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Ziv-Zakai Bound on Time-of-Arrival Estimation with Statistical Channel Knowledge at the Receiver

(Invited Paper)

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Abstract—Time-of-arrival (TOA) based localization plays an important role due to the possibility to exploit the fine delay resolution property when wideband signals are adopted. This paper investigates lower bounds on TOA estimation error for wideband and ultrawide bandwidth (UWB) ranging systems operating in realistic multipath environments. In particular, analytical expressions for the Ziv-Zakai bound (ZZB) on TOA estimation error for multipath channels are provided and discussed under different operating conditions. Using these bounds it is shown how the a priori knowledge about the multipath characteristics can be accounted for in the TOA estimation performance. These bounds serve as useful performance benchmarks for the design of practical TOA estimators.

Index Terms—Ranging, time-of-arrival estimation, Ziv-Zakai bound, UWB, localization

I. INTRODUCTION

In the coming years, we will see the emergence of high-definition situation-aware (HDSA) applications which require localization systems with sub-meter accuracy operating even in harsh propagation environments such as inside buildings and in caves, where Global Positioning System (GPS) typically fails. Among different localization techniques, those based on distance estimation (ranging) are more suitable for high localization accuracy when low complexity devices are available [1]–[3]. One fundamental approach to estimate the distance is through signal time-of-arrival (TOA) measurements. Ranging accuracy is related to signal bandwidth and hence the use of wideband or ultrawide bandwidth (UWB) signals is attractive for ranging applications. In particular, UWB technology offers the potential of achieving high ranging accuracy even in harsh environments [1]–[3], due to its ability to resolve multipath and penetrate obstacles [4]–[6]. In rich multipath channels, the first path is often not the strongest, making estimation of the TOA challenging [7]. Besides specific TOA estimation algorithms, estimation error bounds play a fundamental role since they serve as useful performance benchmarks for the design of TOA estimators. Cramér-Rao bound (CRB) has been used widely as a performance benchmark as it gives the performance limit of any unbiased estimator. It is well known, however, that the CRB is not accurate at low and moderate signal-to-noise ratios (SNRs). In fact, the performance of the TOA estimator, like all non-linear estimators, is characterized by the presence of distinct SNR regions (i.e., low, medium, and high SNRs) corresponding to different modes of operation. This behaviour is referred to as the threshold effect and it is not accounted for by the CRB [8]. Therefore other bounds, which are more complicated but tighter than the CRB, have been proposed in the literature. Among them the Ziv-Zakai bound (ZZB), with its improved versions such as the Bellini-Tartara bound [9] and the Chazan-Zakai-Ziv bound [10], can be applied to a wider range of SNRs.

In [11] the approach developed in [12] for the estimation of the relative delay between two sensors emitting noise-like signals is extended to that emitting UWB signals. The work [13] evaluates the ZZB for Gaussian signals assuming perfect channel knowledge at the receiver. A few results are present for the case where the receiver has a partial or no knowledge about the channel [14], [15]. In particular, in [14] the ZZB using measured data as well as Monte Carlo generated channel pulse responses (CPRs) is investigated, whereas [15] derives the ZZB using second order statistics approach by modeling the received signal as non-stationary Gaussian random process.

Since the a priori channel statistical information can be usefully exploited in the estimator to improve the estimation accuracy, in this paper we derive analytical expressions for the ZZB on TOA estimation in wideband and UWB ranging systems operating in realistic multipath environments assuming the estimator has a priori statistical knowledge on the multipath phenomena.

II. SIGNAL AND CHANNEL MODELS

We assume that a single pulse $p(t)$ with duration $T_p$, energy $E_p$ and normalized auto-correlation function $p_p(\tau)$ is transmitted. Generally, $p(t)$ is a bandpass signal with bandwidth $W$ and central frequency $f_0$.\(^1\) In practice, it is generated starting

\(^1\)In general $p(t)$ can represent a multiple access signal such as direct sequence or time-hopping (TH) [16].
\(^2\)Since a band-limited signal cannot also be time-limited, we assume it can be truncated in time to duration $T_p$ with some small approximation error.
from a real baseband pulse \( g(t) \), that is,

\[
p(t) = \sqrt{2 E_p} g(t) \cos(2\pi f_0 t)
\]

where \( g(t) \) has unitary energy.

When \( p(t) \) is transmitted through a channel with multipath and thermal noise the received signal can be represented by

\[
r(t) = s(t - \tau) + n(t),
\]

where \( s(t) \) is the CPR, \( \tau \) is the TOA of the received signal to be estimated and \( n(t) \) is the additive white Gaussian noise (AWGN) with zero mean and two-sided spectral density \( N_0/2 \).

We consider a typical \( L \)-paths multipath channel model

\[
s(t) = \sqrt{2 E_p} \sum_{i=1}^{L} \mathbb{R} \left\{ \alpha_l g(t - \tau_l) e^{j 2\pi f_0 (t - \tau_l)} \right\},
\]

where \( L \) is the number of multipath components, \( \alpha_i \)'s are the complex path gains and \( \tau_i \)'s are the path delays offsets with respect to the TOA \( \tau \) of the first path (obviously it is \( \tau_i = 0 \)). We consider a tapped delay line channel model where \( \tau_l = \Delta \cdot (l - 1) \) with \( \Delta \) denoting the width of the resolvable time bin, and \( T_\Delta \equiv \Delta \cdot (L - 1) \) is the dispersion of the channel [5], [6]. We further consider resolvable multipath, that is, \( |\tau_i - \tau_l| > T_p \) for \( i \neq l \).\(^3\)

The average power gain of the \( l \)-th path is \( E_l = \mathbb{E} \{ |\alpha_l|^2 \} \), where \( \mathbb{E} \{ \cdot \} \) is the statistical expectation operator. Without loss of generality we assume \( \sum_{l=1}^{L} E_l = 1 \), so that \( E_p \) represents the average received energy per transmitted pulse. By writing \( \alpha_l = v_l + j v_{l+1} \), where \( v_l \) and \( v_{l+1} \) for \( l = 1, 2, \ldots, L \) are, respectively, the real and imaginary parts of \( \alpha_l \), (3) can be rewritten as

\[
s(t) = \sum_{l=1}^{L} v_l p_l(t - \tau_l) + v_{l+1} p_0(t - \tau_l)
\]

where \( p_l(t) \equiv p(t) = \sqrt{2 E_p} g(t) \cos(2\pi f_0 t) \) and \( p_0(t) \equiv -\sqrt{2 E_p} g(t) \sin(2\pi f_0 t) \). For each set of channel parameters \( \mathbf{v} = \{v_1, v_2, \ldots, v_{2L}\} \) we obtain a sample \( s(t) \) of the random process \( s(t) \). We denote \( p_\mathbf{v}(s) \) the (joint) probability distribution function (PDF) of the vector \( \mathbf{v} \).

We will see that the derivation of the bound is strictly dependent on the channel model considered. In general the evaluation of the bound for channel models with generic fading statistics may not be analytically tractable. Therefore we will restrict our analysis to the widely used Rice-Rayleigh channel models. Alternatively, one could adopt Monte Carlo tools and approximate the result considering a finite set of channel realizations as done in [14], [17].

In the Rice-Rayleigh multipath channel model \( v_l/s \) are independent Gaussian random variables with mean

\[
\mu_l = \mathbb{E} \{ v_l \} = \begin{cases} s_l & \text{for } 1 \leq l \leq L, \\ 0 & \text{for } L < l \leq 2L \end{cases}
\]

and variance \( \mathbb{V} \{ v_l \} = \mathbb{V} \{ v_{l+1} \} = \sigma_e^2 \) for \( 1 \leq l \leq L \).

\(^3\)When the multipath is not resolvable, the following analysis gives weaker lower bounds on the achievable performance.

In this case \( p_\mathbf{v}(v) \) is a joint Gaussian PDF. Parameters \( s_l \) and \( \sigma_e^2 \) are, respectively, the amplitude of the specular component (when present) and the gain of the scattered component related to the \( l \)-th path. The average power gain of the \( l \)-th path is \( \Lambda_l = \mu_l^2 + 2 \sigma_e^2 \). The Nakagami-\( m \) fading, extensively adopted to characterize UWB channels, can be well approximated by the Rice statistics with a suitable mapping of parameters [6].

### III. The Ziv-Zakai Lower Bound

The goal is to obtain the estimate \( \hat{\tau} \) of \( \tau \) by observing \( r(t) \) in the interval \( [0, T_{ob}] \) in the presence of signal distortion due to multipath. In the absence of other information, we assume \( \tau \) to be unknown and randomly distributed in the interval \( [0, T_a] \), with \( T_a + T_d < T_{ob} \), so that all the multipath components fall within the observation interval \( T_{ob} \). Several techniques can be adopted to solve this problem (see for example [1] and the references therein). In this paper an improved bound based on the ZZB is derived and discussed considering statistical channel knowledge at the receiver.

The ZZB can be derived starting from the following general identity for mean square error (MSE) estimation\(^4\)

\[
\mathbb{E} \{ \xi^2 \} = \frac{1}{2} \int_{-\infty}^{\infty} z \mathbb{P} \{ |\xi| \geq z/2 \} \, dz
\]

and then by finding a lower bound on \( \mathbb{P} \{ |\xi| \geq z/2 \} \), where \( \xi = \hat{\tau} - \tau \) is the estimation error [9]. In particular, \( \mathbb{P} \{ |\xi| \geq z/2 \} \) is related to the error probability of a classical binary detection scheme with equally probable hypothesis\(^5\)

\[
H_1 : r(t) \sim p \{ r(t)|\tau \}
\]

\[
H_2 : r(t) \sim p \{ r(t)|\tau \}
\]

when using a suboptimum decision rule as described in [10]. It can be shown that (6) can be lower bounded using the error probability corresponding to the optimum decision rule based on the likelihood ratio test (LRT)

\[
\Lambda (r(t)) = \frac{p \{ r(t)|\tau \}}{p \{ r(t)|\tau + z \}}.
\]

When \( \tau \) is uniformly distributed in \( [0, T_a] \), the ZZB is given by [9], [10]

\[
\text{ZZB} = \frac{1}{T_a} \int_0^{T_a} z (T_a - z) P_{\text{min}}(z) \, dz
\]

where \( P_{\text{min}}(z) \) is the error probability corresponding to the optimum decision rule. In an AWGN channel this error probability is

\[
P_{\text{min}}(z) = Q \left( \sqrt{\text{SNR} \cdot (1 - P_{\text{b}}(z))} \right)
\]

where \( Q (\cdot) \) is the Gaussian Q-function and \( \text{SNR} = E_p/N_0 \) [18]. In more complex propagation scenarios, the main challenge in (9) is to design the binary detection scheme based on (8) and to derive a tractable expression for its performance \( P_{\text{min}}(z) \) [17].

\(^4\)Here the expectation is performed with respect to \( \tau \) and \( r(t) \).

\(^5\)The notation \( \sim p \{ \cdot \} \) stands for “with distribution \( p \{ \cdot \} \)."
IV. DESIGN OF THE OPTIMUM DETECTOR

In the presence of multipath \( P_{\min}(z) \) corresponds to the minimum attainable error probability of the binary composite hypothesis testing problem that arises due to the presence of nuisance parameters \( \nu \).

According to the channel model considered, the conditional PDF expression of \( r(t) \), conditioned on \( \nu \), is given by \[ p\{r(t)|\nu, v\} = \eta \exp \left\{ - \int_0^{T_m} [r(t) - s(t - \tau|v)]^2 dt \right\} \] (11)

with \( \eta \) denoting a normalization constant.\(^6\)

Substituting (4) into (11), together with the resolvable multipath assumption, gives

\[ p\{r(t)|\nu, v\} = \eta \exp \left\{ -\text{SNR} \sum_{k=1}^{2L} \left( v_k^2 - 2v_k q_k(\tau) \right) \right\} \] (12)

where we have defined

\[ q_k(\tau) \triangleq \left\{ \begin{array}{ll}
\frac{1}{\sigma_k^2} \int_0^{T_m} r(t) p_k(t) dt & 1 \leq k \leq L \\
\frac{1}{\sigma_k^2} \int_0^{T_m} r(t) p_{k-L}(t) dt & L < k \leq 2L
\end{array} \right. \] (13)

To derive the bound (9) we need the marginal PDF \( p\{r(t)|\nu\} \) that can be evaluated by averaging the conditional PDF \( p\{r(t)|\nu, v\} \) over all nuisance parameters \( \nu \). The marginal PDF for Rayleigh/Rice fading is

\[ p\{r(t)|\nu\} = \mathbb{E}\{p\{r(t)|\nu, v\}\} \approx \prod_{k=1}^{2L} \frac{1}{\sqrt{1 + \text{SNR} 2\sigma_k^2}} \exp\left\{ -\text{SNR} \sum_{k=1}^{2L} \left( \frac{\mu_k}{2\sigma_k^2} + \text{SNR} q_k(\tau) \right)^2 \right\} \] (14)

The corresponding log-likelihood ratio (LLR) is

\[ l(r(t)) = \ln \frac{p\{r(t)|\nu\}}{p\{r(t)|\nu+z\}} = \sum_{k=1}^{2L} A_k \] (15)

where \( A_k \triangleq 2\sigma_k^2/(1 + 2\sigma_k^2 \text{SNR}) \). The detector evaluates \( l(r(t)) \) and, if it is greater than zero, decides for hypothesis \( H_1 \); hypothesis \( H_2 \) otherwise. Note that the LLR (15) leads to the optimal binary pulse position modulation (PPM) (partially coherent) Rake receiver, which scheme is depicted in Fig. 1. To the best of the authors’ knowledge both the structure and the performance of this receiver for Rice fading channels have not been investigated in the literature.

\(^6\)The exact value does not affect the final result.

V. PERFORMANCE OF THE OPTIMUM DETECTOR

The probability of error \( P_{\min}(z) \), required to evaluate the ZZB in (9), is given by

\[ P_{\min}(z) = \frac{1}{2} P_e|_{H_1}(z) + \frac{1}{2} P_e|_{H_2}(z) \] (16)

where \( P_e|_{H_1}(z) \) and \( P_e|_{H_2}(z) \) are the error probabilities conditioned to hypothesis \( H_1 \) and \( H_2 \) respectively. From symmetry, \( P_e|_{H_1}(z) = P_e|_{H_2}(z) \) so that it is sufficient to evaluate \( P_e|_{H_1}(z) \). From (15), under hypothesis \( H_1 \), the decision variable \( y \) is

\[ y = l(r(t)|H_1) = \sum_{k=1}^{2L} A_k \left[ \left( X_k^{(1)} \right)^2 - \left( X_k^{(2)} \right)^2 \right], \] (17)

where

\[ X_k^{(1)} \triangleq \frac{\mu_k}{2\sigma_k^2} + \text{SNR} q_k^{(1)} \] (18)

\[ X_k^{(2)} \triangleq \frac{\mu_k}{2\sigma_k^2} + \text{SNR} q_k^{(2)} \] (19)

and \( q_k^{(1)} \triangleq q_k(\tau)|_{H_1} \) and \( q_k^{(2)} \triangleq q_k(\tau + z)|_{H_1} \) for \( k = 1, 2, \ldots, 2L \). The probability of error required to compute the ZZB in (9) is therefore

\[ P_{\min}(z) = P_e|_{H_1}(z) = \mathbb{P}\{y < 0|H_1\}. \] (20)

To evaluate (20) it is necessary to characterize the statistics of random variable (RV) \( y \). For convenience we write the delay \( z \) as \( z = m \Delta + \delta \), where \( m \) is an integer and \( -\Delta/2 < \delta \leq \Delta/2 \).

It follows from Appendix A that \( X_k^{(1)} \) and \( X_k^{(2)} \) are normally distributed, respectively, with mean

\[ m_k^{(1)} = \frac{\mu_k}{2\sigma_k^2} + \mu_k \text{SNR} \] (21)

\[ m_k^{(2)} = \begin{cases} \frac{\mu_k}{2\sigma_k^2} + \mu_{k+m} \text{SNR} \rho_p(\delta) & 1 \leq k \leq L-m \\ \frac{\mu_k}{2\sigma_k^2} & L-m < k \leq L \\ 2L-m < k \leq 2L \end{cases} \] (22)

and variance

\[ \sigma_k^{(1)} = \text{SNR}^2 \left( \sigma_k^2 + \frac{\sigma_N^2}{\text{SNR}^2} \right) \] (23)

\[ \sigma_k^{(2)} = \begin{cases} \text{SNR}^2 \left( \sigma_{k+m}^2 + \rho_p(\delta)^2 + \sigma_N^2 \right) & 1 \leq k \leq L-m \\ \text{SNR}^2 \sigma_N^2 & L-m < k \leq L \\ 2L-m < k \leq 2L \end{cases} \] (24)

where \( \sigma_N^2 = 1/(2 \text{SNR}) \). The auto-covariance between \( X_k^{(1)} \) and \( X_i^{(2)} \) is

\[ R_{k,i} = \mathbb{E}\left\{ \left( X_k^{(1)} - m_k^{(1)} \right) \cdot \left( X_i^{(2)} - m_i^{(2)} \right) \right\} = \begin{cases} \text{SNR}^2 \rho_p(\delta) \left( \sigma_k^2 + \sigma_i^2 \right) & \text{for } i = k-m \\ 0 & \text{otherwise} \end{cases} \] (25)
It follows that \( y \) in (17) results to be a non central quadratic form in real not identically distributed Gaussian correlated RVs. It will be useful to derive the moment generating function (MGF) of \( y \) defined as \( \Phi_y(s) \triangleq \mathbb{E}\{e^{sy}\} \). The MGF for the general case can be found in [19]. However, considering the particular structure of (25), a more explicit expression of \( \Phi_y(s) \) can be found. We gather all the pairs of correlated RVs in (17), \( X_k^{(1)} \) and \( X_k^{(2)} \) with \( i = k - m \), and form the new RVs \( z_k,i = A_k \left( X_k^{(1)} \right)^2 - A_i \left( X_i^{(2)} \right)^2 \) with MGF \( \Psi_{k,i}(s) \) (in total we obtain \( 2(L - m) \) terms). The remaining terms are independent of each other. It follows that (17) reduces to a weighted sum of independent RVs where their MGFs can be derived as the product MGFs related to each term of the sum

\[
\Phi_y(s) = \prod_{k=1}^m \Phi_k^{(1)}(s) \cdot \prod_{k=m+1}^L \Psi_{k,k-m}(s) \cdot \prod_{k=L+1}^{L+m} \Phi_k^{(1)}(s) \cdot \prod_{k=m+1+L}^{2L} \Psi_{k,k-m}(s) \cdot \prod_{k=2L+1+m}^{L+m} \Phi_k^{(2)}(s)
\]

(26)

where the expressions of \( \Phi_k^{(1)}(s) \), \( \Phi_k^{(2)}(s) \), and \( \Psi_{k,i}(s) \) are given in Appendix B. Several techniques are available to evaluate (20) starting from the MGF (26). The classical way is using the Inversion Theorem

\[
P_{\min}(z) = P\{y < 0\} = \frac{1}{\pi} \Re\left\{ \int_0^\infty \Phi_y(jv) dv \right\}.
\]

(27)

VI. NUMERICAL RESULTS IN THE PRESENCE OF MULTIPATH

In this section, we provide numerical illustrations using analytical results obtained in previous sections. We consider a band-pass pulse with root raised cosine (RRC) envelope with center frequency \( f_0 = 4 \) GHz, roll-off \( \nu = 0.6 \), \( \tau_p = 1 \) ns for our numerical examples.

In Figs. 2 and 3, the root mean square error (RMSE) for the ZZB and the CRB against the SNR are plotted for uniform power delay profile (PDP) (\( \Delta_l = 1 \) for each \( l \), with \( \Delta = 2 \) ns, in different multipath conditions assuming the receiver has a priori statistical channel knowledge. In particular, Fig. 2 considers all paths affected by Rayleigh fading. By increasing the number \( L \) of paths, the ZZB decreases, especially at large SNR, due to the diversity gain the estimator achieves thanks to the a priori statistical channel knowledge. As can be noted, the presence of multipath has a strong effect on the lower bound compared to the AWGN case. This effect is not contemplated by the CRB that appears to be extremely looser also for high SNR.

Fig. 3 considers Ricean fading for the first path with \( K = 9.3 \) dB. Also in this case there is a diversity gain which leads to a rapid convergence of the ZZB to the CRB for large SNR when \( L > 2 \). Again a significant deficiency of the CRB compared to the ZZB in predicting the MSE of TOA estimation is observed for a wide range of practical SNR values.

We now consider a multipath channel with exponential PDP with average path power gains given by

\[
\Lambda_l = \frac{(e^{\Delta_l}/\varepsilon_l - 1)e^{-\Delta_l(1-\varepsilon_l)/\varepsilon_l}}{e^{\Delta_l/\varepsilon_l}(1 - e^{-L\Delta_l/\varepsilon_l})}
\]

(28)

and Nagagami-\( m \) fading with Nakagami severity parameter \( m_l = m_1 e^{-(l-1)/\varepsilon_m} \) for \( l = 1, 2, ..., L \) [5], [6]. The parameter \( \varepsilon_m \) describes the multipath spread of the channel. We consider \( L = 8, \varepsilon_m = 6 \) ns, \( \varepsilon_m = 4 \), and \( m_1 = 3 \). The comparison between different channel models is reported in Fig. 4 where the ZZB in AWGN and in multipath channels with exponential PDP (28), Rayleigh and Rice uniform PDP is shown. It can be noted that the ZZB accounts for the impact of the fading statistic as well as the multipath PDP. The Rayleigh fading provides the worst performance at high SNR.

VII. CONCLUSIONS

Analytical expressions for the ZZB on TOA estimation error for wideband and UWB ranging systems operating in realistic multipath environments are provided by exploiting the a priori knowledge on the multipath phenomena. It is shown that for practical SNR values, the classical CRB gives too optimistic results and it is not able to properly account for the effect of multipath, whereas the ZZB provides improved bounds. Through the analysis of ZZB in the presence of multipath, it is shown how a priori statistical channel knowledge can improve significantly the estimation performance, especially in the presence of Ricean or rich multipath.

APPENDIX A

For resolvable multipath channel, (2), (3), and (13) implies that

\[
q_k^{(1)} = q_k(\tau)|_{\mathcal{H}_k} = \left. \frac{1}{E_p} \int_0^{T_a} \nu_k p_k^2(t - \tau - \tau_k) dt \right. \\
+ \left. \frac{1}{E_p} \int_0^{T_a} \nu(t) p_l(t - \tau - \tau_k) dt = v_k + n_k^{(1)} \right)
\]

(30)

for \( 1 \leq k \leq L \), where \( n_k^{(1)} \) is a Gaussian r.v., independent of \( v_k \), with zero mean and variance \( \sigma^2 \). The parameter \( \varepsilon_l \) is a Gaussian r.v., independent of \( v_k \), with zero mean and variance \( \sigma^2 \). It follows that \( \{q_k^{(1)}\} \), for \( k = 1, 2, ..., L \), results to be a set of independent Gaussian distributed RVs with mean \( \mu_k \) and variance \( \sigma^2 + \sigma^2 \) independent on \( \tau \). The same result holds for \( L < k \leq 2L \).

The evaluation of \( q_k^{(2)} \) is slightly more involved. We first confine our attention to \( 1 \leq k \leq L \). When \( k > L - m \) multipath components fall outside the template of the bottom half correlator in Fig. 1, and hence \( q_k^{(2)} \) contains only the noise term \( n_k^{(2)} \). Instead, for \( k \leq L - m \) we have

\[
q_k^{(2)} = q_k(\tau + z)|_{\mathcal{H}_k} = v_{k+m} \left. \frac{1}{E_p} \int_{-\Delta/2}^{\Delta/2} \nu(t)p(t - \delta) dt \right. + n_k^{(2)} \\
= v_{k+m} \rho_p(\delta) + n_k^{(2)},
\]

(31)
that is, the $k+m$ multipath component falls inside the template of the correlator in Fig. 1 matched to the $k$th path. The same procedure follows for $L < k \leq 2L$ leading to

$$q_k^{(2)} = \begin{cases} v_{k+m} \rho_\delta(\delta) + n_k^{(2)} & 1 \leq k \leq L - m, \\ n_k^{(2)} & L < k \leq 2L - m, \\ L - m < k \leq L, \\ 2L - m < k \leq 2L \end{cases} \quad (32)$$

such that

$$q_k^{(2)} \sim \begin{cases} N\left(\mu_{k+m} \rho_\delta(\delta), \sigma_{k+m}^2 \rho_\delta^2(\delta) + \sigma_k^2\right) & 1 \leq k \leq L - m, \\ N(0, \sigma_k^2) & L < k \leq 2L - m, \\ L - m < k \leq L, \\ 2L - m < k \leq 2L \end{cases} \quad (33)$$

Also $q_k^{(2)}$ does not depend on $\tau$ but it depends on $m$ and $\delta$. It must be observed that RVs $\{q_k^{(2)}\}$ are independent, but each $q_k^{(2)}$ is correlated to $q_i^{(1)}$ for $i = k - m$.

**APPENDIX B**

Since $A_k \left(X_k^{(1)}\right)^2$ and $-A_k \left(X_k^{(2)}\right)^2$ are squares of Gaussian RVs then their MGFs are, respectively, $\Phi^{(1)}(s) = \Phi(s A_k, m_k^{(1)}, w_k^{(1)})$ and $\Phi^{(2)}(s) = \Phi(-s A_k, m_k^{(2)}, w_k^{(2)})$, where [18]

$$\Phi(s, \mu, w) = \frac{\exp\left\{\frac{s \mu}{1 - 2sw}\right\}}{(1 - 2sw)^{\frac{1}{2}}} \quad (34)$$

Considering that $z_{k,i}$ is a weighted sum of the square of two correlated Gaussian RVs, using the result in [19] we obtain the corresponding MGF in (29).

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\[ \Psi_{k,i}(s) = \mathbb{E}\left\{ \exp\left\{ s \left[ A_k \left(X_k^{(1)}\right)^2 - A_i \left(X_i^{(2)}\right)^2 \right] \right\} \right\} \]

\[ = \exp\left\{ -\frac{s A_k m_k^{(1)} + A_k 2s A_i m_k^{(2)} + m_k^{(1)} (m_k^{(1)} - 2s A_i m_k^{(2)} + 2s A_k m_k^{(2)} - 2s A_k R_{k,i} - w_k^{(1)} w_k^{(2)})}{1 - 2s A_k w_k^{(2)}} \right\} \]

\[ \times \left( \sqrt{\frac{w_k^{(1)} w_k^{(2)} (1 - 2s A_k w_k^{(1)} w_k^{(2)})}{w_k^{(1)} w_k^{(2)}}} \right)^{29} \]
Fig. 1. Optimum binary detector in the presence of Rayleigh/Rice fading.

Fig. 2. ZZB and CRB on the RMSE as a function of SNR for the uniform PDP with Rayleigh fading. Band-pass pulse with RRC envelope, $f_0 = 4$ GHz, $\nu = 0.6$, $\tau_p = 1$ ns and $T_a = 100$ ns considered.

Fig. 3. ZZB and CRB on the RMSE as a function of SNR for the uniform PDP with Ricean fading for the first path with $K = 9.3$ dB. Band-pass pulse with RRC envelope, $f_0 = 4$ GHz, $\nu = 0.6$, $\tau_p = 1$ ns and $T_a = 100$ ns considered.

Fig. 4. ZZB and CRB on the RMSE as a function of SNR. Comparison between different channel models. Band-pass pulse with RRC envelope, $f_0 = 4$ GHz, $\nu = 0.6$, $\tau_p = 1$ ns and $T_a = 100$ ns considered.