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Meson widths from string worldsheet instantons

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We show that open strings living on a D-brane which lies outside an AdS black hole can tunnel into the black hole through worldsheet instantons. These instantons have a simple interpretation in terms of thermal quarks in the dual Yang-Mills (YM) theory. As an application we calculate the width of a meson in a strongly coupled quark-gluon plasma which is described holographically as a massless mode on a D7 brane in AdS$_5 \times S_5$. While the width of the meson is zero to all orders in the 1/$\sqrt{\lambda}$ expansion with $\lambda$ the 't Hooft coupling, it receives non-perturbative contributions in 1/$\sqrt{\lambda}$ from worldsheet instantons. We find that the width increases quadratically with momentum at large momentum and comment on potential phenomenological implications of this enhancement for heavy ion collisions. We also comment on how this non-perturbative effect has important consequences for the phase structure of the YM theory obtained in the classical gravity limit.

A heavy quarkonium bound state, like $J/\psi$ or $\Upsilon$, which finds itself in the quark-gluon plasma (QGP), becomes increasingly unstable and eventually dissociates at sufficiently high temperatures. On the one hand this can be attributed to the weakening attraction between a heavy quark and anti-quark in the bound state due to color screening of the medium [1]. On the other hand the bound state can be broken up from collisions with the deconfined quarks and gluons in the medium [2]. Given that the QGP at RHIC and LHC is likely strongly coupled, understanding such medium effects on the propagation of mesons in a strongly coupled plasma is powerful at attacking questions of dynamical origin, such as how the motion of quarkonia relative to the medium affects their various properties. The simplest example of the correspondence is provided by the duality between $N = 4$ $SU(N_c)$ super Yang-Mills (SYM) theory and string theory in AdS$_5 \times S_5$ [3]. Based on a calculation of the potential between a pair of heavy external quark and antiquark moving in the strongly coupled hot $N = 4$ plasma, it has been argued in [4] (see also [5]) that the dissociation temperature $T_{\text{diss}}$ of a heavy quarkonium decreases with their velocity $v$ relative to the medium as $T_{\text{diss}}(v) \approx (1 - v^2)^4 T_{\text{diss}}(v = 0)$. Such a velocity scaling, which can be heuristically understood as due to increased screening from the boosted medium, could provide a significant additional source of quarkonium suppression at nonzero transverse momentum in heavy ion collisions [4].

Rather than drawing inferences from the heavy quark potential, it is also possible to directly study the propagation of mesons in a strongly coupled plasma. While $\mathcal{N} = 4$ SYM theory itself does not contain dynamical mesons, one can obtain a closely related theory which does contain mesons by adding to it $N_f \ll N_c$ fundamental “quarks”, which corresponds to adding some D7-branes to AdS$_5 \times S_5$ in the gravity picture [6]. It was found in [7, 8] that meson dispersion relations are dramatically modified by the plasma and in particular, there exists a limiting velocity $v_{\text{lim}}(T) < 1$, which decreases with increasing temperature. The existence of a subluminal limiting velocity is consistent with the velocity-enhanced screening obtained from the heavy quark potential, as when $v > v_{\text{lim}}(T)$ the quark and anti-quark are completely screened and no bound states are possible.

For a more complete understanding of the dissociation of mesons one needs to study their widths. We will be particularly interested in the momentum dependence of the widths. This has not been possible within the classical gravity approximation developed so far. In this approximation, the mesons are stable (i.e. they have zero width) for $T$ smaller than a dissociation temperature $T_{\text{diss}}$, but completely disappear for $T > T_{\text{diss}}$ [7, 9, 10]. The approximation also has another important drawback: the densities of quarks and antiquarks are identically zero for a range of temperatures and chemical potentials [11] even though they should obey the standard thermal distribution.

In this paper, we discuss a novel tunneling effect on the string worldsheet which gives rise to nonzero quark densities and meson widths for $T < T_{\text{diss}}$. This enables us to calculate explicitly the momentum dependence of the width of a meson in a strongly coupled QGP. We find that the width increases quadratically with momentum at large momentum.

At finite temperature, $\mathcal{N} = 4$ SYM theory can be described by a string theory in the spacetime of a black hole in AdS$_5 \times S_5$, whose metric can be written as

$$ds^2 = -f dt^2 + \frac{1}{f} dx^2 + \frac{\rho^2}{R^2} d\vec{x}^2 + R^2 d\Omega^2_5$$

(1)

where $f = \frac{\rho^2}{R^2} \left(1 - \frac{r^4}{\rho^4}\right)$, $\vec{x} = (x_1, x_2, x_3)$, $d\Omega^2_5$ is the metric on a unit five-sphere $S_5$ which can be written as

$$d\Omega^2_5 = d\theta^2 + \sin^2 \theta d\Omega^2_3 + \cos^2 \theta d\phi^2, \quad \theta \in \left[0, \frac{\pi}{2}\right]$$

(2)
with $d\Omega_3^2$ the metric for a three-sphere $S_3$. The string coupling $g_s$ is related to the Yang-Mills coupling $g_{YM}$ by $g_s = 4\pi g_{YM}$ and the curvature radius $R$ is related to the 't Hooft coupling $\lambda = g_{YM}^2 N_c$ by $\frac{\pi}{\alpha'} \equiv \sqrt{\lambda}$. The perturbative $g_s$ and $\alpha'$ expansions in the bulk string theory are related to the $1/N_c$ and $\frac{1}{\sqrt{\lambda}}$ expansions in the Yang-Mills theory respectively. The temperature $T$ of the YM theory is given by the Hawking temperature of the black hole, $T = \frac{\Lambda}{2\pi R^2}$. Adding $N_f$ fundamental “quarks” can be described in the dual string theory by adding $N_f$ D7-branes in (1) [6]. A fundamental “quark” in the YM connecting the D7-brane to the horizon has a mass in $\mathfrak{r}$ where there lies a 4-dimensional subspace spanned by $\xi$.

The above conclusions can be further illuminated by simple thermodynamic reasoning. From (3), $\beta m_q^{(T)} \propto$
\( \sqrt{\lambda} \), the quark (or anti-quark) density, being proportional to \( e^{-\beta m_q(T)} + \beta \rho \), is then exponentially suppressed in \( \sqrt{\lambda} \) for \( \mu < m_q(T) \). Similarly, since the binding energy \( E_{BB} \) of a meson is \( 2m_q(T) \), thermal effects which destabilize the mesons are also exponentially suppressed in \( \sqrt{\lambda} \). Thus the meson widths and quark densities are not visible in the perturbative expansion in \( 1/\sqrt{\lambda} \) and can only have non-perturbative origins on the worldsheet.

There are indeed non-perturbative corrections in \( \alpha' \) which effectively change the topology of the D7-brane embedding and generate non-vanishing meson widths and quark densities. To see this it is more convenient to analytically continue (1) to Euclidean signature with \( t \to -i \tau \). Then the \( r - \tau \) plane has the topology of a disk. The angular direction \( \tau \) has a period given by the inverse temperature \( \beta \). The center of the disk is located at \( r = r_0 \). Open strings on the D7-brane are described by worldsheet actions with the topology of a disk whose boundary lies on the D7-brane. Denoting the worldsheet coordinates as \( \rho \in [0, 1] \) and \( \sigma \in [0, 2\pi] \), the worldsheet separate into different topological sectors corresponding to the winding number \( m \) of the target space disk \( (r, \tau) \) wrapping around the worldsheet disk \( (\rho, \sigma) \). The DBI action arises from the sector of trivial winding number \( m = 0 \), in which \( (\rho, \sigma) \) is mapped to a single point on the D7-brane. In all the other (nontrivial) sectors, the string worldsheet is mapped to the region in the \( r - \tau \) plane from the location of the D7-brane all the way to the horizon \( r = r_0 \) (see inset of Fig. 1). When analytically continued back to the Lorentzian signature, such a worldsheet describes a tunneling process in which a tiny neck is generated between the D7-brane and the black hole horizon. As a result mesons can leak through the tiny neck into the black hole and dissociate.

As an illustration, we now calculate the contributions from \( m = \pm 1 \) sectors to the quark density and the widths of mesons in (4). We will only be interested in the lowest order terms in the \( \alpha' \) expansion. The relevant spacetime effective action for the D7-brane can be obtained from the worldsheet path integral [15]

\[
S_E[\chi^\phi] = \int_{\text{disk}} DX e^{-I[X]} + \oint_{\partial \mathcal{M}} d\sigma \mu^2 \phi^2 - I_B[\chi^\phi] = \int_{\text{disk}} DX e^{-I[X]} + \oint_{\partial \mathcal{M}} d\sigma \mu^2 \phi^2 - I_B[\chi^\phi] = \int_{\text{disk}} DX e^{-I[X]} + \oint_{\partial \mathcal{M}} d\sigma \mu^2 \phi^2 - I_B[\chi^\phi]
\]

where \( X = (\xi_\alpha, r, \phi) \) denotes collectively the worldsheet fields. \( I[X] \) is the worldsheet action, which for our purpose may be taken to be the Nambu-Goto action for a string propagating in (1)

\[
I[X] = \frac{1}{2\pi^2} \int d\sigma d\rho \sqrt{\det h_{ab}}
\]

with \( h_{ab} = G_{MN}\partial_a X^M \partial_b X^N \) the induced metric on the worldsheet and \( G_{MN} \) the Euclidean version of the metric (1). The second term in the exponential of \( \beta F(\beta, \mu) \) arises from the sector of trivial winding number \( m \) of the target space disk \((r, \tau)\) wrapping around the worldsheet disk \((\rho, \sigma)\). The DBI action is of the form

\[
\int_{\text{disk}} DX e^{-I[X]} + \oint_{\partial \mathcal{M}} d\sigma \mu^2 \phi^2 - I_B[\chi^\phi] = \int_{\text{disk}} DX e^{-I[X]} + \oint_{\partial \mathcal{M}} d\sigma \mu^2 \phi^2 - I_B[\chi^\phi]
\]

where \( \chi^\phi \) is the boundary YM theory. \( I_B[\chi^\phi] = \oint_{\partial \mathcal{M}} d\sigma \frac{R^2}{2\pi^2} \phi^2 \partial_\rho \phi \) is the boundary action which couples the worldsheet to \( \chi^\phi(\xi_\alpha) \). We have suppressed any dependence on spacetime and world sheet fermions. The integral (6) can be evaluated using the saddle point approximation in each topological sector [16], i.e. \( S_E = S_{m=0} + S_{m=+1} + S_{m=-1} + \cdots \), with \( S_{m=0} = S_{DBI} \).

For \( m = \pm 1 \), (7) has a classical solution given by

\[
\tau = \pm \frac{\beta}{2\pi} \sigma, \quad r = \hat{r}(\rho), \quad \theta = 0, \quad \phi = 0, \quad \vec{x} = \vec{x}_0\quad (8)
\]

where \( \vec{x}_0 \) is an arbitrary constant position vector and \( \hat{r}(\rho) \) is chosen so that \( \hat{r}(1) = r_m \) and \( \int d\tau = \int d\rho \propto d\rho^2 + \rho^2 d\sigma^2 \). Eq. (8) represents the worldsheet of a string connecting the tip of the brane to the horizon with the \( \pm \) sign corresponding to opposite orientations. It has a classical action \( I_{\pm} = \beta m_q(T) \) where \( m_q(T) \) is the effective quark mass introduced in (3). One can readily verify that (8) minimizes the action and satisfies the right boundary conditions at the D7-brane. Note that there are only three bosonic zero modes in (8) [19], since it costs energy to move away from \( \theta = 0 \) and the worldsheet time \( \tau \) now coincides with \( r \). With \( \chi^\phi \) set to zero, Eq. (6) then yields

\[
S_{m=\pm 1} = e^{-\beta m_q(T)} e^{\pm \mu^3 g_s} \frac{1}{V_3}
\]

where the \( e^{\pm \mu^3} \) arises from the second term in the exponential of (6), \( V_3 \) is the spatial volume from integrating over the three zero modes in (8), and the \( \frac{1}{V_3} \) factor arises because we are evaluating the disk path integral. \( D \) is a real number coming from Gaussian integration around the saddle point (8) (including worldsheet fermions) whose sign will fix from physical requirements. Identifying the Euclidean action with \( \beta F(\beta, \mu) \) where \( F(\beta, \mu) \) is the free energy, equation (9) leads to a net quark charge density \(-\frac{\beta}{2\pi} e^{-\beta m_q(T)} \sinh \beta \mu \), which in turn requires that \( D \) be should be negative [20]. It is natural to interpret \( S_{m=\pm 1} \) as the contributions from quarks and anti-quarks separately: \( S_{m=\pm 1} = -n_\pm V_3 \), which from (9) leads to a quark and antiquark number density given by

\[
n_\pm = e^{-\beta m_q(T)} e^{\pm \mu^3 g_s} \frac{1}{V_3} (-D)
\]

Note that \( n_\pm \propto 1/g_s \propto N_c \) since quarks come in an \( N_c \)-multiplet.

In our derivation of (9) we have assumed the embedding of the D7-brane is given by that determined by the DBI action. This is justified for \( \mu < m_q(T) \) since the correction to the DBI action is exponentially small. When \( \mu > m_q(T) \), the backreactions from instantons become large and the embedding of Fig. 1 cannot be trusted.

The nonzero quark densities for nonzero \( \mu < m_q(T) \) have important implications for the phase structure of the theory. As discussed in [11] (see also [12]) based on the analysis of the DBI action (which corresponds to \( \lambda = \infty \)), at low temperature there is a phase transition in chemical potential at which the net quark charge density becomes
nonzero. Our results strongly indicate this phase transition (at nonzero temperature) is smoothed to a crossover at any finite value of $\lambda$.

To find the widths of the mesons described by (4), we need to compute (6) to quadratic order in $\chi^\phi$. For simplicity, we will restrict to the $l = 0$ mode on the $S_3$. Near $\theta = 0$, the worldsheet action for $\phi$ is given by $\frac{R^2 n_{\pm}}{2\pi \alpha' \beta^2} \int d^3 x d\tau d\sigma' \left( \chi^\phi(\tau, \bar{x}_0) \tilde{G}_D(\sigma, \sigma') \chi^\phi(\tau', \bar{x}_0) \right)_{\theta = 0}$, where $\tilde{G}_D(\sigma, \sigma') = \lim_{\rho \to 1, \rho' \to 1} \partial_\rho \partial_{\rho'} G_D(\rho, \sigma; \rho', \sigma')$ with $G_D(\rho, \sigma; \rho', \sigma')$ the Dirichlet propagator for a canonically normalized massless field on the unit disk and $\sigma = \frac{2\pi}{\beta} \tau$.

Note that (10) only depends on the value of $\chi^\phi$ at the tip of the brane and is nonlocal in the Euclidean time direction.

Treating (10) as a small perturbation to (4), one can compute the corrections to the Euclidean two-point function of the (meson) operator dual to $\chi^\phi$ in the boundary YM theory, from which the corrections to the Lorentzian retarded function can be found by analytic continuation. One finds that the poles of the retarded function now occur at nonzero imaginary part. Alternatively one can directly obtain the Lorentzian counterpart of (10) by analytically continuing the worldsheet disk to Lorentzian signature with $\sigma = i\eta = i2\pi t/\beta$, which gives the part of Rindler spacetime $ds^2 = d\rho^2 - \rho^2 dt^2$ with $\rho \leq 1$. The Lorentzian spacetime effective action can be obtained using the Schwinger-Keldysh contour, giving the Lorentzian equation of motion [21]

$$\partial_\sigma \left( \sqrt{-g} G_{\phi\phi} \partial^\sigma \chi^\phi \right) = -\frac{R^2 n_{\pm}}{2\pi \alpha' \beta^2} \int dt' \tilde{G}_R(\eta - \eta') \chi^\phi(\tau')$$

(11)

where $\tilde{G}_R(\eta - \eta') = \lim_{\rho \to 1, \rho' \to 1} \partial_\rho \partial_{\rho'} G_R(\rho, \eta; \rho', \eta')$ with $G_R(\rho, \eta; \rho', \eta')$ the retarded propagator for a massless field in the Rindler spacetime with Dirichlet boundary condition at $\rho = 1$. Fourier transforming (11) to momentum space and using $\int d\eta e^{i\nu\eta} \tilde{G}_R(\eta) = i\nu$, one finds that (5) is modified to

$$\tilde{H}(\tilde{k}, l = 0) \psi - \frac{\omega^2}{4\pi^3 \alpha' \mu^2 A} \delta(\theta) \psi(\theta) = \omega^2 \psi(\theta)$$

(12)

with $A = \sqrt{-g(\tilde{k})}$. Writing the dispersion relation as $\omega = \omega_n - \frac{1}{2} \Gamma_n$ where $n$ denotes the excitation number, and using first order perturbation theory in $n_{\pm}$ we find [22]

$$\Gamma_n^{(\pm)} = \frac{32\pi^3 \sqrt{N}}{N_c m_q^2} |\psi_n(\theta = 0)|^2 n_{\pm}$$

(13)

with $\psi_n(\theta = 0)$ eigenfunctions of (5) evaluated at the tip of the brane. Recall that $n_{\pm}$ are thermal densities for quarks and antiquarks and are proportional to $N_c$.

The ratio

$$\frac{\Gamma_n(k)}{\Gamma_n(0)} = \frac{\psi_n(\theta = 0; \tilde{k})^2}{\psi_n(\theta = 0; k = 0)^2}$$

(14)

can be evaluated numerically and the results are shown in Fig. 2. For large $k$, the asymptotic form of the wave function, found in [8], can be used to show that the width (14) scales like $k^2$ for large $k$: $\Gamma_n(k)/\Gamma_n(0) = R_n[T/M](k/M)^2 + O(k)$ for some function $R_n[T/M]$. Furthermore for temperatures $T \ll M$ and $k \gg M^2/\pi^2$ one finds the closed form expression $\Gamma_n(k)/\Gamma_n(0) \approx \frac{2(4\pi)^3}{(n+2)(n+3/2)} \frac{T^4 k^2}{M^2}$. Fig. 2 has the interesting feature that the width is roughly constant for small $k$, but turns up quadratically around $k/M = 0.52(T_{\text{diss}}/T)^2$, which is roughly the momentum at which the meson approaches its limiting velocity $v_c(T)$. This is consistent with the conclusions based on the velocity dependence of the screened quark potential found in [4]. Note that the width as defined here is in the rest frame of the plasma, so the $k^2$ behavior at large $k$ should be contrasted with the $1/k$ behavior of a relativistic decay width which comes from the usual time-dilation argument.

The plots here also share some similarities with those in [13] for momentum-dependence of meson widths obtained for $\mu > m_q(T)$ with $\lambda = \infty$ where the relevant D7 brane embedding resembles a long spike reaching down to the horizon.

![FIG. 2: The behavior of the width as a function of $k$ for $T/T_{\text{diss}} = .99, .71, .3, .13$ from left to right. The dashed lines are analytic results for large momenta.](image)

![FIG. 3: Schematic diagrams of the relevant thermal processes contributing to the meson width. For large $\lambda$ the first process is dominant, coming from the single instanton sector.](image)

Our result (13) has a very simple physical interpretation as shown in the left plot of Fig. 3, in which a meson...
breaks apart by colliding with a quark (or anti-quark) in the thermal bath. There are also processes corresponding to a meson breaking up by colliding with gluons in the thermal bath, shown in the right plot of Fig. 3. For such a process to happen the gluon should have an energy above the binding energy of the meson. The density of such gluons is thus suppressed by $e^{-2\beta M(T)}$ and should be described by an instanton and anti-instanton so that the resulting worldsheet has trivial topology. We expect that contributions from such processes are also controlled by the value of the meson wave function at the tip of the brane, and possibly have similar growth with momentum.

Our method should be generic to any theory with a holographic dual. While the precise value of the width is clearly model dependent, it is conceivable that the momentum dependence may apply in a wider context including QCD. In particular, our result highlights the contributions to quarkonium suppression from the collisions with medium quarks and gluons in the large transverse momentum region in heavy ion collisions.

Consider the effect of such a momentum scaling on $J/\psi$ with $M \approx 3\text{GeV}$. The dissociation temperature from the gravity set-up is $T_{\text{diss}} = 0.122 M \approx 370\text{MeV}$ in fairly good agreement with lattice data \cite{17} for QCD $T_{\text{diss}} \approx 2.1 T_c$ for $T_c = 170\text{MeV}$ \cite{11}. If we take the RHIC temperature of $T = 250\text{MeV}$ (this corresponds to the second curve from the left in Fig. 2) then a moving $J/\Psi$’s width will increase by a factor of 2(10) at a momentum $k = 6(13)\text{GeV}$. When the width of a meson approaches the spacing between different meson states, the meson can no longer be considered as a well-defined quasi-particle. The momentum scaling thus implies a maximal momentum beyond which the meson no longer exists. As an illustration, suppose the width for the $J/\psi$ in the QGP at zero momentum is about 200 MeV (which is not known) then one expects a maximal momentum around 7 GeV.

Finally, we expect the worldsheet instantons found here to have many other applications to various aspects of flavor physics in AdS/CFT.

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\bibitem{17} J. Mas, J. P. Shock, J. Tarrio and D. Zoakos, arXiv:0805.2601 [hep-th].
\bibitem{21} Above $\mu > m_0^q$, a new phase, where the D7 brane falls into the horizon, is thermodynamically preferred. This phase exhibits both meson widths and finite quark density [11–14].
\bibitem{22} In contrast, the $n = 0$ sector has eight zero modes corresponding to all directions on the D7-brane. There are also no fermionic zero modes here.
\bibitem{23} There are a few other indications that $D$ should be negative. The instanton action also induces a tadpole for the $r$ component of the transverse fluctuations. One finds for $D$ negative the tadpole pulls the brane toward the horizon as required by physical consistency. Also only for $D$ negative, do the meson widths we calculated below have the correct sign.
\bibitem{24} Eq. (11) can also be obtained directly from the Euclidean
action (10) using the following general prescription: write down the equation of motion following from the Euclidean action; replace the Euclidean worldsheet time by the Lorentzian worldsheet time and the Euclidean propagator by the corresponding retarded propagator in the Lorentzian signature.

[22] We normalize the eigenfunctions $\psi_n$ of $\hat{H}(\vec{k}, l)$ as

$$\frac{1}{L^2 R^4} \int_0^{\pi/2} d\theta \sqrt{-g} G_{\phi\phi} \left( -g^{tt} \right) |\psi_n(\vec{k}, l; \theta)|^2 = 1$$

so that $\psi_n$ is dimensionless and has a smooth zero temperature limit. It depends on ratios of $T, M$ and $k$ but not explicitly on $\lambda$. 