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Dynamic-Range Analysis and Maximization of Micropower 
$G_m$–$C$ Bandpass Filters by Adaptive Biasing

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Abstract—We analyze and present an input gain-varying scheme for maximizing dynamic range in a well-known $G_m$–$C$ bandpass filter by both minimizing noise for small input signals, and by achieving balanced swing levels at all filter nodes for large input signals. A micropower bandpass filter suitable for use in cochlear implants and other power-constrained biomedical applications was implemented and tested in subthreshold CMOS. At a center frequency of 1.4kHz and quality factor of 4, the filter has 70dB of dynamic range (57dB maximum SNR, 2.5% total harmonic distortion (THD)) and consumes 2.55mW of power.

I. INTRODUCTION

Low-power, wide-dynamic-range bandpass filters tunable in the audio-frequency range are an important component in cochlear implant processors and other biomedical devices [1], [2]. The power consumption of $G_m$–$C$ filters increases with their dynamic range of operation, such that achieving a large dynamic-range of operation is expensive in power. $G_m$–$C$, as well as other types of filters, are often designed with a balanced internal signal swing condition [3], [4]. The balanced condition ensures that when the input is large and the output node operates at the maximum available swing, no other internal node attempts to exceed this maximum swing. While this approach maximizes the largest signal that can be handled by the filter, it does not improve the filter’s noise performance. Other design approaches that seek to minimize noise [5] do not discuss how noise reduction affects the filter’s balanced-swing condition at the upper end of its dynamic range.

To address these issues, we consider the effects of bias current choices on the filter’s overall dynamic range, taking into account both noise and the maximum allowable signal. We find that different biasing strategies within the same filter topology are needed to achieve either minimal noise or maximal signal swing as the input amplitude changes, without altering the filter’s time constant or quality factor. Thus, by adaptively biasing the filter as the level of the input signal changes, gains in overall dynamic range may be realized.

II. FILTER ANALYSIS

A. Transfer Function Analysis

Fig. 1 shows a $G_m$–$C$ implementation of the bandpass filter described in this work. The topology itself is well known [6]. We will begin by analytically determining dynamic range of the filter in completely general terms, allowing the transconductances of each OTA to take on any value. We shall assume that white noise is dominant in all OTAs, which we later experimentally demonstrate to be a good approximation for evaluating dynamic range.

Fig. 2 shows a block diagram of the same filter, with noise sources included. First, we ignore the noise sources and compute the transfer function of the filter:

\[ v_{out} \over v_{in} = G_{m1} \left( C_{1} C_{2} s^2 + G_{m4} C_{2} s + 1 \right) \]

where expressions for $A$, $\tau$, and $Q$ are given below:

\[ A = \frac{G_{m1}}{G_{m4}} \]

\[ \tau = \sqrt{C_{1} C_{2}} \]

\[ Q = \sqrt{\frac{C_{1}}{C_{2}}} \]

Eqn. 1 can be rewritten in a more standard, canonical form, as follows:

\[ \frac{v_{out}}{v_{in}} = A \cdot \frac{\tau s}{\tau^2 s^2 + \frac{\tau s}{Q} + 1} \]

The transfer function from the input to the intermediate node, $v_1$, is given by:

\[ \frac{v_{1}}{v_{in}} = G_{m1} \left( \frac{G_{m4} C_{2}}{G_{m3} r^2} + \frac{G_{m4} C_{2}}{G_{m3} r^2 + 1} \right) \]
\[ \frac{v_1}{v_{in}} = \frac{v_{out}}{v_{in}} \frac{G_{m2}}{C_2} = \frac{\frac{G_{m1}}{G_{m3}}}{\frac{\tau^2}{Q} + 1} \quad (2) \]

B. Noise Analysis

To determine the effect of noise, we ignore the input, \( v_{in} \), and consider the output noise currents of each OTA. The noise source \( \bar{I}_{n,134}^2 \) denotes the combined output noise power spectral densities (PSD) of OTAs \( G_{m1}, G_{m3}, \) and \( G_{m4} \), while \( \bar{I}_{n^2} \) denotes the PSD of \( G_{m2} \).

To determine expressions for \( \bar{I}_{n,134}^2 \) and \( \bar{I}_{n^2} \), we will assume that each transconductor operates in subthreshold at some bias current \( I_{bias} \). Assuming the transconductors are made with differential pairs, then each transistor operates at a nominal bias current of \( I_{bias}/2 \).

Therefore, the total shot noise current for each transconductor is \( N^2qI_{bias}/2 \), where \( N \) is the effective number of noisy devices. We can also define the linear range of each transconductor, \( V_L = I_{bias}/G_m \). We can now rewrite the shot noise current for each of the two sources as:

\[ \bar{I}_{n,134}^2 = NqV_L (G_{m1} + G_{m3} + G_{m4}) \]
\[ \bar{I}_{n^2} = NqV_L G_{m2} \]

The total integrated voltage squared output noise of the filter is found by integrating the current noise PSDs multiplied by the squared noise transfer functions to the output node obtained from the block diagram (Fig. 2). The integration procedure is described in [7].

\[ \bar{v}_{no}^2 = \bar{v}_{in}^2 \bar{v}_{out}^2 + \bar{v}_{out}^2 = NqV_L \left( \frac{1 + \frac{G_{m1}}{G_{m4}} + \frac{G_{m3}}{G_{m4}}}{C_1} + \frac{G_{m3}}{C_2} \right) \quad (3) \]

C. Optimal Filter Parameter Selection

Let us define the following parameters which are useful for describing the filter’s noise and power performance: \( \tau = C/G_r \), where \( 1/\tau \) is the filter’s center frequency, \( C_1 = C/\beta, C_2 = C/\beta, G_{m2} = G_r \alpha, \) and \( G_{m3} = G_r/\alpha \). If we divide \( \bar{v}_{no}^2 \) by \( A^2 \), we can refer the output noise to the input for in-band signals, as shown below:

\[ \bar{v}_{in}^2 = \frac{NqV_L}{4\beta A^2} \left[ 1 + A \frac{Q}{\alpha} \left( \frac{1}{\beta} + \beta \right) \right] \quad (4) \]

In this parameterized form, one can make several observations about how to lower the noise of the filter. Most obviously, one can try to reduce \( N \) (by changing the OTA topology, for example), or increase the value of \( C \). Lowering \( V_L \) will also reduce the noise power, but this will reduce the maximum signal power by \( V_L^2 \). Therefore \( V_L \) should be increased as much as possible (ideally to \( V_{DD}/2 \)) to maximize dynamic range. We also observe that the input-referred noise does not depend on the center frequency, but on the output. Altering \( \beta \) may somewhat lower noise, but this advantage is negated by the excessive amounts of chip area required for \( \beta \neq 1 \). Increasing \( \alpha \) always lowers noise, especially if \( Q \) is large, but this strategy also increases power consumption. The same is also true for increasing \( \alpha \), which should have the most substantial effect on lowering the input-referred noise.

Since it appears that increasing \( \beta \) will not help lower noise significantly, and can result in large area losses, we set \( \beta = 1 \). Now, to determine the best value of \( \alpha \), we compute a figure-of-merit, namely the input-referred noise power times the power consumed in the filter.

If we assume subthreshold operation, then an OTA’s transconductance is directly proportional to its bias current, and therefore its power consumption. The total power consumed by the filter is therefore:

\[ P_n = G_r V_L V_{DD} \left[ \left( \alpha + \frac{1}{\alpha} \right) + \frac{\beta}{Q} (A + 1) \right] \quad (5) \]

Fig. 3 shows the values of \( \alpha \) which minimizes the product of Eqs. 4 and 5 for various values of \( A \) and \( Q \). We see that \( \alpha \) should be typically between 2 and 3 for the best noise-power consumption tradeoff. However, increasing \( \alpha \) or \( A \) beyond 1, while lowering noise, also lowers dynamic range at the upper end because the maximum input signal level is reduced. Referring to Eqn. 1, the filter transfer function to the output \( v_{out}/v_{in} \), we see that the gain of the filter at resonance is simply \( A \), implying that increasing \( A \) beyond 1 is only helpful for small input signals. The effect of increasing \( \alpha \) beyond 1 is more subtle. Since the output is independent of \( \alpha \), it appears that increasing \( \alpha \) to its optimal level is the best strategy. However, the filter transfer function to the intermediate node, \( v_{out}/v_{in} \) is no longer independent of \( \alpha \). Referring to Eqn. 2, we see that the gain at resonance is \( Q/G_{m1}/G_{m3} \). This quantity can also be expressed as \( \beta \alpha A \). Hence, like \( A \), increasing \( \alpha \) should be done when the input signal is small to avoid premature saturation of the OTAs at the intermediate node.

If tuned correctly by adjusting \( G_{m1} \), \( A \) has no effect on the filter \( Q \) or center frequency. Intuitively, this is because \( G_{m1} \) is not technically part of the filter, but just a voltage-to-current converter that feeds the remaining OTAs and capacitors. Similarly, adjusting \( \alpha \) also has no effect on the filter \( Q \) or center frequency. To adjust \( \alpha \), the value of \( G_{m2} \) is raised by a factor of \( \alpha \) and \( G_{m3} \) is reduced by a factor of \( \alpha \). Because \( G_{m2} \) and \( G_{m3} \) form a gyrator loop, only the product of their transconductances, designed to depend only on \( \tau \), matters to determine \( v_{out}/v_{in} \). However, the same is not true for the intermediate node \( v_{out} \) or the output noise, which are affected by \( \alpha \).

The best way to take advantage of the filter sensitivity scaling in \( A \) is to use the filter like an automatic gain control (AGC) circuit. For example, placing the filter in a feedback loop with an envelope detector [8] that senses when the input signal is small and increases \( A \) appropriately gives payoffs in improved dynamic range. \( A \) should always be 1 once the signal is large. A similar, parallel loop can be used for \( \alpha \). For large signals, \( \alpha \) should also be 1. Setting \( A \) and \( \alpha \) both to 1 ensures that all OTAs receive input signals of the same magnitude, thus preventing any single OTA from limiting performance.

Standard implementations of this filter maximize the allowable input signal amplitude by choosing \( A = \alpha = \beta = 1 \). The input noise of the filter then simplifies to:

\[ \bar{v}_{out}^2 = \frac{NqV_L}{4\beta C} [2 + 2Q] \quad (6) \]

which is approximately proportional to \( Q \) for \( Q \gg 1 \).

III. IMPLEMENTATION

The bandpass filter of Fig. 1 was implemented in a 0.5 \( \mu \)m CMOS process. OTAs were built with well-inputs, source degeneration, and bump linearization [9] to increase \( V_L \) to an estimated 1.15V.
Capacitors $C_1$ and $C_2$ were 10pF each, setting $\beta = 1$. All OTA transconductances could be tuned by varying external bias current sources.

Single-ended and fully-differential versions of the same filter were built. The fully-differential version uses a similar OTA design as the single-ended version. A high-input-impedance common-mode feedback (CMFB) circuit using a wide-linear range differential-difference transconductor (DD-OTA) [10] and no passive components detects the common-mode output voltage of each differential OTA and feeds back an appropriate bias current to servo it to a fixed reference voltage. Fig. 4 shows one differential OTA with CMFB. The load capacitors are arranged such that each CMFB half-circuit sees a load capacitance of $C_A$ while each differential half-circuit sees $C = C_A + 2C_B$, where $C = \pi G_v$.

Fig. 4. High-level schematic of a fully-differential OTA with CMFB provided by a DD-OTA.

IV. MEASUREMENTS

A. Single-Ended Filter

A variety of measurements were made on the single-ended band-pass filter to experimentally evaluate the effect of varying $A$, $\alpha$, $G_v$, and $Q$ on the power, noise and dynamic range.

First, to establish programmability of the filter, parameters $A$, $f_0 = 1/(2\pi\tau)$, $Q$, and $\alpha$ were adjusted, strictly based on design, by varying the 4 OTA bias currents. Actual measured values of $Q$ and $f_0$ vs. their expected values are shown in Fig. 5. The center frequency rises more slowly with bias current than expected because the OTAs begin to operate above-threshold, where transconductance is no longer proportional to the bias current. Varying $A$ has almost no effect on $f_0$ or $Q$, as it should not because $G_m1$ controls only the gain. Varying $\alpha$ tended to cause variability in the measured $f_0$ and $Q$. In principle, the product of $G_m2$ and $G_m3$, on which $f_0$ and $Q$ in part depend, is constant for any value of $\alpha$. However, if above-threshold and on-chip current mirror mismatch effects are considered, the $G_m2G_m3$ product could vary with $\alpha$ and cause deviations from the expected values of $f_0$ and $Q$.

Fig. 5. (left) Set vs. measured center frequency at $Q = 4$. (right) Set vs. measured $Q$ at $f_0 = 1.4$ kHz. Both plots are taken for various values of $A$ and $\alpha$.

Noise PSDs at the filter output were measured and integrated to find the total output noise power at a variety of settings of $A$, $f_0$, $Q$, and $\alpha$. Fig. 6 shows noise PSDs for 3 different values of $I_r$, the bias current which ultimately determines $G_v$, and by extension, $f_0$. The $Q$ setting was 4 and $\alpha$ and $A$ settings were 1. It is apparent that a $1/f$ noise contribution is present. As $I_r$ increases, the thermal noise floor is lowered, causing the $1/f$ corner frequency to increase, as seen in the data and reported in [9]. The $1/f$ noise is primarily due to $G_m2$ since $G_m2$’s noise transfer function to the output is lowpass, as can be determined from the block diagram in Fig. 2. Fortunately, the noise current contributions of $G_m1$, $G_m3$, and $G_m4$ are all bandpass to the output, making their $1/f$ noise contributions negligible.

Fig. 6. Output noise power spectral densities of the filter at multiple center frequencies.

Fig. 7 shows the aggregate results of many noise measurements, taken at a constant $f_0$ of 1.4kHz. In these plots, the integrated voltage noise shown has been input-referred, or divided by the filter gain $A$. Consider the left panel of Fig. 7. As the set value of $A$ was adjusted between 1 and 6 (the actual value of $A$ was close to the set value), the input-referred noise dropped as expected. At a particular value of
A, increasing Q also increased noise, as expected. Finally, for any given values of A and Q, increasing α caused the noise to drop, all in accordance with theory. The most dramatic reductions in noise as α was increased were associated with high values of Q, also noted from theory. However, it is clear that far more significant wins in noise are possible by adjusting A rather than α, as α has little effect on the noise except when A = 1.

For example, suppose α = 1 and Q = 8. When A = 1, the input-referred noise is about 435 μV. When A = 6, this decreases to about 90 μV, or by approximately a factor of 4.8 = 13.7dB. Theory predicts the noise ratio should be about 5.3 or 14.5dB. The small discrepancy between theory and practice is likely caused by 1/f noise encountered in measurement but not included in the theory. Similar discrepancies exist for varying α, but it is clear that A is much more effective at reducing input-referred noise than α.

The right panel of Fig. 7 includes the effect of power consumption on the \( \sqrt{\text{power}} \times \text{noise figure-of-merit} \) as A and α are increased. The plot shows that increasing A offers better performance value than increasing α. However, if one desires to vary α, α = 2 offers optimal performance, similar to what was predicted by theory.

At \( f_0 = 1.4kHz \), Q = 4, α = 1, the filter had 70dB of dynamic range. The noise floor was 90μV with A = 6 and the maximum signal amplitude was 255mV\(_{\text{m}}\) with A = 1. The upper end of the dynamic range was limited by THD, set at 2.5% maximum. With A = 1, the maximum SNR was 57dB. Power consumption ranged from 1.63 to 2.55μW as A increases from 1 to 6. To realize the full 70dB dynamic range, adaptation of A with changes in input level are necessary and were implemented manually. Future versions could incorporate an adaptive-biasing gain-control loop as in [7] to automate the process. A die microphotograph of the chip is shown in Fig. 8.

\[ \text{Fig. 7. Total input-referred noise voltage (left) and total input-referred noise voltage } \times \sqrt{V_{\text{m}}}(\text{right}) \text{ at various settings of } A, Q, \text{ and } \alpha. \text{ The center frequency was 1.4kHz.} \]

### B. Fully-Differential Filter

At \( f_0 = 1.4kHz \), Q = 4, α = 1, the fully-differential filter achieved 58dB of SNR with only 1% THD with approximately twice the power consumption of the single-ended filter. This is a lower quoted THD than the single-ended filter while providing 1 dB additional SNR. It is also possible to extend dynamic range of this filter as with the single-ended filter through A control. However, this capability was not characterized.

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**Fig. 8.** Die microphotograph of the bandpass filter in 0.5μm CMOS. The outline of the single-ended filter (area = 55,000μm\(^2\)) is shown. The size of the die is 1.25 × 1.25mm.

### V. Conclusion

We have presented a general scheme for optimizing the dynamic range in a \( G_m-C \) bandpass filter by both minimizing noise for small input signals, and by achieving balanced swing levels at all filter nodes for large input signals. We built and measured single-ended and fully-differential versions of the filter in a 0.5μm CMOS technology to confirm our analysis. At \( f_0 = 1.4kHz \) and Q = 4, the fully-differential version had 58dB dynamic range at 1% THD. By configuring the single-ended filter to realize an input amplitude dependent adaptive gain between 1 and 6, we were able to extend the dynamic range of the single-ended filter from 57dB to 70dB (THD = 2.5%) at \( f_0 = 1.4kHz \) and Q = 4 on 2.55μW of power.

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**References**


