Diameter Dependence of the Dielectric Constant for the Excitonic Transition Energy of Single-Wall Carbon Nanotubes

The MIT Faculty has made this article openly available. Please share how this access benefits you. Your story matters.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>As Published</td>
<td><a href="http://dx.doi.org/10.1103/PhysRevLett.103.146802">http://dx.doi.org/10.1103/PhysRevLett.103.146802</a></td>
</tr>
<tr>
<td>Publisher</td>
<td>American Physical Society</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Accessed</td>
<td>Mon Dec 31 02:38:35 EST 2018</td>
</tr>
<tr>
<td>Citable Link</td>
<td><a href="http://hdl.handle.net/1721.1/51761">http://hdl.handle.net/1721.1/51761</a></td>
</tr>
<tr>
<td>Terms of Use</td>
<td>Article is made available in accordance with the publisher’s policy and may be subject to US copyright law. Please refer to the publisher’s site for terms of use.</td>
</tr>
<tr>
<td>Detailed Terms</td>
<td></td>
</tr>
</tbody>
</table>
Diameter Dependence of the Dielectric Constant for the Exciton Transition Energy of Single-Wall Carbon Nanotubes

P. T. Araujo, A. Jorio, M. S. Dresselhaus, K. Sato, and R. Saito

Departamento de Física, Universidade Federal de Minas Gerais, Belo Horizonte, MG, 30123-970 Brazil

Divisão de Metrologia de Materiais, Instituto Nacional de Metrologia, Normalização e Qualidade Industrial (INMETRO), Duque de Caxias, RJ, 25250-020, Brazil

Department of Electrical Engineering and Computer Science, and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139-4307, USA

Department of Physics, Tohoku University, Sendai 980-8578, Japan

(Received 14 April 2009; published 30 September 2009)

The measured optical transition energies $E_{ii}$ of single-wall carbon nanotubes are compared with bright exciton energy calculations. The $E_{ii}$ differences between experiment and theory are minimized by considering a diameter-dependent dielectric constant $\kappa$, which comprises the screening from the tube and from the environment. Different $\kappa$ dependencies are obtained for $(E_{11}^0, E_{22}^0, E_{11}^M)$ relative to $(E_{33}^0, E_{44}^0)$. A changing environment changes the $\kappa$ diameter dependence for $(E_{11}^0, E_{22}^0, E_{11}^M)$, but for $(E_{33}^0, E_{44}^0)$ the environmental effects are minimal. The resulting calculated exciton energies reproduce experimental $E_{ii}$ values within $\pm 70$ meV for a diameter range ($0.7 < d_t < 3.8$ nm) and $1.2 < E_i < 2.7$ eV, thus providing a theoretical justification for $E_{ii}$, environmental effects and important insights on the dielectric screening in one-dimensional structures.

DOI: 10.1103/PhysRevLett.103.146802 PACS numbers: 73.22.-f, 61.48.De, 78.20.-e, 78.67.Ch

The last decade has been marked by an impressive development in understanding the nature of the optical transition energies in quasi-one-dimensional single-wall carbon nanotubes (SWNTs) [1], called $E_{ii}$, where $i = 1, 2, 3, \ldots$ denotes the intersubband transitions between the $i$th valence and the $i$th conduction band for a given SWNT. While the interest in excitons and dielectric screening in one-dimensional structures dates from research in $\pi$-conjugated polymers, in carbon nanotubes the large attention started in 2003 with the so-called “ratio problem” [2]. Strong debate still exists about the strength of the exciton binding energy, mostly related to the complex dielectric screening in one-dimensional materials. In 2007, Araujo et al. [3] and Michel et al. [4] showed that the scaling law for the exciton energies explaining the ratio problem [2] breaks down for transitions higher than $E_{11}^M$. These results lead to the discussion of the exciton nature for higher energy levels, where quantum-chemistry calculations and solid-state physics (tight binding and first-principles) calculations give contradictory pictures [3,5]. Now, the accumulated knowledge in SWNTs, both theoretical and experimental [1], makes it possible to accurately determine the effect of the general dielectric constant $\kappa$ on $E_{ii}$. By “general” we mean $\kappa$ comprises the screening from the tube and from the environment. In this work we show a $d_t$-dependent effective $\kappa$ values for the exciton calculation that are needed to reproduce the experimental $E_{ii}$ values consistently. The results thus obtained are important for the physics of quasi and truly one-dimensional materials and can be used in interpreting optical experiments and environment effects.

Figure 1 shows a map of previously reported experimental $E_{ii}$ values (black dots) [7–9] from a SWNT sample grown by the water-assisted (“super-growth”) chemical vapor deposition method [10–13]. The resulting data for the $E_{ii}$ transition energies are plotted as a function of the radial breathing mode frequencies $\omega_{RBM}$, as obtained by resonance Raman spectroscopy (RRS) [7–9,14]. This sample was chosen for the initial analysis developed in this paper because the sample has a homogeneous environment and a large variety of SWNT diameters, as measured by RRS. Furthermore, this sample shows the following features: (i) Experimental observation of the fundamental relation $\omega_{RBM} = 227/d_t$, for which the constant term $B$ is zero in the relation $\omega_{RBM} = A/d_t + B$. The constant term $B$ comes from the interaction of SWNTs with their environment, which is negligible for the present low-density SWNT example [7]. (ii) The highest $E_{ii}$ values are obtained for these SWNTs when compared to other samples in the literature [8]. This result indicates that the tubes are surrounded by the lowest environmental dielectric constant ($\kappa_{env} \rightarrow 1$) reported in the literature [1,3,4,15–21]. (iii) The previously elusive high energy...
E\text{ii} of samples in the literature. In Fig. 1, the experimental values of perturbation for metallic tubes as compared to other compared with the calculated bright exciton energies observed energy-dependent E\text{ii} vs \omega\text{RBM} curves. Due to time consumption, only 1, 2, 3, 4 and 5 stands for metallic SWNTs.

E\text{ii} transitions [18–20] can be observed for metallic SWNTs in these samples [14], indicating the lowest degree of perturbation for metallic tubes as compared to other samples in the literature. In Fig. 1, the experimental values of E\text{ii} vs \omega\text{RBM} for the super-growth sample E\text{ii}\text{exp} are compared with the calculated bright exciton energies E\text{ii}\text{cal} (open circles and stars). Although E\text{ii}\text{cal} include SWNT curvature and many-body effects [6], clearly the E\text{ii}\text{exp} values are redshifted when compared with theory, and the redshift depends on \omega\text{RBM}, i.e., on d, and on the optical levels (i in E\text{ii}).

The E\text{ii} values can be renormalized in the calculation by explicitly considering the dielectric constant \kappa in the coulomb potential energy V(q)/\varepsilon(q)\kappa [22]. Here, \kappa represents the screening of the e-h (electron-hole) pair by core (1s) and \sigma electrons (\kappa\text{tube}) and by the surrounding materials (\kappa\text{env}). Here \varepsilon(q) explicitly gives the polarization function for \pi-electrons calculated within the random phase approximation (RPA) [6,22,23]. To fully account for the observed energy-dependent E\text{ii} redshift, we fit the total \kappa values (1/\kappa = C_{\text{env}}/\kappa_{\text{env}} + C_{\text{tube}}/\kappa_{\text{tube}}) to minimize E\text{ii}\text{exp} - E\text{ii}\text{cal}. The bullets in Fig. 2 show the fitted \kappa values as a function of p/d, which reproduce each experimental E\text{ii} value for the assigned (n, m) SWNTs for the super-growth SWNT sample. The stars stand for a different SWNT sample, named “alcohol-assisted” SWNTs [24], and they will be discussed below in the next paragraph. The integer p corresponds to the distance ratio of the cutting lines from the K point, where p = 1, 2, 3, 4 and 5 stands for E\text{ii}^\text{cal} with \omega\text{RBM} = 227 d, [7]. The dashed and dotted curves are given by Eq. (1) with C\text{cal} = 0.75 and 1.02, respectively. (b) E\text{ii}^\text{cal} (black) and E\text{ii}^\text{cal} (dark gray). The dashed curve is for Eq. (1) with C\text{cal} = 0.49.

To gain more insight into the \kappa influence on the optical transition energies, Fig. 3 shows a comparison between the E\text{ii}\text{exp} from the super-growth SWNT sample (bullets) [8] and

FIG. 2. The calculated \kappa, which are fitted to the experimental E\text{ii} values from the super-growth (bullets) [8,9] and alcohol-assisted (stars) [3,9] samples. (a) E\text{ii}^S (black) and E\text{ii}^M (dark gray). The dashed and dotted curves are given by Eq. (1) with C\text{cal} = 0.75 and 1.02, respectively. (b) E\text{ii}^S (black) and E\text{ii}^S (dark gray). The dashed curve is for Eq. (1) with C\text{cal} = 0.49.

FIG. 3. E\text{ii}\text{exp} vs \omega\text{RBM} results obtained for the super-growth (bullets) [8,9] and alcohol assisted (open circles) [3,9] SWNT samples.
from the “alcohol-assisted” SWNT samples (open circles) [3]. The alcohol-assisted SWNT sample is chosen for comparison for three reasons: (i) this sample also has a broad diameter distribution (0.7 < d < 2.3 nm); (ii) the observed \( E_{ii} \) are similar to many other samples in the literature [8]; (iii) the sample is morphologically similar to the super-growth sample (both are carpetlike free standing SWNTs). From Fig. 3, we see that the \( E_{ii}^{exp} \) values from the “alcohol-assisted” SWNTs are generally redshifted with respect to those from the super-growth SWNTs. Assuming that \( \kappa_{tub} \) does not change from sample to sample, since the structure of a given \((n, m)\) tube should be the same, these results indicate that the alcohol-assisted SWNTs are surrounded by a larger \( \kappa_{env} \) value, thus increasing the effective \( \kappa \) and decreasing \( E_{ii} \).

Looking at Fig. 2 we can observe the difference in the \( \kappa \) values resulting from fitting the \( E_{ii}^{exp} \) to the super-growth (bullets) in comparison to alcohol-assisted (stars) SWNT samples. For \( E_{22}^S \) and \( E_{11}^M \) [Fig. 2(a)], we see a clear difference for \( \kappa \) up to \( p = 3 \) when comparing the two samples. However, for \( E_{33}^S \) and \( E_{44}^S \) [Fig. 2(b)], no difference in \( \kappa \) between the two samples can be seen. This means that the electric field of the \( E_{33}^S \) and \( E_{44}^S \) excitons do not extend much outside the SWNT volume, in contrast to the \( E_{22}^S \) and \( E_{11}^M \) excitons for which the \( \kappa_{env} \) effect is significant. Since the effect of \( \kappa_{env} \) is relatively small for energies above \( E_{11} \), it is possible to assign the \((n, m)\) values from \( E_{33} \) and \( E_{44} \) if the dielectric constant of the environment is not known, even though the \( E_{33}^S \) and \( E_{44}^S \) values are seen within a large density of dots in the Kataura plot. In constructing Fig. 2, we used the relation

\[
\kappa = C_\kappa \left( \frac{p}{d} \right)^{1.7},
\]

where the exponent 1.7 was found to work for all \( E_{ii}^{exp} \), but different \( C_\kappa \) parameters are needed. For \( E_{11}^S \), \( E_{22}^S \) and \( E_{11}^M \), \( C_\kappa = 0.75 \) for the super-growth SWNTs and \( C_\kappa = 1.02 \) for the alcohol-assisted SWNTs (dashed and dotted curves in Fig. 2(a), respectively). The \( E_{33}^S \) and \( E_{44}^S \) are fitted using \( C_\kappa = 0.49 \) for both samples, as shown by the dashed line in Fig. 2(b).

Figure 4 summarizes the effect of our \( \kappa \)-based renormalization of \( E_{ii} \). In Fig. 4 we plot the energy difference \( \Delta E_{ii} = E_{ii}^{exp} - E_{ii}^{cal} \) as a function of \( d \), for (a) \( \kappa \) values fixed to be equal to 1 \((\kappa = 1)\), (b) \( \kappa \) values fixed to be equal to 2.22 \((\kappa = 2.22)\), broadly used in the literature [1] and (c) the diameter-dependent \( \kappa \) values using the function of Eq. (1), including both the super-growth SWNTs and the alcohol-assisted SWNTs. From Figs. 4(a) and 4(b) we observe that the fitting results using fixed values for \( \kappa \) are not good enough from a physical point of view, because the data in these figures are spread over a window of about 400 meV, which is greater than the observed resonance window for a single \((n, m)\) tube [1]. Besides, the spread of the \( \Delta E_{ii} \) points in Figs. 4(a) and 4(b) increases for small \( d \) tubes. To address this physical inconsistency, we analyze the energy difference \( \Delta E_{ii} = E_{ii}^{exp} - E_{ii}^{cal} \), where \( E_{ii}^{cal} \) is now calculated by considering \( \kappa \) given by Eq. (1). When the resulting \( \Delta E_{ii} \) is plotted vs \( d \), for all \( E_{ii} \) \((p = 1, \ldots, 5)\), using the \( C_\kappa \) parameters given in the text, all the \( \Delta E_{ii} \) in Fig. 4(c) deviate from zero by less than 70 meV, for both the super-growth and the “water-assisted” SWNT samples, over the entire \( d \) region.

Qualitatively, the origin of the diameter dependence presented by Eq. (1) consists of: (i) the exciton size and (ii) the amount of electric field “feeling” the dielectric constant of the surrounding material. These two factors are connected and the development of an electromagnetism model is needed to fully rationalize this equation. Interestingly, the similarity between the \( \kappa \) values found for \( E_{22}^S \) and \( E_{11}^M \) shows that the difference between metallic and semiconducting tubes is satisfactorily accounted by the
RPA in \( e(q) \) [5]. Also interesting is the different \( \kappa \) behavior observed for higher levels \((p > 3)\), where \( C_\kappa \) is smaller than for \( E_{ii} \) with \( p \leq 3 \), and it is independent of the sample environment. Again, two pictures can be given: (1) the more localized exciton wave function (a larger exciton binding energy) for \( E^{i}_{33} \) and \( E^{i}_{44} \) compared with \( E^{ii}_{11} \) and \( E^{ii}_{22} \), leads to smaller \( \kappa \) and a lack of a \( \kappa_{\text{env}} \) dependence of the wave functions for the \( E^{i}_{33} \) and \( E^{i}_{44} \) excitons; (2) stronger tube screening \((\kappa_{\text{tube}})\) leads to an independence in \( \kappa_{\text{env}} \) and, consequently, a smaller effective \( \kappa \).

In summary, we have shown that the diameter-dependent dielectric constants following Eq. (1) reproduce the measured \( E_{ii} \) values well for a large region of energy \((1.2-2.7 \text{ eV})\) and tube diameter \((0.7-3.8 \text{ nm})\). The present treatment for \( \kappa \) is sufficiently accurate for assigning both the \( 2n + m \) family numbers and the \((n, m)\) SWNTs belonging to each family for different SWNT samples, for the supergrowth and alcohol-assisted samples used for establishing the model system of this Letter. All the observed \( E_{ii} \) vs \((n, m)\) values are now theoretically described within experimental precision, considering use of the extended-tight binding model along with many-body corrections plus a diameter-dependent dielectric constant \( \kappa \), as established in the present work [Eq. (1)]. The empirical exponent 1.7 is not yet fully understood, and theoretical modeling considering the role of effective mass is under way. The results presented here are also consistent with the previously published empirical methodology of Ref. [3], and therefore provide the theoretical justification for it.

Having justified this empirical methodology, the findings of the present work have the following implications on taking account of environmental effects for fitting the measured \( d_i \) dependence of \( \omega_RBM \) for an arbitrary experimental sample. Using the results of the empirical methodology of Ref. [3], the measured \( \omega_RBM \) for an arbitrary experimental sample is fitted to the formula

\[
\omega_RBM = \left( \frac{227}{d_i} \right) \sqrt{1 + C_\epsilon \times d_i^2}, \tag{2}
\]

where only a single parameter \( (C_\epsilon) \) is used to account for the observed \( d_i \)-dependent environmental effects. For the case of the super-growth SWNTs we have a fit to the \( E_{ii}^{\exp} \) data for \( C_\epsilon = 0 [7] \), while for the alcohol-assisted sample, the fit is accomplished by \( C_\epsilon = 0.057 \). However, for an arbitrary sample measured in the laboratory, a least squares fit is made by the researchers themselves to determine the best fit value for \( C_\epsilon \) in Eq. (2) to their experimental data. In this way the results of this paper can be extended to yield a fit of \( \omega_RBM \) to arbitrary samples in a metrological sense, based on a single fitting parameter \( C_\epsilon \) to account for the experimental environmental effect measured for their sample by the resonance Raman effect.

P.T.A. and A.J. acknowledge financial support by FAPEMIG, Rede de pesquisa em nanotubos de carbono MCT/CNPq and AFOSR. M. S. D. acknowledges support from NSF grant DMR07-04197. K. S. is supported by a JSPS research grant (No. 20-4594). R. S. acknowledges support from MEXT Grants (No. 20241023).


[9] See EPAPS Document No. E-PRLTAO-103-022942 for an example of the resonant Raman spectral profile of the carpetlike water-assisted CVD grown SWNTs ranging from 65 cm\(^{-1}\) to 4550 cm\(^{-1}\). In addition, file Epaps-Araujo-EiiKappaTable.xls brings the experimental \( \omega_RBM \), \( E_{ii} \) and the \( \kappa \) values fitting \( E_{ii} \) for the samples analyzed in our Letter. For the alcohol-assisted sample we present experimental results for 84 different \((n, m)\) SWNTs, including \( E_{ii} \) values for \( E^{33}_{22} \), \( E^{33}_{11} \) (lower branch), \( E^{44}_{33} \) and \( E^{44}_{44} \). For the water-assisted sample we present experimental values for 176 different \((n, m)\) SWNTs, including \( E_{ii} \) values for \( E^{33}_{22} \), \( E^{44}_{33} \), \( E^{44}_{44} \), \( E^{33}_{11} \) (lower branch), \( E^{33}_{11} \) (higher branch), \( E^{44}_{33} \), \( E^{44}_{44} \) (lower branch), \( E^{44}_{33} \) (higher branch), \( E^{44}_{44} \). For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.


