1/2 (e[^2]/h) Conductance Plateau without 1D Chiral Majorana Fermions

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One promising direction in building a quantum computer is topological quantum computation [1], which can be realized using non-Abelian topological orders that contain Ising non-Abelian anyons, or other more general non-Abelian anyons [2,3]. Although Ising non-Abelian anyons cannot perform universal topological quantum computation [4], they can be realized by noninteracting fermion systems, such as the vortex in a \( p + ip \) 2D superconductor [5–7].

In 1993 [8], it was predicted that some non-Abelian fractional quantum Hall states [2,3] can have 1D chiral Majorana fermions on the edge. (1D chiral Majorana fermions are fermions with only fermion-number-parity conservation [9,10] that propagate only in one direction in 1D space.) In fact, the appearance of an odd number of 1D chiral Majorana fermion modes on the edge implies the appearance of non-Abelian defects in the bulk [8,11]. The non-Abelian states may have already been realized in experiments [12–14]. In particular, the recently observed half quantized thermal Hall conductance [15] from the quantum Hall edge states [8,16,17] provides conclusive evidence of 1D chiral Majorana fermions and its “parent” non-Abelian fractional quantum Hall states. In 2000 [5], 1D chiral Majorana fermions were predicted to exist on the edge of a \( p + ip \) 2D superconductor. More recently, 1D chiral Majorana fermions were predicted to exist on the interface of a ferromagnet and superconductor on the surface of a topological insulator [7], and on the edge of an integer quantum Hall (IQH) film covered by a superconducting (SC) film [18,19].

In Refs. [18,19], it was shown that 1D chiral Majorana fermions can give rise to a \( \frac{1}{2} (e^2/h) \) conductance plateau for a two terminal conductance \( \sigma_{12} \) across a Hall bar covered by a superconductor. Recently, Ref. [20] observed such a conductance plateau, which was regarded as a “distinct signature” of 1D chiral Majorana fermions. This leads to the claimed discovery of 1D chiral Majorana fermions. The discovered Majorana fermions were named “angel particles,” and have attracted a lot of attention.

However, observing a \( \frac{1}{2} (e^2/h) \) conductance plateau may not imply the existence of 1D chiral Majorana fermions. For example, in Fig. 4(a) of the very same paper [20], a \( \frac{1}{2} (e^2/h) \) conductance was observed in a stacked IQH film and a metal film without the Majorana fermions. Similarly, Refs. [20,21] pointed out that a \( \frac{1}{2} (e^2/h) \) conductance can appear when the Hall bar under the SC film is in a metallic state without the Majorana fermions.

Such an explanation was discarded in Refs. [20,21] since it was thought to be inconsistent with the observed magnetic field \( B \) dependence of \( \sigma_{12} \) [Fig. 2(c) and Fig. 4(a) in Ref. [20]]. In the experiment, \( \sigma_{12}(B) \) is found to be \( \frac{1}{2} (e^2/h) \) when the magnetic field \( B \) is high and thereby the topped film is in the normal metallic state. Then, it increases up to \( e^2/h \), as \( B \) is reduced and the topped film becomes SC. As \( B \) is reduced further, \( \sigma_{12} \) drops to a \( \frac{1}{2} (e^2/h) \) plateau near \( B_c \), and then to near 0.

**Result.** In this Letter, we study the Majorana-fermionless mechanism for the \( \frac{1}{2} (e^2/h) \) conductance plateau in detail. We find that it can explain the whole observed magnetic field \( B \) dependence of \( \sigma_{12} \) very well. The \( \frac{1}{2} (e^2/h) \) conductance plateau can be a general feature of a good electric contact between the IQH and the SC films, regardless of whether the 1D chiral Majorana fermions exist or not.
chemical potential also opposite: \( \sigma \) on the two outgoing edge channels of the Hall bar are seen that the conductance is quantized conductance of \( \sigma \), where the IQH edge channel fills the sample and its trajectory length \( L_\text{edge} \) is long. As a result, the contact resistance \( 1/\sigma \) is low. A microscopic calculation of the contact conductance \( \sigma \) between the superconductor and an IQH edge channel.—We first assume the SC film and IQH bulk are clean enough that they are both fully gapped. Thus, only Andreev scattering along the edge contributes to \( \sigma \). To include the effects of charge conserving inelastic scattering, we first divide the IQH edge channel into many segments each of length \( L_\text{edge} \)—the dephasing length. Each segment is coupled to a superconductor [see Fig. 2(a)], which induces the coherent Andreev scattering: free electrons up to a chemical potential \( \mu \) can be coherently scattered and come out as holes. The incoming edge state is an equilibrium state with an incoming chemical potential \( \mu \), while the outgoing edge state out of one SC segment is not an equilibrium state. Charge conserving inelastic scattering equilibrates the outgoing edge state, which now has a chemical potential \( \mu' \). From \( \mu - \mu' \), we can determine \( \sigma \) for the segment.

To analyze the change in \( \mu \) after passing a single SC layer, let us start with the equation of motion for a free chiral fermion:

\[
\dot{\psi} = v_f (-i \partial_x - k_F)\psi + \frac{\hbar}{2} (v_\text{sc} \partial_x c^\dagger + \partial_x (v_\text{sc} c^\dagger)),
\]

where \( v_f \) is the velocity of the chiral fermion, \( k_F \) is the Fermi momentum at \( \mu = E_F = 0 \), and \( v_\text{sc}(x) \) is the SC coupling coefficient, which depends on \( x \) (\( v_\text{sc} = 0 \) for an edge not under the superconductor). We treat \( (c, c^\dagger) = (\psi_1, \psi_2) = \psi^T \) as independent fields. For a mode with frequency \( \omega \), the equation of motion becomes

\[
\omega \psi = \left( v_f (-i \partial_x - k_F) + \frac{\hbar}{2} (v_\text{sc} \partial_x + \partial_x v_\text{sc}) \right) \psi
\]
or (up to linear $v_{qe}$ order)

$$- v_f \left( \frac{r_{sc}}{x_f} \right)^{-1} i \partial_x \left( \frac{r_{sc}}{x_f} \right)^{-1} \psi$$

and, to the linear order in $v_{sc}$, we can rewrite the above as

$$M(x) = \begin{pmatrix} 1 & \frac{v_{sc}(x)}{v_f} k_F & 0 \\ \frac{v_{sc}(x)}{v_f} k_F & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Let $\tilde{\psi} = \left( \frac{v_{sc}/(2v_f)}{1} \right)^{-1} \psi$, we can rewrite the above as

$$- i \partial_x \tilde{\psi}(x) = M(x) \tilde{\psi}(x),$$

$$S = P[\hat{\phi} e^{i \int_0^L dx M(x)}] \tilde{\psi}(0),$$

where $P$ is the path ordering. Now we assume that $v_{sc}(x) = 0$ for $x < 0$ and $x > l$, and $v_{sc}(x)$ is a constant for $x \in [0, l]$. We find $\psi(l) = S \psi(0)$, where the unitary matrix $S$ is given by

$$s_{SC-Hall} = - \frac{\delta v_{sc}}{v_f} = \left( \frac{\mu}{\Delta} \right)^2 \frac{\delta L_{edge}}{l},$$

with $1/\Delta = \sqrt{2 |v_{sc}|/(\hbar v_f^2 k_F)}$, where we have replaced $\sin^2(k_F l)$ by its average $1/2$. Interestingly, $s_{SC-Hall}$ is proportional to $\mu^2$, or rather, non-Ohmic.

In the high temperature limit,

$$\langle a_k^\dagger a_k \rangle = \frac{1}{e^{(\Delta - \mu k_F) T} + 1},$$

$$\langle b_k^\dagger b_k \rangle = \cos^2(\theta(\mu, k)) + \sin^2(1 - \cos(\mu, -k))$$

Keeping to the first order of $\mu$, we reach

$$\mu' = \mu \left[ 1 - \frac{2\pi}{3} \left( \frac{v_{sc}}{v_f} \frac{\sin(k_F l)}{k_F} \right)^2 \right].$$

From this we obtain, for a length $\delta L_{edge}$ edge,

$$s_{SC-Hall} = \gamma \frac{\delta L_{edge}}{l},$$

with $\gamma = (\pi^2/3)|v_{sc}|k_B T/(\hbar v_f^2 k_F)^2$. In this case, $s_{SC-Hall}$ is independent of $\mu$ and is Ohmic.

If either the SC film or IQH bulk is not clean enough and has gapless electronic states that couple to the chiral edge channel, we can take into account those gapless states by assuming the superconductor to be a gapless superconductor. In this case, $s_{SC-Hall}$ will in addition receive a contribution from the electron tunneling into the quasiparticle states in the gapless superconductor. We expect such a contribution to be Ohmic and $s_{SC-Hall}$ can be modeled by Eq. (12) over the entire temperature range.

In the following, we will separately calculate $s_{12}(B)$, using the non-Ohmic (9) or Ohmic (12) $s_{SC-Hall}$.
Non-Ohmic case.—From Eq. (9), we see that the contact resistance can be much bigger than \( h/e^2 \), as long as \( \mu^2 \delta L_{\text{edge}} \) is small enough. The current \( \delta I = \sigma_{\text{SC-Hall}} \mu \) flowing from the edge to the superconductor will cause a drop in the chemical potential \( \mu \) along the edge:

\[
d\mu(x) = -\sigma_{\text{SC-Hall}} \mu = -\frac{\mu^3(x)}{l_{\phi} \Delta^2} dx. \tag{13}
\]

Solving the above equation, we find \( \mu = \mu(L_{\text{edge}}) = \mu_0 \sqrt{2(\mu_0^2/\Delta^2 l_{\phi})L_{\text{edge}}} + 1 \) for an edge of length \( L_{\text{edge}} \).

Therefore, for \( B > B_c \) [see Fig. 1(a)]

\[
\sigma_{12} = \frac{\mu_0 + \mu}{2\mu_0} = \frac{\sqrt{2 \mu_0^2 \Delta^{-1} L_{\text{edge}}}}{2 \mu_0}.
\tag{14}
\]

In a percolation cluster of size \( \xi \), the edge length is \( (\xi^2/a) \), where \( a \) is the cutoff length scale of the percolation model. The total edge length is \( L_{\text{edge}} = (l_{\text{edge}}/\xi)(\xi^2/a) = l_{\text{edge}}(\xi^2/a) \). The linear size of the percolation cluster \( \xi \) scales as

\[
\xi = a \left( \frac{B - B_0}{B_0} \right)^{-\nu} + a, \quad \nu = 1.33. \tag{15}
\]

With the above choice, we see that \( L_{\text{edge}}(\sigma_{12}) \rightarrow (l_{\text{edge}}, 1) \) as \( B \rightarrow \infty \) (assuming \( (2l_{\phi}/h^2)(l_{\text{edge}}) \) is small), and \( L_{\text{edge}}(\sigma_{12}) \rightarrow (\infty, \frac{1}{2}) \) as \( B \rightarrow B_c \).

But \( \xi \) can only increase up to \( l_{\text{edge}} \), the width of superconductor covered Hall bar, beyond which \( \xi \) remains as \( l_{\text{edge}} \) in the metallic phase in Fig. 1(b). To model such a behavior, we choose

\[
L_{\text{edge}} = a^{-1} \xi l_{\text{edge}} \Theta(B - B_c) \Theta(l_{\text{edge}} - \xi)
+ a^{-1} l_{\text{edge}}^2 \Theta(\xi - l_{\text{edge}})
+ a^{-1} l_{\text{edge}}^2 e^{(l_{\text{edge}} - \xi)/\xi} \Theta(B_c - B) \Theta(l_{\text{edge}} - \xi). \tag{16}
\]

where \( \Theta(x) = 1 \) if \( x > 0 \) and \( \Theta(x) = 0 \) if \( x < 0 \). When \( B > B_c \), the above gives \( L_{\text{edge}} = a^{-1} \xi l_{\text{edge}} \) or \( a^{-1} l_{\text{edge}}^2 \) near \( B_c \) [see Fig. 2(b)]. When \( B \) is much less than \( B_c \), we also assign \( L_{\text{edge}} \) a very large value to make \( \mu_0 \sqrt{2(\mu_0^2/\Delta^2 l_{\phi})L_{\text{edge}}} + 1 \) vanish. This allows us to combine the \( B > B_c \) and \( B < B_c \) results together later. For \( B < B_c \) [see Fig. 1(c)]

\[
\sigma_{12} = \frac{\mu_0 - \mu}{2\mu_0} = \sqrt{\frac{2 \mu_0^2}{\Delta^2 l_{\phi}} L_{\text{edge}}^\perp + 1},
\]

\[
L_{\text{edge}}^\perp = a^{-1} \xi l_{\text{edge}} \Theta(B_c - B) \Theta(l_{\text{edge}} - \xi)
+ a^{-1} l_{\text{edge}}^2 \Theta(\xi - l_{\text{edge}})
+ a^{-1} l_{\text{edge}}^2 e^{(l_{\text{edge}} - \xi)/\xi} \Theta(B - B_c) \Theta(l_{\text{edge}} - \xi). \tag{17}
\]

We can combine the \( B > B_c \) and \( B < B_c \) cases:

\[
\sigma_{12} = \frac{1}{2} \left( 1 + \frac{1}{\sqrt{2 \mu_0^2 \Delta^{-1} L_{\text{edge}}}} - \frac{1}{\sqrt{2 \mu_0^2 \Delta^{-1} L_{\text{edge}}^\perp}} \right). \tag{18}
\]

With the above design of \( L_{\text{edge}}^\perp \) and \( L_{\text{edge}} \), only one of the two terms in \( 1/\sqrt{2(\mu_0^2/\Delta^2 l_{\phi})L_{\text{edge}}} + 1 - 1/\sqrt{2(\mu_0^2/\Delta^2 l_{\phi})L_{\text{edge}}^\perp + 1} \) contributes in either the \( N_{\text{Chern}} = 1 \) phase or the \( N_{\text{Chern}} = 0 \) phase. In the metallic phase [see Fig. 1(b)], both terms are small, and their difference makes the contribution even smaller. This gives rise to the \( \frac{1}{2} \) quantized two terminal conductance. The above result is plotted in Fig. 3(a). Such a result is very close to what was observed in Ref. [20]. But it has a very different mechanism than what was proposed in Refs. [18,19]. In our non-Ohmic case, the \( \sigma_{12} = \frac{1}{2} (e/\hbar) \) plateau roughly corresponds to the metallic phase in Fig. 1 where \( \xi/l_{\text{edge}} \approx 1 \), with no need to introduce a 1D chiral Majorana fermion on the edge.

Ohmic case.—From Eq. (12), we see that the contact resistance can be much bigger than \( h/e^2 \), if \( \gamma \delta l_{\text{edge}} l_{\phi} \) is small enough. From the equation \( d\mu(x) = -\gamma(\phi dvl_{\phi})\mu(x) \) and for a given total length of the edge channel \( L_{\text{edge}} \), we find \( \mu = \mu_0 e^{-\gamma L_{\text{edge}} l_{\phi}} \). Therefore, for \( B > B_c \) [see Fig. 1(a)]

\[
\sigma_{12} = (\mu_0 + \mu)/(2\mu_0) = (1 + e^{-\gamma l_{\text{edge}} l_{\phi}/2}),
\]

FIG. 3. Two terminal conductance \( \sigma_{12} \) as a function of the magnetic field \( B \). (a) Non-Ohmic case (18), with \( l_{\text{edge}}/a = 70 \) and \( 2\mu_0^2 l_{\text{edge}}/(\Delta^2 l_{\phi}) = 0.12 \). Deviation of \( \sigma_{12} \) from \( e^2/h \) and 0 will have a clear voltage \( V = \mu_0 e \) dependence. (b) Ohmic case (19), with \( \gamma(l_{\text{edge}} l_{\phi}) = 1/14 \). The curve for the Ohmic case is independent of the percolation cutoff length scale \( a \).
where \( L_{\text{edge}} = (l_{\text{edge}}/\xi)(\zeta^2/a) = l_{\text{edge}}(\zeta/a) \). With \( \xi \) given in Eq. (15), we see that \( L_{\text{edge}} \rightarrow l_{\text{edge}} \) as \( B \rightarrow \infty \) and \( L_{\text{edge}} \rightarrow 0 \) as \( B \rightarrow B_c \). Similarly, for \( B < B_c \) [see Fig. 1(c)], \( \sigma_{12} = (\mu_0 - \mu)/(2\mu_0) = (1 - e^{-\gamma l_{\text{edge}}/a}/2) \). We can combine the \( B > B_c \) and \( B < B_c \) cases together:

\[
\sigma_{12} = \frac{1 + \text{sgn}(B - B_c)e^{-(B_c/B - B_c^2)/(\gamma l_{\text{edge}}/a)}}{2}. \tag{19}
\]

The above result is plotted in Fig. 3(b). Such a result for the Ohmic case is also very close to what was observed in Ref. [20]. But for the Ohmic case, the \( \sigma_{12} \) is very close to what was observed in Ref. [20]. The above result is plotted in Fig. 3(b). Such a result for the Ohmic case is also very close to what was observed in Ref. [20].

**Summary.**—In the percolation model, we considered two possible cases, the Ohmic case and the non-Ohmic case; both can explain the \( \sigma_{12}(B) \) curve in the experiment of Ref. [20]. More experiments are needed to see which case applies. If an Ohmic contact conductance is observed, it will indicate that either the SC and/or IQH bulks have gapless electronic states, or the electron temperature is high.

If a non-Ohmic contact conductance \( \sigma_{SC-Hall} \) between the superconductor and the IQH edge channel is observed near \( \sigma_{12} \approx 0 \) or \( \sigma_{12} \approx 1 \), it will indicate the SC and IQH bulks are fully gapped. Therefore, observing a non-Ohmic contact conductance is a sign of clean samples, which is necessary for further strong quantum coherent phenomena. For instance, on such samples at a low enough temperature, the dephasing length can become large, and 1D chiral Majorana fermions can appear.

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**Note added.**—Recently, another paper [22] appeared where the same conclusion was reached via a similar consideration. And in another recent paper [23], the dephasing length \( l_\phi \) is assumed to be larger than the “\( p + ip \) SC coherence length” \( \xi_{p+ip} \) (to put it another way, the minimum width of a \( p + ip \) SC stripe is such that 1D chiral Majorana fermions on the two edges are well separated). In this case, the 1D chiral Majorana edge mode can be well defined, and gives rise to a \( \frac{1}{2} (e^2/h) \) plateau in \( \sigma_{12} \). In this Letter, we consider the opposite limit \( l_\phi < \xi_{p+ip} \) without a coherent 1D chiral Majorana edge mode, and show that there is still a \( \frac{1}{2} (e^2/h) \) plateau. Furthermore, the \( B \) dependence of \( \sigma_{12} \) can be made to agree with the experiment very well, with a proper choice of some parameters. In particular, if we choose \( B_0 \approx 200 \) mT, the plateau width will be about 20 mT (see Fig. 3).