First observation of the decay \( B_{c}^{+} \) to \( J/K^{+} \).
First observation of the decay $B_c^+ \rightarrow J/\psi K^+$

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Abstract: The decay $B_c^+ \rightarrow J/\psi K^+$ is observed for the first time using a data sample, corresponding to an integrated luminosity of 1.0 fb$^{-1}$, collected by the LHCb experiment in $pp$ collisions at a centre-of-mass energy of 7 TeV. A yield of 46 ± 12 events is reported, with a significance of 5.0 standard deviations. The ratio of the branching fraction of $B_c^+ \rightarrow J/\psi K^+$ to that of $B_c^+ \rightarrow J/\psi \pi^+$ is measured to be 0.069 ± 0.019 ± 0.005, where the first uncertainty is statistical and the second is systematic.

Keywords: Hadron-Hadron Scattering, Branching fraction, B physics

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The $B_c^+$ meson is composed of two heavy valence quarks, and has a wide range of expected decay modes [1–10]. Prior to LHCb taking data, only a few decay channels, such as $B_c^+ \rightarrow J/\psi \pi^+$ and $B_c^+ \rightarrow J/\psi \mu^+\nu$ had been observed [11, 12]. For $pp$ collisions at a centre-of-mass energy of 7 TeV, the total $B_c^+$ production cross-section is predicted to be about 0.4 μb, one order of magnitude higher than that at the Tevatron [13, 14]. LHCb has thus been able to observe new decay modes, such as $B_c^+ \rightarrow J/\psi \pi^+\pi^-\pi^+$ [15], $B_c^+ \rightarrow \psi(2S)\pi^+$ [16] and $B_c^+ \rightarrow J/\psi D_s^{(*)+}$ [17], and to measure precisely the mass of the $B_c^+$ meson [18].

In this paper, we report the first observation of the decay channel $B_c^+ \rightarrow J/\psi K^+$ (inclusion of charge conjugate modes is implied throughout the paper). The $J/\psi$ meson is reconstructed in the dimuon final state. The branching fraction is measured relative to that of the $B_c^+ \rightarrow J/\psi \pi^+$ decay mode, which has identical topology and similar kinematic properties, as shown in figure 1. No absolute branching fraction of the $B_c^+$ meson is known to date. The predicted ratio of branching fractions $\mathcal{B}(B_c^+ \rightarrow J/\psi K^+)/\mathcal{B}(B_c^+ \rightarrow J/\psi \pi^+)$ is dominated by the ratio of the relevant Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{ud}/V_{us}|^2 \approx 0.05$ [19]. However, after including the decay constants, $f_{K^+/(\pi^+)}$, the ratio is enhanced,

$$\frac{\mathcal{B}(B_c^+ \rightarrow J/\psi K^+)}{\mathcal{B}(B_c^+ \rightarrow J/\psi \pi^+)} \approx \left| \frac{V_{us}f_{K^+}}{V_{ud}f_{\pi^+}} \right|^2 = 0.077,$$

(1)

where the values of $f_{K^+/(\pi^+)}$ are given in ref. [19]. Taking into account the contributions of the $B_c^+$ form factor and the kinematics, the theoretical predictions for the ratio of branching fractions lie in the range from 0.054 to 0.088 [2, 3, 5–7, 9, 10]. The large span of these predictions is due to the various models and the uncertainties on the phenomenological parameters. The measurement of $\mathcal{B}(B_c^+ \rightarrow J/\psi K^+)/\mathcal{B}(B_c^+ \rightarrow J/\psi \pi^+)$ therefore provides a test of the theoretical predictions of hadronisation.

The analysis is based on a data sample, corresponding to an integrated luminosity of 1.0 fb$^{-1}$ of $pp$ collisions, collected by the LHCb experiment at a centre-of-mass energy of 7 TeV. The LHCb detector [20] is a single-arm, forward spectrometer covering the pseudorapidity range $2 < \eta < 5$ and is designed for precise measurements in the $b$ and $c$ quark sectors. The detector includes a high precision tracking system consisting of a silicon-strip vertex detector surrounding the $pp$ interaction region, a large area silicon-strip detector located upstream of a dipole magnet with a bending power of about 4 Tm, and three stations of silicon-strip detectors and straw drift tubes placed downstream. The combined tracking system has momentum resolution $\Delta p/p$ that varies from 0.4% at 5 GeV/c to 0.6% at 100 GeV/c, and impact parameter (IP) resolution of 20 μm for tracks with high transverse momentum ($p_T$). Charged hadrons are identified using two ring-imaging Cherenkov (RICH) detectors and good kaon-pion separation is achieved for tracks with momentum between 5 GeV/c and 100 GeV/c [21]. Photon, electron and hadron candidates are identified by a calorimeter system consisting of scintillating-pad and preshower detectors, an electromagnetic calorimeter and a hadronic calorimeter. Muons are identified by a system composed of alternating layers of iron and multiwire proportional chambers. The trigger system [22] consists of a hardware stage, based on information from the calorimeter and
muon systems, followed by a two-stage software trigger that applies event reconstruction and reduces the event rate from 1 MHz to around 3 kHz.

In the hardware trigger, events are selected by requiring a single muon with $p_T > 1.48 \text{ GeV}/c$ or a dimuon candidate with the product of their $p_T$ larger than $1.68 \text{ (GeV}/c)^2$. In the first stage of the software trigger, events are selected by requiring either a single muon with $p_T > 1 \text{ GeV}/c$ and $p > 8 \text{ GeV}/c$, or a dimuon candidate with invariant mass larger than $2.7 \text{ GeV}/c^2$, constructed from two muons with $p_T > 0.5 \text{ GeV}/c$ and $p > 6 \text{ GeV}/c$.

In the second stage of the software trigger, dimuon candidates are selected with invariant mass within $120 \text{ MeV}/c^2$ of the known $J/\psi$ mass [19] and with decay length significance greater than 3 with respect to the associated primary vertex (PV). For events with several PVs, the one with the smallest $\chi^2_{\text{IP}}$ is chosen, where $\chi^2_{\text{IP}}$ is defined as the difference in $\chi^2$ of a given PV reconstructed with and without the considered particle.

For the offline selection, the bachelor hadrons ($K^+$ for $B_c^+ \rightarrow J/\psi K^+$ and $\pi^+$ for $B_c^+ \rightarrow J/\psi \pi^+$ decays) are required to be separated from the $B_c^+$ PV and have $p_T > 0.5 \text{ GeV}/c$.

The $B_c^+$ candidates are required to have good vertex quality with vertex fit $\chi^2_{\text{Vtx}}$ per degree of freedom less than 5, and mass within $500 \text{ MeV}/c^2$ of the world average value of the $B_c^+$ mass [19].

A boosted decision tree (BDT) [23] is used for the final event selection. The BDT is trained using a simulated $B_c^+ \rightarrow J/\psi \pi^+$ sample as a proxy for signal and the high-mass sideband ($m_{J/\psi \pi^+} > 6650 \text{ MeV}/c^2$) in data for background. The BDT cut value is optimised to maximise the expected $B_c^+ \rightarrow J/\psi K^+$ signal significance. In the simulation, $pp$ collisions are generated using PYTHIA 6.4 [24] with a specific LHCb configuration [25]. The $B_c^+$ meson production is simulated with the dedicated generator BCVEGPY [26]. Decays of hadronic particles are described by EVTGEN [27], in which final state radiation is generated using PHOTOS [28]. The interaction of the generated particles with the detector and its response are implemented using the GEANT4 toolkit [29, 30] as described in ref. [31].

The BDT takes the following variables into account: the $\chi^2_{\text{IP}}$ of the bachelor hadron and $B_c^+$ mesons with respect to the PV; the $B_c^+$ vertex quality; the distance between the $B_c^+$ decay vertex and the PV; the $p_T$ of the $B_c^+$ candidate; the $\chi^2$ from the $B_c^+$ decay vertex refit [32], obtained with a constraint on the PV and the reconstructed $J/\psi$ mass; and the cosine of the angle between the momentum of the $B_c^+$ meson and the direction vector from the PV to the $B_c^+$ decay vertex. These variables are chosen as they discriminate the signal from the background, and have similar distributions for $B_c^+ \rightarrow J/\psi K^+$ and $B_c^+ \rightarrow J/\psi \pi^+$ decays,

Figure 1. Diagram for a $B_c^+ \rightarrow J/\psi \pi^+(K^+)$ decay.
ensuring that the systematic uncertainty due to the relative selection efficiency is minimal. After the BDT selection, no event with multiple candidates remains.

The branching fraction ratio is computed as

$$\frac{B(B_c^+ \to J/\psi K^+)}{B(B_c^+ \to J/\psi \pi^+)} = \frac{N(B_c^+ \to J/\psi K^+)}{N(B_c^+ \to J/\psi \pi^+)} \cdot \frac{\epsilon(B_c^+ \to J/\psi K^+)}{\epsilon(B_c^+ \to J/\psi \pi^+)} \cdot \epsilon(B_c^+ \to J/\psi \pi^+)$$

(2)

where $N$ is the signal yield of $B_c^+ \to J/\psi K^+$ or $B_c^+ \to J/\psi \pi^+$ decays and $\epsilon$ is the total efficiency, which takes into account the geometrical acceptance, detection, reconstruction, selection and trigger effects.

An unbinned maximum likelihood fit is used to determine the yields from the $J/\psi K^+$ mass distribution of the $B_c^+$ candidates, under the kaon mass hypothesis. The total probability density function for the fit has four components: signals for $B_c^+ \to J/\psi K^+$ and $B_c^+ \to J/\psi \pi^+$ decays; the combinatorial background; and the partially reconstructed background.

To discriminate between pion and kaon bachelor tracks, the quantity

$$\text{DLL}_{K\pi} = \ln \mathcal{L}(K) - \ln \mathcal{L}(\pi)$$

(3)

is used, where $\mathcal{L}(K)$ and $\mathcal{L}(\pi)$ are the likelihood values provided by the RICH system under the kaon and pion hypotheses, respectively. Since the momentum spectra of the bachelor pions and kaons are correlated with the DLL hypotheses, respectively. Since the momentum spectra of the bachelor pions and kaons are correlated with the DLL, the shapes of the mass distribution used in the fit vary as a function of DLL$_{K\pi}$. To reduce this dependence and separate the two signals, the DLL$_{K\pi}$ range is divided into four bins, DLL$_{K\pi} < -5$, $-5 < \text{DLL}_{K\pi} < 0$, $0 < \text{DLL}_{K\pi} < 5$ and DLL$_{K\pi} > 5$. The ratio of the total signal yields is defined as

$$R_{K^+/\pi} = \sum_{i=1}^{4} N_{i}^{j/\psi K^+} / \sum_{i=1}^{4} N_{i}^{j/\psi \pi^+},$$

where $N_{i}^{j/\psi K^+/(\pi^+)}$ is the signal yield in each DLL$_{K\pi}$ bin $i$. Due to the limited sample size of the $B_c^+ \to J/\psi K^+$ signal in the bins with DLL$_{K\pi} < -5$ and $-5 < \text{DLL}_{K\pi} < 0$, their signal yields are fixed, respectively, to be zero and $P \times \sum_{i=1}^{4} N_{i}^{j/\psi K^+}$ where the $P$ is the probability that the kaon from the $B_c^+ \to J/\psi K^+$ decay has $-5 < \text{DLL}_{K\pi} < 0$, as estimated from simulation.

Figure 2 shows the invariant mass distributions of the $B_c^+$ candidates, calculated with the kaon mass hypothesis in the four DLL$_{K\pi}$ bins. In the fit to the $B_c^+$ mass spectrum, the shape of the $B_c^+ \to J/\psi K^+$ signal is modelled by a double-sided Crystal Ball (DSCB) function [33] as

$$f(x; M, \sigma, a_l, n_l, a_r, n_r) = \begin{cases} 
    e^{-\frac{x^2}{2}} \left( \frac{n_l}{a_l} \right)^{n_l} \left( \frac{n_l}{a_l} - \frac{x - M}{\sigma} \right)^{-n_l} & \frac{x - M}{\sigma} < -a_l \\
    \exp \left[ -\frac{1}{2} \left( \frac{x - M}{\sigma} \right)^2 \right] & -a_l \leq \frac{x - M}{\sigma} \leq a_r \\
    e^{-\frac{x^2}{2}} \left( \frac{n_r}{a_r} \right)^{n_r} \left( \frac{n_r}{a_r} - \frac{x - M}{\sigma} \right)^{-n_r} & \frac{x - M}{\sigma} > a_r
\end{cases}$$

(4)

where the peak position is fixed to that from an independent fit to the $B_c^+ \to J/\psi \pi^+$ mass distribution, and the tail parameters $a_{l,r}$ and $n_{l,r}$ on both sides are taken from simulation.
As the decay $B_c^+ \rightarrow J/\psi \pi^+$ is reconstructed with the kaon mass replacing the pion mass, the signal is shifted to higher mass values and is modelled by another DSCB function whose shape and the relative position to the $B_c^+ \rightarrow J/\psi K^+$ signal are also derived from simulation. Two corrections are applied to the $B_c^+ \rightarrow J/\psi \pi^+$ simulation sample. Firstly, since the resolution of the detector is overestimated, the momenta of charged particles are smeared to make the resolution on the $B_c^+$ mass in the $B_c^+ \rightarrow J/\psi \pi^+$ simulation sample the same as that of the $J/\psi \pi^+$ mass distribution of the $B_c^+$ candidates in the data sample. Secondly, the shapes of the $B_c^+ \rightarrow J/\psi \pi^+$ mass distribution in the four DLL$_{K\pi}$ bins depend on the DLL$_{K\pi}$ distribution, which is different in data and simulation. To reduce the effect of this difference, each simulated event is reweighted by a DLL$_{K\pi}$ dependent correction factor, which is derived from a linear fit to the ratio of the DLL$_{K\pi}$ distribution in background-subtracted data, to that of the simulation sample. The background subtraction [34] is performed with the $J/\psi \pi^+$ mass distribution of the $B_c^+$ candidates in the data sample with the pion mass hypothesis.
The combinatorial background is modelled as an exponential function with a different freely varying parameter in each DLL\textsubscript{\(K\pi\)} bin. The contribution of the partially reconstructed background is modelled by an ARGUS function \cite{35}. The contribution of the partially reconstructed background is dominated by events with bachelor pions, which are suppressed in the high-value DLL\textsubscript{\(K\pi\)} bins, therefore the number of the partially reconstructed events in the DLL\textsubscript{\(K\pi\)} > 5 bin is assumed to be zero. All parameters of the partially reconstructed background are allowed to vary. The observed \(B_c^+ \to J/\psi K^+\) signal yield is 46 ± 12 and the ratio of yields is

\[
R_{K^+/\pi^+} = \frac{N(B_c^+ \to J/\psi K^+)}{N(B_c^+ \to J/\psi \pi^+)} = 0.071 \pm 0.020 \text{ (stat)}.
\]

The ratio of the total efficiencies computed over the full DLL\textsubscript{\(K\pi\)} range is

\[
\frac{\epsilon(B_c^+ \to J/\psi K^+)}{\epsilon(B_c^+ \to J/\psi \pi^+)} = 1.029 \pm 0.007,
\]

which is determined from simulation and the uncertainty is due to the finite size of the simulation samples.

The \(B_c^+ \to J/\psi \pi^+\) signal has a long tail that may extend into the high mass region. A systematic uncertainty is assigned due to the choice of fit range, and is determined to be 0.9\% by changing the mass window from 6000-6600 MeV/c\(^2\) to 6200-6700 MeV/c\(^2\) and comparing the results. To estimate the systematic uncertainty due to the potentially different performance of the BDT on data and simulation, the BDT cut values have been varied in the range 0.21-0.24, compared to a default value of 0.22. The resulting branching fraction ratios have a spread of 5.7\%, which is taken as the corresponding systematic uncertainty.

To estimate the uncertainty due to the choice of signal shape, an alternative \(B_c^+ \to J/\psi \pi^+\) mass shape is used, which is determined from the data sample by subtracting the background in the \(J/\psi \pi^+\) mass distribution of the \(B_c^+\) candidates with the pion hypothesis. A 2.7\% difference with the ratio obtained with the nominal signal shape is observed.

To estimate the systematic uncertainty due to the partially reconstructed background shape in each DLL\textsubscript{\(K\pi\)} bin, the shape is modelled with the ARGUS function convolved with a Gaussian function. The observed 2.3\% deviation from the default fit is assigned as the systematic uncertainty.

For the \(B_c^+ \to J/\psi K^+\) yields in the two bins with DLL\textsubscript{\(K\pi\)} < 0, half of the probability estimated from the simulation, namely 1.8\%, is taken as systematic uncertainty.

To estimate the uncertainty due to the choice of the DLL\textsubscript{\(K\pi\)} binning, two other binning choices are tried: DLL\textsubscript{\(K\pi\)} < -6, -6 < DLL\textsubscript{\(K\pi\)} < -1, -1 < DLL\textsubscript{\(K\pi\)} < 4, DLL\textsubscript{\(K\pi\)} > 4 and DLL\textsubscript{\(K\pi\)} < -4, -4 < DLL\textsubscript{\(K\pi\)} < 1, 1 < DLL\textsubscript{\(K\pi\)} < 6, DLL\textsubscript{\(K\pi\)} > 6. The average value of the
results with these two binning choices has a 1.2% deviation from the default value, which is taken as the systematic uncertainty.

There is a systematic uncertainty due to the different track reconstruction efficiencies for kaons and pions. Since the simulation does not describe hadronic interactions with detector material perfectly, a 2% uncertainty is assumed, as in ref. [36].

An uncertainty of 0.7% arises from the statistical uncertainty of the ratio of the total efficiencies, which is due to the finite size of the simulation sample.

The systematic uncertainties are summarised in table 1. The total systematic uncertainty, obtained as the quadratic sum of the individual uncertainties, is 7.5%.

The asymptotic formula for a likelihood-based test $\sqrt{-2 \ln(L_B/L_{S+B})}$ is used to estimate the $B_c^+ \to J/\psi K^+$ signal significance, where $L_B$ and $L_{S+B}$ stand for the likelihood of the background-only hypothesis and the signal and background hypothesis respectively. A deviation from the background-only hypothesis with 5.2 standard deviations is found when only the statistical uncertainty is considered. When taking the systematic uncertainty into account, the total significance of the $B_c^+ \to J/\psi K^+$ signal is 5.0 $\sigma$.

In summary, a search for the $B_c^+ \to J/\psi K^+$ decay is performed using a data sample, corresponding to an integrated luminosity of 1.0 fb$^{-1}$ of $pp$ collisions, collected by the LHCb experiment. The signal yield is 46 $\pm$ 12 candidates, and represents the first observation of this decay channel. The branching fraction of $B_c^+ \to J/\psi K^+$ with respect to that of $B_c^+ \to J/\psi \pi^+$ is measured as

$$\frac{B(B_c^+ \to J/\psi K^+)}{B(B_c^+ \to J/\psi \pi^+)} = 0.069 \pm 0.019 \pm 0.005 ,$$

where the first uncertainty is the statistical and the second is systematic. The measurement is in agreement with the theoretical predictions [2, 3, 5–7, 9, 10].

<table>
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<tr>
<th>Source</th>
<th>Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>BDT selection</td>
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<tr>
<td>$B_c^+ \to J/\psi K^+$ signal model</td>
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<tr>
<td>$B_c^+ \to J/\psi \pi^+$ signal model</td>
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<tr>
<td>Choice of signal shape</td>
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<tr>
<td>Partially reconstructed background shape</td>
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<td>$B_c^+ \to J/\psi K^+$ signals in DLL$_{K\pi}$ &lt; 0 bins</td>
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<tr>
<td>Total</td>
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</tr>
</tbody>
</table>

Table 1. Relative systematic uncertainties on the ratio of branching fractions.
Assuming factorisation holds, the naïve prediction of the ratio $\mathcal{B}(B_c^+ \to J/\psi K^+)/\mathcal{B}(B_c^+ \to J/\psi \pi^+)$ can be compared to other $B$ meson decays with a similar topology

$$
\frac{\mathcal{B}(B \to DK^+)}{\mathcal{B}(B \to D\pi^+)} = \begin{cases} 
0.0646 \pm 0.0043 \pm 0.0025 & \text{for } B_s^0 \to D_s^- K^+(\pi^+) \\
0.0774 \pm 0.0012 \pm 0.0019 & \text{for } B^+ \to \bar{D}^0 K^+(\pi^+) \\
0.074 \pm 0.009 & \text{for } B^0 \to D^- K^+(\pi^+) 
\end{cases}
$$

(5)

taken from ref. [19, 37, 38]. Hence, this measurement of $\mathcal{B}(B_c^+ \to J/\psi K^+)/\mathcal{B}(B_c^+ \to J/\psi \pi^+)$ is consistent with naïve factorisation in $B$ decays.

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