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Extended Coherence Length and Depth Ranging Using a Fourier-Domain Mode-Locked Frequency Comb and Circular Interferometric Ranging

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Fourier-domain mode locking has been a popular laser design for high-speed optical-frequency-domain imaging (OFDI), but achieving long coherence lengths, and therefore imaging range, has been challenging. The narrow linewidth of a Fourier-domain mode-locked (FDML) frequency-comb (FC) laser could provide an attractive platform for high-speed as well as long-range OFDI. Unfortunately, aliasing artifacts arising from signals beyond the principal measurement depth of the free spectral range have prohibited the use of an FDML FC laser for imaging so far. To make the increased coherence length of an FDML FC laser available, methods to manage such artifacts are required. Recently, coherent circular ranging that uses frequency combs for imaging in much-reduced rf bandwidths has been demonstrated. Here we revisit circular ranging as a tool for making the long coherence length of an FDML FC laser and its use for tissue imaging accessible. Using an acousto-optic frequency shifter (AOFS), we describe an active method to mitigate signal aliasing that is both stable and wavelength independent. We show that an FDML FC laser increases the coherence length by an order of magnitude compared with traditional FDML-laser designs without requiring precise dispersion engineering. We discuss design parameters of a frequency-stepping laser resonator as well as aliasing from a frequency comb and AOFS in OFDI with numerical simulations. The use of circular ranging additionally reduces acquisition bandwidths 15-fold compared with traditional OFDI methods. The FDML FC–AOFS design offers a convenient platform for long-range and high-speed imaging as well as for exploring signal- and image-processing methods in circular ranging.

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I. INTRODUCTION

Wavelength-swept laser sources used for optical-frequency-domain imaging (OFDI) have been the focus of intense research to increase their repetition rate and coherence length over the past decade [1–3]. The Fourier-domain mode-locked (FDML) laser has become increasingly attractive for high-speed imaging [2,4–9]. It uses a long optical fiber delay line and a fiber Fabry-Perot tunable (TFP) filter with a sweep rate that is synchronized with the round-trip time of the cavity, thereby increasing output power and tuning rate. However, its depth range has been limited to only a few millimeters. Current strategies to extend the depth range in FDML lasers rely on precise dispersion engineering of the cavity using a set of chirped, broadband optical fiber Bragg gratings that have provided marginal extensions of the imaging range to 5 mm at a repetition rate (sweep rate) of 1.6 MHz (30 THz/μs) and to 1 cm at a repetition rate (sweep rate) of 200 kHz (3.6 THz/μs) [4,5]. Recently, extremely precise (very high-order) dispersion control was implemented by application of a temperature gradient (with resolution of approximately 0.1 °C) along a fiber Bragg grating with simultaneous control of the temperature of the gain medium, tunable spectral filter, and housing, achieving an imaging range of 10 cm at a repetition rate of 3 MHz and a sweep bandwidth of 77 nm (40 THz/μs) for the first time [9]. While impressive in achievement, this involves an environmentally sensitive laser cavity, and imaging at full performance [high speed, broad sweeping bandwidth, long imaging range] requires sampling at tens of gigasamples per second using state-of-the-art real-time oscilloscopes, unbalanced and high-bandwidth photodetectors with low signal gain, and hundreds of gigabyte of memory for a single frame, which is not practical. Frequency combs (FCs) can provide an alternative strategy to increase coherence length. FDML FC lasers have been proposed but the FC induced a degeneracy in the
ranging measurements that resulted in imaging artifacts. These artifacts effectively limited the imaging range to a few millimeters, and thus no significant improvement was achieved [10].

Recently, circular-ranging optical coherence tomography has been introduced to reduce bandwidth requirements without compromising range, speed, or axial resolution [11,12]. In this approach, the depth space of the interferometric signals is folded, leveraging sparsity within the imaged field to reduce the number of measurements needed to capture the sample properties. Circular ranging is implemented in the optical domain by combining a frequency-comb source with an interferometric system capable of discriminating between positive and negative differential delays. While this technique has been proposed for the primary purpose of bandwidth reduction, it also provides a means for overcoming degeneracy artifacts induced by use of a FC source. In this work, we demonstrate that these methods from circular ranging can be adopted to avoid artifacts that have limited the use of FDML FC lasers. Using an acousto-optic frequency shifter (AOFS), we discriminate positive from negative delay and mitigate aliasing signals outside the principal measurement range, making the long coherence length of FDML FC lasers accessible and its application for tissue imaging possible.

The combination of FDML FC lasers and circular ranging offers coherence length increased by 1 order of magnitude compared with standard FDML sources [2,8], without the temperature control and accurate high-order dispersion compensation that was previously required. We demonstrate imaging results obtained over centimeter ranges at a repetition rate of 488 kHz and a wavelength-sweep range of 105 nm (18 THz). In addition, the use of circular ranging reduces acquisition bandwidths and memory requirements 15-fold compared with a traditional FDML laser. Moreover, the active AOFS approach is stable and wavelength independent, yielding an advantage over the previously proposed passive polarization-demodulation method used for circular ranging [11,13]. We provide new insight for the design parameters of a frequency-stepped laser and discuss aliasing signals arising from a frequency-comb source in OFDI using numerical simulations.

This FDML-AOFS circular-ranging architecture offers high-speed, long-range imaging and provides a convenient and accessible platform for developing and optimizing the novel and unique aspects of circular ranging. In addition, it demonstrates a broader strategy for converting existing swept-wavelength laser designs to operate as stepped-frequency-comb sources for circular ranging to overcome signal-acquisition limitations.

II. FDML FC LASER

By incorporation of an intracavity Fabry-Perot etalon into the extended cavity of an FDML laser, the nested cavity produces a frequency comb that is superimposed onto the FDML longitudinal modes (frequency comb is independent of the FDML cavity) (Fig. 1). Although every mode-locked laser produces a spectrum comprising a comblike structure rather than a smooth, continuous distribution, the longitudinal mode spacing of the large FDML-laser cavity is on the order of hundreds of kilohertz and has limited impact on imaging performance. The “nested” frequency comb generated by the intracavity etalon, however, has a much broader line spacing and can be used to implement circular ranging. The 1.68-km fiber (Corning SMF28) resonator includes a 1310 ± 40
nm semiconductor optical amplifier (SOA; Covega, BOA-7875) as the gain medium, a fixed Fabry-Perot (FFP) etalon (LightMachinery Inc.) with a free spectral range (FSR) $\Delta \nu_{\text{FFP}}$ of 80 GHz and a finesse of 80, and a tunable Fabry-Perot filter (TFP) filter (Micron Optics Inc.) with a FSR $\Delta \nu_{\text{TFP}}$ of 36 THz and a finesse of approximately 900. The TFP filter is driven at the resonator’s fundamental resonance of 122 kHz. This frequency is closely matched to the second harmonic of the mechanical resonance frequency of the TFP filter. The SOA is current modulated, with use of a 2-$\mu$s electrical pulse, to suppress lasing during the long- to short-wavelength (backward) sweep, and to enable lasing for 50% of each forward sweep. This results in a lasing bandwidth of 105 nm (18 THz) and an average sweep speed of 52 nm/$\mu$s (9 THz/$\mu$s). In some configurations, a second SOA is included to increase output power and reduce laser noise. The laser output is directed through delay lines to generate four time-shifted copies before amplification. This increases the duty cycle from 25% to nearly 100% and the repetition rate from 122 to 488 kHz.

Figure 2 shows the laser output in the optical-frequency and time domains. The frequency-comb source extends across a bandwidth of 105 nm (18.5 THz). Individual comb lines are visible in a magnified view showing a line spacing of 80 GHz (approximately 0.45 nm). The time trace shows five consecutive sweeps at 25% duty cycle obtained directly at the laser output [Fig. 2(b), top] and a full duty cycle with a repetition rate of 488 kHz after the delay line and amplification [Fig. 2(b), bottom]. The magnified view depicts a single sweep. Pulsation is clearly visible, and each pulse corresponds to a single frequency-comb line. Near the center wavelength of the sweep, the time between two consecutive pulses is 7.2 ns, reflecting the wavelength sweeping speed and fixed-etalon FSR as $\Delta t = \Delta \nu_{\text{FFP}} / v$, where $v$ is sweeping speed of the light source (in hertz per second). The measured pulse width is 4.5 ns with a modulation depth of approximately 0.6. The pulsation induced by the FFP etalon can be used to self-clock acquisition at the optical Nyquist rate (i.e., one digitization per frequency-comb line). The measured coherence length is 4 cm as shown in Fig. 3(a). Each data point is recorded at integer multiples of the baseband length, $L_B = c / (2 \Delta \nu_{\text{FFP}})$, to avoid confounding coherence length with cyclic variations in signal strength across the baseband. The coherence length is smaller than that predicted by the transmission-peak linewidth of the FFP etalon (approximately 10 cm) due to nonlinear linewidth broadening in the SOA. Figure 3(b) shows the measured coherence length when two SOAs are used in the cavity. As expected, the additional nonlinear interactions in the second SOA reduce the coherence length. However, we observe that two SOAs reduces laser noise. This can also be appreciated from the insets in Figs. 3(a) and 3(b), where the pulsation is clearly visible in the case when two SOAs are used.

III. SIMULATIONS

In this section, we discuss the design parameters of a stepped frequency comb by considering an extended-cavity semiconductor laser with numerical simulations. We consider a fiber ring cavity that includes a SOA and intracavity TFP and FFP filters [Fig. 4(a)]. For simplicity, the numerical model simulates laser performance with short cavity round-trip times. A detailed description can be found in the Appendix. Laser parameters are selected...
partly on the basis of imaging requirements and include choosing a FFP FSR ($\Delta\nu_{\text{FFP}} = 80 \text{ GHz}$) to define a circular depth range ($L_B = 1.88 \text{ mm}$). We begin by choosing a TFP bandwidth of $\delta\nu_{\text{TFP}} = 40 \text{ GHz}$, yielding a filter ratio of $\delta\nu_{\text{TFP}}/\Delta\nu_{\text{FFP}} = 0.5$ as is the case in our experiments. For circular ranging, a frequency comb with narrow linewidths provides long coherence lengths and correspondingly long depth ranges. A portion of the optical spectrum of a stepped frequency comb at the laser output is simulated in Fig. 4(b) for increasing FFP finesse. The insets in (d) show linear plots of the FFP and TFP (FWHM of 40 GHz) transmission profiles (spanning $\pm150 \text{ GHz}$) for $F_{\text{FFP}} = 1$, 2, and 10. (e) Simulated frequency comb linewidth at the laser output for increasing FFP finesse (solid black line). The dashed red line depicts the FFP bandwidth. FFP FSR is 80 GHz, TFP linewidth is 40 GHz, and sweep speed is 2 nm/µs. Fig. 4. (a) The frequency-stepped, external-cavity, frequency-comb laser used for simulations. (b) Simulated optical spectra of the output of the stepped-frequency-comb laser with different values of the FFP finesse. (c) Three selected optical spectra for a FFP finesse of 1 (blue), 2 (red), and 10 (green). (d) Simulated wavelength per round trip (one round trip is equivalent to 15 ns) at the laser output for three values of the FFP finesse. The insets in (d) show linear plots of the FFP and TFP transmission profiles (spanning $\pm150 \text{ GHz}$) for $F_{\text{FFP}} = 1$, 2, and 10. (e) Simulated frequency comb linewidth at the laser output for increasing FFP finesse (solid black line). The dashed red line depicts the FFP bandwidth. FFP FSR is 80 GHz, TFP linewidth is 40 GHz, and sweep speed is 2 nm/µs.

yields quasi-frequency-stepping, where neighboring wavelengths appear to compete for gain, which could lead to artifacts and reduced coherence length [Fig. 4(d), red]. A sufficiently high finesse yields frequency stepping with a nearly constant wavelength until discretely switching to the next comb line [Fig. 4(d), green]. Figure 4(e) shows the average frequency-comb linewidth at the laser output as a function of FFP finesse. The output linewidth is a function of, but does not directly match, the set FFP linewidth (dashed red line). Specifically, for low finesse, linewidths narrower than the FFP bandwidth ($\delta\nu_{\text{TFP}}$) are observed, while for high finesse, linewidths broader than $\delta\nu_{\text{TFP}}$ (dashed red line) occur. While a high finesse is important for frequency stepping, we observe increased laser noise when $\delta\nu_{\text{FFP}}$ becomes too small. The region where linewidths are larger than $\delta\nu_{\text{FFP}}$ seen in Fig. 4(e) could indicate a transition into this noisy regime. This suggests that optimum performance may be achieved at more moderate finesse values, where the narrow FFP bandwidth does not compete with the nonlinear spectral broadening induced by the SOA. Importantly, the linewidth broadening with respect to the set FFP linewidth is similar to that in the experiments shown in Fig. 3. The experimental linewidth broadening factor is 2.5 with an average laser-output linewidth of 2.5 GHz (single SOA), while the simulated broadening factor is 4 with a linewidth of 4 GHz. The TFP bandwidth ($\delta\nu_{\text{TFP}}$) should be selected to avoid simultaneous lasing of multiple comb lines. As the TFP bandwidth narrows, amplitude pulsation at the output of a stepped-frequency-comb laser becomes more pronounced. Figure 5(a) shows time traces of the laser output for different ratios between TFP bandwidth, $\delta\nu_{\text{TFP}}$, and FFP FSR, $\Delta\nu_{\text{FFP}}$. The TFP loss (finesse) of the FFP is kept constant, while the bandwidth of the TFP is increased. TheFig. 5. (a) Simulated time traces for selected ratios, $\delta\nu_{\text{TFP}}/\Delta\nu_{\text{FFP}}$, of 0.5 (red), 1 (blue), and 2 (green) between the bandwidth of the TFP filter ($\delta\nu_{\text{TFP}}$) and the FSR of the FFP etalon ($\Delta\nu_{\text{FFP}}$). Pulsation is clearly visible. (b) Simulated pulse modulation depth as a function of filter ratio. The inset shows the pulse modulation depth for the same filter ratios versus finesse of the FFP etalon. The red circle shows the filter ratio used in our experiments.
laser output shows strong pulsation for low filter ratios (small $\delta v_{\text{FFP}}$). Modulation depth visibly reduces and transitions into a cw regime for increasing filter ratio (increasing $\delta v_{\text{FFP}}$), which is clearly illustrated in Fig. 5(b). Of note, the modulation depth is not influenced by the FFP finesse for $F_{\text{FP}} > 2$ [Fig. 5(b), inset]. For large TFP bandwidths ($\delta v_{\text{TFP}}/\Delta v_{\text{FFP}} > 1$), a lasing comb line competes with neighboring comb lines and again challenges the required wavelength stepping shown in Fig. 4(d). By adjustment of the TFP bandwidth with respect to FFP FSR, the pulsation amplitude can be controlled, which determines the rf content of signal harmonics described in Sec. IV. The experimentally observed pulse modulation depth is 0.6 for a filter ratio of 0.5, which compares with a modulation depth of 0.7 in the simulation [indicated by the red circle in Fig. 5(b)]. The pulses directly correlate to the underlying frequency-comb structure in the spectral domain and thus the FSR. This opens the possibility, in future work, to generate an acquisition clock signal directly from the laser output. This optical clock would be intrinsically synchronized to the arrival time of the laser pulses even in the presence of a nonlinear sweep, and would have a rate equal to the optical Nyquist rate (i.e., one digitization per comb line).

**IV. ARTIFACT SUPPRESSION IN FREQUENCY-SHIFTED CIRCULAR RANGING**

The circular-ranging method requires that the positive and negative delay spaces around the zero path delay are resolved independently; that is, that the sign of the delay is measured in addition to the absolute value of the delay [12]. In prior reports of circular ranging, this was achieved by passive polarization-based optical quadrature demodulation [11] and acousto-optic frequency shifting [12]. Here, we adopt the acousto-optic-frequency-shifting approach. While straightforward to implement, the acousto-optic-frequency-shifting method can introduce complex conjugate artifacts that have not been described in detail. Here we explain the origin of these signals, and discuss laser-source and system design factors that can mitigate their impact. To this end, we first model and analyze the interference fringes generated by a time-stepped frequency-comb source. The model assumes a perfect frequency-comb spectrum with a constant 80-GHz FSR. In the time domain, each comb line is represented by a single finite pulse parameterized by the modulation depth $m$. The rf content of the fringes is calculated for detection with either dual quadrature I,Q outputs (i.e., like those generated in Refs. [13,14] and allowing discrimination of positive and negative radio frequencies) or with a single real output (i.e., the I output without the Q output, which, as a result, does not distinguish positive and negative radio frequencies). All fringe simulations assume a single mirror reflection. In these simulations, the location of the mirror is parameterized by its circular delay rather than its physical delay; the circular delay range is given by $\Delta \tau = 1/\Delta v$, where $\Delta v$ is the FSR of the frequency comb. This corresponds to a circular depth range $L_B = c\Delta \tau/2$.

First we examine the rf content with I,Q outputs and without an AOFS [Fig. 6(a)]. In circular ranging a single reflection generates a set of rf harmonics. These result from the discontinuous wavelength stepping and amplitude pulsing of the source. A baseband signal (zero order) is located within the window defined as $-F_s/2$ to $+F_s/2$, where $F_s = 1/\Delta \tau = v/\Delta v_{\text{FP}}$. Higher-order rf harmonics are located in adjacent, nonoverlapping rf windows (i.e., positive first order from $F_s/2$ to $3F_s/2$). These higher rf orders contain the same information as the baseband signal and do not need to be captured for imaging. As such, detection and acquisition can be optimized for the baseband (zero-order) window with a sampling rate (on each of the I and Q channels) of only $S = F_s = 1/\Delta \tau = 2\nu L_B/c$. With quadrature detection as shown in Fig. 6(a), low-pass analog filters can remove the higher-order signals before digitization.

Next we investigate the use of an AOFS. Although the AOFS is used to avoid the need for I,Q detection, we first simulate the rf response with both an AOFS and I,Q detection [Fig. 6(b), upper panel] to more clearly illustrate the effect of the AOFS relative to the results in Fig. 6(a). As shown, the AOFS shifts the rf signals by the applied optical frequency shift $f_{\text{AOM}}$ in the rf domain. A frequency shift of $f_{\text{AOM}} = F_s/2$ therefore shifts the baseband signals entirely into the positive rf space. In principle, because these baseband signals of interest are located within the positive rf space, they can be captured by a single channel (I only). Figure 6(b) (lower panel) illustrates the rf content when detection is with a single channel. Here we can see that negative harmonics (which are still located in the negative rf space) generate artifacts in the baseband. Thus, while the AOFS is effective at placing the full range of baseband signals into an entirely positive (or negative) rf space, this does not prevent the rf harmonics created by the frequency-comb source from creating artifacts in the baseband. With $f_{\text{AOM}} = F_s/2$, the artifacts result from the negative first-order harmonics. Higher values of $f_{\text{AOM}}$ can be used to translate the baseband signals to higher radio frequencies and to increase the order of the harmonics that alias into the baseband window. Figure 6(c) illustrates imaging with $f_{\text{AOM}} = F_s$, which selects the negative second-order harmonics. It is important to note that the rf harmonic amplitudes decay with order and the pulse shape influences the decay rate. Therefore, through manipulation of the pulse shape and the frequency shift, $f_{\text{AOM}}$, the extinction between the baseband signal and these artifacts can be controlled. This is shown in Fig. 7(a) as a function of $f_{\text{AOM}}$ for $m = 0$ and in Fig. 7(b) as a function of $m$ for a fixed $f_{\text{AOM}} = F_s$. The red curve in Fig. 7(b) shows the artifact suppression for $m = 0.5$ while also taking into account the group
velocity dispersion of amorphous material transmitting infrared radiation of a typical AOFS ($\beta_2 = 820 \text{ ps}^2/\text{km}$ at $
abla 2$ $\beta_2 = 820 \text{ ps}^2/\text{km}$ at $(a)$

FIG. 6. Radio-frequency harmonic generation and resulting artifacts in time-stepped frequency-comb circular ranging. (a) The rf content generated by a single reflection with the denoted circular delay is shown for I,Q demodulation and without an AOFS. The signals are located in the baseband centered at zero frequency (DC), and rf harmonics of varying orders occur at higher positive and negative radio frequencies. The amplitude of the rf harmonics drops with order and is a function of the pulse shape. (b) Including an AOFS at $F_s/2$, while retaining I,Q outputs, shifts the baseband signals to the positive rf space (upper panel). However, use of a single channel (I only) causes the negative first-order harmonic to appear within the baseband (denoted by an asterisk in the lower panel). (c) Increasing the AOFS frequency shift to $F_s$ places the baseband further from DC, and shifts artifacts to the negative second order, which have even lower amplitudes (denoted by two asterisks in the lower panel).

$\lambda_0 = 1.3 \mu\text{m}, 10$-$\text{mm}$ length). Numerical correction of dispersion for the baseband signals in the positive rf space causes broadening in the negative rf space, and further reduces the artifact peak. This broadening, however, does not reduce the integrated power of the artifact signal. Artifact suppression of better than 30 dB can be achieved. Higher values of $f_{AOFS}$ also increase the digitization rate and therefore force a compromise between reducing acquisition bandwidth (through lower values of $f_{AOFS}$) and reducing artifacts (through higher values of $f_{AOFS}$).

V. CIRCULAR IMAGING

The full-imaging and data-acquisition system is shown in Fig. 1. An AOFS with a frequency shift of 100 MHz (approximately $F_s$) is used. The laser provides output pulses described by $m \approx 0.6$. We observe artifact suppression of more than 30 dB (Fig. 8). This includes the influence of dispersion mismatch in the interferometer, which is compensated numerically. The magnitude of the point-spread function decays by approximately 4 dB toward the baseband edge. This decay rate depends on the pulse shape and pulse modulation depth as well as the circular depth range (FSR of FFP etalon). This roll-off is stable and is compensated for numerically in the generated images. Imaging is performed with lenses with focal lengths of

FIG. 7. Simulated artifact suppression within zero order baseband ($L_B$). (a) Artifact suppression for different frequency shifts $\pm f_{AOFS}$ corresponding to a depth translation of a fraction of the baseband depth. (b) Artifact suppression for a frequency shift of $F_s$ and different pulse modulation depths, $m$. The red curve takes into account the group velocity dispersion (GVD) of amorphous material transmitting infrared radiation ($\beta_2 = 820 \text{ ps}^2/\text{km}$) and numerical dispersion compensation for $m = 0.5$.

FIG. 8. The zero-order baseband. Artifact suppression is greater than 30 dB across the baseband.
data volume of 39 GB (assuming 1000 × 1000 volume, 14 bit digitization, $\lambda_0 = 1310$ nm). This is faster than current state-of-the-art acquisition boards. The use of circular ranging as shown in this work reduces this 15-fold (319 MS/s and 2.6 GB). The 1.88-mm circular depth range produced by the 80-GHz FFP FSR can capture the full detectable signal for most biological samples. A shorter circular depth range (i.e., larger FFP FSR) may be adequate for some samples, and would increase the data compression. Reducing self-phase modulation in the SOA is likely to increase the coherence length and thereby further extend the imaging range. Additional intracavity dispersion compensation would enhance the FDML condition across the entire sweep range and may further increase the coherence length. Choosing a lower FFP finesse could improve laser noise performance as is indicative from the numerical simulation. The imaging speed is limited by the mechanical resonance frequency of the tunable spectral filter. Use of different filters or higher harmonics of the filter’s mechanical resonance frequency will allow higher repetition rates. Current acousto-optic frequency shifters support repetition rates of several megahertz. For example, a repetition rate of 2 MHz requires a frequency shift of approximately 400 MHz (assuming $\Delta v_{\text{FFP}} = 80$ GHz, $v = 35$ THz/µs). Higher frequency shifts are available but have low diffraction efficiencies. The ability to use the full potential of the FDML laser (speed, range, resolution) by the use of a nested frequency comb and circular ranging overcomes traditional limitations and could enable new degrees of freedom where cameralike, three-dimensional imaging with wide fields of view can be beneficial, including surgical guidance and diagnostics.

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APPENDIX: NUMERICAL SIMULATION

We describe here the model we use to simulate a stepped-frequency-comb laser in Sec. III. For our analysis we consider a fiber ring cavity that includes a SOA, a tunable spectral bandpass filter, and a coupler for optical feedback and output coupling. A frequency comb is generated considering a bulk Fabry-Perot etalon. The electric field inside a SOA is described by the coupled differential equations [15–19]

$$\frac{dE}{dz} + \frac{1}{v_g} \frac{dE(\tau,z)}{d\tau} = \frac{1}{2} [(1 + i\alpha_H)g - a_{\text{SOA}}]E + N_c(\tau,z),$$

(A1)

$$\frac{dh}{d\tau} = \frac{g_0l - h}{\tau_c} - \frac{|E(\tau)|^2}{E_{\text{sat}}} \{ \exp[h(\tau)] - 1 \},$$

(A2)
where the carrier-induced index change is accounted for through the linewidth enhancement factor $\alpha_1$, $g_0$ is the small signal gain, $\tau_c$ is the carrier lifetime, $\alpha_{SOA}$ is the SOA losses, $E_{sat} = P_{sat}\tau_c$ is the saturation energy (where $P_{sat}$ is the saturation power), $v_g$ is the group velocity inside the SOA, $N_c$ represents the addition of a noise spectrum due to amplified spontaneous emission, and $\tau$ is the time reference frame. Furthermore, $h(\tau)$ is the time-dependent gain over SOA length $l$, $h(\tau) = \int_0^{\tau} g(z, \tau) dz$, and represents the integrated gain at each point of the temporal profile. Equation (A1) is solved with use of the split-step Fourier method, where the field propagation with group velocity $v_g$ is accounted for in the frequency domain and the SOA gain [right-hand side of Eq. (A1)] is computed in the time domain by solving Eq. (A2) numerically. Moreover, the solutions for the SOA output are constrained by the boundary condition
\[
E(\omega, z = 0) = E(\omega, z = l) \sqrt{\eta(1 - \alpha_c)} F_{TFP}(\omega) \times F_{FFP}(\omega) \exp[-i\beta(\omega)L], \tag{A3}
\]
where $E(\omega) = 0$ is the electric field at the SOA input, $E(z = l)$ is the field at the SOA output, $\eta$ is the cavity feedback, $\alpha_c$ is the cavity losses, $L$ is the cavity length, $\omega = 2\pi v$ is the angular frequency, and $\beta(\omega)$ is the fiber propagation constant, which can be expressed as a Taylor-series expansion around a center frequency, $\beta(\omega) = \sum_{k=0}^{\infty} (\beta_k/k!)(\omega - \omega_0)^k$, with $\beta_k = (d^k\beta/d\omega^k)|_{\omega=\omega_0}$, where we consider only second-order chromatic dispersion. Furthermore, $F_{TFP}(\omega) = [1/(1 + i2\omega/(2\pi \delta_{TFP}))^2]$ is the tunable optical bandpass filter with a filter bandwidth $\delta_{TFP}$ (FWHM). The transmission through the fixed etalon is accounted for by $F_{FFP} = 1/[1 + F_{TFP}\sin^2(\delta/2)]$, with $\delta = \omega_0 2\pi F_{FP} L_{FFP} \cos \theta/c$. The propagation angle, $\theta$, is assumed to zero and the etalon length is given by $L_{FFP} = c/(2n_{FFP}\Delta_{VFP})$, with $\Delta_{VFP}$ being the FSR and $n_{FFP}$ the etalon refractive index. The simulations are performed in the swept-filter reference frame starting at a center wavelength of 1310 nm. Starting from spontaneous emission noise, we compute iteratively the gain and the optical field until a steady-state regime is observed. Unless stated in the text or figures, the following values are chosen for the simulations: $L = 3 \text{ m}$, $G_0 = \exp(g_0 L) = 29 \text{ dB}$, $E_{sat} = 7.92 \text{ pJ}$, $v_g = 8.4272 \times 10^7 \text{ m/s}$, $\alpha_H = 5$, $\tau_c = 440 \text{ ps}$, $\alpha_c = 0.6$, $\eta = 0.1$, $\beta_2 = 1 \text{ ps}^2/\text{km}$, $\Delta_{VFP} = 40 \text{ GHz}$, $\eta_{FFP} = 80 \text{ GHz}$, $F_{TFP} = 80$, and $n_{FFP} = 1.45$.

