The Calibration and Performance of a Non-homothetic CDE Demand System for CGE Models

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The Calibration and Performance of a Non-homothetic CDE Demand System for CGE Models

BY Y.-H. HENRY CHEN

In computable general equilibrium modeling, whether the simulation results are consistent with a set of valid own-price and income demand elasticities that are observed empirically remains a key challenge in many modeling exercises, since functional forms that are not fully flexible can only allow a limited subset of elasticities. While not fully flexible, the Constant Difference of Elasticities (CDE) demand system has enough free parameters to match own-price and income elasticities in some cases, leading to its adoption by some models since the 1990s. However, perhaps due to complexities of the system, the applications of CDE demand in other models are less common. Furthermore, how well the system can represent the given elasticities is rarely discussed or examined in the existing literature. This study aims to fill this gap by revisiting calibration strategies for the CDE demand system and exploring conditions where the calibrated elasticities of the system can better match a set of valid target elasticities. Results show that the calibrated elasticities can be matched to the target ones more precisely if the sectoral expenditure shares are lower, the target own-price demand elasticities are lower, and target income demand elasticities are relatively higher. The study also incorporates a CDE demand into the GTAP in GAMS model and verifies that for the revised model with a CDE demand system, the model can successfully replicate the calibrated elasticities under various price and income shocks.

JEL codes: C6, C8, D5, R1

Keywords: Computable general equilibrium modeling; Constant difference of elasticities; Demand system flexibility; Calibration.

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1. Introduction

In Computable General Equilibrium (CGE) modeling, price and income elasticities of demand are crucial in determining the sectoral growth pattern and economic impacts of various policies (Hertel, 2012). This suggests the widely used Constant Elasticity of Substitution (CES) utility function (Sancho, 2009; Annabi et al., 2006; Elsenburg, 2003), is likely unsatisfactory due to its unitary income elasticities of demand, and relative price inflexibility. Indeed, in a single-nest CES setting, after applying the Cournot aggregation, the sectoral expenditure shares will fully determine the variation in own-price elasticities of demand.

To capture the observed non-homothetic preferences with income elasticities of demand diverging from unity, one approach is to use the Linear Expenditure System (LES) derived from Stone-Geary preferences (Geary, 1950; Stone, 1954). The LES system can be calibrated to income elasticities of demand compatible to a valid demand system, although it only allows for calibration to a single price elasticity of demand. In addition, with a special multi-nest structure, the calibrated own-price elasticities of demand can be matched perfectly to any valid elasticities (Perroni and Rutherford, 1995). The shortcoming of LES, however, is that due to constant marginal budget shares with respect to income, the limit property of LES is still homotheticity, and therefore the underlying income elasticities of demand will approach one as income grows and subsistence expenditures dwindle.

An alternative option to model non-homotheticity is to utilize the Constant Difference of Elasticities (CDE) demand system proposed by Hanoch (1975). With implicit additivity, a \( N \)-commodity CDE demand system has \( N \) expansion parameters and \( N \) substitution parameters to achieve a more general functional form than the single nest CES case. The \( N \) expansion parameters make it possible to incorporate various income elasticities of demand across commodities/sectors, and the income elasticities will remain at their given levels as income changes ("commodity" and "sector" are used interchangeably in this study). On the other hand, compared to a single-nest CES setting, the \( N \) substitution parameters allow modelers to come up with a somewhat better match with target own-price demand elasticities. This led Hertel et al. (1991) to propose the use of the CDE functional form in CGE models as a means of bridging the gap between fully flexible forms and the restrictive, LES/CES functions.

One caveat of CDE applications, paradoxically, comes from the relative stability of each income elasticity regardless of income levels. Specifically, if a good is a luxury it remains a luxury (Yu et al., 2003). While this limitation might not severely contradict empirical evidence for developed countries, existing studies have found that, for instance, income elasticities of some food items in developing countries

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1 While Perroni and Rutherford (1995) focuses on homothetic preferences, it points out that the multi-nest strategy achieving a perfect match in own-price elasticities calibration also works for non-homothetic preferences.
tend to decrease as income grows (Haque, 2005; Chern et al., 2003). In some cases, economic growth may turn luxury goods into necessities (Zhou et al., 2012). One strategy to address this limitation is to recalibrate CDE parameters over time so that target income demand elasticities could be adjusted based on a given projection for economic development (Anderson and Strutt (2012); Woltjer et al. (2014)). With this treatment within-period welfare comparison can be done when the CDE parameters remain unchanged. However, between-period welfare comparison is no longer possible, since changing CDE parameters means changing preferences, and in that case equivalent variation will not be well-defined (Chen et al., 2016). To overcome this, Rimmer and Powell (1996) propose an implicit directly additive demand system (AIDADS) that allows income elasticities of demand to vary logistically as the marginal budget shares at subsistence income and very high income are separately estimated. Nevertheless, because of the implicit additivity assumption, AIDADS only allows a limited range of substitution possibilities across goods, and due to theoretical and computational reasons, AIDADS applications have thus far been limited to 10 commodities/sectors (Reimer and Hertel, 2004). As a result, CGE applications with AIDADS are less common and more project-specific. In contrast, despite some limitations, the CDE demand system seems to be more broadly applicable as a generic setting for modeling non-homothetic preferences with variation in the price-responsiveness of demand. For instance, with the CDE demand, modelers have more degrees of freedom in choosing the desired sectoral aggregation level that fits their research purposes.

While CGE models such as GTAP (Hertel and Tsigan, 1997), MAGNET (Woltjer and Kuiper, 2014), GTEM (ABARE/DFAT, 1995; ABARE, 1996), and ENVISAGE (van der Mensbrugge, 2008) have been using the CDE demand system in modeling final consumption behaviors, perhaps due to the complexities in both calibration and implementation, other CDE applications are less common so far. More importantly, when studying the responses of CGE models with non-homothetic preferences, besides examining the implications of income elasticities of demand on future projection, the roles of own-price elasticities of demand are crucial as well since they can also influence projections if relative prices or income levels change. Existing literature also points out that to ensure the regularity of a well-behaved demand function, calibrating a CDE demand system to the target elasticities that are valid might be infeasible (Hertel, 2012; Huff et al., 1997). How well the system can match those elasticities is beyond the discussion of most existing literature. One exception is Liu et al. (1998), which presents the differences between target and calibrated elasticities. Nevertheless, exploring sources of differences between calibrated and target elasticities is beyond the scope of that study.

Before studying how well the calibrated elasticities of a demand system can match a set of target elasticities, one needs to ensure that under a given
expenditure share structure, the target elasticities are valid, i.e., they are conformable to aggregation conditions and a negative semi-definite Slutsky matrix. Therefore, the demand system under consideration will only be calibrated to a set of valid target elasticities. With that in mind, the study will answer the question both analytically and numerically: given a set of valid target own-price demand elasticities, income demand elasticities and expenditure shares, under what conditions will the calibrated elasticities of a CDE demand system most closely approximate the target values? The findings of this study can help modelers who implement a CDE demand system in explaining how well the target elasticities are represented in their models, and provide information for choosing an appropriate sectoral aggregation so that, if possible, at least target elasticities of interesting sectors can be better matched.

Following Chen (2015) this paper also presents strategies for putting the CDE demand system into GTAPinGAMS (Rutherford, 2012; Lanz and Rutherford, 2016), a global CGE model which is written in GAMS and MPSGE and which employs the GTAP database (Narayanan et al., 2012; Aguiar et al., 2016). MPSGE is a subsystem of GAMS (Rutherford, 1999), and earlier it was sometimes thought that despite being a powerful tool that handles the calibration of CES functions automatically, MPSGE can only be applied to models with CES or LES utility functions (Konovalchuk, 2006; Hertel et al., 1991). Perhaps the misconception is because, until recently, CGE models built by MPSGE were largely characterized by either CES or LES preferences. This study provides an example of extending the application of MPSGE beyond the CES or LES preferences. The revised GTAPinGAMS with a CDE demand system is tested with income and price shocks to verify the model response is consistent to the calibrated elasticities. The programs for the CDE calibration and the revised GTAPinGAMS with a CDE demand system are provided in the Appendix, so readers can use them for verification or research purposes.

The rest of the paper is organized as follows: Section 2 briefly reviews the theories and settings of the CDE demand system; Section 3 presents the calibration, performance, and implementation of the CDE demand system; and Section 4 provides a conclusion.

2. Theoretical Background

To understand what constitutes a regular (i.e., valid) demand response, the section will briefly review the economic considerations for a regular demand system. In the subsequent analysis, this paper will focus on the following question: how can one evaluate the performance of a regular demand system in representing a set of valid own-price and income demand elasticity targets? To explore this, the section will discuss a demand system’s flexibilities in own-price and income demand elasticities calibration, introduce the settings of CDE demand system, and
finally examine the implications of CDE regularity conditions on the calibration performance of the system.

2.1 Regularity and Flexibility of a Demand System

Let us denote a cost (or expenditure) function by $C(p, u)$ where $p$ is a $N$-dimensional price vector and $u$ is the utility. For $C$ to be considered as well-behaved, $\partial C/\partial p$, which is the Hicksian demand vector $q(p, u)$, is nonnegative and homogeneous of degree zero in $p$, and $[\partial^2 C/\partial p_i \partial p_j]_{N \times N}$, which is the Slutsky matrix, is negative semi-definite (NSD). The intuition of a NSD Slutsky matrix is: for a given utility level $u$, when a good becomes more expensive, it will be replaced by other cheaper alternatives; as a result, the cost increase with the new consumption bundle after the price increase will never exceed the cost increase when the bundle cannot be altered.

The Slutsky matrix $[\partial^2 C/\partial p_i \partial p_j]_{N \times N}$, or equivalently $[\partial q/\partial p]_{N \times N}$, is symmetric and each term of the matrix is:

$$\frac{\partial q_i(p,u)}{\partial p_j} = \frac{\partial_x i(p,w)}{\partial p_j} + \frac{\partial_x j(p,w)}{\partial w} x_j(p,w)$$

Equation (1) is the Slutsky equation, which decomposes the impacts of a price change on the uncompensated demand $x_i(p,w)$ into the income effect and substitution effect, where $w$ is the income (or expenditure) level. With some algebra, the Slutsky equation can also be expressed as

$$\sigma_{ij}^c = \sigma_{ij}^m + \eta_i \theta_j$$

where $\sigma_{ij}^c$, $\sigma_{ij}^m$, $\eta_i$, and $\theta_j$ are compensated demand elasticity of commodity $i$ with respect to the price of commodity $j$, uncompensated demand elasticity of $i$ with respect to the price of $j$, income demand elasticity of $i$, and expenditure share of $j$, respectively. If both sides of (2) are divided by $\theta_j$, one can come up with a Slutsky matrix $[\sigma_{ij}]_{N \times N}$ in the form of Allen-Uzawa elasticity of substitution (AUES) (Allen and Hicks, 1934; Uzawa, 1962) with

$$\sigma_{ij} = \sigma_{ij}^m / \theta_j + \eta_i$$

It can be shown that $[\sigma_{ij}]_{N \times N}$ is also symmetric, and the matrix is NSD if and only if $[\partial q/\partial p]_{N \times N}$ is NSD. Therefore, a demand system is regular means 1) the Slutsky/AUES matrix $[\sigma_{ij}]_{N \times N}$ is NSD; and 2) the Hicksian demand $q$ is nonnegative. For CGE modeling, it is necessary to ensure that the demand system is globally regular (i.e., it should remain regular everywhere in the domain of price). This is because the algorithm of the solver for finding equilibria may begin from an initial point of price and quantity combination that is far from the equilibrium levels, and in the process of solving the model, the algorithm might fail if the

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2 For example, see p.59 and p.933 in Mas-Colell et al. (1995).
demand system is not globally regular, even the system is locally regular at the equilibrium points (Perroni and Rutherford, 1998).

Perroni and Rutherford (1995) defined a regular-flexible demand system as the one that is globally regular and can locally represent any valid configuration of compensated demands and the AUES matrix \( \sigma_{ij} \). Based on an inductive argument, Perroni and Rutherford proved that a demand system derived from a special version of the non-separable n-stage CES function is regular-flexible. Nevertheless, in general, testing whether other demand systems are regular-flexible would need to identify the domain of a regular flexible demand system first, which is beyond the scope of the current research. Instead of matching the entire AUES matrix under a given expenditure share structure, this study will focus on the ability of a demand system to match a valid combination of own-price demand elasticities, income demand elasticities, and expenditure shares.

Own-price and income demand elasticities are usually of first-order importance in characterizing the model responses to exogenous policy or productivity shocks, and are also the most common behavioral parameters available for calibrating a demand system. In particular, this study will examine whether a global regular demand system under consideration is own-price and income flexible, or equivalently, if the system can be calibrated to a valid combination of \((\sigma_{ii}^m, \eta_i, \theta_i)\) — the combination that is consistent to any well-behaved cost function (i.e., the aggregation conditions are satisfied, and the AUES matrix is NSD). Ideally, the functional form of a demand system used in a CGE model should not become a constraint in matching any valid combination of \((\sigma_{ii}^m, \eta_i, \theta_i)\). However, usually that is not the case. Note that each of the three components of \((\sigma_{ii}^m, \eta_i, \theta_i)\) is a N-dimensional vector, and these components \((\sigma_{ii}^m, \eta_i, \theta_i)\) are interdependent. For instance, Pigou’s Law states that under certain assumptions on preferences, when the sectoral expenditure share \(\theta_i\) is negligible, there is a proportional relationship between the income and uncompensated own-price demand elasticities (Pigou, 1910; Snow and Warren, 2015). Because of this interdependency, identifying the domain of \((\sigma_{ii}^m, \eta_i, \theta_i)\) is numerically challenging, unless one is willing to consider very few sectors under a given distribution of \(\{\theta_i\}\), such as examples shown in Perroni and Rutherford (1998). Therefore, in the numerical examples for the CDE calibration provided later,

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3 Based on the Slutsky equation (see Equation (2)), if the following information is given: 1) expenditure share \(\theta_i\), 2) target income demand elasticity \(\eta_i^t\), and 3) any one of the uncompensated own-price demand elasticity \(\sigma_{ii}^{mt}\), compensated own-price demand elasticity \(\sigma_{ii}^{mc}\), or compensated own-price demand elasticity target in AUES form \(\sigma_{ii}^t\), then targeting any of the aforementioned three versions of own-price demand elasticities is equivalent to targeting another. What matters is to identify the form of the target elasticities clearly when doing the calibration job. For example, if one calibrates the compensated elasticities to the uncompensated elasticity targets, that would be incorrect.
rather than identifying the full space of valid target elasticities and sectoral shares, this study will begin by checking whether the target elasticities aggregated from the GTAP database with various sectoral resolutions and expenditure share structures actually constitute a theoretically valid demand configuration.

2.2 The CDE Demand System

Let us consider the expenditure function $C$ with a price vector $p$ and a Hicksian demand vector $q$, i.e., $c_0 = C(p_0, u) \equiv \{\min p_0 q_0 : f(q_0) \geq u\}$ where the subscript 0 denotes the benchmark condition. If the function is normalized by $c_0$, it becomes $C(p_0/c_0, u) \equiv 1$. With this normalization, Hanoch (1975) proposes the expenditure function of a CDE demand system as follows:

$$C \left( \frac{p}{c_0}, u \right) = \sum_i \beta_i u^{e(1-\alpha_i)} \left( \frac{p_i}{c_0} \right)^{1-\alpha_i} \equiv 1$$ (4)

where $\alpha_i$ and $e_i$ are the substitution parameter and expansion parameter, respectively. In this setting, the utility $u$ is only implicitly defined, and in general there is no reduced form representation for $u$. The Hicksian demand for commodity $i$ based on this setting is:

$$q_i = \frac{\beta_i e_i^{(1-\alpha_i)} (1-\alpha_i) \left( \frac{p_i}{c_0} \right)^{1-\alpha_i}}{\sum_j \beta_j e_j^{(1-\alpha_j)} (1-\alpha_j) \left( \frac{p_j}{c_0} \right)^{1-\alpha_j}}$$ (5)

For the CDE demand system, the substitution elasticity $\sigma_{ij}$ in AUES form is presented in Equation (6), where the expenditure share is denoted by $\theta_i$, and $\delta_{ij} = 1$ if $i = j$, otherwise $\delta_{ij} = 0$. The income elasticity of demand $\eta_i$ is presented in Equation (7):

$$\sigma_{ij} = \alpha_i + \alpha_j - \sum_k \theta_k e_k \alpha_k - \frac{\delta_{ij} \alpha_i}{\theta_i}$$ (6)

$$\eta_i = (\sum_k \theta_k e_k)^{-1}[e_i (1 - \alpha_i) + \sum_k \theta_k e_k \alpha_k] + (\alpha_i - \sum_k \theta_k \alpha_k)$$ (7)

It can be shown that both Cournot aggregation and Engel aggregation conditions hold for these elasticities, i.e., $\sum_i \theta_i \sigma_{ij} = 0$ and $\sum_i \theta_i \eta_i = 1$. Note that for each off-diagonal term, the difference between the substitution elasticities, $\sigma_{ij} - \sigma_{ik} = \alpha_j - \alpha_k$, is invariant to $i$. Hence the name CDE since the demand system has a constant difference of (substitution) elasticities. The regularity condition for the system presented in Hanoch (1975) includes: $\beta_i > 0$; $e_i > 0$; $0 < \alpha_i < 1$ or $\alpha_i \geq 1$ $\forall i$ and $\alpha_i > 1$ for some $i \in I$. It is worth noting that with the regularity condition, each own-price elasticity of demand $\sigma_{ii}^c$ is always negative. This is because from Equation (6) and $\sigma_{ii}^c = \sigma_{ii} \theta_i$, we have

$$\sigma_{ii}^c = -\alpha_i (1 - \theta_i)^2 - \theta_i \sum_{k \neq i} \theta_k \alpha_k$$ (8)

For a given vector of budget shares, $\theta_i$, the requirement that all $\alpha_i$s should lie on the same side of one imposes a constraint in choosing the vector of $\alpha_i$s such that $\sigma_{ii}^c$ can match the target own-price demand elasticity. For instance, some sectors
may have a very small expenditure share \((\theta_i \to 0)\) and so for those sectors \(\sigma_i^c \to -\alpha_i\). However, for those sectors, if some target own-price elasticities do not lie on the same side of one, it would be impossible to match every single \(\sigma_i^c\) with the target elasticity value no matter what regulatory condition on \(\alpha_i\) is chosen. Therefore, the CDE demand system is not own-price flexible. Further, the requirement of \(e_i \geq 0\) also suggests that some compromise has to be made in calibrating income elasticities of demand.

3. Calibration, Performance, and Implementation

Two CDE calibration methods have been presented. The first is the three-step sequential approach documented in Hertel et al. (1991) and Huff et al. (1997). In this approach, own-price demand elasticities are calibrated to target levels first. Taking parameters determined in the first step as given, income elasticities of demand are calibrated to target levels next, and scale parameters of the system are specified last. The second method is the maximum entropy approach presented by Surry (1997) and Liu et al. (1998). Rather than calibrating the system sequentially, the idea of this approach is finding all parameters simultaneously by maximizing an objective function that considers matching both own-price and income elasticities of demand. This section will analytically examine the performance of CDE calibration (i.e., how well target elasticities can be matched by their calibrated counterparts), and then provide numerical examples based on both calibration methods. It will also demonstrate how to put the CDE demand system into GTAPinGAMS and verify the model response is consistent to the calibrated elasticities.

3.1 Calibration: sequential approach

- **Step 1: Calibrating the own-price elasticity of demand.**

  Let us denote the target compensated own-price elasticity of demand by \(\sigma_i^{ct}\). The purpose of this step is to choose \(\alpha_i\) so that the objective function \(g(\alpha_i) = -\sum_i \sigma_i^c(\alpha_i)[\ln(\sigma_i^c(\alpha_i)/\sigma_i^{ct}) - 1]\) is minimized. Note that \(g(\alpha_i)\) is convex in \(\alpha_i\), and the function achieves its minimal value when \(\sigma_i^c(\alpha_i) = \sigma_i^{ct}\) for every \(i\). The problem can be formulated as:

\[
\min_{\alpha_i} g(\alpha_i) \text{ s. t. } \alpha_i \in (0, 1) \text{ or } \alpha_i \geq 1 \forall i \text{ and } \alpha_i > 1 \text{ for some } l \in i
\]  

where \(\sigma_i^c(\alpha_i) = -\alpha_i(1 - \theta_i)^2 - \theta_i \sum_{k \neq i} \theta_k \alpha_k\) (see Equation (8)).

- **Step 2: Calibrating the income elasticity of demand.**

\[
\frac{\partial g}{\partial \alpha_i} = \frac{\partial g}{\partial \sigma_i^c} \cdot \frac{\partial \sigma_i^c}{\partial \alpha_i} = \left(-\ln \sigma_i^c + \ln \sigma_i^{ct}\right) \cdot \left(-(1 - \theta)^2\right) \cdot \frac{\partial^2 g}{\partial \alpha_i^2} = \frac{(1 - \theta)^2}{\sigma_i^{ct}} > 0.
\]
Let us denote the target income elasticity of demand by \( \eta^*_i \) (\( \eta^*_i \) must satisfy the Engel aggregation). Given \( \alpha_i \) determined in the previous step, by choosing \( e_i \), the goal is to calibrate \( \eta_i \) to \( \eta^*_i \), if possible. Similar to the idea of Step 1, the following problem is solved:

\[
\min_{e_i|\alpha_i} \sum_i \theta_i (\eta_i - \eta_i^*)^2 \quad \text{s.t.} \quad \sum_i \theta_i \eta_i = 1; \quad (\eta_i - 1)(\eta_i^* - 1) > 0 & e_i > 0 \forall i \quad (10)
\]

The condition \( \sum_i \theta_i \eta_i = 1 \) is to ensure the calibrated elasticities satisfy the Engel aggregation, and as noted in Huff et al. (1997), the second condition is to ensure the calibrated elasticities lie on the same side of one as the target values.

- **Step 3: Calibrating scale coefficients holding the utility level equals one.**

With the calibrated \( \alpha_i \) and \( e_i \), and the normalization \( u = 1, p_{0i} = 1, \) and \( q_{0i} = \theta_i \) (since \( c_0 = \sum i p_{0i} q_{0i} = 1 \)), the \( N \) scale parameters \( \beta_i \) can be solved by using (4) and (5):

\[
\beta_i = \frac{q_{0i}}{1 - \alpha_i} / \sum_k \frac{q_{0k}}{1 - \alpha_k} \quad (11)
\]

Because the calibration is done sequentially, how well the income elasticities of demand can be matched to target levels is also affected by the calibration of own-price elasticities of demand. Appendix A provides the program for the sequential approach. The program is written in GAMS, and each minimization problem in the program is formulated as a nonlinear programming (NLP) problem.

### 3.2 Calibration: maximum entropy approach

Following the notation used before, in this approach the substitution parameters \( \alpha_i \) and the expansion parameters \( e_i \) are chosen simultaneously by maximizing the objective function, which is the entropy relative to the unknown parameters of the CDE demand system. As in the sequential approach, the scale parameters \( \beta_i \) can be calculated once \( \alpha_i \) and \( e_i \) are determined (see Equation (11)). To provide more details, let us denote the cross entropy of the substitution parameter and the cross entropy of the expansion parameter as \( \alpha_{etp} \) and \( e_{etp} \), respectively. The two entropy measures are defined as:

\[
\alpha_{etp} = -\sum_k \theta_k \left( \alpha_k \ln \frac{\alpha_k}{\alpha} + (1 - \alpha_k) \ln \frac{1 - \alpha_k}{1 - \alpha} \right) \quad (12)
\]

\[
e_{etp} = -\sum_k \theta_k e_k \ln e_k \quad (13)
\]

Since the calibrated elasticities may deviate from target levels, let us define the penalty for errors in the substitution parameters and that for errors in the expansions parameters as \( \alpha_{pnt} \) and \( e_{pnt} \), respectively:

\[
\alpha_{pnt} = \sum_k \theta_k \cdot (\sigma_{kk}^\alpha - \sigma_{kk}^{mt})^2 \quad (14)
\]

\[
e_{pnt} = \sum_k \theta_k \cdot (\eta_k - \eta_k^*)^2 \quad (15)
\]
The maximum entropy approach is to solve the following problem:

$$\max_{\alpha_i, \epsilon_i} \left\{ \left( e_{etp} + \alpha_{etp} \right) - \left( e_{pnt} + \alpha_{pnt} \right) \right\} \text{ s.t. } \sum_k \theta_k e_k = 1 \tag{16}$$

Interested readers may also refer to Hertel et al. (2014) for details of this approach. Note that while results from both calibration methods will be presented later, comparing one method with another or assessing which one is preferable is beyond the scope of this study, which seeks instead to simply provide numerical examples for a set of propositions aimed to guide those seeking to assess the performance of CDE calibrations (see Section 3.3). Appendix B presents the GAMS program that implements the maximum entropy approach, which is also formulated as a nonlinear programming (NLP) problem.

### 3.3 Performance

Before putting the system into a CGE model, two interesting questions arise: under what circumstances does the calibration become more accurate, and how well are those target elasticities represented? The following analysis seek to shed light on these questions.

**Proposition 3.3.1:**

The lower the expenditure share, the larger the influence of the own-sector substitution parameter in determining the calibrated own-price elasticity of demand. On the other hand, the higher the expenditure share, the greater the influence of other sectors’ substitution parameters in determining the calibrated elasticity.

**Proof:**

$$\sigma_{ii} = -\alpha_i (1 - \theta_i)^2 - \theta_i \sum_{k \neq i} \theta_k \alpha_k; \text{ note that } (1 - \theta_i)^2 \text{ is decreasing but } \theta_i \text{ is increasing on } \theta_i \in (0, 1), \text{ respectively. Also, note that: } \lim_{\theta_i \rightarrow 0} \sigma_{ii} = -\alpha_i \text{ and } \lim_{\theta_i \rightarrow 1} \sigma_{ii} = -\sum_{k \neq i} \theta_k \alpha_k \text{.} \tag*{∎}$$

The compensated own-price elasticities of demand presented in GTAP 8 (Narayanan et al., 2012), from which this study, and many others, draw their data, lie between −1 and 0. Therefore, based on the discussion above, it appears that the regularity condition with \( \alpha_i \in (0,1) \) produces more accurate calibration results for sectors with smaller expenditure shares. With a higher sectoral resolution, more commodities/sectors will have smaller expenditure shares, and thus having \( \alpha_i \in (0,1) \) will make it possible for producing a better match between calibrated and target levels for each individual sector.

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5 The author is grateful to Erwin Corongan and Thomas Hertel from the Center for Global Trade Analysis at the Department of Agricultural Economics in Purdue University for sharing the CDE calibration code that implements the maximum entropy approach. The code presented in Appendix B follows exactly the same setting as their code, except for the fact that some variable names are changed, which makes it easier to read data from GTAP in GAMS.
Proposition 3.3.2:
When $\alpha_i \in (0, 1)$, calibrating the income elasticity of demand to a larger value is less likely to violate $e_i > 0$, which is part of the CDE regularity conditions. On the other hand, when $\alpha_i > 1$, calibrating the elasticity to a lower level is less likely to violate $e_i > 0$.

Proof:
From Equation (7), $e_i = \frac{\sum_k \theta_k e_k [\eta_i - (\alpha_i - \sum_k \theta_k \alpha_k)] - \sum_k \theta_k e_k \alpha_k}{1 - \alpha_i}$. Since $\frac{\partial e_i}{\partial \eta_i} = \frac{\sum_k \theta_k e_k}{1 - \alpha_i}$, $\therefore \frac{\partial e_i}{\partial \eta_i} > 0$ if $\alpha_i \in (0, 1)$, and $\frac{\partial e_i}{\partial \alpha_i} < 0$ if $\alpha_i > 1$.

If one considers $\alpha_i \in (0, 1)$, the second proposition suggests that matching the target income elasticities for the demand of agricultural products in developed countries might be trickier, since in general these products tend to have lower income elasticity values; as a result, the calibrated income demand elasticities for these products might end up with levels higher than the target numbers. Nevertheless, the values of $\alpha_i$ may also affect how well the target income elasticities of demand are met, as will be explored in the next proposition.

Proposition 3.3.3:
When $\alpha_i \in (0, 1)$, calibrating the income elasticity of demand to a target level is less likely to violate $e_i > 0$ with a smaller $\alpha_i$. On the other hand, when $\alpha_i > 1$, calibrating the elasticity to the target level is less likely to violate $e_i > 0$ with a larger $\alpha_i$.

Proof:
$e_i = (\sum_k \theta_k e_k [\eta_i - (\alpha_i - \sum_k \theta_k \alpha_k)] - \sum_k \theta_k e_k \alpha_k)/(1 - \alpha_i)$. Since $\frac{\partial e_i}{\partial \alpha_i} = \frac{1}{1 - \alpha_i} [(-1 + \theta_i) \sum_k \theta_k e_k - \theta_i e_i]$, $\therefore \frac{\partial e_i}{\partial \alpha_i} < 0$ if $\alpha_i \in (0, 1)$, and $\frac{\partial e_i}{\partial \alpha_i} > 0$ if $\alpha_i > 1$.

Continuing our previous example for commodities with low income elasticities of demand and with $\alpha_i \in (0, 1)$, while Proposition 3.3.2 says that for given values of $\alpha_i$, it is harder to calibrate the income elasticity of demand to a lower value, Proposition 3.3.3 suggests that if the calibrated $\alpha_i$ is small enough, it is still possible to calibrate the income elasticity of demand to a lower level.

Proposition 3.3.4:
Commodities with substitution parameters $\alpha_i$ close to one will have similar calibrated income elasticities of demand.

Proof:
From Equation (7), $\lim_{\alpha_i \to 1} \eta_i = \sum_k \theta_k e_k \alpha_k / \sum_k \theta_k e_k + 1 - \sum_k \theta_k \alpha_k = \lim_{\alpha_j \to 1} \eta_j$.

Proposition 3.3.4 shows that the calibrated $\alpha_i$ may work against the calibration of income elasticities of demand. For instance, if there are two commodities with $\alpha_i$ and $\alpha_j$ both approaching unity, according to the proposition, the calibrated income elasticities of demand $\eta_i$ and $\eta_j$ will be very close to each other, even if their target values $\eta_i^t$ and $\eta_j^t$ are quite different. The four propositions presented above are summarized in Table 1.
Table 1. Summary of Propositions relating to CDE Calibration.

<table>
<thead>
<tr>
<th>Proposition</th>
<th>Argument</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3.1</td>
<td>The lower the expenditure share, the larger the influence of own-sector substitution parameter in determining the calibrated own-price elasticity of demand. On the other hand, the higher the expenditure share, the greater the influence of other sectors’ substitution parameters in determining the calibrated elasticity. When $\alpha_i \in (0, 1)$, calibrating the income elasticity of demand to a larger value is less likely to violate $e_i &gt; 0$, which is part of the CDE regularity condition. On the other hand, when $\alpha_i &gt; 1$, calibrating the elasticity to a lower level is less likely to violate $e_i &lt; 0$.</td>
</tr>
<tr>
<td>3.3.2</td>
<td>When $\alpha_i \in (0, 1)$, calibrating the income elasticity of demand to a target level is less likely to violate $e_i &gt; 0$ with a smaller $\alpha_i$. On the other hand, when $\alpha_i &gt; 1$, calibrating the elasticity to the target level is less likely to violate $e_i &lt; 0$ with a larger $\alpha_i$.</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Commodities with substitution parameters $\alpha_i$ close to one will have similar calibrated income elasticities of demand.</td>
</tr>
</tbody>
</table>

Source: The author’s summary for the propositions presented in Section 3.3.

To show how different sectoral aggregation levels could affect the accuracy of elasticity calibration, the study considers several different aggregation levels (Table 2). The mapping between the GTAP sector numbers shown in Table 2 and their abbreviations are presented in Appendix C. For demonstration purposes, all GTAP regions are combined into a single region using the aggregation routine of GTAPinGAMS. In particular, wherever needed, target elasticities are aggregated based on expenditure shares. It is worth noting that the 10-sector income demand elasticity estimates based on an AIDADS system were mapped to and used as the target income demand elasticities of the original GTAP database, and following Zeitsch et al. (1991), income demand elasticities are then used to compute the own-price demand elasticities of the database, as documented in Hertel et al. (2014).6

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6 Interested readers may refer to page 14-6, Table 14.5 and Table 14.6 in Hertel et al. (2014) for details.
Table 2. Settings for calibration exercises with various sectoral aggregation levels.

<table>
<thead>
<tr>
<th>Aggregation level</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>1r3s2f</td>
<td>3 sectors: combine GTAP sector 1 (g01) to sector 14 (g14) &amp; sector 22 (g22) to sector 26 (g26) into s01 (agriculture); g15 to g21 &amp; g27 to g46 into s02 (manufacturing); and g47 to g57 into s03 (service)</td>
</tr>
<tr>
<td>1r4s2f</td>
<td>4 sectors: similar to “1r3s2f” except for the fact that the service sector in “1r3s2f” is disaggregated into trade and transport sector (g47 to g51) and service sector (g52 to g57).</td>
</tr>
<tr>
<td>1r5s2f</td>
<td>5 sectors: combine g01 to g17 into s01; g18 to g27 into s02; …; g48 to g57 into s05</td>
</tr>
<tr>
<td>1r8s2f</td>
<td>8 sectors: combine g01 to g15 into s01; g16 to g21 into s02; …; g52 to g57 into s08</td>
</tr>
<tr>
<td>1r16s2f</td>
<td>16 sectors: combine g01 to g12 into s01; g13 to g15 into s02; g16 to g18 into s03; …; g55 to g57 into s16</td>
</tr>
<tr>
<td>1r29s2f</td>
<td>29 sectors: combine g01 &amp; g02 into s01; g03 &amp; g04 into s02; …; g55 &amp; g56 into s28; g57 becomes s29</td>
</tr>
<tr>
<td>1r57s2f</td>
<td>57 sectors: keep the original GTAP sectors (g01 to g57)</td>
</tr>
</tbody>
</table>

Notes: All settings have one aggregated region and two aggregated primary factors: labor and capital.

To assess the calibration performance for each type of elasticity, in addition to a one-by-one comparison between calibrated and target numbers for each commodity, it is informative to have an index for measuring how far the point of calibrated elasticities is from the point of target elasticities as follows:

\[
d = \sqrt{\frac{\sum_{i=1}^{N} \theta_i \cdot (x_i - x_i^t)^2}{\sum_{i=1}^{N} x_i}}
\]  

(17)

Depending on the type of elasticity evaluated, \(x_i\) in Equation (17) could be either the own-price elasticity of demand \(\sigma_i^o\) or the income elasticity of demand \(\eta_i\), while the superscript \(x_i^t\) denotes target value.

When the 57 GTAP sectors are aggregated into a 3-sector setting, even the smallest sectoral expenditure share, denoted by \(\theta_{min}\) approximates 12%, and with this setting the largest share \(\theta_{max}\) exceeds 63%. As the sectoral resolution increases, the absolute difference between \(\theta_{max}\) and \(\theta_{min}\) is reduced. In the most disaggregated case where all 57 GTAP sectors are kept, \(\theta_{max}\) is slightly above 17% and \(\theta_{min}\) is only 0.0002% (Table 3). Per compensated own-price demand elasticity targets, the range between the largest one \(\sigma_i^{ct_{max}}\) and the smallest one \(\sigma_i^{ct_{min}}\) increases as the sectoral resolution gets higher, since more disaggregation means extreme
values are more likely to appear. In general, \( \sigma_{\text{max}}^{ct} \) becomes larger (\(|\sigma_{\text{max}}^{ct}| \) becomes smaller, i.e., less elastic) and \( \sigma_{\text{min}}^{ct} \) becomes smaller (\(|\sigma_{\text{min}}^{ct}| \) becomes larger, i.e., more elastic) as the sectoral resolution increases. The same story applies to the income demand elasticity targets—with more disaggregated sectors, the range between \( \eta_{\text{max}}^{ct} \) and \( \eta_{\text{min}}^{ct} \) increases as \( \eta_{\text{min}}^{ct} \) becomes smaller (less elastic) and/or \( \eta_{\text{max}}^{ct} \) becomes larger (more elastic).

When trying to calibrate the CDE demand system to the target own-price demand elasticities, it is important to verify if there exists an AUES matrix that is NSD and is compatible to those elasticity targets. For instance, with the 3-sector setting, based on Cournot aggregation, the three off-diagonal terms of the AUES matrix are fully determined once the own-price demand elasticities in AUES form (i.e., the diagonal terms of the matrix) are given, and hence the whole AUES matrix is identified. However, given the target own-price demand elasticities in this particular example, one cannot find an AUES matrix that is NSD, which means the target own-price demand elasticities are invalid, and one cannot claim the CDE demand system is not own-price flexible based on this setting. On the other hand, in the 4-sector, 5-sector, 8-sector, and 16-sector settings, it can be shown that under each setting, the target own-price demand elasticities are compatible to an AUES matrix that is NSD, and therefore the target elasticities are valid. More specifically, if one denotes the number of sectors/commodities by \( n \), there will be \( n \cdot (n - 1)/2 \) variables (cross-price demand elasticities in AUES form) that are off-diagonal terms in an AUES matrix, and after considering \( n \) constraints imposed by the Cournot aggregation, there will be \( n \cdot (n - 1)/2 - n \) variables that can be assigned by using random number generators, provided that the diagonal terms (compensated own-price demand elasticities in AUES form) are given. The remaining \( n \) variables can be solved based on the aforementioned \( n \) constraints. The task can be done iteratively, and if a NSD AUES matrix can be found, then the target own-price demand elasticities are valid. The MATLAB subroutine for doing this job is presented in Appendix D. For income demand elasticity targets, on the other hand, they are valid as long as the Engel aggregation is satisfied.

Since sectoral own-price demand elasticity targets are all between 0 and 1, in all cases, to calibrate the CDE demand system, similar to Huff et al. (1997), the study chooses \( \alpha_i \in (0, 1) \), a setting that produces a more accurate own-price demand elasticity calibration when the sectoral resolution becomes higher or the sectors under consideration have smaller expenditure shares, based on Proposition 3.3.1. It is worth noting that since the CDE demand system is regular, under each sectoral aggregation setting presented in Table 2, the calibrated own-price and cross-price demand elasticities always constitute a valid AUES matrix, regardless of whether the target own-price demand elasticities can form a valid AUES matrix. For instance, while the target own-price demand elasticities under the 3-sector setting do not constitute a valid AUES matrix, one can still try to calibrate the CDE demand system to those targets, and although the calibrated own-price demand
elasticities will not (and should not) match those invalid targets, they (the calibrated elasticities) will produce a valid AUES matrix (calibration results with the 3-sector setting are included in Table 3). As mentioned, what one cannot do is to use those invalid targets to assess the performance of CDE calibration.

The study finds that with the 4-sector, 5-sector, 8-sector, and 16-sector settings, although the target elasticities are valid, under both calibration methods, the calibrated own-price demand elasticities cannot match their target levels since the corresponding distance measure $d_\sigma$ is nonzero for each case. Nevertheless, in general, $d_\sigma$ gets smaller as the sectoral resolution increases (Table 3). Indeed, if one moves further to the 29-sector or 57-sector settings, a perfect match between the calibrated own-price demand elasticities and their target levels is possible since in both sectoral settings $d_\sigma = 0$ under the sequential approach and $d_\sigma \rightarrow 0$ under the maximum entropy approach. Also, as sectoral shares get smaller, the calibrated own-price demand elasticity $\sigma_{ij}$ will be closer to $-\alpha_i$ (Appendix E and Appendix F). These findings can also be explained by Proposition 3.3.1.

The results also show that for both calibration methods, the calibrated income elasticities of demand are matched to their target levels more precisely than the cases for calibrated own-price demand elasticities (Table 3). According to Propositions 3.3.2 and 3.3.3, under the same $\alpha_i$, a bigger $\eta_{ii}$ is more achievable, and under the same $\eta_{ii}$, a smaller $\alpha_i$ makes match $\eta_{ii}$ easier. For both calibration methods, if one looks at the substitution parameter $\alpha_i$ and the income demand elasticity target $\eta_{ii}$, they are strongly positively correlated (Figure 1), which means under the given data, a bigger $\alpha_i$ may be less of a problem since the relevant income demand elasticity target $\eta_{ii}$ also tends to be higher, and at the same time a smaller $\eta_{ii}$ also tends to be coupled with a smaller $\alpha_i$, which may raise the possibility of a precise income demand elasticity match. Another finding is that under both calibration methods, in general, $\eta_{ii}$ is matched more precisely when sectors become more disaggregated. This is mainly due to the fact that with more disaggregated sectors, the given data tend to yield smaller $\alpha_i$, the substitution parameters of the CDE demand, while there is no obvious trend for the income demand elasticity targets $\eta_{ii}$ (Figure 1), and Proposition 3.3.3 explains why $\eta_{ii}$ can be matched better in this case. In fact, under the sequential approach with 29-sector and 57-sector settings, a perfect match between calibrated and target income demand elasticity can be achieved. Finally, regardless of calibration methods, when there are multiple sectors with their own $\alpha_i$ close to 1, the calibrated income demand elasticities will converge to the same level, despite the fact that the target

---

7 The strong positive correlation between $\alpha_i$ and the $\eta_{ii}$ goes back to the fact that in the GTAP database, only the income elasticities of demand are estimated. The own-price demand elasticities are obtained from a Frisch parameter, using the assumption of additivity, as shown in Zeitsch et al. (1991). Interested readers may refer to p.14-6 in Hertel et al. (2014) for details.
elasticity levels are different (Appendix E and Appendix F). Proposition 3.3.4 provides an explanation to this observation.

<table>
<thead>
<tr>
<th>Table 3. Summary statistics, calibration performance, and validity of the AUES matrix.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Setting</td>
</tr>
<tr>
<td>Number of sectors</td>
</tr>
</tbody>
</table>

Summary statistics of targets

**Sectoral expenditure share**

- \( \theta_{\text{max}} \) 63.430\% 39.553\% 46.242\% 39.553\% 26.481\% 20.440\% 17.186\%
- \( \theta_{\text{min}} \) 11.779\% 11.779\% 3.432\% 2.279\% 0.92\% 0.017\% 0.0002\%

**Own-price demand elasticity**

- \( \sigma_{\text{max}} \) -0.4294 -0.4294 -0.2056 -0.1942 -0.1669 -0.0936 -0.0711
- \( \sigma_{\text{min}} \) -0.7658 -0.7800 -0.7608 -0.7800 -0.7974 -0.7957 -0.8095
- \( \sigma_{\text{avg}} \) -0.6201 -0.6542 -0.5807 -0.6022 -0.6093 -0.5331 -0.5294
- \( \sigma_{\text{std}} \) 0.1410 0.1363 0.2064 0.1813 0.1634 0.2269 0.2220

**Income demand elasticity**

- \( \eta_{\text{max}} \) 1.0502 1.0543 1.0513 1.0543 1.0987 1.0916 1.1190
- \( \eta_{\text{min}} \) 0.7300 0.7300 0.5504 0.5387 0.4874 0.3382 0.2704
- \( \eta_{\text{avg}} \) 0.9267 0.9569 0.8947 0.9181 0.9457 0.8851 0.8970
- \( \eta_{\text{std}} \) 0.1406 0.1326 0.1920 0.1708 0.1547 0.2344 0.2272

**Calibration: sequential**

- Match each \( \sigma_u \)? no no no no no yes yes
- Match each \( \eta_u \)? no no no no no yes yes

| \( d_s \) | 0.3526 | 0.1321 | 0.1879 | 0.1441 | 0.0406 | 0.0000 | 0.0000 |
| \( d_\eta \) | 0.2363 | 0.0080 | 0.0041 | 0.0081 | 0.0131 | 0.0000 | 0.0000 |

**Calibration: max entropy**

- Match each \( \sigma_u \)? no no no no no no no
- Match each \( \eta_u \)? no no no no no no no

| \( d_s \) | 0.3578 | 0.1322 | 0.1856 | 0.1427 | 0.0405 | 0.0024 | 0.0017 |
| \( d_\eta \) | 0.0815 | 0.0382 | 0.0971 | 0.0842 | 0.0243 | 0.0070 | 0.0051 |

**Validity of the AUES matrix based on target elasticities**

- \( \sigma_u^{\text{vec}} \) compatible to a NSD AUES? no yes yes yes yes yes yes
Figure 1. Summary statistics for income demand elasticity targets and substitution parameters.

Notes: “avg” and “std” stand for mean and standard deviation, respectively.

Source: The author’s calculation based on the GTAP 8 database.

3.4 Implementation

With the calibrated parameters, the study demonstrates how to put the CDE demand system into the multi-region and multi-sector CGE model of GTAP in GAMS. The original CGE model is constructed based on CES technologies for both production and final consumption. It includes a series of mixed complementary problems (MCP) (Mathiesen, 1985; Rutherford, 1995; Ferris and Peng, 1997) written in MPSGE, a subsystem of GAMS (Rutherford, 1999). To implement the CDE demand system, the CES expenditure function is dropped, and by declaring auxiliary variables and equations in MPSGE to formulate relevant MCP, three sets of conditions below are incorporated into the revised model:

- The equation for total expenditure. The total expenditure $c$ for purchasing one unit of utility (Equation (4)) is added into the model to form a MCP
with a complementarity variable \( c \). Note that in Equation (4), \( c \) is only implicitly defined. The purpose of this problem is to determine \( c \) jointly with other conditions. As previously mentioned, in the benchmark, both the utility level and price indices of commodities are normalized to unity.

- **The equation for final demand.** This equation (Equation (5)) is coupled with its complementarity variable, the activity level of final demand, to form a MCP. The problem is incorporated into the model to solve for the final demand of each commodity.

- **The zero profit condition for utility.** Let us denote the marginal cost and marginal revenue of utility (i.e., price of utility) by \( mcu \) and \( pu \), respectively. The zero profit condition of utility and the activity level of utility compose another MCP:

\[
mcu \geq pu; u \geq 0; (mcu - pu) \cdot u = 0; mcu = \frac{c \sum_i \beta_i e_i (1-a_i) u_i e_i (1-a_i) - 1 \left( \frac{p_i}{z} \right)^{1-a_i}}{\sum_i \beta_i (1-a_i) u_i e_i (1-a_i) \left( \frac{p_i}{z} \right)^{1-a_i}}
\]

Condition (18) states that in equilibrium, if the supply of utility \( u \) is positive, the marginal cost of utility \( mcu \) must equal the marginal revenue \( pu \), and if \( mcu \) is higher than \( pu \) in equilibrium, \( u \) must be zero.

With the commodity price being a complementarity variable, the market clearing condition of each commodity is also formulated as a MCP by comparing the commodity supply (determined by its zero profit condition) with the final demand shown above plus the intermediate demand derived from a CES cost function as the original GTAP in GAMS. Similarly, with the price of utility being the complementarity variable, the supply of utility combined with the demand for utility (income/\( pu \)) make up the MCP for the market clearing condition of utility.

The model code is provided in Appendix G, and interested readers may refer to Rutherford (1999) and Markusen (2013) for details of MPSGE.

For demonstration purposes, the study considers a setting with the aggregation level of two regions, four sectors, and one primary factor, and denotes this setting by “2r4s1f.” The two regions are USA and the rest of the world (ROW); four sectors are agriculture (agri), manufacturing (man), trade and transport (tran), and service (serv), following the sectoral classification for the setting “1r4s2f” presented in Table 2; and the only one primary factor is the aggregation of all primary factors of GTAP8. As before, prior to conduct and evaluate the CDE calibration, one

---

8 \( mcu \) in Condition (18) can be derived by taking the total derivative of Equation (4) with respect to \( u \) and \( c \) at a given commodity price vector.

9 With a very extreme sectoral expenditure share distribution (such as the final consumption structure of 0.0007, 30.7722, 0.00002, and 191.5950 billion US$ for coal, gas, crude oil, and refined oil products for the U.S. extracted from GTAP 8), as of GAMS version 23.7.3, the MCP solvers may encounter numerical issues in solving the model.
needs to check if the target elasticities under this setting (2r4s1f) are consistent to an AUES matrix that is NSD, and it can be shown that this is indeed the case (the NSD AUES matrix can be found numerically based on the subroutine presented in Appendix D). Taking the sequential approach as an example, Table 4 presents the calibration performance for the CDE demand system under the 2-region and 4-sector setting.

**Table 4. Performance of the CDE Calibration under the setting “2r4s1f”**

<table>
<thead>
<tr>
<th>Region: USA</th>
<th>( \theta_i )</th>
<th>( \alpha_i )</th>
<th>( \epsilon_i )</th>
<th>( \sigma_{ii}^R )</th>
<th>( \sigma_{ii}^M )</th>
<th>( \eta_i^R )</th>
<th>( \eta_i^M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>agri</td>
<td>0.04909</td>
<td>0.70623</td>
<td>2.00000</td>
<td>-0.67034</td>
<td>-0.68528</td>
<td>0.81292</td>
<td>0.99981</td>
</tr>
<tr>
<td>manu</td>
<td>0.18381</td>
<td>0.99999</td>
<td>0.00000</td>
<td>-0.82044</td>
<td>-0.81353</td>
<td>0.99514</td>
<td>1.00000</td>
</tr>
<tr>
<td>tran</td>
<td>0.20250</td>
<td>0.99999</td>
<td>0.00000</td>
<td>-0.85294</td>
<td>-0.79457</td>
<td>1.01152</td>
<td>1.00000</td>
</tr>
<tr>
<td>serv</td>
<td>0.56460</td>
<td>0.99999</td>
<td>3.37090</td>
<td>-0.85273</td>
<td>-0.42725</td>
<td>1.01372</td>
<td>1.00002</td>
</tr>
<tr>
<td>Distance</td>
<td>0.32082</td>
<td>0.04303</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Region: ROW</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>agri</td>
<td>0.14694</td>
<td>0.38159</td>
<td>3.95186</td>
<td>-0.39520</td>
<td>-0.39795</td>
<td>0.71822</td>
<td>0.71822</td>
</tr>
<tr>
<td>manu</td>
<td>0.27510</td>
<td>0.87414</td>
<td>3.04606</td>
<td>-0.62097</td>
<td>-0.63376</td>
<td>1.00104</td>
<td>1.00104</td>
</tr>
<tr>
<td>tran</td>
<td>0.25415</td>
<td>0.99999</td>
<td>3.38127</td>
<td>-0.70506</td>
<td>-0.71395</td>
<td>1.05431</td>
<td>1.07114</td>
</tr>
<tr>
<td>serv</td>
<td>0.32380</td>
<td>0.99999</td>
<td>14.65377</td>
<td>-0.72614</td>
<td>-0.63556</td>
<td>1.08436</td>
<td>1.07115</td>
</tr>
<tr>
<td>Distance</td>
<td>0.05218</td>
<td>0.01133</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: The author’s calculation based on the GTAP 8 database.

Let us parameterize the revised CGE model of GTAPinGAMS, based on calibrated parameters in Table 4. In the model, the aggregated primary factor along with the choice of the numeraire, which is the price for the aggregated primary factor, facilitate the identification of income effect. Now, to verify whether the CDE demand system is correctly implemented, the study will test if the outputs of the CGE model are consistent to the underlying calibrated elasticities under given price or income shocks. For example, with the shock on the price of agricultural product in the U.S., the first exercise changes the cost of final consumption for agricultural product in the U.S. exogenously to create the considered price shock. The goal is to calculate the uncompensated (Marshallian) own-price arc elasticity for the demand of agricultural product based on the model response, and see if the realized elasticity from the model output is consistent to the calibrated level.

Table 4 shows that while the target own-price elasticity of demand for the agricultural product is  \( \sigma_{ii}^{ct} = -0.6703 \), the calibrated own-price demand elasticity is  \( \sigma_{ii}^{ct} = -0.6853 \). The target is not matched precisely due to the fact that CDE demand is not own-price flexible. Besides, since with a nontrivial price shock

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10 For instance, in the revised CGE model of GTAPinGAMS, a 10% increase in the price of agricultural product is achieved by multiplying both vdfm(“agri”, c, “usa”) and vifm(“agri”, c, “usa”) in GTAPinGAMS by 1.1. Note that in GTAPinGAMS, both vdfm and vifm are redefined in a way such that both intermediate and final consumption are considered. For example, vdfm(i,g,r) means the domestically produced good i is used by g in region r, where g includes both users from industrial sectors and from final consumption.
imposed on the CGE model, it is more convenient to derive a “realized” uncompensated arc demand elasticity based on the model’s output, for comparison purposes, the study will also convert the calibrated own-price demand elasticity $\sigma_{ii}^{c}$, which is a compensated point elasticity, into an uncompensated arc demand elasticity with the same price shock so one can easily compare the realized level to the calibrated one. Note that while changes in expenditure share may change the point elasticity levels of both own-price and income elasticities of demand, for simplicity, this study uses the point elasticity level under the original expenditure share to derive the arc elasticity since changes in the structure of expenditure share are relatively moderate under the considered shocks. Besides, it is the calibrated income demand elasticity, rather than its target level, that is used in the calculation of the uncompensated own-price demand elasticity.

The calibrated uncompensated own-price demand elasticity, $\sigma_{ii}^{m} = -0.7344$ (a point elasticity), can be derived from $\sigma_{i}^{c}$, $\eta_{i}$, and $\theta_{i}$ based on the Slutsky equation presented in Equation (2). Let us consider the quantity index $\tilde{q}_{i} = q_{i}/\theta_{i}$ with the benchmark level $\tilde{q}_{i0} = 1$ since $q_{i0} = \theta_{i}$ (see Step 3 in Section 3.1). Because the percentage change in $\tilde{q}_{i}$ is equivalent to the percentage change in $q_{i}$, $\tilde{q}_{i}$ can replace $q_{i}$ in deriving the uncompensated (Marshallian) arc demand elasticity $\sigma_{ii}^{ma}$ — with both price and quantity indices normalized to unity, the arc elasticity $\sigma_{ii}^{ma}$ can be expressed as:

$$\sigma_{ii}^{ma} = \frac{\sigma_{i}^{c}}{\tilde{q}_{i}} \cdot \frac{\theta_{i}}{\eta_{i}}$$

(19)

where $p_{i}$ is the after-shock price level. When various price shocks of agricultural product are in place, the values for $\sigma_{ii}^{ma}$ (the calibrated Marshallian arc demand elasticity) and the realized arc elasticity levels $\sigma_{ii}^{mar}$ (derived from the model output) are both presented in Figure 2. Note that with the exogenous price shocks in agricultural product, in the new equilibrium, one may also observe changes in prices of other commodities relative to their pre-shock levels, and this will in turn affect the equilibrium food consumption level due to the existence of cross-price elasticities of food demand. The exogenous price shock may also induce an income effect as reflected by the change in total (final) expenditure level. Therefore, to calculate $\sigma_{ii}^{mar}$, the consumption index $\tilde{q}_{i}$ is adjusted such that it is net of the cross-price and income effects.12 The result in Figure 2 shows that, as expected, the larger

---

11 $\sigma_{ii}^{m} = \frac{dq_{i}}{q_{i}} / \frac{dp_{i}}{p_{i}}$. Therefore $\int q_{i}^{0} dq_{i} / q_{i} = \int \sigma_{i}^{m} dp_{i} / p_{i}$, and so $q_{i} = p_{i} \sigma_{ii}^{m}$.

12 To calculate the “realized” Marshallian arc own-price demand elasticity from the CGE model’s output, $\sigma_{ii}^{mar}$, the following steps are done: 1) step 1: calculate the substitution effect due to changes in all prices but the own-price based on prices change and the theoretical cross-price arc demand elasticity $\sigma_{ij}^{ma}$. The calculation is conditional on the original income level and the new (after shock) own-price level; 2) step 2: calculate the
the price shock, the more the arc elasticity deviates from the point elasticity $\sigma_{ii}^{m}$, which is the calibrated level without any price shock in the figure. Figure 2 also verifies that the uncompensated arc demand elasticity $\sigma_{ii}^{\text{mar}}$ calculated from the model output replicates its calibrated counterpart $\sigma_{ii}^{\text{ma}}$.

![Diagram](image)

**Figure 2.** Own-price arc elasticity for the demand of agricultural product in the U.S.

The study continues to examine the model response under various income shocks in the U.S. The shocks are created by changing the quantity of the aggregated primary factor of the U.S., which is just the real GDP level of the U.S. Since GDP is not only spent on private consumption, to calculate the income elasticities of various commodities based on the model response, one needs to use the percentage change in the portion of income dedicated to private consumption, or equivalently, the percentage change in total expenditure on private consumption. Following the same logic as Equation (19), the income arc demand elasticity can be written as:

$$\eta_{i}^{a} = \frac{c_{\eta_{i}+1}}{c_{\eta_{i}-1}} \cdot \frac{c+1}{c_{\eta_{i}+1}}; \ c \text{ is the after-shock income level} \ (20)$$

Under various levels of income shock, Equation (20) is used to convert the calibrated point elasticity into the calibrated arc elasticity, which serves as the benchmark for the comparison between the realized arc elasticity from model outputs and the calibrated level the model is given. Finally, as the previous example, the new equilibrium with an income shock will generally accompany changes in price levels of various commodities. This means that the resulting consumption levels will be contaminated by changes in prices, although these

---

income effect on top of changes in all prices; based on the theoretical elasticities $\eta_{i}^{a}$ and $\sigma_{ii}^{\text{mar}}$; 3) step 3: calculate the adjusted demand net of cross-price effect calculated from step 1 and income effect calculated from step 2, and then calculate $\sigma_{ii}^{\text{mar}}$ accordingly.
changes are usually small. The study accounts for this price effect and removes it from the consumption levels, and then for each commodity, uses the percentage change of the adjusted consumption level as the numerator of the income demand elasticity. The study accounts for this price effect and removes it from the consumption levels, and then for each commodity, uses the percentage change of the adjusted consumption level as the numerator of the income demand elasticity. Figure 3 demonstrates that for the final consumption of agricultural product, the realized income arc demand elasticity levels, as expected, replicate their calibrated counterparts. The two exercises presented here can be extended to other sectors and regions. For instance, with this 2-region and 4-sector setting, most of the calibrated income demand elasticities are close to one, although the target income demand elasticities can significantly deviate from one (Table 4). One exception is the income demand elasticity for the agricultural product in the rest of the world, $\eta_i^r = 0.7182$ (Table 4). For this elasticity, the calibrated elasticity and the realized numbers are matched as well (Figure 4).

![Figure 3. Income arc elasticity for the agricultural product demand in the U.S.](image)

13 To calculate the “realized” income arc demand elasticity from the model’s output, $\eta_i^{ar}$, the following steps are done: 1) step 1: calculate the expected quantity level due to pure income effect, based on changes in income level and the theoretical income arc demand elasticity $\eta_i^e$; 2) step 2: under the expected quantity derived from the pure income effect calculated in step 1, calculate the quantity changes due to changes in all prices, based on prices changes and the theoretical own-price and cross-price arc demand elasticity $\sigma_{ij}^{ma}$; 3) step 3: subtract the quantity change due to price effect calculated in step 2 from the observed quantity of model output, and then calculate $\eta_i^{ar}$ accordingly.
4. Conclusion

This paper provides the first comprehensive investigation of the circumstances under which the calibrated own-price and income elasticities of demand in a CDE demand system can be matched more accurately to their target levels. It finds that while the system is neither own-price nor income flexible, the elasticity match improves with smaller sectoral expenditure shares (i.e., higher sectoral resolution), lower target own-price demand elasticities, and higher target income demand elasticities. In any case, to understand the extent to which the elasticity targets are correctly represented in a CGE model, it is crucial to check whether the target elasticities are valid (i.e., compatible to aggregation conditions and a NSD AUES matrix), and disclose how well the calibrated elasticities match their target counterparts. Without having these inspections, when the calibrated elasticities deviate from target levels, it will not be possible to determine if that is due to targeting elasticity levels that are invalid, or if the inflexibility of the demand system is indeed the cause of the mismatch. For modelers who need to make sure the target income or own-price demand elasticities are reasonable, they can apply the program presented in Appendix D, which tests if the Engel aggregation is satisfied to ensure income demand elasticity targets are valid, and conditional on the satisfaction of Cournot aggregation, it also checks if a NSD AUES matrix can be found to verify the legitimacy of own-price demand elasticity targets.

In addition, using GTAPinGAMS, the study also incorporates the CDE demand system into a global CGE model written in MPSGE. Furthermore, price and income shocks are imposed on this revised GTAPinGAMS, and the model outputs successfully replicate the calibrated elasticities of the CDE demand system. Since implementing a CDE demand could be complicated and error-prone, future studies may examine if other CGE applications with the CDE demand can produce results in line with the calibrated elasticities, or they may investigate the flexibility and calibration performance of other demand systems—these issues are rarely
studied yet essential because a more flexible demand system can better represent a set of valid target elasticities observed empirically, and allows CGE models to produce results more consistent to the underlying characteristics of the economy.

Acknowledgements

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References


Appendix A. The CDE calibration program: sequential approach\textsuperscript{14}

\begin{verbatim}
$title Calibrate a CDE Demand System using GTAP data

$if not set ds $set ds g20
$if not set datadir $set datadir .\input\n$if not set wt $set wt 0
$include gtap8data_old

set     info    Information about this calibration /
       ds      "%ds%",
       datadir "%datadir%",
       workdir "%gams.workdir%"
       date    "%system.date%"
       time    "%system.time%" /;

alias(i,j,k);

set rr(r) dynamic subset of r;
rr(r) = no;

parameters
  z(i,r)        normalized price
  theta(i,r)    value share in final demand
  vafm(i,r)     Aggregate final demand,
  delta(i,j,r)  diagonal-one off-diagonal-zero
  sigma(i,j,r)  Allen partial elasticity of substitution
  epsilon_(i,r) targeted own-price elasticity of demand
  eta_(i,r)     targeted income elasticity of demand
  p0(i,r)       benchmark price index
  q0(i,r)       benchmark consumption level
  c0(r)         expenditure level
  mc0(r)        marginal cost when u is one
  weight(i,r)   weight for the square distance
  beta(i,r)     scale coefficient
  
; 

  vafm(i,r) = vdfm(i,"c",r)*(1+rtfd0(i,"c",r))+vifm(i,"c",r)*(1+rtfi0(i,"c",r));
  theta(i,r) = vafm(i,r) / (vom("c",r)*(1-rto("c",r)));
abort$sum(r, round(abs(1-sum(i,theta(i,r))),5)) "Shares do not add up."

  epsilon_(i,r)            = epsilon(i,r);
  eta_(i,r)                = eta(i,r);

$ontext
  theta(i,r)               = data(r,i,"shr");
  epsilon_(i,r)            = data(r,i,"vt");
  eta_(i,r)                = data(r,i,"eta_"");
$offtext

  p0(i,r)                  = 1;

\end{verbatim}

\textsuperscript{14} To run this program, one needs: 1) the GTAP 8 data in the gdx format (created by GTAPinGAMS); 2) the subroutine “gtap8data.gms,” which is also included in GTAPinGAMS, that reads data needed in the calibration program; 3) to type “gams cdecalib” under the DOS command prompt—this will use the default database “2r4s1f.gdx.” The environment variable “ds” can be used to overwrite the default database setting. Similarly, to run the maximum entropy approach calibration shown in Appendix B, one can type “gams cdeetp --ds=2r4s1f”.

\footnote{To run this program, one needs: 1) the GTAP 8 data in the gdx format (created by GTAPinGAMS); 2) the subroutine “gtap8data.gms,” which is also included in GTAPinGAMS, that reads data needed in the calibration program; 3) to type “gams cdecalib” under the DOS command prompt—this will use the default database “2r4s1f.gdx.” The environment variable “ds” can be used to overwrite the default database setting. Similarly, to run the maximum entropy approach calibration shown in Appendix B, one can type “gams cdeetp --ds=2r4s1f”.

28
\[ q_0(i,r) = \frac{\theta(i,r)}{p_0(i,r)}; \]
\[ c_0(r) = \sum_i p_0(i,r) \cdot q_0(i,r); \]
\[ \delta(i,j,r) = 0; \]
\[ \delta(i,j,r) \text{sameas}(i,j) = 1; \]
\[ \text{weight}(i,r) = \theta(i,r) \text{wt} 0 + (1/\text{card}(j)) \text{wt} ne 0; \]

* Finish reading data

*---------------------------------------------------------

variables
ALPHA(i,r) substitution coefficient
V(i,r) own-price elasticity of demand
E(i,r) expansion coefficient
ETAV(i,r) income elasticity of demand
OBJONE objective value for own-price elasticity calibration
OBJTWO objective value for income elasticity calibration
OBJTHR objective value for the dummy
OBJFOR objective value for the dummy
U(r) utility

* The equation "e_engel" deals with the case where eta from data doesn't
* satisfy the Engel aggregation equations

\[ e_v(i,rr) \text{ .. } V(i,rr) \text{theta}(i,rr) = \theta(i,rr) \cdot (2 \cdot \text{ALPHA}(i,rr) - \sum_k \theta(k,rr) \cdot \text{ALPHA}(k,rr)) - \text{ALPHA}(i,rr); \]
\[ e_objone \text{ .. } \text{OBJONE} = \sum_i V(i,rr) \cdot (\log(V(i,rr)/\epsilon(i,rr)) - 1); \]

model demandelas / e_v, e_objone /;

loop(r,
  rr(r) = yes;
  ALPHA.L(i,rr) = 0.5;
  ALPHA.UP(i,rr) = 0.999999;
  ALPHA.LO(i,rr) = 0.00001;
  V.L(i,rr) = \epsilon(i,rr);
  OBJONE.L = 0;
  solve demandelas using nlp minimizing OBJONE;
  sigma(i,j,r)\theta(i,r) = ALPHA.L(i,r) + ALPHA.L(j,r) - \sum_k \theta(k,r) \cdot \text{ALPHA}(L(k,r)) - \delta(i,j,r) \cdot \text{ALPHA}(L(i,r)/\theta(i,r));
  rr(r) = no;
);

* Step 2: Calibrating the income elasticity of demand

\[ e_eta(i,rr) \text{ .. } \text{ETAV}(i,rr) = \frac{1}{\text{sum}(k,\theta(k,rr) \cdot \text{E}(k,rr))} \cdot (\text{E}(i,rr) \cdot (1 - \text{weight}(i,r)) \cdot \text{weight}(i,r) \cdot \text{weight}(i,r) \cdot \text{weight}(i,r) \cdot \text{weight}(i,r)); \]
\( \text{ALPHA.L}(i,rr) + \sum(k, \theta(k,rr) \cdot E(k,rr) \cdot \text{ALPHA.L}(k,rr)) + \{ \text{ALPHA.L}(i,rr) - \sum(k, \theta(k,rr) \cdot \text{ALPHA.L}(k,rr)) \} \)

\( \text{e_objtwo} .. \)
\( \text{OBJTWO} = \sum(i,rr) , \text{weight}(i,rr) \cdot (\text{ETAV}(i,rr) - \text{eta}(i,rr)) \cdot (\text{ETAV}(i,rr) - \text{eta}(i,rr)) \)

\( \text{e_engel}(rr) .. \)
\( \sum(i, \theta(i,rr) \cdot \text{ETAV}(i,rr)) = 1; \)

\( \text{e_etaside}(i,rr) .. \)
\( (\text{ETAV}(i,rr) - 1) \cdot (\text{eta}(i,rr) - 1) \geq 0; \)

model incomeelas / e_objtwo, e_engel, e_eta, e_etaside /

loop(r,
   rr(r) = yes;
   E.LO(i,rr) = 1e-6;
   E.L(i,rr) = 1;
   ETAV.L(i,rr) = \eta(i,r);
   OBJTWO.L = 0;
);

* Step 3: Calibrating the scale coefficient BETA holding the utility level equals one

\( \text{beta}(i,r) = (q0(i,r)/(1-\text{ALPHA.L}(i,r))) / \sum(j, q0(j,r)/(1-\text{ALPHA.L}(j,r))); \)

\( \text{U.FX}(r) = 1; \)

parameter epsilon00(i,r) \text{ EPSILONV solved by the CDE calibration routine,} \)
parameter etav00(i,r) \text{ ETAV solved by the CDE calibration routine} \)
parameter alpha00(i,r) \text{ ALPHA solved by the CDE calibration routine} \)
parameter e00(i,r) \text{ E solved by the CDE calibration routine} \)
parameter u00(r) \text{ U solved by the CDE calibration routine} \)
parameter beta00(i,r) \text{ beta solved by the CDE calibration routine} \)
parameter mc00(r) \text{ Marginal cost} \)

\( \text{epsilon00}(i,r) = V.L(i,r); \)
\( \text{etav00}(i,r) = \text{ETAV.L}(i,r); \)
\( \text{alpha00}(i,r) = \text{ALPHA.L}(i,r); \)
\( \text{e00}(i,r) = \text{E.L}(i,r); \)
\( \text{u00}(r) = \text{U.L}(r); \)
\( \text{beta00}(i,r) = \text{beta}(i,r); \)
\( \text{mc00}(r) = \)

\( c0(r) \cdot \sum(i, \text{beta}(i,r) \cdot E(L(i,r) \cdot (1-\text{ALPHA.L}(i,r)) \cdot (U.L(r) \cdot (E.L(i,r) \cdot (1-\text{ALPHA.L}(i,r)))) \cdot (p0(i,r)/c0(r)) \cdot (1-\text{ALPHA.L}(i,r))) \cdot (U.L(r) \cdot (E.L(i,r) \cdot (1-\text{ALPHA.L}(i,r)))) \cdot (p0(i,r)/c0(r)) \cdot (1-\text{ALPHA.L}(i,r))) \)

parameter data model output;
\( \text{data}(i,r, "\text{theta}") = \text{theta}(i,r); \)
\( \text{data}(i,r, "\text{epsilon}") = \text{epsilon00}(i,r); \)
\( \text{data}(i,r, "\text{epsilonv00}") = \text{epsilon00}(i,r); \)
\( \text{data}(i,r, "\text{eta}") = \text{eta}(i,r); \)
\( \text{data}(i,r, "\text{etav}") = \text{ETAV.L}(i,r); \)
\( \text{data}(i,r, "\text{alpha}") = \text{ALPHA.L}(i,r); \)
data(i,r,"e") = E.L(i,r);
data(i,r,"beta") = beta(i,r);
data("mc",r,"mc") = mc00(r);
data(i,r,"weight") = weight(i,r);

execute_unload "..\output\cdecalib_%ds%_lnobj.gdx";
Appendix B. The CDE calibration program: maximum entropy approach

$title Calibrate a CDE Demand System using GTAP data

$if not set ds $set ds g20
$if not set datadir $set datadir .\input\n$if not set wt $set wt 0
$include gtap8data_old

set     info    Information about this calibration /
ds      "%ds%",
datadir "%datadir%",
workdir "%gams.workdir%"
date    "%system.date%"
time    "%system.time%" /;

alias(i,j,k);

set rr(r) dynamic subset of r;
rr(r) = no;

parameters
z(i,r)        normalized price
theta(i,r)    value share in final demand
vafm(i,r)     Aggregate final demand,
delta(i,j,r)  diagonal-one off-diagonal-zero
sigma(i,j,r)  Allen partial elasticity of substitution
epsilon_(i,r) realized compensated own-price elasticity of demand
p0(i,r)       benchmark price index
q0(i,r)       benchmark consumption level
c0(r)         expenditure level
mc0(r)        marginal cost when u is one
weight(i,r)   weight for the square distance
beta(i,r)     scale coefficient
uncelas(i,r)  targeted uncompensated own-price demand elasticity
incelas(i,r)  targeted income demand elasticity
bound         to avoid zero division
regind        index for region
result(r)     the real objective value for each r
;

bound  = 0.000001;
tt     = 1000;

vafm(i,r) = vdfm(i,"c",r)*(1+rtd0(i,"c",r))+vifm(i,"c",r)*(1+rto(i,"c",r));
theta(i,r) = vafm(i,r) / (vom("c",r)*(1-rto("c",r)));
abort$sum(r, round(abs(1-sum(i,theta(i,r))),5)) "Shares do not add up."
uncelas(i,r) = epsilon(i,r)-eta(i,r)*theta(i,r);
incelas(i,r) = eta(i,r);

p0(i,r) = 1;
q0(i,r) = theta(i,r)/p0(i,r);
c0(r) = sum(i,p0(i,r)*q0(i,r));
delta(i,j,r)$sameas(i,j) = 1;
weight(i,r) = theta(i,r)$%(wt% eq 0) + (1/card(j))$%(wt% ne 0);

*      Finish reading data
*      ---------------------------------------------------------

variables

ALPHA(i,r)        substitution coefficient
V(i,r)         own-price elasticity of demand
E(i,r)         expansion coefficient
ETAV(i,r)      income elasticity of demand
ALPHAETP(r)   entropy of ALPHA
EETP(r)        entropy of E
ALPHAPNT(r)   penalty for deviations in ALPHA
EPNT(r)        penalty for deviations in E
AHAT(i,r)      deviation of uncompensated own-price demand elasticity
AHATI(i,r)     deviation of income demand elasticity
UNCELASAC(i,r) actual uncompensated price elasticity
ALPHAMEAN(r)   mean substitution coefficient

OBJ;

equations
objective(r)
aalphaetpeq(r)
eeetpeq(r)
alphapnteq(r)
epnteq(r)
icmels(i,r)
ahatleq(i,r)
epsmnl(r)
uelasaceq(i,r)  actual price elasticity
ahateq(i,r)     error in elasticity
alphameaneq(r);

* Objective function: maximize the entropy relative to the unknown parameters
  of the cde function
OBJECTIVE(r)$((ord(r) eq regind)
  OBJ =E= -TT*(EPNT(R) + ALPHAPNT(R)) + EETP(R) + ALPHAETP(R);

* Penalty for errors in the expansion parameter
EPNTEQ(r)$((ord(r) eq regind)
  EPNT(r) =E= sum(i, theta(i,r)*sqr(AHAT1(i,r)));

* Deviation of income elasticity
AHAT1EQ(i,r)$((ord(r) eq regind)
  AHAT1(i,r) =E= ETAV(i,r) - incelas(i,r);

* Income elasticity expression found in Hanoch (1975) or Hertel et al (1990)
icmels(i,r)$((ord(r) eq regind)
EAVAV(i,r) =E= (1/(sum(j,theta(j,r)*E(j,r))))*(E(i,r)*((1-
ALPHA(i,r)))/sum(j,theta(j,r)*E(j,r)*ALPHA(j,r)))+
(ALPHA(i,r)-sum(j,theta(j,r)*ALPHA(j,r))));

* Penalty for errors in the substitution parameter
alphapnteq(r)$((ord(r) eq regind)
  ALPHAPNT(r) =E= sum(i, theta(i,r)*sqr(AHAT1(i,r)));

* Deviation of uncompensated own-price demand elasticity
ahateq(i,r)$((ord(r) eq regind)
  AHAT(i,r) =E= UNCELASAC(i,r) - uncelas(i,r);

* This last constraint pertains to the uncompensated direct price elasticities
uelasaceq(i,r)$((ord(r) eq regind)
  UNCELASAC(i,r) =E= -(1-theta(i,r))*ALPHA(i,r) - theta(i,r)*E(i,r)
  + theta(i,r)*(ALPHA(i,r)*E(i,r) -
  SUM(j,theta(j,r)*ALPHA(j,r)*E(j,r)));

* Cross entropy of the expansion parameter
\[
\text{EETP}(r) = -\sum(i, \theta(i,r) \cdot E(i,r) \cdot \log(E(i,r)))
\]

* Normalize the expansion parameter

\[
\sum(i, \theta(i,r) \cdot E(i,r)) = 1;
\]

* Cross entropy of the substitution parameter

\[
\text{ALPHAETP}(r) = -\sum(i, \theta(i,r)) \cdot (\alpha(i,r) \cdot \log(\alpha(i,r)/\alpha\text{MEAN}(r)) + (1-\alpha(i,r)) \cdot \log(1-\alpha(i,r)));
\]

* Mean substitution parameter

\[
\alpha\text{MEAN}(r) = \sum(i, \theta(i,r) \cdot \alpha(i,r));
\]

* Variable bounds

\[
\alpha.LO(i,r) = \text{bound};
\alpha.L(i,r) = 0.5;
\alpha.UP(i,r) = 1.0 - \text{bound};
\alpha\text{MEAN}.L(r) = 0.5;
E.LO(i,r) = \text{bound};
E.L(i,r) = 1.0;
\text{UNCELASAC}.L(i,r) = \text{uncelas}(i,r);
ETAV.L(i,r) = \text{incelas}(i,r);
\]

alias (r, rreg);

model cdent /all/;
loop (rreg,
regind = ord(rreg);
solve cdent using nlp maximizing obj;
display obj.l;
result(rreg) = obj.l + \text{tt} \cdot \left( \sum(i, \theta(i,rreg) \cdot \text{power}(\text{ahat}.l(i,rreg), 2) + \sum(i, \theta(i,rreg) \cdot \text{power}(\text{ahat}.1(i,rreg), 2)) \right);
\epsilon_{i,r} = \text{UNCELASAC}.L(i,r) + \text{ETAV}.L(i,r) \cdot \theta(i,r);
);
execute_unload ".\output\cdeetp_ids%.gdx";
### Appendix C. Sectors in GTAP 8 database

<table>
<thead>
<tr>
<th>Notation used in Table 2</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>g01</td>
<td>PDR</td>
<td>Paddy rice</td>
</tr>
<tr>
<td>g02</td>
<td>WHT</td>
<td>Wheat</td>
</tr>
<tr>
<td>g03</td>
<td>GRO</td>
<td>Cereal grains nec</td>
</tr>
<tr>
<td>g04</td>
<td>V_F</td>
<td>Vegetables, fruit, nuts</td>
</tr>
<tr>
<td>g05</td>
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*Source: GTAP (2015).*
Appendix D. The program (in MATLAB) checking if elasticity targets are valid

```matlab
% Read EXCEL input: share; eps_target; eps_calib; eta_target; 
% eta_calib
data = xlsread('.\input\elastheta.xlsx','4x4','B3:F6');

% data in the worksheet "4x4"
sector share eps_target eps_calib eta_target eta_calib 
s01 0.1178 -0.4294 -0.4657 0.7300 0.8442 
s02 0.2479 -0.6650 -0.7201 0.9997 1.0000 
s03 0.3955 -0.7800 -0.5767 1.0543 1.0289 
s04 0.2388 -0.7424 -0.7445 1.0435 1.0289 
%
% Declare dimension
n = 4;

% Check Engel aggregation (variable engel = 1 must hold)
eta_target = data(1:n, 4:4);
theta = data(1:n, 1:1);
engel = theta'*eta_target;

% Create a diagonal matrix with diagonal terms being the own-
% price AUES elasticities
eps_target = data(1:n, 2:2);
theta_diag = diag(theta);
aues_diag = diag(inv(theta_diag)*eps_target);

% Initialize the determinants for checking ND (sa stores values 
of various determinants)
sa = zeros(n,1);
for i = 1:n-1
    sa(i) = (-1)^(i+1);
end

while sa(1)>0|sa(2)<0|sa(3)>0|abs(sa(4))>0.00000001
    % Empty aues from the previous run
    aues_off = zeros(n,n);
    % For each row create random variables no larger than the 
    % diagonal term/n
    for i = 1:n-3
        offi = (-1+2*rand)*abs(aues_diag(i,i))/n;
        for j = i+1:n-1
            aues_off(i,j) = offi;
            aues_off(j,i) = aues_off(i,j);
    end
```

36
aues = aues_diag + aues_off;

% Create the "A" (LHS coefficient) matrix for solving the unknowns
A = zeros(n,n);
for i = 1:n-1
    A(i,i) = theta(n,1);
    A(n,i) = theta(i,1);
end
A(n-2,n) = theta(n-1,1);
A(n-1,n) = theta(n-2,1);

% This incomplete aues matrix is suitable for finding "C" (RHS coefficient) matrix
C = -aues*theta;

% The unknowns are in "B" and are solved by A*B = C
B = inv(A)*C;

% Assign "B" to unknowns in aues, and now all aues unknowns are found
for i = 1:n-1
    aues(i,n) = B(i,1);
end
aues(n-2,n-1) = B(n,1);

% Assign the solved AUES unknowns (i,j) to their corresponding (j,i) elements
for i = 1:n
    for j= 1:n
        aues(j,i) = aues(i,j);
    end
end

% Check Cournot aggregation
cournot = aues*theta;

% Check NSD
for i = 1:n
    sa(i) = det(aues(1:i, 1:i)/10);
end
Appendix E. CDE calibration results: sequential approach

<table>
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<tr>
<th>Ir3s2f</th>
<th>θ</th>
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Appendix F. CDE calibration results: maximum entropy approach

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Appendix G. The CGE model with a CDE demand in GTAPinGAMS

$title Read GTAP8 Base data and Replicate the Benchmark in MPSGE
* To run the model, type, for example: gams mrtmge_cde --start=0.1 --end=20 -- step=0.1

* The following pre-assignment for ds will be used in a $gdxin command in gtap8data.gms
$if not set ds $set ds 2r4s1f
$if not set wt $set wt 0

* Sets, parameters declarations and assignments are done in gtap8data.gms
$include ..\build\gtap8data

set c(g) private consumption /c/;
set e(g) exogenous consumption /g, i/;

parameters
esub(g) Top-level elasticity in demand /C 1/
vcm(i,c,r) Tax included Armington good i for private consumption,
data(*,*,*) Output from cdecalib,
cde CDE calibration,
chkd(i,r) Check final expenditure D;

* Aggregate final demand (Armington good)
vcm(i,c,r) = vdfm(i,c,r)*(1+rtfd0(i,c,r))+vifm(i,c,r)*(1+rtfi0(i,c,r));

* Read the CDE coefficients
*execute_load "\input\cdecalib \ds% \wt%.gdx" data = data;
execute_load "\input\cdecalib \ds% lnobj.gdx" data = data;
cde(i,r,"alpha") = data(i,r,"alpha")
cde(i,r,"e") = data(i,r,"e")
cde(i,r,"beta") = data(i,r,"beta")
cde("utility",r,"u") = 1;
cde("mc",r,"mc") = data("mc",r,"mc")
$cde(i,r,"alpha")$(cde(i,r,"alpha") eq eps) = 0;
cde(i,r,"e")$(cde(i,r,"e") eq eps) = 0;
cde(i,r,"beta")$(cde(i,r,"beta") eq eps) = 0;
cde("utility",r,"u")$(cde("utility",r,"u") eq eps) = 0;
cde("mc",r,"mc")$(cde("mc",r,"mc") eq eps) = 0;

$ontext
$model:gtap8
$ssectors:
y(g,r)$ (not c(g) and vom(g,r)) ! Supply
m(i,r)$vim(i,r) ! Imports

To run this MPSGE program “mrtmge_cde.gms,” one needs to 1) place it inside the subdirectory “model” of GTAPinGAMS; 2) set either price shock or income shock within the loop; 3) set the output file name that distinguishes price shock from income shock; and 4) type, for example, “gams mrtmge_cde --start=0.1 --end=20 --step=0.1” under the DOS command prompt. With the default setting, this will produce 20 different price shocks for the agricultural product—the first shock will be created by multiplying both vdfm("agri",c,"usa") and vifm("agri",c,"usa") by 0.1, and for each following shock, the multiplicand increases by 0.1 compared to that in the previous shock.
$y_t(j) v_{tw}(j)$ ! Transportation services
$ft(f,r) (sf(f) and evom(f,r))$ ! Specific factor transformation
$yc(i,c,r) v_{cm}(i,c,r)$ ! Private consumption by commodity

$\texttt{commodities:}$
$p(g,r) v_{om}(g,r)$ ! Domestic output price
$pm(j,r) v_{im}(j,r)$ ! Import price
$pt(j) v_{tw}(j)$ ! Transportation services
$pf(f,r) v_{vm}(f,r)$ ! Primary factors rent
$ps(f,g,r) (sf(f) and vfm(f,g,r))$ ! Sector-specific primary factors
$pc(i,c,r) v_{cm}(i,c,r)$ ! Private consumption price

$\texttt{consumers:}$
$ra(r)$ ! Representative agent

$\texttt{auxiliary:}$
$TC(r)$ ! Expenditure for the CDE system
$U(r)$ ! Activity level of Utility
$D(i,r) v_{cm}(i,"c",r)$ ! Activity level of final consumption

* Sectoral output
$\texttt{prod:y(j,r) v_{om}(j,r)} s:esub(j)$ i.tl:esubd(i) va:esuba(j)
o:pt(j) q:vtw(j) a:ra(r) t:rto(j,r)
i:p(j,r) q:vdfm(i,j,r) p:(1+rtfd0(i,j,r)) i.tl: a:ra(r)
t:rtfd(i,j,r)
i:pm(i,r) q:vifm(i,j,r) p:(1+rtfi0(i,j,r)) i.tl: a:ra(r)
t:rtfi(i,j,r)
i:pf(sf,j,r) q:vfm(sf,j,r) p:(1+rtf0(sf,j,r)) va: a:ra(r)
t:rtf(sf,j,r)
i:pm(f,j,r) q:vdfm(i,f,j,r) p:(1+rtfd0(i,f,j,r)) i.tl: a:ra(r)
t:rtfd(i,f,j,r)
i:pm(mf,j,r) q:vdfm(i,mf,j,r) p:(1+rtfd0(i,mf,j,r)) i.tl: a:ra(r)
t:rtfd(i,mf,j,r)
i:pm(i,r) q:vdfm(i,c,r) p:(1+rtfd0(i,c,r)) i.tl: a:ra(r)
t:rtfd(i,c,r)
i:pm(i,r) q:vifm(i,c,r) p:(1+rtfi0(i,c,r)) i.tl: a:ra(r)
t:rtfi(i,c,r)
i:pm(i,r) q:vcm(i,c,r) p:(1+rtfd0(i,c,r)) a:ra(r)
t:rtfd(i,c,r)
i:pm(i,r) q:vifm(i,c,r) p:(1+rtfi0(i,c,r)) a:ra(r)
t:rtfi(i,c,r)
i:pm(i,r) q:vcm(i,c,r) p:(1+rtfd0(i,c,r)) a:ra(r)
t:rtfd(i,c,r)

* Government consumption and investment (exogenous consumption)
$\texttt{prod:y(e,r) v_{om}(e,r)} s:esub(e)$ i.tl:esubd(i)
o:pt(e) q:vtw(e) a:ra(r) t:rto(e,r)
i:p(e,r) q:vdfm(i,e,r) p:(1+rtfd0(i,e,r)) i.tl: a:ra(r)
t:rtfd(i,e,r)
i:pm(i,r) q:vifm(i,e,r) p:(1+rtfi0(i,e,r)) i.tl: a:ra(r)
t:rtfi(i,e,r)

* Private consumption: new
* Level 1: Armington good of commodity i
$\texttt{prod:yc(i,c,r) v_{cm}(i,c,r)} s:esubd(i)$
o:pc(i,c,r) q:vcm(i,c,r) a:ra(r)
t:rto(c,r)
i:p(i,r) q:vdfm(i,c,r) p:(1+rtfd0(i,c,r)) a:ra(r)
t:rtfd(i,c,r)
i:pm(i,r) q:vifm(i,c,r) p:(1+rtfi0(i,c,r)) a:ra(r)
t:rtfi(i,c,r)

* Level 2: Aggregate various goods to a single consumption good c. This is where we need to work on for CDE.
* Let's temporarily remove the declaration of y(c,r), and move the sources and sinks in this block to demand block.
* This strategy is similar to linking the top-down and bottom-up.
* Now this is moved to the representative agent block.

$\texttt{prod:yt(j) v_{tw}(j)} s:1$
o:pt(j) q:vtw(j)
i:p(j,r) q:vst(j,r)
$\texttt{prod:m(i,r) v_{im}(i,r)} s:esubm(i)$ s.tl:0
o:pm(i,r) q:vim(i,r)
\[ \begin{align*}
i: & p(i,s) \quad q: vxm(i,s,r) \quad p: pvxmd(i,s,r) \quad s.t.: \quad a: ra(s) \quad t:= \\neg \quad rtxs(i,s,r) \quad a: ra(r) \quad t: = \quad rtms(i,s,r) \\
i: & pt(j) \quad q: vtw(j,i,s,r) \quad p: pvtrw(i,s,r) \quad s.t.: \quad a: ra(r) \quad t:= \\neg \quad rtms(i,s,r)
\end{align*} \]

\[
\text{prod:} ft(sf,r) \quad evom(sf,r) \quad t: etrae(sf) \\
o: ps(sf,j,r) \quad q: vfm(sf,j,r) \quad i: pf(sf,r) \quad q: evom(sf,r)
\]

\[
\text{demand:} ra(r) \\
d: p(\text{"c"},r) \quad q: vom(\text{"c"},r) \\
e: p(\text{"g"},r) \quad q: \{\neg vom(\text{"g"},r)\} \\
e: p(\text{"i"},r) \quad q: \{\neg vom(\text{"i"},r)\} \\
e: pf(f,r) \quad q: evom(f,r) \\
e: p(c,r) \quad q: vom(c,r) \quad r: U(r) \\
e: pc(i,c,r) \quad q: \{\neg vom(i,c,r)\} \quad r: D(i,r)
\]

\[
\text{constraint:} TC(r) \\
\sum(i,cde(i,r,\text{"beta"})*(cde(\text{"utility"},r,\text{"u"})*U(r))***(cde(i,r,\text{"e"})*(1-cde(i,r,\text{"alpha"}))))**
(PC(i,"c",r)/TC(r))**(1-cde(i,r,\text{"alpha"}))) = etrae(sf)
\]

\[
\text{constraint:} U(r) \\
TC(r)* \sum(i,cde(i,r,\text{"beta"})*cde(i,r,\text{"e"})*(1-cde(i,r,\text{"alpha"}))*\{U(r)**(cde(i,r,\text{"e"})*(1-cde(i,r,\text{"alpha"}))-1)\})*PC(i,"c",r)/TC(r)***(1-cde(i,r,\text{"alpha"})}) \\
e= \text{data("mc",r,"mc")}*\text{(cde(i,r,\text{"beta"})*(1-cde(i,r,\text{"alpha"})})*PC(i,"c",r)/TC(r)***(1-cde(i,r,\text{"alpha"})})
\]

\[
\text{constraint:} D(i,r)$ \quad \text{vcm(i,"c",r)}/\text{vom(\text{"c"},r)}*D(i,r)*\sum(j,cde(j,r,\text{"beta"})*\{U(r)**(1-cde(j,r,\text{"alpha"})})*cde(j,r,\text{e")})*\{1-cde(j,r,\text{"alpha"})})*PC(j,"c",r)/TC(r)***(1-cde(j,r,\text{"alpha"})}) \\
e= \text{(cde(i,r,\text{"beta"})*(1-cde(i,r,\text{"alpha"})})*cde(i,r,\text{e")})**(1-cde(i,r,\text{"alpha"})})*PC(i,"c",r)/TC(r)***(1-cde(i,r,\text{"alpha"})})
\]

\[
\text{offtext}
\]

\[
\text{sysinclude mpsgeset gtap8}
\]

\[
\begin{align*}
TC.L(r) & = 1; \\
TC.Lo(r) & = 0.000001; \\
U.L(r) & = 1; \\
U.Lo(r) & = 0.000001; \\
D.L(i,r) & = 1; \\
D.Lo(i,r) & = 0.000001; \\
PF.FX("primary",\"usa") & = 1;
\end{align*}
\]

\[
\text{gtap8.workspace} = 128; \\
\text{gtap8.iterlim} = 0; \\
\text{sysinclude gtap8.gen}
\]

\[
solve \text{gtap8 using mcp;}
\]

\[
chkd(i,r) = vcm(i,"c",r)/\text{vom(\text{"c"},r)}*D.L(i,r) \\
- \{cde(i,r,\text{"beta"})*(U.L(r)**((1-cde(i,r,\text{"alpha"})})*cde(i,r,\text{"e"})})**(1-cde(i,r,\text{"alpha"})})*PC.L(i,"c",r)/TC.L(r)***(1-cde(i,r,\text{"alpha"})}) \\
/\sum(j,cde(j,r,\text{"beta"})*\{U.L(r)**((1-cde(j,r,\text{"alpha"})})*cde(j,r,\text{e")})*\{1-cde(j,r,\text{"alpha"})})*PC.L(j,"c",r)**(1-cde(j,r,\text{"alpha"})})*TC.L(r)**(1-cde(i,r,\text{"alpha"})}
\]
execute_unload ".\output\mrtmge_cde_ref_ds=%ds%.gdx"

* The code below is for testing whether the model's realized elasticities equal
the calibrated levels it is given to

$if not set step $set step 0
set x shock level /1*
parameters
step step of the shock level,
start initial shock coefficient,
vdfm0 vdfm value from GTAP,
vifm0 vifm value from GTAP,
evom0 evom value from GTAP,
pfx realized PF with shock level x,
pxc realized PC over PF with shock level x,
dx realized D with shock level x,
theta_i final consumption expenditure share,
eta_i calibrated income demand point elasticity,
priexp total private expenditure,
priexpi total private expenditure index,
etai_a calibrated arc income demand elasticity,
sigma calibrated AUES price demand elasticity (point elasticity),
delta(i,j,r) diagonal-one off-diagonal-zero,
sigma_c calibrated compensated price demand elasticity (point elasticity),
sigma_m calibrated Marshallian price demand elasticity (point elasticity),
sigma_mar calibrated Marshallian price demand elasticity (arc elasticity),
dxn realized D with shock level x net of prices & income effects,
sigma_mar realized Marshallian price elasticity (arc elasticity),
dxn realized D with shock level x net of prices & income effects,
dxn realized D with shock level x net of prices & income effects,
dxn realized D with shock level x net of prices & income effects,
eta_i_ar realized arc income demand elasticity;
alias(i,k);

* Assign start and step in the command line using environment variables
start = %start%;
step = %step%;

* Read the shares and calibrated elasticities
theta_i = data(i,r,theta);
eta_i = data(i,r,etav);

* Store the original vdfm, vifm, and evom in GTAP
vdfm0 = vdfm(i,c,r);
vifm0 = vifm(i,c,r);
evom0 = evom(f,r);

* Step 1: Calculate the Marshallian price demand elasticity (point elasticity)
delta(i,j,r) = 0;
delta(i,j,r)$sameas(i,j) = 1;
sigma(i,j,r) = cde(i,r,\text{"alpha"})+cde(j,r,\text{"alpha"})-sum(k,
theta_i(k,r,\text{"theta"})*cde(k,r,\text{"alpha"}))
-delta(i,j,r)*cde(i,r,\text{"alpha"})/theta_i(i,r,\text{"theta"});
sigma_c(i,j,r) = sigma(i,j,r)*theta_i(i,r,\text{"theta"});
sigma_m(i,j,r) = sigma_c(i,j,r);
etai(i,r,etav)*theta_i(i,r,\text{"theta"});

loop(x,
    * Consumer's price shock:
vdffm("agri",c,"usa") = vdfm0("agri",c,"usa")*(start+(ord(x)-1)*step);
vdffm("agri",c,"usa") = vdfm0("agri",c,"usa")*(start+(ord(x)-1)*step);

* Endowment shock:
   *evom(f,"usa") = evom0(f,"usa")*(start+(ord(x)-1)*step);

* Avoid raise 0 by a negative number in the third auxiliary equation
   PC.LO(i,"c",r) = 0.000001;

$include gtap8.gen
solve gtap8 using mcp;

* Step 2: Within the loop, derive the calibrated arc elasticities associated with the shock

** Marshallian price demand elasticity (arc elasticity)
   pfx(r,x) = PF.L("primary",r);
pfx(i,r,x) = PC.L(i,"c",r)/pfx(r,x);
sigma_ma(i,j,r,x)$(pcx(j,r,x) ne 1) = (pcx(j,r,x)**sigma_m(i,j,r)+1)/(pcx(j,r,x)**sigma_m(i,j,r)+1);
sigma_ma(i,j,r,x)$(pcx(j,r,x) eq 1) = sigma_m(i,j,r);

** Income demand elasticity (arc elasticity)
   priexp(r,x) = sum(i,pcx(i,r,x)*D.L(i,r)*vcm(i,"c",r));
priexpi(r,x) = priexp(r,x)/sum(i,vcm(i,"c",r));
   eta_i_a(i,r,x)$(priexpi(r,x) ne 1) = (priexpi(r,x)**eta_i(i,r,"etav")-1)/(priexpi(r,x)-1)
   * (priexpi(r,x)+1)/(priexpi(r,x)**eta_i(i,r,"etav")+1);
   eta_i_a(i,r,x)$(priexpi(r,x) eq 1) = eta_i(i,r,"etav");

* Step 3-1: Calculate the substitution effect due to changes in PC-others|original income; after shock PC-own
   dx(i,r,x) = D.L(i,r);
cds(i,r,x) = sum(i,j$(not sameas(i,j)), (pcx(j,r,x)-1)/((pcx(j,r,x)+1)/2)*sigma_ma(i,j,r,x)*pcx(j,r,x)**sigma_m(i,j,r)+1)/(pcx(j,r,x)**sigma_m(i,j,r)+1);

* Step 3-2: Calculate the income effect on top of changes in all PC
   cdi(i,r,x) = (priexpi(r,x)**eta_i(i,r,"etav")-1)/(pcx(i,r,x)**eta_i(i,r,"etav")+1);

* Step 3-3: Calculate the adjusted demand net of cross-price effect and income effect
   dxn(i,r,x) = dx(i,r,x)-cds(i,r,x)-cdi(i,r,x);
sigma_mar(i,j,r,x)$(pcx(j,r,x) ne 1) = (dxn(i,r,x)-1)/((dxn(i,r,x)+1)/2)
   /((pcx(j,r,x)-1)/((pcx(j,r,x)+1)/2));

* Step 3-4: Calculate expected quantity level due to pure income effect
   eqi(i,r,x) = (priexpi(r,x)**eta_i(i,r,"etav");

* Step 3-5: Based on the expected quantity derived from pure income effect, calculate the quantity changes due to changes in prices
   cqp(i,r,x) = sum(j, (pcx(j,r,x)-1)/((pcx(j,r,x)+1)/2)*sigma_ma(i,j,r,x)*eqi(i,r,x));

* Step 3-6: Subtract quantity change due to price effect from the observed quantity
   dxi(i,r,x) = dx(i,r,x) - cqp(i,r,x);

* Step 3-7: Calculate the realized income demand elasticity
eta_{i,r}(i,r,x) = \frac{(dx_i(i,r,x)-1)}{((dx_i(i,r,x)+1)/2)}/\frac{(pri_{exp}(r,x)-1)}{((pri_{exp}(r,x)+1)/2)};

execute_unload "./output/mrtmge_cde_policy_ds-%ds%_priceshock-%step%.gdx";
*execute_unload "./output/mrtmge_cde_policy_ds-%ds%_incomeshock-%step%.gdx";
};