**Terascale Physics Opportunities at a High Statistics, High Energy Neutrino Scattering Experiment: NuSOnG**

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Terascale Physics Opportunities at a High Statistics, High Energy Neutrino Scattering Experiment: NuSOnG

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This article presents the physics case for a new high-energy, ultra-high statistics neutrino scattering experiment, NuSOnG (Neutrino Scattering on Glass). This experiment uses a Tevatron-based neutrino beam to obtain over an order of magnitude higher statistics than presently available for the purely weak processes $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$ and $\nu_\mu + e^- \rightarrow \nu_e + \mu^-$. A sample of Deep Inelastic Scattering events which is over two orders of magnitude larger than past samples will also be obtained. As a result, NuSOnG will be unique among present and planned experiments for its ability to probe neutrino couplings to Beyond the Standard Model physics. Many Beyond Standard Model theories predict a rich hierarchy of TeV-scale new states that can correct neutrino couplings to gauge boson propagators. These corrections are generic in theories of extra dimensions, extended gauge symmetries, supersymmetry, and more. The sensitivity of NuSOnG to this new physics extends beyond 5 TeV mass scales. This article reviews these physics opportunities.

I. INTRODUCTION

Exploring for new physics at the “Terascale” – energy scales of $\sim 1$ TeV and beyond – is the highest priority for particle physics. A new, high energy, high statistics neutrino scattering experiment running at the Tevatron at Fermi National Accelerator Laboratory can look beyond the Standard Model at Terascale energies by making precision electroweak measurements, direct searches for novel phenomena, and precision QCD studies. In this article we limit the discussion to precision electroweak measurements; QCD studies and their impact on the precision measurements are explored in ref. [1][2]. The ideas developed in this article were proposed within the context of an expression of interest for a new neutrino experiment, NuSOnG (Neutrino Scattering On Glass) [1].

A unique and important measurement of the NuSOnG physics program is the ratio of neutral current (NC) and charged current (CC) neutrino-electron scattering, which probes new physics. The leading order Feynman diagrams for these processes are shown in Fig. [1]. The NC process, $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$, called “elastic” scattering or ES, provides the sensitivity to the Terascale physics. This process can explore new physics signatures in the neutrino sector which are not open to other, presently planned experiments. The CC process, called “inverse muon decay” or IMD, $\nu_\mu + e^- \rightarrow \nu_e + \mu^-$, is well understood in the Standard Model due to precision measurement of muon decay [3]. Since the data samples are collected with the same beam, target and detector at
the same time, the ratio of ES to IMD events cancels many systematic errors while maintaining a strong sensitivity to the physics of interest. Our measurement goal of the ES to IMD ratio is a 0.7% error, adding systematic and statistical errors in quadrature. The high sensitivity which we propose arises from the combined high energy and high intensity of the NuSOnG design, leading to event samples more than an order of magnitude higher than past experiments.

Normalizing the ES to the IMD events represents an important step forward from past ES measurements, which have normalized neutrino-mode ES measurements to the antineutrino mode, $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$. The improvement is in both the experimental and the theoretical aspects of the measurement. First, the flux contributing to IMD and $\nu$ ES is identical, whereas neutrino and antineutrino fluxes are never identical and so require corrections. Second, the ratio of $\nu$ ES to $\bar{\nu}$ ES cancels sensitivity to Beyond Standard Model (BSM) physics effects from the NC to CC coupling ratio, $\rho$, which are among the primary physics goals of the NuSOnG measurement. In contrast, there is no such cancellation in the ES to IMD ratio.

The design of this experiment, described in Sec. II, is driven both by requiring sufficient statistics to make precision neutrino-electron scattering measurements and by the need for a neutrino flux which does not extend below the IMD threshold. The threshold for IMD events is

$$E_\nu \geq E_\mu \geq \frac{m_\mu^2}{2m_e} = 10.9 \text{ GeV},$$

where we have dropped the small $m_e^2$ term for simplicity. The functional form above threshold, shown in Fig. 2, is given by $(1 - m_\mu^2/E_{cm}^2)^2$, where $E_{cm}$ is the center of mass energy. Thus a high energy neutrino beam is required to obtain a high statistics sample of these events. The flux design should provide a lower limit on the beam energy of about 30 GeV, still well above the IMD threshold.

Sec. III describes the Standard Model Physics of neutrino electroweak scattering, for both electron and quark targets. In this section, the value of the normalization of the ES to IMD events is further explored. The very high statistics will also permit an electroweak measurement using the deep inelastic scattering (DIS) data sample from NuSOnG, via the “Paschos Wolfenstein method” (PW) [6]. The best electroweak measurement using DIS events to date comes from the NuTeV experiment, which has observed an anomaly. The status of this result is reviewed below. Making conservative assumptions concerning systematic improvements over NuTeV, our measurement goal using this technique is a 0.4% error on $\sin^2 \theta_W$, adding statistical and systematic errors in quadrature.

In Sec. IV we discuss NuSOnG’s potential to discover or constrain new physics through indirect probes, by making precision measurements of SM processes to look for deviations from SM predictions. We first frame the issue by considering in turn several model-independent parameterizations of possible new physics and asking what constraints will be imposed on new physics in the event NuSOnG agrees with the SM. (1) Oblique correction parameters describe the effects of heavy new states in vector boson loops. (2) New states may induce higher-dimensional effective operators involving neutrinos. Finally, (3) new states may modify the couplings of the gauge bosons to neutrinos and leptons, including possibly violating lepton universality. In each case we consider the ability of NuSOnG to detect or constrain these types of deviations from the SM.

In Sec. V we examine specific models for new physics. We begin by presenting the sensitivity to a set of new physics models. In particular, we consider

- typical $Z'$ models,
- non-degenerate leptoquark models,
- R-parity violating SUSY models,
- extended Higgs models.

The models were selected because they are often used as benchmarks in the literature. While this list is not exhaustive, it serves to illustrate the possibilities. For each case, we consider how NuSOnG compares to other measurements and note the unique contributions. We end this section by approaching the question from the opposite view, asking: how could the results from NuSOnG clarify the underlying physics model, should evidence of new physics emerge from LHC in the near future?
is essentially no flux below 30 GeV, hence all neutrinos
are well above the IMD threshold. It is sign-selected: in
neutrino mode, 98.2% of neutrino interactions were due
to $\pi^+$ and $K^+$ secondaries, while in antineutrino mode
97.3% came from $\pi^-$ and $K^-$. The “wrong sign” content
was very low, with a 0.03% antineutrino contamination
in neutrino mode and 0.4% neutrino contamination in
antineutrino mode. The electron-flavor content was 1.8%
in neutrino mode and 2.3% in antineutrino mode. The
major source of these neutrinos is $K^\pm$ decay, representing
1.7% of the total flux in neutrino mode, and 1.6% in
antineutrino mode.

Redesign of the beamline for NuSOnG is expected to
lead to modest changes in these ratios. For example,
if the decay pipe length is 1.5 km rather than 440 m,
as in NuTeV, the $\pi^-/K^-$ ratio increases by 20% and the
fractional $\nu_e$ content is reduced.

With respect to Tevatron running conditions, we will
assume that twenty times more protons on target (POT)
per year can be produced for NuSOnG compared to
NuTeV. This is achieved through three times higher in-
tensity per pulse (or “ping”). Nearly an order of mag-
itude more pulses per spill are provided. Our studies
assume that twenty times more protons on target (POT)
per year can be produced for NuSOnG compared to
NuTeV. This is achieved through three times higher in-
tensity per pulse (or “ping”). Nearly an order of mag-
itude more pulses per spill are provided. Our studies
assume 4 \times 10^{19} POT/year, with 5 years of running.

Preliminary studies supporting these goals are provided
in ref. [8].

The event rates quoted below are consistent with
1.5 \times 10^{20} protons on target in neutrino running and
0.5 \times 10^{20} protons on target in antineutrino running.
The choice to emphasize neutrino running is driven by obtain-
ing high statistics ES, which has a higher cross section for
neutrino scattering, and to use the IMD for normalization –
this process only occurs in neutrino scattering. The Stan-
ard Model forbids an IMD signal in antineutrino mode.
However, some antineutrino running is required for the
physics described in the following sections, especially the
PW electroweak measurement.

The beam from such a design is highly forward di-
rected. NuTeV was designed so that 90% of the neutrinos
from pion decay were contained within the detector face,
where the detector was located at 1 km. For NuSOnG,
which will use a 5 m detector, \sim 90% of the neutrinos
from pion decay are contained at \sim 3 km.

The optimal detector is a fine-grained calorimeter
for electromagnetic shower reconstruction followed by a
toroid muon spectrometer. This allows excellent re-
construction of the energy of the outgoing lepton from
charged current events. We employ a Charm II style de-
sign [9], which uses a glass target calorimeter followed
by a toroid. We assume one inch glass panels with ac-
tive detectors interspersed for energy and position mea-
surement. Glass provides an optimal choice of density,
low enough to allow electromagnetic showers to be well
sampled, but high enough that the detector length does
not compromise acceptance for large angle muons by the
toroid. Approximately 10% of the glass will be doped
with scintillator to allow for background studies, as dis-
cussed in Sec. [10].

The design introduces four identical sub-detectors of

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{flux.png}
\caption{The assumed energy-weighted flux, from the NuTeV
Experiment [7], in neutrino mode (left) and antineutrino
mode (right). Black: muon neutrino, red: muon antineutrino,
blue: electron neutrino and antineutrino flux.}
\end{figure}

Two further studies which can be performed by Nu-
SOnG are QCD measurements and direct searches. The
very large (~ 600 million event) DIS sample will allow
the opportunity for precision studies of QCD. There are
many interesting measurements which can be made
in their own right and which are important to NuSOnG’s
Terascale physics program. The very high flux will also
permits direct searches for new physics. Those which
complement the physics discussed in this paper include:

- non-unitarity in the light neutrino mixing matrix;
- wrong-sign inverse muon decay (WSIMD), $\bar{\nu}_\mu + 
  e^- \rightarrow \mu^- + \bar{\nu}_e$;
- decays of neutrissimos, i.e., moderately-heavy
  neutral-heavy-leptons, with masses above 45 GeV.

For more information on these studies, see refs. [1-2].

II. CONCEPTUAL DESIGN FOR THE
EXPERIMENT

In order to discuss the physics case for a new high en-
ergy, high statistics experiment, one must specify certain
design parameters for the beam and detector. The beam
and detector should marry the best aspects of NuTeV
[7], the highest energy neutrino experiment, and Charm
II [9], the experiment with the largest ES sample to date.
The plan presented here is not optimized, but provides a
basis for discussion. The final design of the NuSOnG de-
tector will be based on these concepts, and is still under
development.

In this section, we present, but do not justify, the de-
sign choices. Later in this article, we discuss the reason-
ing for the choices, particularly in Secs. [II-C] and [II-D].

We will assume a beam design based on the one used by
the NuTeV experiment [7], which is the most recent high
energy neutrino experiment. This experiment used 800
GeV protons on target. The beam flux, shown in Fig. [3]
is ideal for the physics case for several reasons. There
is essentially no flux below 30 GeV, hence all neutrinos

\begin{itemize}
  \item non-unitarity in the light neutrino mixing matrix;
  \item wrong-sign inverse muon decay (WSIMD), $\bar{\nu}_\mu + 
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  \item decays of neutrissimos, i.e., moderately-heavy
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nu, CC Deep Inelastic Scattering
nu, NC Deep Inelastic Scattering
nu, electron NC elastic scatters (ES)
nu, electron CC quasi-elastic scatters (IMD)
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nu, electron CC quasi-elastic scatters (WSIMD)

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<td>600M</td>
<td>nu, CC Deep Inelastic Scattering</td>
</tr>
<tr>
<td>190M</td>
<td>nu, NC Deep Inelastic Scattering</td>
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<td>75k</td>
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<td>700k</td>
<td>nu, electron CC quasi-elastic scatters (IMD)</td>
</tr>
<tr>
<td>33M</td>
<td>nu, CC Deep Inelastic Scattering</td>
</tr>
<tr>
<td>12M</td>
<td>nu, NC Deep Inelastic Scattering</td>
</tr>
<tr>
<td>7k</td>
<td>nu, electron NC elastic scatters (ES)</td>
</tr>
<tr>
<td>0k</td>
<td>nu, electron CC quasi-elastic scatters (WSIMD)</td>
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TABLE I: Rates assumed for this paper. NC indicates “neutral current” and CC indicates “charged current.”

this glass-calorimeter and toroid design, each a total of 29 m in length (including the toroid). Between each sub-detector is a 15 m decay region for direct searches for new physics. The total fiducial volume is 3 ktons.

The NuSOnG run plan, for reasons discussed in Sec. III B and III C, concentrates on running in neutrino mode. This design will yield the rates shown in Table I. These rates, before cuts, are assumed throughout the rest of the discussion. We can compare this sample to past experiments. The present highest statistics sample for \( \nu_e \) and \( \bar{\nu}_e \) ES is from CHARM II, with 2677 ± 282 events in neutrino mode and 2752 ± 88 events in antineutrino mode [5]. Thus the proposed experiment will have a factor of 30 (2.5) more \( \nu(\bar{\nu}) \)-electron events. As an example, after cuts, the first method of analysis described in Sec. III D retains 63% of the \( \nu \) sample. For deep inelastic scattering, 600M and 190M events are expected in neutrino and antineutrino modes, respectively. After minimal cuts to isolate DIS events [10], NuTeV had 1.62M DIS (NC+CC) events in neutrino mode and 0.35M in antineutrino mode; thus NuSOnG has orders of magnitude more events.

The detector will incorporate several specialized regions. A region of fine vertex-tracking facilitates measurements of the strange sea relevant for the electroweak analysis, as described in ref. [2]. Two possibilities are under consideration: an emulsion detector or a silicon detector of the style of NOMAD-STAR [11]. Both are compact and easily accommodated. For further QCD studies, it will also be useful to intersperse alternative target materials: C, Al, Fe, and Pb [2].

III. ELECTROWEAK MEASUREMENTS IN NEUTRINO SCATTERING

Neutrino neutral current (NC) scattering is an ideal probe for new physics. An experiment like NuSOnG is unique in its ability to test the NC couplings by studying scattering of neutrinos from both electrons and quarks. A deviation from the Standard Model predictions in both the electron and quark measurements would present a compelling case for new physics.

The exchange of the Z boson between the neutrino \( \nu \) and fermion \( f \) leads to the effective interaction:

\[
\mathcal{L} = -\sqrt{2}G_F \left[ \bar{\nu}\gamma_\mu (g'_\nu - g'_\nu \gamma_5) \nu \right] \left[ \bar{f} \gamma^\mu (g'_f - g'_f \gamma_5) f \right]
\]

This rate, before cuts, is assumed throughout the rest of the discussion. We can compare this sample to past experiments. The present highest statistics sample for \( \nu_e \) and \( \bar{\nu}_e \) ES is from CHARM II, with 2677 ± 282 events in neutrino mode and 2752 ± 88 events in antineutrino mode [5]. Thus the proposed experiment will have a factor of 30 (2.5) more \( \nu(\bar{\nu}) \)-electron events. As an example, after cuts, the first method of analysis described in Sec. III D retains 63% of the \( \nu \) sample. For deep inelastic scattering, 600M and 190M events are expected in neutrino and antineutrino modes, respectively. After minimal cuts to isolate DIS events [10], NuTeV had 1.62M DIS (NC+CC) events in neutrino mode and 0.35M in antineutrino mode; thus NuSOnG has orders of magnitude more events.

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The differential cross section for \( \nu_e \) and \( \bar{\nu}_e \) ES, defined using the coupling constants described above, is:

\[
\frac{d\sigma}{dT} = \frac{2G_F^2 m_e}{\pi} \left[ (g'_{L}g'_{\nu} \pm g'_{L}g'_{A})^2 
+ (g'_{L}g'_{\nu} \pm g'_{L}g'_{A}) \left( 1 - \frac{T}{E_\nu} \right)^2 
- \left( (g'_{L}g'_{\nu})^2 - (g'_{L}g'_{A})^2 \right) \frac{m_e T}{E_\nu^2} \right].
\]

The upper and lower signs correspond to the neutrino and anti-neutrino cases, respectively. In this equation, \( E_\nu \) is the incident \( \nu_e \) energy and \( T \) is the electron recoil kinetic energy.

More often in the literature, the cross section is defined in terms of the parameters \( (g'_{\nu}^e, g'_{A}^e) \), which are defined as

\[
g'_{\nu}^e \equiv (2g'_{L}g'_{\nu}) = \rho \left( \frac{1}{2} + 2\sin^2 \theta_W \right),
\]
\[ g_A^{\nu e} = (2g_L g_A^e) = \rho \left( -\frac{1}{2} \right) , \] (7)

In terms of these parameters, we can write:

\[
\frac{d\sigma}{dT} = \frac{G_F^2 m_e}{2\pi} \left[ (g_{\nu e}^{\nu e} \pm g_{\nu A}^{\nu e})^2 
+ (g_{\nu e}^{\nu e} + g_{\nu A}^{\nu e})^2 \left( 1 - \frac{T}{E_\nu} \right)^2 
- \frac{1}{3} (g_{\nu e}^{\nu e} - g_{\nu A}^{\nu e})^2 \right] \left( \frac{m_e T}{E_\nu^2} \right) .
\] (8)

When \( m_e \ll E_\nu \), as is the case in NuSOnG, the third term in these expressions can be neglected. If we introduce the variable \( y = T/E_\nu \), then

\[
\frac{d\sigma}{dy} = \frac{G_F^2 m_e E_\nu}{2\pi} \left[ (g_{\nu e}^{\nu e} \pm g_{\nu A}^{\nu e})^2 + (g_{\nu e}^{\nu e} + g_{\nu A}^{\nu e})^2 (1 - y)^2 \right] .
\] (9)

Integrating, we obtain the total cross sections which are

\[
\sigma = \frac{G_F^2 m_e E_\nu}{2\pi} \left[ (g_{\nu e}^{\nu e} \pm g_{\nu A}^{\nu e})^2 + \frac{1}{3} (g_{\nu e}^{\nu e} + g_{\nu A}^{\nu e})^2 \right] .
\] (10)

Note that

\[
(g_{\nu e}^{\nu e} + g_{\nu A}^{\nu e})^2 = \rho^2 \left( -1 + 2\sin^2 \theta_W \right)^2
= \rho^2 \left( 1 - 4\sin^2 \theta_W + 4\sin^4 \theta_W \right) ,
\]
\[
(g_{\nu e}^{\nu e} - g_{\nu A}^{\nu e})^2 = \rho^2 \left( 2\sin^2 \theta_W \right)^2
= \rho^2 \left( 4\sin^4 \theta_W \right) .
\] (11)

Therefore,

\[
\sigma(\nu_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} \rho^2 \left[ 1 - 4\sin^2 \theta_W + \frac{16}{3} \sin^4 \theta_W \right] ,
\]
\[
\sigma(\bar{\nu}_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} \rho^2 \frac{1}{3} \left[ 1 - 4\sin^2 \theta_W + 16\sin^4 \theta_W \right] .
\] (12)

The ratio of the integrated cross sections for neutrino to antineutrino electron ES is

\[
R_{\nu/\bar{\nu}} = \frac{\sigma(\nu_\mu e)}{\sigma(\bar{\nu}_\mu e)} = 3 \frac{1 - 4\sin^2 \theta_W + \frac{16}{3} \sin^4 \theta_W}{1 - 4\sin^2 \theta_W + 16\sin^4 \theta_W} .
\] (13)

Fig. 4 (top) shows the results for \( \sin^2 \theta_W \) from past experiments which have used this “\( \nu/\bar{\nu} \) ES ratio.”

In the ratio, \( R_{\nu/\bar{\nu}} \), the dependence on \( \rho \) canceled. This directly extracts \( \sin^2 \theta_W \). The relationship between the error on the ratio and the error on \( \sin^2 \theta_W \), which for convenience we abbreviate as \( z \), is:

\[
\delta z = \frac{32z - 12}{16z^2 - 4z + 1} + \frac{48z^2 - 144z - 512z^3 + 12}{48z^2 - 8z - 128z^3 + 256z + 1} \delta R_{\nu/\bar{\nu}}
= -0.103 \delta R_{\nu/\bar{\nu}};
\] (14)
\[
\delta z/z = -0.575 \delta R_{\nu/\bar{\nu}}/R_{\nu/\bar{\nu}} .
\] (15)

FIG. 4: Measurements of \( \sin^2 \theta_W \) from past experiments. Top: neutrino-electron elastic scattering experiments. Bottom: neutrino DIS experiments. All DIS results are adjusted to the same charm mass (relevant for experiments not using the PW method). The Standard Model value, indicated by the line, is 0.2227 [12].

for \( z = 0.2227 \) (or \( R_{\nu/\bar{\nu}} = 1.242 \)). Roughly, the fractional error on \( \sin^2 \theta_W \) is 60% of the fractional error on \( R_{\nu/\bar{\nu}} \).

B. A New Technique: Normalization Through IMD

An experiment such as NuSOnG can make independent measurements of the electroweak parameters for both \( \nu_\mu \) and \( \bar{\nu}_\mu \)-electron scattering. We can achieve this via ratios or by direct extraction of the cross section. In the case of \( \nu_\mu \)-electron scattering, we will use the ratio of the number of events in neutrino-electron elastic scattering to inverse muon decay:

\[
\frac{N(\nu_\mu e^- \rightarrow \nu_\mu e^-)}{N(\bar{\nu}_\mu e^- \rightarrow \mu^- e^-)} = \frac{\sigma_{NC}^{\nu e} \times \Phi_\nu}{\sigma_{IMD}^{\nu e} \times \Phi_\nu} .
\] (16)

Because the cross section for IMD events is well determined by the Standard Model, this ratio should have low errors and will isolate the EW parameters from NC scattering. In the discussion below, we will assume that the systematic error on this ratio is 0.5%.

In the case of \( \bar{\nu}_\mu \) data, the absolute normalization is more complex because there is no equivalent process to inverse muon decay (since there are no positrons in the detector). One can use the fact that, for low exchange energy (or “nu”) in Deep Inelastic Scattering,
the cross sections in neutrino and antineutrino scattering approach the same constant, $A$. This is called the “low $\nu$ method” of flux extractions. For DIS events with low energy transfer and hence low hadronic energy ($5 \lesssim E_{\text{had}} \lesssim 10$ GeV), $N_{\nu \text{DIS}}^{\text{low $E_{\text{had}}$}} = \Phi\nu A$ and $N_{\bar{\nu} \text{DIS}}^{\text{low $E_{\text{had}}$}} = \Phi\bar{\nu} A$. The result is that the electroweak parameters can be extracted using the ratio

$$\frac{N_{\text{low $E_{\text{had}}$} \nu \text{DIS}}}{N_{\text{low $E_{\text{had}}$} \bar{\nu} \text{DIS}}} \times \frac{N(\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-)}{N(\nu_\mu e^- \to \mu^- \nu_e)} = \frac{\Phi\nu}{\Phi\bar{\nu}} \times \frac{\sigma_{\text{NC}}}{\sigma_{\text{IMD}}} \times \frac{\Phi\nu}{\Phi\bar{\nu}}. \quad (17)$$

The first ratio cancels the DIS cross section, leaving the energy-integrated $\nu$ to $\bar{\nu}$ flux ratio. The IMD events in the denominator of the second term cancel the integrated $\nu$ flux. The NC elastic events cancel the integrated $\bar{\nu}$ flux.

Because of the added layer of complexity, the antineutrino ES measurement would have a higher systematic error than the neutrino ES scattering measurement. The potentially higher error is one factor leading to the plan that NuSOnG concentrate on neutrino running for the ES studies.

As shown in Fig. 2, IMD events have a kinematic threshold at 10.9 GeV. These events also have other interesting kinematic properties. The minimum energy of the outgoing muon in the lab frame is given by

$$E_{\mu \text{, lab}}^{\text{min}} = \frac{m_\mu^2 + m_e^2}{2m_e} = 10.9 \text{ GeV}. \quad (18)$$

In the detector described above, muons of this energy and higher will reach the toroid spectrometer without ranging-out in the glass. An interesting consequence is that, independent of $E_\nu$, the energy transfer in the interaction has a maximum value of

$$y_{\text{max}} = 1 - \frac{10.9 \text{ GeV}}{E_\nu}. \quad (19)$$

Thus at low $E_\nu$, the cutoff in $y$ is less than unity, as shown in Fig. 5 (left). The direct consequence of this is a strong cutoff in angle of the outgoing muon, shown in Fig. 5 (right). In principle, one can reconstruct the full neutrino energy in these events:

$$E_\nu^{\text{IMD}} = \frac{1}{2} \frac{m_e - m_e^2 - m_\mu^2}{m_e - m_\mu} + p_\mu \cos \theta_\mu. \quad (20)$$

This formula depends on $\theta_\mu$, which is small. The reconstructed $E_\nu$ is smeared by resolution effects as seen in Fig. 6. While the analysis can be done by summing over all energies, these distributions indicate that an energy binned analysis may be possible. This is more powerful because one can fit for the energy dependence of backgrounds. For the illustrative analyzes below, however, we do not employ this technique.

The error on $\sin^2 \theta_W$ extracted from this ratio, $R_{\text{ES/IMD}}$, assuming a Standard Model value for $\rho$, is the same as the error on the ratio:

$$\frac{\delta(\sin^2 \theta_W)}{\sin^2 \theta_W} \approx \delta R_{\text{ES/IMD}} / R_{\text{ES/IMD}}. \quad (21)$$
Ref. [14] provides a useful summary of radiative corrections for the ES and IMD processes, which were originally calculated in Ref. [15]. The error from radiative corrections is expected to be below 0.1%. It is noted that to reduce the error below 0.1%, leading two-loop effects must be included. A new evaluation of the radiative corrections is underway [16].

C. IMD Normalization vs. $\bar{\nu}$ Normalization

NuSOnG can measure both the $\nu/\bar{\nu}$ ES ratio, as in the case of past experiments shown in Eq. (13), as well as the ES/IMD ratio. In the case of the former, to obtain the best measurement in a 5 year run, one would choose a 1:3 ratio of run time in $\nu$ versus $\bar{\nu}$ mode. In the latter case, one would maximize running in $\nu$ mode. The result of the two cases is a nearly equal error on $\sin^2 \theta_W$, despite the fact that the error on the $\nu/\bar{\nu}$ ES is nearly twice that of the ES/IMD ratio. To understand this, compare Eq. (15) to Eq. (21). However, the ES/IMD ratio is substantially stronger for reasons of physics. Therefore, our conceptual design calls for running mainly with a $\nu$ beam. In this section we explore the issues for these two methods of measurement further. We also justify why the precision measurement requires high energies, only available from a Tevatron-based beam.

1. Comparison of the Two Measurement Options

From the point of view of physics, the ES/IMD ratio is more interesting than the $\nu/\bar{\nu}$ ES ratio. This is because $\rho$ has canceled in the $\nu/\bar{\nu}$ ES ratio of Eq. (13), leaving the ratio insensitive to physics which manifests itself through changes in the NC coupling. Many of the unique physics goals of NuSOnG, discussed in Sec. IV, depend upon sensitivity to the NC coupling.

An equally important concern was one of systematics. The $\nu$ and $\bar{\nu}$ fluxes for a conventional neutrino beam are substantially different. For the case of NuSOnG, the fluxes are compared in Fig. 3. Predicting the differences in these fluxes from secondary production measurements and simulations leads to substantial systematic errors. For beams at high energies ($>30$ GeV), such as NuSOnG, the “low $\nu$” method [13] for determining the ratio of the neutrino to antineutrino fluxes from Deep Inelastic events, developed by CCFR and NuTeV and described in Sec. III B, can be employed. However, this leads to the criticism that one has introduced a new process into the purely-lepton analysis.

Neither criticism is relevant to the ES/IMD ratio. The sensitivity to the new physics through the couplings does not cancel. Because both processes are in neutrino mode, the flux exactly cancels, as long as the neutrino energies are well above the IMD threshold (this will be illustrated in the analysis presented in Sec. III D). This ratio has the added advantage of needing only neutrino-mode running, which means that very high statistics can be obtained. This is clearly the more elegant solution.

It should be noted that nothing precludes continued running of NuSOnG beyond the 5-year plan presented here. This run-length was selected as “reasonable” for first results. If interesting physics is observed in this first phase, an extended run in antineutrino mode may be warranted, in which case both the ES/IMD and $\nu/\bar{\nu}$ ES ratios could be measured. The latter would then constrain $\sin^2 \theta_W$ in a pure neutrino measurement and the former is then used to extract $\rho$.

To measure the ES/IMD ratio to high precision, there must be little low energy flux. This is because the IMD has a threshold of 10.9 GeV, and does not have substantial rate until $\sim30$ GeV. The low-energy cut-off in the flux (see Fig. 3) coming from the energy-angle correlation of neutrinos from pion decay, is ideal.

2. Why a Tevatron-based Beam is Best for Both Options

The ES/IMD measurement is not an option for the planned beams from the Main Injector at Fermilab. For both presently planned Main Injector experiments at Fermilab [17] and for the proposed Project-X DUSEL beam [18], the neutrino flux is peaked at $\sim5$ GeV. The majority of the flux of these beams is below 5 GeV, and most of the flux is below the 10.9 GeV IMD threshold. Because of this, one simply cannot use the IMD events to normalize.

In principle, the $\nu/\bar{\nu}$ ES ratio could be used. However, in practice this will have large systematics. The $\nu$ and $\bar{\nu}$ fluxes for a horn beam are significantly different. First principles predictions of secondary mesons are not sufficient to reduce this error to the precision level. The energy range is well below the deep inelastic region where the “low $\nu$” method can be applied to accurately extract a $\bar{\nu}/\nu$ flux ratio. Other processes, such as charged-current quasi-elastic scattering, could be considered for normalization, but the differences in nuclear effects in neutrino and antineutrino scattering for these events is not sufficiently well understood to yield a precision measurement.

Lastly, the ES rates for the present Main Injector beams are too low for a high statistics measurement. This is because the cross section falls linearly with energy. Event samples on the order of 10k may be possible with extended running in the Project X DUSEL beam in the future. From the point of view of statistics, even though two orders of magnitude more protons on target are supplied in such a beam, the Tevatron provides a substantially higher rate of ES per year of running.

Compared to the Main Injector beam, a Tevatron-based beam does not face these issues. The choice of running in neutrino mode provides the highest precision measurement while optimizing the physics.
### TABLE II: Resolutions and systematic uncertainty estimates used in the parameterized Monte Carlo studies. The NuTeV estimates are based on Ref. [19] and the CHARM II estimates from Ref. [9]. Units for angles are radians and energies are in GeV.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Assumed Value</th>
<th>Uncertainty</th>
<th>Source of Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy Resolution</td>
<td>$\delta E/E = 10%$</td>
<td>2.5%</td>
<td>NuTeV testbeam measurement</td>
</tr>
<tr>
<td>Energy Scale Error</td>
<td>$E_{	ext{rec}} = 1.0 \times E_{	ext{true}}$</td>
<td>0.5%</td>
<td>NuTeV testbeam measurement</td>
</tr>
<tr>
<td>Angular Resolution</td>
<td>$\delta \theta = 0.011/E^{0.96}$ rad</td>
<td>2.5%</td>
<td>Multiple scattering fit simulation</td>
</tr>
<tr>
<td>Electron</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy Resolution</td>
<td>$\delta E/E = 0.23/E^{0.5}$</td>
<td>1.0%</td>
<td>Same as CHARM II</td>
</tr>
<tr>
<td>Energy Scale Error</td>
<td>$E_{	ext{rec}} = 1.0 \times E_{	ext{true}}$</td>
<td>1.0%</td>
<td>Scaled from CHARM II with NuSOnG statistics</td>
</tr>
<tr>
<td>Angular Resolution</td>
<td>$\delta \theta = 0.008/E^{0.5}$ rad</td>
<td>2.5%</td>
<td>2 better than CHARM II due to sampling</td>
</tr>
<tr>
<td>Flux</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normalization</td>
<td>1.0</td>
<td>3%</td>
<td>Current total cross section uncertainty</td>
</tr>
<tr>
<td>Shape Uncertainty</td>
<td>1.0</td>
<td>1%</td>
<td>Similar to NuTeV low-nu method</td>
</tr>
<tr>
<td>Backgrounds</td>
<td>$\nu_e$ CCQE</td>
<td>1.0</td>
<td>Extrapolated from NuTeV</td>
</tr>
<tr>
<td>$\nu_\mu$ CCQE</td>
<td>1.0</td>
<td>3%</td>
<td>Extrapolated from CHARM II</td>
</tr>
</tbody>
</table>

**D. A 0.7% Measurement Goal for the ES to IMD Ratio**

Achieving 0.7% precision on the ES/IMD measurement depends on reducing the backgrounds to an acceptable level without introducing significant systematics and while maintaining high signal statistics. Many of the systematic uncertainties will tend to cancel. The most important background for both the $\nu$-$e$ neutral current and IMD events comes from charged current quasi-elastic (CCQE) or neutral current (NC) processes. The NC background has a much broader $Q^2$ distribution as compared to the signal processes and, therefore, can be partially eliminated by kinematic cuts on the outgoing muon or electron. Initial cuts on the scattering angle and energy of the outgoing muon or electron can easily reduce the CCQE background by factors of 60 and 14 respectively while retaining over 50% of the $\nu$-$e$ neutral current and IMD signal. This leaves events with very forward scatters and outgoing scattered protons of low kinetic energy.

Because the NuSOnG design is at the conceptual stage and in order to be conservative, we have developed two different strategies for achieving a 0.7% error. This serves as a proof of principle that this level of error, or better, can be reached. The first method relies on detecting protons from the quasi-elastic scatter. The second method uses the beam kinematics to cut the low energy flux which reduces the CCQE background.

These methods were checked via two, independently written, parameterized Monte Carlos. The parameterized Monte Carlos made the assumptions given in Table [11] where both the assumed values and uncertainties are presented. These estimates of resolutions and systematic errors are based on previous experimental measurements or on fits to simulated data. One Monte Carlo used the Nuance event generator [20] to produce events, while the other was an independently written event generator. Both Monte Carlos include nuclear absorption and binding effects.

The first strategy uses the number of protons which exit the glass to constrain the total rate of the background. In ~33% of the events, a proton will exit the glass, enter a chamber and traverse the gas. This sample of protons at all energies and $Q^2$ since the interactions occur uniformly throughout the glass. After initial cuts, the protons are below 100 MeV, and therefore highly ionizing. If we define 1 MIP as the energy deposited by a single minimum ionizing particle, like a muon, then the protons consistently deposit greater than 5 MIPs in the chamber. Thus, one can identify CCQE events by requiring $>4$ MIPs in the first chamber. The amount of remaining CCQE background after this requirement can be measured if a fraction such as 10% of the detector is made of scintillating glass that can directly identify CCQE events from light associated with the outgoing proton. A wide range of scintillating glasses have been developed [21] for nuclear experiments. These glasses are not commonly used in high energy physics experiments because the scintillation time constant is typically on the order of 100 ns. In a neutrino experiment, which has inherently lower rates than most particle experiments, this is not an issue. CCQE events can be identified by the scintillation light from the proton assuming reasonable parameters for the glass and readout photomultiplier tubes: 450 photons/MeV, an attenuation length of 2 m, eight phototubes per glass sheet, quantum efficiency of the tubes of 20%. Using the identified CCQE events from the instrumented glass, the uncertainty in the residual background can be reduced to 2.0% for the IMD measurement. For the CCQE background to the $\nu_\mu$-$e$ current measurement, the uncertainty is assumed to be 3% for the Monte Carlo prediction. Combining all the systematic errors leads to a $\sim$0.7% accuracy on the $\nu$-$e$ measurement as shown in Tab. [13].

In Tab. [13], the cancellation of the flux errors should be
noted. This occurred because we use the ES/IMD ratio, as discussed in the previous section.

The second strategy involves reducing the relative CCQE background to signal by using a harder flux for the analysis. This study used the same Monte Carlos, with the resolutions listed in Tab. I as the first analysis. The total systematic and statistical error achieved was 0.6%. Below, we explain how a harder flux is obtained for the analysis. Then, we explain how this flux improves the signal-to-background in both the ES and IMD analyzes.

The strong correlation between energy and angle at the NuTeV beam and detector lay-out, this paper we take the conservative approach of assuming able to achieve a better than 0.7% error. However, for tor moves from a conceptual to real design, we will be preliminary studies, we feel confident that as the detec-

gories and can, in principle, be combined. Given these improved with a high energy flux. This allows looser cuts fluxes above 1 GeV. This signal-to-background is greatly rising with energy. In contrast, the which shows the threshold effects. The IMD signal is also the improvement in the IMD analysis, consider Fig. 2, the energy region due to the combination of a flat y distribu-
ted above 50 GeV. On the other hand, the outgoing electron in ES events tends to populate the low visible energy region due to the combination of a flat y distribution for the process convoluted with the incident neutrino energy spectrum. The result is that a cut on the visible energy less than 50 GeV reduces the error from the νe CCQE events produced by the harder flux populate the visible energy range above 50 GeV. On the other hand, the outgoing neutrino quark scattering, which compares neutral-
current (NC) to charged-current (CC) scattering to ex-
tract sin^2 θW. However, these experiments are subject to issues of modeling in the quark sector. Fig. bottom) reviews the history of these measurements.

The lowest systematic errors come from implement-
ing a “Paschos-Wolfenstein style” analysis. This PW technique would be used by any future experiment, including NuSonG. This requires high purity ν and ¯ν beams, for which the following ratios of DIS events could be formed:

\begin{align}
R^\nu &= \frac{\sigma_{NC}^{\nu}}{\sigma_{CC}^{\nu}} \\
R^\bar{\nu} &= \frac{\sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^{\bar{\nu}}}
\end{align}

Paschos and Wolfenstein recast these as:

\begin{align}
R^- &= \frac{\sigma_{NC}^{\nu} - \sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^{\nu} - \sigma_{CC}^{\bar{\nu}}} = \frac{R^\nu - R^\bar{\nu}}{1 - r},
\end{align}

where \( r = \sigma_{CC}^{\nu} / \sigma_{CC}^{\bar{\nu}} \). In \( R^- \) many systematics cancel to first order, including the effects of the quark and antiquark seas for u, d, s, and c. Charm production only enters through \( d_{valence} \) (which is Cabibbo suppressed) and at high \( x_f \); thus the error from the charm mass is greatly reduced. The cross section ratios can be written in terms of the effective neutrino-quark coupling parameters \( g_L^2 \) and \( g_R^2 \) as:

\begin{align}
R^\nu &= g_L^2 + r g_R^2 \\
R^\bar{\nu} &= g_L^2 + \frac{1}{r} g_R^2 \\
R^- &= g_L^2 - g_R^2 = \rho^2 \left( 1 - \sin^2 \theta_W \right),
\end{align}

in which

\begin{align}
g_L^2 &= (2g_L^\nu g_L^\bar{\nu})^2 + (2g_L^\nu g_L^d)^2 = \rho^2 \left( \frac{1}{2} - \sin^2 \theta_W + \frac{5}{9} \sin^4 \theta_W \right) \\
g_R^2 &= (2g_R^\nu g_R^\bar{\nu})^2 + (2g_R^\nu g_R^d)^2 = \rho^2 \left( \frac{1}{9} \sin^4 \theta_W \right).
\end{align}

In a variation on the PW idea, rather than directly form \( R^- \), NuTeV fit simultaneously for \( R^\nu \) and \( R^\bar{\nu} \) to extract \( \sin^2 \theta_W \), obtaining the value \( \sin^2 \theta_W = 0.2277 \pm 0.00162 \). Events were classified according to the length of hits in the scintillator planes of the NuTeV detector, with long events identified as CC interactions and short events as NC. An important background in the CC sample came from pion decay-in-flight, producing a muon in
a NC shower. Significant backgrounds in the NC sample came from muons which ranged out or exited and from $\nu_e$ CC scatters which do not have a muon and thus are classified as “short.”

In this paper, we present the sensitivity of NuSOnG to new physics if the NuTeV errors are reduced by a factor of $\sim 2$. This is a very conservative estimate, since most of the improvement comes from higher statistics. Only a 90% improvement in the systematics is required to reach this goal. Tab. IV argues why a 90% reduction in systematic error should be straightforward to achieve. It is likely that the NuSOnG errors will be lower, but this requires detailed study.

In Table IV we list the errors which NuTeV identified in their original analysis and indicate how NuSOnG will improve each error. Many of the largest experimental systematics of NuTeV are improved by introducing a fine-grained sampling calorimeter. The NuTeV detector had four inches of iron between unsegmented scintillator planes and eight inches between drift chamber planes. Better lateral segmentation and transverse detection will improve identification of scatters from intrinsic $\nu_e$ in the beam and separation of CC and NC events by improved three-dimensional shower shape analyses. The NuTeV analyzes of the intrinsic $\nu_e$ content\cite{22} and the CC/NC separation for the $\sin^2 \theta_W$ analysis which relied strictly on event length. With this said, the power of classifying by event length is shown by the fact that the NuTeV intrinsic $\nu_e$ analysis was sensitive to a discrepancy in the predicted intrinsic $\nu_e$ rate which was recently resolved with a new measurement of the $K_{e3}$ branching ratio that was published in 2003. Details of these issues are considered in the next section.

### F. The NuTeV Anomaly

From Fig. 4 it is apparent that the NuTeV measurement is in agreement with past neutrino scattering results, although these have much larger errors; however, in disagreement with the global fits to the electroweak data which give a Standard Model value of $\sin^2 \theta_W = 0.2227$\cite{25}. Expressed in terms of the couplings, NuTeV measures:

$$g^2_{L} = 0.30005 \pm 0.00137$$
$$g^2_{R} = 0.03076 \pm 0.00110,$$

which can be compared to the Standard Model values of $g^2_{L} = 0.3042$ and $g^2_{R} = 0.0301$, respectively.

NuTeV is one of a set of $Q^2 \approx m^2_Z$ experiments measuring $\sin^2 \theta_W$. It was performed at $Q^2 = 1$ to 140 GeV$^2$, $\langle Q^2 \rangle = 26$ GeV$^2$, $\langle Q^2 \rangle = 15$ GeV$^2$, which is also the expected range for NuSOnG. Two other precision low $Q^2$ measurements are from atomic parity violation\cite{20} (APV), which samples $Q^2 \sim 0$; and SLAC E158, a Moller scattering experiment at average $Q^2 = 0.026$ GeV$^2$\cite{27}. Using the measurements at the Z-pole with $Q^2 = M_Z^2$ to fix the value of $\sin^2 \theta_W$, and evolving to low $Q^2$\cite{28}, the APV and SLAC E158 are in agreement with the Standard Model. However, the radiative corrections to neutrino interactions allow sensitivity to high-mass par-
<table>
<thead>
<tr>
<th>Source</th>
<th>NuTeV Error</th>
<th>Method of reduction in NuSOnG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics</td>
<td>0.00135</td>
<td>Higher statistics</td>
</tr>
<tr>
<td>$\nu_e, \bar{\nu}_e$ flux prediction</td>
<td>0.00039</td>
<td>Improves in-situ measurement of $\bar{\nu}_e$ CC scatters, thereby constraining prediction, due to better lateral segmentation and transverse detection. Also, improved beam design to further reduce $\bar{\nu}_e$ from $K^0$.</td>
</tr>
<tr>
<td>Interaction vertex position</td>
<td>0.00030</td>
<td>Better lateral segmentation.</td>
</tr>
<tr>
<td>Shower length model</td>
<td>0.00027</td>
<td>Better lateral segmentation and transverse detection will allow more sophisticated shower identification model.</td>
</tr>
<tr>
<td>Counter efficiency and noise</td>
<td>0.00023</td>
<td>Segmented scintillator strips of the type developed by MINOS [23] will improve this.</td>
</tr>
<tr>
<td>Energy Measurement</td>
<td>0.00018</td>
<td>Better lateral segmentation.</td>
</tr>
<tr>
<td>Charm production, strange sea</td>
<td>0.00041</td>
<td>In-situ measurement [1][2]</td>
</tr>
<tr>
<td>$R_L$</td>
<td>0.00032</td>
<td>In-situ measurement [1][2]</td>
</tr>
<tr>
<td>$\sigma^*/\sigma^0$</td>
<td>0.00022</td>
<td>Likely to be at a similar level.</td>
</tr>
<tr>
<td>Higher Twist</td>
<td>0.00014</td>
<td>Recent results reduce this error [24].</td>
</tr>
<tr>
<td>Radiative Corrections</td>
<td>0.00011</td>
<td>New analysis underway, see text below.</td>
</tr>
<tr>
<td>Charm Sea</td>
<td>0.00010</td>
<td>Measured in-situ using wrong-sign muon production in DIS.</td>
</tr>
<tr>
<td>Non-isoscalar target</td>
<td>0.00005</td>
<td>Glass is isoscalar</td>
</tr>
</tbody>
</table>

TABLE IV: Source and value of NuTeV errors on $\sin^2 \theta_W$, and reason why the error will be reduced in the PW-style analysis of NuSOnG. This paper assumes NuSOnG will reduce the total NuTeV error by a factor of two. This is achieved largely through the improved statistical precision and requires only a 90% reduction in the overal NuTeV systematic error. This table argues that a better than 90% reduction is likely, but further study, once the detector design is complete, is required.

ticles which are complementary to the APV and Møller-scattering corrections. Thus, these results may not be in conflict with NuTeV. The NuSOnG measurement will provide valuable additional information on this question.

Since the NuTeV result was published, more than 300 papers have been written which cite this result. Several “Standard-Model” explanations have been suggested. While some constraints on these ideas can come from outside experiments, it will be necessary for any future neutrino scattering experiment, such as NuSOnG, to be able to directly address these proposed solutions. Also various Beyond Standard Model explanations have been put forward; those which best explain the result require a follow-up experiment which probes the neutral weak couplings specifically with neutrinos, such as NuSOnG. Here, we consider the explanations which are “within the Standard Model” and address the Beyond Standard Model later.

Several systematic adjustments to the NuTeV result have been identified since the result was published but have not yet been incorporated into a new NuTeV analysis. As discussed here, the corrections due to the two new inputs, a new $K_{e3}$ branching ratio and a new strange sea symmetry, are significant in size but are in opposite direction – away and toward the Standard Model. So a re-analysis can be expected to yield a central value for NuTeV which will not change significantly. However, the error is expected to become larger.

In 2003, a new result from BNL865 [29] yielded a $K_{e3}$ branching ratio which was $2.3\sigma$ larger than past measurements and a value of $|V_{us}|^2$ which brought the CKM matrix measurements into agreement with unitarity in

FIG. 7: Effect of various “Standard Model” explanations on the NuTeV anomaly. The y-axis is the deviation ($\delta \sin^2 \theta_W = \sin^2 \theta_W^{SM} - \sin^2 \theta_W^{NuTeV}$). The solid line is the published NuTeV deviation. Thick black lines extending from the NuTeV deviation show the range of possible pulls from NLO QCD and various isospin violation models. Note that the isospin violation models are mutually exclusive and so should not be added in quadrature. They are, from left to right, the full bag model, the meson cloud model, and the isospin QED model.
the first row. The measurement was confirmed by CERN NA48/2. The resulting increased Kγ3 branching ratio increases the absolute prediction of intrinsic νs in the NuTeV beam. This does not significantly change the error because the error on $K\gamma$ was already included in the analysis. However, it introduces a correction moving the NuTeV result further away from the Standard Model, since it implies that in the original analysis, NuTeV under-subtracted the νs background in the NC sample. The shift in $\sin^2\theta_W$ can be estimated in a back of envelope calculation to be about ~0.001 away from the Standard Model.

The final NuTeV measurement of the difference between the strange and anti-strange sea momentum distributions, was published in 2007. This “strange sea asymmetry" is defined as

$$xs^-(x) \equiv xs(x) - x\bar{s}(x),$$

Because of mass suppression for the production of charm in CC scatters from strange quarks, a difference in the momentum distributions will result in a difference in the CC cross sections for neutrinos and antineutrinos. Thus a correction to the denominator of Eq. (25) would be required. The most recent next-to-leading order analysis finds the asymmetry, integrated over $x$ is $0.00195 \pm 0.00055 \pm 0.00138$. An integrated asymmetry of 0.007 is required to explain the published NuTeV result, and so one can estimate that this is a shift of about 0.0014 in $\sin^2\theta_W$ toward the Standard Model. In this case, the errors on the NuTeV result will become larger because this effect was not originally considered in the analysis. A very naive estimate of the size of the increase can be derived by scaling the error on the integrated strange sea, quoted above, and is about 0.001 toward the Standard Model. If this naive estimate of the systematic error is borne out, then this could raise the NuTeV error on $\sin^2\theta_W$ from 0.0016 to 0.0018. NuSOnG will directly address the strange sea asymmetry in its QCD measurement program, as described in ref. [2].

In ref. [34], additional electromagnetic radiative corrections have been suggested as a source of the discrepancy. However, this paper only considered the effect of these corrections on $R^e$ and not $R^\nu$ and for fixed beam energy of $E_\nu = 80$ GeV. The structure of the code from these authors has also made it difficult to modify for use in NuTeV. This has prompted a new set of calculations by other authors which are now under way [16]. There are, as yet, only estimates for the approximate size of newly identified effects, which are small.

The NuTeV analysis was not performed at a full NLO level in QCD; any new experiment, such as NuSOnG will need to undertake a full NLO analysis. This is possible given recently published calculations, including those on target mass corrections [37]. On Fig. [7] we show an early estimate of the expected size and direction of the pull. On this plot, the solid horizontal line indicates the deviation of NuTeV from the Standard Model. The thick vertical lines, which emanate from the NuTeV deviation, show the range of pulls estimated for various explanations. The range of pull for the NLO calculation is shown on the left.

The last possibility is that there is large isospin violation (or charge symmetry violation) in the nucleus. The NuTeV analysis assumed isospin symmetry, that is, $u(x)^P = d(x)^n$ and $d(x)^P = u(x)^n$. Isospin violation can come about from a variety of sources and is an interesting physics question in its own right. NuSOnG’s direct constraints on isospin violation are discussed in ref. [2], which also considers the constraints from other experiments. Various models for isospin violation have been studied and their pulls range from less than 1σ away from the Standard Model to ~1σ toward the Standard Model.

We have chosen three examples for illustration on Fig. [7] the full bag model, the meson cloud model, and the isospin QED model. These are mutually exclusive models, so only one of these can affect the NuTeV anomaly.

IV. THE TERASCALE PHYSICS REACH OF NUSONG

Even when new states are too heavy to be produced at resonance in collisions they can make their presence known indirectly, as virtual particles which affect SM processes through interference with SM contributions to amplitudes. The new heavy states induce small shifts in observables from SM predictions, and conversely precise measurements of these observables can constrain or detect new physics at mass scales well above the energies of the colliding particles. In this way the precision neutrino scattering measurements at NuSOnG will place TeV-scale indirect constraints on many classes of new physics, or perhaps detect new physics by measuring deviations from SM predictions. The effects of new high-scale physics may be reduced to a small number of effective operators along with corresponding parameters which may be fit to data. Although the particular set of operators used depends on broad assumptions about the new physics, the approach gives a parameterization of new physics which is largely model-independent.

For concreteness we will assume that NuSOnG will be able to measure the neutrino ES/IMD ratio to a precision of 0.7%, $\sigma(\nu_e, e)$ (normalized as per Sec. [11]) to 1.3%, and that NuSOnG will be able to halve the errors on NuTeV’s measurement of DIS effective couplings, to $\Delta q^2_L = 0.0007$ and $\Delta q^2_R = 0.0006$ (where $g_L$ and $g_R$ were defined in Eqs. [29] and [30]).

We first parameterize new physics using the oblique parameters $S_T$, which is appropriate when the important effects of the new physics appear in vacuum polarizations of gauge bosons. We next assume new physics effects manifest as higher-dimensional operators made of SM fermion fields. We separately consider the possibility that the gauge couplings to neutrinos are modified.
Realistic models usually introduce several new operators with relations among the coefficients; we consider several examples. A summary of the contributions of NuSOng to the study of Terascale Physics is provided in Table [V].

### A. Oblique corrections

For models of new physics in which the dominant loop corrections are vacuum polarization corrections to the $SU(2)_L \times U(1)_Y$ gauge boson propagators ("oblique" corrections), the $STU$ parameterization provides a convenient framework in which to describe the effects of new physics on precision electroweak data. Differences between the predictions of a new physics model and those of a reference Standard Model (with a specified Higgs boson and top quark mass) can be expressed as nonzero values of the oblique correction parameters $S$, $T$ and $U$. $T$ and $U$ are sensitive to new physics that violates isospin, while $S$ is sensitive to isospin-conserving physics. Predictions of a Standard Model with Higgs or top masses different from the reference Standard Model may also be subsumed into shifts in $S$ and $T$ (in many models $U$ is much smaller than $S$ and $T$ and is largely unaffected by the Higgs mass, so it is often omitted in fits). Within a specific model of new physics the shift on the $ST$ plot away from the SM will be calculable [42]. For example,

- A heavy Standard Model Higgs boson will make a positive contribution to $S$ and a larger negative contribution to $T$.
- Within the space of $Z'$ models, a shift in almost any direction in $ST$ space is possible, with larger shifts for smaller $Z'$ masses.
- Models with a fourth-generation of fermions will shift $S$ positive, and will shift $T$ positive if there are violations of isospin.

In constructing models incorporating several types of new physics the corresponding shifts to $S$ and $T$ combine; if contributions from different sectors are large, then they must conspire to cancel.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Contribution of NuSOng Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oblique Corrections</td>
<td>Four distinct and complementary probes of $S$ and $T$. In the case of agreement with LEP/SLD: $\sim 25%$ improvement in electroweak precision.</td>
</tr>
<tr>
<td>Neutrino-lepton NSIs</td>
<td>Order of magnitude improvement in neutrino-electron effective couplings measurements. Energy scale sensitivity up to $\sim 5$ TeV at 95% CL.</td>
</tr>
<tr>
<td>Neutrino-quark NSIs</td>
<td>Factor of two improvement in neutrino-quark effective coupling measurements. Energy scale sensitivity up to $\sim 7$ TeV at 95% CL.</td>
</tr>
<tr>
<td>Mixing with Neutrissimos</td>
<td>$30%$ improvement on the $e$-family coupling in a global fit. $75%$ improvement on the $\mu$-family coupling in a global fit.</td>
</tr>
<tr>
<td>Right-handed Couplings</td>
<td>Complementary sensitivity to $g_R/g_L$ compared to LEP. Order of magnitude improvement compared to past experiments.</td>
</tr>
</tbody>
</table>

TABLE V: Summary of NuSOng's contribution to general Terascale physics studies.

![FIG. 8: The impact of NuSOng on the limits of $S$ and $T$. The reference SM is $m_t = 170.9$ GeV, and $m_H = 115$ GeV. $1\sigma$ bands due to NuSOng observables are shown against the 90\% contour from LEP/SLD. The central ellipses are the 68\% and 90\% confidence limit contours with NuSOng included. See Eqs. (29) and (30) for the definitions of $g_L$ and $g_R$.](image)

The constraints on $S$ and $T$ from the full set of precision electroweak data strongly restrict the models of new physics which are viable. The strongest constraints are from LEP/SLD, which give a current bound of

$$S = -0.02 \pm 0.11, \quad T = +0.06 \pm 0.13,$$

$$\text{Corr}(S, T) = 0.91. \quad (34)$$

The ES and DIS measurements from NuSOng provide four distinct and complementary probes of $S$ and $T$, as shown in Fig. 8. If the target precision is achieved, and assuming the NuSOng agree with SM predictions, NuSOng will further reduce the errors on $S$ and $T$ from the LEP/SLD values to

$$S = -0.05 \pm 0.09, \quad T = +0.02 \pm 0.10,$$
The $\sim 25\%$ reduction in the errors is primarily due to the improved measurement of $g^f_L$. We note that the error $g^f_L$ is likely to be further reduced (see Sec. III E), and so the this is conservative estimate of NuSOnG’s contribution to the physics.

### B. Non-standard interactions

NuSOnG will probe new physics that modifies neutrino-quark and neutrino-electron scattering. If the masses associated to the new degrees of freedom are much larger than the center of mass energy ($s = 2m_e E_{\text{beam}} < 0.5 \text{ GeV}^2$) then modifications to these processes are well-described by higher-dimensional effective operators. In the context of neutrino reactions, these operators are also referred to as non-standard interactions (NSI’s). In a model-independent effective Lagrangian approach these effective operators are added to the SM effective Lagrangian with arbitrary coefficients. Expressions for experimental observables can be computed using the new effective Lagrangian, and the arbitrary coefficients can then be constrained by fitting to data. Typically, bounds on the magnitude of the coefficients are obtained using only one or a few of the available effective operators. This approach simplifies the analysis and gives an indication of the scale of constraints, although we must be mindful of relationships among different operators that will be imposed by specific assumptions regarding the underlying physics.

To assess the sensitivity of NuSOnG to “heavy” new physics in neutral current processes, we introduce the following effective Lagrangian for neutrino-fermion interactions [44, 48, 49]:

$$
\mathcal{L}_{\text{NSI}} = -\sqrt{2} G_F \left[ \bar{\nu}_\alpha \gamma_\sigma P_L \nu_\beta \right] \left[ \varepsilon^{fV}_\alpha \bar{f}_L \gamma^\sigma f - \varepsilon^{fA}_\alpha \bar{f}_L f \gamma^\sigma \gamma_5 f \right] + \varepsilon^{fL}_\alpha \bar{f}_L f \gamma^\alpha \gamma_5 P_L f + \varepsilon^{fR}_\alpha \bar{f}_R f \gamma^\alpha P_R f \right].
$$

(36)

where $\alpha, \beta = e, \mu, \tau$ and $L, R$ represent left-chiral and right-chiral fermion fields. If $\alpha \neq \beta$, then the $\alpha \leftrightarrow \beta$ terms must be Hermitian conjugates of each other, i.e. $\varepsilon^{*}_{\alpha\beta} = \varepsilon^{*}_{\beta\alpha}$. NuSOnG is sensitive to the $\beta = \mu$ couplings. This effective Lagrangian is appropriate for parameterizing corrections to neutral current processes; an analysis of corrections to charged-current processes requires a different set of four-fermion operators.

Assuming $\varepsilon^{*}_{\alpha\beta} = 0$ for $\alpha \neq \beta$ we need consider only the terms $\varepsilon^{\mu\nu}_{\mu\nu}$ ($* = V, A, L, R$). If we rewrite Eq. (2) as

$$
\mathcal{L} = -\sqrt{2} G_F \left[ \bar{\nu}_\mu P_L \nu_\nu \right] \left[ g^{\nu\nu}_L \bar{f}_L \gamma^\mu f - g^{\nu\nu}_R \bar{f}_R f \gamma^\mu \right] + \varepsilon^{fL}_\mu \bar{f}_L f \gamma^\nu P_L f + \varepsilon^{fR}_\mu \bar{f}_R f \gamma^\nu P_R f \right],
$$

(37)

then we see that adding Eq. (36) to the SM Lagrangian will simply shift the effective couplings:

$$
g^{\nu\nu}_V \rightarrow g^{\nu\nu}_V + \varepsilon^{fV}_\mu \ ,
$$

$$
g^{\nu\nu}_A \rightarrow g^{\nu\nu}_A + \varepsilon^{fA}_\mu \ ,
$$

$$
g^{\nu\nu}_L \rightarrow g^{\nu\nu}_L + \varepsilon^{fL}_\mu \ ,
$$

$$
g^{\nu\nu}_R \rightarrow g^{\nu\nu}_R + \varepsilon^{fR}_\mu .
$$

(38)

Consequently, errors on the $g^{\nu\nu}_f$’s translate directly into errors on the $\varepsilon^{f\nu}_{\mu\nu}$’s. $P = V, A$ or $P = L, R$.

#### 1. Neutrino-lepton NSI

A useful review of present constraints on non-standard neutrino-electron interactions can be found in ref. [45]. As this paper states, and as we show below, an improved measurement of neutrino-electron scattering is needed.

The world average value for neutrino-electron effective couplings, dominated by CHARM II, is

$$
g^{\nu\nu}_V = -0.040 \pm 0.015 ,
$$

$$
g^{\nu\nu}_A = -0.507 \pm 0.014 ,
$$

$$
\text{Corr}(g^{\nu\nu}_V, g^{\nu\nu}_A) = -0.05 .
$$

(40)

The current $1\sigma$ bounds from CHARM II, Eq. (41) translates to $|\varepsilon^{\nu\nu}_V| < 0.01$. $P = L, R$ with a correlation of 0.07 [44]. At the current precision goals, NuSOnG’s $\nu_{\mu}$ and $\nu_{\tau}$ will significantly reduce the uncertainties on these NSIs, to

$$
|\varepsilon^{\nu\nu}_V| < 0.0036 ,
$$

$$
|\varepsilon^{\nu\nu}_A| < 0.0019 ,
$$

$$
\text{Corr}(\varepsilon^{\nu\nu}_V, \varepsilon^{\nu\nu}_A) = -0.57 .
$$

(41)

or in terms of the chiral couplings,

$$
|\varepsilon^{\nu\nu}_L| < 0.0015 ,
$$

$$
|\varepsilon^{\nu\nu}_R| < 0.0025 ,
$$

$$
\text{Corr}(\varepsilon^{\nu\nu}_L, \varepsilon^{\nu\nu}_R) = 0.64 .
$$

(42)

Even in the absence of a $\sigma(\bar{\nu}_\mu e)$ measurement $\varepsilon^{\nu\nu}_{\mu\mu}$ and $\varepsilon^{\nu\nu}_{\mu\tau}$ can be constrained from the $\nu_{\mu}e$ scattering data alone through a fit to the recoil electron energy spectrum (see Eq. 6).

We first consider the constraint on $\varepsilon^{\nu\nu}_{\mu\mu}$ and $\varepsilon^{\nu\nu}_{\mu\tau}$ from the total cross section $\sigma(\nu_{\mu}e)$. It is convenient to recast the effective interaction slightly, as

$$
\text{Corr}(S, T) = 0.87 .
$$

(35)
The new physics is parameterized by two coefficients $\Lambda$ and $\theta$. $\Lambda$ represents the broadly-defined new physics scale while $\theta \in [0, 2\pi]$ defines the relative coupling of left-chiral and right-chiral electrons to the new physics. As an example, a scenario with a purely “left-handed” $Z'$ that couples to leptons with coupling $g'$ would be described by $\Lambda \propto M_{Z'}/g'$ and $\theta = 0$ or $\theta = \pi$, depending on the relative sign between $g'$ and the electroweak couplings. $\Lambda$ and $\theta$ are related to the NSI parameters in Eq. (36) by

$$\varepsilon_{\alpha\mu}^{L} = -\frac{\cos \theta}{2G_F \Lambda^2}, \quad \varepsilon_{\alpha\mu}^{R} = -\frac{\sin \theta}{2G_F \Lambda^2}. \quad (44)$$

Note that we have generalized from our assumption of the previous section and not taken $\alpha = \mu$ necessarily. At NuSoN, new physics modifies (pseudo)elastic neutrino–electron scattering. Here we use the word “pseudo” to refer to the fact that we cannot identify the flavor of the final-state neutrino, which could be different from the incoming neutrino flavor in the case of flavor changing neutral currents.

The shift in the total cross section is

$$\frac{\delta \sigma(\nu_\mu e)}{\sigma(\nu_\mu e)} = \left\{ \frac{2 g^\nu e \varepsilon_{\alpha\mu}^{L} + (\varepsilon_{\alpha\mu}^{L})^2}{g^\nu e} \right\}^2 \frac{1}{4} \left\{ \frac{2 g^\nu e \varepsilon_{\alpha\mu}^{R} + (\varepsilon_{\alpha\mu}^{R})^2}{g^\nu e} \right\}^2 \approx -\left( \frac{516 \text{GeV}}{\Lambda} \right)^2 \cos(\theta - \phi) + 0.096 \left( \frac{516 \text{GeV}}{\Lambda} \right)^4 \left( 1 + 2 \cos^2 \theta \right) \cdot (45)$$

where

$$\tan \phi = \frac{g^\nu e}{3g^{\nu e}_L} \approx -0.28 \cdot (46)$$

When $O(\varepsilon^2)$ terms are negligible, a 0.7% measurement of $\sigma(\nu_\mu e)$ translates into a 95% confidence level bound of

$$\Lambda > (4.4 \text{ TeV}) \times \sqrt{|\cos(\theta - \phi)|} \quad (47)$$

from elastic scattering.

The measurement of the electron recoil energy will allow us to do better. Fig. 9 (darker line) depicts the 95% confidence level sensitivity of NuSoN to new heavy physics described by Eq. (43) when $\nu_\alpha = \nu_\mu$ (higher curve) and $\nu_\alpha \neq \nu_\mu$ (lower curve). (CLOSED CONTOURS) NuSoN measurement of $\Lambda$ and $\theta$, at the 95% level, assuming $\nu_\alpha = \nu_\mu$, $\Lambda = 3.5$ TeV and $\theta = 2\pi/3$ (higher, solid contour) and $\nu_\alpha \neq \nu_\mu$, $\Lambda = 1$ TeV and $\theta = 4\pi/3$ (lower, dashed contour). Note that in the pseudoelastic scattering case ($\nu_\alpha \neq \nu_\mu$) $\theta$ and $\pi + \theta$ are physically indistinguishable.

![FIG. 9: (DARK LINES) 95% confidence level sensitivity of NuSoN to new heavy physics described by Eq. (43) when $\nu_\alpha = \nu_\mu$ (higher curve) and $\nu_\alpha \neq \nu_\mu$ (lower curve). (CLOSED CONTOURS) NuSoN measurement of $\Lambda$ and $\theta$, at the 95% level, assuming $\nu_\alpha = \nu_\mu$, $\Lambda = 3.5$ TeV and $\theta = 2\pi/3$ (higher, solid contour) and $\nu_\alpha \neq \nu_\mu$, $\Lambda = 1$ TeV and $\theta = 4\pi/3$ (lower, dashed contour). Note that in the pseudoelastic scattering case ($\nu_\alpha \neq \nu_\mu$) $\theta$ and $\pi + \theta$ are physically indistinguishable.](image-url)
severely constrained by electron–electron scattering and rare muon and tau decays. One way around such constraints is to postulate that the operators in Eq. (43) are dimension-eight operators proportional to $LH^c \gamma_\alpha LH$, where $L$ is the left-chiral lepton doublet and $H$ is the Higgs scalar doublet. In this case, $1/\Lambda^2$ should be replaced by $v^2/\Lambda^4$, where $v = 246$ GeV is the scale of electroweak symmetry breaking.

Finally, another concern is whether modifications to the charged current neutrino–electron (pseudo)quasi-elastic scattering (i.e., (pseudo)IMD, $\nu_\mu e \rightarrow \nu_\alpha \mu$) can render the translation of NuSOnG data into constraints or measurements of $\theta$ and $\Lambda$ less straightforward. This turns out not to be the case, since new physics contributions to $\nu_\mu e \rightarrow \nu_\alpha \mu$ are already very well constrained by precision studies of muon decay. Hence, given the provisos of the two previous paragraphs, Eq. (43) is expected to capture all “heavy” new physics effects in (pseudo)elastic neutrino electron scattering.

2. Neutrino-quark NSI

We next consider the $f = u, d$ case. The change in the parameters $g_L^2$ and $g_R^2$ (see Eqs. (29,30)) due to the NSI's is

$$
\Delta g_L^2 = 2g_{L\mu}^\nu \varepsilon_{\mu\nu} + 2g_{L\mu}^\nu \varepsilon_{\mu\nu},
\Delta g_R^2 = 2g_{R\mu}^\nu \varepsilon_{\mu\nu} + 2g_{R\mu}^\nu \varepsilon_{\mu\nu},
$$

so only these linear combinations are constrained. The bounds from NuTeV (rescaled to $1\sigma$ bounds from ref. [44]) are:

$$
|\varepsilon_{\mu\nu}| < 0.0053 \pm 0.0020 ,
|\varepsilon_{\mu\nu}| < 0.0043 \pm 0.0016 ,
|\varepsilon_{\mu\nu}| < 0.0035 ,
|\varepsilon_{\mu\nu}| < 0.0073 .
$$

These bounds are obtained by setting only one of the parameters be non-zero at a time. If NuSoN reduces the errors on the NuTeV measurement of $g_L^2$ and $g_R^2$ by a factor of 2, the $1\sigma$ bounds on the NSI parameters are similarly reduced:

$$
|\varepsilon_{\mu\nu}| < 0.001 ,
|\varepsilon_{\mu\nu}| < 0.0008 ,
|\varepsilon_{\mu\nu}| < 0.0002 ,
|\varepsilon_{\mu\nu}| < 0.0004 .
$$

In terms of a new physics scale defined as $\Lambda = 1/\sqrt{2} G_F \varepsilon$, these constraints range from $\Lambda > 3$ TeV to $\Lambda > 7$ TeV.

We note that neutrino-quark scattering will also be sensitive to NSIs which correct CC interactions. These interactions are not included in Eq. (39).

C. Neutrissimos, Neutrino Mixing and Gauge Couplings

![Figure 10: Potential constraint on $\epsilon_\mu$ and $\epsilon_e$ from NuSOnG](image)

In those classes of models which include moderately heavy electroweak gauge singlet (“neutrissimo”) states, with masses above 45 GeV, the mixing of the $SU(2)_L$-active neutrinos and the sterile states may lead to a suppression of the neutrino-gauge couplings. The resulting pattern of modified interactions is distinct from those of the previous section since they will also induce correlated shifts to the charged-current coupling. For example, Ref. [46] presents models with one sterile state per active neutrino flavor and intergenerational mixing among neutrinos. In these models the flavor eigenstates are linear combinations of mass eigenstates, and those mass eigenstates too heavy to be produced in final states result in an effective suppression of the neutrino-gauge boson coupling. This suppression may be flavor-dependent depending on the structure of the neutrino mixing matrix. If the mass matrix contains Majorana terms, such models permit both lepton flavor violation and lepton universality violation.

Neutrinos couple to the $W$ and the $Z$ through interactions described by:

$$
\mathcal{L} = \frac{g}{\sqrt{2}} W^-_\mu \bar{\ell}_L \gamma^\mu \nu_{\ell L} + \frac{g}{\sqrt{2}} W^+_\mu \bar{\nu}_{\ell L} \gamma^\mu \ell_L
$$
\[ + \frac{e}{2\sqrt{2}} Z_\mu \tilde{\nu}_L \gamma^\mu \nu_{\ell L}, \]  
where \( \ell = e, \mu, \tau \). If the neutrinos mix with gauge singlet states so that the 2\(SU(2)_L\) interaction eigenstate is a superposition of mass eigenstates \(\nu_{\ell,\text{light}}\) and \(\nu_{\ell,\text{heavy}}\)

\[ \nu_{\ell L} = \nu_{\ell,\text{light}} \cos \theta_\ell + \nu_{\ell,\text{heavy}} \sin \theta_\ell, \]  
then the interaction of the light states is given by

\[ \mathcal{L} = \left( \frac{g}{\sqrt{2}} W^-_\mu \tilde{\nu}_L \gamma^\mu \nu_{\ell,\text{light}} + \frac{g}{\sqrt{2}} W^+_\mu \tilde{\nu}_L \gamma^\mu \nu_{\ell,\text{light}} \right) \cos \theta_\ell + \left( \frac{e}{2\sqrt{2}} Z_\mu \tilde{\nu}_L \gamma^\mu \nu_{\ell,\text{light}} \right) \cos^2 \theta_\ell. \]  

Defining

\[ \epsilon_\ell = 1 - \cos^2 \theta_\ell, \]  
the shift in the Lagrangian due to this mixing is

\[ \delta \mathcal{L} = -\left( \frac{g}{\sqrt{2}} W^-_\mu \tilde{\nu}_L \gamma^\mu \nu_{\ell,\text{light}} + \frac{g}{\sqrt{2}} W^+_\mu \tilde{\nu}_L \gamma^\mu \nu_{\ell,\text{light}} \right) \frac{\epsilon_\ell}{2} - \left( \frac{e}{2\sqrt{2}} Z_\mu \tilde{\nu}_L \gamma^\mu \nu_{\ell,\text{light}} \right) \epsilon_\ell, \]  

where we have dropped the subscript “light” from the neutrino fields.

Lepton universality data from W decays and from charged current \(\pi, \tau\) and \(K\) decays [17] constraint differences \(\epsilon_\ell - \epsilon_j\). LEP/SLD and other precision electroweak data will impose additional constraints on \(\epsilon_\ell\) in combination with the oblique parameters, as will NuSOnG. A fit to all the charge current decay data and LEP/SLD with \(S, T, \epsilon_e\) and \(\epsilon_\mu\) yields

\[ S = -0.05 \pm 0.11, \]  
\[ T = -0.44 \pm 0.28, \]  
\[ \epsilon_e = 0.0049 \pm 0.0022, \]  
\[ \epsilon_\mu = 0.0023 \pm 0.0021. \]  

If we now included hypothetical data from NuSOnG, assuming NuSOnG achieves its precision goals and measures central values consistent with the Standard Model, we see the constraints on \(\epsilon_\mu\) and \(\epsilon_e\) are substantially improved. In this case, the fit yields

\[ S = 0.00 \pm 0.10, \]  
\[ T = -0.01 \pm 0.12, \]  
\[ \epsilon_e = 0.0030 \pm 0.0017, \]  
\[ \epsilon_\mu = 0.0001 \pm 0.0012. \]  

Fig. [10] shows the two dimensional cross section in the \(\epsilon_\ell - \epsilon_\mu\) plane of the four dimensional fit. The likelihood contours are 2D projections. Though not obvious from the figure, it is NuSOnG’s improved measurement of \(g_L^\mu\) which contributes the most to strengthening the bounds on the \(\epsilon_\ell\).

In models of this class lepton flavor violating decays such as \(\mu \rightarrow e\gamma\) impose additional constraints on products \(\epsilon_\ell \epsilon_j\). For example, the strong constraint from \(\mu \rightarrow e\gamma\) implies \(\epsilon_e \epsilon_\mu \approx 0\). This type of model has been proposed as a solution to the NuTeV anomaly. If we take only one of \(\epsilon_e\) or \(\epsilon_\mu\) to be nonzero (to respect the constraint from \(\mu \rightarrow e\gamma\)), the NuTeV value of \(g_L^\mu\) is accommodated in the fit by best-fit values of \(\epsilon\) that are large and positive and best-fit values of \(T\) are large and negative (consistent with a heavy Higgs).

D. Right-handed coupling of the neutrino to the Z

In the Standard Model, neutrino couplings to the W- and Z-bosons are purely left-handed. The fact that the neutrino coupling to the W-boson and an electron is purely left-handed is, experimentally, a well-established fact (evidence includes precision measurements of pion and muon decay, nuclear processes, etc.). By contrast, the nature of the neutrino coupling to the Z boson is, experimentally, far from being precisely established [50]. The possibility of a right-handed neutrino–Z-boson coupling is not included in the previous discussions, and is pursued separately in this subsection.

The best measurement of the neutrino coupling to the Z-boson is provided by indirect measurements of the invisible Z-boson width at LEP. In units where the Standard Model neutrino–Z-boson couplings are \(g_L^\nu = 0.5\), \(g_R^\nu = 0\), the LEP measurement [21] translates into 

\[ (g_L^\nu)^2 + (g_R^\nu)^2 = 0.2487 \pm 0.0010. \]

Note that this result places no meaningful bound on \(g_R^\nu\).

Precise, model-independent information on \(g_L^\nu\) can be obtained by combining \(\nu_\mu + e\) scattering data from CHARM II and LEP and SLD data. Assuming model-independent couplings of the fermions to the Z-boson, \(\nu_\mu + e\) scattering measures \(g_L^\nu = \sqrt{\rho}/2\), while LEP and SLD measure the left and right-handed couplings of the electron to the Z. The CHARM II result translates into

\[ |g_L^\nu| = 0.502 \pm 0.017 \]  

assuming that the charged-current weak interactions produce only left-handed neutrinos. In spite of the good precision of the CHARM II result (around 3.5%), a combination of all available data allows

\[ |g_R^\nu/g_L^\nu| \sim 0.4 \]  

at the two \(\sigma\) confidence level [50].

Significant improvement in our understanding of \(g_R^\nu\) can only be obtained with more precise measurements of \(\nu^+ e^-\) scattering, or with the advent of a new high intensity \(e^+ e^-\) collider, such as the ILC. By combining ILC running at the Z-boson pole mass and at \(\sqrt{s} = 170\) GeV, \(|g_R^\nu/g_L^\nu| \lesssim 0.3\) could be constrained at the two \(\sigma\) level after analyzing \(e^+ e^- \rightarrow \gamma +\text{missing energy events}\) [50].

Assuming that \(g_L^\nu\) can be measured with 0.7% uncertainty, Fig. [11] depicts an estimate of how precisely \(g_R^\nu\) could be constrained once NuSOnG “data” is combined with LEP data. Fig. [11](left) considers the hypothesis that the Standard Model expectations are correct. In this case, NuSOnG data would reveal that \(g_R^\nu/g_L^\nu\) is less than 0.2 at the two sigma level. On the other hand, if \(g_R^\nu/g_L^\nu = 0.25\) in good agreement with the current CHARM II and LEP data – NuSOnG data should reveal that \(g_R^\nu \neq 0\) at more than the two sigma level, as depicted
FIG. 11: Precision with which the right-handed neutrino–Z-boson coupling can be determined by combining NuSOnG measurements of \( g_R^\nu \) with the indirect determination of the invisible Z-boson width at LEP if (left) the \( \nu + e \) scattering measurement is consistent with the Standard Model prediction \( g_L^\nu = 0.5 \) and (right) the \( \nu + e \) scattering measurement is significantly lower, \( g_L^\nu = 0.485 \), but still in agreement with the CHARM II measurement (at the one sigma level). Contours (black, red) are one and two sigma, respectively. The star indicates the Standard Model expectation.

The capability of performing this measurement in other experiments has been examined. The NuSOnG measurement compares favorably, and complements, the ILC capabilities estimated in [50]. Ref [52] studied measurements using other neutrino beams, including reactor fluxes and beta beams. NuSOnG’s reach is equivalent to or exceeds the most optimistic estimates for these various neutrino sources.

V. SPECIFIC THEORETICAL MODELS AND EXPERIMENTAL SCENARIOS

If NuSOnG’s measurements agree with the SM within errors, we will place stringent constraints on new physics models; if they disagree, it will be a signal for new physics. In the latter case the availability of both DIS and ES channels will improve our ability to discriminate among new physics candidates. NuSOnG will also provide an important complement to the LHC. The LHC will provide detailed information about the spectrum of new states directly produced. However, measurements of the widths of these new states will provide only limited information about their couplings. NuSOnG will probe in multiple ways the couplings of these new states to neutrinos and to other SM particles. In this section we provide several case studies of NuSOnG sensitivity to specific models of new physics. These include several typical Z’ models, leptoquark models, models of R-parity violating supersymmetry, and models with extended Higgs sectors. We examine how these will affect \( \nu_\mu e \) ES and \( \nu_\mu N \) DIS at tree-level. Our list is far from exhaustive but serves to illustrate the possibilities. We summarize our contributions in Table V.

The opposite way to approach this problem is to ask: in the face of evidence for new Terascale Physics, how can we differentiate between specific models? NuSOnG has the potential to discover new physics through indirect probes, in the event that one or more of its measurements definitively contradicts SM predictions. We discuss several possible patterns of deviation of model-independent parameters from SM predictions and some interpretations in terms of particular models. This is presented in the context of various expectations for LHC to illustrate how NuSOnG enhances the overall physics program. Since the NuTeV reanalysis is ongoing, and since the ES constraints from CHARM-II are weak, it is prudent that we commit to no strong assumptions about the central value of the NuSOnG measurements but instead consider all reasonable outcomes.

A. Sensitivity in the Case of Specific Theoretical Models

We next consider the constraints imposed by the proposed NuSOnG measurements on explicit models of BSM physics. An explicit model provides relations among effective operators which give stronger and sometimes better-motivated constraints on new physics than is obtained from bounds obtained by considering effective op-
Contribution of NuSOnG Measurement

Typical $Z'$ \( (B-xL),(q-xu),(d+xu) \) at the level of, and complementary to, LEP II bounds.

Extended Higgs Sector at the level of, and complementary to $\tau$ decay bounds.

$\Lambda$-parity Violating SUSY Sensitivity to masses $\sim 2$ TeV at 95\% CL.

Intergenerational Leptoquarks with non-degenerate masses

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<table>
<thead>
<tr>
<th>Model</th>
<th>Contribution of NuSOnG Measurement</th>
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<tr>
<td>Typical $Z'$</td>
<td>At the level of, and complementary to, LEP II bounds.</td>
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<tr>
<td>Extended Higgs Sector</td>
<td>At the level of, and complementary to $\tau$ decay bounds.</td>
</tr>
<tr>
<td>$\Lambda$-parity Violating SUSY</td>
<td>Sensitivity to masses $\sim 2$ TeV at 95% CL.</td>
</tr>
<tr>
<td>Intergenerational Leptoquarks with non-degenerate masses</td>
<td>Accesses unique combinations of couplings.</td>
</tr>
</tbody>
</table>

TABLE VI: Summary of NuSOnG’s contribution in the case of specific models

<table>
<thead>
<tr>
<th>$\nu_{\mu L}, e_L, e_R$</th>
<th>$U(1)_{B-xL}$</th>
<th>$U(1)_{q+xu}$</th>
<th>$U(1)_{d+xu}$</th>
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<tr>
<td>$\nu_{\mu L}, e_L$</td>
<td>$-x$</td>
<td>$-1$</td>
<td>$x/3$</td>
</tr>
<tr>
<td>$e_R$</td>
<td>$-x$</td>
<td>$-(2+x)/3$</td>
<td>$-1/3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(-1+x)/3$</td>
</tr>
</tbody>
</table>

TABLE VII: Charges of $\nu_{\mu L}, e_L, e_R$ under 4 phenomenologically viable classes of $U(1)'$ symmetries. Each value of $x$ corresponds to a different $U(1)'$ symmetry that is considered.

If the $Z'$ mass is sufficiently large, its exchange is well-described at NuSOnG energies by the effective operator of Eq. (43). In this case, the new physics scale is related to the $Z'$ model by $\Lambda \sim M_{Z'}/g_{Z'}$, the ratio of the $Z'$ mass to its gauge-coupling. Further model-dependence shows up in the ratio of fermion charges under the $U(1)'$ symmetry associated with the $Z'$, and the presence of any $Z-Z'$ mixing. With reasonable theoretical assumptions, the absence of new sources of large flavor-changing neutral currents, the consistency of Yukawa interactions, and anomaly cancellation with a minimal number of exotic fermions, the number of interesting models can be reduced substantially, to four discrete families of generic $U(1)'$ models each containing one free parameter, $x$ \cite{54}. In Table VII, we indicate the charges of $\nu_{\mu L}, e_L, e_R$ under these families of $U(1)'$ symmetries.

Using the sensitivity of NuSOnG to the scale $\Lambda$ in $\nu_{\mu}$ scattering shown in Figure 9, we can bound the combination $M_{Z'}/g_{Z'}$ for the four families of $Z'$ models as a function of $x$. It is important to note that these bounds are competitive with the LEP-II bounds found in \cite{54}, which are based on $Z'$ decays to all fermions, not just electrons and neutrinos.

There are $Z'$ models which distinguish among generations and affect neutrino scattering. These will be probed by NuSOnG at the TeV scale \cite{55,56,57,58,59}. Among these, $B-3L_{\mu}$ was suggested as a possible explanation for the NuTeV anomaly \cite{60,61}, however, we show here that this is not the case. Nevertheless, it remains an interesting example to consider.

In the gauged $B-3L_{\mu}$, the $Z'$ modifies $\nu_{\mu}$N DIS. The exchange of the $Z'$ between the $\nu_{\mu}$ and the quarks induces operators with coefficients

$$\varepsilon_{\mu \mu} = \varepsilon_{\mu \mu}^R = \varepsilon_{dL} = \varepsilon_{dR}$$
which with coefficient
\[ \varepsilon^{\mu\ell}_{\mu\mu} = - \frac{1}{\sqrt{2} G_F} \frac{|\lambda_{\mu\mu}|^2}{M_h^2} , \quad \varepsilon^{\mu R}_{\mu\mu} = 0. \] (62)

From Eq. (42), the 95% bound is:
\[ \frac{M_h}{|\lambda_{\mu\mu}|} > 5.2 \text{ TeV}, \] (63)

competitive with current bound from \( \tau \)-decay of 5.4 TeV.

3. \( R \)-parity violating SUSY

Assuming the particle content of the Minimal Supersymmetric Standard Model (MSSM), the most general \( R \)-parity violating superpotential (involving only tri-linear couplings) has the form \[ |64| \]
\[ W_R = \frac{1}{2} \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k + \frac{1}{2} \lambda''_{ijk} \hat{U}_i \hat{D}_j \hat{D}_k , \] (64)

where \( \hat{L}_i, \hat{E}_i, \hat{Q}_i, \hat{D}_i, \) and \( \hat{U}_i \) are the left-handed MSSM superfields defined in the usual fashion, and the subscripts \( i, j, k = 1, 2, 3 \) are the generation indices. \( SU(2)_L \) gauge invariance requires the couplings \( \lambda_{ijk} \) to be anti-symmetric in the first two indices:
\[ \lambda_{ijk} = - \lambda_{jik} , \] (65)

The purely baryonic operator \( \hat{U}_i \hat{D}_j \hat{D}_k \) is irrelevant to neutrino scattering, so only the 9 \( \lambda_{ijk} \) and 27 \( \lambda'_{ijk} \) couplings are of interest.

From the LLE part of the Eq. (64) slepton exchange will contribute to \( \nu_\mu \rightarrow \nu_\mu \) ES at NuSOnG. These induce four-fermion operators appearing in Eq. (36) with corresponding coefficients
\[ \varepsilon^{\mu\ell}_{\mu\mu} = - \frac{1}{4 \sqrt{2} G_F} \sum_{k=1}^{3} \frac{|\lambda_{2ik}|^2}{M_{\hat{E}_k}^2} , \]
\[ \varepsilon^{\mu R}_{\mu\mu} = \frac{1}{4 \sqrt{2} G_F} \sum_{j=1}^{3} \frac{|\lambda'_{j12}|^2}{M_{\hat{D}_j}^2} . \] (66)

If we place bounds on the sleptons one at a time, then Eq. (42) translates to the 2\( \sigma \) bounds shown in Table VIII presented for masses of 100 GeV. To rescale to different masses, use \( \frac{M}{M_{\hat{E}_k}} \). This can be compared to current bounds Ref. \[ 65 \]. NuSOnG improves all of these bounds.

From the LQD part of Eq. (64), squark exchange will contribute to NC \( \nu_\mu \rightarrow \nu_\mu \) DIS and CC \( \nu_\mu \rightarrow N \) DIS. The resulting shifts in \( g_L^2 \) and \( g_R^2 \) are
\[ \delta g_L^2 = 2 \left[ g_L^2 \varepsilon^{\mu L}_{\mu\mu} - g_L^2 \varepsilon^{\mu c}_{\mu\mu} \right] , \]
\[ \delta g_R^2 = 2 \left[ g_R^2 \varepsilon^{\mu R}_{\mu\mu} - g_R^2 \varepsilon^{\mu c}_{\mu\mu} \right] , \] (67)

where
\[ \varepsilon^{\mu L}_{\mu\mu} = - \frac{1}{4 \sqrt{2} G_F} \sum_{k=1}^{3} \frac{|\lambda_{2ik}|^2}{M_{\hat{E}_k}^2} , \]
\[ \varepsilon^{\mu R}_{\mu\mu} = - \frac{1}{4 \sqrt{2} G_F} \sum_{j=1}^{3} \frac{|\lambda'_{j12}|^2}{M_{\hat{D}_j}^2} . \] (68)

2. Models with extended Higgs sectors

In the Zee \[ 62 \] and Babu-Zee \[ 63 \] models, an isosinglet scalar \( h^+ \) with hypercharge \( Y = +1 \) is introduced, which couples to left-handed lepton doublet \( \ell \) as
\[ \mathcal{L}_h = \lambda_{ab} \left( \overline{\ell} \sigma_{\mu\nu} i \sigma_2 \ell \right) h^+ + h.c. , \] (61)

where \( (ab) \) are flavor indices: \( a, b = e, \mu, \tau \). The exchange of a charged Higgs induces the effective operator from Eq. (36) which with coefficient
\[ \varepsilon^{\mu\ell}_{\mu\mu} = - \frac{1}{\sqrt{2} G_F} \frac{|\lambda_{\mu\mu}|^2}{M_h^2} , \quad \varepsilon^{\mu R}_{\mu\mu} = 0. \] (62)

From Eq. (42), the 95% bound is:
\[ \frac{M_h}{|\lambda_{\mu\mu}|} > 5.2 \text{ TeV}, \] (63)

competitive with current bound from \( \tau \)-decay of 5.4 TeV.
<table>
<thead>
<tr>
<th>Coupling</th>
<th>95% NuSOnG bound</th>
<th>current 95% bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{121}$</td>
<td>0.03</td>
<td>0.05 (V_{ud})</td>
</tr>
<tr>
<td>$\lambda_{122}$</td>
<td>0.04</td>
<td>0.05 (V_{ud})</td>
</tr>
<tr>
<td>$\lambda_{123}$</td>
<td>0.04</td>
<td>0.05 (V_{ud})</td>
</tr>
<tr>
<td>$\lambda_{231}$</td>
<td>0.05</td>
<td>0.07 (\tau decay)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coupling</th>
<th>95% NuSOnG bound</th>
<th>current 95% bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{121}$</td>
<td>0.05</td>
<td>0.06 (\pi decay)</td>
</tr>
<tr>
<td>$\lambda_{122}$</td>
<td>0.06</td>
<td>0.06 (\pi decay)</td>
</tr>
<tr>
<td>$\lambda_{123}$</td>
<td>0.06</td>
<td>0.06 (\pi decay)</td>
</tr>
<tr>
<td>$\lambda_{231}$</td>
<td>0.07</td>
<td>0.21 (D meson decay)</td>
</tr>
<tr>
<td>$\lambda_{231}$</td>
<td>0.07</td>
<td>0.45 (Z \rightarrow \mu^+\mu^-)</td>
</tr>
</tbody>
</table>

TABLE VIII: Potential bounds on the R-parity violating LLE (top) and LQD (bottom) couplings from NuSOnG, assuming that only one coupling is non-zero at a time for each set. All squark and slepton masses are set to 100 GeV. To obtain limits for different masses, rescale by \(\frac{M}{100 \text{ GeV}}\). Current bounds are from Ref. [69].

\[
\varepsilon_{\mu}^{dR} = -\frac{1}{4\sqrt{2} G_{F}} \sum_{j=1}^{3} \frac{|\lambda_{ijj}|^2}{M_{d_{L}}} ,
\]

\[
\varepsilon_{\mu}^{dL} = -\frac{1}{4\sqrt{2} G_{F}} \sum_{k=1}^{3} \frac{3 |\lambda_{i1k}|^2}{M_{d_{R}}} = -\varepsilon_{\mu}^{dL} , \quad (68)
\]

\[
\varepsilon_{\mu}^{c} = \frac{1}{4\sqrt{2} G_{F}} \sum_{j=1}^{3} \frac{|\lambda_{ijj}|^2}{M_{d_{L}}} ,
\]

The shifts in \(g_{L}^{2}\) and \(g_{R}^{2}\) are:

\[
\delta g_{L}^{2} = 2 (g_{L}^{\mu d} + g_{L}^{\nu d}) \varepsilon_{\mu}^{dL} ,
\]

\[
\delta g_{R}^{2} = 2 g_{R}^{\mu d} \varepsilon_{\mu}^{dL} + 2 g_{R}^{\nu d} \varepsilon_{\mu}^{dR} . \quad (70)
\]

Assuming the projected precision goals for NuSOnG on \(g_{L}^{2}\) and \(g_{R}^{2}\), and allowing only one of the couplings to be nonzero at a time, the 2\sigma bounds are given in Table VIII mass of 100 GeV, in all cases. To obtain limits for different masses, one simply rescales by \(\frac{M}{100 \text{ GeV}}\). NuSOnG’s measurements are competitive with \(\pi\) decay bounds, and improves the current bounds on the 221 and 231 couplings by factors of 3 and 5, respectively.

4. Intergenerational leptoquark models

Measurements of \(g_{L}^{2}\) and \(g_{R}^{2}\) are sensitive to leptoquarks. Because the exchange of a leptoquark can interfere with both \(W\) and \(Z\) exchange processes, we cannot use the limits on the NSI’s of Eq. (36), since we must also include the effects of the four-fermion operators associated with charged-current processes. Instead, the interactions of leptoquarks with ordinary matter can be described in a model-independent fashion by an effective low-energy Lagrangian as discussed in Refs. [66, 68] for generation-universal leptoquark couplings. For leptoquarks to contribute to \(\nu_{\mu}N\) DIS, they must couple second generation leptons to first generation quarks, so we use the more general Lagrangian of \([67, 69]\), which allows the coupling constants to depend on the generations of the quarks and leptons that couple to each leptoquark. We summarize the quantum numbers and couplings of the various leptoquarks fields in Table IX; our notation conventions are those of Ref. [69].

The four-fermion operators induced by leptoquark exchange will affect NC and/or CC processes, and at NuSOnG the effect manifests itself in shifts \(g_{L}^{2}\) and \(g_{R}^{2}\). Assuming degenerate masses within each iso-multiplet, the shifts in \(g_{L}^{2}\) and \(g_{R}^{2}\) can be written generically as

\[
\delta g_{L}^{2} = C_{L} \frac{|\lambda_{LQ}^{ij} 2/M_{LQ}|^2}{g_{L}^2/M_{W}^2} = C_{L} \frac{|\lambda_{LQ}^{ij} 2/M_{LQ}|^2}{4\sqrt{2} G_{F} M_{LQ}^2} ,
\]

\[
\delta g_{R}^{2} = C_{R} \frac{|\lambda_{LQ}^{ij} 2/M_{LQ}|^2}{g_{R}^2/M_{W}^2} = C_{R} \frac{|\lambda_{LQ}^{ij} 2/M_{LQ}|^2}{4\sqrt{2} G_{F} M_{LQ}^2} , \quad (71)
\]

where \(\lambda_{LQ}^{ij}\) denotes the \((ij) = (12)\) coupling of the leptoquark and \(M_{LQ}\) is its mass. In Table X, we list what they are, and in figure 14 we plot the dependence of \(\delta g_{L}^{2}\) and \(\delta g_{R}^{2}\) on the ratio \(|\lambda_{LQ}^{ij} 2/M_{LQ}|^2\). Table X also lists the projected NuSOnG bounds on the coupling constants [70]. Existing bounds on \(S_{1}, S_{3}, V_{1}\), and \(V_{3}\) couplings from \(R_{\bar{\nu}} = Br(\pi \rightarrow e\nu)/Br(\pi \rightarrow \mu\nu)\) are already much stronger, but could be circumvented for \(S_{3}\) and \(V_{3}\) if the masses within the multiplet are allowed to be non-degenerate.

B. Interplay with LHC to Isolate the Source of New Physics

By the time NuSOnG runs, the LHC will have accumulated a wealth of data and will have begun to change the particle physics landscape. The message from LHC data may be difficult to decipher, however. As discussed below, NuSOnG will be able to help elucidate the new physics revealed at the LHC. The discovery of a Higgs along with the anticipated measurement of the top mass to 1 GeV precision would effectively fix the center of the ST plot and will enhance the power of the precision electroweak data as a tool for discovering new physics. If additional resonances are discovered at the LHC, it is still likely that little will be learned about their couplings.

The NuSOnG experiment provides complementary information to LHC. Rather than generalize, to illustrate the power of NuSOnG, two specific examples are given here. We emphasize that these are just two of a wide range of examples, but they serve well to demonstrate the point. Here we have chosen examples from typical new physics models other than \(Z'\) models which were discussed above, in order to demonstrate the physics range which can be probed by NuSOnG.
<table>
<thead>
<tr>
<th>Leptoquark</th>
<th>Spin</th>
<th>$F$</th>
<th>SU(3)$_C$</th>
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<th>$Y$</th>
<th>$Q_{em}$</th>
<th>Allowed Couplings</th>
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</thead>
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<tr>
<td>$S_1$</td>
<td>$S_1^+$</td>
<td>0</td>
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<td>$3$</td>
<td>0</td>
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<td>$g_{1L}(u_L^c e_L - d_L^c e_L), g_{1R}(u_R^c e_R)$</td>
</tr>
<tr>
<td>$S_1$</td>
<td>$S_1^-$</td>
<td>0</td>
<td>$-2$</td>
<td>$3$</td>
<td>0</td>
<td>$\frac{i}{2}$</td>
<td>$g_{1L}(d_R^c e_R)$</td>
</tr>
<tr>
<td>$V_{2\mu}$</td>
<td>$V_{2\mu}^+$</td>
<td>1</td>
<td>$-2$</td>
<td>$3$</td>
<td>$\frac{1}{2}$</td>
<td>$-\frac{1}{2}$</td>
<td>$g_{2L}(d_R^c e_L), g_{2R}(d_R^c e_R)$</td>
</tr>
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<td>$V_{2\mu}^-$</td>
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<td>$-2$</td>
<td>$3$</td>
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<td>$-\frac{1}{2}$</td>
<td>$g_{2L}(u_R^c e_R)$, $g_{2R}(u_R^c e_R)$</td>
</tr>
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<td>$S_3$</td>
<td>$S_3^+$</td>
<td>0</td>
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<td>$+1$</td>
<td>$\frac{i}{3}$</td>
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<td>$3$</td>
<td>$+1$</td>
<td>$\frac{i}{3}$</td>
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<tr>
<td>$S_2$</td>
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<td>$0$</td>
<td>$3$</td>
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<td>$h_{2L}(u_R^c e_L), h_{2R}(u_R^c e_R)$</td>
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<td>$0$</td>
<td>$3$</td>
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<td>$\frac{i}{3}$</td>
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<td>1</td>
<td>$0$</td>
<td>$3$</td>
<td>$0$</td>
<td>$\frac{i}{2}$</td>
<td>$h_{1L}(u_R^c e_L) + d_R^c e_L), h_{1R}(d_R^c e_R)$</td>
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<td>$0$</td>
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<td>$h_{3L}(u_R^c e_L) - d_R^c e_L), \sqrt{2}h_{3L}(d_R^c e_L)$</td>
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<td>$0$</td>
<td>$3$</td>
<td>$+1$</td>
<td>$\frac{i}{3}$</td>
<td>$\sqrt{2}h_{3L}(d_R^c e_L)$</td>
</tr>
</tbody>
</table>

Table IX: Quantum numbers of scalar and vector leptoquarks with SU(3)$_C \times$ SU(2)$_L \times$ U(1)$_Y$ invariant couplings to quark-lepton pairs ($Q_{em} = I_S + Y$). \[72\]

| LQ  | $C_L$  | $C_R$  | $|\lambda_{LQ}|^2$ | NuSOnG 95% bound | 95% bound from $R_\pi$ |
|-----|-------|-------|-----------------|-------------------|---------------------|
| $S_1$ | $s^2 (\frac{1}{2} - \frac{3}{2} s^2)$ | $-\frac{1}{2} s^2$ | $g_{1L}^2$ | 0.0036 | 0.0037 |
| $S_3$ | $+\frac{1}{2} s^3$ | $+\frac{3}{2} s^3$ | $g_{1L}^2$ | 0.010 | 0.0008 |
| $S_3$ | $-\frac{1}{2} s^2$ | $\frac{1}{2} s^2$ | $g_{1L}^2$ | 0.004 | N/A |
| $V_3$ | $s^2 (\frac{3}{2} - \frac{3}{2} s^2)$ | $-\frac{3}{2} s^2$ | $g_{1L}^2$ | 0.0044 | N/A |
| $V_3$ | $-4 s^2 (1 - \frac{3}{2} s^2)$ | $+4 s^2 (1 - \frac{3}{2} s^2)$ | $g_{1L}^2$ | 0.0011 | 0.0004 |
| $V_2$ | $0$ | $-\frac{1}{2} s^2$ | $g_{1L}^2$ | 0.0026 | N/A |
| $V_2$ | $0$ | $\frac{3}{2} s^2$ | $g_{1L}^2$ | 0.0013 | N/A |

Table X: Potential and existing 95% bounds on the leptoquark couplings squared when the leptoquark masses are set to 100 GeV. To obtain the limits for different leptoquark masses, multiply by $(M_{LQ}/100\text{GeV})^2$. Existing bounds on the $S_1$, $S_3$, $V_1$, and $V_3$ couplings from $R_\pi = Br(\pi \rightarrow e\nu)/Br(\pi \rightarrow \mu\nu)$ are also shown.

First, extend the Standard Model to include a non-degenerate SU(2)$_L$ triplet leptoquark ($\tilde{S}_3$ or $\tilde{V}_3$) in the notation of [69], with masses in the 0.5-1.5 TeV range. At the LHC these leptoquarks will be produced primarily in pairs through gluon fusion, and each leptoquark will decay to a lepton and a jet [72]. The peak in the lepton-jet invariant mass distribution will be easily detected over background. This will provide the leptoquark masses but yield little information about their couplings to fermions. The leptoquarks will also shift the neutrino-nucleon effective coupling $g^2_{\nu}$ in a way that depends sensitively on both the leptoquark couplings and masses. Such a leptoquark-induced shift could provide an explanation for the NuTeV anomaly [61, 67, 73]. In this scenario, NuSOnG would find that isospin and the strange sea can be constrained to the point that they do not provide an explanation for the NuTeV anomaly, thus the NuTeV anomaly is the result of new physics. The NuSOnG PW measurement of sin2$\theta_W$ will agree with NuTeV; $g^2_{\nu}$ and the $\nu e$ and $\nu \bar{e}$ elastic scattering measurements will agree with LEP. Fig. 13 illustrates this example. NuSOnG’s measurement of $g^2_{\nu}$ would provide a sensitive measurement of the leptoquark couplings when combined with the LHC mass measurements as inputs.

A second example is the existence of a fourth generation family. A fourth family with non-degenerate masses (i.e. isospin violating) is allowed within the LEP/SLD constraints [49]. As a model, we choose a fourth family with mass splitting on the order of $\sim 75$ GeV and a 300 GeV Higgs. This is consistent with LEP at $1\sigma$ and perfectly consistent with $M_W$, describing the point (0.2,0.19) on the ST plot. In this scenario, LHC will measure the Higgs mass from the highly enhanced $H \rightarrow ZZ$ decay. An array of exotic decays which will be difficult to fully reconstruct, such as production of 6 W’s and 2 b’s, will be observed at low rates. In this scenario, isospin
violation explains the NuTeV anomaly, thus the NuTeV PW and the NuSOnG PW measurements agree with the $\nu e$ES measurements. These three precision neutrino results, all with “LEP-size” errors, can be combined and will intersect the one-sigma edge of the LEP measurements. Fig. 16 illustrates this example. From this, the source, a fourth generation with isospin violation, can be demonstrated.

Lastly, while it seems unlikely, it is possible that LHC will observe a Standard Model Higgs and no signatures of new physics. If this is the case, it is still possible for NuSOnG to add valuable clues to new physics. This is because the experiment is uniquely sensitive to the neutrino sector. If a situation such as is illustrated on Fig. 17 arose, the only explanation would be new physics unique to neutrino interactions.

VI. SUMMARY AND CONCLUSIONS

NuSOnG is an experiment which can search for new physics from keV through TeV energy scales, as well as make interesting QCD measurements. This article has focussed mainly on the Terascale physics which can be accessed through this new high energy, high statistics
neutrino scattering experiment. The case has been made that this new neutrino experiment would be a valuable addition to the presently planned suite of experiments with Terascale reach.

The NuSOnG experiment design draws on the heritage of the CHARM II and CCFR/NuTeV experiments. A high energy, flavor-pure neutrino flux is produced using 800 GeV protons from the Tevatron. The detector consists of four modules, each composed of a finely-segmented glass-target (SiO$_2$) calorimeter followed by a muon spectrometer. In its five-year data acquisition period, this experiment will record almost one hundred thousand neutrino-electron elastic scatters and hundreds of millions of deep inelastic scattering events, exceeding the current world data sample by more than an order of magnitude. This experiment can address concerns related to model systematics of electroweak measurements in neutrino-quark scattering by direct constraints using in-situ structure function measurements.

NuSOnG will be unique among planned experiments for its ability to probe neutrino couplings to Beyond Standard Model physics. This experiment offers four distinct and complementary probes of $S$ and $T$. Two are of high precision with the proposed run-plan, and the precision of the other two would be improved by a follow-up five-year antineutrino run. Neutrino-lepton non-standard interactions can be probed with an order of magnitude improvement in the measured effective couplings. Neutrino-quark non-standard interactions can be probed by an improvement in the measured neutrino-quark effective couplings of a factor of two or better. The experiment is sensitive to new physics up to energy scales $\sim 5$ TeV at 95% CL. The measurements are sensitive to universality of the couplings and an improvement in the $c$-family of 30% and $\mu$-family of 75% will allow for probes of neutrissimos. As a unique contribution, NuSOnG measures $g_R/g_L$, which is not accessible by other near-future experiments. This article described NuSOnG’s physics contribution under several specific models. These included models of $Z'$s, extended Higgs models, leptoquark models and $R$-parity violating SUSY models. We also considered how, once data are taken at LHC and NuSOnG, the underlying physics can be extracted. The opportunity for direct searches related to these indirect electroweak searches was also described. The conclusion of our analysis is that a new neutrino experiment, such as NuSOnG, would substantially enhance the presently planned Terascale program.

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