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Economic framework for net power density and levelized cost of electricity in pressure-retarded osmosis

Hyung Won Chung, Jaichander Swaminathan, Leonardo D. Banchik, John H. Lienhard

Rohsenow Kendall Heat Transfer Laboratory, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge MA 02139-4307 USA

Abstract

Economic analysis is necessary to ascertain the practical viability of a pressure-retarded osmosis (PRO) system for power production, but high complexity and the lack of large scale data has limited such work. In this study, a simple yet powerful economic framework is developed to relate the lower bound of levelized cost of electricity (LCOE) to net power density. A set of simplifying assumptions are used to develop an inverse linear relationship between net power density and LCOE. While net power density can be inferred based on experimentally measured power density, LCOE can be used to judge the economic viability of the PRO system. The minimum required net power density for PRO system to achieve an LCOE of $0.074/kWh (the capacity-weighted average LCOE of solar PV in the U.S.) is found to be 56.4 W/m$². Using this framework, we revisit the commonly cited power density of 5 W/m$² to conclude that it is not economically viable because net power density would be even lower. Finally, we demonstrate that fundamental difference exists between power density and net power density, and as a result we recommend using net power density as a performance metric for PRO system.

Keywords: Pressure-retarded osmosis; Economic analysis; Levelized cost of electricity; net power density


*Corresponding author: lienhard@mit.edu
Nomenclature

\(i\) \quad \text{Interest rate}
\(n\) \quad \text{Payback period}
\(A_m\) \quad \text{Membrane area, m}^2
\(w_d\) \quad \text{Draw salinity, w/m}^2
\(\tilde{C}_{\text{CapEx}} (A_m, w_d)\) \quad \text{Total capital expenditure as a function of } A_m \text{ and } w_d, \$\n\(\tilde{C}_m (A_m, w_d)\) \quad \text{Total membrane cost as a function of } A_m \text{ and } w_d, \$
\(\tilde{C}_{\text{labor}} (A_m, w_d)\) \quad \text{Total labor cost as a function of } A_m \text{ and } w_d, \$
\(\tilde{C}_{\text{chem}} (A_m, w_d)\) \quad \text{Total chemical cost as a function of } A_m \text{ and } w_d, \$
\(\tilde{C}_{\text{parts}} (A_m, w_d)\) \quad \text{Total parts replacement cost as a function of } A_m \text{ and } w_d, \$
\(\tilde{C}_{\text{CapEx}} (A_m, w_{d,\text{min}})\) \quad \text{Total capital expenditure as a function of } A_m \text{ at } w_{d,\text{min}}, \$
\(\tilde{C}_m (A_m, w_{d,\text{min}})\) \quad \text{Total membrane cost as a function of } A_m \text{ at } w_{d,\text{min}}, \$
\(\tilde{C}_{\text{labor}} (A_m, w_{d,\text{min}})\) \quad \text{Total labor cost as a function of } A_m \text{ at } w_{d,\text{min}}, \$
\(\tilde{C}_{\text{chem}} (A_m, w_{d,\text{min}})\) \quad \text{Total chemical cost as a function of } A_m \text{ at } w_{d,\text{min}}, \$
\(\tilde{C}_{\text{parts}} (A_m, w_{d,\text{min}})\) \quad \text{Total parts replacement cost as a function of } A_m \text{ at } w_{d,\text{min}}, \$
\(C_{\text{CapEx}}\) \quad \text{Proportionality constant for linear approximation of capital expenditure, } $/m^2$
\(C_m\) \quad \text{Membrane cost per unit membrane area, $/m^2$
\(C_{\text{labor}}\) \quad \text{Proportionality constant for linear approximation of labor cost, } $/\text{year-m}^2$
\(C_{\text{chem}}\) \quad \text{Proportionality constant for linear approximation of chemical cost, } $/\text{year-m}^2$
\(C_{\text{parts}}\) \quad \text{Proportionality constant for linear approximation of parts replacement cost, } $/\text{year-m}^2$
\(\text{CRF}\) \quad \text{Capital recovery factor}
\(L_m\) \quad \text{Membrane life, years}
\(\text{LCOE}_{\text{CapEx}}\) \quad \text{Capital expenditure per unit membrane area, $/m^2$
\(\text{LCOE}_m\) \quad \text{Membrane cost per unit membrane area, $/m^2$
\(\text{LCOE}_{\text{labor}}\) \quad \text{Labor cost per unit membrane area, $/m^2$
\(\text{LCOE}_{\text{chem}}\) \quad \text{Chemical cost unit membrane area, $/m^2$
\(\text{LCOE}_{\text{parts}}\) \quad \text{Parts replacement cost per unit membrane area, $/m^2$
\(W_{\text{module}}\) \quad \text{Module (gross) power output, W}
\(W_{\text{net}}\) \quad \text{Net power, kW}
\(W_{\text{pump}}\) \quad \text{Pump power consumption, W}
\(W_{\text{pt}}\) \quad \text{Pretreatment power consumption, W}
\(W_{\text{aux}}\) \quad \text{Auxiliary power consumption, W}
\(P_{\text{den, module}}\) \quad \text{Module (gross) power density, W/m}^2
\(P_{\text{den, net}}\) \quad \text{Net power density, W/m}^2
\(P_{\text{den, pump}}\) \quad \text{Pump power density, W/m}^2
\(t_{\text{op}}\) \quad \text{Operating days of a year, hr/year}
1. Introduction

Given the rapidly increasing need for a non-intermittent source of renewable energy, pressure-retarded osmosis (PRO) has continued to receive significant interest even after the first commercial pilot plant by Statkraft stopped operation in 2014 [1]. Japan’s Megaton Project has integrated PRO with seawater reverse osmosis (SWRO) to lower capital cost and harness the high salinity of RO brine [2]. In South Korea, PRO was integrated with RO and membrane distillation (MD) systems to process the brine at even higher salinity. These leading pilot plants and recent studies on PRO ([3, 4]) suggest that the most popular salinity pairing of seawater and river water is not feasible and that a more saline draw stream is necessary to make PRO viable. Although these studies identified the need for high salinity, the power density needed for economic viability remains unclear. In 2008, Statkraft, the Norwegian company that constructed a PRO power production pilot plant producing 2–4 kW of electricity, reported that a power density of at least 5 W/m$^2$ is necessary for PRO to be economically viable [5]. Since then, this number has been widely quoted in the literature [6, 7, 8, 9, 10, 11], yet the economic basis for this value of minimum power density remains unclear. Using a new economic framework, we revisit the minimum power density for PRO and conclude that 5 W/m$^2$ is an order-of-magnitude lower than the required power density for economic viability.

Most PRO studies report power density to quantify the performance of PRO because it is an easily measurable quantity. We will sometimes use “module power density” to differentiate power density from net power density. To our knowledge, 60 W/m$^2$ is the highest module power density achieved in the literature [12], using a coupon-sized system. Coupon-sized experiments do not account for the streamwise variations of salinity found in larger, commercial-scale elements, and consequently their power densities are significantly higher than can be achieved at a practical scale. Also, net power density, which accounts for the necessary power input to the pumps, was not reported by Straub et al. [12]. As will be discussed in Sec. 5, the net power density even for a coupon sized system is much lower than the module power density.

Although the power density is a metric that accounts for both the energy production (related to OpEx) and the membrane area (related to CapEx), maximizing the power density directly may not correspond to minimizing the overall cost. This ambiguity arises because a clear relationship between the power density and an economic metric is missing in the literature. The economic framework developed in the present study directly relates the levelized cost of electricity (the most widely used economic metric for power production) to net power density. Using this model, we can calculate the minimum required net power density to achieve the target LCOE. Another way to use this model, when a laboratory measurement of the net power density is available, is to calculate the minimum LCOE based on scaling up the the laboratory PRO system.

2. Economic analysis

The goal of this section is to develop a direct relationship between LCOE and net power density that does not depend on a particular use case or system design (e.g., salinity or area) of the PRO system.

2.1. Breakdown of CapEx data

Since no commercial PRO plant is operating as of 2017, the capital expenditure (CapEx) for a full PRO plant is difficult to estimate. Therefore, we benchmark using CapEx data for reverse osmosis plants. This approach is similar to that of Loeb [13, 14] in that SWRO data was used to approximate the PRO cost. However, our approach is different in that we do not attempt to accurately model the PRO cost with SWRO cost because significant uncertainty may arise in doing so. Instead, whenever SWRO and PRO have fundamental differences, we exclude such cost factors so that the resulting cost is lower than an actual cost would be.

A detailed breakdown of CapEx and operating expenditures (OpEx) data for SWRO CapEx is available from DesalData.com [15]. Each CapEx value from DesalData.com is given as a function of pure water production capacity of the RO plant ($/-day/m$^3$), which we convert to a per-membrane area basis by using an average RO water flux of 14 L/m$^2$-hr [16]. From the CapEx data, we exclude the membrane cost because the membrane replacement cost is taken into account by the OpEx. In this paper, we assume that PRO membranes have the same price as RO membranes, which makes the resulting PRO cost lower than the actual cost, assuming that a PRO membrane is likely to be more expensive than an RO membrane. We also
exclude intake and outfall cost. This is to account for the possibility that a PRO plant can be added to an existing SWRO plant, in which case the intake and outfall systems are already present.

In typical SWRO with 50% recovery (with the salinity range of 35–70 g/kg), the highest pressure is around 70 bar. For PRO system operating with similar salinity range (e.g., draw solution is the SWRO brine and diluted to 35 g/kg), the highest pressure involved is lower. Typically, the draw stream is pressurized to half the osmotic pressure difference at the inlet condition. Hence, the cost of pressure vessels and pumps was excluded to avoid SWRO CapEx overestimating the PRO CapEx. Finally, we also excluded the CapEx associated with the pretreatment system because the best practice for pretreatment is not well-understood for PRO operation. Note that these assumption are consistent in that they all tend to lower the cost of PRO thus giving a lower bound on PRO’s cost. All these exclusions resulted in 31% reduction in CapEx (averaged over plant capacity) relative to SWRO CapEx. This CapEx level should, therefore, serve as a lower bound for the CapEx of PRO.

2.2. Lower bound of LCOE

Both CapEx and OpEx contribute to LCOE. The OpEx contribution can be further broken down into labor cost, chemical and parts replacement costs:

\[
\text{LCOE} = \text{LCOE}_{\text{CapEx}} + \text{LCOE}_{\text{OpEx}} = \text{LCOE}_{\text{CapEx}} + \text{LCOE}_{m} + \text{LCOE}_{\text{labor}} + \text{LCOE}_{\text{chem}} + \text{LCOE}_{\text{parts}}
\]  

(1)

We can work with each term separately starting with the \(\text{LCOE}_{\text{CapEx}}\). In most infrastructure projects, a loan is made so that the CapEx can be paid in annual installments over some period of time. The annuity of the CapEx (i.e., constant yearly payment) can be calculated using the capital recovery factor (CRF), which is obtained by dividing the annuity by the sum of the annuity over the loan period \(n\) in present value. In other words, the annuity is the product of the total CapEx and CRF. A 25 year loan period and 8% interest rate \([17]\) were used to evaluate the CRF. CRF can be calculated as:

\[
\text{CRF} = \frac{i (1 + i)^n}{(1 + i)^n - 1}
\]

(2)

In general, CapEx is a function of system size \((A_m)\) and draw salinity \((w_d)\). To capture these functional dependences, let \(\tilde{C}_{\text{CapEx}}' (A_m, w_d)\) be the total CapEx in $ such that \(\tilde{C}_{\text{CapEx}}' (A_m, w_d) \times \text{CRF}\) is the uniform annuity. We will drop tilde (associated with functional dependence on \(A_m\)) and prime (associated with \(w_d\)) notation as we simplify the model. With appropriate conversion of units, the contribution of CapEx to LCOE (\(\text{LCOE}_{\text{CapEx}}\)) is obtained as:

\[
\text{LCOE}_{\text{CapEx}} = \frac{\tilde{C}_{\text{CapEx}}' (A_m, w_d) \times \text{CRF}}{W_{\text{net}} t_{\text{op}}}
\]

(3)

The numerator has a unit of $/year, and the denominator has a unit of kWh/year, resulting in $/kWh. Here, \(t_{\text{op}}\) is the utilization rate in hrs/year with an assumed 90% plant up-time (330 days). We define the net power, \(W_{\text{net}}\), to be the total power production minus all the power input to the system including pumping (\(W_{\text{pump}}\)), pretreatment (\(W_{\text{pt}}\)), and auxiliary consumption (\(W_{\text{aux}}\)), as defined in Eq. (4):

\[
W_{\text{net}} = W_{\text{module}} - W_{\text{pump}} - W_{\text{pt}} - W_{\text{aux}}
\]

(4)

Next, the contribution of the membrane replacement cost to the LCOE is given by Eq. (5):

\[
\text{LCOE}_m = \frac{\tilde{C}_m' (A_m, w_d)}{L_m W_{\text{net}} t_{\text{op}}}
\]

(5)

---

1 The choice of 50% of inlet osmotic pressure difference is not theoretically justified unless the system size is very small. See Sec. [14] for detail
2 Operating expenditures related to pretreatment energy cost or other energy input are captured in \(W_{\text{net}}\). So we should not double count these.
3 CapEx also depends on feed salinity but in this paper we assume that feed salinity is fixed at low level (e.g., 1 g/kg). PRO system with higher feed salinity is unlikely to be viable. \([3]\)
where $\tilde{C}'_{\text{w}}(A_m, w_d)$ is the membrane cost in $ as a function of membrane area and draw salinity, and $L_m$ is the membrane lifetime. A membrane average life of 4 years was assumed.

A similar approach to CapEx can be used to evaluate LCOE$_{\text{lab}}$, LCOE$_{\text{chem}}$, and LCOE$_{\text{parts}}$.

\[
\text{LCOE}_{\text{lab}} = \frac{\tilde{C}'_{\text{lab}}(A_m, w_d)}{W_{\text{net}t_{\text{op}}}}
\]

(6)

\[
\text{LCOE}_{\text{chem}} = \frac{\tilde{C}'_{\text{chem}}(A_m, w_d)}{W_{\text{net}t_{\text{op}}}}
\]

(7)

\[
\text{LCOE}_{\text{parts}} = \frac{\tilde{C}'_{\text{parts}}(A_m, w_d)}{W_{\text{net}t_{\text{op}}}}
\]

(8)

With a slight abuse of notation $\tilde{C}'_{\text{lab}}(A_m, w_d)$ is the annual labor cost in $/year as a function of membrane area and draw salinity. $\tilde{C}'_{\text{chem}}(A_m, w_d)$ and $\tilde{C}'_{\text{parts}}(A_m, w_d)$ are defined in a similar way.

Substituting Eqs. (3), (5), (6), (7) and (8) into Eq. (1) results in

\[
\text{LCOE} = \text{LCOE}_{\text{CapEx}} + \text{LCOE}_{\text{m}} + \text{LCOE}_{\text{lab}} + \text{LCOE}_{\text{chem}} + \text{LCOE}_{\text{parts}}
\]

\[
= \frac{\tilde{C}'_{\text{CapEx}}(A_m, w_d) \times CRF}{W_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}'_{\text{m}}(A_m, w_d)}{L_mW_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}'_{\text{lab}}(A_m, w_d)}{W_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}'_{\text{chem}}(A_m, w_d)}{W_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}'_{\text{parts}}(A_m, w_d)}{W_{\text{net}t_{\text{op}}}}.
\]

(9)

Equation (9) is a general form that takes into account functional dependence of LCOE on draw salinity and system size (quantified by $A_m$). The problem is that the $\tilde{C}'(A_m, w_d)$ functions are unknown. Here we use $\tilde{C}'_{\text{w}}(A_m, w_d)$ (without subscript) to describe common characteristics shared by $\tilde{C}'_{\text{CapEx}}(A_m, w_d)$, $\tilde{C}'_{\text{m}}(A_m, w_d)$, etc. Nevertheless, we can make a few observations to understand these functions better.

First, for a fixed $A_m$, we define the domain of interest of $\tilde{C}'$ functions to be $[w_d^{\text{min}}, w_d^{\text{max}}] \subset \mathbb{R}$, where $w_d^{\text{min}}$ and $w_d^{\text{max}}$ are the minimum and maximum draw salinity of interest. For example, $w_d^{\text{max}} = 260$ g/kg will be appropriate if one is modeling NaCl draw solution because 260 g/kg corresponds to NaCl saturation. (the choice of a numerical value does not affect the subsequent analysis). Next we assume that if the operating salinity level is higher, then any cost (CapEx, membrane cost, etc) is higher than or equal to the cost of lower salinity operation. This assumption is reasonable because as draw salinity increases the construction materials would need to be stronger to withstand the higher hydraulic pressure, chemically stable to tolerate the higher salinity, etc. Mathematically, this means that, for a fixed $A_m$, $\tilde{C}'$ functions are monotonically non-decreasing function of $w_d$. Then the following inequality holds

\[
\tilde{C}'(A_m, w_d) \geq \tilde{C}'(A_m, w_d^{\text{min}}) \triangleq \tilde{C}(A_m), \quad \forall w_d \in [w_d^{\text{min}}, w_d^{\text{max}}]
\]

(10)

where $\triangleq$ symbol refers to a definition. So by using the cost associated with the minimum draw salinity, the functional dependence of $\tilde{C}'$ on draw salinity is eliminated. Applying this result to each term in Eq. (9), we have the following simplification

\[
\text{LCOE} = \text{LCOE}_{\text{CapEx}} + \text{LCOE}_{\text{m}} + \text{LCOE}_{\text{lab}} + \text{LCOE}_{\text{chem}} + \text{LCOE}_{\text{parts}}
\]

\[
= \frac{\tilde{C}_{\text{CapEx}}(A_m, w_d) \times CRF}{W_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}_{\text{m}}(A_m, w_d)}{L_mW_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}_{\text{lab}}(A_m, w_d)}{W_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}_{\text{chem}}(A_m, w_d)}{W_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}_{\text{parts}}(A_m, w_d)}{W_{\text{net}t_{\text{op}}}}
\]

\[
\geq \frac{\tilde{C}_{\text{CapEx}}(A_m) \times CRF}{W_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}_{\text{m}}(A_m)}{L_mW_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}_{\text{lab}}(A_m)}{W_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}_{\text{chem}}(A_m)}{W_{\text{net}t_{\text{op}}}} + \frac{\tilde{C}_{\text{parts}}(A_m)}{W_{\text{net}t_{\text{op}}}}.
\]

(11)

We proceed to further simplify $\tilde{C}(A_m)$ to get rid tilde (or to drop the dependence on $A_m$) by approximating $\tilde{C}(A_m)$ with a linear function, i.e.,

\[
\tilde{C}(A_m) \approx kA_m, \quad \text{for some } k \in \mathbb{R}^+.
\]

(12)

The use of a linear (not affine) function will be justified at the end of this section. Instead of any linear approximation, we make lower bound linear approximation such that this is consistent with other assumptions.
\[ \tilde{C}(A_m) \geq kA_m, \quad \text{for some } k \in \mathbb{R}^+, \quad \forall A_m \in (0,A_m^{\text{max}}] \]  
\hspace{1cm} (13)

where \( A_m^{\text{max}} \) is the maximum membrane area of interest. This maximum corresponds to the maximum membrane area of SWRO data from DesalData.com \[18\]. In fact the approximation can be restricted more by making tight linear lower bound approximation

\[ C \doteq \max k \quad \text{s.t.} \quad \tilde{C}(A_m) \geq kA_m, \quad \forall A_m \in (0,A_m^{\text{max}}] \]  
\hspace{1cm} (14)

where we defined \( C \) (without tilde) as a proportionality constant. For example, \( C_{\text{CapEx}} \) is a constant that has unit of \$/m^2.

A unique solution exists for this maximization if the following two assumptions are made. First, we assume that economies of scale apply to PRO. This assumption is reasonable as similar technologies (e.g., reverse osmosis) exhibit such characteristics. Mathematically, this implies that \( \tilde{C}(A_m) \) is concave in \( A_m \). Next we assume that \( \lim_{A_m \to 0} \tilde{C}(A_m) = 0 \). In other words, the cost approaches zero as the system size approaches zero.

The unique solution satisfying these two assumptions can be graphically illustrated as shown in Fig. 1 for the case of CapEx. The red curve is the actual CapEx data for SWRO \[18\] excluding the costs discussed in Sec. 2.1. Note that no line that passes through the origin (i.e., satisfies \( \lim_{A_m \to 0} \tilde{C}(A_m) = 0 \)) while being smaller than equal to the red curve has a value larger than the blue line at any \( A_m \in (0,A_m^{\text{max}}] \). Hence, the blue line is the solution to the maximization in Eq. (14). The blue line has a slope \( C_{\text{CapEx}} = \$239/m^2 \).

![Figure 1: RO CapEx excluding the cost factors mentioned in Sec. 2.1 as a function of membrane area with a linear curve fit. \( C_{\text{CapEx}} \) is the slope of the linear fit line.](image)

We can perform the same analysis for the labor cost. Figure \[2\] shows that the tight linear lower bound approximation is not as good as the CapEx. One might argue that a higher order polynomial approximation better fits the labor cost. However, we need to use a linear fit in order to derive the desired result, as will be shown later in the section. Annual labor cost per unit membrane area is \( C_{\text{labor}} = \$5.44/\text{year-m}^2 \).
For the chemical and parts data provided by DesalData.com have linear relation with the membrane area as shown in Fig. 3. Hence, in these cases we do not need to make further assumptions and just read off the slope of the lines to find the solutions to maximization in Eq. (14).

Applying the tight linear lower bound approximation to each terms in Eq. (11) and rearranging the terms,
A key step is isolating net power density \((\tilde{P}_{\text{den},\text{net}}})\) to approximate etc., the inequality shows a direct relationship between net power density and LCOE. We can be certain that the system will not be economically viable. 

The lower bound can be justified without this quantification. In particular, if the lower bound of LCOE quantifies how tight the lower bound is because of lack of large scale PRO plants. However, the utility of estimate the LCOE. Since \(LCOE\) is not measurable. Hence, it is easier to think in terms of net power density, an experimentally measurable quantity. Rearranging the inequality \((15)\) and substituting a target LCOE in place of LCOE, we can find a lower bound on the minimum required net power density \(P_{\text{den},\text{net}}^\text{min}\).

\[
P_{\text{den},\text{net}}^\text{min} \geq \frac{1}{\text{LCOE}_{\text{target}} t_{\text{top}}} \left( C_{\text{CapEx CRF}} + \frac{C_m}{L_m} + C_{\text{labor}} + C_{\text{chem}} + C_{\text{parts}} \right).
\]  

This is why we needed to use linear approximation. Basically, we sacrifice the tightness of the lower bound so as to obtain a more compact way of expressing a PRO system.

### 2.3. Remarks on the lower bound

A lower bound of LCOE was derived. But how tight is this lower bound? Unfortunately, we can’t quantify how tight the lower bound is because of lack of large scale PRO plants. However, the utility of the lower bound can be justified without this quantification. In particular, if the lower bound of LCOE suggested by this model is lower than the LCOE achievable with a PRO system under investigation, then we can be certain that the system will not be economically viable.

In practice, LCOE is not measurable. Hence, it is easier to think in terms of net power density, an experimentally measurable quantity.
A PRO system operator can measure the net power density and compare that to the minimum required net power density to check if the current system has potential to achieve the same level of LCOE as the target technology. If the measured net power density is lower than the minimum required net power density, the system will not achieve the target LCOE. This is a simple way to evaluate the economic viability of a complicated system without actually calculating LCOE, which is prone to significant error. This analysis is especially useful because the current state of the art PRO system has much lower net power density than the minimum required to compete with other renewable technology such as solar. Hence, without being able to quantify how tight the bound is, the inequality can be used. The detail of this analysis are in Sec. 3.

3. Required net power density to achieve a target LCOE

The economic competitiveness of PRO is evaluated by comparing its cost to the LCOE of a competing renewable energy source. We choose solar PV as the competing technology (i.e., \( \text{LCOE}_{\text{target}} = \text{LCOE}_{\text{solar}} \)) in the inequality (18), to illustrate the framework but the same analysis applies to any other power generation technology including non-renewable ones. The data for solar PV is available from the U.S. Energy Information Administration. As of 2017, solar power had the capacity-weighted national (U.S.) average LCOE of \$0.074/kWh\[19\].

Figure 4 shows the power density required as a function of membrane specific cost with \( C_m \) constants obtained from Sec. 2.2. For a PRO system with membrane cost of \$15/m^2, the net power density required for PRO to have LCOE of \$0.074/kWh is 56.4 W/m^2. This net power density is clearly much higher than the current state-of-the-art. The detail will be given in Sec. 4.

The utility of the economic framework stated by Eq. (15) is manifold. For example, we can easily test the commercial viability of the seawater and river water pairing. The classical zero-dimensional maximum power density \[20\] \( P_{\text{den,0D}} = \frac{A \Delta \Pi}{4} \) for seawater/river water pair is 10.9 W/m^2 using seawater property data \[21\]. The classical model does not account for any loss mechanisms, such as local concentration polarization and the streamwise decrease of flux driving force \[22\]. Hence, power density from the zero-dimensional model serves as an upper bound on power density achievable in reality. Even this upper-bound 0D power density, however, is much lower than the required net power density in Fig. 4. With the aggressive assumption of \$0/m^2 membrane cost, the required minimum net power density is 50 W/m^2 which is more than 4 times

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4 Although we make this comparison, we note that non-storage PV is intermittent whereas PRO is not.
the classical 0D limiting value of 10.9 W/m². In addition, the latter is power density while the former is net power density. So the difference in net power density will be even larger. Hence, we can argue that this most famous PRO configuration, pairing seawater with river water, is not viable because of low net power density. A high salinity draw stream is crucial, as has been previously observed on a strictly energetic basis by others [4, 23].

In addition, we can check the validity of the previously reported target value of 5 W/m². Although Statkraft did not explicitly state what salinity pairing this 5 W/m² refers to, we may safely assume that it is for seawater/river water pairing. If we take 5 W/m² as the net power density (which itself is a generous assumption as we neglect the energy input), and plug into the inequality (15), the resulting lower bound of LCOE for the PRO system would be $0.83/kWh, clearly indicating that 5 W/m² is not economically viable.

4. Evaluation of PRO literature through the lens of power density

Now that we have seen how much net power density is required for PRO to have a potential to be competitive (in terms of LCOE), we can evaluate the state of the art PRO systems reported in the literature. In fact the evaluation is impossible because to our knowledge no one has reported experimentally determined net power density. Instead we summarize the major publications from the perspective of power density. Skilhagen et al. [5] from Statkraft wrote “In early estimations, it was established that the PRO membranes needed to generate at least 4 W/m² net to produce power at a sensible cost.” But as to how this number was established, no detail was provided. In the same year, Gerstandt et al. [24], in collaboration with Statkraft, reported that “To make PRO profitable, the power density of the membrane was determined to be between 4–6 W/m²”. Again, no detail was given on how that number was determined. They also stated “...5 W/m², which is the target value for the membrane to make PRO commercial attractive”. This 5 W/m² has since been used as the target power density throughout PRO literature. Given this widespread use, it is unfortunate that Gerstandt et al. did not mention the word “net” which had been mentioned in the earlier paper by Skilhagen et al.

In 2014, Straub et al. [12] achieved power density of 60 W/m² with 7.7 cm long and 2.6 cm wide (exposed) membrane area, using 175 g/kg NaCl draw solution. As it will be shown in Sec. 5 even though the module power density is high, this system likely had negative net power density.

In 2018, Kurahara et al. [25] from Japan’s Megaton Project achieved power density of 13.3 W/m² with 10 in long pilot scale module and 17.1 W/m² with 5 in long lab scale module pairing SWRO brine and wastewater.

Also in 2018, Lee et al. [26] from South Korea’s GMVP project achieved power density of 6–8 W/m² pairing 70 g/kg NaCl draw and wastewater.

Lastly, Wan et al. [27] reported 38 W/m² also with 70 g/kg NaCl draw paired with deionized feed.

Other than the number reported by Straub et al. [12], all other reported power density values are lower than the minimum required net power density of 56.4 W/m² to achieved LCOE of $0.074/kWh with membrane cost of $15/m². Should we consider the achievement of Straub et al. as a sign that we are close to achieving economic viability of PRO? We may conclude so if net power density and power density are close to each other. So far, we have been somewhat vague about the difference between power density and net power density. Unfortunately, even though similarity in their names suggests such proximity, there exists a fundamental difference between them. This will be the topic of the next section.

5. Difference between net power density and module power density

To motivate the comparison of net power density and gross power density, we may consider how desalination differs from PRO. In desalination, one of the major costs is energy input, and the useful output is fresh water production. Usually, there is a tradeoff between increasing the energy efficiency and increasing the fresh water production per unit membrane area (flux): larger area system (with smaller flux) generally has higher energy efficiency than a smaller system [28] (at higher flux). So, one needs to find an optimal flux and energy efficiency combination that minimizes the cost. That process is not simple.

In contrast, the energetic cost and the output per unit membrane area are both energies in the case of PRO, as shown in Fig. 5. Hence, net power density captures both cost and useful output of the system,
simplifying the objective of PRO design. However, in the PRO literature, researchers have most often focused on module power density ($P_{\text{den}}$), not net power density.

Figure 5: Net power density captures both cost ($\dot{W}_{\text{in}}/A_m$) and useful output ($\dot{W}_{\text{out}}/A_m$) of the PRO process.

We will demonstrate this difference mathematically for the asymptotic case of $A_m \to 0$. Then we will expand the idea to the finite area case.

Figure 6 shows a typical PRO system with counterflow geometry.

Equation (4) shows that net power is the total power production minus pumping, pretreatment and auxiliary power consumption. In this section, we assume that the pretreatment and auxiliary power consumption are negligible in order to emphasize the relationship between the net power and pumping power. Then the net power density can be expressed as:

$$P_{\text{den},\text{net}} = \frac{\dot{W}_{\text{module}}}{A_m} - \frac{\dot{W}_{\text{pump}}}{A_m}$$

$$(19)$$

As a function of $A_m$, $P_{\text{den,module}}$ is maximized as $A_m \to 0$ where the flux is maximum and there is only an infinitesimal axial variation in feed and draw salinities [6 29 22]. We can analyze $P_{\text{den,module}}$, $P_{\text{den,pump}}$ and $P_{\text{den,net}}$ in this limit. We start with $P_{\text{den,module}}$. In the limit, module power density can be expressed in terms of inlet osmotic pressure using the solution diffusion model

$$\lim_{A_m \to 0} P_{\text{den,module}} = J \Delta P$$

$$= A(\Delta \Pi_{\text{in}} - \Delta P)\Delta P$$

$$(20)$$

where $\Delta \Pi_{\text{in}}$ is the inlet osmotic pressure difference. Note that this $P_{\text{den,module}} = J \Delta P = A(\Delta \Pi_{\text{in}} - \Delta P)\Delta P$ is only valid in the limit $A_m \to 0$. For any finite area system, $\Delta \Pi_{\text{in}} > \Delta \Pi_{\text{avg}}$. Hence incorrectly using
\( P_{\text{den, module}} = J \Delta P = A(\Delta \Pi_{\text{in}} - \Delta P) \Delta P \) for a finite area system results in overestimation of the resulting power density.

Since \( P_{\text{den, module}} \) (in the limit) is a concave function of \( \Delta P \), there exists a unique maximum achieved by \( \Delta P^* = \Delta \Pi_{\text{in}} \), and the resulting module power density is

\[
\lim_{A_m \to 0} P_{\text{den, module}} = A \frac{\Delta \Pi_{\text{in}}^2}{4}.
\]  

(21)

Again, this result is justified only for the limit \( A_m \to 0 \). This result means that the power density has a finite limit which in turn implies that

\[
\lim_{A_m \to 0} \dot{W}_{\text{module}} = \lim_{A_m \to 0} P_{\text{den, module}} A_m = 0
\]  

(22)

Next we can analyze \( P_{\text{den, pump}} \). By the energy balance on the pressure exchanger, the pump power can be expressed as

\[
\dot{W}_{\text{pump}} = Q_d (P_{d, \text{in}} - P_p) = Q_d [P_{d, \text{in}} - P_0 - \eta_{\Pi X} (P_{d, \text{out}} - P_0)].
\]  

In the limit \( A_m \to 0 \), the viscous pressure drop in the module vanishes and the draw inlet pressure approaches the outlet pressure. So the draw pressure can be expressed as single pressure value i.e., \( P_{d, \text{in}} \to P_{d, \text{out}} = P_d \). Then the resulting limit of the pumping power is

\[
\lim_{A_m \to 0} \dot{W}_{\text{pump}} = Q_d (1 - \eta_{\Pi X}) (P_d - P_0).
\]  

(24)

In this case, pumping power instead of density converges to a finite limit. Then, the limiting pumping power density is

\[
\lim_{A_m \to 0} P_{\text{den, pump}} = \lim_{A_m \to 0} - \frac{\dot{W}_{\text{pump}}}{A_m} = -\infty.
\]  

(25)

That the pumping power density diverges implies that net power density diverges as well.

\[
\lim_{A_m \to 0} P_{\text{den, net}} = \lim_{A_m \to 0} P_{\text{den, module}} + P_{\text{den, pump}} = -\infty
\]  

(26)

By comparing Eqs. (22) and (24), we can see that \( P_{\text{den, pump}} \) does not diverge if \( \dot{W}_{\text{pump}} \to 0 \). This can happen if \( Q_d = 0 \), \( \eta_{\Pi X} = 1 \) or \( P_d = P_0 \). The first case means no flow at all. The last case means that the draw pressure is the ambient pressure, and the resulting turbine power is zero. So the only practically viable way to prevent \( P_{\text{den, pump}} \) from diverging is to have \( \eta_{\Pi X} = 1 \), which is impossible in practice.

A financial analogy may help illustrate the implication of this analysis better. Module power density is like a revenue whereas net power density is the net income or profit. With this analogy, we can summarize the asymptotic analysis: maximizing revenue (power density) requires making the PRO system as small (or short) as possible. But in doing so, profit (net power density) approaches negative infinity.

Now we can extend the analysis to finite area case. As we increase the system size, module power density decreases because the average driving force decreases. This trend agrees with the results from Kurihara et al. \cite{25} who reported power density reduction from 17.1 to 13.3 as the module length decreased from 10 in to 5 in. If we assume that the \( P_{d, \text{in}} \) and flow rate are constant, pumping power density will increase as system size increases\footnote{Or one could increase the system size normalized by flow rate, in which case flow rate does not have to be held constant. But the result of the analysis will be the same.}.

Net power density, which is the sum of module power density and pumping power density, then increases at a slower rate until it plateaus. The existence of peak was found by Chung et al. \cite{3}; the optimum length that minimizes LCOE ranges from 5 m to 7.3 m for draw salinity ranging from 70 g/kg to 260 g/kg with feed salinity at 1 g/kg. This trend of power densities is graphically illustrated in Fig. \ref{fig:power_dens}. Hence, increasing power density leads to decrease in net power density. With this in mind, we strongly recommend the use of net power density to evaluate the performance of PRO system designs.
Figure 7: Qualitative plot of module power density, pumping power density, and net power density. Optimum membrane area (or length) exists if net power density is used as a performance metric.

We can briefly revisit the high power density achieved in the literature. The small scale system (7.7 cm long) that achieved 60 W/m² reported by Straub et al. [12] will probably have large negative net power density. This is because 7.7 cm length is short enough that the asymptotic analysis of this section can be applied. Although it is not possible to calculate net power density due to the lack of information (i.e., flow velocity, channel geometry, pump efficiency, etc.) for larger systems such as Megaton Project and GMVP Project, net power density is always lower than power density because of the finite energy input.

Since the current state-of-the-art PRO systems are characterized by far lower net power density than the minimum required net power density, we argue that the lower bound on net power density derived in this work, albeit somewhat aggressive, serves as a useful performance threshold for any PRO system.

6. Conclusions

A novel framework for an economic analysis of pressure-retarded osmosis is established using a set of simplifying assumptions. Our primary conclusions are as follow:

- We derived a lower bound on LCOE, which can be rearrange to yield minimum required net power density to achieve a target LCOE.
- Although the tightness of the lower bound could not be quantified, the bound is still useful because no one achieved the minimum value asserted by the lower bound.
- The required net power density for PRO to break-even with solar PV is 56.4 W/m² with membrane cost of $15/m².
- Net power density should be used as a single performance metric to evaluate the economic viability of a PRO system for any specific feed-draw salinity pairing.
- Net power density and power density fundamentally differ because of the inherent overhead associated with the PRO process in the form of irrecoverable pumping energy input.
- By using net power density as the performance metric, we found that an optimal length exists that maximizes the net power density and hence minimizes LCOE.
7. Future work

Now that net power density is established as the performance metric, a natural direction of future work is on improving net power density. The methods of improvement can be broadly classified into two categories: improving the membrane and system-level optimization. We finish this paper with a brief discussion of these two methods.

For the membrane improvement, Chung et al. [3] found that reducing the structural parameter $S$ of the membrane is a much more effective way to reduce LCOE than improving the membrane permeability $A$. Since the analysis of this paper links LCOE to net power density, we can infer that reducing $S$ is a more effective way to maximize net power density.

To increase net power density by system-level optimization, we should identify extra degrees of freedom (DOF) and manipulate these DOFs to maximize the net power density. From a thermodynamic perspective, multistaging is a natural direction. Because PRO involves two input streams as opposed to one for desalination systems, multistaging can be done in various ways. In the PRO literature, researchers have tried different ways of multistaging but the direct effect on net power density has not been investigated. Also a systematic analysis of different multistaging schemes is missing.

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