The Politics of Compromise

By Alessandro Bonatti and Heikki Rantakari

An organization must select among competing projects that differ in their payoff consequences for its members. Each agent chooses a project and exerts effort affecting its completion time. When one or more projects are complete, the agents select which one to adopt. The selection rule for multiple projects that maximizes ex post welfare leads to inefficiently high polarization; rules that favor later proposals improve upon ex post optimal selections. The optimal degree of favoritism increases in the cost of effort and discount rate. This trade-off informs the design of process rules in standard-setting organizations and helps explain their performance. (JEL C78, D23, D71, D72, D83, L15)

Many organizations rely on their members to develop solutions to specific problems. Universities establish search committees to hire at the senior level or to recommend changes to the curriculum. In a very similar fashion, standards bodies routinely form working groups to define the properties of a new technological standard. In both settings, there are no readily available solutions (i.e., candidates, curricula, standards) from which to select. Instead, the members of the organization must invest time and effort developing potential solutions. Furthermore, different members may have conflicting preferences over the feasible alternatives: which candidate to hire or which courses or patents to include in the curriculum or standard, respectively. Finally, as decision rights are typically shared, members must ultimately come to an agreement over which proposed solution to adopt.

The following problem is at the heart of all these examples. Because developing a proposal is costly, the first agent who presents a concrete proposal acquires considerable bargaining power. The other agents can avoid further development costs by approving his project and, hence, are willing to endorse projects that are not ideal from their perspective. When agents have conflicting preferences over potential

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projects, this creates scope for rent-seeking behavior, i.e., agents exert effort to develop projects that are biased in their favor and to preempt others from presenting alternatives that they like much less.

The development of projects that are highly skewed toward one member or group can be detrimental to an organization if solutions that compromise among members’ goals are more efficient. The challenge for the organization is then to provide members with incentives to develop moderate rather than highly polarized projects. However, the more a member is motivated to compromise on project selection, the less interested he is in the development of his own project, leading to inefficient effort and, hence, to inefficient delay.

In this paper, we are interested in the design of organizational decision-making processes that strike a balance between compromise in project choices and equilibrium effort levels. We explore this trade-off in a dynamic model consisting of a development phase and an adoption phase. Loosely speaking, the development phase integrates the classic patent race and team production frameworks with a choice of “research direction.” Two agents continuously choose which project to pursue and how much effort to exert. Each project’s completion requires a single stochastic breakthrough, and each agent affects the probability distribution of its arrival time through costly effort. Agents have conflicting interests, and compromise is efficient. In particular, there exists a continuum of potential projects that generate different payoffs for each agent, forming a strictly concave Pareto frontier. Therefore, “intermediate” or “compromise” projects are socially desirable.

Once a project is complete, the agents must decide whether to adopt it or wait until the second project is developed. While projects can be ranked in terms of their payoffs for the two agents, the space of their underlying characteristics can be quite complex. Therefore, we do not allow the agents to adopt convex combinations of projects with different characteristics. Further, we do not allow agents to write contracts that condition payments or decision rights on the characteristics of the projects developed.

The decision to adopt an agent’s project requires the acquiescence of the other agent. For example, if a member of a hiring committee finds a given candidate sufficiently unattractive, he can delay the adoption decision and continue to search for an alternative candidate. A consensus requirement can thus limit the scope for rent-seeking and induce each agent to shift away from his ideal project to ensure the other agent’s support. However, a consensus requirement alone does not determine a particular level of compromise. In the hiring example, each committee member’s incentives to block or adopt the first candidate depend not only on the value of that candidate but also on which alternative candidates he prefers and which ones he expects to generate consensus. In other words, the option value of blocking a project depends on continuation play and, thus, on the selection rule dictating which project

\[1\] In the context of a business school, examples include senior faculty candidates who can interact with heterogeneous groups and core curricula for MBA students that are not dominated by one subject area. In the context of standardization, consider technological solutions that minimize the total switching costs faced by firms and consumers.

\[2\] The complexity of the projects suggests that it can be exceedingly difficult to describe them in a contract. Similarly, the existence of complementarities within a given project can make combining two distinct projects unprofitable, if not unfeasible. See Brynjolfsson and Milgrom (2013) for a discussion. Finally, while theoretically attractive, ex ante payments are often impractical, if not illegal, in many applications. We discuss this assumption within the context of standards-setting in greater detail in the next subsection and in Section VI.
is adopted when two projects have been developed. Solving backward, the selection rule influences the types of projects developed in equilibrium and their completion times.

Our main results are as follows. Under the selection rule that adopts the project with the highest ex post social value, the agents exert efficient effort levels conditional on their chosen projects, but they pursue excessively polarized projects. In other words, we uncover a trade-off between ex post welfare and the incentives for compromise in the initial project choices. To improve ex ante welfare (i.e., the sum of the agents’ utilities) relative to the ex post optimal criterion, the selection rule must be distorted in favor of the project developed later. For instance, the selection rule can allow each agent to respond to the project that is developed first with a more polarized (and, hence, more selfish) project. This ex post distortion “levels the playing field” by increasing the option value of blocking the first project, and it forces both agents to compromise more in their initial project choices.

The optimal (second-best) combination of projects and effort levels can be induced in equilibrium by adopting the project that maximizes a weighted sum of the agents’ payoffs. Consistent with the option value logic, the optimal Pareto weights are skewed in favor of the agent who develops the second project. Furthermore, as the agents’ costs of effort and discount rates increase, the option value of blocking the first project decreases, and the optimal degree of favoritism must consequently increase.

We finally consider how the model applies to standard-setting organizations. To capture some features of their environment, we consider a slightly modified setting in which voting and active agents coexist. Specifically, we introduce a continuum of voters with heterogeneous preferences over projects. A simple voting procedure implements the optimal combination of projects and effort levels: projects are voted on sequentially as they are completed; a qualified majority is required for approval; and any project that fails to gain approval is removed from further consideration. In particular, the ability to discard projects that have been voted down is necessary to strike the balance between compromise and effort. Conversely, a procedure that saves voted-down projects for a tie-breaking “runoff” vote leads to excessively polarized initial project choices.

This tension is reflected in the experiences of many standard-setting organizations, to which we now turn.

Standard-Setting Organizations.—We focus on the workings of voluntary standard-setting organizations (SSOs) as an application. This is a suitable application for three reasons: (i) economic relevance; (ii) the trade-off between free-riding and rent-seeking; and (iii) data availability. In particular, the inter-firm nature of SSOs facilitates finding evidence relative to intra-firm applications.

Following Simcoe (2014, p. 103), we define an SSO as a “multilateral organization that governs some key piece of a shared technology platform.” SSOs provide forums (with voluntary participation) for the development and establishment

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3 From this perspective, the SSOs for the governance of the Internet are especially relevant for our model. These organizations include the Institute for Electrical and Electronics Engineers (IEEE), the Internet Engineering Task Force (IETF), and the World Wide Web Consortium (W3C).
of broad consensus on standards prior to their adoption. Thus, the main economic advantage of SSOs is to solve the potential coordination failures that arise under unfettered market competition. In markets with network externalities, de jure standardization saves duplication costs, stimulates specific investments by complementary products, and avoids the risk of a standards war.

This process has been used to establish a multitude of voluntary consensus standards. However, shared interest in establishing a standard to realize the benefits of network economies often conflicts with the vested interests of each participant. Overall, the combination of free-riding, distributional conflicts, and consensus requirements makes reaching an agreement quite challenging. Consistent with our approach, Simcoe (2014) and Baron, Meniere, and Pohlmann (2014) argue that the process of developing and adopting a standard must balance free-rider problems and rent-seeking behaviors. In Section VI, we illustrate the decision-making procedures that SSOs use to manage each element of this trade-off.

Finally, ex ante contracting may alleviate many of the inefficiencies faced by SSOs, including those examined by our model. Recent theoretical work, e.g., Lerner and Tirole (2015), suggests ex ante price commitments as a means of improving efficiency, and some SSOs have taken steps toward such commitments. However, given that price negotiations open the door to litigation, most SSOs do not encourage price commitments, and some explicitly forbid firms from negotiating licensing agreements at the standard-setting stage. Consequently, we rule out ex ante transfers. In the working paper version, Bonatti and Rantakari (2015), we allow agents to offer payments in exchange for support for their projects. We show that policy compromise remains a crucial means of building consensus even when money is available and that ex post transfers may, in fact, be detrimental to compromise.

**Related Literature.**—At a broad level, this paper is part of a growing literature adopting the political view of organizational decision-making initiated by March (1962) and Cyert and March (1963), which is summarized by Pfeffer (1981) as follows, “to understand organizational choices using a political model, it is necessary to understand who participates in decision making, […] what determines each actor’s relative power, and how the decision process arrives at a decision.” See Gibbons, Matouschek, and Roberts (2013) for a survey.

At a more detailed level, this paper is related to several strands of more recent research. First, our model can be viewed as an analysis of real authority and project choice in organizations. The most closely related papers in this field are Aghion and Tirole (1997) and Rantakari (2012) in their focus on ex ante incentives and Armstrong and Vickers (2010) in their analysis of endogenous proposals.

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4 Examples of Internet-related standards adopted by the SSOs referenced above include the 802.11 standard for wireless communication in the IEEE, the HTTP protocol in the IETF, and the URL standard for the World Wide Web in the W3C.

5 Llanes and Poblete (2014) note that in the IEEE “participants should never discuss the price at which compliant products may or will be sold, or the specific licensing fees, terms, and conditions being offered by the owner of a potential Essential Patent Claim.” The European Telecommunication Standards Institute (ETSI) has similar rules in place.

6 Other papers have examined the impact of organizational structure on information flows inside the organization. For instance, Dessein (2002); Alonso, Dessein, and Matouschek (2008); and Rantakari (2008) consider the impact of the allocation of decision rights on strategic communication and decision-making.
Second, our work ties into a large literature focused on conflict resolution within a committee. For instance, Dewatripont and Tirole (1999) and Che and Kartik (2009) analyze the value of conflict for information acquisition in committees. In contrast, we focus on the roles of ex ante conflict and ex post negotiation in achieving equilibrium compromises on project choices. More closely related to our application are Farrell and Saloner (1988) and Farrell and Simcoe (2012), who study consensus decision-making in standard-setting organizations. In their setting, the organization must select one of two exogenously developed projects when information on project quality is asymmetric and the decision structure is fixed. In contrast, in our model, the development phase precedes the adoption phase. The development phase is closely related to the R&D and patent race models of Reinganum (1982) and Doraszelski (2008). Relative to these papers, we integrate choices over research directions and negotiations over project adoption.

Third, our paper is related to the dynamic provision of public goods, e.g., Admati and Perry (1991) and Marx and Matthews (2000). In these models, as well as in ours, each agent conditions his contributions on the type of public good provided by the other agents. An innovative feature of our framework is that it allows agents to choose which type of public good they wish to provide.

Finally, our paper is part of a recent political economy literature on policy contests. Callander and Harstad (2015) develop a two-period model of policy experimentation along both horizontal (ideology) and vertical (effort or quantity) dimensions. In an application to federal systems, they compare decentralized decision-making to a regime of progressive centralization. Hirsch and Shotts (2015) consider a related (two-dimensional) static model of competing policy proposals with a fixed decision structure.

I. Model

We model an organization consisting of two agents $i = 1, 2$. There exists a continuum of feasible projects indexed by $x \in [0, 1]$. For a project to yield payoffs to the agents, it must be first developed and then adopted, as described below.

Time is continuous, and the horizon is infinite. Both agents are impatient and discount the future at rate $r$. To develop a project, agents exert costly effort. The development (or “completion”) of each project is stochastic and requires the arrival of a single breakthrough that follows a Poisson process: if agent $i$ were to choose a constant project $x_i$ and exert a constant effort $a_i$ over some time interval $dt$, then the delay until the development of project $x_i$ would be distributed exponentially over that time interval with parameter $\lambda a_i$. Without loss of generality, we normalize $\lambda$ to one. For tractability, the instantaneous cost to agent $i$ of exerting effort $a_i \in \mathbb{R}_+$ is independent of the project chosen. It is given by $c_i(a_i) = c_i \cdot a_i^2/2$ for some constant $c_i > 0$.

The chosen projects and effort levels are assumed to be noncontractible and unobtainable to the other player. Once a project has been developed, it can be adopted. The selection of a project is irreversible and ends the game. We analyze various

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7 Simcoe (2012) estimates a complete-information, stochastic model of bargaining by exploiting variation in the nature of the projects submitted to the IETF. The decision structure is then fixed by construction.
procedures for selecting the project that is, in fact, adopted. In all of our settings, an outcome of the game consists of (i) the measurable functions $a_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and $x_i : \mathbb{R}_+ \rightarrow [0, 1]$, where $a_{i,t}$ is the level of effort exerted by $i$ at time $t$ toward the development of project $x_{i,t}$; (ii) the set of projects $x_{i,\tau}$ developed by either agent $i$ at any time $\tau$; and (iii) at most one project $x_{i,\tau}$ adopted at time $\tau' \geq \tau$.

We also assume that each agent can freely modify his choice of project $x_{i,t}$ during the game, that each agent $i$ can develop at most one project and that the development of any project is publicly observable. We discuss these assumptions in Section V.

Adoption of project $x$ yields a net present value of $v_i(x)$ for each agent $i$. As long as no project has been adopted, agents reap no benefits from any project. If project $x$ is adopted at time $\tau$, the discounted payoff to agent $i$ is given by

$$ V_i = e^{-r \tau} v_i(x) - \int_0^\tau e^{-rt} c_i(a_{i,t}) \, dt. $$

The payoff functions $v_i(x)$ are monotone, differentiable, and strictly concave. In particular, $v_1(x)$ is increasing, and $v_2(x)$ is decreasing, with $v_1(1) = v_2(0) = 1$ and $v_1(0) = v_2(1) = 0$. Thus, agents have conflicting preferences over projects: $x = 1$ is agent 1’s preferred project and $x = 0$ is agent 2’s preferred project. Moreover, compromise is efficient: the agents’ payoffs $(v_1(x), v_2(x))$ form a continuously differentiable and strictly concave payoff frontier. We maintain the following assumptions throughout the paper.

**ASSUMPTION 1** (Symmetry):

(i) The agents’ cost functions are identical, i.e., $c_1 = c_2 = c$.

(ii) The payoff frontier is symmetric, i.e., for all $x \in [0, 1]$,

$$ v_1(x) = v_2(1-x). $$

The costs associated with developing a project, i.e., the delay itself and the cost of effort incurred during that delay, can often be summarized by the effective discount rate

$$ \rho \triangleq c \cdot r. $$

Under Assumption 1, we denote the payoff frontier as a strictly decreasing and strictly concave function $v_j = \phi(v_i)$ [Figure 1] provides an illustration.

The *development phase* is followed by an *adoption phase*. In the adoption phase, once one or more projects have been developed, negotiations to select which one is adopted take place. Adoption decisions require consensus, i.e., both agents must agree to adopt a project. More formally, suppose that agent 1 developed the first project $x_1$ at time $\tau$. Agent 2 can choose to adopt agent 1’s project at any time $t \geq \tau$. As long as no project has been adopted, agent 2 can attempt to develop a different project $x_2$, i.e., agent 2 can de facto veto agent 1’s initial project by delaying its adoption until he has developed a competing project.
Naturally, the incentives to adopt or to veto a project depend on the outcome that agent 2 expects once he has developed his own project. We must then understand how negotiations unfold in the subgame that begins once two projects $x_1$ and $x_2$ have been developed. Our model seeks to capture two crucial aspects of the bargaining process: (i) agents are able to condition their play on the public history prior to the adoption phase, and (ii) because developed projects are publicly observable, each agent $i$ can anticipate the outcome of the adoption phase as a function of which project is developed at which time. We would like our approach to be insensitive to the details of the bargaining process. Thus, we do not analyze a specific extensive form game. Instead, we follow the approach to (re)negotiation introduced by Tirole (1986) in the context of procurement, i.e., we posit a selection function

$$\xi(x, \tau) \in \{x_1, x_2\}$$

that indicates which project is adopted if $x = (x_1, x_2)$ were developed at $\tau = (\tau_1, \tau_2)$.

With this formulation, we are focusing on deterministic, ex post Pareto-efficient selection functions, i.e., a project is adopted immediately with probability one. As an illustration, suppose that negotiations unfold as a complete information war of attrition in continuous time: each agent $i$ can “concede” at any time, leading to the adoption of project $x_j$. Under this protocol, $\xi(x, \tau) = x_j$ selects the Pareto-efficient equilibrium in which agent $j$ concedes immediately. The war of attrition and other bargaining games also admit inefficient equilibria, such as those with costly delays.
characterized by Hendricks, Weiss, and Wilson (1988). In Section V, we discuss other selection functions.

II. Benchmark Projects

Before analyzing the full model, we consider a benchmark model with the following characteristics: each agent $i$ works on an exogenously given project $x_i$, the first project to be developed is adopted immediately, and effort levels are chosen noncooperatively. The goal of this section is twofold: to derive how the equilibrium effort levels depend on the project characteristics $(x_1, x_2)$, which is instrumental to characterizing on-path effort when projects are endogenously chosen, and to identify the second-best projects $(x_1^*, x_2^*)$ that would be developed if agents could contract ex ante on project characteristics. We describe our results informally and refer the reader to Bonatti and Rantakari (2015) (henceforth, the working paper) for the details.

Given projects $(x_1, x_2)$, each agent $i$ chooses a measurable function $a_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ to maximize his expected discounted payoff $V_{i,0}$. Because the hazard rate of the first breakthrough is given by $a_1, t + a_2, t$, each agent’s expected payoff at time $t = 0$ is given by

$$ V_{i,0} = \int_0^{\infty} e^{-\int_0^t (r + a_1, v_i(x_i) + a_2, v_i(x_j) - c(a_i, t)) dt}. $$

This game has a unique Nash equilibrium for any pair of projects $(x_1, x_2)$ such that each agent prefers his own project to the other agent’s, i.e., $x_1 > x_2$. (This is the relevant range for the benchmark projects described below.) In equilibrium, agents use stationary strategies, i.e., constant effort levels $a_{i,t} = a_i$. In the case of symmetric projects, i.e., $x_2 = 1 - x_1$, this equilibrium is also symmetric, and the equilibrium effort level depends negatively on $c$ and positively on the distance $x_1 - x_2$, on the value of each agent’s own project $v_i(x_i)$, and on the discount rate $r$. Finally, if $x_1 < x_2$ and the projects are symmetric, the game has a unique symmetric equilibrium with these same properties.\footnote{When each agent prefers his opponent’s project to his own, i.e., $x_1 < x_2$, multiple stationary and nonstationary equilibria may exist due to an extreme free-rider problem in the provision of effort.}

Each agent increases the probability of achieving a breakthrough by exerting more effort. Agent $i$’s incentives at time $t$ are then driven by the value of ending the game with a payoff of $v_i(x_i)$. Under stationary strategies, we can write agent $i$’s problem recursively as

$$ rV_i = \max_{a_i} \left[ a_i(v_i(x_i) - V_i) + a_j(v_i(x_j) - V_j) - ca_i^2 / 2 \right]. $$

Each agent’s first-order condition relates the incentives for effort to the gains from developing his own project over and above his continuation value:

$$ ca_i^* = v_i(x_i) - V_i. $$

8 When each agent prefers his opponent’s project to his own, i.e., $x_1 < x_2$, multiple stationary and nonstationary equilibria may exist due to an extreme free-rider problem in the provision of effort.
Formulation (4) suggests that agent \( j \)’s effort may impose a positive or negative externality on agent \( i \), depending on whether the payoff of agent \( i \) from his opponent’s project \( v_i(x_j) \) is higher or lower than his own continuation value \( V_i \). Intuitively, agent \( j \)’s effort has two effects on agent \( i \)’s payoff: on the one hand, agent \( j \) is more likely to generate positive benefits \( v_i(x_j) \) for agent \( i \) and allow him to save on further development costs; on the other hand, agent \( i \) is now less likely to realize the benefits \( v_i(x_i) \) that accrue from developing his project first.

The characteristics of the two projects \( x_1 \) and \( x_2 \) determine the nature of the externality that each player’s actions impose on the other player. In particular, when the two projects are sufficiently different, agent \( j \)’s effort imposes a negative externality on agent \( i \). Suppose, for example, that the agents pursue their favorite projects \( x_1 = 1 \) and \( x_2 = 0 \). Agent 2’s effort imposes a negative externality on agent 1 because the payoff \( v_1(0) = 0 \) falls short of the equilibrium continuation value \( V_1 \), which is strictly positive because agent 1 has a positive probability of developing and adopting his own project \( x_1 \). The opposite holds when the two projects are very similar and \( v_1(x_1) \approx v_1(x_2) \). In this case, the payoff \( v_1(x_2) \) exceeds the continuation value \( V_1 \), which accounts for costly effort and delay.

An increase in agent \( j \)’s effort may then either motivate or discourage high effort levels by agent \( i \). Specifically, the nature of the payoff externality imposed by one agent’s effort on the other agent also determines whether the game has the strategic properties of a patent race, where the agents want to preempt each other by working harder, or of a moral hazard in teams problem, where the agents have an incentive to free ride. Consequently, the effort levels in the noncooperative solution may be above or below the levels that would maximize the agents’ joint surplus. Figure 2 illustrates the free-riding and racing regions for two different values of \( \rho \) as well as the benchmark projects described below.

If both the effort levels and the project characteristics were contractible, then the first-best solution would have each agent develop project \( x = 1/2 \) and choose first-best effort levels. However, as discussed above, when the effort levels are noncontractible, pursuing these projects yields inefficiently low effort levels. For any pair of symmetric projects, i.e., \( x_1 = 1 - x_2 \), we refer to the value \( v_j(x_i) \) that each agent’s project generates for the other agent as the degree of compromise. We now characterize two pairs of benchmark projects.

Efficient-Effort Projects.—There exists a unique pair of projects \( (x_1^E, x_2^E) \) that yield the first-best effort levels, i.e., the levels that maximize the agents’ joint surplus, conditional on the projects pursued. The efficient-effort projects satisfy

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v_i(x_j^E) = V_i(x_i^E, x_j^E).
\]

In other words, under efficient-effort projects, each agent \( i \) is indifferent between receiving the payoff from agent \( j \)’s completed project \( v_j(x_j) \) and continuing the game with an expected value \( V_i(x_i, x_j) \). Thus, agent \( i \) is indifferent to whether \( j \) develops his project. Additionally, because agent \( j \)’s choice of effort \( a_j \) then affects his own

\cite{Beath, Katsoulacos, Ulph} and \cite{Doraszelski} obtain analogous results for R&D races with imperfect patent protection.
payoff only, it is efficient. Moreover, if \( x_1 - x_2 > x_1^E - x_2^E \), effort levels are strategic complements, and the equilibrium effort levels are inefficiently high; inversely, if \( x_1 - x_2 < x_1^E - x_2^E \), effort levels are strategic substitutes and inefficiently low in equilibrium. As either the discount rate \( r \) or the cost of effort \( c \) increases, the payoff distance between the two projects \( x_1^E \) and \( x_2^E \) increases, i.e., agents’ efforts are strategic substitutes for a wider choice of projects. If an agent is very impatient or finds effort to be very costly, he is more likely to benefit from the other agent developing his project and hence to free ride on the other agent’s effort. As either \( c \) or \( r \) grows without bound, equation (6) implies \( v_1(x_i^E) \rightarrow 0 \), meaning that \( x_1^E \rightarrow 1 \) and \( x_2^E \rightarrow 0 \).

While projects \( x_1^E \) and \( x_2^E \) elicit efficient effort levels, they do not maximize the agents’ ex ante payoffs. Starting from the efficient effort levels, inducing a higher degree of compromise entails a second-order loss due to reduced effort but a first-order gain due to the increased social value of the adopted project.

**Second-Best Projects.**—There exists a unique pair of projects \((x_1^*, x_2^*)\) that maximize the ex ante sum of the agents’ utilities \( V_{1,0} + V_{2,0} \) when effort levels are chosen noncooperatively and the first project is adopted. The second-best projects \( x_1^* \) and \( x_2^* \) are symmetric, and they trade-off the expected cost of delay and the total value of the adopted projects. More formally, the sum of the agents’ payoffs is quasiconcave in the degree of compromise with a single peak at \((x_1^*, x_2^*)\).

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10Throughout the paper, we adopt the utilitarian criterion to assess welfare. Even if decisions in the model are not contractible (because we have ruled out monetary transfers ex post), it is sufficient that agents be able to contract on process rules for the ex ante choice of decision structure to be guided by the utilitarian criterion. Finally, maximizing the sum of the agents’ payoffs is a simple second-best goal for the organization. Murphy and Yates (2009, p. 16) note that the utilitarian criterion is cited as part of the International Standards Organization (ISO) mission statement: “to unify the needs of industry and thus bring about the greatest good for the greatest number.”
The second-best projects $x_1^*$ and $x_2^*$ induce a game of strategic substitutes with equilibrium effort levels below the first best. In other words, the distance between the second-best projects satisfies $x_1^* - x_2^* < x_1^E - x_2^E$ for any positive marginal cost and discount rate. While the second-best projects always lie in the region of strategic substitutes, the exact characteristics of these projects depend on the discount rate and the cost of effort. As either $c$ or $r$ increases, the second-best projects become more distant because a higher degree of conflict stimulates effort when the development of a project is more urgent or more costly. However, in the limit, and in contrast to efficient-effort projects, second-best projects do not approach $(0, 1)$; even as the agents become arbitrarily impatient or inefficient, it is always optimal to induce some positive degree of compromise.

Finally, one may wonder whether the sum of ex ante payoffs could be improved by relaxing the assumption that agents pursue the second-best projects at all times and the first project completed is immediately adopted. We therefore consider arbitrary paths $(x_1, t, x_2, t)$ and more general adoption rules that allow for deadlines, delays, and other distortions. The optimal mechanism in this class is quite simple.

**LEMMA 1 (Optimal Mechanism):** In the optimal symmetric mechanism, agents pursue the second-best projects $x_i^*$ at all $t \geq 0$, and the first project developed is adopted immediately.

Intuitively, expected payoffs can only be improved relative to pursuing the second-best projects by inducing higher effort levels at the early stages of the game. This can be achieved by distorting future project and adoption choices. However, as (5) makes clear, increasing effort requires lowering continuation payoffs (pursuing less valuable projects). This tension between payoffs and incentives prevents ex post distortions from improving upon the best stationary mechanism.

### III. Equilibrium Project Selection

When project choice is not contractible, agents choose which projects to pursue based on their expectations of how the game will unfold when one or both projects have been completed. If only agent $j$ has completed his project, the outcome is determined by agent $i$’s choice to either accept the proposal or continue his development efforts, given the expected payoff of continued development. The expectation over the outcome of the negotiations with two complete projects is, in turn, captured by the selection function $\xi(x, \tau)$.

We must then compute agent $i$’s expected value from continuing the game. In particular, agent $i$ can choose a new project $x_i'$ after agent $j$ develops project $x_j$, even if agent $i$ had previously been pursuing project $x_i$. However, developing his own project is costly for agent $i$ in terms of both effort and time. Thus, agent $i$ adopts project $x_j$ immediately if and only if its value $v_i(x_j)$ exceeds his continuation value under the selection function.

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11 As the agents become arbitrarily patient or efficient, both $x_i^E$ and $x_i^*$ converge to $1/2$. 
We will refer to this continuation value as the option value of blocking the first project. The option value is crucially determined by the set of projects that agent $j$ can expect to develop and has adopted. In particular, let $u(w)$ denote the value that an agent assigns to earning $0 \leq w \leq 1$ upon the development of his project. This value is given by

$$
u(w) \equiv \max_{a_{i,t}} \int_0^\infty e^{-\int_0^t (r + a_{i,t}) \, ds} (a_{i,t} w - ca_{i,t}^2/2) \, dt = w + \rho - \sqrt{\rho^2 + 2wp}.
$$

Solving backward, both agents have an incentive to engage in preemptive compromise: by proposing a project that is sufficiently attractive to the other agent, the first agent is able to guarantee immediate adoption. This avoids a deadlock wherein the second agent develops his own project and then negotiations take place over the two proposed alternatives. The remainder of this section analyzes how the agents’ project choice is influenced by the selection function (and thus by the option value of blocking) and the conditions under which it is possible to induce the agents to pursue second-best projects.

### A. Ex Post Optimal Selection

We begin our analysis by considering an intuitive setting wherein the agents select the more socially valuable project whenever two projects have been developed. Ties are broken in favor of the project developed later to ensure the existence of a best response once one project has been developed.

We define the ex post optimal selection function as follows: let $\xi(x, \tau) = x_1$ if $\sum_i v_i(x_1) > \sum_i v_i(x_2)$ or $\sum_i v_i(x_1) = \sum_i v_i(x_2)$ and $\tau_1 > \tau_2$. The ex post optimal selection function can be easily implemented, e.g., through an impartial mediator. We now show how this selection function determines the option value of blocking a project and, thus, the degree of compromise in the initial project necessary to achieve immediate acceptance.

Suppose that agent 1 completes project $x_1 > 1/2$. To prevail in the adoption phase, agent 2 must develop a project $x_2$ that gives the sum of the agents at least as much as under the standing project $x_1$. With this continuation play, the best project agent 2 can develop and adopt is $x_2 = 1 - x_1$, i.e., a project that yields the same level of total surplus as $x_1$ and grants agent 2 exactly as much as agent 1 would receive under the original project $x_1$. Figure 3 illustrates the equilibrium outcome under the ex post optimal selection function.

However, developing the second project is costly for agent 2, and thus, he is willing to accept projects that yield less than $v_2(x_2)$. More generally, each agent $i$ immediately adopts any project $x_j$ that satisfies

$$v_j(x_j) \geq u(v_i(1 - x_j)),$$

where the value of the single-agent problem $u(\cdot)$ is defined in (7). Intuitively, each agent $j$ initially chooses to pursue a project that makes agent $i$’s acceptance constraint (8) bind. Proposition 1 characterizes the projects developed in equilibrium.
PROPOSITION 1 (Ex Post Optimal Selection): The ex post optimal selection function yields a unique equilibrium outcome. Agents develop the efficient-effort projects \((x_1^E, x_2^E)\), and the first project is immediately adopted.

PROOF:

Suppose that agent 1 develops the first project \(x_1\), where \(v_1(x_1) = v_1\). Agent 2 will adopt it if and only if its option value of developing the second project is less than \(\phi(v_1)\). If he decides to reject it, he will develop a project \(x_2 = 1 - x_1\). By backward induction, agent 1 will develop a project \(x_1\) such that agent 2 is just indifferent between adopting it and rejecting it: any project that yields a higher value \(v_1\) will be rejected, and any project that yields a higher value \(v_2\) will be adopted but yields a strictly lower payoff to agent 1. Therefore, the first project to be developed on the equilibrium path is adopted immediately. Furthermore, agent \(i\)’s initial project \(x_i\) coincides with the competing project \(1 - x_j\) he could develop and successfully adopt in response to agent \(j\) developing project \(x_j\). Thus, agent \(i\) receives from agent \(j\)’s project \(x_j\) a payoff \(v_i(x_j)\) equal to his continuation value \(V_i(x_i, x_j)\). As we argued in condition (6) above, this characterizes the unique level \(x_i^E\) that imposes neither positive nor negative externalities in effort, so it is exactly \(x_i^E\).

The natural selection function that adopts the best available project thus highlights a tension between the value of the projects adopted ex post and the incentives for
ex ante compromise: if the organization insists on the ex post optimal selection, the alternatives that are generated will exhibit excessive polarization.

B. Ex Ante Optimal Selection

The development of the second-best projects along the equilibrium path must then be supported by the selection of the less socially valuable project (or other forms of ex post value burning) off the equilibrium path. In particular, the selection function must sufficiently favor the agent with the later project to discipline both agents’ initial project choices.

An extreme level of favoritism allows each agent $j$ to “counter” the initial project $x_i$ with his most preferred project (worth zero to agent $i$) and have it adopted. We shall refer to this case as second-mover authority. Then, to be adopted by agent $j$, each agent $i$ must develop an initial project $x_i$ that yields to agent $j$ at least $u(1)$. We define the project $\bar{x}_i$ such that $v_j(\bar{x}_i) = u(1)$ as agent $i$’s maximum-compromise project. Because the value of the single-agent problem in (7) is strictly decreasing in $\rho$, the value of each agent’s own maximum-compromise project $v_j(\bar{x}_i(\rho))$ is strictly increasing in $\rho$ and goes to 1 in the limit.

We now introduce a class of simple selection functions that incorporates ex post optimal selection and second-mover authority. Consider selection functions that maximize a weighted sum of the two agents’ payoffs, where $\alpha \in [0, 1]$ denotes the weight assigned to the agent who develops the first project. Thus, if agent 1 develops the first project, the weighted-utilitarian selection function $\xi(x, \tau, \alpha)$ adopts project $x_1$ if

$$\alpha v_1(x_1) + (1 - \alpha) v_2(x_1) > \alpha v_1(x_2) + (1 - \alpha) v_2(x_2),$$

and $x_2$ otherwise. (Ties are again broken in favor of the second project.) Within this class, $\alpha = 1/2$ yields the ex post optimal selection and $\alpha = 0$ yields second-mover authority.

Intuitively, the more costly the development and the more impatient the agent, the lower the option value of blocking and the less the first agent needs to compromise to induce acceptance. We define the threshold $\bar{\rho} > 0$ as the (unique) discount rate for which the second-best and maximum-compromise projects coincide, i.e.,

$$x_i^*(\bar{\rho}) = \bar{x}_i(\bar{\rho}).$$

We now restrict our attention to the case of $\rho \leq \bar{\rho}$. In other words, we assume that the second-best projects are more polarized than the maximum-compromise projects, i.e., $v_i(\bar{x}_i) \leq v_i(x_i^*)$. We discuss the case where $\rho > \bar{\rho}$ informally at the end of this section, and we characterize the equilibrium payoff set for any $\rho$ in the working paper.
PROPOSITION 2 (Weighted-Utilitarian Selection):

(i) The selection function \( \xi(x, \tau, \alpha) \) yields a unique symmetric equilibrium outcome. Agent \( i \) develops project \( x_i(\alpha, \rho) \), and the first completed project is immediately adopted.

(ii) The degree of compromise \( v_j(x_i(\alpha, \rho)) \) is decreasing in \( \alpha \) (weakly) and in \( \rho \) (strictly).

(iii) There exists \( \bar{\alpha}(\rho) \in (0, 1/2) \) such that agents develop the maximum-compromise projects \( \bar{x}_i(\rho) \) if and only if \( \alpha \leq \bar{\alpha}(\rho) \).

(iv) There exists a unique \( \alpha^*(\rho) \in [\bar{\alpha}(\rho), 1/2] \) that induces the second-best projects \( x_i^*(\rho) \). The weight \( \alpha^*(\rho) \) is decreasing in \( \rho \), with \( \alpha^*(0) = 1/2 \) and \( \alpha^*(\rho^*) = \bar{\alpha}(\rho^*) \).

The results in Proposition 2 reflect the logic of option values: the more an agent expects to earn from the adoption phase with two projects on the table, the more the other agent’s project must compromise to ensure immediate adoption. As illustrated in panel A of Figure 4 (and as discussed above), under the ex post optimal selection function \( (\alpha = 1/2) \), each agent \( j \) can develop and adopt his current project \( x_j^E \) in response to agent \( i \)’s equilibrium project \( x_i^E \). With weights \( \alpha < 1/2 \), each agent can switch to a more polarized project after the first equilibrium project is developed, increasing the option value of blocking. Both agents must then choose initial projects with a higher degree of compromise.

The equilibrium degree of compromise is increasing in the weight placed on the second agent’s payoff. For sufficiently small \( \alpha \), the second agent is able to pursue and adopt his favorite project. Thus, all \( \alpha \leq \bar{\alpha} \) are equivalent to second-mover authority and induce the agents to initially pursue the maximum-compromise project. The degree of compromise is also decreasing in the effective discount rate \( \rho \). Intuitively, as the agents become more impatient or the cost of development increases, they are willing to accept increasingly less attractive projects to avoid the costs of continued development efforts.

Finally, there exists an optimal weight \( \alpha^*(\rho) \) that supports the second-best projects. Consistent with the result in part (2.), the optimal selection function must favor the agent with the later project to increase the degree of compromise above \( v_j(x_i^E) \). Further, the optimal degree of favoritism increases with the effective discount rate. Intuitively, as the agents become more willing to acquiesce, their option value must increase to maintain the incentive to compromise. Thus, \( \alpha^* \) is decreasing in \( \rho^* \), despite the fact that second-best projects become more polarized as \( \rho \) increases.12

These equilibrium levels of compromise are illustrated in panel B of Figure 4.

Different selection functions, including stochastic or ex post inefficient ones, can also implement the second-best projects. One such function randomizes ex post, adopting the project that was completed first with probability \( \alpha \). While the value of

12 Conversely, the option value \( u(1; \rho) \) decreases sufficiently rapidly with \( \rho \) that the maximum-compromise projects are supported by a wider range of \( \alpha \), i.e., the threshold \( \bar{\alpha}(\rho) \) is increasing in \( \rho \).
the optimal $\alpha^*$ will differ from the weighted-utilitarian case, both selection functions generate the correct level of the option value on the equilibrium path.

However, no selection function can yield a higher degree of compromise than the second-mover authority. More generally, Pareto-inefficient or stochastic selection functions cannot enlarge the set of projects supported in equilibrium. This is a consequence of the agents’ lack of commitment: each agent can always adopt the other agent’s project when it is completed, and the selection function only governs equilibrium behavior when two projects are complete. Because the option value of developing a second project is bounded by $u(1)$, no agent $i$ develops a project $x_i$ with $v_i(x_i) < v_i(x_\bar{i})$ as part of an equilibrium outcome. Therefore, only projects $(x_1, x_2)$ ranging from “purely selfish” $(1, 0)$ to maximum compromise $(\bar{x}_1, \bar{x}_2)$ can be supported in equilibrium. By a similar logic, the second-best projects are not attainable with any selection function when the effective discount rate $\rho$ exceeds the threshold $\bar{\rho}$ defined in (9) above.

IV. Implementation through Voting

If agents can commit to decision-making procedures, they can implement the optimal decision rule in several ways. In the working paper, we explore a protocol wherein either agent can adopt the first complete project or permanently eliminate it, in which case he faces an exogenous deadline to develop his own. The correct deadline uniquely implements the optimal mechanism. Moreover, this procedure

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13 For instance, suppose that negotiations with two complete projects unfold as in the mixed-strategy equilibrium of a war of attrition. Then, each agent’s option value of developing the second project is equal to the value of adopting the first project (less the costs of additional effort and delay). This leads to the immediate adoption of any project and, therefore, to the development of each agent’s favorite project on the equilibrium path, i.e., $x_1 = 1$ and $x_2 = 0$. However, this outcome can already be achieved with a (Pareto-efficient) selection function ($\alpha = 1$) described above.
may be appealing in settings with a small number of relevant participants, such as university hiring committees or research joint ventures.\[^{14}\]

SSOs, in contrast, encourage broad participation, and proposals are voted on by participants who are not linked to a specific project. With this context in mind, we describe a setting in which actively participating agents (“firms”) and simply voting agents (“users”) coexist and how voting rules influence the equilibrium project choices.

Consider a continuum of users of mass one, whose types θ are distributed on [0, 1] according to some continuous and symmetric density f(θ). A user of type θ derives value \( w(\Delta(\theta, x)) \) if project x is adopted, where \( \Delta(\theta, x) \equiv |\theta - x| \) is the distance of the project from the user’s ideal project and \( w \) is a strictly decreasing and concave function. The two firms have identical preferences to those of the potential users. Their types are given by \( \theta_2 = 0 \) and \( \theta_1 = 1 \), representing the two extremes of the preference spectrum.\[^{15}\]

The timing of the game is similar to that of the baseline model: each firm chooses a project to work on; upon completion of the first project, the second firm can decide whether to “endorse” the project, ending the game with its adoption;\[^{16}\] if the second firm does not endorse, the first firm calls for a vote on whether its project should be adopted, and the voting rule requires that a qualified majority \( \gamma \geq 1/2 \) vote in favor for it to be adopted.

If the first complete project is rejected by the voters, two alternative procedures are especially relevant for our application to SSOs. In the first, the rejected project is removed from further consideration. The second firm can continue its development efforts until its project is complete, at which point a binary vote is held between the second project and the status quo (which is worth zero to all agents). In the second, the rejected project is set aside until the second project is complete, after which a binary “runoff” vote determines which alternative is adopted. Proposition 3 summarizes the key properties of the equilibrium outcome, restricting attention to the case when \( \rho \leq \bar{\rho} \).

**PROPOSITION 3 (Voting and Equilibrium Compromise):**

\(^{(i)}\) Under either voting procedure, any supermajority requirement \( \gamma \geq 1/2 \) induces a unique equilibrium outcome in which the first project developed is immediately adopted.

\(^{(ii)}\) If \( \gamma \in \left[1/2, \gamma(\rho)\right] \), each firm develops its favorite project. The threshold \( \gamma(\rho) \) is increasing in \( \rho \) with \( \gamma(0) = 1/2 \) and \( \lim_{\rho \to \infty} \gamma(\rho) = 1 \).

\[^{14}\] In the hiring example, it prescribes that the committee loses the hiring slot if a member vetoes a candidate and fails to suggest an alternative candidate in a reasonable time. Note, however, that imposing a fixed deadline for filling the slot is never optimal. This follows from Lemma 1 and differentiates our result from Bonatti and Hörner (2011) and Campbell, Ederer, and Spinnewijn (2014).

\[^{15}\] As we show in the online Appendix, the logic of our results generalizes to both interior preferences for the developing firms and to the case in which the firms are partly (or fully) profit driven, with the profits being generated through the collection of licensing fees from the users.

\[^{16}\] More generally, a binary vote is held between the newly developed project and the status quo, with the knowledge that if the second firm endorsed the first project, it also terminated its development efforts.
(iii) If $\gamma > \gamma(\rho)$, the equilibrium degree of compromise is strictly positive and weakly increasing in $\gamma$ and in $\rho$.

(iv) If the first project is removed from consideration upon a negative vote, there exists a unique $\gamma^*(\rho) \in (\gamma(\rho), 1)$ that induces the second-best projects.

(v) If the first project is set aside until a runoff vote, the highest degree of equilibrium compromise is given by the efficient-effort projects.

The supermajority requirement $\gamma$ plays a similar role to the Pareto weight $\alpha$ in Proposition 2, with some important differences. Most strikingly, simple majority and small supermajority requirements are unable to induce any compromise. The pivotal voter knows that if the first project developed is rejected, the second project will be either fully selfish (if the former is eliminated), or only incrementally better than the first from her perspective (if a runoff is held). Hence, she votes in favor of the first project developed to avoid the costs of delay. This allows firms to pursue their favorite projects.\(^{17}\) Conversely, under the ex post optimal selection function, the first project must satisfy an agent (i.e., the other firm) who is willing to wait for a competing project. This forces each agent to offer a positive degree of compromise.

Thus, while a simple majority is effective at selecting among preexisting alternatives, it performs considerably worse in inducing the development of attractive projects. Indeed, the only ways to induce compromise under majority voting are to delay decisions until both projects are complete or to use a predetermined date on which all complete projects are put to a vote.\(^{18}\)

As we increase the supermajority requirement $\gamma$ above $\gamma(\rho)$, the pivotal voter becomes increasingly aligned with the firm that does not develop the first project. She is thus more willing to bear the cost of delay and both firms must offer a positive degree of compromise to induce immediate adoption. However, for sufficiently high $\gamma$, the second firm becomes willing to halt its development efforts and endorse the first project. This may occur even if a voter who bears no cost of development prefers to wait for the second project.\(^{19}\)

The voting procedure following rejection of the first project faces a similar trade-off to the one identified in Propositions 1 and 2. If the first project is removed from further consideration, a negative vote gives the second firm free reign to develop and adopt its preferred project. The second firm is thus willing to cease its development efforts if offered the maximum-compromise project described in Section IIIB. Additionally, because the level of compromise is monotonically increasing in $\gamma$ up to the maximum-compromise project, there exists an interior supermajority requirement that induces the second-best projects.

\(^{17}\) A simple majority rule is unable to induce mutual compromise even when the distribution of user types is asymmetric: if the median voter falls in an intermediate range, both firms pursue fully selfish projects; and if the median voter’s preferences are highly skewed, only the favored firm does.

\(^{18}\) The fixed-date procedure is reminiscent of the policy contests in the models of Callander and Harstad (2015) and Hirsch and Shotts (2015). We examine this alternative procedure in the online Appendix, and we show that the outcome is less efficient than if players can vote upon completion of the first project.

\(^{19}\) The threshold $\bar{\gamma}(\rho)$ at which this occurs depends on the procedure followed after the rejection of the first project.
If the first project is not eliminated, the second firm must develop a project that is preferred by a majority of the voters to win the runoff vote. Thus, firm \( j \) develops project \( x_j = 1 - x_i \) as a competing project. Turning to the initial project choice, the equilibrium degree of compromise is increasing in \( \gamma \). However, for sufficiently high \( \gamma \), the second firm endorses the same projects as under the ex post optimal selection function. Both firms then pursue the efficient-effort projects \( x_i^E \) and the second-best projects are not attainable.

Therefore, inducing the second-best level of compromise requires both a supermajority requirement and the threat of inefficient continuation if the first project is rejected.

V. Discussion

We now discuss the robustness of our results to the main assumptions of our model. We begin with the role of quadratic effort costs, and then address other assumptions on the technology and the monitoring structure.

A. General Cost Functions

The quadratic cost function makes the analysis in Section II quite tractable. In particular, under symmetric projects, it yields the equilibrium effort levels and payoffs in closed form. This facilitates the subsequent analysis of specific selection functions and voting procedures. Throughout this subsection, we consider an arbitrary (strictly convex) cost of effort \( c(a) \), and we restrict attention to symmetric projects and to symmetric, stationary effort levels. In the online Appendix, we derive regularity conditions that guarantee existence and uniqueness of an equilibrium in such strategies. We then show that our main results do not rely on a specific cost function.

The key observation is that, under any weighted-utilitarian selection function, each agent \( i \) develops the best project that induces immediate acceptance by agent \( j \). Furthermore, the selection function alone determines the set of projects that agent \( j \) can develop as competing proposals. The cost function \( c(a) \) only affects the level of agent \( j \)’s option value, which imposes bounds on his acceptance set. Thus, while the projects developed in equilibrium depend on the cost function quantitatively, the qualitative effect of the selection function on the equilibrium outcome is independent of the cost of effort.

In particular, under the ex post optimal selection, each agent is indifferent between adopting the first project and developing its “mirror image” as a competing project. As explained in the proof of Proposition 1, this implies the first agent’s effort level imposes no externalities on the second agent, and both agents develop the efficient-effort projects regardless of the cost function. In fact, the same argument establishes that Proposition 1 holds as stated even if development costs \( c(a, x) \) depend explicitly on the project chosen.

Furthermore, increasing the degree of compromise improves equilibrium payoffs unambiguously if effort levels are (weakly) above first best. This result holds quite generally: more compromise reduces effort and increases the total value of the projects developed, both of which improve welfare. Therefore, the ex post optimal
selection function yields insufficient compromise, and the optimal rule must favor later proposals.\footnote{If the cost of effort \( c(a, x) \) depends on the project chosen, most of the results in Proposition 2 require imposing more structure on the problem. The main difficulty lies in the characterization of the second-best projects, which now depend on the specific cost function.}

The comparative statics result for the optimal weights \( \alpha^*(\rho) \) in Proposition 2 relies on closed-form expressions and on quadratic costs. While a more general analytical result seems out of reach, numerical examples suggest that the optimal degree of favoritism must increase as the agents become more impatient for a broader class of cost functions. In the online Appendix, we illustrate the case of power cost functions, \( c(a) = a^b/b \), where \( b > 1 \).

The main results in our voting model (Proposition 3) can also be generalized quite easily. In particular, the impossibility of compromise under majority voting is driven only by the similarity in the preferences of the pivotal voter in each round. More generally, supermajority requirements create option value and induce equilibrium compromise to the extent that they drive a wedge in the preferences of the pivotal voter across the voting rounds. Finally, the option value of blocking the first project is lower in the shadow of a runoff vote than when the first project can be eliminated. The runoff vote thus reduces the scope for equilibrium compromise independently of the specific cost function.

\section*{B. Disclosure, Switching Costs, Learning by Doing}

Even though we have shown that the optimal selection function favors later proposals, the observability assumption for complete projects does not affect our main results. In particular, the online Appendix establishes that the equilibrium under the optimal weight \( \alpha^*(\rho) \) is robust to allowing agents to verifiably but voluntarily disclose project completion. Thus, agents cannot exploit the degree of favoritism built into the optimal selection function by developing a more extreme project and then concealing it until the other agent has completed his project.

The model can also be extended to accommodate a fixed cost for switching projects. Switching costs limit the retaliatory ability of the second agent and reduce the option value of developing the second project. For sufficiently high costs, the second agent does not modify his project choice off the equilibrium path. In this case, the highest degree of compromise supported in equilibrium is given by the efficient-effort projects \( x_i^E \).

Finally, the logic of option values extends to nonstationary environments. For example, under learning by doing, the option value of the agent without a project increases over time, and the optimal weight \( \alpha^*_t(\rho) \) consequently increases. Conversely, if agents face a fixed deadline for adopting a project, the optimal weight \( \alpha^*_t(\rho) \) decreases over time.

\section*{VI. Standard-Setting Organizations}

The trade-offs described in our model parallel closely the challenges faced by SSOs. A fundamental problem in standards-setting is the public good nature of a
technological standard. Crafting proposed standards and participating in SSO processes can be quite costly, creating incentives to free ride.\textsuperscript{21} To counter free-riding, SSOs rely on participating firms’ vested interests. Indeed, the value of a standard to a firm depends on its provisions: which patents it includes, what the licensing conditions are, and how compatible it is with each firm’s systems, to name a few. At the same time, however, solutions that disproportionately favor one firm are typically less effective (from a utilitarian perspective) than more integrated compromise solutions.\textsuperscript{22} Thus, compromise is efficient and desired by the SSO, but it may be challenging to achieve if participants have conflicting interests.

A. SSO Processes

SSOs vary significantly in their size, focus, and rules for participation and voting. The basic features of the process itself are, however, fairly common. First, a need for a standard is identified. Second, the relevant SSO forms a working group composed of member firm and organization representatives. The working group then reviews the existing technology and develops new solutions “with individual members often proposing specific alternatives based on their firm’s proprietary intellectual property” Layne-Farrar and Padilla (2011). Finally, the members vote on the proposed options per the rules of the particular SSO.

Most working groups use Robert’s Rules of Order (Robert 2008) to govern their procedures, with special provisions added according to the circumstances. The votes on proposals are based on motions and are therefore taken sequentially. After a proposal has been discussed, there is a motion to put that proposal to a vote. If the motion passes (generally requiring a vote in itself), a vote for endorsement takes place. Nearly every SSO requires a supermajority to qualify for “consensus” and, thus, for endorsement of the standard.\textsuperscript{23} Proposals that fail to meet the supermajority requirement after further debate are supposed to be removed from further consideration. The process used by SSOs is thus qualitatively similar to the second-best voting mechanism discussed in Proposition 3.

While the basic process and the related challenges are relatively homogeneous, the SSOs differ in the more specific rules. These differences lie in the requirements that must be met for the acceptance of a standard, in their appeals and arbitration procedures, in the allocation of voting rights, and in how deadlocks and proposals that have failed to meet the supermajority requirement are treated.\textsuperscript{24} All of these

\textsuperscript{21}Formulating a proposal may require research and development of a new technology or combining multiple existing technologies (not all of which are proprietary) into a well-functioning solution. The administrative costs of participating in the standardization process can also be substantial. Weiss and Toyofuku (1996) study free-rider problems in the development of the 10BaseT Ethernet standard in the IEEE, uncovering a large number of noncontributing members.

\textsuperscript{22}Consistent with this view, Lehr (1992) notes that “a firm may find it profitable to promote a standard which promises to increase its market share even if total surplus declines. Those who benefit from standards do not always bear the full costs of adoption. For example, new component manufacturers who may benefit from lower entry costs may fail to share the switching costs faced by incumbent firms and their customers.”

\textsuperscript{23}The American National Standards Institute (ANSI) defines consensus as “general agreement, but not necessarily unanimity, [with] a process for attempting to resolve objections by interested parties.”

\textsuperscript{24}The data in Baron and Spulber (2015) provide an overview of some of the above dimensions. Of the 36 SSOs they surveyed, 11 operate under majority and 10 under unanimity rules; supermajority requirements range from 66 percent to 75 percent, including or excluding abstentions. All SSOs attempt to induce consensus by requiring negative votes to be accompanied by detailed motivations. Finally, some SSOs have specific rules for participation that
dimensions influence which projects are proposed and eventually endorsed by the SSO. In Table 1, we summarize the approval requirements and tie-breaking procedures for each SSO we mention in the paper.\textsuperscript{25}

B. SSO Experiences

The actual experiences of SSOs reflect the main themes of our model. First, the option value of blocking a project determines the required degree of compromise in the proposed standards. Second, SSOs face a trade-off between ex post optimal project selection and ex ante incentives for compromise. Below, we articulate these themes further.

**Remark 1:** Supermajority requirements are able to induce substantive compromise when competing parties do not yet have working and ready-to-market solutions. Conversely, when a technology is mature, compromise becomes more difficult.

When a technology is new and the cost of developing a project is high, a sufficiently balanced proposal can receive support from other members and lead to a relatively rapid adoption of a standard. The case of Local Area Networks (LANs) provides a good example. The IEEE began to standardize LANs in 1980. Xerox offered an open networking standard to convince computer manufacturers to adopt the Ethernet interface for their printers. The support of 3Com, Digital, and Intel convinced IEEE working group 802 to adopt Ethernet as an open standard in 1982 (Shapiro and Varian 1999). Similarly, Lucent and Intersil proposed a compromise standard for first-generation wireless networks (WLANs) in 1999, which was quickly adopted by the IEEE as standard 802.11b. In both examples, the initiator offered enough compromise to gain sufficient support and preempt the development of alternatives.\textsuperscript{26}

limit the number of representatives (or votes) from any given firm or interest group. See Yates and Murphy (2015) for a discussion of the balance requirement in standards bodies.

\textsuperscript{25} To the best of our knowledge, the only studies to systematically examine variation across SSO rules are Chiao, Lerner, and Tirole (2007) and Baron and Spulber (2015). For more details on the adoption processes, including quorum requirements and appeals, see Table 4 in Baron and Spulber (2015).

\textsuperscript{26} A supermajority requirement is also able to block single proposals that do not offer sufficient compromise. A recent example is provided by the W3C’s Tracking Protection Group. The only proposal for a “Do Not Track” standard that has been voted on so far was presented by the Digital Advertising Alliance (an industry association). The proposal was mostly silent on data collection, regulating only its use for targeted advertising. Due to the opposition of consumer protection groups, the proposal was voted down in 2013, forcing the group to look for a new solution.
Once the WLAN technology matured and firms developed advanced proprietary solutions, such preemption became more difficult, and negotiations over multiple standard proposals became a reality. In 2001, the negotiations over the 802.11g standard witnessed a standoff between two proposals, neither of which was able to gain the 75 percent majority required for acceptance. The resulting stalemate led to the de facto adoption of multiple standards.27

Indeed, the risk of deadlocks in the future lowers the option value of blocking initial proposals, and that can allow parties to exploit the fear of deadlock to pursue more selfish projects. For instance, during the negotiations for the 802.11n standard in 2006, the process stalled again. In this case, DeLacey et al. (2006) describe how “a group of influential semiconductor companies formed a third group, taking on the name Enhanced Wireless Consortium (EWC) […] they banded together and began promoting their own specifications for the standard, working outside IEEE approval.” Given this pressure, the IEEE adopted the EWC’s draft specification in 2007 with very few concessions.28

Remarking 2: In practice, devising rules for breaking deadlocks is an important aspect of SSO procedures. These rules must balance the benefits of a supermajority requirement in inducing ex ante compromise and the risks of ex post impasse and delay.

Delays in calling a vote, private information, and other exogenous elements can cause multiple projects to be completed prior to any vote. As seen above, in such cases a supermajority requirement can lead to a deadlock, the risk of which can reduce the equilibrium degree of compromise. Intuitive solutions to break a deadlock consist of turning to an impartial mediator, or having a runoff vote between any remaining candidates to determine the final choice. For example, in the IETF, the Working Group chair is charged with establishing whether “rough consensus” has been achieved, and in the W3C, the chair has the power to call a majority vote to break the deadlock.29

An alternative is to introduce more elaborate voting rules for resolving deadlock. These rules are often the objects of contentious negotiations prior to commencing work on a given standard. For instance, during the negotiations over the IEEE 802.11g standard, the chair formulated detailed selection criteria, including a “down vote” to eliminate multiple proposals: “Rounds of voting will be held that successively eliminate one candidate proposal at a time. On each round of voting, the candidate proposal that receives the least number of votes shall be eliminated from consideration.”

The key tension faced by the selection criteria is that (following Proposition 3) ex post optimal selection, such as achieved through a runoff vote or mediator, may

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27 DeLacey et al. (2006, p. 12) note that “As the 802.11g standard was being developed, several companies began producing pre-11g devices that did not interoperate effectively.”

28 Layne-Farrar and Padilla (2011, p. 23) describe how “this outside group, a sort of hybrid de facto/cooperative alliance, forced IEEE’s hand and a consensus standard that combined the breakout group’s proposal with elements of the proposal that had bogged down in IEEE committee finally emerged through the SSO.”

29 Further, since the IETF is somewhat unique in that it has no formal votes, determining rough consensus almost always falls on the Working Group chair.
induce insufficient compromise. Instead, achieving sufficient compromise can require even good proposals to be eliminated from consideration following a failure to garner sufficient support. The requirement of such ex post value burning leads us to our final remark.

**Remark 3:** All selection criteria face the challenge of credibly and permanently removing failed projects from consideration.

Proposed standards that are voted down are, in fact, often eliminated from consideration, sometimes even leading the SSO to disband a working group. Nevertheless, eliminating a proposal in the absence of a clearly superior alternative is difficult for several reasons.

First, the actual implementation of any selection procedure is often contentious. Indeed, the deadlock in the 802.11g negotiations essentially resulted from the interpretation of the down vote, whereby the chair insisted on the elimination of any proposal that failed to meet the 75 percent requirement (leading to rejection of all proposals), while Intersil (the majority proposal) argued that the last vote should be between the last remaining proposal and “doing nothing” (Liu 2001). Indeed, the latter solution is optimal ex post, but the approach of the chair could be necessary for inducing sufficient compromise in the project choices themselves.

Second not approving a standard is rarely a realistic outcome: upon prolonged disagreement, earlier proposals might be brought back to the table. Lehr (1996) describes the case the ANSI-accredited X3 committee (now INCITS), where a simple majority was required to discuss and vote on a working document. In 1987, IBM was able to delay the proceedings on the Fiber Distributed Data Interface (FDDI) by repeatedly submitting alternative proposals that contained minor design differences from the leading project.

Third, a better technology can be reintroduced by other means, whether in another SSO (“forum shopping”) or into the market directly. For example, as the process for 802.11n stalled, Eisenmann and Barley (2006) suggest that the trade association Wi-Fi Alliance emerged as a competing de facto standards body, marketing partial solutions such as 802.11i and 802.11e, prior to the approval of an IEEE standard. Again, the ability to circumvent the supermajority requirement is valuable to limit ex post inefficiencies in the implementation of a standard, but inhibits compromise in the standards pursued in the first place.

**C. Discussion**

We have explicitly focused on a particular aspect of SSOs, namely the effect of the technological environment and the rules and bylaws on the types of proposal brought fore by the participants. There are, of course, several aspects of SSOs that

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30 This was the case, for example, of the IEEE’s group for Ultra Wide Band standards in 2006.

31 The committee eventually approved the leading proposal and modified its rules by introducing a two-thirds supermajority requirement to reopen a discussion (Architecture Technology Corporation 1991).

32 Similarly, in June 2004, a web standards proposal by Mozilla and Opera was turned down by the W3C. They subsequently formed the Web Hypertext Application Technology Working Group (WHATWG). In 2007, they convinced the W3C to adopt WHATWG’s Web Forms 2.0 as part of the HTML standard.
our model does not capture. Even if projects are not contractible, ex post payments are realistic in SSOs for at least two reasons. Quite literally, the payoff from any technological proposal can be amended by bargaining over the licensing terms. More importantly, many SSO members interact repeatedly in the same industry. Thus, logrolling multiple standards negotiations effectively forms a repeated-game (relational) transfer. In the working paper, we examine the possibility of agents offering payments in exchange for support for their projects.

We also adopt a reduced-form approach toward many of the institutional details that address ex post opportunism. These include disclosure requirements for standard-essential patents as well as the extent and nature of licensing commitments regarding the intellectual property incorporated into the standard. We do not compare the choice of technology in market-based versus committee-based standardization, which is the focus of Farrell and Saloner (1988) and Llanes and Poblete (2015). Finally, we abstract from competition among standards bodies and the related issue of “forum shopping” Lerner and Tirole (2006).

VII. Conclusions

We have analyzed a collective decision-making problem in which members of an organization develop projects and negotiate over adoption decisions. When the agents have conflicting preferences over the outcomes, a key trade-off emerges between the total value of the projects they pursue and the incentives to exert effort toward their development. The agents’ expectations over future negotiations influence the specific projects pursued. An agreement to adopt the more socially efficient project induces insufficient compromise. The second-best combination of compromise and effort levels can be achieved in equilibrium, provided that the selection criterion favors later projects, or that the organization adopts supermajority voting rules and discards failed proposals.

At a broader level, our paper relates to the organization of research and development efforts. An intuitive approach suggests letting “a thousand flowers bloom” prior to adopting a project. In contrast, we have shown that when project choice is endogenous, a dynamic model of decision-making can yield an ex ante efficient outcome by utilizing the preemption motive, even in the absence of any costs of discerning among completed projects. However, our model is quite stylized. Introducing a stochastic element to project quality leads to sequential sampling, which makes adopting the first complete project unlikely to be optimal. Similarly, additional information is often learned during the development process, creating benefits to collecting multiple projects before making a final selection. The benefits of dynamic competition are, however, likely to remain even in models with a richer structure, and examining the optimal termination of alternative projects in such richer environments is a promising avenue of further research.

33 Ganglmair and Tarantino (2014) study the incentives to conceal a standard-essential patent from the SSO to subsequently hold up other members.

34 Some consortia (e.g., W3C) require ex ante commitment to royalty-free licensing, as opposed to the more common and vague requirement of ex post “reasonable and non-discriminatory” licensing terms. See Lemley (2002) and Layne-Farrar and Padilla (2011) for details on intellectual property in SSOs.
Appendix

Proof of Proposition 2:
We begin by establishing some properties of the second-best projects \((x_1^*, x_2^*)\).
Under symmetric effort levels, each agent’s equilibrium payoff is given by (up to a multiplicative constant)

\[
V(a(v), v) \propto 2v + \phi(v) + \rho - \sqrt{(v - \phi(v) - \rho)^2 + 6\rho v}.
\]

We then maximize (10) with respect to \(v\). The first-order condition yields the inverse function \(\rho^*(v)\) that characterizes the second-best projects:

\[
\rho^*(v) = \frac{1 + 2\phi'(v)}{2(2 + \phi'(v))} \frac{(v - \phi(v))^2}{v + \phi(v) + \nu\phi(v)}.
\]

Note that (11) implies \(v^*(0) = \phi(v^*(0))\), i.e., \(x_i^*(0) = 1/2\). Therefore, \(\phi'(v^*(\rho)) \to -1\) as \(\rho \to 0\). For small enough \(\rho\), we then have \(\phi'(v^*(\rho)) > -2\) and \(v + \phi(v) + \nu\phi'(v) > 0\). As \(v\) increases, both terms in the numerator increase in absolute value, and both terms in the denominator decrease (because \(\phi'(v) < -1\)). Therefore, \(\rho^*(v)\) is strictly increasing in \(v\). Hence, the value of the second-best projects \(v^*(\rho)\) is given by the unique solution to the equation \(\rho^E(v) = \rho\).
Moreover, \(\rho^*(v)\) grows without bound as \(v\) approaches the root of \(v + \phi(v) + \nu\phi'(v)\), which is itself bounded away from 1. Thus, \(v^*(\rho)\) satisfies \(2 + \phi'(v^*) > v^* + \phi(v^*) + \nu\phi'(v^*) > 0\) for all \(\rho \geq 0\).

We now consider the maximum-compromise projects \((\bar{x}_1, \bar{x}_2)\). Using the definition of \(u(w)\), these projects satisfy

\[
\phi(\bar{v}) = 1 + \rho - \sqrt{\rho^2 + 2\rho}.
\]

Solving (12) for \(\rho\) we obtain the inverse function

\[
\bar{\rho}(v) = \frac{(1 - \phi(v))^2}{2\phi(v)}.
\]

Both \(\rho^*(v)\) and \(\bar{\rho}(v)\) are strictly increasing in \(v\), where \(v^*(0) > \bar{v}(0) = 0\). Below, we establish that the ratio \(\rho^*(v)/\bar{\rho}(v)\) is increasing and, hence, that \(\bar{\rho}(v)\) crosses \(\rho^*(v)\) only once (from above). Therefore, we have \(v^*(\rho) > \bar{v}(\rho)\) for \(\rho \in [0, \bar{\rho}]\). The threshold \(\bar{\rho}\) is the unique level for which \(\rho^*(v) = \bar{\rho}(v)\). Inversely, for \(\rho > \bar{\rho}\), the maximum-compromise projects are more polarized than the second-best projects, i.e., \(v^*(\rho) < \bar{v}(\rho)\).

The ratio \(\rho^*(v)/\bar{\rho}(v)\) can be written as

\[
\frac{\rho^*(v)}{\bar{\rho}(v)} = \frac{1 + 2\phi'(v)}{2 + \phi'(v)} \frac{\phi(v)}{v + \phi(v) + \nu\phi'(v)} \frac{(v - \phi(v))^2}{(1 - \phi(v))^2}.
\]
Because the last term is strictly increasing in \(v\), denote the product of the first two terms as

\[
(14) \quad z(v) := \frac{1 + 2\phi'(v)}{2 + \phi''(v)} \frac{\phi(v)}{v + \phi(v) + v\phi'(v)}.
\]

We show that \(z\) is strictly increasing. Indeed, we have

\[
z'(v) = \frac{(\phi'(v) + 1)(2\phi'(v) + 1)(\phi(v) - v\phi'(v))}{(\phi'(v) + 2)(v\phi'(v) + \phi(v) + v)^2} - \frac{\phi(v)(3\phi(v) + v - 2(\phi'(v) + 1)\phi'(v)\phi''(v))}{(\phi'(v) + 2)^2(v\phi'(v) + \phi(v) + v)^2}.
\]

Because \(v + \phi(v) + v\phi'(v) > 0\), we know that \(v + 3\phi(v) - 2v\phi'(v)(\phi'(v) + 1) > 0\) and, therefore, \(z'(v) > 0\). This implies that the ratio \(\rho^*(v)/\bar{p}(v)\) is strictly increasing.

(i) Consider the selection function \(\xi(x, \tau, \alpha)\). Let \(x_1\) be the first project developed, where \(v_1(x_1) = v\). Agent 2 can adopt \(x_1\) or develop a competing project worth \(w\) such that

\[
(15) \quad (1 - \alpha)w + \alpha\phi(w) \geq (1 - \alpha)\phi(v) + \alpha v.
\]

Let \(w(v, \alpha) = \min\{1, \hat{w}\}\), where \(\hat{w}\) denotes the largest solution to (15) holding with equality. The value to agent 2 of developing the second project is given by \(u(w(v, \alpha))\). Therefore, agent 2 adopts project \(x_1\) if and only if \(\phi(v) \geq u(w(v, \alpha))\).

On the equilibrium path, \(w(v, \alpha)\) is increasing in \(v\). First, the right-hand side of (15) is decreasing in \(v\): suppose that agent 1 chooses a project \(v\) for which \((1 - \alpha)\phi(v) + \alpha v\) is increasing; agent 2 would adopt it because \(w(v, \alpha) = \phi(v)\); agent 1 could then increase his payoff \(v\) and still induce agent 2 to adopt his project. Second, the left-hand side of (15) is decreasing in \(w\) because agent 2’s choice of second project maximizes his payoff subject to (15). Therefore, agent 2 adopts agent 1’s project if \(v\) is sufficiently low. Agent 1’s payoff is then increasing in \(v\) as long as agent 2 adopts it, but decreasing in \(v\) if he does not. Therefore, the constraint (15) binds on the equilibrium path, and the first project developed is adopted.

(ii) Each agent pursues a project \(x_i(\alpha, \rho)\) with value \(v_i(x_i(\alpha, \rho)) = v\) such that

\[
(16) \quad \phi(v) = u(w(v, \alpha), \rho).
\]

We now show that \(v_i(x_i(\alpha, \rho))\) is increasing in \(\alpha\). Suppose instead that \(\alpha\) increases and \(v_i(x_i(\alpha, \rho))\) decreases. Because \(v_i(x_i(\alpha, \rho))\) satisfies (16) and (15) holds with equality, this means that \(w(v, \alpha)\) must decrease, but then, the
left-hand side of (16) increases while the right-hand side (which depends on \( \alpha \) only through \( w \)) decreases, yielding a contradiction. A similar argument establishes that \( v_i(x_i(\alpha, \rho)) \) is increasing in \( \rho \).

(iii) The threshold \( \bar{\alpha}(\rho) \) satisfies the condition

\[
1 - \alpha = (1 - \alpha) \phi(\bar{v}) + \alpha \bar{v}.
\]

Because the frontier \( \phi(v) \) is symmetric, solving for \( \alpha \) yields

\[
\bar{\alpha}(\rho) = \frac{\sqrt{\rho^2 + 2\rho - \rho}}{\sqrt{\rho^2 + 2\rho - \rho + \phi(1 + \rho - \sqrt{\rho^2 + 2\rho})}},
\]

which is increasing in \( \rho \). Furthermore, if we let \( x := 1 + \rho - \sqrt{\rho^2 + 2\rho} \), we obtain

\[
\bar{\alpha}(x) = \frac{1 - x}{1 - x + \phi(x)} \leq \frac{1}{2}.
\]

(iv) Let \( v^*(\rho) \) denote the value of the second-best project \( v_i(x_i^*(\rho)) \). The optimal weight \( \alpha^*(\rho) \) satisfies the following equation

\[
(1 - \alpha) w(v^*(\rho), \alpha) + \alpha \phi(w(v^*(\rho), \alpha)) = (1 - \alpha) \phi(v^*(\rho)) + \alpha v^*(\rho),
\]

where

\[
w(v^*(\rho), \alpha) + \rho - \sqrt{\rho^2 + 2\rho w(v^*(\rho), \alpha)} = \varphi(v^*(\rho)).
\]

We then consider the two functions

\[
\rho(\alpha, v) = \frac{(w(v, \alpha) - \phi(v))^2}{2\phi(v)}
\]

and

\[
\rho^*(v) = \frac{(w^*(v) - \phi(v))^2}{2\phi(v)},
\]

We know from the proof of Proposition 1 that \( v_i(x_i(1/2, \rho)) = v^E(\rho) \) and from the previous argument that \( v^E(\rho) \geq v^*(\rho) \geq \bar{v}(\rho) \) for all \( \rho \in [0, \bar{\rho}] \). Finally, because we know from part (ii) that \( v_i(x_i(\alpha, \rho)) \) is increasing in \( \alpha \), we conclude that \( \alpha^*(\rho) \in [\bar{\alpha}(\rho), 1/2] \). It then follows by construction that \( \alpha^*(\bar{\rho}) = \alpha(\bar{\rho}) \).

To show that the optimal weight \( \alpha^*(\rho) \) is decreasing in \( \rho \), we rewrite the two conditions for the optimal weight \( \alpha^*(\rho) \) as

\[
(1 - \alpha) w(v) + \alpha \phi(w(v)) = (1 - \alpha) \phi(v) + \alpha v,
\]

\[
w^*(v) = \phi(v) + \sqrt{2\rho^*(v) \phi(v)}.
\]

We then consider the two functions

\[
\rho(\alpha, v) = \frac{(w(v, \alpha) - \phi(v))^2}{2\phi(v)}
\]

and

\[
\rho^*(v) = \frac{(w^*(v) - \phi(v))^2}{2\phi(v)},
\]
where $w(v, \alpha)$ is given in (15) and $w^*(v)$ in (17). Because $w(v, \alpha)$ is decreasing in $\alpha$, so is $\rho(\alpha, v)$. Thus, both $\rho^*(v)$ and $\rho(v, \alpha)$ are increasing in $v$. We now show that $\rho^*(v)$ crosses $\rho(\alpha, v)$ only from below, i.e., if $v$ satisfies $\rho^*(v) = \rho(v, \alpha)$, then it must hold that

$$\frac{d w^*(v)}{dv} > \frac{\partial w(v, \alpha)}{\partial v}. \tag{18}$$

To show this, totally differentiate condition (15), and let $\beta := \alpha / (1 - \alpha)$. We obtain

$$\frac{\partial w(v, \alpha)}{\partial v} = \frac{\phi'(v) + \beta}{\beta \phi'(w) + 1}.$$

Notice that as $\rho \to 0$, $w(v) \to \phi(v)$ and $\phi'(v) \to -\beta$. Because the frontier $\phi$ is symmetric, it holds that $\phi'(\phi(v)) = 1 / \phi'(v)$ and, therefore,

$$\lim_{\phi'(v) \to -\beta} \frac{\partial w(v, \alpha)}{\partial v} = -\phi'(v).$$

Because $w(v) \geq \phi(v)$, we can bound the right-hand side of (18) as

$$\frac{\partial w(v, \alpha)}{\partial v} \leq -\phi'(v).$$

Now consider the value $w^*(v)$; substitute the definition of $\rho^*(v)$ from (11) into (17); and obtain

$$\frac{d w^*(v)}{dv} = \phi'(v) + (1 - \phi'(v)) \sqrt{z(v)} + (v - \phi(v)) \frac{z'(v)}{2 \sqrt{z(v)}},$$

where $z(v)$ is defined in (14). Because $z$ is strictly increasing, it suffices to show that

$$2\phi'(v) + (1 - \phi'(v)) \sqrt{z(v)} \geq 0. \tag{19}$$

Recall the definition of $z(v)$ in (14); define $c := \phi(v) / v$ and $x := \phi'(v) \in [-1 - c, -1]$; and consider the function

$$C(x) := 2x + (1 - x) \sqrt{\frac{1 + 2x}{2 + x} \frac{c}{1 + c + x}}.$$  

Because $C(-1) = 0$ and $C'(x) < 0$ if $C(x) = 0$, we have $C(x) \geq 0$, which implies that the left-hand side of (19) is strictly positive, and that (18) is satisfied. □
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