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Detailed Terms
Violation of the Leggett-Garg Inequality in Neutrino Oscillations

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The Leggett-Garg inequality, an analogue of Bell’s inequality involving correlations of measurements on a system at different times, stands as one of the hallmark tests of quantum mechanics against classical predictions. The phenomenon of neutrino oscillations should adhere to quantum-mechanical predictions and provide an observable violation of the Leggett-Garg inequality. We demonstrate how oscillation phenomena can be used to test for violations of the classical bound by performing measurements on an ensemble of neutrinos at distinct energies, as opposed to a single neutrino at distinct times. A study of the MINOS experiment’s data shows a greater than $6\sigma$ violation over a distance of 735 km, representing the longest distance over which either the Leggett-Garg inequality or Bell’s inequality has been tested.

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Perhaps one of the most counterintuitive aspects of quantum mechanics is the principle of superposition, which stipulates that an entity can exist simultaneously in multiple different states. Bell and others indicated how experiments could distinguish between classical systems and those that demonstrate quantum superposition [1,2]. Bell’s inequality concerns correlations among measurements on spatially separated systems. Leggett and Garg developed an analogous test that concerns correlations among measurements performed on a system at different times, and they extended this test to apply to macroscopic entities [3]. Sometimes referred to as the “time analogue” of Bell’s inequality, the Leggett-Garg inequality (LGI) allows for a complementary test of quantum mechanics while potentially avoiding some of the difficulties involved in performing a truly loophole-free test of Bell’s inequality [4–7]. See [8] for a recent review.

The original goal of LGI tests was to demonstrate macroscopic coherence—that is, that quantum mechanics applies on macroscopic scales up to the level at which many-particle systems exhibit decoherence [3,8–12]. For this reason, a major focus of recent LGI research has been scaling up to tests with macroscopic systems.

LGI tests have another purpose: to test “realism,” the notion that physical systems possess complete sets of definite values for various parameters prior to, and independent of, measurement. Realism is often encoded in hidden-variable theories, which allow for systems that are treated as identical according to quantum mechanics to be fundamentally distinguishable through a hidden set of parameters that they possess, such that any measurement on a system reveals a preexisting value [13]. LGI violations imply that such hidden-variable (or “realistic”) alternatives to quantum mechanics cannot adequately describe a system’s time evolution. Experiments using few-particle systems can test realism even if they do not directly address macrorealism [13–19].

Neutrino flavor oscillations, which are coherent in the few-particle limit, provide an interesting system with which to test the LGI. Neutrinos have been detected in three distinct “flavors,” which interact in particular ways with electrons, muons, and tau leptons, respectively. Flavor oscillations occur because the flavor states are distinct from the neutrino mass states; in particular, a given flavor state may be represented as a coherent superposition of the different mass states [20,21]. Neutrino flavor oscillations may be treated with the same formalism that is typically used to describe systems that classically disrupt the system, such as squeezed atomic states [22]. The major difference between neutrinos and these familiar systems, however, is that the coherence length of neutrino oscillations—the length over which interference occurs and oscillations may be observed—extends over vast distances, even astrophysical scales [23]. A LGI experiment using neutrino oscillations therefore presents a stark contrast to other types of LGI tests, which typically use photons, electrons, or nuclear spins, for which coherence distances are much more constrained [8].

Experimental violations of the LGI can lead to definitive conclusions about realism only if the measurement outcomes represent the underlying time evolution of the system. Invasive measurements, characterized either by wave function collapse or by experimental imperfections that classically disrupt the system, would prevent an experimenter from ruling out realistic alternatives to quantum mechanics, even in the face of an apparent violation of the LGI. Several experiments have worked to bypass this limitation by using indirect or weak measurements to probe the system [11,14,15].

In the case of neutrino flavor oscillations, it is possible to circumvent the problems posed by invasivity by performing measurements on members of an identically prepared ensemble; this obviates the issue of whether individual measurements influence one another. When combined with
a separate assumption of “stationarity”—such that the correlations between different measurements depend only
on the durations between them rather than on their individual times—the prepared-ensemble condition allows
one to test particular classes of realistic alternatives to quantum mechanics [8,9,13,17]. Although the idea of
testing the LGI and related measures of quantum entanglement using neutrino oscillations has been proposed in the
literature [24–26], we believe this is the first such empirical test to be performed.

Formalism and assumptions.—We consider a dichoto-
mic observable \( \hat{Q} \) (with realizations \( \pm 1 \)) that may be
measured at various times \( t_i \). The correlation between
measurements at times \( t_i \) and \( t_j \) can be written
\[
C_{ij} \equiv \langle \hat{Q}(t_i)\hat{Q}(t_j) \rangle ,
\]
where \( \langle \ldots \rangle \) indicates averaging over many trials. For
measurements at \( n \) distinct times, we may define the
Leggett-Garg parameter \( K_n \) as
\[
K_n \equiv \sum_{i=1}^{n-1} C_{i,i+1} - C_{n,1} .
\]
Realistic systems obey the Leggett-Garg inequality [3,8],
which for \( n \geq 3 \) is given by \( K_n \leq n - 2 \).

We may calculate an expected value for \( K_n \) according to
quantum mechanics, \( K_n^0 \), by time-evolving \( \hat{Q} \) under the
unitary operator \( U(t) \): \( \hat{Q}(t_i) = U(t_i)\hat{Q}U(t_i) \). When it is
possible to represent \( \hat{Q}(t_i) \) as \( \hat{Q}(t_i) = \hat{b}_i \cdot \hat{\sigma} \), where
\( \hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \) is a vector of two-by-two Pauli matrices,
then, for any complete set of normalized states \( |\phi\rangle \), the two-
time correlation function may be written
\[
C_{ij} = \frac{1}{2} \langle \phi | \{ \hat{Q}(t_i), \hat{Q}(t_j) \} |\phi\rangle = \hat{b}_i \cdot \hat{b}_j .
\]
We use the anticommutator \( \{ \hat{A}, \hat{B} \} \equiv \hat{A} \hat{B} + \hat{B} \hat{A} \) to avoid possible
time-ordering ambiguities, since \( \hat{Q}(t_i) \) need not commute with \( \hat{Q}(t_j) \).

In a quantum-mechanical system, violations of the LGI
arise due to the nonvanishing commutators of the operators
\( \hat{Q}(t_i) \) and \( \hat{Q}(t_j) \). If one artificially imposes that \( \hat{Q}(t_i) \) and
\( \hat{Q}(t_j) \) must commute (i.e., taking the limit \( \hbar \to 0 \)), then one
recovers the classical prediction for \( K_n \), which we denote
\( K_n^C \) and which has the compact expression
\[
K_n^C = \sum_{i=1}^{n-1} C_{i,i+1} - \prod_{i=1}^{n-1} C_{i,i+1} .
\]
Given that \( C_{ij} \) is real and \( |C_{ij}| \leq 1 \), we see that \( K_n^C \)
satisfies the LGI, whereas \( K_n^0 \leq n \cos(\pi/n) \) may violate the
LGI for particular angles \( \theta_j \equiv \arccos(\hat{b}_i \cdot \hat{b}_{i+1}) \). The
discrepancy between these two predictions provides an
opening for experimental testing and verification [8].

The original derivation of the LGI assumed that mea-
surements of \( \hat{Q} \) at various times \( t_i \) are made in a noninva-
sive manner [3]. The LGI may be derived instead under the
assumption of stationarity, such that the correlation func-
tions \( C_{ij} \) depend only on the time difference \( \tau = t_j - t_i \)
between measurements [8,9,13,17]. In this case, the bound
\( K_n \leq n - 2 \) applies to the class of realistic models that are
Markovian, for which the evolution of the system after
some time \( t \) is independent of the means by which the
system arrived in a given state at \( t \) [8]. We may then
consider measurements performed on distinct members of
an identically prepared ensemble, each of which begins in
some known initial state.

Stationarity allows for measurements made on distinct
ensemble members to mimic a series of measurements
made on a single time-evolving system. For example, in
order to construct \( K_3 \), we take advantage of the fact that \( C_{23} \)
in one system is equivalent to the correlation between
measurements separated by time \( \tau = t_3 - t_2 \) on a different
member of the ensemble.

The combination of the prepared-ensemble and station-
arity conditions therefore acts as a substitute for meas-
surement schemes intended to be noninvasive (e.g., weak
measurements), because wave function collapse and
classical disturbance in a given system do not influence
previous or subsequent measurements on distinct members
of the ensemble [10,13]. Unlike the assumption of
noninvasive measurability, moreover, the stationarity
condition may be subjected to independent testing
[9,13,17]. As we will see, both the prepared-ensemble
and stationarity conditions may be fulfilled in measure-
ments of neutrino flavor oscillations. Furthermore, these
two conditions enable us to analyze measurements on
separate groups of particles (directly analogous to mea-
surements on spatially separated systems in tests of Bell's
inequality), circumventing the recent criticism of the LGI
whereby measurements on a single system at later times
may be influenced by the outcomes of earlier measure-
ments on that same system [27].

LGI violation using neutrinos.—The standard model
includes three distinct neutrino flavors. However, the
energies and distances on which we focus single out
oscillations almost entirely between two flavor states.
Hence, we adopt a two-state approximation. In the rela-
tivistic limit, oscillations between these two states may be
treated with the Blöch sphere formalism [28], which
geometrically represents the space of pure states of a
generic two-level system. The observable \( \hat{Q} \) measures
the neutrino flavor as projected along a particular axis
(which we take to be \( \pm \)): \( \hat{Q} \equiv \sigma_z \), with eigenvalues
\( \hat{Q}|\nu_\mu\rangle = |\nu_\mu\rangle \) and \( \hat{Q}|\nu_\tau\rangle = -|\nu_\tau\rangle \), for muon-
and electron-flavor neutrino states, respectively.
The Hamiltonian for neutrino propagation in the two-flavor limit is given by (setting \( \hbar = c = 1 \)) [20,21]
\[
\mathcal{H} = \left( p + \frac{m_1^2 + m_2^2}{4p} + \frac{V_C}{2} + V_N \right) \mathds{1} + \frac{1}{2} \left( \begin{array}{cc}
V_C - \omega \cos \theta & \omega \sin \theta \\
\omega \sin \theta & \omega \cos \theta - V_C
\end{array} \right)
\equiv \mathcal{H}_{osc} \equiv \mathbf{r} \cdot \mathbf{\sigma} / 2,
\]
where \( \theta \) is the neutrino vacuum mixing angle, \( m_1 \) and \( m_2 \) label the distinct mass states, \( \omega \equiv (m_2^2 - m_1^2) / 2p \) is the oscillation frequency, and \( p = E \) is the relativistic neutrino momentum-energy. The term \( V_{C(N)} = \sqrt{2} G_F n_{e(o)} \) is the charged (neutral) current potential due to coherent forward scattering of neutrinos with electrons (neutrons) in matter, and \( G_F \) is the Fermi coupling constant. The term in Eq. (5) proportional to \( \mathds{1} \) affects all flavor states identically and therefore does not contribute to flavor oscillations.

For neutrinos of a given energy \( E_a \), the time evolution of flavor states is governed by the unitary operator \( \mathcal{U} \), which is related to \( \mathcal{H}_{osc} \equiv \mathbf{r} \cdot \mathbf{\sigma} / 2 \) via
\[
\mathcal{U}(\omega_a; t_1, t_j) \equiv \mathcal{U}(\psi_{a;ij}) = \exp \left( -i \int_{t_i}^{t_j} \mathcal{H}_{osc}(\omega_a) dt \right) = \cos(\psi_{a;ij}) \mathds{1} - i \sin(\psi_{a;ij}) (\mathbf{r}(\omega_a) \cdot \mathbf{\sigma}),
\]
where \( \omega_a \) is the oscillation frequency for energy \( E_a \) and \( \psi_{a;ij} \equiv |\mathbf{r}(\omega_a)| (t_j - t_i) / 2 \) is the phase accumulated while propagating from \( t_i \) to \( t_j \) with energy \( E_a \). In the limit in which matter effects remain negligible,
\[
\psi_{a;ij} = \frac{\omega_a}{2} (t_j - t_i) = \frac{1}{4E_a} (m_2^2 - m_1^2)(t_j - t_i).
\]
A neutrino’s time evolution depends only on the accumulated phase \( \psi_{a;ij} \) rather than the individual times \( t_i \) and \( t_j \). Moreover, the phases obey a sum rule: For a given energy \( E_a \), we have \( \psi_{a;12} + \psi_{a;23} + \psi_{a;13} = 0 \), or, more generally,
\[
\sum_{i=1}^{n-1} \psi_{a;i+1} = \psi_{a;1n}.
\]
Given the unitary operator defined in Eq. (6), for neutrinos propagating with energy \( E_a \), we find the evolution of the operator \( \mathcal{Q}(t_j - t_i) = \mathcal{U}^\dagger(\psi_{a;ij}) \mathcal{Q} \mathcal{U}(\psi_{a;ij}) = \mathcal{U}(\psi_{a;ij}) \mathcal{Q} \mathcal{U}^\dagger(\psi_{a;ij}) \equiv \mathcal{B}_{a;ij} \cdot \mathbf{\sigma} \).
The observable is defined only along the \( z \) projection, for which \( \mathcal{B}_{a;ij} \cdot \mathbf{z} = 1 - 2 (\mathbf{r} \cdot \mathbf{z})^2 \sin^2 \psi_{a;ij} \), and hence the correlation \( C_{ij}(\omega_a) \) defined in Eq. (3) simplifies to
\[
C_{ij}(\omega_a) = 1 - 2 \sin^2 \theta \sin^2 \psi_{a;ij}.
\]
The evolution of a given state depends only on the phase \( \psi_{a;ij} \). Hence, we may probe the LGI by exploiting differences in phase that come from the spacetime separation between measurements. For a pair of measurements that depend on an oscillation frequency \( \omega_a \) and a time interval \( \tau = t_j - t_i \), the overall phase is \( \psi_{a;ij} = \omega_a \tau / 2 \), consistent with the stationarity condition. Furthermore, for an experimental arrangement in which measurements occur at a fixed distance \( \delta L \) from the neutrino source, we have \( \tau = \delta L \) in the relativistic limit. In that case, the phase varies only with the energy \( E_a \); that is, \( \psi_{a;ij} \approx \psi_a = \omega_a \delta L / 2 \).

Assuming a beam that begins in the pure \( |\nu_\mu\rangle \) state and is subjected to measurement at two fixed locations separated by \( \delta L \), the correlation term in Eq. (9) simplifies to the difference between the neutrino survival probability and oscillation probability:
\[
C(\omega_a) = P_{\mu\mu}(\psi_a) - P_{\mu\mu}(\psi_a) = 2P_{\mu\mu}(\psi_a) - 1,
\]
over a time interval \( \tau = t_j - t_i = \delta L \). In the limit in which matter effects remain negligible, the survival probability (and thus each correlation function) depends only on the neutrino energy \( E_a \). It is therefore possible to construct the Leggett-Garg parameter \( K_n^\Omega \) as a sum of measured neutrino survival probabilities \( P_{\mu\mu}(\psi_a) \) for fixed \( \delta L \):
\[
K_n^\Omega = (2 - n) + 2 \sum_{a=1}^{n-1} P_{\mu\mu}(\psi_a) - 2P_{\mu\mu} \left( \sum_{a=1}^{n-1} \psi_a \right).
\]
For nonzero mixing angles \( \theta \), violations of the K \( n \leq (n - 2) \) limit are expected in neutrino oscillations.

Results.—In order to test for violations of the LGI, we use the data gathered by the MINOS neutrino experiment, which extends from Fermi National Accelerator Laboratory in Batavia, Illinois, to Soudan, Minnesota [29]. MINOS measures the survival probabilities of oscillating muon neutrinos produced in the NuMI accelerator complex. The accelerator provides a source of neutrinos with a fixed baseline and an energy spectrum that peaks at a point corresponding to \( \delta L / E_a \sim 250 \text{ km/GeV} \), close to the region where the survival probability \( P_{\mu\mu} \) reaches its first minimum. This experimental design provides an ideal phase space to test for LGI violations.

The MINOS Near Detector at Fermilab measures a beam of neutrinos, more than 98% of which are found to be in the \( |\nu_\mu\rangle \) state [29], consistent with the identically prepared flavor state assumption. Moreover, the MINOS experimental data exhibit stationarity, as verified by tests of Lorentz invariance in neutrino oscillations. Violation of Lorentz invariance would lead to a time-dependent alteration of the oscillation parameters, caused by the relative velocity of Earth as it orbits around the Sun. Tests of Lorentz violation using the same MINOS data we use here reveal no observed
violation [30,31], which indicates that the MINOS oscillation data depend on $\tau$ but not on $t_i$ or $t_j$ separately.

The MINOS Collaboration recently released preliminary oscillation results as a function of neutrino energy [32]. For their baseline distance of 735 km, the MINOS experiment covers the energy interval 0.5–50 GeV, which corresponds to a phase range of $\sim (0, 3\pi/2]$, within which LGI violations are expected to be near maximal for a quantum-mechanical system. As Fig. 1 illustrates, the data are readily consistent with the existing quantum-mechanical model of neutrino oscillations [34]. To test or constrain alternative explanations, we use survival probabilities measured at different energies $E_a$ and, thus, at different phases $\psi_a$.

To construct $K_3$, we select all pairs of measured points on the Fig. 1 oscillation curve $a \geq b$ such that the projected sum of phases $\psi_a + \psi_b$ given by Eq. (8) falls within $0.5\%$ of a third measured phase value $\psi_c$. A total of 82 correlation triples $(\psi_a, \psi_b, \psi_c)$ satisfy the phase condition $\psi_a + \psi_b \in \psi_c \pm 0.5\%$, 64 of which explicitly violate the LGI bound, yielding $K_3 > 1$. In order to properly account for the strong statistical correlations which exist between different empirical values of $K_3$, we generate a large sample of pseudodata based on the observed $P_{\mu\mu}$ values. These data points are modeled as normal distributions, with their means and variances matched to those of the observed probabilities. Each simulated measurement thus yields an artificial number of values for $K_3$, from which one can determine the probability that the system represented by the given data set violates the LGI. The modeling and parameter extraction is executed using the STAN Markov simulation package [35].

Because of statistical fluctuations present in the oscillation data, some fraction of the observed $K_3$ values may fluctuate above the classical bound, even if the underlying distribution is itself classical or realistic. To determine the frequency with which classical distributions give false-positive LGI violations, we use the same Markov chain statistical sampling method to construct a classical distribution of $K_3^C$. This allows us to make a quantitative comparison between classical and quantum predictions: The observed number of points above the classical bound may be directly compared to the predictions from classical [Eq. (4)] and quantum [Eq. (11)] rules. The impact of the systematic uncertainties from the amplitude and phases, as best estimated from Ref. [36], are also included in our construction of $K_3^C$.

To estimate the degree to which these results are inconsistent with a hidden-variable or realistic model, we fit the distribution of the number of expected LGI violations from the classical model [Eq. (4)] to a beta-binomial function, so as to account for the heteroscedasticity of the underlying distribution. The observed number of LGI violations (64 out of 82) represents a $6.2\sigma$ deviation from
the number of violations one would expect to arise from an underlying classical distribution. In addition, the quantum-mechanical model described by Eq. (11) shows generally good agreement with the observed values of $K_4$ ($K_4^Q = 104.8$ for 81 degrees of freedom). Changing the phase correlation criterion value from 0.5% to either 0.05% or 1% still yields a $\gg 5\sigma$ deviation from predictions consistent with realism.

A similar test was performed for $K_4$. Using the criteria and methods described above, a total of 577 (out of 715) violations of the LGI were observed for $K_4$. As Fig. 2 illustrates, there is a clear discrepancy between the observed number of violations and the classical prediction. The $K_4$ data are inconsistent with the realistic prediction at confidence 7σ.

Discussion.—The results discussed above strongly constrain alternatives to quantum mechanics, such as classical Markovian models. The original LGI was derived under an assumption that measurements may be performed in a noninvasive manner [3]. Within the context of realistic hidden-variable models, any disturbance would be considered of classical origin and could lead to a violation of the LGI [8]. Therefore, a determined “realist” could criticize previous experiments that had aimed to minimize quantum disturbances (such as wave function collapse) by performing weak measurements. We pursue a complementary method to address the “clumsiness loophole,” akin to Refs. [9,13,17], exploiting stationarity and the prepared-ensemble condition rather than using weak measurements. Our method makes use of projective measurements on individual neutrinos from the ensemble, minimizing the opportunity for one measurement to affect the evolution of other, independent neutrinos, in either a quantum-mechanical or a classical manner.

We have shown that neutrino oscillations clearly demonstrate a violation of the classical limits imposed by the Leggett-Garg inequality. This violation occurs over a distance of 735 km, providing the longest range over which a Bell-like test of quantum mechanics has been carried out to date. The observation stands as a further affirmation that quantum coherence can apply broadly to microscopic systems, including neutrinos, across macroscopic distances.

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[34] Each measured $P_{\mu\mu}(E_a)$ value represents a correlation between separate neutrino flavor measurements at the MINOS Near and Far Detectors. In order for a hidden-variable theory to replicate the curve in Fig. 1, each neutrino measured at the Far Detector would need to have access to most of the measurement outcomes on an ensemble of neutrinos at the Near Detector—including measurements at the Near Detector that had not yet been performed. This separation between sets of measurements is akin to the “no signaling in time” condition identified in Ref. [27].
