Odd-Parity Superconductivity near an Inversion Breaking Quantum Critical Point in One Dimension

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Odd-Parity Superconductivity near an Inversion Breaking Quantum Critical Point in One Dimension

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We study how an inversion-breaking quantum critical point affects the ground state of a one-dimensional electronic liquid with repulsive interaction and spin-orbit coupling. We find that regardless of the interaction strength, the critical fluctuations always lead to a gap in the electronic spin sector. The origin of the gap is a two-particle backscattering process, which becomes relevant due to renormalization of the Luttinger parameter near the critical point. The resulting spin-gapped state is topological and can be considered as a one-dimensional version of a spin-triplet superconductor. Interestingly, in the case of a ferromagnetic critical point, the Luttinger parameter is renormalized in the opposite manner, such that the system remains nonsuperconducting.

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Introduction.—Enhancement of superconductivity in the vicinity of a quantum critical point (QCP) has been studied extensively in strongly correlated materials [1–4]. Theoretically, this phenomenon was studied for the case of a nematic [5–9], charge ordering [10], and antiferromagnetic QCP [11–13]. Recently, it was proposed that the quantum fluctuations associated with an inversion-breaking QCP will also lead to superconductivity in systems with strong spin-orbit coupling [14,15]. The simplest example would be a paraelectric metal undergoing a quantum transition into an electrically polarized state (i.e., a “ferroelectric metal” [16,17]). This kind of transition has been observed in metallic compounds such as SrTiO$_3$ [18] and Cd$_2$Re$_2$O$_7$ [19–21], and was theoretically associated with superconductivity [14,15,22]. The question is how the inversion-breaking QCP affects the electronic states in these materials.

Fluctuations near an inversion-breaking QCP have been proposed to give rise to odd-parity superconductivity [14,15]. An odd-parity superconductor is formed when the pair wave function is odd under inversion, which typically leads to topological superconductivity [23,24], for example, the $B$ phase of He-3 [25].

Evidence for odd-parity superconductivity were found in doped Bi$_2$Se$_3$ [26–29], via NMR [30] and specific heat [31] measurements, which agree with theoretical proposals [23,32,33]. The superconducting state was shown to be correlated with structural transitions induced by external pressure [34], which may also be induced internally by the doping process [35]. Thus, it is interesting to understand the interplay between structural transitions and superconductivity in topological materials.

In this Letter we study the impact of an inversion breaking structural transition on metallic states in one dimension. The advantage of one dimension is that we have a good description of the interacting electronic state in terms of a Luttinger liquid (LL). It is important to note that one-dimensional superconductivity differs from higher dimensional superconductivity. Fluctuations prevent the establishment of true long-range order; instead, the order parameter exhibits only power law correlations, while spin excitations become gapped. This state is known as a Luther-Emery liquid (LEL) [36].

For concreteness we study a specific model. The model consists of two electronic wires coupled to a soft transverse optical phonon that undergoes a transition into a polarized state, where it breaks inversion between the wires. We note, however, that the low-energy theory we obtain is generic and describes any spin-orbit coupled system, which is coupled to an inversion breaking transition in one dimension. We tune the phonon through an inversion-breaking QCP and find that despite the repulsive electron-electron interactions there is always a region, close enough to the critical point, where a LEL is formed. The electron-electron interactions are crucial for the formation of the gap; they provide a backscattering term, which becomes relevant near the critical point. We also show that the most divergent superconducting order parameter is odd under inversion, and, therefore, we identify this state as the one-dimensional version of an odd-parity superconductor. Interestingly, this type of LEL was shown to be a gapless topological state protected by time-reversal symmetry [37], and is also related to gapped topological superconducting phases with time-reversal symmetry [38–41]. Note that an analogous construction with a ferromagnetic QCP, where time-reversal symmetry is broken instead of inversion, does not lead to superconductivity.

Model.—We consider a minimal model for a one-dimensional metal with inversion symmetry and spin-orbit coupling. The model consists of two wires, which are interchanged under inversion and thus have Rashba spin-orbit coupling of opposite signs, as shown in Fig. 1(a). The structural inversion-breaking transition is considered to be due to an ionic distortion between these wires, represented by the gray sites in Fig. 1(a). The electrons are coupled to...
the ions through the local deformation potential generated by the distortion.

We now elaborate on each ingredient of the model starting with the two fermionic wires, which are described by the Lagrangian

$$L_f = \sum_{j=1,2} \psi_j^\dagger \left( \partial_t \psi_j - \frac{\partial^2}{2m} + \alpha(-i\partial_t)\sigma^\nu - \mu \right) \psi_j$$

$$- t_{\perp}(\psi_1^\dagger \psi_3 + \psi_3^\dagger \psi_1) + V_{\text{int}},$$

(1)

where $$\psi_1 = (\psi_{1^\uparrow}, \psi_{1^\downarrow})^T$$ and $$\psi_2 = (\psi_{2^\uparrow}, \psi_{2^\downarrow})^T$$ are the fermionic fields of the two wires, $$m$$ is the band mass, $$\alpha$$ is the strength of Rashba spin-orbit coupling, and $$\pm$$ corresponds to $$j = 1, 2$$, respectively. $$\mu$$ is the chemical potential. The term $$V_{\text{int}}$$ describes a generic $$S_z$$ conserving local repulsive interaction, and $$\sigma^\nu$$ are the Pauli matrices in the spin space.

Tunneling between the wires $$t_{\perp}$$ opens a gap at the $$\Gamma$$ point, as shown in Fig. 1(b). We consider the case $$|\mu| < t_{\perp}$$, where only two points cross the Fermi energy. These two modes are denoted by their helicity $$\nu = \pm$$, which is interchanged under inversion.

Next, we consider the phonon mode that becomes soft at the QCP. This mode describes the motion of the ions located in between the two fermionic wires, as shown in Fig. 1. These ions are localized, but may fluctuate around their equilibrium positions. We consider tunneling these ions to a critical point where they become soft and condense in a different configuration, which breaks the inversion symmetry between the wires [16].

Among the three (one longitudinal and two transverse) phonon modes, only the transverse modes become soft at the transition [42]. The transverse motion is decomposed into two inversion-breaking polarizations, $$\hat{y}$$ and $$\hat{z}$$ [see Fig. 1(c)]. The electronic wires, lying in the $$xz$$ plane, lift the degeneracy between these modes, such that one can focus on the lower energy one, which becomes soft near the QCP. This mode can be described by a scalar field

$$\mathcal{L}_{\text{ph}} = -\lambda \varphi(\psi_1^\dagger \psi_1 - \psi_2^\dagger \psi_2).$$

(3)

In what follows we consider only this polarization. We note that in the case of the $$\hat{y}$$ polarization, shown on the right panel of Fig. 1(c), the lattice distortion generates a Rashba-like effect for tunneling between the wires, leading to a coupling of the form $$\mathcal{L}'_{\text{ph}} = -i\lambda \varphi(\psi_1^\dagger \sigma^\nu \psi_2 - \psi_2^\dagger \sigma^\nu \psi_1)$$. This expression can be reduced to Eq. (3) by a unitary transformation, so the case of $$\hat{y}$$ polarization leads to the same results.

**Analysis of the model near the QCP.—**To analyze the model given by Eqs. (1)–(3) we bosonize the two fermionic modes crossing the Fermi energy $$\varphi_{\varepsilon,\nu} = (F_{\varepsilon}/\sqrt{2\pi a}) \exp(iK_{\nu}x) \exp[-i(e\theta_{\nu} - \varphi_{\varepsilon})]$$, where $$F_{\varepsilon}$$ are Klein factors, $$a$$ is the short distance cutoff, $$\varepsilon = R, L$$ and $$\nu = \pm$$ denote the chirality and helicity of the modes, respectively ($$R, L$$ correspond to $$\nu = +, -,$$ respectively). Bosonic fields $$\theta_{\nu}$$ and $$\varphi_{\nu}$$ obey commutation relations $$[\theta_{\nu}(x), \varphi_{\nu'}(x')] = i\pi \delta_{\nu \nu'} \Theta(x - x')$$, where $$\Theta(x)$$ is the Heaviside step function. In these notations, $$\rho_{\nu} = -(1/\pi)\partial_x \theta_{\nu}$$ and $$J_{\nu} = (1/\pi)\partial_y \varphi_{\nu}$$ are the charge and current densities of the helical band $$\nu$$, respectively (the uniform part of density is omitted). Because of the helical structure of the bands, spin density $$\rho^s_{\nu}$$ and spin current density $$J^s_{\nu}$$ can be expressed through charge components as $$\rho^s_{\nu} = -\nu J_{\nu}$$ and $$J^s_{\nu} = \nu J_{\nu}$$.

After the bosonization, the fermionic Lagrangian (1) becomes a sum of two decoupled LLs describing charge and spin degrees of freedom:

$$\mathcal{L}_f = \mathcal{L}_\rho + \mathcal{L}_\sigma = \sum_{\eta = \rho, \sigma} \frac{\phi_\eta^\dagger G_\eta^{-1} \phi_\eta}{2(2\pi a)^2} \cos \sqrt{8} \theta_\eta,$$

(4)

where $$\phi_\eta^\dagger = 1/\pi \left( \begin{array}{c} -u_{\eta} K_\rho \partial_x^2 + i \partial_x \partial_y \\ i \partial_x \partial_y - u_{\eta} K_\sigma \partial_y^2 \end{array} \right)$$ is the bare Green’s function and $$\Phi_\eta = (\theta_\eta, \varphi_\eta)^T$$. Here we defined “charge” and “spin” variables $$\theta_\rho \equiv (1/\sqrt{2}) (\theta_+ + \theta_-)$$, $$\theta_\sigma \equiv (1/\sqrt{2}) (\theta_+ - \theta_-)$$, $$\phi_\rho \equiv (1/\sqrt{2}) (\phi_+ + \phi_-)$$, $$\phi_\sigma \equiv (1/\sqrt{2}) (\phi_+ - \phi_-)$$, and $$\phi_\theta \equiv (1/\sqrt{2}) (\theta_+ - \theta_-)$$ that obey the same commutation relations as $$\theta_\rho$$ and $$\phi_\rho$$. Coefficients $$K_\rho$$ and $$K_\sigma$$ are Luttinger parameters for the charge and spin sectors.
respectively. We consider repulsive fermionic interaction, which generically corresponds to $K_\rho < 1$ and $K_\sigma > 1$ [43]. Coefficients $u_\rho$ and $u_\sigma$ are velocities of charge and spin excitations. Expressions for $u_\eta$ and $K_\eta$ in terms of microscopic parameters of Eq. (1) can be found in Ref. [44].

The cosine in Eq. (4) originates from the BCS scattering processes, i.e., from the term $\psi_{R,+}^\dagger \psi_{L,+}^\dagger \psi_{L,-}^\dagger \psi_{R,-} + \text{H.c.}$, which describes pair hopping between the fermionic wires. For small bare $g$, the behavior of this term in the renormalization group (RG) sense is determined by the Luttinger parameter $K_\sigma$. In the case of repulsive interactions, $K_\sigma > 1$, the cosine term is irrelevant and flows to zero. Attractive interaction (corresponding to $K_\sigma < 1$), on the contrary, guarantees that $g$ flows to the strong coupling regime, and pins the variable $\theta_\sigma$ to one of the minima of the cosine. This state is known as the LEL [36] and is characterized by a spin gap.

We consider repulsive electron-electron interaction, so one could naively expect that the cosine term becomes irrelevant. However, as we show, close enough to the critical point the cosine always becomes relevant rendering the spin sector gapped, regardless of the strength of the repulsive interaction. To demonstrate it explicitly, we rewrite the coupling term (3) in the bosonized form

$$\mathcal{L}_b = -\sqrt{2} \pi \bar{\phi} \partial_x \phi_\sigma. \quad (5)$$

As expected, the phonon field couples to the total spin current density $J_\sigma = (\sqrt{2}/\pi) \partial_x \phi_\sigma$. It is important to note that the coupling to spin current, Eq. (5), vanishes in the absence of spin-orbit coupling [14]. In this case, the gaplessness of the spin sector of the fermions is protected by SU(2) symmetry [43].

Now all ingredients are ready for constructing the effective theory that elucidates the main results of this work. First, we integrate out the massive phonon mode Eq. (2). Formally, this procedure is equivalent to the shifting $\bar{\phi} = \phi - (\sqrt{2}/\pi) \bar{G}_\sigma \partial_x \phi_\sigma$, which demonstrates that the phonon field is locked to the spin current that is odd under inversion. As a result, the Lagrangian of the spin sector in Eq. (4) becomes

$$\mathcal{L}_\sigma \rightarrow \mathcal{L}_\sigma - \frac{\lambda^2}{\pi^2} \partial_x \phi_\sigma \bar{G}_\sigma \partial_x \phi_\sigma. \quad (6)$$

Note that because the real ordering transition occurs at a finite value of the tuning parameter $r$, the bosonic field $\phi$ is massive and, therefore, can be integrated out without generating nonanalyticities.

At long wavelength and low energy, the resulting effective theory (6) has the same form as Eq. (4) with a renormalized spin stiffness, which is suppressed by the critical phonon fluctuations. Consequently, the Luttinger parameter $K_\sigma$ and spin velocity $u_\sigma$ also become significantly suppressed:

$$\tilde{K}_\sigma(r) = K_\sigma \sqrt{1 - \frac{r_c}{r}} , \quad \tilde{u}_\sigma(r) = u_\sigma \sqrt{1 - \frac{r_c}{r}},$$

where $r_c \equiv 2 \lambda^2 / \pi u_\sigma K_\sigma$.

We identify two critical values of mass term $r$:

(i) First, for values of $r$ smaller than $r_\ast \equiv r_c / (1 - K_\sigma^2)$, the renormalized Luttinger parameter $\tilde{K}_\sigma(r)$ becomes smaller than 1. Therefore, $r_\ast$ marks the Berezinskii-Kosterlitz-Thouless (BKT) transition point between the gapped LEL and the gapless LL. We note that $r_\ast$ is the critical value for a vanishing $g$. The case of finite $g$ appears in the Supplemental Material [45].

We emphasize that, as opposed to the conventional scenario where the LEL is realized in systems with attraction between electrons (implying $K_\sigma < 1, g < 0$), in our theory it naturally appears in the presence of repulsive interaction, i.e., $g > 0$. This type of spin-gapped liquid was studied in detail in Refs. [48,49]. More recently, it was pointed out in Ref. [37] that a LEL with $g > 0$ is a gapless symmetry protected topological state. In this case, the most divergent superconducting correlations are of spin triplet type. To see this, one may compare the correlations of the triplet order parameter, $O_t = \psi_{R,+}^\dagger \psi_{L,+}^\dagger \psi_{L,-}^\dagger \psi_{R,-} + \text{H.c.}$, with those of the singlet, $O_s = \psi_{R,+}^\dagger \psi_{L,+}^\dagger + \psi_{L,+}^\dagger \psi_{R,-}^\dagger \propto e^{-i \sqrt{2} \phi_g} \sin \sqrt{2} \theta_\sigma$. For $g > 0$, the field $\theta_\sigma$ is pinned to $(\pi/\sqrt{8}) + (n\pi/\sqrt{2})$ ($n$ is the integer), such that the correlations of $O_s$ are exponentially suppressed while correlations of $O_t$ exhibit quasi-long-range order (note that here the operators are written in the helical basis where spin is locked to momentum). It should be noted, however, that in this case the most divergent order parameter is the spin-density wave $O_{SDW} = \psi_{R,+}^\dagger \psi_{L,-} - \psi_{R,-}^\dagger \psi_{L,+} \propto e^{i \sqrt{2} \phi_g} \sin \sqrt{2} \theta_\sigma$. This is due to the correlations in the charge sector which are controlled by $K_\rho < 1$ (repulsive interactions) [48,49]. However, in the last part of this Letter we couple an array of wires and show that the two-dimensional version of our model can realize odd-parity superconductivity.

(ii) Next, if the tuning parameter $r$ is further decreased, the system undergoes the second phase transition. Indeed, at $r \leq r_c$, the stiffness of the field $\phi_\sigma$ becomes negative, meaning that $r_c$ marks the classical transition point to the inversion-breaking phase where the system develops a spontaneous spin current. To stabilize the theory, as usual, the higher order term, $(\partial_x \phi_\sigma)^4$, which has been neglected in the derivation of Eq. (4) must be taken into account. This term may have different microscopic origin [50].

To understand the nature of the transition at $r = r_c$, we assume that in the vicinity of $r_c$, the system is already deep in the spin-gapped state, so that the cosine term in Eq. (6) can be expanded near one of its minima, leading to an effective gap $\Delta$. We will justify this assumption later, by explicitly calculating $\Delta$ using the variational principle.
In this case we can integrate out the field $\theta_\sigma$ in Eq. (6) and express the resulting Lagrangian in terms of the spin current, $J_\sigma = (\sqrt{2}/\pi) \partial_\tau \phi_\sigma$. In the long wavelength limit (which implies $r \gg q, \omega$ and $\Delta \gg u_\sigma q$), it becomes

$$
\mathcal{L}_\text{Ising} = \frac{1}{2} J_\sigma \left( -\frac{\partial_\tau^2}{\zeta} - \rho_\sigma \partial_\tau^2 + \alpha \right) J_\sigma + V J_\sigma^2,
$$

where $\zeta = 2 \Delta^2 r^2 / \pi u_\sigma K_\sigma (r^2 + \Delta^2 r_c)$, $\rho_\sigma = \pi u_\sigma K_\sigma r_c / 2r^2$, and $\alpha = \pi u_\sigma K_\sigma (1 - r_c/r)/2$. The Lagrangian (8) describes the Ising transition in 1 + 1 dimensions. Here $\alpha$ plays the role of the tuning parameter. The term $V$ is responsible for quantum fluctuations, which shift the transition point. We note that in the absence of particle-hole symmetry the Ising theory, Eq. (8), is also coupled to the charge sector by $\mathcal{L}_{\rho \sigma} = (\lambda_{\rho \sigma}/\pi) \partial_\tau \theta_\sigma^2$, which modifies the nature of the transition [51,52]. However, it couples the gapless-charge to gapped-Ising modes and thus does not shift the transition point or change the nature of the Ising phases.

We now contrast the results we have obtained for inversion-breaking transition with the ferromagnetic case considered in Ref. [53]. In the latter, the ferromagnetic fluctuations couple to the spin density as opposed to spin current, implying that $\partial_\tau \theta_\sigma$ substitutes $\partial_\tau \phi_\sigma$ in Eq. (5). The renormalized Luttinger parameter then equals $K_\sigma(r) = K_\sigma/\sqrt{1 - (r_c K_\sigma^2/r)}$, which is only enhanced in the vicinity of the QCP, making the cosine term even more irrelevant. Therefore, in the case of a ferromagnetic transition, the fermionic spin sector remains gapless all the way to the QCP, which is characterized by the dynamical exponent $z = 2$, as opposed to the Ising transition in our case. In this case the superconducting correlations are not divergent.

**Calculation of the spin gap.**—We now turn to explicitly calculate the magnitude of the spin gap, $\Delta$, as a function of the tuning parameter $r$. For this purpose we employ the variational principle, which is known to capture the qualitative behavior of the gapped LEL [43]. We introduce the variational action $\mathcal{L}_\nu = \frac{1}{2} \Phi^T \phi^T \phi^T \phi^T$, which represents the gap of spin excitations [the same $\Delta$ appears in Eq. (8)]. Therefore, we minimize the free energy corresponding to Eq. (6) with respect to the $\Delta$ [43,45]. On the ordered side of the transition $(r < r_c)$ the spin stiffness is negative. In this case we expand around the broken symmetry state, taking into account the higher order term presented in Eq. (8). Consequently, another BKT transition occurs on the ordered side, at $r = r_{es} \equiv r_c/[1 + (2 K_\sigma^2)^{-1}]$ [45].

We note that the variational breaks down at two points: near $r = r_s$ and near $r = r_c$. In the former, the gap vanishes like $\Delta \approx \sqrt{r} \exp(-\pi/\sqrt{1 - r/r_c})$, where $A$ is defined in the Supplemental Material [45]. In the latter, the quartic term $V$ in Eq. (8), which we have neglected in this calculation, becomes important in the regime $r - r_c \sim \rho_\sigma V/\zeta$ [54].

The results of the variational calculation [45] are summarized in Fig. 2, where we plot the gap $\Delta$ as a function of the tuning parameter $r$. Starting from the disordered side and reducing the tuning parameter $r$ towards the critical value $r_c$, the fermionic sector first undergoes a BKT transition into the LEL at $r = r_s$. Then, at $r = r_s$ the Ising transition occurs and inversion becomes spontaneously broken, where the gap reaches its maximal value. Finally, at $r = r_{es}$ there is another BKT transition to the gapless LL state.

**2D wire construction.**—We now extend our results to two dimensions by discussing the phase diagram of an array of wires, which are individually described by Eqs. (6) and the gapless charge sector. Close to the QCP and in the weak coupling limit, $\theta_\sigma$ is pinned on each wire and there is a finite gap $\Delta$. In this case, for weakly coupled wires we left with an array of LL in the charge sector, $L_j = \frac{1}{2} (\phi^T_\sigma)^2$, interacting by

$$
\mathcal{L}_{j,j+1} = \frac{1}{2 \pi} (W_\phi \partial_\tau \phi_\sigma \partial_\tau \phi_\sigma + W_\theta \partial_\tau \theta_\sigma \partial_\tau \theta_\sigma) + g_\phi \cos \sqrt{2} (\phi_\sigma^k - \phi_\sigma^{k+1}) + g_\theta \cos \sqrt{2} (\theta_\sigma^k - \theta_\sigma^{k+1}).
$$

$W_\phi$ and $W_\theta$ describe forward scattering (for repulsive interaction $W_\phi < W_\theta$). The cosine term $g_\phi$ is proportional to the strength of pair hopping between the wires and $g_\theta$ results form wire-number conserving processes.

This model was studied in Refs. [55–58]. It was shown that a superconducting phase, where $g_\phi$ is the most relevant perturbation, exists over a wide range of parameters in the case of spin-gapped wires [57]. In Fig. 3 we plot the phase diagram based on the scaling equations for $g_\phi$ and $g_\theta$ [57]. However, since the coupling $g$ in Eq. (4) is positive the spin variable $\theta_\sigma$ is locked to $\pm \pi/\sqrt{8}$. As a result the two
FIG. 3. Phase diagram obtained from the RG equations in Refs. [55,56]. The blue region denotes odd-parity superconductivity, white silver denotes the gapless sliding LL state, and pink denotes a spin ordered state.

possible gapped phases are the spin-density wave and odd-parity superconductor rather than the charge density wave and even-parity superconductor. The novel aspect of this result is that inversion breaking clearly enhances the odd-parity channel over the even-parity one.

Conclusions.—We have shown that a one-dimensional electronic liquid with strong spin-orbit coupling and repulsive interactions coupled to an inversion breaking QCP develops a spin gap close enough to the transition due to the establishment of a paired state. We have argued that this state indicates the emergence of odd-parity superconductivity near inversion breaking QCP. We have also shown that inversion breaking is distinct from the ferromagnetic transition, where the spin sector remains gapless.

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[50] This term originates from higher order interaction terms. For similar discussion see Ref. [53].