions, such as “How much would home prices have risen if the Fed had not cut interest rates from 2000 to 2003?”, are not addressed in our simulations. Instead, our narrower reduced-form focus has been to gauge the magnitude of the refinancing ratchet effect on mortgage lenders. A more formal general-equilibrium analysis of these markets would begin with optimizing households from which the demand for housing and mortgages are derived, aggregated, and equilibrated with optimizing home builders and mortgage lenders that supply the homes and mortgages, respectively, to households. While computable general equilibrium models have become considerably more sophisticated in recent years (see, for example, Dixon and Rimmer, 2002), the dynamic and stochastic nature of the demand and supply decisions are sufficiently complex, even for a single agent, that constructing a true stochastic dynamic general equilibrium model of the entire U.S. housing market seems computationally impractical. Nevertheless, some
useful insights may be gleaned from considering special cases of such optimizing behavior and equilibrium, e.g., Pliska (2006), Fortin et al. (2007), and Agarwal, Driscoll, and Laibson (2008), which may be worth pursuing further.

6.2 Lending Behavior

Any analysis of the Financial Crisis of 2007–2009 would not be complete without some discussion of the behavior of mortgage lenders and associated businesses. Our simulations assume that all household demand for mortgages and refinancing is satisfied at prevailing historical rates, i.e., the supply of funds to borrowers is infinitely elastic at all times. While this may have been a reasonable approximation to reality during the decade prior to the peak of the housing market in 2006, our motivation for this simplifying assumption is to gauge the impact of the refinancing ratchet effect in isolation. However, supply shocks certainly must have had an impact on systemic risk in recent years as well. Therefore, an important open question is how lenders behaved during the course of our simulations, and what economic or behavioral forces led them to engage in such behavior.

A tractable and empirically plausible model of lending behavior is beyond the scope of our current simulation, and it deserves a separate set of simulation studies in its own right (one possible starting point is Thurner, Farmer, and Geanakoplos, 2009). However, it is not difficult to speculate about the factors those simulations might include. In addition to modeling the behavior of banks, which are the traditional sources of home loans, such a simulation must also account for a host of financial innovations that have emerged only recently, including securitized debt (e.g., CDOs and CDO-squareds), credit default swaps and related insurance products, Internet-based marketing of consumer-finance products, the growth of the “shadow banking industry” and illiquidity, and the globalization of financial markets. Chan et al. (2006), Rajan (2006), Gorton (2008, 2009), Brunnermeier (2009), and Gorton and Metrick (2009) provide overviews of some of these developments. In addition, these simulations must incorporate the impact of rating agencies, GSEs, and broader government policies in promoting cheap financing for would-be homeowners, as well as the increasing competition for yield among asset-managers and asset-owners. Collectively, these developments contributed to the enormous supply of funds available to homeowners during the past decade, but further analysis is needed before we can determine the relative importance of each.
The challenge in constructing a simulation with all of these features is the fact that there is precious little history on which to calibrate many of the parameters. In contrast to typical simulations that assume a statistically stationary environment, simulating the supply of funds for residential real-estate purchases involves the historically unique financial innovations described above. This simulation may provide a clue as to the magnitude of the current crisis, as well as its apparent uniqueness in recent history. The mechanism discussed in this paper is surely relevant when comparing impact of housing-price crashes across countries. As discussed in Hubbard and Mayer (2010), many countries experienced similar housing-price growth driven by comparable trends in real interest rates. In a country where cash-out refinancing is easier or more common—perhaps because of familiarity or other social characteristics—a larger portion of the increase in homeowner’s equity is extracted by the owner. This would, in turn, cause more synchronization in homeowners’ leverage and more severe losses for lenders in the wake of home-price declines.

More importantly, the main thrust of our analysis is that the refinancing ratchet effect is a wholly separate mechanism that operates irrespective of the supply of credit, and one that must be considered a potential source of systemic risk in its own right.

6.3 Market Risk vs. Systemic Risk

While the $1.7 trillion figure seems imposing, large financial losses do not necessarily imply significant systemic risk. For example, on April 14, 2000, the CRSP value-weighted stock market index (excluding dividends) declined by 6.63%, implying an aggregate one-day loss of approximately $1.04 trillion to corporate America. While certainly unfortunate, this event was not particularly memorable, nor was it a cause for national alarm or emergency government intervention. Market risk is distinct from systemic risk; the latter arises when large financial losses affect important economic entities that are unprepared for and unable to withstand such losses, causing a cascade of failures and widespread loss of confidence. This element of surprise lies at the heart of the recent financial crisis. The fact that the three conditions that cause the refinancing ratchet effect—rising house prices, falling interest rates, and easy access to refinancing opportunities—are individually innocuous and are often viewed as signs of economic growth and prosperity creates the element of surprise. Therefore, not only is the magnitude of losses caused by the refinancing ratchet effect large, but these losses are also more likely to come as a surprise to the parties involved, resulting in systemic
risk to the financial system.

6.4 Welfare Implications

Although much has already been written about the Financial Crisis of 2007–2009, its welfare implications for homeowners, lenders, and intermediaries are not yet fully understood. While many homeowners have been adversely affected by higher interest rates, foreclosures, and falling property values, there are other satisfied and solvent homeowners who are homeowners only because of the business practices, government policies, and economic circumstances that contributed to the refinancing ratchet effect. Eliminating or otherwise restricting these business practices and policies may benefit some groups, but it will no doubt disadvantage others. Moreover, as discussed above, we have not attempted to model the supply side of the refinancing industry, which no doubt contributed to the growth of home prices, leverage ratios, and systemic risk. Many have criticized the role of securitization, insurance, and financial innovation in creating the crisis, but during the decade leading up to the peak of the housing market in 2006, these developments were responsible for the low-interest-rate and easy-credit environment that was so conducive to global economic growth and the “ownership society.” Any policy recommendations with respect to the Financial Crisis of 2007–2009 must balance these myriad trade-offs between individual and institutional stakeholders.

7 Conclusions

During periods of rising house prices, falling interest rates, and increasingly competitive and efficient refinancing markets, cash-out refinancing is like a ratchet, incrementally increasing homeowner leverage as real-estate values appreciate without the ability to symmetrically decrease leverage by increments as real-estate values decline. Using a numerical simulation calibrated to the basic time-series properties of U.S. residential housing market, we show that this ratchet effect is capable of generating the magnitude of losses suffered by mortgage lenders during the Financial Crisis of 2007–2009. During normal times, and in the absence of cash-out refinancing, the cross-sectional distribution of leverage among homeowners is relatively heterogeneous, with newer homeowners more highly leveraged than those who have had the opportunity to build more equity. Heterogeneity of leverage in the cross section implies fewer correlated defaults among borrowers and lower systemic risk.
However, during periods of falling interest rates and rising house prices, most homeowners will have an incentive to refinance. If the refinancing market is so competitive and efficient that homeowners refinance frequently, this pattern of behavior has an effect on systemic risk similar to the one that would occur if these homeowners all purchased their homes at the same time, at peak prices, with newly issued mortgages at the highest allowable LTV ratios. A coordinated increase in leverage among homeowners during good times will lead to sharply higher correlations in defaults among those same homeowners in bad times. Our simulations show that this effect alone is enough to generate $1.7 trillion in losses for mortgage-lending institutions since June 2006.

These observations have important implications for risk management practices and regulatory reform. The fact that the refinancing ratchet effect arises only when three market conditions are simultaneously satisfied demonstrates that the recent financial crisis is subtle and may not be attributable to a single cause. Moreover, a number of the activities that gave rise to these three conditions are likely to be ones that we would not want to sharply curtail or outright ban because they are individually beneficial. While excessive risk-taking, overly aggressive lending practices, pro-cyclical regulations, and political pressures surely contributed to the recent problems in the U.S. housing market, our simulations show that even if all homeowners, lenders, investors, insurers, rating agencies, regulators, and policymakers behaved rationally, ethically, and with the purest of motives, financial crises could still occur. Therefore, we must acknowledge the possibility that no easy legislative or regulatory solutions may exist. As Reinhart and Rogoff (2008a,b) have documented, financial crises occur on a regular basis throughout the world and are often tied to economic growth, capital inflows, and financial liberalization and innovation. Successfully managing systemic risk will require flexible, creative, and well-trained professionals who understand the fundamental drivers of such risk, not static rules meant to prevent history from repeating.
A Appendix

In this Appendix, we describe the components of Greenspan and Kennedy’s (2005) gross equity extraction time series used in our analysis (Section A.1), our methods for constructing all of the variables used as inputs in calibrating our simulations (Section A.2), calibration results for two alternative refinancing intensity specifications (Section A.3), and some additional sensitivity analysis for our option-pricing analysis of the embedded put options in non-recourse mortgages (Section A.4).

A.1 Components of Gross Equity Extractions Series

Greenspan and Kennedy (2005) (hereafter “GK”) propose a method for disaggregating the net change in outstanding home mortgage debt into its constituent gross flows. For our purposes, the most important series produced by their approach is the Gross Equity Extractions series, and for completeness, we provide a brief overview of this series in this section. Please see their paper for further details.

GK define Gross Equity Extractions as “extraction of equity on existing homes as the discretionary initiatives of home owners to convert equity in their homes into cash by borrowing in the home mortgage market.” To calculate gross equity extractions, they hypothesize that the change in home mortgage debt outstanding in the absence of discretionary initiatives would have been equal to the mortgage origination to purchase new homes minus the scheduled amortization. Accordingly, they define gross equity extractions as the difference between the actual change in total home mortgage debt outstanding and this quantity. More precisely, they use the following relationship:

\[
\text{Gross Equity Extractions} \equiv \text{Change in home mortgage debt outstanding excluding construction loans} \\
- \text{Origination for new homes} \\
+ \text{Scheduled amortization}.
\]

Using a variety of sources, they are able to estimate this quantity at a quarterly frequency since 1968Q1. This series is one of the two primary reference time series that we use to calibrate our simulations.

With a more detailed set of data sources that are only available since 1991, GK further decompose gross equity extractions into the following three components from 1991Q1 to 2008Q4:

\[
\text{Turnover Extractions} \equiv \text{Origination to purchase existing homes} \\
- \text{Cancellation of home-seller’s mortgage} \\
\text{Gross Cash-Out} \equiv \text{Origination for refinancing} \\
- \text{Cancellation of refinanced loans} \\
\text{Net Change in Home Equity Loans} \equiv \text{Change in home equity loans outstanding}
\]
Figure A.1 plots these three components at a quarterly frequency from 1991Q1 to 2008Q4.

Figure A.1: Three components of the Gross Equity Extractions series of Greenspan and Kennedy (2005), from 1991Q1 to 2008Q4. The three components are: “Turnover Extractions” (originations to purchase existing homes minus cancellation of home-seller’s mortgage), “Gross Cash-Out” (defined as originations for refinancing minus cancellation of refinanced loans), and “Net Change in Home Equity Loans” (defined as the change in home equity loans outstanding minus unscheduled repayments). See Greenspan and Kennedy (2005) for further details.

Our choice to calibrate our simulations to the Gross Equity Extractions series rather than to one of the three subcomponents is motivated by our focus on systemic risk. In particular, the probabilistic refinancing rule (1) is meant to capture the aggregate effects of all three components of the Gross Equity Extractions series. As we discussed in Section 4.1, this broader focus is more relevant for aggregate risk measurement because all three components contribute to the total leverage of the residential housing market. In particular, in the example of Section 4.1 in which a homeowner decides to sell his home and rent thereafter, he would fall into the “Turnover Extraction” case, but the buyer of his home presumably finances the purchase with a similarly leveraged loan, yielding virtually the same impact on aggregate leverage as if the original homeowner continued owning after engaging in a cash-out refinancing (place him in the “Gross Cash-Out” category). Therefore, for our purposes, combining the three GK disaggregated series seems more appropriate.

A.2 Construction of Input Series

In this section, we describe the steps we followed to construct the various time series used as inputs to our simulation:
1. **Home Price Appreciation.** We use three sources to assemble this series. Prior to 1975Q1, we use the nominal home price index collected by Robert Shiller. From 1975Q1 to 1986Q4, we use the national house price index from the FHFA.\(^{15}\) Given the importance of this variable for our simulation, we considered two other home price series using different data in the more recent period, but because the results did not differ significantly from those based on HPI\(_t\), we have omitted them to conserve space.\(^{16}\) For the most recent history (since January 1987), we use the S&P/Case-Shiller Composite-10 Home Price Index. For the simulation in which we introduce geographical heterogeneity in home price appreciation, we use the 10 individual components of the S&P/Case-Shiller Index. We use the same weighting scheme followed by the Composite-10 index, given in Table 1.\(^{17}\)

2. **New Homes Entering the Mortgage System.** We construct this time series from a variety of sources. The time series of “New One-Family Houses Sold” available from the U.S. Census Bureau is the starting point.\(^{18}\) This series is available monthly since January 1963. However, it includes only homes built for sale and excludes homes built by homeowners and contractors. To take such cases into account, we use data collected by the U.S. Census Bureau on the intent of completed home constructions.\(^{19}\) This construction data separates the completed units by their intent—units in the “Built for Sale” category correspond to homes that will be reported in the “New One-Family Houses Sold” upon the completion of a sale transaction. We take the sum of construction numbers reported under the “Contractor-Built,” “Owner-Built,” and “Multi-Units Built for Sale” categories, and use the ratio of this sum to the number of “One-Family Units Built for Sale” to adjust the “New One-Family Houses Sold”

\(^{15}\)See [http://www.fhfa.gov/Default.aspx?Page=87](http://www.fhfa.gov/Default.aspx?Page=87). The relevant data may be found in the “All-Transactions Indexes” section. These two series are only available at a quarterly and annual frequency, respectively, and to be consistent with the rest of our simulation, we convert them into monthly series assuming geometric growth. Specifically, for months other than March, June, September, and December, HPI\(_t\) is computed as:

\[
HPI_t = \exp\left[\log(HPI_{Q^-}) + (t - t_{Q^-}) \log\left(\frac{HPI_{Q^+}}{HPI_{Q^-}}\right)\right]
\]

where HPI\(_{Q^-}\) denotes the quarterly index value from the previous quarter, and HPI\(_{Q^+}\) denotes the quarterly index value from the current quarter. The approach for interpolating monthly observations from annual data is similar.

\(^{16}\)Specifically, we define CSNAT-HPI\(_t\) and NAR-HPI\(_t\) using the same data as HPI\(_t\) for the earlier part of the simulation period, but CSNAT-HPI\(_t\) uses the S&P/Case-Shiller Composite-10 Home Price Index price since 1987Q1, and NAR-HPI\(_t\) uses the appreciation in the median price of existing homes sold as reported by the National Association of Realtors, which is available since January 1999. Because the Case-Shiller Index is only available at a quarterly frequency, we construct monthly observations for this variable via interpolation assuming geometric growth. Simulation results based on these home price series are available from the authors upon request.

\(^{17}\)See the [S&P/Case-Shiller Home Price Indices Index Methodology](http://www.standardandpoors.com/) document on Standard & Poor’s website.


\(^{19}\)See [http://www.census.gov/const/www/newresconstindex_excel.html](http://www.census.gov/const/www/newresconstindex_excel.html). The relevant data may be found in “Quarterly Housing Completions by Purpose of Construction and Design Type”. We use the annual series since it is available since 1974 (quarterly data only goes back to 1999).
<table>
<thead>
<tr>
<th>Geographical Region</th>
<th>Case-Shiller Symbol</th>
<th>Weight (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston</td>
<td>BOXR</td>
<td>7.4</td>
</tr>
<tr>
<td>Chicago</td>
<td>CHXR</td>
<td>8.9</td>
</tr>
<tr>
<td>Denver</td>
<td>DNXR</td>
<td>3.7</td>
</tr>
<tr>
<td>Las Vegas</td>
<td>LVXR</td>
<td>1.5</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>LXXR</td>
<td>21.2</td>
</tr>
<tr>
<td>Miami</td>
<td>MIXR</td>
<td>5.0</td>
</tr>
<tr>
<td>New York</td>
<td>NYXR</td>
<td>27.2</td>
</tr>
<tr>
<td>San Diego</td>
<td>SDXR</td>
<td>5.5</td>
</tr>
<tr>
<td>San Francisco</td>
<td>SFXR</td>
<td>11.8</td>
</tr>
<tr>
<td>Washington, D.C.</td>
<td>WDXR</td>
<td>7.8</td>
</tr>
</tbody>
</table>

Table A.1: Weights of the regional indexes within the S&P/Case-Shiller Composite-10 Home Price Index. Source: S&P/Case-Shiller Home Price Indices Index Methodology.

series. For example, in 1974 this ratio is 1.06, therefore we multiply the monthly “New One-Family Houses Sold” by a factor of $1 + 1.06 = 2.06$ in each month during 1974 to estimate the total number of units entering the mortgage system that year. For the period from 1963 to 1973, this ratio is not available (see footnote 19), so we will use the average of the adjustment factor from 1974 to 1983 to make the adjustments prior to 1974. This yields values of NH$_{t}$ back to January 1963.

However, the useful life of a typical home is often greater than 46 years (1963 to 2008), hence we may be omitting a significant fraction of homes with current mortgages if NH$_{t}$ only starts in January 1963. Based on data from the 2007 American Housing Survey, approximately 93% of homes surveyed were built after 1919. Therefore, we chose to extend NH$_{t}$ back to January 1919 to yield a more realistic time series for the stock of U.S. residential real estate in more recent years. We now describe the statistical model used to “backfill” the new home sales time series from January 1919 to December 1962.

We begin by hypothesizing that the growth rate in NH$_{t}$ is related to the growth in population. Higher values of NH$_{t}$ also seem to be correlated with periods of high real home price appreciation such as the earlier part of this decade. Accordingly, we first collect data for annual new home sales, population, and real home prices. The population data are obtained from two sources: data from 1900 to 1999 are obtained

---

20 We have excluded the “Multi-Units Built for Rent” category because our focus is the mortgage liability of the Personal sector. Mortgages for multi-units built for rent, such as large apartment buildings, are typically held outside of the Personal sector. However, some of these units may eventually be converted into condominiums and sold to individual buyers, which will not be captured in our simulations.

from

http://www.census.gov/popest/archives/1990s/popclockest.txt

and data from 2000 to 2008 are obtained from


The Real Home Price Index is obtained from Robert Shiller at


We then transform each of these series into growth rates by taking the first difference of the natural logarithms of the original time series, i.e.,

\[ \Delta NH_t = \log(NH_{t+1}) - \log(NH_t) \quad \text{for} \quad t \in \{1963, \ldots, 2008\} \]

\[ \Delta POP_t = \log(POP_{t+1}) - \log(POP_t) \quad \text{for} \quad t \in \{1919, \ldots, 2008\} \]

\[ \Delta HPI_t = \log(HPI_{t+1}) - \log(HPI_t) \quad \text{for} \quad t \in \{1919, \ldots, 2008\} \]

We then estimate the following linear model:

\[ \Delta NH_t = \alpha + \beta \Delta POP_t + \gamma \Delta HPI_t. \quad (A.1) \]

The estimated parameters, \( \hat{\alpha} \), \( \hat{\beta} \), and \( \hat{\gamma} \), and the data for \( \Delta POP_t \) and \( \Delta HPI_t \) from 1919 to 1962 are used to construct the left-hand-side variable \( \hat{\Delta} NH_t \) for this period. We use \( NH_{1963} \) and \( \hat{\Delta} NH_t \) to backfill \( NH_t \) from 1919 to 1962 through the following calculation:

\[ \log(\hat{NH}_t) = \log(NH_{1963}) - \sum_{i=t}^{1962} \Delta NH_i \quad \text{for} \quad t \in \{1919, \ldots, 1962\}. \quad (A.2) \]

In the final step, we use the empirical distribution of monthly new home sales estimated using monthly data from 1963 to 2008 to construct monthly estimates based on the annual estimates of \( \hat{NH}_t \) constructed using the method above. Given \( NH_{i,t} \), i.e., the sales in month \( i \) of year \( t \) for years 1963 to 2008, we estimate the proportion of houses constructed in month \( i \) using the following estimator:

\[ P_i = \frac{1}{2008 - 1963 + 1} \sum_{t=1963}^{2008} \frac{NH_{i,t}}{NH_t}. \quad (A.3) \]

Combining (A.2) and (A.3), we set the number of new houses entering the mortgage
system in month \( i \) of year \( t \) over the period from 1919 to 1962 to be:

\[
\hat{N}_{i,t} = \hat{N}_t \times P_i \quad \text{for} \quad t \in \{1919, \ldots, 1962\}.
\] (A.4)

Figure A.2 shows the actual and estimated time series of the number of new units entering the mortgage system from January 1919 to December 2008. This approach implies that 101.5 million units enter our simulation. Of this total, 52.6 million are based on actual data from January 1963 to December 2008, and 48.9 million are based on the estimation approach outlined above for the period from January 1919 to December 1962. The total of 101.5 million seems reasonable given our objective of capturing more than 90% of the homes in the U.S.

Figure A.2: Time series of the number of units of new homes entering the mortgage system since January 1919. This series is used as an input to our simulations. The data since January 1963 is available from the U.S. Census Bureau, and we have extrapolated the data back to 1919 based on a linear-regression model using population growth and a real-estate price index as regressors.

### 3. New Home Purchase Price

We begin by constructing a time series for the average home price since 1919. We will use the average home price available from the U.S. Census Bureau for “New One-Family Houses Sold” in the period from January 1975 to December 2008. For the period from January 1963 to January 1975, the average price is not available. However, the Census reports the median sale price for this period, which we will use as our starting point. From January 1975 to December 2008, when both mean and median home prices are available, we observe that the mean price is typically higher than the median by approximately 5%, and the ratio has increased in more recent history. To make our simulations more accurate, we multiply the reported
median prices in the January-1963-to-December-1974 sample by 1.05, i.e., we inflate the median by 5%, and use the resulting values to complete the NHP\textsubscript{t} series. From January 1919 to December 1962, we will use the growth rate of HPI\textsubscript{t} to backfill sales prices, starting with the sales price in January 1963.

We then use data from the 2007 American Housing Survey to create a distribution around the average price series. The details of these calculations are given in Table A.2. We start with the price range and number of homes in each given price range from the 2007 American Housing Survey (columns 1 and 2 of Table A.2). We then assign a single price level to each price range, given in column 3, and calculate the average price using the assigned price level and counts. In this case, the average was calculated to be $125,420. Then we computed the “Weight” (column 4) as the number of homes in each price range as a percentage of the total number of homes surveyed in the American Housing Survey, and the “Price Multiplier” (in column 5) as the ratio of the assigned price level of each price range to the calculated mean purchase price of $125,420. We use the calculated weights and price multiplier to create a distribution around the time series of average purchase price we constructed in the previous steps.

<table>
<thead>
<tr>
<th>American Housing Survey Range</th>
<th>American Housing Survey Count</th>
<th>Price Assigned to This Bin</th>
<th>Weight (%)</th>
<th>Price Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than $10,000</td>
<td>2,897</td>
<td>$10,000</td>
<td>4.0</td>
<td>0.08</td>
</tr>
<tr>
<td>$10,000 to $19,999</td>
<td>4,143</td>
<td>$15,000</td>
<td>7.0</td>
<td>0.12</td>
</tr>
<tr>
<td>$20,000 to $29,999</td>
<td>3,837</td>
<td>$25,000</td>
<td>6.0</td>
<td>0.20</td>
</tr>
<tr>
<td>$30,000 to $39,999</td>
<td>3,645</td>
<td>$35,000</td>
<td>6.0</td>
<td>0.28</td>
</tr>
<tr>
<td>$40,000 to $49,999</td>
<td>3,184</td>
<td>$45,000</td>
<td>5.0</td>
<td>0.36</td>
</tr>
<tr>
<td>$50,000 to $59,999</td>
<td>3,129</td>
<td>$55,000</td>
<td>5.0</td>
<td>0.44</td>
</tr>
<tr>
<td>$60,000 to $69,999</td>
<td>3,115</td>
<td>$65,000</td>
<td>5.0</td>
<td>0.52</td>
</tr>
<tr>
<td>$70,000 to $79,999</td>
<td>3,009</td>
<td>$75,000</td>
<td>5.0</td>
<td>0.60</td>
</tr>
<tr>
<td>$80,000 to $99,999</td>
<td>5,563</td>
<td>$90,000</td>
<td>8.0</td>
<td>0.72</td>
</tr>
<tr>
<td>$100,000 to $119,999</td>
<td>4,216</td>
<td>$110,000</td>
<td>6.0</td>
<td>0.88</td>
</tr>
<tr>
<td>$120,000 to $149,999</td>
<td>6,320</td>
<td>$135,000</td>
<td>10.0</td>
<td>1.08</td>
</tr>
<tr>
<td>$150,000 to $199,999</td>
<td>7,581</td>
<td>$175,000</td>
<td>12.0</td>
<td>1.40</td>
</tr>
<tr>
<td>$200,000 to $249,999</td>
<td>4,522</td>
<td>$225,000</td>
<td>7.0</td>
<td>1.79</td>
</tr>
<tr>
<td>$250,000 to $299,999</td>
<td>2,820</td>
<td>$275,000</td>
<td>4.0</td>
<td>2.19</td>
</tr>
<tr>
<td>$300,000 or more</td>
<td>7,483</td>
<td>$300,000</td>
<td>10.0</td>
<td>2.39</td>
</tr>
</tbody>
</table>

|                         | Average | $125,420 |

Table A.2: Data from the 2007 American Housing Survey (Table 3–14) used in calculating the distribution of home purchase price around the average-price time series.
4. **Initial Loan-to-Value Ratio.** Finding actual data about the initial LTV ratio is surprisingly difficult. In lieu of actual data, we have assumed that the initial LTV ratio is uniformly distributed between 75% and 95%.

5. **Initial Mortgage Maturity.** Another input to our simulations is the initial maturity and type of mortgage used. As discussed in the main text, we have assumed that all mortgages are standard fully-amortizing mortgages over a fixed interval in our simulations. Based on the data from Table 3–15 of the 2007 American Housing Survey, out of the 41,567 participants that reported their type of mortgage, 37,876 or 91.1% reported using “fixed payment, self-amortizing” mortgages. Therefore our assumption of standard mortgages in our simulation seems to be a plausible starting point.

Table A.3 contains some data from the 2007 American Housing Survey on initial maturities. In our simulations, we assume 80% of the mortgages have a 30-year maturity period at origination, with the remaining 20% having a 15-year maturity.

<table>
<thead>
<tr>
<th>Term of Primary Mortgage at Origination or Assumption</th>
<th>Count</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 8 years</td>
<td>1,243</td>
<td>3</td>
</tr>
<tr>
<td>8 to 12 years</td>
<td>1,340</td>
<td>3</td>
</tr>
<tr>
<td>13 to 17 years</td>
<td>6,594</td>
<td>14</td>
</tr>
<tr>
<td>18 to 22 years</td>
<td>2,573</td>
<td>6</td>
</tr>
<tr>
<td>23 to 27 years</td>
<td>912</td>
<td>2</td>
</tr>
<tr>
<td>28 to 32 years</td>
<td>32,641</td>
<td>70</td>
</tr>
<tr>
<td>33 years or more</td>
<td>1,092</td>
<td>2</td>
</tr>
<tr>
<td>Variable</td>
<td>66</td>
<td>0</td>
</tr>
</tbody>
</table>

Table A.3: Data from Table 3–15 of the 2007 American Housing Survey on mortgage maturities at origination.

6. **Long-Term Risk-Free Rate.** We use the yield on 30-year constant-maturity U.S. Treasury securities, which is available from February 1977 to December 2008, but with a gap between March 2002 to January 2006. We fill this gap using yields on 20-year constant maturity Treasury securities. For the period prior to February 1977, we will use the “Long Rate” collected by Robert Shiller, which is only available annually, so we use linear interpolation to obtain monthly observations.

7. **Mortgage Rates** For 30-year fixed-rate mortgage rates, we use the series constructed

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by Freddie Mac, which starts in April 1971. For the earlier period, we simply add 150 bps to the long-term risk-free rates $RF_t$ (see above), which is approximately the average spread between 30-year mortgage rates and $RF_t$ for the period from April 1971 to December 2008. For 15-year mortgage rates, we use the data from Freddie Mac which starts in September 1991. For the earlier period, we backfill this series by subtracting 46 bps from 30-year mortgage rates, which is approximately the difference between 30-year and 15-year mortgage rates in the post-September-1991 period for which we have access to data for both series from Freddie Mac.

### A.3 Calibration Results for Alternative Refinancing Rules

In this section we provide calibration results for two alternative refinancing intensity functions in the context of our probabilistic refinancing rule (1). In Table A.4, we present results for a refinancing intensity function that starts at zero and linearly increases through time reaching its maximum level in 2008, as given by the first element of each row. Table A.5 provides results of a similar calibration exercise for a rule that is uniform before and after 1988 but undergoes a level shift in 1988. This form for the refinancing intensity is motivated by Bennett, Peach, and Peristiani (2001). The level of refinancing intensity before and after 1988 are given by the first two numbers listed in each row. Figure A.3 compares the total mortgages outstanding and the cumulative equity extraction under the properly calibrated rule from each of these two specifications.

### A.4 Additional Sensitivity Analysis of Option-based Risk Metrics

Tables A.6–A.8 show the sensitivity of estimated delta, vega, and gamma values, respectively, to different volatility and rent yield assumptions. These tables are comparable to Table 5 which shows the sensitivity of the estimated put values to different volatility and rent yield assumptions.

\[24\text{See } \text{http://www.freddiemac.com/dlink/html/PMMS/display/PMMSOutputYr.jsp. The section “30-Year Fixed-Rate Historic Tables” contains the relevant data.}\]
### Table A.4: Mean Absolute Deviations (MAD), as defined in (2), between the simulated total mortgages outstanding time series vs. the Total Mortgage Liability time series from the *Flow of Funds Accounts* data, and the cumulative equity extractions vs. the series produced by Greenspan and Kennedy (2005), for three different time periods. The cash-out refinancing takes place according to probabilistic rule (1), where the Base Refinancing Rate is 0.1% per month and the refinancing intensity increases linearly from 1919 to 2008 and reaches a peak value given by the first element of each row of this table.

<table>
<thead>
<tr>
<th>Linear</th>
<th>MAD of Mortgages Outstanding (%)</th>
<th>MAD of Cumulative Equity Extractions (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80-08</td>
<td>90-08</td>
</tr>
<tr>
<td>3.00%</td>
<td>21.29</td>
<td>19.00</td>
</tr>
<tr>
<td>3.25%</td>
<td>20.16</td>
<td>17.37</td>
</tr>
<tr>
<td>3.50%</td>
<td>19.22</td>
<td>16.02</td>
</tr>
<tr>
<td>3.75%</td>
<td>18.17</td>
<td>14.54</td>
</tr>
<tr>
<td>4.00%</td>
<td>17.31</td>
<td>13.32</td>
</tr>
<tr>
<td>4.25%</td>
<td>16.63</td>
<td>12.40</td>
</tr>
<tr>
<td>4.50%</td>
<td>16.05</td>
<td>11.63</td>
</tr>
<tr>
<td>4.75%</td>
<td>15.65</td>
<td>11.13</td>
</tr>
<tr>
<td>5.00%</td>
<td>15.31</td>
<td>10.71</td>
</tr>
</tbody>
</table>

### Table A.5: Mean Absolute Deviations (MAD), as defined in (2), between the simulated total mortgages outstanding time series vs. the Total Mortgage Liability time series from the *Flow of Funds Accounts* data, and the cumulative equity extractions vs. the series produced by Greenspan and Kennedy (2005), for three different time periods. The cash-out refinancing takes place according to probabilistic rule (1), where the Base Refinancing Rate is 0.1% per month and the refinancing intensity is constant between 1919 and 1988, and then jumps to a higher level in 1988 and remains at that new level until 2008. The initial level and the level after the jump in 1988 are given by the first element of each row of this table.

<table>
<thead>
<tr>
<th>Uniform with Break in 1988</th>
<th>MAD of Mortgages Outstanding (%)</th>
<th>MAD of Cumulative Equity Extractions (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80-08</td>
<td>90-08</td>
</tr>
<tr>
<td>2.50%-3.00%</td>
<td>18.91</td>
<td>15.83</td>
</tr>
<tr>
<td>2.75%-3.25%</td>
<td>17.79</td>
<td>14.23</td>
</tr>
<tr>
<td>3.00%-3.50%</td>
<td>16.78</td>
<td>12.82</td>
</tr>
<tr>
<td>3.25%-3.75%</td>
<td>15.95</td>
<td>11.67</td>
</tr>
<tr>
<td>3.50%-4.00%</td>
<td>15.30</td>
<td>10.79</td>
</tr>
<tr>
<td>3.75%-4.25%</td>
<td>14.83</td>
<td>10.21</td>
</tr>
<tr>
<td>4.00%-4.50%</td>
<td>14.47</td>
<td>9.78</td>
</tr>
<tr>
<td>4.25%-4.75%</td>
<td>14.31</td>
<td>9.57</td>
</tr>
<tr>
<td>4.50%-5.00%</td>
<td>14.07</td>
<td>9.34</td>
</tr>
</tbody>
</table>
Figure A.3: The simulated times series of total mortgages outstanding and cumulative equity extraction compared to the Total Mortgage Liability series from Flow of Funds Accounts data and the cumulative equity extractions series produced by Greenspan and Kennedy (2005). The cash-out refinancing takes place according to probabilistic rule (1), where the base refinancing rate is 0.1% per month and the refinancing intensity follows a calibrated Linear or Uniform-with-Structural-Break-in-1988 profile as given in Tables A.4 and A.5. Specifically, the refinancing intensity is either linearly increasing from 1919 to 2008 and reaching a peak of 4.5%, or the refinancing intensity is constant except for a structural break in 1988 where the levels are 3.75% and 4.25% before and after the break, respectively.
Table A.6: Estimated sensitivities, in $billions, of the total value of guarantees extended to home buyers by mortgage lenders to a 1% drop in home prices (option deltas) under different assumptions for the volatility of home prices (Vol) and rent yield (RY). The estimated values are based on a simulation scenario with the Uniform refinancing rule as calibrated in Table 2.
Table A.7: Estimated second-order sensitivities, in $millions, of the total value of guarantees extended to home buyers by mortgage lenders to a 1% drop in home prices (option gammas) under different assumptions for the volatility of home prices (Vol) and rent yield (RY). The estimated values are based on a simulation scenario with the Uniform refinancing rule as calibrated in Table 2.
Table A.8: Estimated sensitivities, in $billions, of the total value of guarantees extended to home buyers by mortgage lenders to a 1% increase in home price volatility (option vegas) under different assumptions for the volatility of home prices (Vol) and rent yield (RY). The estimated values are based on a simulation scenario with the Uniform refinancing rule as calibrated in Table 2.
References


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