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Tailoring and cancelling dispersion of slow or stopped and subwavelength surface-plasmonodielectric-polaritonic light

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A new physical mechanism enabling simultaneous cancellation of group-velocity and attenuation dispersion to extremely high orders for subwavelength light of any small positive, negative or zero value of the group velocity is introduced. It exploits the unique dispersive properties of the surface polaritons supported by a novel proposed material platform of multilayered Surface-PlasmoDielectric-Polaritonic (SPDP) systems. These single-polarization broadband-slow- or stopped-light systems are thus essentially free from all kinds of dispersion and could therefore have great impact on the technology of integrated nanophotonics, for example in the design of efficient and very compact delay lines and active devices. The same dispersion-manipulation mechanism can be employed to invent a large variety of exotic slow-light dispersion relations.

INTRODUCTION

One fundamental aspect of photonic technology has always been the quest for the perfect light-guiding system, which would exhibit, over a large frequency bandwidth, subwavelength modes of controllable (with special interest lately on small [1]) group velocity and small attenuation, both devoid of frequency dispersion [2]. If this were possible, a temporally and spatially tiny wavepacket would basically propagate without changing shape but only with slowly uniformly decreasing size. Such a system is yet not known to exist in nature, as none of the existing material platforms can achieve simultaneously all of the above attributes, but at most only a subset. All-dielectric structures [3] cannot support highly-subwavelength light propagation, which can be attained by exploiting (bulk or surface) polaritons in plasmonic [4–14] or other resonant-material (e.g., atomic, excitonic, phononic [15, 16]) waveguiding structures, which typically suffer though from high absorption losses. The one problem, however, that is commonly shared among all existing photonic systems is modal dispersion. In particular, for slow- [17–25] and stopped- [26–28] light systems, dispersion is the major reason there is a limitation on their achievable so-called ‘bandwidth-delay product’ [29–34]. This fact has thus motivated the recent invention of a few advanced dispersion-cancellation schemes, which make use of coupled geometric [35] or gain-material [36] resonances or a fine balance of dispersion with nonlinearities [37]. It was also pointed out recently [38] that layered axially-uniform plasmonic-dielectric-hybrid waveguiding systems can guide broadband slow and subwavelength light, but the proposed systems were still dispersion-limited. In this Article, we show that such multilayered Surface-PlasmoDielectric-Polaritonic (SPDP) systems allow for a new physical mechanism, which enables their inherently-single-polarization surface-polaritonic modes to additionally have - for small positive, negative or zero group-velocity - the dispersion coefficients of simultaneously both the group velocity and the attenuation systematically cancelled to unusually high orders, thus leading to the first linear passive system in nature, known to us, that essentially is dispersionless and breaks the ‘bandwidth-delay product’ limitation. By arguing [38] that, in the absence of disorder, attenuation may also, in principle, be reducible by cooling, these material systems approach the ideal slow-light-guiding system. Furthermore, they can also be tailored to invent a variety of novel intricate dispersion relations with multiple points of zero group velocity. The applications of this class of guiding systems in the technological realm of nanophotonics could be substantial.

THEORY AND IMPLEMENTATION METHODS

Surface-Plasmon-Polaritons (SPP) are well known [4] electromagnetic waves that propagate along the interface between a plasmonic material (e.g., metal) and a single dielectric material of permittivities $\epsilon_p$ and $\epsilon$. A SPP exists only for TM polarization (magnetic field parallel to the interface) and its $\omega$-$k$ dispersion relation is $k \equiv |k| = \omega / c \cdot \sqrt{\epsilon_\infty \cdot \epsilon_p (\omega) / (\epsilon + \epsilon_p (\omega))}$, where $k$ is in the plane of two-dimensional (2d) translational symmetry. For example, assuming for now lossless materials and using the Drude model $\epsilon_p (\omega) = \epsilon_\infty - \omega_p^2 / \omega^2$, where $\epsilon_\infty$ is the permittivity at very high frequencies and $\omega_p$ is the bulk plasma frequency, the condition $\epsilon_p < -\epsilon < 0$ for propagation leads to a high-frequency cutoff at $\omega_c (\epsilon) = \omega_p / \sqrt{\epsilon_\infty + \epsilon}$, to which the SPP asymptotes for large wavevectors $k$, as it is then tightly confined on the interface, while the SPP asymptotes to the light-line of the dielectric $k = \omega / c \cdot \sqrt{\epsilon}$ for small wavevectors $k$, as it then extends far into the dielectric.

Intricate dispersion relations

Several extensions of this simple structure in terms of planar plasmonic layers have been examined in the past [39, 40]. Instead, let us keep the plasmonic substrate infinite and insert, between this and the dielectric, a planar dielectric layer of permittivity $\epsilon_1 > \epsilon$ and thickness $d_1$. This structure has been analyzed in Ref. [38] (and Fig. 1 curve C therein), where it was seen that: for small $k$, the Surface-PlasmoDielectric-Polariton (SPDP) mode extends a lot into the $\epsilon$-dielectric and does not ‘see’ much...
the $\epsilon_1$-dielectric layer, thus it asymptotes to a SPP on a $\epsilon_p$-$\epsilon$ interface, while, for large $k$, the SPDP mode is tightly confined on the $\epsilon_p$-$\epsilon_1$ interface, thus it asymptotes to a SPP on such an interface; since both these SPPs have positive group velocity and $\epsilon_1 > \epsilon \Rightarrow \omega_c (\epsilon_1) < \omega_c (\epsilon)$, the limiting $k$-regions must be connected through a regime of negative group velocity, with two boundaries of zero group velocity (zgv), one with negative curvature at a small $k_{n,1}$ and one with positive curvature at a large $k_{p,1} > k_{n,1}$. Note that these boundaries are circles in the 2d $k$-plane (projected as points in $\omega$-$k$ plots) and that Surface-Polaritonic systems are the only systems in nature with 2d continuous translational symmetry known so far to exhibit such zgv points at a non-zero wavevector.

Let us further insert, between the plasmonic substrate and the $\epsilon_1$-dielectric layer, another planar dielectric layer of permittivity $\epsilon_2 < \epsilon_1$ and thickness $d_2 < d_1$. This structure has never, to our knowledge, been analyzed before and behaves as follows: for small $k$, the behavior will not change, but, for large $k$, the SPDP mode must now asymptote to a SPP on a $\epsilon_p$-$\epsilon_2$ interface; since $\epsilon_2 < \epsilon_1 \Rightarrow \omega_c (\epsilon_2) > \omega_c (\epsilon_1)$, the large-$k_{p,1}$ zgv point will move to a smaller wavevector $k_{p,2} < k_{p,1}$ and two zgv points will
remain at $\sim k_{n,1}$ and $k_{p,2}$, provided $d_2$ is small enough that $k_{n,1} < k_{p,2}$.

This process of inserting additional layers of smaller and smaller thicknesses $\{d_i\}$ of various dielectrics $\{\epsilon_i\}$ onto the plasmonic substrate can be continued arbitrarily. The general rules for the resulting SPDP modal dispersion relation upon insertion of the $i^{th}$ layer are that always $\omega (k \rightarrow \infty) = \omega_c (\epsilon_i)$ and typically (for smaller enough thicknesses $d_i < d_{i-1}$) the following occur: (A) $\epsilon_i < \epsilon_{i-1} < \epsilon_{i-2}$ $\Rightarrow$ no new zgv point, (B) $\epsilon_i < \epsilon_{i-1} > \epsilon_{i-2}$ $\Rightarrow$ no new zgv point and the large-$k_{p,i}$ zgv point moves to $k_{p,i}$ (where $k_{n,1} < k_{p,1} < k_{p,i-1}$), (C) $\epsilon_i > \epsilon_{i-1} < \epsilon_{i-2}$ $\Rightarrow$ two new zgv points at $k_{n,i}$ and $k_{p,i}$, and (D) $\epsilon_i > \epsilon_{i-1} > \epsilon_{i-2}$ $\Rightarrow$ either two new zgv points at $k_{n,i}$ or $k_{p,i}$ and $k_{p,i}$, if $d_i$ is really small, or no new zgv point and the large-$k_{p,i}$ zgv point moves to another large wavevector $k_{p,i}$, if $d_i$ is only a little smaller than $d_{i-1}$. These rules provide ample degrees of freedom to create very interesting dispersion relations with many zgv points. An example, which was calculated using a standard transfer-matrix method, is shown in Fig. 1.

**Group-velocity dispersion cancellation**

An even larger variety of exotic dispersion relations can be created, if we don’t restrict the dielectric layers to be progressively thinner from top to bottom. One fascinating example is that of exceptionally-high-order cancellation of group-velocity dispersion...
for a slow subwavelength light mode. To demonstrate how this can be accomplished, let us use a structure of the type shown in Fig. 1 as a starting point. Consider a small group velocity \( v_{go} \), indicated in Fig. 1(ii) by a dotted line, so that the function \( v_{g} - v_{go} \) has several roots, depending on the chosen \( v_{go} \); by then modifying the structure of Fig. 1 so as to increase (decrease) the thicknesses of the bottom (top) layers, the roots of \( v_{g} - v_{go} \) at large (small) wavevectors will now move to smaller (larger) wavevectors, and, with the appropriate choice of the thicknesses \( \{d_1\} \), multiple (let’s say N) of these roots can be forced to coincide at the same wavevector of intermediate value \( k_o \) and at frequency \( \omega_o \). Such a root of multiplicity N means that dispersion has been cancelled up to \( N^{th} \) order for the SPDP mode; locally its dispersion relation looks like

\[
k - k_o = v_{go}^{-1}(\omega - \omega_o) + \frac{e^{-1} \omega_o^{-N}}{(N + 1)!} D_{k,N+1} (\omega - \omega_o)^{N+1},
\]

or equivalently

\[
\omega - \omega_o = v_{go} (k - k_o) + \frac{\omega_o^{-N}}{(N + 1)!} D_{\omega,N+1} (k - k_o)^{N+1},
\]

where \( D_{k,N} \equiv \omega_o^{N-1} \frac{\partial \omega_o}{\partial k} \left( \frac{\partial k(\omega_o)}{\partial k_o} \right) \), \( D_{\omega,N} \equiv \omega_o^{N-1} \frac{\partial \omega_o}{\partial \omega_o} \left( \frac{\partial \omega(\omega_o)}{\partial \omega_o} \right) \) are the normalized dimensionless dispersion constants of \( N^{th} \) order, and, when all are zero up to \( N^{th} \) order as in Eqs. (1) and (2), then \( D_{\omega,N+1}/D_{k,N+1} = -(v_{go}/c)^{N+2} \). Examples for such high-order dispersion cancellation are given in Figs. 2(a), 2(b) and 3 for three values of \( v_{go} \): positive, negative and zero respectively. To our knowledge, there is no other system in nature, whose photonic dispersion relation can be such a long straight-line segment and which is not a plane-wave.

Plasmonic-dielectric layered material systems even allow the parameters \( \omega_o, k_o, v_{go} \) and \( N \) of this phenomenal dispersion cancellation to be tailored at will, to a great extent:

- These systems have the unique property of allowing independent control of the temporal and spatial scales of their photonic modes: their dispersion curve must lie in the \( \omega\cdot k \) plane “allowed region” between the two curves for simple SPPs on a \( \epsilon_p \cdot \min \{\epsilon_1, \epsilon_p\} \) interface and a \( \epsilon_p \cdot \max \{\epsilon_1, \epsilon_p\} \) interface (in particular, \( \omega_o (\max \{\epsilon_1, \epsilon_p\}) < \omega < \omega_o (\min \{\epsilon_1, \epsilon_p\}) \) and \( k > \omega/c \cdot \sqrt{\epsilon} \) in the subwavelength regime of interest); then \( \omega_o \) can be dictated mainly by the proper choice of materials for the plasmonic substrate \( \{\omega_p, \epsilon_\infty\} \) and the dielectric layers \( \{\epsilon_1, \epsilon_p\} \), while \( k_o \), quite independently, by the choice of the layer thicknesses \( \{d_1\} \).

Similarly there is substantial freedom of choice for \( v_{go} \), with the sole restriction, imposed by the general nature of plasmonic systems, that the maximum attainable \( |v_g| \) decreases rapidly as \( k \) increases, with an apparent scaling \( \max |v_g| \sim 1/k \) as can be seen from Fig. 1(ii), namely the same scaling as for the phase velocity \( v_p = \omega/k \), which is typically much larger (\( v_p \ll v_p \)) for these systems in the subwavelength regime.

- Given desired (and compatible under the above restriction) \( \omega_o, k_o \) and \( v_{go} \), the recipe for an \( N^{th} \) order dispersion cancellation is then as follows: Since the roots of the \( v_{g} - v_{go} \) function are the essential tool of the cancellation mechanism, an oscillatory behavior of this function is needed and thus the choice of materials for the layers should be such that higher- and lower-index layers alternate (cases B and C discussed earlier), since each such pair of consecutive layers generates an additional oscillation with two zgy points (and hopefully, depending on the chosen \( v_{go} \), two roots of \( v_{g} - v_{go} \) at controllable \( \omega \)- and \( k \)-values; therefore, with this design, a minimal number of \( N \) layers are needed for creating \( N \) roots of \( v_{g} - v_{go} \), and then their \( N \)-fold coincidence can be accomplished at the desired frequency \( \omega_o \) and subwavelength wavevector \( k_o \), by choosing and/or fine-tuning the parameters \( \{\omega_p, \epsilon_\infty, \epsilon_1, \epsilon_p, d_1\} \).

In principle, one can keep adding pairs of layers to increase \( N \) and whether this can be increased arbitrarily depends on how rapid a change in group velocity within the dispersion curve does the plasmonic material system allow. Unfortunately, due to its complexity, the existence and estimation of such an upper limit on the achievable \( N \) remains an open theoretical question. We can simply argue that a larger \( N \) is easier to get as \( |v_{go}| \) gets smaller and \( k_o \), increases, since a longer straight-line segment can then fit inside the “allowed region” in the \( \omega\cdot k \) plane described earlier or, seen differently from Fig. 1(ii), \( v_{g} - v_{go} \) can have more roots more easily. In practical applications though, this is not even an issue, since \( N \) does not need to be too large, as will be seen in examples later.

This scheme can very easily take into account also the potential material dispersion of the dielectrics, since this dispersion is much weaker than the strong geometric dispersion enforced by the design.

**Attenuation dispersion cancellation**

All plasmonic materials in reality exhibit absorption losses, whose effects on the suggested dispersion manipulation mechanisms need to be studied. For any material system, intrinsic absorption losses are usually quantified as losses-per-unit-distance \( \alpha \) (also called ‘propagation losses’) or losses-per-unit-time \( \Gamma \) (related to the ‘quality factors’ \( Q = \omega/2\Gamma \)); for small loss, using the analyticity of the \( \omega\cdot k \) relation and the Cauchy-Riemann equation \( \frac{\partial \Re k}{\partial \Re \omega} = \frac{\partial \Im k}{\partial \Im \omega} = 1/v_g \), these relate by \( \alpha \approx \frac{\partial \Re k}{\partial \Re \omega} \cdot \Gamma = \alpha v_g \approx \Gamma \), so \( \alpha \) is not a good measure in regimes close to zgy points, where ‘propagation’ lacks physical meaning and \( \alpha \sim 1/v_g \to \infty \) (see Fig. 1(ii)), but \( \Gamma \) is more appropriate. Using accordingly the appropriate loss measure, the group-velocity dispersion-characteristics (\( \Re \omega - \Re k \) relation) for light within a system remain almost unaffected in the
presence of material absorption, its basic effect being just field attenuation. This means that, in particular for our proposed plasmonic-dielectric material structures, our dispersion manipulation and cancellation methods are still easily applicable in the presence of material absorption, its basic effect being just field attenuation. This means that, in particular for our proposed

To account for absorption and calculate losses, we use the simple Drude model for plasmonic materials \( \epsilon_p(\omega) = \epsilon_\infty - \omega_p^2 / (\omega^2 + i\gamma\omega) \) (where the tilde denotes a complex quantity), which has the unique property that, for small \( \gamma/\omega_p \) (\( \sim 10^{-2} \) at room temperature for metals), \( \epsilon_p(\omega) \approx \epsilon_p(\omega + i\gamma/2) \). Then, denoting as \( n_p(\omega) \equiv c\partial k / \partial \omega = c/v_p = n_p(\omega) + \omega d\alpha_p(\omega) / d\omega \) the group index of a lossless plasmonic-dielectric structure, when we add loss only to the plasmonic material as above, the complex propagation constant becomes \( c\tilde k = \omega\tilde n_p(\omega) = \omega n_p(\omega + i\gamma/2) = \omega n_p(\omega) + \omega d\alpha_p(\omega) / d\omega \cdot i\gamma/2 \), namely \( \alpha \approx (n_g - n_p)\gamma/2 \) and, since \( n_G \approx n_p \Gamma \), we conclude that: in the small-\( k \) regime, where most of the light is localized outside the lossy plasmonic material and \( n_g \approx n_p \Rightarrow v_g \approx v_p \), we naturally find \( \alpha \approx \Gamma \approx 0 \), while in the subwavelength large-\( k \) regime of interest, where there is strong light confinement inside the plasmonic material and typically \( n_g \gg n_p \Rightarrow v_g \ll v_p \), as seen earlier, we get \( \alpha v_g \approx \Gamma \approx \gamma/2 \), namely a constant independent of frequency and wavevector (see part (ii) of Figs. 1, 2, 3). This has the remarkable implication that, at points \( \omega_o, k_o \) where group-velocity dispersion (gvd) has been cancelled to \( N^{th} \) order \( (D_{k,n} = 0 \text{ for } n = 2 \ldots N) \), also the attenuation dispersion (ad) has been cancelled to \( (N - 1)^{th} \) order \( (D_{\alpha,n} \equiv \omega_o^2 \partial \alpha / \partial \omega_o = \omega_o^2 \partial \alpha / \partial \omega_o \cdot \gamma/2\omega_p = D_{k,n+1} \cdot \gamma/2\omega_p = 0 \text{ for } n = 1 \ldots N - 1) \).
... and similar considerations as before hold for the range of the simultaneously achievable $\omega_o$, $\beta_o$, $v_{go}$ and $N$.

$\epsilon$

where

Figure 4: (i) $\omega - \beta$ dispersion diagram for the plasmonic-dielectric layered waveguide structure shown in the inset with $\epsilon = \epsilon_c = 5$, $\epsilon_i = \{2, 5, 2, 5, 1.75, 5\}$, $d_i/\lambda_p = \{0.0066136, 0.0170872, 0.0021628, 0.008271, 0.000923, 0.000333\}$, $w/\lambda_p = 0.1$ and $\epsilon_\infty = 1$: note the extremely linear regime around $\omega_o/\omega_p = 0.449$, $\beta_o/k_p = 15.3$; using a finite-element calculation (at $\omega_o$ giving $\beta_o/k_p \approx 15.48$), the associated transverse modal profile is plotted in the inset for the $H$-component parallel to the plasmonic interface. (ii) $\omega - D_{\beta,n}$ group-velocity (gvd) dispersion diagrams for the same structure (namely derivatives of (i)); note that gvd orders $n = 2 \ldots N$ are cancelled at $\omega_o$ (dotted line), where $v_{go}/c = D_{\beta,1}^{-2} = D_{\omega,1} = 0.001 > 0$, $N = 5$, $D_{\beta,6} = 1.82 \cdot 10^{12} \Rightarrow D_{\omega,6} = -1.66 \cdot 10^{-5}$, and thus the inset depicts the Eq. (1)-type behavior.

Finally, since these Surface-Polaritonic modes exist inherently only for a single (TM) polarization, they do not suffer from Polarization Mode Dispersion [2] either. Clearly, this triple simultaneous dispersion cancellation is an amazing and unique feature of layered plasmonic-dielectric material structures. Its main consequence, discussed in detail in a later section, is the freedom from all kinds of pulse-shape distortion in these systems.

**Linear waveguides**

The currently proposed plasmonic-dielectric platform and the mechanisms to design complicated dispersion relations, which include many zero-group points and/or extremely-high-order dispersion cancellation, can be extended to linear waveguides. Consider the suggested layered dielectric structure, on top of the plasmonic substrate, to be axially uniform in the waveguiding direction but to have finite width $w$ in the transverse direction, being surrounded by a homogeneous dielectric of permittivity $\epsilon_c$, as shown in the inset of Fig. 4(i). Then, using the effective-index method, described in detail in Refs. [38, 41], we conclude that multiple guided modes exist, as long as the dispersion curve of the initial layered plasmonic structure of infinite width lies in the $\omega$-$k$ plane to the right of a SPP on a $\epsilon_p$-$\epsilon_c$ interface. This condition can be satisfied for a wide range of permittivities $\epsilon_c$, including at least $\epsilon_c \leq \min \{\epsilon_i, \epsilon\}$ and $\epsilon_c \geq \max \{\epsilon_i, \epsilon\}$ (e.g. for $\epsilon_c = \epsilon = 1$). Then, denoting $\beta$ the conserved wavevector along the one-dimensional (1d) translationally-invariant guiding direction, the same manipulations as before (e.g. choosing and/or fine-tuning $\{\omega_p, \epsilon_\infty, \epsilon_i, \epsilon_c, \epsilon, d_i\}$ for an $N^{th}$ order cancellation) can be implemented on the $\omega$-$\beta$ dispersion of the first few of these guided SPDP modes (typically the very first one would be chosen), since it digresses only a little from that of the infinite-width structure. An example is shown in Fig. 4: the structural parameters of the final design were calculated using the efficient but approximate semi-analytical effective-index method, however, finite-element [42] numerical calculations were also performed on the final structure to confirm the existence (the achievable discretization led to 1% convergence accuracy, not enough to also calculate dispersion coefficients) of the guided mode, shown in the inset of (i); note that guidance occurs, counter-intuitively, in the region of lower average dielectric permittivity [38]. Again, the method can take the weak material dispersion of the dielectrics into account, and similar considerations as before hold for the range of the simultaneously achievable $\omega_o$, $\beta_o$, $v_{go}$ and $N$.
APPLICATIONS

This unusually-high-order gvd and ad cancellation enabled by the currently proposed slow-wave plasmonic platform can have significant applications in the world of nanophotonics:

First of all, this material system can be designed to support slow and subwavelength propagation of short pulses that do not suffer any type of distortion (phase or amplitude) as they travel. This is a very important feature required for the design of very compact and efficient optical buffers and memory, for use in optical telecommunications and computing. The common figure-of-merit for the characterization of optical delay lines is the so-called ‘bandwidth-delay product’, which has been shown to be fundamentally limited by dispersion [29–34]. Indeed, using the procedure in Ref. [2], we find that: after propagation on length $L$ of a line with only gvd as in Eq. (1), Gaussian bit-pulses of initial width (standard deviation) $T_P (0)$, consisting of not-too-few ($\gg D_{k,N+2}/D_{k,N+1}$) optical cycles so that the $(N + 2)^{th}$-order dispersion can be ignored, are broadened to width

$$T_P^2 (L) = T_P^2 (0) \cdot \left[1 + \frac{D_{k,N+1} (\omega_p L/c)}{(\omega_p T_P (0))^{N+1} s_{N+1}}\right]^2,$$

where $s_{N+1} = \sqrt{(2N-1)!!/(N!)^2 - \text{mod}(N+1, 2)/(N!)^2}/2^N$, so, for bitrate $B = 1/T_B$, the optimal $T_P (0)$ and the maximum $L$ for distortionless ($T_P (L) / T_B \leq 1/4$) propagation are

$$T_P (0) / T_B = \frac{1}{4} \sqrt{\frac{N}{N+1}},$$

$$L/\lambda_0 = \frac{S_{N+1} (\omega_p T_B)^{N+1}}{2\pi |D_{k,N+1}|},$$

where $S_{N+1} = \sqrt{N^N/(N+1)^{N+1}/s_{N+1}/2^{2N+2}}$, and then the figure-of-merit, expressed as the number $M$ of bits that the line can store (namely the largest possible product of the bitrate $B$ times the distortionless time delay $\tau = L/|v_{go}|$), is

$$M = \tau / T_B = \frac{S_{N+1} (\omega_p T_B)^N}{|D_{k,N+1}| |v_{go}| / c}.$$

The huge benefit from having a delay line with a large $N$ is obvious from Eq. (7), since even the shortest possible pulse has at least one optical-cycle duration ($\omega_p T_P \geq 2\pi \Rightarrow \omega_p T_B \geq 8\pi$) and by allowing for slightly broader pulses (smaller bitrate) the N-polynomial increase in $M$ is then tremendous. Similarly, for a line with only ad as in Eq. (3), under the assumption $D_{\alpha,N} (\omega_p L/c) / (\omega_p T_P (0))^{N} \ll 1$, the leading-order contribution to pulse-broadening comes for $N$ even ($N = 2, 4, 6...$) and is

$$T_P^2 (L) = T_P^2 (0) \cdot \left[1 + \frac{D_{\alpha,N} (\omega_p L/c)}{(\omega_p T_P (0))^{N+1} s_{N}}\right]^2,$$

where $s_{N} = [(N - 1)!!/(N - 1)!]/2^{N-1}$, while for the combined effect of both gvd and ad mechanisms one needs to add the contributions from Eqs. (4) and (8), but then simple analytical formulas for $L$ and $M$ cannot be derived. To test the performance of our proposed plasmonic-dielectric structures, first note that, since $D_{\alpha,N} = D_{k,N+1} \cdot \gamma/2\omega_p$ and $\gamma/\omega_p \leq 1/100 \ll 1$, these structures are limited only by gvd for pulses of not-too-many ($\ll \omega_p/\gamma$) optical cycles, namely ad can be ignored and Eqs. (6) and (7) provide safe performance estimates. Using then the results of Figs. 2(a), 2(b) and 4, we find that these delay lines could, for example, hold respectively $63200$, $61700$ and $21400$ undistorted bits of 50-optical-cycle pulses ($\sim 1$Tbps at $\lambda_0 \equiv 2\pi c/\omega_0 = 1.3\mu m$ using Eq. (5)) with line lengths of respectively $\sim 45200\lambda_0$, $\sim 27000\lambda_0$ and $\sim 4700\lambda_0$, or that a $\sim 29$Gbps at $\lambda_0 = 1.3\mu m$ data-stream could travel undistorted (albeit significantly attenuated) on the delay line of Fig. 4 all the way from Boston, USA to Bangkok, Thailand (13730km), the same distance as a typical fiber at this zero-dispersion wavelength [2], only with $\sim 10^3$ times slower speed! This slow-light performance is orders of magnitude better than what has been demonstrated ever before [32], thus we can claim that this material system essentially does not suffer from any practical limit on the ‘bandwidth-delay product’.

Furthermore, the presently proposed system could greatly enhance the performance of a large variety of active optical devices (nonlinear, electronic, thermal etc). The rate of the underlying interaction depends strongly on the number of the participating photonic states (as suggested for example by Fermi’s Golden Rule [43]); therefore, if $\Delta\omega$ is the frequency bandwidth of the interaction, we will define here, as the figure-of-merit for a photonic structure, the enhancement ratio $M$ of the number of
states within $\Delta \omega$ for this structure compared to a uniform medium, whose refractive index equals the effective index of the photonic structure at the center frequency of the interaction. Then, using the standard procedure [43] and accounting for only one polarization state, we find that: for a dispersion relation as in Eq. (2) with $v_{go} = 0$, the associated density of 1d and 2d Surface-Politonic states close to $\omega_0$ (in particular so that $k - k_0 \ll k_0$) of a system with 1d-length $L$ and 2d-area $A$ respectively, is

$$g_{1d,2d}(\omega) = S \frac{\omega_0^2 k_0 A}{2 \pi c (N + 1)} \left[ \frac{(N + 1)!}{\omega_0 |D_{\omega,N+1}|} \right]^{\frac{1}{N+1}} |\omega - \omega_0|^{-\frac{N}{N+1}},$$

where $S = \text{mod}(N, 2) + \text{sign}(D_{\omega,N+1} : (\omega - \omega_0)) \in \{0, 1, 2\}$. It can be seen from Eq. (9) that both 1d and 2d densities of states interestingly have the same frequency dependence, which for $N > 1$ exhibits a new type of singularity (not classified by van Hove [44]) and for $N \gg 1$ approaches the non-integrable $|\omega - \omega_0|^{-1}$, as $\omega \to \omega_0$. Therefore, by integrating Eq. (9) over $\Delta \omega$ and then dividing, for each dimensionality, by the well-known [43] number-of-states result (accounting for both polarizations) inside a uniform medium of the same dimensionality and index $ck_0/\omega_0$, we find the enhancement factor

$$M = \frac{\omega_0}{ck_0} \left[ \frac{(N + 1)!}{2 |D_{\omega,N+1}|} \right]^{\frac{1}{N+1}} \left( \frac{\omega_0}{\Delta \omega} \right)^{\frac{N}{N+1}}.$$  

This result is again the same for both 1d and 2d, and shows that, as the interaction bandwidth $\Delta \omega/\omega_0 \ll 1$ decreases, for a system with a large $N$ the enhancement $M$ can be extremely large. To test our proposed plasmonic-dielectric structures, first note that, for the specific case of a zgv mode there is, with certainty, no theoretical upper limit on $N$ and dispersion can be cancelled to arbitrarily high order, since $k_0$ can in theory be arbitrarily large, leading to an extraordinary straight-horizontal-line segment of arbitrary $k$-length as the associated $\omega$-$k$ dispersion curve. Using now the results of Fig. 3, we find that the $\Delta \omega$-independent prefactor in Eq. (10) is $\sim 2.5$ and thus, for example, for atoms with typical due-to-collisions homogeneously-broadened linewidths $\Delta \omega/\omega_0 \approx 10^{-8}$ [45], the figure-of-merit is $M \sim 6.2 \cdot 10^6$, and the radiative lifetime of the atoms would be reduced approximately by this factor compared to that in a uniform medium, so even a dipole-forbidden transition could have a lifetime $\sim 200\text{ps}$ instead of its typical $\sim 1\text{ms}$ [45]! This enhancement is orders of magnitude better than for any other known translationally-invariant material system, and should therefore make the currently proposed plasmonic-dielectric platform a strong candidate for future active devices, while it could possibly enable also a variety of novel applications.

**PRACTICAL LIMITATIONS**

The practical limitations of the currently proposed plasmonic-dielectric structures need also to be discussed:

The most significant problem of integrated plasmonic structures is absorption and scattering loss. Although attenuation dispersion induced by plasmonic-material-type absorption losses was shown earlier in this article to be cancellable, unfortunately, for metals in the optical frequency regime and at room temperatures $\gamma/\omega_p \approx 1/100$, so the attenuation rate itself is strong ($Q \approx 100$ in time or $\alpha \approx 10\log_{10}(c/|v_{g}| \cdot \pi/100)$ dB/λ) in space, the latter increasing prohibitively with the wavevector, since $\min |\alpha| \approx (\gamma/2)/(\max |v_{g}|) \sim k$, as seen earlier and in Fig. 1(ii); these intrinsic losses may possibly be reducible by lowering temperature, as discussed in Ref. [38]. A significant cause for scattering losses ($\sim 1/|v_{g}|$) and spurious reflections ($\sim 1/v_{g}^2$) is geometrical imperfections and disorder [46], which are unavoidable and dictated by the fabrication tolerances: current atomic layer-by-layer deposition techniques (such as atomic layer deposition and molecular beam epitaxy) and nm-scale lithographic techniques (such as deep ultraviolet immersion lithography) are promising for high-quality mass-producible plasmonic structures with operation at not-too-large $k$-values, such as those in Figs. 2,3,4. Moreover, since slow light results in a significant reduction of the required device sizes [47], the overall suffered loss by a plasmonic device might still be in tolerable levels.

Even under the assumption of perfect materials and geometry, there is also the issue of coupling light efficiently into such a broadband slow and subwavelength system, since any common material system would be highly $k$-mismatched. This problem could be addressed for positive-group-velocity slow structures by adiabatically changing the dimensions of the layered structure, so as to always have dispersion cancellation at a fixed frequency, while adiabatically increasing group velocity, until it can be similar to that of a common medium. For negative-group-velocity structures also adiabatic transitions in time would be required.

**EXTENSIONS**

The dispersion-manipulation methods presented so far here can be applied also to different geometries and material systems: Even though this light-manipulation scheme was shown for rectangular waveguides on a plasmonic substrate, it should be applicable straightforwardly also to cylindrical or elliptical fibers with a plasmonic core.
Moreover, instead of many adjacent dielectric layers on a single plasmonic material, one could also use one or many dielectrics \( \{ \epsilon_i \} \) on multiple adjacent plasmonic layers \( \{ \omega_{p,j}, \epsilon_{\infty,j} \} \) to vary the limiting \( \omega_c = \omega_{p,j}/\sqrt{\epsilon_{\infty,j}+\epsilon_i} \) at will. In fact, instead of discrete layers, SPDP modes from continuous \( \epsilon > 0 \) and \( \epsilon_p < 0 \) distributions (stuck together on some interface) can, in principle, also be designed.

Finally, devices operating in lower frequency regimes can be designed by exploiting Surface-AtomDielectric-, ExcitonDielectric- or PhononDielectric-Polaritons on interfaces between dielectrics and materials with permittivities such as \( \tilde{\epsilon}_p(\omega) = \epsilon_{\infty} - \omega_p^2/(\omega^2 - \omega_0^2 + i\gamma \omega) \), where \( \omega_0 \) is the atomic, excitonic or phononic resonance frequency. In this case, some of the above problems are mitigated, since these polaritonic materials often exhibit smaller intrinsic losses (smaller \( \gamma/\omega_p \)) and, at lower frequencies, also large \( k \)-values could translate to fabricatable devices with relatively small surface disorder.

**CONCLUSION**

In conclusion, in this Article we are proposing a novel dielectric-plasmonic material platform, which allows the design of optical waveguiding systems, in which the dispersion relation of slow subwavelength light can be tailored to exhibit a large variety of exotic behaviors, such as multiple zero-group-velocity points with intermediate regions of positive and negative group velocity or simultaneous cancellation of group-velocity and attenuation dispersion to extremely high orders for positive, negative or zero group velocity. Therefore, this platform attacks systematically a fundamental aspect of optical physics (dispersion), potentially enabling thus great technological achievements in nanophotonics (such as efficient and compact delay lines and active devices).

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