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Qubit Protection in Nuclear-Spin Quantum Dot Memories

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We present a mechanism to protect quantum information stored in an ensemble of nuclear spins in a semiconductor quantum dot. When the dot is charged the nuclei interact with the spin of the excess electron through the hyperfine coupling. If this coupling is made off-resonant, it leads to an energy gap between the collective storage states and all other states. We show that the energy gap protects the quantum memory from local spin-flip and spin-dephasing noise. Effects of nonperfect initial spin polarization and inhomogeneous hyperfine coupling are discussed.

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An essential ingredient for quantum computation and long-distance quantum communication is a reliable quantum memory. Nuclear spins in semiconductor nanostructures are excellent candidates for this task. With a magneton 3 orders of magnitude weaker than electron spins, they are largely decoupled from their environment, and the hyperfine interaction with electron spins allows one to access ensembles of nuclear spins in a controlled way [1–10]. In particular, the quantum state of an electron spin can be mapped onto the nuclear spins, giving rise to a long-term memory [3–7]. Nevertheless, memory lifetimes are limited, e.g., by dipole-dipole interactions among the nuclei. In this Letter, we demonstrate that the presence of the electron spin in the quantum dot substantially reduces the decoherence of this collective memory associated with surrounding nuclear spins. The virtual transitions between electronic and nuclear states can be used to produce an energy shift proportional to the number of excitations in the storage spin-wave mode. This isolates the storage states energetically and protects them against nuclear-spin flips and spin diffusion.

Consider a quantum dot charged with a single excess electron as indicated in Fig. 1. The electron spin \( \hat{S} \) is coupled to the ensemble of underlying nuclear spins \( \hat{I} \) by the Fermi contact interaction,

\[
\hat{H}_{\text{hf}} = \mathcal{A} \sum_j q_j \left[ \hat{I}_z \hat{S}_z + \frac{1}{2} (\hat{I}_+ \hat{S}_- + \hat{I}_- \hat{S}_+) \right],
\]

where \( \mathcal{A} \) is the average hyperfine interaction constant, \( \mathcal{A} = 90 \mu \text{eV} \) for GaAs, and \( q_j \) is proportional to the electron density at the position of the \( j \)th nucleus, \( \sum_j q_j = 1 \). For convenience, we introduce the collective operators \( \hat{A} = \sum_j q_j \hat{I}_j \). The first term in Eq. (1) yields the Overhauser field, an effective magnetic field for the electron, and also the Knight shift for each nuclei. The flip-flop terms in Eq. (1), \( \hat{H}_{\text{IC}} = \frac{\mathcal{A}}{2} (\hat{A}_+ \hat{S}_- + \hat{A}_- \hat{S}_+) \), can be used to polarize the nuclear spins [1,2], and to map the electron’s spin state into a collective spin mode of the nuclei [3–5]. As will be shown here, the same can be used to provide a protective energy gap.

**Fully polarized nuclei.**—We start by reconsidering the storage of a qubit in collective nuclear states [3]. In the case when all the nuclear spins are initially polarized in the \(-z\) direction (zero temperature limit), the \( |0\rangle \) and \( |1\rangle \) spin states of the electron are mapped onto the nuclear-spin states \( |0\rangle = |I, -I, \ldots, -I\rangle \) and

\[
|1\rangle = \frac{\mathcal{A}}{\Omega} \hat{A}_+ |0\rangle \propto \sum_j q_j |-I, \ldots, -(I+1), \ldots, -I\rangle,
\]

respectively. \( \hat{H}_{\text{IC}} \) couples the state \( |0\rangle \) to \( |1\rangle \) with an angular frequency \( \Omega = \mathcal{A} (\sum_j q_j^2 2I)^{1/2} \). The detuning between these two states, \( \delta = \delta_{\text{OH}} + \delta_{\text{el}} \), comes from the Overhauser field, \( \delta_{\text{OH}} = -\mathcal{A} I, \) and from the electron’s intrinsic energy splitting \( \delta_{\text{el}} \) due to, e.g., an external magnetic field or a spin-state dependent Stark laser pulse [11]. Coherent flip-flops between the electron and nuclear spins can be brought into resonance (\( \delta \ll \Omega \)) through \( \delta_{\text{el}} \). Then \( |0\rangle (|0\rangle + \beta |1\rangle) \) is rotated by \( (\alpha |0\rangle \beta |1\rangle) \), and the

![FIG. 1 (color online). Left: Charged quantum dot with a single, polarized excess electron. Right: Spectrum of the effective nuclear Hamiltonian in the presence of a polarized electron. Off-resonant hyperfine coupling results in a gap \( \Delta_{\text{gap}} \) between the storage state \( |1\rangle \) and the nonstorage states \( |0\rangle \). \( \Delta_{\text{K}} \) denotes the Zeeman shift due to the effective magnetic field associated with the electron spin (Knight shift).](image-url)
quantum information is transferred from the electron to the nuclear-spin ensemble and back [3,4].

Assume that, after the qubit has been written into the nuclei, the polarized electron is not removed from the dot but the hyperfine flipflops are tuned off-resonant (δ ≫ Ω). Now, real transitions can no longer take place between |0⟩ and |1⟩. However, the residual virtual transitions repel the two states from each other, in analogy to the dynamic Stark effect. As a result, after adiabatic elimination of the electronic states, the energy of state |1⟩ gets shifted by Δgap = −Ωz2/4δ. The other, orthogonal states also having exactly one spin flipped (denoted by |1⟩q) in Fig. 1) are “subradiant,” i.e., are not coupled via the Knight shift. This is the origin of the energy gap.

To understand the protection scheme, let us introduce nuclear-spin waves. For highly polarized nuclei, one can introduce bosonic operators through the Holstein-Primakoff transformation: āj = ℏJ / √2 and ā†j = ℏJ + ℏJ. This allows us to define the bosonic spin waves

\[ \Phi_q = \sum_j \eta_{qj} \hat{a}_j, \quad \Phi^\dagger_q = \sum_j \eta^*_{qj} \hat{a}_j^\dagger, \]

where the unitary matrix ηqj describes the mode functions. We identify the storage mode q = 0 as the one given by

\[ \eta_{0j} = \sqrt{2\mathcal{B}} \frac{\hat{a}_j}{\sqrt{\sum_j \hat{a}_j^2}}, \]

and write |1⟩ = Φ^0_0|0⟩. This is the mode which is directly coupled to the electron spin. In fact, \( \hat{H}_{JC} = \frac{\Omega}{2} (\hat{F}_0^\dagger \hat{S}_- + \hat{F}_0 \hat{S}_+) \) is a Jaynes-Cummings coupling in the bosonic approximation. After adiabatically eliminating the electronic states, \( \hat{H}_{JC} \) reduces to \( \hat{H}_{gap} = -\frac{\Omega^2}{4\delta} \hat{A}_+ \hat{A}_- \), which is a coherent interaction closer to resonance. A nonzero external magnetic field or laser induced ac Stark shifts [11] can partially cancel the Overhauser field, such that δ ≪ δel = −δOH = Ω. (Of course, δ should be kept sufficiently large so that the hyperfine coupling remains off-resonant). The requirement of separation of time scales implies ζ ≪ |Δgap| ≪ Ω ≪ |δ|. To estimate the orders of magnitude of the different energies, we take an oblate Gaussian electron density of ratio (1, 1, 1/3) and spin-1/2 nuclei. Then we find that ΔK and ζ are inversely proportional to the number of nuclei N, whereas Ω, Δgap ∝ N^{-1/2} only [Fig. 2(a)].

To analyze the decoherence suppression, we first consider a simplistic noise model where the nuclear spins are coupled to fluctuating, classical fields. The corresponding interaction Hamiltonian is \( \hat{V} = \sum_j \mathcal{B} \cdot \hat{V} \). We assume isotropic Gaussian noise with zero mean and \( \mathcal{B}^\mu(i) \mathcal{B}^\nu(r') = \delta_{\mu\nu} C e^{-|r-r'|} \) for μ, ν = x, y, z, where \( \mathcal{B}^j \) specifies the spatial correlations of the noise acting on different nuclei. For simplicity, the noise spectrum is assumed to be Lorentzian with a width Π, although similar results hold for other spectra with high-frequency cutoff.

Let us first discuss the dephasing part of the noise,

\[ \hat{V}_z = \sum_j \mathcal{B}_z \hat{a}_j^\dagger \hat{a}_j = \sum_{pq} \left( \mathcal{B}_z \eta_{pq} \eta_{qj} \right) \hat{F}^\dagger_p \hat{F}^\dagger_q. \]

FIG. 2. Hyperfine Rabi frequency (Ω), protective energy gap (Δgap), Knight shift difference between the logical states (ΔK), symmetry breaking couplings due to inhomogeneities (ζ and ζ), and qubit decoherence rates due to dipolar spin diffusion without (ΓD) and with (ΓD) protection. (a) The fully polarized (zero temperature) case shown versus the number of spin-i nuclei (N) taking part in the storage, i.e., located within 3σ of the oblate Gaussian electron distribution with in-plane variance σ. (b) Estimated energies in dark states |D⟩, with n spins flipped from the fully polarized state for N = 10^5. Energy units are chosen to match GaAs. Δgap is obtained by taking δ = 10 Ω.
expressed by the bosonic spin-wave operators (3). Dephasing of individual nuclear spins means transfer of excitations between different spin-wave modes. Especially, it leads to both real and virtual transitions from |1⟩ to a nonstorage state |1p⟩. As the latter state is “subradiant” and, thus, equivalent to |0⟩ when the memory is read out, this process essentially results in damping (for real transitions) and dephasing (for virtual transitions) of the stored logical qubit [12]. Assuming the zero temperature limit with all nonstorage modes \( \hat{\Phi}_{q=0} \) in the vacuum state and formally eliminating them in the Markov approximation together with the classical fields, we derive a master equation for the storage mode: 

\[
\frac{d}{dt} \hat{\rho} = i[\hat{\rho}, E_\perp \hat{\Phi}_0^+ \hat{\Phi}_0] + \mathcal{L}_\perp(\hat{\rho}),
\]

with energy shift 

\[
E_\perp = (1 - \Xi)C\Delta\text{gap}/(\Gamma^2 + \Delta^2\text{gap})
\]

and

\[
\mathcal{L}_\perp(\hat{\rho}) = \gamma_1(2\hat{\Phi}_0^+ \hat{\rho} \hat{\Phi}_0 - \hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\rho} - \hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\rho}) + \gamma_2(2\hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\rho} \hat{\Phi}_0 - \hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\Phi}_0 \hat{\rho} - \hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\Phi}_0 \hat{\rho} - \hat{\rho} \hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\Phi}_0 \hat{\rho}).
\]

Here, \( \gamma_1 \) is the damping rate of the stored qubit while \( \gamma_2 \) describes its dephasing. The rates are given by

\[
\gamma_1 = \frac{C\Gamma}{\Gamma^2 + \Delta^2\text{gap}}(1 - \Xi), \quad \gamma_2 = \frac{C}{\Gamma^2}(1 - \Xi),
\]

where we have introduced the dimensionless parameter 

\[\Xi = \sum_j \xi_{jk}^2/\sum_j \xi_j^2\]

containing the spatial part of the noise correlator.

When the correlation length of the classical noise is smaller than the internuclear distance (local uncorrelated noise, \( \xi_{jk} \sim \delta_{jk} \)), \( \Xi \) scales inversely with the number of nuclei (Fig. 3). In this case, the dephasing rate \( \gamma_2 \) vanishes as \( 1/N \), which is an effect of the collective nature of the storage states [12]. The storage is based on encoding the logical qubit states in a large, delocalized ensemble of \( N \) physical spins. As the decoherence has strongly local character, it has only a very small effect on the dephasing of the qubit. Secondly, the loss of the stored qubit is associated with a change in the number of excitations in the storage mode. Such transitions are strongly suppressed, and the damping rate \( \gamma_1 \) is decreased if \( \Delta\text{gap} \) is large compared to the width of the noise spectrum \( \Gamma \) (or the corresponding cutoff frequency). Finally, the opposite limit of infinite spatial correlation length \( (\xi_{jk} = 1) \) corresponds to a homogeneous random field resulting, e.g., from a global external source. In that case, \( \Xi = 1 \) (see Fig. 3), and there is no protection against dephasing.

Following a similar but slightly more involved procedure, we can discuss the spin-flip part \( \hat{V}_{xy} = \frac{i}{2} \sum_j (B_j k \hat{\Phi}_j^+ \hat{\Phi}_j - \hat{\Phi}_j^+ \hat{\Phi}_j k \hat{\Phi}_j^+ \hat{\Phi}_j) \) of the noise. When deriving a master equation for this case, we need to keep higher order terms in the Holstein-Primakoff approximation: in the next order, \( \hat{\Phi}_j \approx \sqrt{2\Gamma(1 - \lambda\hat{a}_j^+ \hat{a}_j)}\hat{a}_j \) with \( \lambda = 1 - (1 - 1/(2\Gamma))^{1/2} \). Here, we have neglected the probability of double or more excitations on the same site \( j \), which is reasonable in the high polarization \( (T = 0) \) limit and exact for spin-\( \frac{1}{2} \) nuclei. The Lindbladian describing decoherences due to spin flips reads, in leading order of \( 1/N \),

\[
\mathcal{L}_{xy}(\hat{\rho}) = (\gamma_3 + \gamma_4)(2\hat{\Phi}_0^+ \hat{\rho} \hat{\Phi}_0 - \hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\rho} - \hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\rho}) + \gamma_5(2\hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\rho} \hat{\Phi}_0 - \hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\Phi}_0 \hat{\rho} - \hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\Phi}_0 \hat{\rho} - \hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\Phi}_0 \hat{\rho} - \hat{\rho} \hat{\Phi}_0^+ \hat{\Phi}_0 \hat{\Phi}_0 \hat{\rho}).
\]

which describes decay with rate \( \gamma_3 \), dephasing with rate \( \gamma_4 \), and additionally thermalization (relaxation to the identity matrix) with rate \( \gamma_5 \). The rates read

\[
\gamma_3 = \frac{CT\Xi'}{\Gamma^2 + (\Delta\text{gap} + \Delta K)^2}, \quad \gamma_4 = \frac{2CT\lambda^2}{\Gamma^2 + (\Delta\text{gap} - \Delta K)^2}, \quad \gamma_5 = \frac{4CT\lambda^2}{\Gamma^2 + (\Delta K)^2}, \quad \gamma_6 = \frac{\sum_j \xi_j^2}{\sum_j \xi_j^2}.
\]

In the limit of vanishing spatial correlations of the spin-flip noise, \( \Xi' = \sum_j \xi_j^2/\sum_j \xi_j^2 \) tends to 1 (Fig. 3), and we have protection against thermalization (\( \gamma_5 \)) by the separation of \(|0⟩\) and \( |1⟩ \) by \( \Delta\text{gap} + \Delta K \). The decay corresponding to \( \gamma_4 \) is due to spin-flip induced transitions between \( |1⟩ \) and \( |1p, 1q⟩ \) (the latter containing a total of two excitations but none in the storage mode), and the energy to bridge is in the order of \( \Delta\text{gap} - \Delta K \) (see Fig. 1). Finally, the last factor in the dephasing rate \( \gamma_6 \) scales as \( 1/N \), indicating that it is the collective nature of the storage that leads to protection. Note that the nonlinearity of the Holstein-Primakoff representation is responsible for this dephasing: the virtual nonstorage excitations are interacting with the storage mode.

Another potential source of decoherence is nuclear-spin diffusion due to dipole-dipole interaction between nuclear spins [13]. The energy gap gives protection against this effect, too. The dipolar interaction between the pairs of spins is described in the secular approximation by

\[
\hat{H}_D = \sum_{j \neq k} B_{jk} (\hat{P}_j \hat{P}_k - \hat{P}_j \hat{P}_k) = 2i \sum_{j \neq k} B_j \hat{a}_j^+ \hat{a}_k.
\]

FIG. 3. The parameters \( \Xi \) and \( \Xi' \) describing the effects of spatial correlations in the classical noise \( \xi_{jk} = e^{-r/\xi} \) for different number of nuclei. The same family of Gaussian electron densities was used as in Fig. 2. The bullets on the curves denote the linear size of the dot given by the variance \( \sigma \).
where \( B_{jk} \propto (3\cos^2\theta_{jk} - 1)/f_{jk} \), \( r_{jk} = r_j - r_k \) is the distance between two nuclei, \( \theta_{jk} \) is the zenith angle of the vector \( r_{jk} \), and we used the first order Holstein-Primakoff approximation. At full polarization, we can rewrite Eq. (9) using the bosonic spin-wave mode operators (3) as \( \hat{H}_D = \sum_{pq} B_{pq} \hat{\Phi}_p^{\dagger} \hat{\Phi}_q \). As if it were a central spin coupled to a mesoscopic spin bath [14,15], the storage mode is coupled to a bath of nonstorage modes that present a fluctuating, effective transversal magnetic field. If the electron were not present, these fluctuations would lead, in the mean-field approximation, to a decoherence rate \( \Gamma_D \sim (2\sum_{q=0} B_{0q}^2)^{1/2} \), which is numerically found to be in the order of 100 Hz for GaAs [Fig. 2(a)]. With the protective gap, however, the storage-mode operator \( \hat{\Phi}_0 \) rotates rapidly with respect to the other ones, and the above coupling averages out. The strength of the remaining coupling between the storage mode and mode \( q \) is proportional to \( \Delta_{gap}^{-1} \sum_{r+0} \hat{B}_r \hat{B}_{rq} \), and the corresponding fluctuations yield a reduced decoherence rate of \( \Gamma_D \sim \Delta_{gap}^{-1} (2\sum_{q=0} B_{0q}^2)^{1/2} \) as indicated in Fig. 2(a).

Depending on the dot size, the effects of spin diffusion can be suppressed by several orders of magnitude.

**Nonperfect nuclear-spin polarization.**—It has been shown that partially polarized nuclei (at finite temperature) can also be used for storing a qubit state [4]. Instead of the shown that partially polarized nuclei (at finite temperature) can also be used for storing a qubit state [4].

The explicit form of the inhomogeneous dark states [4] allows us to estimate these values [see Fig. 2(b)]. We expect that the storage mode is still protected as long as \( \omega_n \) and \( \zeta_n \) are much smaller than \( \Delta_{gap,n} \). Our simulation suggests that even for a polarization of 80\% (\( n = 10^4 \)), the gap is more than 5 times larger than \( \omega_n \) and \( \zeta_n \).

In summary, we have demonstrated the suppression of spin dephasing and spin flips in a quantum memory consisting of a delocalized ensemble of nuclear spins in a quantum dot if the noise has a highly local character and the spectral width of the noise spectrum is small compared to the energy gap. We have shown that the memory can be protected against nuclear-spin diffusion mediated by dipole-dipole interaction. We have also analyzed the effects of inhomogeneous hyperfine couplings and imperfect initial nuclear-spin polarization.

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