Measurement of the shape of the $[^0_\text{b}] [^+_\text{c}] [\text{bar over } [^\text{subscript } \text{c}]]$ differential decay rate

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I. INTRODUCTION

In the Standard Model (SM) of particle physics, quarks participate in a rich pattern of flavor-changing transitions. The relevant couplings form a complex $3 \times 3$ matrix, known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix, characterized by just four independent parameters [1]. A vast body of measurements of individual CKM elements exists, and thus the overall consistency of the SM picture of charged current interactions is highly constrained. Decades of experimental work have demonstrated the impressive consistency of experimental data with the CKM paradigm [2,3]; nonetheless, the motivation to probe the CKM matrix remains strong. Effects of physics beyond the SM may be subtle; thus, more precise measurements are necessary to unveil them. Semileptonic decays of heavy-flavored hadrons are commonly used to measure CKM parameters, as they involve only one hadronic current, parametrized in terms of scalar functions known as form factors. The number of form factors needed to describe a particular decay depends upon the spin of the initial- and final-state hadrons [4,5]. A precise calculation of these form factors has been elusive for many years as it is not possible in perturbative QCD. Heavy-Quark Effective Theory (HQET) provides the framework to systematically include nonperturbative corrections in computations involving hadrons containing heavy quarks. In particular, in the limit of infinite heavy-quark mass, all the form factors describing the semileptonic decay of a heavy-flavored hadron are proportional to a universal function, known as the Isgur-Wise (IW) function [6]. Lattice QCD, namely the use of lattice formulations of QCD in large scale numerical simulations, has emerged in recent years as a technique with well-defined and systematically improvable uncertainties which can be applied to a wide range of processes and physical quantities [7]. Predictions from the infinite heavy-quark mass limit are useful as a check of several lattice QCD calculations [8].

The decay $\Lambda_b^0 \to \Lambda_c^+ \mu^- \bar{\nu}_\mu$ is described by six form factors corresponding to the vector and axial-vector components of the flavor-changing charged current [9]. In HQET, $\Lambda_b^0$ decays are particularly simple, as the light $ud$ quark pair has total spin $j = 0$, and thus the chromomagnetic corrections, which are of the order of a few percent for $B$ mesons, are not present [10]. In the static approximation of infinite heavy-quark masses, the six form factors characterizing the baryonic semileptonic decay $^1 \Lambda_b^0 \to \Lambda_c^+ \mu^- \bar{\nu}_\mu$ can be expressed in terms of the elastic heavy-baryon Isgur-Wise function $\xi_B(w)$ [11]. The scalar invariant $w \equiv v_{\Lambda_b^0} \cdot v_{\Lambda_c^+}$ is related to the squared four-momentum transfer between the heavy baryons, $q^2$, by

$$w = \left( m_{\Lambda_b^0}^2 + m_{\Lambda_c^+}^2 - q^2 \right) / \left( 2 m_{\Lambda_b^0} m_{\Lambda_c^+} \right),$$

where $v_{\Lambda_b^0}$ and $v_{\Lambda_c^+}$ are the four-velocities of the $\Lambda_b^0$ and $\Lambda_c^+$ baryons, respectively, and $m_{\Lambda_b^0}$ and $m_{\Lambda_c^+}$ are the corresponding invariant masses. Nonperturbative corrections to the static limit can be expressed in terms of an expansion in powers of $1/m_c$ and $1/m_b$, where $m_c$ and $m_b$ represent the $c$- and $b$-quark masses, respectively. It has been shown in Ref. [12] that the $1/m_c$ term can be expressed in terms of $\xi_B(w)$ and one dimensionful constant. Moreover, partial cancellations lead to small first-order corrections near $w = 1$ [13].

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In the static approximation, the differential decay width of the $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ decay is given by

$$\frac{d\Gamma}{dw} = G(w)\xi_B(w), \quad (2)$$

where the constant factor $G$ is given by

$$G = \frac{2}{3(2\pi)^4} |V_{cb}|^2 (m_{\Lambda_b^0})^4 r^2 \quad \text{with} \quad r = m_{\Lambda_c^+}/m_{\Lambda_b^0}, \quad (3)$$

where $G_F$ represents the Fermi coupling constant [14], $|V_{cb}|$ is the magnitude of the matrix element describing the coupling of the $c$ quark to the $b$ quark, and the kinematic factor $K(w)$ is given by

$$K(w) = m_{\Lambda_c^+} \sqrt{w^2 - 1} \left[3w(1-2rw + r^2) + 2r(w^2 - 1)\right]. \quad (4)$$

The function $\xi_B(w)$ cannot be determined from first principles in HQET, but calculations based on a variety of approaches exist. The kinematic limit $w = 1$ is special in HQET, as only modest corrections in the $(1/m_b, 1/m_c)$ expansion are expected, due to the absence of hyperfine corrections [15]. Thus, it is interesting to express $\xi_B$ as a Taylor series expansion

$$\xi_B(w) = 1 - \rho^2 (w-1) + \frac{1}{2} \sigma^2 (w-1)^2 + \cdots, \quad (5)$$

where $\rho^2$ is the magnitude of the slope of $\xi_B$ and $\sigma^2$ is its curvature at $w = 1$. Sum rules provide constraints on $\rho^2$ and $\sigma^2$. In particular, they require the slope at the zero recoil point to be negative and give bounds on the curvature and higher-order derivatives [16,17]. In addition, they predict $\sigma^2 \geq 3/5 \rho^2 + (\rho^2)^2$ [18] and $\rho^2 \geq 3/4$. Table I summarizes theoretical predictions for $\rho^2$ from quenched lattice QCD, QCD sum rules, and a relativistic quark model.

Recently, state-of-the-art calculations of the six form factors describing the decay $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ have been obtained using lattice QCD with $2 + 1$ flavors of dynamical domain-wall fermions [19]. These form factors are calculated in terms of $q^2$. More details on this formalism are given in Appendix A. The resulting theoretical uncertainty attached to a measurement of $|V_{cb}|$ using this form-factor prediction is about 3.2%. The precision of this calculation makes this approach an appealing alternative to the ones currently used, all based on $B$-meson semileptonic decays such as $B^0 \rightarrow D^+ \mu^- \bar{\nu}_\mu$. Thus, it is important to examine the model’s agreement with measured quantities such as the shape of the $d\Gamma/dq^2$ spectrum.

The experimental knowledge of $\Lambda_b^0$ semileptonic decays is quite sparse, as this baryon is too heavy to be produced at the $e^+e^-B$-factories. The only previous experimental study of $\xi_B(w)$ was performed by the DELPHI experiment at LEP, which obtained $\rho^2 = 2.03 \pm 0.46(\text{stat})^{+0.72}_{-1.00}(\text{syst})$, with an overall uncertainty of the order of 50% [20].

In this paper, we describe a determination of the shape of the $w$ or $q^2$ spectrum of the decay $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ and compare it with functional forms related to a single form factor, inspired by HQET, and the lattice QCD prediction of Ref. [19]. Section II presents the experimental procedure and simulated samples, while Sec. III describes the method employed to reconstruct $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ candidates and to estimate the corresponding kinematic variables $w$ and $q^2$. Section IV describes the method adopted to isolate the signal, the unfolding procedure used to account for experimental resolution effects, and the efficiency corrections. The fit results for $\xi_B(w)$ corresponding to different functional forms are summarized in Sec. V. The same analysis procedure is used in Sec. VI to derive the shape of the differential decay width $d\Gamma/dq^2(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu)$ and compare with the predictions of Ref. [19]. These data are also fitted with a single form-factor parametrization that corresponds to the HQET infinite heavy-quark mass limit.

### TABLE I. Predictions for the slope at zero recoil of the baryonic Isgur-Wise function $\xi_B$. The evaluation from Ref. [21] includes first-order corrections in HQET.

<table>
<thead>
<tr>
<th>$\rho^2$</th>
<th>Approach</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.35 \pm 0.13$</td>
<td>QCD sum rules</td>
<td>[22]</td>
</tr>
<tr>
<td>$1.2^{+0.8}_{-1.1}$</td>
<td>Lattice QCD (static approximation)</td>
<td>[23]</td>
</tr>
<tr>
<td>$1.51$</td>
<td>HQET + relativistic wave function</td>
<td>[21]</td>
</tr>
</tbody>
</table>

II. EXPERIMENTAL METHOD

The data used in this analysis were collected with the LHCb detector at the Large Hadron Collider at CERN and correspond to 1 fb$^{-1}$ of integrated luminosity collected at a center-of-mass energy of 7 TeV in 2011 and 2 fb$^{-1}$ collected at a center-of-mass energy of 8 TeV in 2012.

The LHCb detector [24,25] is a single-arm forward spectrometer designed for the study of particles containing $b$ or $c$ quarks. The detector covers the pseudorapidity range $2 < \eta < 5$, where $\eta$ is defined in terms of the polar angle $\theta$ with respect to the beam direction as $-\ln(\tan(\theta/2))$. The detector includes a high-precision tracking system consisting of a silicon-strip vertex detector surrounding the $pp$ interaction region [26], a large-area silicon-strip detector located upstream of a dipole magnet with a bending power of about 4 Tm, and three stations of silicon-strip detectors and straw drift tubes [27] placed downstream of the magnet. The tracking system provides a measurement of the momentum, $p$, of charged particles with a relative uncertainty that varies from 0.5% at low momentum to 1.0% at 200 GeV.\(^2\) The minimum distance of a track to a

\(^2\)Natural units with $c = \hbar = 1$ are used throughout.
primary vertex, the impact parameter (IP), is measured with a resolution of \((15 + 29/p_T) \text{ mm}\), where \(p_T\) is the component of the momentum transverse to the beam, in GeV. Different types of charged hadrons are distinguished using information from two ring-imaging Cherenkov detectors (RICH) [28]. Photons, electrons, and hadrons are identified by a calorimeter system consisting of scintillating-pad and preshower detectors, an electromagnetic calorimeter, and a hadronic calorimeter. Muons are identified by a system composed of alternating layers of iron and multiwire proportional chambers [29]. The online event selection is performed by a trigger [30], which consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction.

Muon candidates are first required to pass the hardware trigger that selects muons with a transverse momentum \(p_T > 1.6 \text{ (1.8) GeV}\) for the 2011 (2012) data taking period. In the subsequent software trigger, events with one particle identified as a muon are selected if at least one of the final-state particles has both \(p_T > 0.8 \text{ GeV}\) and IP larger than 100 \(\mu\text{m}\) with respect to all of the primary \(pp\) interaction vertices (PVs) in the event. In the offline selection, trigger signals are associated with reconstructed particles. Selection requirements can therefore be made on the trigger selection itself and on whether the decision was due to the signal candidate, other particles produced in the pp collision, or a combination of both. This classification of trigger selections can be used for data-driven efficiency determination. Finally, the tracks of two or more of the final-state particles are required to form a vertex that is significantly displaced from the PVs.

Our study makes use of simulated semileptonic decays, where \(pp\) collisions are generated using Pythia [31] with a specific LHCb configuration [32]. Decays of hadronic particles are described by EVTGEN [33], in which final-state radiation is generated using PHOTOS [34]. The interaction of the generated particles with the detector, and its response, are implemented using the Geant4 toolkit [35] as described in Ref. [36].

### III. EVENT RECONSTRUCTION

To isolate a sample of \(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu X\) semileptonic decays, where \(X\) represents the undetected particles produced with the \(\Lambda_c^+\) in the \(c\)-quark hadronization, we combine \(\Lambda_c^+ \rightarrow pK^-\pi^+\) candidates with tracks identified as muons. We consider candidates where a well-identified muon passing the hardware and software trigger algorithms with momentum greater than 3 GeV is found. Charmed baryon candidates are formed from hadrons with momenta greater than 2 GeV and transverse momenta greater than 0.3 GeV. In addition, we require that the average of the magnitudes of the transverse momenta of the hadrons forming the \(\Lambda_c^+\) candidate be greater than 0.7 GeV. Kaons, pions, and protons are identified using the RICH system. Each track’s IP significance with respect to the associated primary vertex is required to be greater than 9.\(^3\) Moreover, the selected tracks must be consistent with coming from a common vertex: the \(\chi^2\) per degree of freedom (DOF) of the vertex fit must be smaller than 6. In order to ensure that the direction of the parent \(\Lambda_b^0\) is well measured, the \(\Lambda_c^+\) vertex must be distinct from the primary \(pp\) interaction vertex. To this end, we require that the flight-distance significance of the \(\Lambda_c^+\) candidate (defined as the measured flight distance divided by its uncertainty) with respect to the associated PV be greater than 100.

Partially reconstructed \(\Lambda_b^0\) baryon candidates are formed combining \(\mu^-\) and \(\Lambda_c^+\) candidates that are consistent with coming from a common vertex, and we require that the angle between the direction of the momentum of the \(\Lambda_c^+\mu^-\) candidate and the line from the associated PV to the \(\Lambda_c^+\mu^-\) vertex be less than 45 mrad. As the \(\Lambda_c^+\) baryon is a \(\Lambda_b^0\) decay product with a small but significant lifetime, we require that the difference in the component of the decay vertex position of the charmed hadron candidate along the beam axis and that of the beauty candidate be positive. We explicitly require that the \(\Lambda_b^0\) hadron candidate pseudorapidity be between 2 and 5. We measure \(\eta\) using the line defined by connecting the associated PV and the vertex formed by the \(\Lambda_c^+\) and the \(\mu^-\) lepton. Finally, the invariant mass of the \(\Lambda_c^+\mu^-\) system must be between 3.3 and 5.3 GeV. These selection criteria ensure that the \(\Lambda_c^+\) candidates are decay products of \(\Lambda_b^0\) semileptonic decays. In particular, the background from directly produced \(\Lambda_c^+\) (prompt \(\Lambda_c^+\)) is highly suppressed. This is quantified by an unbinned extended maximum likelihood fit to the two-dimensional \(pK^-\pi^+\) invariant mass and \(\ln(IP/mm)\) distributions of the \(\Lambda_c^+\) candidates, where “\(mm\)” refers to the length unit used to measure the IP. The \(\ln(IP/mm)\) component allows us to determine the small prompt \(\Lambda_c^+\) background. The parameters of the IP distribution of the prompt sample are found by examining directly produced charm hadrons, as described in Ref. [37]. An empirical probability density function (PDF) derived from simulation is used for the \(\Lambda_c^+\) from \(\Lambda_b^0\) component. We find \((2.74 \pm 0.02) \times 10^6 \Lambda_c^+ \rightarrow pK^-\pi^+\) candidates, which can be interpreted as \(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu X\) decays, and we determine the prompt \(\Lambda_c^+ \rightarrow pK^-\pi^+\) fraction to be 1.5%, which can be neglected. The corresponding fit is shown in Fig. 1.

Our goal is the study of the ground-state semileptonic decay \(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu\); thus, we need to estimate the contributions from \(\Lambda_c^+\) decaying into \(\Lambda_b^0\) states. Theoretical predictions suggest that the inclusive rate \(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu X\) is dominated by the exclusive channel \(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu\) [38,39]. The residual contribution is expected to be accounted for by

\(^3\)The associated primary vertex to a \(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu X\) candidate is selected as the primary vertex which minimizes the IP significance of the \(\Lambda_c^+\mu^-\) system.
the $\Lambda_b^0 \to \Lambda_c(2595)^+ \mu^− \bar{\nu}_\mu$ and $\Lambda_b^0 \to \Lambda_c(2625)^+ \mu^− \bar{\nu}_\mu$ channels. Other modes, such as $\Lambda_b^0 \to \Sigma_c^+ \mu^− \bar{\nu}_\mu$, are suppressed in the static limit and to order $1/m_Q$, where $m_Q$ represents the heavy-quark mass ($m_c$ or $m_b$) [40], with an additional stronger suppression factor of the order $(m_d - m_u)/m_c$ rather than $(m_d - m_u)/m_{QCD}$ [9].

We use $\Lambda_b^0 \to \Lambda_c^+ \pi^+ \mu^− \bar{\nu}_\mu$ decays to infer contributions from the excited $\Lambda_c^+$ modes, where the $\Lambda_c^+$ candidates are selected as $pK^−\pi^+$ combinations of which the invariant mass is within $\pm 20$ MeV of the nominal $\Lambda_c^+$ mass. The $\Lambda_c^+ \pi^+ \mu^− \bar{\nu}_\mu$ candidates are combined with pairs of opposite-charge pions that satisfy criteria similar to those used to select the pions from the $\Lambda_c^+$ decay. The minimum transverse momentum of these pions is required to be 0.2 GeV, and the transverse momentum of the $\Lambda_c^+ \pi^+\pi^−$ system is required to be greater than 1.5 GeV. Lastly, the $\chi^2$ per degree of freedom of the vertex fit for the $\Lambda_c^+ \pi^+\pi^−$ system must be smaller than 6.

The resulting spectrum, measured as the mass difference $m(pK^−\pi^+\pi^−) - m(pK^−\pi^+)$ added to the known $\Lambda_c^+$ mass [14], is shown in Fig. 2. We see peaks corresponding to the $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, $\Lambda_c(2765)^+$, and $\Lambda_c(2880)^+$ resonances. The $\Lambda_c(2595)^+$ is only a few MeV above the kinematic threshold, and thus it is not well described by a Breit-Wigner function. The baseline fit for this resonance uses a PDF consisting of the sum of two bifurcated Gaussian functions. As a check, we use an S-wave relativistic Breit-Wigner convolved with a Gaussian function with standard deviation $\sigma = 2$ MeV that accounts for the detector resolution. While the second parametrization is more accurate, the fits to the invariant mass spectra in different kinematic bins are more stable with the baseline parametrization. We fit the $\Lambda_c(2625)^+$ signal with a double Gaussian PDF with shared mean, as the natural width is expected to be well below the measured detector resolution. The shape of the combinatoric background PDF is inferred from wrong-sign (WS) candidates, where a $\pi^+\pi^+$ or $\pi^-\pi^-$ pair is combined with $\Lambda_c^+$ instead of $\pi^+\pi^-$. In addition, we observe peaks corresponding to two higher-mass resonances, with masses and widths consistent with the $\Lambda_c(2765)^+$ and $\Lambda_c(2880)^+$ baryons [14]. In order to determine their yields, we fit the two signal peaks with single Gaussian PDFs with unconstrained masses and widths. The measured yields for the four $\Lambda_c^+$ final states, as well as the $\Lambda_c^+ \mu^− \bar{\nu}_\mu X$ final state, are presented in Table II.

![Figure 1](image1.png)

**FIG. 1.** (a) The ln(IP/mm) distribution and (b) $pK^−\pi^+$ invariant mass for $\Lambda_c^+$ candidate combinations with a muon. The red (dashed-dotted) curves show the combinatorial $\Lambda_c^+$ background, the green (dashed) curves show the $\Lambda_c^+$ from $\Lambda_b^0$, and the blue-solid curves show the total yields.

![Figure 2](image2.png)

**FIG. 2.** The mass difference $m(pK^−\pi^+\pi^−) - m(pK^−\pi^+)$ added to the known $\Lambda_c^+$ mass, $m_{PDG}(\Lambda_c^+)$ [14], for candidates with $pK^−\pi^+$ invariant mass within $\pm 20$ MeV of the known $\Lambda_c^+$ mass in candidate semileptonic decays for the entire $w$ range: data are shown as black dots, the combinatoric background is shown as a blue solid line, and the gray histogram shows the WS spectrum, obtained by combining a $\pi^+\pi^+$ or $\pi^-\pi^-$ pair with $\Lambda_c^+$ instead of $\pi^+\pi^-$. The signal fits are identified as follows: (a) for $m < 2700$ MeV, the $\Lambda_c(2595)^+$ as a magenta dashed line and the $\Lambda_c(2625)^+$ as a green long-dashed line; (b) for $m > 2700$ MeV, the $\Lambda_c(2765)^+$ as a magenta dashed line and the $\Lambda_c(2880)^+$ as a green long-dashed line.
MEASUREMENT OF THE SHAPE OF THE …

IV. SPECTRAL DISTRIBUTION

dN_{corr}/dw(Λ^+_b → Λ_c^+ μ^- ν_μ)

The Λ^+ b → Λ_c^+ μ^- ν_μ X candidates are separated into 14 equal-size bins of reconstructed w in the full kinematic range 1 ≤ w ≤ 1.43. The parameters of the PDFs describing the signal and background components are determined from the fit to the overall pK^-π^+ mass spectrum. The contributions from semileptonic decays including higher-mass baryons in the final state is evaluated by fitting the Λ_c^+ π^+ π^- mass spectra with two different methods. In the first, we fit for the four resonances shown in Fig. 2 using a PDF derived from the WS sample to model the background and then use the simulation to correct for efficiency. In the second, we determine the signal yields of the Λ_c^+ states by subtracting the WS background and treating the residual smooth component of the spectrum as originating from a semileptonic decay Λ^+ b → Λ_c^+ μ^- ν_μ X. The second method provides an estimate of the systematic uncertainty introduced by the contribution from nonresonant Λ_c^+ π^+ π^- components of the hadron spectrum, as the smooth component of this spectrum is likely to comprise also the combinatoric background.

Next, we correct the raw Λ_c^+ μ^- ν_μ X and Λ_c^+ π^+ π^- μ^- ν_μ X signal yields for the corresponding software trigger efficiencies, which are derived with a data-driven method [30], based on the determination of Λ_c^+ μ^- ν_μ X events where a positive trigger decision is provided by the signal candidates and events where the trigger decision is independent of the signal. Then, we subtract the raw yields reported in Table II, scaled by the corresponding efficiency ratios $\epsilon_d(Λ_c^+ μ^- ν_μ X) / \epsilon_d(Λ_c^+ π^+ π^- μ^- ν_μ X)$, from the corrected Λ_c^+ μ^- ν_μ X yields. These ratios are derived from Λ^+ b → Λ_c(2595) + μ^- ν_μ and Λ^+ b → Λ_c(2625) + μ^- ν_μ simulations. The higher-mass yields are scaled by an average of these two corrections, as no model for these semileptonic decays is available. These corrections account for the efficiency loss due to the reconstruction of the additional pion pairs, as well as for the unseen Λ^+ b → Λ_c^+ π^+ π^- μ^- ν_μ X decay, and are only mildly dependent upon the invariant mass of the final state. The expectation is that Λ^+ b → Λ_cπ^+ π^- μ^- ν_μ X accounts for two-thirds of the inclusive dipion final state. We have checked this prediction by studying the inclusive final states Σ_c^+ μ^- ν_μ X, Σ_c^+ μ^- ν_μ X, and Σ_c^+ μ^- ν_μ X. Taking into account the difference in the Λ_c^+ π^+ π^- μ^- ν_μ X and Λ_c^+ π^+ π^- μ^- ν_μ X detection efficiencies, estimated with simulations, we measure the ratio $R = N(Λ^+_c π^+ π^-)/N(Λ^+_c π^+ π^- + Λ^+_c π^+ π^-)$ with

$$R = \frac{N(Σ_c^+ μ^- ν_μ X) + N(Σ_c^+ π^- ν_μ X)}{N(Σ_c^+ μ^- ν_μ X) + N(Σ_c^+ π^- ν_μ X)} \cdot \frac{e(Λ_c^+ π^+ π^- μ^- ν_μ X)}{e(Λ_c^+ π^+ π^- μ^- ν_μ X)}$$

(7)

where $N(Σ_c^+ μ^- ν_μ X)$ and $N(Σ_c^+ π^- ν_μ X)$ are the detected yields for the final states Σ_c^+ π^- μ^- ν_μ and Σ_c^+ π^- μ^- ν_μ. $N(Σ_c^+)$ is the detected yield for
the final state $\Sigma^+ c\mu X$, and $e(\Lambda^+_c \pi^+ \pi^- \mu)/e(\Lambda^+_c \pi^0 \mu)$ is the ratio between the reconstruction efficiencies of these final states calculated with simulation. A simulation study gives $e(\Lambda^+_c \pi^+ \pi^- \mu)/e(\Lambda^+_c \pi^0 \mu) = 25.9 \pm 2.7$, where the uncertainty reflects the limited sample size of the simulation. We obtain $R = 0.63 \pm 0.14$, where the statistical uncertainty is due to limited $\pi^0$ reconstruction efficiency, consistent with the expectation $R = 2/3$, and a negligible $\Sigma^+_c \mu^- \bar{\nu}_\mu$ component in the denominator of Eq. (7).

The $\Lambda_b \rightarrow \Lambda^+_c \mu^- \bar{\nu}_\mu$ spectrum $dN_{\text{meas}}/dw$ is then unfolded to account for the detector resolution and other $w$ smearing effects such as the possible choice of the wrong solution of Eq. (6). The procedure adopted is based on the single value decomposition (SVD) method [41] using the ROOUNFOLD package [42]. We choose to divide the unfolded spectrum $dN_{\text{meas}}/dw$ into seven $w$ bins, to be consistent with the recommendation of Ref. [43] to divide the measured spectrum into a number of bins at least twice as many as the ones in the corrected spectrum. The SVD method includes a regularization procedure that depends upon a parameter $k$ [41], ranging between unity and the number of degrees of freedom, in our case 14. Simulation studies demonstrate that $k = 4$ is optimal in our case. Variations associated with different choices of $k$ have been studied and are included in the systematic uncertainties. We have performed closure tests with different simulation models of the $\Lambda^0_b \rightarrow \Lambda^+_c \mu^- \bar{\nu}_\mu$ dynamics and verified that this unfolding procedure does not bias the reconstructed distribution. The spectra before and after unfolding are shown in Fig. 3. Finally, using simulated samples of signal events, we correct the unfolded spectrum for $w$-dependent acceptance and selection efficiency to obtain the distribution $dN_{\text{corr}}/dw$. Various kinematic distributions have been studied in simulation and data, and we find that they are all in good agreement.

V. SHAPE OF $\xi_B(w)$ FOR $\Lambda^0_b \rightarrow \Lambda^+_c \mu^- \bar{\nu}_\mu$ DECAYS

In order to determine the shape of the Isgur-Wise function $\xi_B(w)$, we use the square root of $dN_{\text{corr}}/dw$ divided by the kinematic factor $K(\langle w \rangle)$, defined in Eq. (4), evaluated at the midpoint in the seven unfolded $w$ bins. We derive the IW shape with a $\chi^2$ fit, where the $\chi^2$ is formed using the full covariance matrix of $dN_{\text{corr}}/dw$.

We use various functional forms to extract the slope, $\rho^2$, and curvature, $\sigma^2$, of $\xi_B(w)$. The first functional form is motivated by the $1/N_c$ expansion [44], where $N_c$ represents the number of colors, and has an exponential shape parametrized as

$$\xi_B(w) \propto e^{-\rho^2 (w-1)/2}.$$
TABLE III. Summary of the values for the slope and curvature of the Isgur-Wise function with different parametrizations. The quoted uncertainties are statistical only. The models marked with “*” have only the slope at zero recoil as a free parameter; thus, the curvature is derived from the fitted $\rho^2$.

<table>
<thead>
<tr>
<th>Shape</th>
<th>$\rho^2$</th>
<th>$\sigma^2$</th>
<th>Correlation coefficient</th>
<th>$\chi^2$/DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential*</td>
<td>1.65 ± 0.03</td>
<td>2.72 ± 0.10</td>
<td>100%</td>
<td>5.3/5</td>
</tr>
<tr>
<td>Dipole*</td>
<td>1.82 ± 0.03</td>
<td>4.22 ± 0.12</td>
<td>100%</td>
<td>5.3/5</td>
</tr>
<tr>
<td>Taylor series</td>
<td>1.63 ± 0.07</td>
<td>2.16 ± 0.34</td>
<td>97%</td>
<td>4.5/4</td>
</tr>
</tbody>
</table>

\[ \xi_B(w) = \exp[-\rho^2(w - 1)]. \] (8)

The second functional form, the so-called dipole IW function, which is more consistent with sum-rule bounds [17], is given by

\[ \xi_B(w) = \left( \frac{2}{w + 1} \right)^{2\rho^2}. \] (9)

Finally, we can use a simple Taylor series expansion of the Isgur-Wise function and fit for the slope and curvature parameters using the Taylor series expansion introduced in Eq. (5). Figure 4 shows the measured $\xi_B(w)$ and the fit results with this parametrization. Table III summarizes the slope and curvature at zero recoil obtained with the three fit models. Note that the curvature is an independent parameter only in the last fit, while in the first two models, it is related to the second derivative of the IW function.

As the slope of the IW function is the most relevant quantity to determine $|V_{cb}|$ in the framework of HQET [13], we focus our studies on the systematic uncertainties on this parameter. We consider several sources of systematic uncertainties, which are listed in Table IV. The first two are determined by changing the fit models for $\Lambda^+_c$ and $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ signal shapes in the corresponding candidate mass spectra. The software trigger efficiency uncertainty is estimated by using an alternative procedure to evaluate this efficiency using the trigger emulation in the LHCb simulation. In order to assess systematics associated with the bin size, we perform the

TABLE IV. Summary of the systematic uncertainties on the slope parameter $\rho^2$. The total uncertainty is obtained by adding the individual components in quadrature.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sigma(\rho^2)$</th>
</tr>
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<tbody>
<tr>
<td>Signal fit for $\Lambda^+_c$</td>
<td>0.02</td>
</tr>
<tr>
<td>Signal PDF for $\Lambda_c^{++}$</td>
<td>0.02</td>
</tr>
<tr>
<td>Software trigger efficiency</td>
<td>0.02</td>
</tr>
<tr>
<td>$w$ binning</td>
<td>0.03</td>
</tr>
<tr>
<td>SVD unfolding regularization</td>
<td>0.03</td>
</tr>
<tr>
<td>Phase space averaging</td>
<td>0.03</td>
</tr>
<tr>
<td>$\Lambda^0_b \rightarrow \Lambda^+<em>c \mu^- \bar{\nu}</em>\mu$ modeling</td>
<td>0.03</td>
</tr>
<tr>
<td>$\Lambda^0_b \rightarrow \Lambda_c^{++} \mu^- \bar{\nu}_\mu$ modeling</td>
<td>0.03</td>
</tr>
<tr>
<td>Additional components of the semileptonic spectrum</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Lambda^0_b$ kinematic dependencies</td>
<td>0.02</td>
</tr>
<tr>
<td>Total</td>
<td>0.08</td>
</tr>
</tbody>
</table>

analysis with different binning choices. The sensitivity to the $\Lambda^0_b \rightarrow \Lambda^+_c \mu^- \bar{\nu}_\mu$ form-factor modeling is assessed by reweighting the simulated $w$ spectra to correspond to different $\xi_B$ functions with slopes ranging from 1.5 to 1.7. The “phase space averaging” sensitivity is estimated by comparing the fit to the expression for $dN_{corr} / dw$ with the fit to $1/K(\langle w \rangle) \sqrt{dN_{corr} / dw}$. The uncertainty associated with the $\Lambda^0_b \rightarrow \Lambda_c^{++} \mu^- \bar{\nu}_\mu$ modeling is evaluated by changing the relative fraction of $\Lambda^+_c \pi^- \pi^-$ versus $\Lambda^+_c \pi^0 \pi^0$ of the $\Lambda^+_c$ spectrum by ±20%. Finally, we use the alternative evaluation of the fraction of $\Lambda^0_b \rightarrow \Lambda^+_c \pi^- \pi^- \mu^- \bar{\nu}_\mu$ which includes the maximum possible nonresonant component to assess the sensitivity to residual $\Lambda_c^{++}$ components in the subtracted spectrum. The total systematic uncertainty in $\rho^2$ is 0.08.

The value of $\rho^2$ obtained from the Taylor series expansion is

$$\rho^2 = 1.63 \pm 0.07 \pm 0.08,$$

which is consistent with lattice calculations [23], QCD sum rules [22], and relativistic quark model [21] expectations. The measured slope is compatible with theoretical predictions and with the bound $\rho^2 \geq 3/4$ [16]. The measured curvature $\sigma^2$ is compatible within uncertainties with the lower bound $\sigma^2 \geq 3/5[\rho^2 + (\rho^2)^2]$ [18].

VI. COMPARISON WITH UNQUEACHED LATTICE PREDICTIONS

The lattice QCD calculation in Ref. [19] uses a helicity-based description of the six form factors governing $\Lambda^0_b \rightarrow \Lambda$ transitions introduced in Ref. [45]. The calculation uses state-of-the-art techniques encompassing the entire $q^2$ region. The stated uncertainties on the predicted width are therefore larger than what is expected in a high-$q^2$ region but remain rather small, namely 6.3%. This corresponds to a 3.2% theoretical uncertainty on $|V_{cb}|$, thus raising the prospect of an additional precise independent determination of $|V_{cb}|$.

The simplest check on this theoretical prediction consists of a comparison of the predicted shape of $d\Gamma/dq^2$ and the measured data. Thus, we measure the distribution $dN_{corr}/dq^2$ with the same procedure adopted to derive $dN_{corr}/dw$, including efficiency corrections and the unfolding procedure, with the same number of bins used to determine the raw and unfolded yields. We produce
seven corrected yields and their associated covariance matrix, where the nondiagonal terms are related to the unfolding procedure. We then perform a $\chi^2$ fit to the seven experimental $dN_{\text{corr}}/dq^2$ data points using the theoretical functional shape given in Eq. (85) of Ref. [19], which also provides the nominal values of the form-factor parameters, and thus we leave only the relative normalization floating. This fit uses a covariance matrix that combines experimental and theoretical uncertainties, which yields a $\chi^2$ equal to 1.32 for 6 degrees of freedom and a corresponding p-value of 97%. This shows that the predicted shape is in good agreement with our measurement.

The form-factor decomposition in Ref. [19] does not allow a straightforward extrapolation to the HQET limit of infinite heavy-quark masses. However, we know that in the static limit all the form factors are proportional to a single universal function. In order to assess how well our data are consistent with the static limit, we perform a second $\chi^2$ fit assuming that all the form factors are proportional to a single $z$-expansion function [46]. Fits with different pole masses used in the six form factors determined in Ref. [19] are performed. The overall shape does not change appreciably; the pole mass of 6.768 GeV is preferred. The two fit masses used in the six form factors determined in Ref. [19] can be found in the Appendix.

A precise measurement of the shape of the Isgur-Wise function describing the semileptonic decay $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ has been performed. The measured slope is consistent with theoretical models and the bound $\rho^2 \geq 3/4$ [16]. The measured curvature $\sigma^2$ is consistent with the lower-bound constraint $\sigma^2 \geq 3/5(p^2 + (p^2)^2)$ [18]. The shape of $d\Gamma/dq^2$ is studied and found to be well described by the unquenched lattice QCD prediction of Ref. [19], as well as by a single form-factor parametrization. Further studies with a suitable normalization channel will lead to a precise independent determination of the CKM parameter $|V_{cb}|$.

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APPENDIX A: ANALYTICAL EXPRESSION FOR $d\Gamma/dq^2$

This Appendix describes the formalism used in the $d\Gamma/dq^2$ fits. In particular, we give the expression of $d\Gamma/dq^2$ in terms of the form-factor basis chosen in Ref. [19], the so-called helicity form factors. In addition, we show the corresponding expression used to model the static limit.

The lattice QCD calculations reported in Ref. [19] predict the differential decay width $d\Gamma(\Lambda^0_b \rightarrow \Lambda^+_c \mu^- \bar{\nu}_\mu)/dq^2$ as follows,

![FIG. 5. Comparison between the fit to the seven experimental data points using either the lattice QCD calculation of Ref. [19], shown as gray points with a shaded area corresponding to the binned 1σ theory uncertainty, or a single form-factor fit in the z-expansion, shown as the solid blue curve. The data points, modulo a scale factor, are shown as black points with error bars.](image)
\[
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2 \sqrt{s_+s_-}}{768\pi^3 m_{\Lambda_0}^3} \left( 1 - \frac{m_e^2}{q^2} \right)^2 \\
\times \left\{ 4(m_e^2 + 2q^2)(s_+[g_+ (q^2)]^2 + s_-[f_+ (q^2)]^2) \\
+ 2\frac{m_e^2 + 2q^2}{q^2}(s_+[(m_{\Lambda_0} - m_X)g_+(q^2)]^2 + s_-[(m_{\Lambda_0} + m_X)f_+(q^2)]^2) \\
+ \frac{6m_e^2}{q^2}(s_+[(m_{\Lambda_0} - m_X)f_0(q^2)]^2 + s_-[(m_{\Lambda_0} + m_X)g_0(q^2)]^2) \right\},
\]

(A1)

where \(g_+, f_+, g_-, f_0, \) and \(f_0\) represent the six form factors necessary to describe this decay, \(X \equiv \Lambda_c\) denotes the final-state baryon, \(m_e\) represents the mass of the muon, \(q^2\) is the squared four-momentum transfer between the heavy baryons, and

\[
s_{\pm} = (m_{\Lambda_0} \pm m_X)^2 - q^2.
\]

The six form factors are cast in terms of the \(z\)-expansion [46] up to first order and have the functional form

\[
f(q^2) = \frac{1}{1 - q^2 / (m_{\text{pole}}^2)^2} \\
\times \left[ a_0^f + a_1^f z^f(q^2) \right],
\]

(A3)

where \(z^f(q^2)\) is given by

\[
z^f(q^2) = \sqrt{t_+ - q^2} - \sqrt{t_+ - t_0},
\]

(A4)

and \(t_0 = (m_{\Lambda_0} - m_X)^2\).

and \(t_+^f\) is given by

\[
t_+^f = (m_{\text{pole}}^f)^2.
\]

(A6)

and the pole masses used in the calculations are shown in Table V. The parameters \(a_0^f\) and \(a_1^f\) for the six form factors describing this decay are given in Table VIII of Ref. [19].

\begin{table}[h]
\centering
\caption{Masses of the relevant form-factor poles in the physical limit (in GeV).}
\begin{tabular}{lll}
\hline
\(f\) & \(J^P\) & \(m_{\text{pole}}^f (\Lambda_0^0 \to \Lambda_c)\) (GeV) \\
\hline
\(f_+, f_-\) & 1\(^-\) & 6.332 \\
\(f_0\) & 0\(^+\) & 6.725 \\
g_+, g_- & 1\(^+\) & 6.768 \\
g_0 & 0\(^-\) & 6.276 \\
\hline
\end{tabular}
\end{table}

In the static limit, all the helicity form factors are proportional to a single universal function. Thus, we use a common \(z\)-expansion parametrization

\[
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2 \sqrt{s_+s_-}}{768\pi^3 m_{\Lambda_0}^3} \left( 1 - \frac{m_e^2}{q^2} \right)^2 \left[ 4(m_e^2 + 2q^2)(s_+ + s_-) \\
+ \frac{4}{q^2}[(m_{\Lambda_0} - m_X)^2 + (m_{\Lambda_0} + m_X)^2] [2m_e^2 + q^2] \right],
\]

(A7)

where the choice of \(g_\perp\) reflects the choice of the pole mass used in the single \(z\)-expansion fit given in Sec. VI. We performed the fits with various choices of pole masses and examined the effects on the shape \(d\Gamma/dq^2\) and found that the pendulars defining \(g_\perp\) yielded the optimal fit. In this case, the fit parameters defining \(a_0^f\) and \(a_1^f\) in the \(z\)-expansion parametrization of \(g_\perp(q^2)\), which has the form shown in Eq. (A3).

**APPENDIX B: MEASURED NORMALIZED SPECTRA \(dN_{\text{corr}}/dq^2\) AND ASSOCIATED COVARIANCE MATRIX**

In this Appendix, we report the seven measured data points \(dN_{\text{corr}}/dq^2\) and the corresponding covariance matrix, shown in Tables VI and VII, respectively.

**TABLE VI.** Measured normalized yields \(dN_{\text{corr}}(\Lambda_0^0 \to \Lambda_c^0 \mu^- \nu_\mu)/dq^2\).

<table>
<thead>
<tr>
<th>(q^2) (GeV(^2))</th>
<th>(dN_{\text{corr}}/dq^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>1.50 ± 0.10</td>
</tr>
<tr>
<td>2.38</td>
<td>1.80 ± 0.10</td>
</tr>
<tr>
<td>3.97</td>
<td>2.04 ± 0.10</td>
</tr>
<tr>
<td>5.56</td>
<td>2.23 ± 0.08</td>
</tr>
<tr>
<td>7.14</td>
<td>2.35 ± 0.07</td>
</tr>
<tr>
<td>8.73</td>
<td>2.28 ± 0.05</td>
</tr>
<tr>
<td>10.32</td>
<td>1.50 ± 0.04</td>
</tr>
</tbody>
</table>
TABLE VII. Covariance matrix of the measured normalized yields Cov\(\{dN_{\text{cor}}(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu)/dq^2\}\).

<table>
<thead>
<tr>
<th>(q^2 (\text{GeV}^2))</th>
<th>(dN_{\text{cor}}/dq^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>0.0103</td>
</tr>
<tr>
<td>2.38</td>
<td>0.0052</td>
</tr>
<tr>
<td>3.97</td>
<td>-0.0032</td>
</tr>
<tr>
<td>5.56</td>
<td>-0.0035</td>
</tr>
<tr>
<td>7.14</td>
<td>-0.0009</td>
</tr>
<tr>
<td>8.73</td>
<td>0.0004</td>
</tr>
<tr>
<td>10.32</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

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