Modelling of a bridge-shaped nonlinear piezoelectric energy harvester

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Modelling of a bridge-shaped nonlinear piezoelectric energy harvester

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\textbf{Abstract.} Piezoelectric MicroElectroMechanical Systems (MEMS) energy harvesting is an attractive technology for harvesting small magnitudes of energy from ambient vibrations. Increasing the operating frequency bandwidth of such devices is one of the major issues for real world applications. A MEMS-scale doubly clamped nonlinear beam resonator is designed and developed to demonstrate very wide bandwidth and high power density. In this paper a first complete theoretical discussion of nonlinear resonating piezoelectric energy harvesting is provided. The sectional behaviour of the beam is studied through the Classical Lamination Theory (CLT) specifically modified to introduce the piezoelectric coupling and nonlinear Green-Lagrange strain tensor. A lumped parameter model is built through Rayleigh-Ritz Method and the resulting nonlinear coupled equations are solved in the frequency domain through the Harmonic Balance Method (HBM). Finally, the influence of external load resistance on the dynamic behaviour is studied. The theoretical model shows that nonlinear resonant harvesters have much wider power bandwidth than that of linear resonators but their maximum power is still bounded by the mechanical damping as is the case for linear resonating harvesters.

1. Introduction

Piezoelectric MEMS is an attractive technology for harvesting small magnitudes of energy from ambient vibrations. This technology promises to eliminate the need for replacing chemical batteries or complex wiring in microsensorsmicrosystems, moving us closer toward battery-less autonomous sensors and networks by harvesting the environmental energy on-site to fulfil their tasks. New developments in electronics have considerably reduced the power consumption of devices which can be now powered by MEMS harvesters if they can robustly generate about 100 μW of continuous power from ambient vibrations [1]. In addition, harvesters should be small enough to be integrated with the electronics and the manufacturing cost should be sufficiently low for mass scale deployment. For MEMS-scale (smaller than 0.5 cm\textsuperscript{3}) energy harvesters, piezoelectric transduction is the most appropriate scenario since standard MEMS thin-film processes are available for many piezoelectric materials assuring high efficiency, high energy density and scalability. Operating frequency, frequency bandwidth, excitation level, power density and size are the key design requirements. At the present time, most of the devices reported in the literature do not meet the desired requirements. Cantilever
laminated beams show reasonable power generation but the frequency bandwidth is impractically small and big device size is required to have low operating frequencies. Moreover, the full strain capability of piezoelectric materials is only exploited at high (and impractical) proof mass displacements. A MEMS-scale nonlinear resonator is developed for ultra-wide-bandwidth (UWB) operation [3]. By utilizing a doubly clamped beam resonator, the stretching strain triggered at large deflection stiffens the beam and transforms the dynamics to nonlinear regime and increases the bandwidth. Nonlinear resonant harvesters can also easily achieve the full strain use of PZT since higher strains can be obtained at lower displacements than is needed by a linear resonator. In this paper a first comprehensive theoretical discussion on nonlinear resonating piezoelectric energy harvesting is provided. Firstly, the electrical damping is considered as a linear dashpot added to the classical mechanical damper. In the second part, a more accurate model is built to consider the influence of an external load resistance to the dynamic behaviour of the nonlinear oscillator. The modelling shows that the power generation of the nonlinear resonant harvesters is bounded by the mechanical damping of the system and the maximum power generation can be obtained when the electrical damping inject to the system equals the mechanical one as was reported for linear resonant harvesters [5].

2. Simple model: linear electrical damping

A general nonlinear piezoelectric resonant energy harvester is modelled as a classical Duffing oscillator with an additional linear dashpot \( c_E \) to include the dissipation due to energy harvesting. By classical methods [4] the nonlinear dimensionless Frequency Response Function (FRF) is:

\[
Y = \left[ -\Omega_M^2 + i \frac{3}{4} \alpha Y^2 \right] + i \left[ 2 \left( \zeta_E + \zeta_M \right) \Omega_M \right]^j
\]

where \( \alpha = \frac{k_N}{k_L} \cdot W_0^2 \) is a measure of the nonlinearity of the system, \( k_L \) and \( k_N \) are the linear and nonlinear stiffness, \( W_0 = \frac{F}{k_L \cdot k_N} |_{\omega=0} \) is the linear static displacement, \( F \) is the external inertial force, \( Y = \frac{W}{W_0} \) is the dimensionless amplitude, \( \Omega_M = \frac{\omega}{\omega_n} \) is the dimensionless excitation frequency, \( \omega \) the excitation frequency, \( \omega_n \) the linear natural frequency and \( \zeta_M = c_M / 2 \omega_m m_{TIP} \) and \( \zeta_E = c_E / 2 \omega_m m_{TIP} \) are the electrical and mechanical damping ratio, \( c_M \) and \( c_E \) are the mechanical and electrical damping coefficient and \( m_{TIP} \) the tip mass.

The amplitude of the oscillation is obtained by solving equation (1) and plotted in figure 1 \( (\zeta_M = 0.0056, \alpha = 0.048, \omega = 0.0016) \) where it is shown the influence of the damping on the amplitude response. The higher is the damping the lower the jump-down frequency and the relative amplitude. The linear electric dashpot represents the amount of damping injected in the system by the piezoelectric material, thus the generated power is dissipated by the dashpot. The power generation increases until the system jumps down, where the power reaches the maximum. When the electrical damping is zero, no power is harvested, on the other hand when the damping is too high the oscillator does not move and no power is then scavenged. An optimum lies in the middle. Providing the power generation for all the values of electrical damping, the envelope of all peaks is obtained and shown in Figure 2. It is shown that nonlinear harvesters have a much larger bandwidth than linear harvesters which are efficient only near the resonance frequency of the system. A close form expression of the maximum power is obtained computing the energy dissipated by the electrical dashpot:

\[
P_d = \frac{\zeta_E m |\vec{y}|^2}{4 \omega_0 \left( \zeta_M + \zeta_E \right)^2}
\]

Equation (2) is the same expression that is obtained for linear harvesters [5] and the peak power occurs when \( \zeta_E = \zeta_M \) as it can be seen in figure 2. In conclusion, the previous expression can be considered as a theoretical upper bound to the power generation for all kind of resonant energy harvesters. However, this does not mean that \( \zeta_E = \zeta_M \) is always the optimal condition for harvesting power. Indeed, this is true
only for $\Omega=\Omega_{d|\varepsilon=M}$ (for linear systems it is true only at resonance) while for other frequencies the optimal $c_E$ can be higher ($\Omega<\Omega_{d|\varepsilon=M}$) or lower ($\Omega>\Omega_{d|\varepsilon=M}$) than $c_M$. This means that the optimal $c_E$ depends on the excitation frequency. Linear systems harvest high power at resonance because inertial and restoring forces compensate each other and the solicitation is in phase with the velocity (so with the voltage); in that condition the optimum is obtained for $c_E=c_M$. Out of resonance a phase shift between the force and the velocity (and the voltage) is introduced. Thus, the electrical damping must be calibrated to compensate the shifting and assure the optimal power generation. In nonlinear harvesters the same thing happens, the electrical damping must be calibrated at each frequency to assure the maximum power generation even if the peak power is still obtained for $c_E=c_M$.

3. Accurate model

A more accurate model needs to be discussed since a linear dashpot is not suitable to describe the completely coupled piezoelectric behaviour [5]. This second model allows studying the influence of an external load resistance ($R$) on the electromechanical behaviour of the harvester. A doubly clamped beam resonator is presented in figure 3. The $x_1$-coordinate originates in the neutral axis and is directed downwards while $x_3$-coordinate lies along the beam axis. A PZT layer is placed on top of the beam substrate and is activated in d33-mode when the beam deflects.

3.1. Dynamic equilibrium equations

The beam’s final stack is not homogeneous since different deposited layers are employed. The mechanical response of the layered beam can be obtained by means of a number of theories, examined in [6]. A very thin beam must be designed to activate nonlinear stretching. Therefore Bernoulli hypothesis and CLT is adopted herein, resulting significant improvement by introducing the piezoelectric coupling in PZT layer and the Green-Lagrange nonlinear strain tensor. The lumped
dynamic equilibrium equations are then obtained through Rayleigh Ritz method by supposing a cubic displacement and an alternate (between IDT electrodes) electric field. Additional coupling terms proportional to the amplitude arise because of the nonlinear strain as shown below:

\[
\begin{align*}
    m\dddot{w} + c_m\ddot{w} + \left( k_l + k_r \right) w + k_N w^3 & - \Theta_p v + \Theta_m w v = -m\dddot{y}_{ext} \\
    k_E \dddot{v} + \Theta_p w - \Theta_m w v + R^T v &= 0
\end{align*}
\]  

(3)

where \( m \) is the total mass; \( k_l, k_r \) and \( k_N \) are the linear elastic, the residual stress and the nonlinear stiffness; \( k_E \) is the internal capacitance of PZT while \( \Theta_p \) and \( \Theta_m \) are the linear and nonlinear coupling coefficients. All these coefficients take into account both the integration in the thickness (achieved by the CLT) and along the length (achieved by the Rayleigh Ritz method) [7].

3.2. Solution: Harmonic Balance Method (HBM)

The frequency response of the oscillator to harmonic excitations is studied through HBM. Nonlinear systems do not respond to a monoharmonic signal with a monoharmonic at the same frequency but all harmonics are involved in the response [8]. In this case the amplitude is well described by a single harmonic while two harmonics are required to get the good voltage response since stretching mode has twice the frequency of bending one. Moreover, at large amplitude the linear response (described by the first harmonic) can be neglected and only the second harmonic survives. Neglecting linear coupling terms and substituting in equation (3) the suitable trial solutions the FRF is computed. An implicit expression of the equivalent electrical damping and of the power generation is computed starting from equation (4) and using the definition of power:

\[
Y = \left[ 1 - \Omega_M^2 \left( 1 + \frac{3}{4} \alpha + \frac{\kappa^2 \Omega_M^2}{2 \left( 4 \Omega_M^2 + \Omega_E^2 \right)} \right) + \frac{1}{2} \left( 2 \zeta_M + \frac{\kappa^2 \Omega_E}{4 \left( 4 \Omega_M^2 + \Omega_E^2 \right)} \right) \right]^{1/2} \Omega_M
\]

\[
P = \frac{1}{2} c_E \left| \ddot{w} \right|^2 = m \omega_n \Omega_M^2 \left( 1 + \frac{\kappa^2 \Omega_E}{8 \left( 4 \Omega_M^2 + \Omega_E^2 \right)} \right) \Omega_M
\]

\[
(5a,b)
\]

where \( \Omega_E = 1 / (R k_E \theta_{\text{nr}}) \) is the dimensionless cut-off frequency of the circuit and \( \kappa_{\text{nr}} = W_0 \theta_{\text{nr}} / (k_E k_l)^{1/2} \) the effective piezoelectric nonlinear coupling coefficient.

Figure 6 and Figure 7 plot the solution of Equations (4) and (5b) while Figure 8 shows the normalized electrical damping and the normalized power generation. In short circuit (S.C., \( \Omega_E \rightarrow \infty \)) and open circuit (O.C., \( \Omega_E = 0 \)) conditions, no electrical damping is introduced in the system, and the amplitude is higher while no power is generated.
Starting from O.C. condition the damping introduced in the system increases as $\Omega_E$ increases, until a maximum is reached (and the maximum amplitude is the lowest). After this point, increasing $\Omega_E$ reduces the electrical damping until the S.C. conditions is reached. The peak power is not obtained when the electrical damping is the maximum because in that condition the high damping excessively restrains the movement of the oscillator. A trade-off between high damping (required to produce power) and low damping (required to not stop the motion of the oscillator) has to be found. As the simpler model suggests and figure 8 confirms the optimum is obtained for $c_E = c_M$.

![Figure 8. Normalized electrical damping ($c_E / c_M$; upper) and Normalized power generation ($P / P_{MAX}$; bottom) vs. dimensionless frequency ($\Omega_M$; left) and dimensionless cut-off frequency ($\Omega_M$; right)](image)

4. Conclusion

A theoretical bounding of the power generation for nonlinear resonating harvesters has been studied through two different models. A simpler model shows that the electrical damping due to piezoelectric coupling should be equal to the mechanical damping for the system to produce the maximum power. This result is the same obtained for linear resonating harvesters. However, the accurate model shows that the near maximum production of power is spread out over a wider bandwidth in the nonlinear harvesters.

5. References