Measurements of time-dependent CP asymmetries in
$B^0 \rightarrow D(\ast)^+ D(\ast)^-$
decays...
Measurements of time-dependent $CP$ asymmetries in $B^0 \to D^{(*)} + D^{(*)}$ decays

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We present new measurements of time-dependent CP asymmetries for $B^0 \rightarrow D^{(*)+}D^{(*)-}$ decays using $(467 \pm 5) \times 10^6 \ BB$ pairs collected with the BABAR detector located at the PEP-II B Factory at the Stanford Linear Accelerator Center. We determine the CP-odd fraction of the $B^0 \rightarrow D^{(*)+}D^{(*)-}$ decays to be $R_\perp = 0.158 \pm 0.028 \pm 0.006$ and find CP asymmetry parameters $S_+ = -0.76 \pm 0.16 \pm 0.04$ and $C_+ = +0.00 \pm 0.12 \pm 0.02$ for the CP-even component of this decay and $S_\perp = -1.80 \pm 0.70 \pm 0.16$ and $C_\perp = +0.41 \pm 0.49 \pm 0.08$ for the CP-odd component. We measure $S = -0.63 \pm 0.36 \pm 0.05$ and $C = -0.07 \pm 0.23 \pm 0.03$ for $B^0 \rightarrow D^{(*)+}D^{(*)-}$, $S = -0.62 \pm 0.21 \pm 0.03$ and $C = +0.08 \pm 0.17 \pm 0.04$ for $B^0 \rightarrow D^{(*)+}D^{-}$, and $S = -0.73 \pm 0.23 \pm 0.05$ and $C = +0.00 \pm 0.17 \pm 0.03$ for $B^0 \rightarrow D^{+}D^{-}$. For the $B^0 \rightarrow D^{(*)+}D^{-}$ decays, we also determine the CP-violating asymmetry $A = +0.008 \pm 0.048 \pm 0.013$. In each case, the first uncertainty is statistical and the second is systematic. The measured values for the asymmetries are all consistent with the standard model.

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I. INTRODUCTION

In the standard model (SM), CP violation is described by the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, $V$ [1,2]. In particular, an irreducible complex phase in the $3 \times 3$ mixing matrix is the source of all SM CP violation. Both the BABAR [3] and Belle [4] collaborations have measured the CP parameter $\sin2\beta$, where $\beta = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$, in $b \rightarrow (c\bar{c})s$ processes.

The leading-order diagrams contributing to $B^0 \rightarrow D^{(*)+}D^{(*)-}$ decays are shown in Fig. 1, where the color-favored tree-diagram of Fig. 1(a) dominates. When neglecting the penguin (loop) amplitude in Fig. 1(b), the mixing-induced CP asymmetry of $B^0 \rightarrow D^{(*)+}D^{(*)-}$, denoted $S$, is also determined by $\sin2\beta$ [5]. The effect of neglecting the penguin amplitude has been estimated in models based on factorization and heavy quark symmetry, and the corrections are expected to be a few percent [6,7]. Large deviations of $S$ in $B^0 \rightarrow D^{(*)+}D^{(*)-}$ decays with respect to $\sin2\beta$ determined from $b \rightarrow (c\bar{c})s$ transitions could indicate physics beyond the SM [8–10].

The CP asymmetries of $B^0 \rightarrow D^{(*)+}D^{(*)-}$ decays have been studied by both the BABAR [11,12] and Belle [13–15] collaborations. In the SM, the direct CP asymmetry $C$, defined in Sec. IV, for the $B^0 \rightarrow D^{(*)+}D^{(*)-}$ decays is expected to be near zero. The Belle Collaboration has observed a 3.2 sigma deviation of $C$ from zero in the $B^0 \rightarrow D^{+}D^{-}$ channel [15]. This has not been observed by BABAR nor has it been seen in other $B^0 \rightarrow D^{(*)+}D^{(*)-}$ decay modes, which involve the same quark-level diagrams. As was pointed out in [9], understanding any possible asymmetries in these decays is important to constraining theoretical models.

In this article, we update the previous measurements of CP asymmetry parameters in $B^0 \rightarrow D^{(*)+}D^{(*)-}$ decays [11,12], including the CP-odd fraction for $B^0 \rightarrow D^{(*)+}D^{-}$, using the final BABAR data sample. Charge conjugate decays are included implicitly in expressions throughout this article unless otherwise indicated.

II. DETECTOR, DATA SAMPLE, AND RECONSTRUCTION

A. The BABAR detector

The data used in this analysis were collected with the BABAR detector [16] operating at the PEP-II B Factory located at the Stanford Linear Accelerator Center (SLAC). The BABAR dataset comprises $(467 \pm 5) \times 10^6 \ BB$ pairs collected from 1999 to 2007 at the center-of-mass (CM) energy $\sqrt{s} = 10.58$ GeV, corresponding to the $Y(4S)$ resonance. We use GEANT4-based [17] Monte Carlo (MC)
simulation to study backgrounds and to validate the analysis procedures. The asymmetric energies of the PEP-II beams provide an ideal environment to study time-dependent CP phenomena in the $B^0 - B^0$ system by boosting the $\Upsilon(4S)$ in the laboratory frame, thus making possible precise determination of the decay vertices of the two $B$ meson daughters. BABAR employs a five-layer silicon vertex tracker (SVT) close to the interaction region to provide precise vertex measurements and to track low momentum charged particles. A drift chamber (DCH) provides excellent momentum measurement of charged particles. Particle identification of kaons and pions is primarily derived from ionization losses in the SVT and DCH and from measurements of photons produced in the fused silica bars of a ring-imaging Cherenkov light detector (DIRC). A CsI(Tl) crystal-based electromagnetic calorimeter enables reconstruction of photons and identification of electrons. All of these systems operate within a 1.5 T superconducting solenoid, whose iron flux return is instrumented to detect charged particles. These improvements have increased the sensitivity of the measurements in this paper has increased more than would be expected by just the increment in luminosity.

B. Candidate reconstruction and selection

The candidates used in this analysis are formed from oppositely charged $D^{(*)}$ mesons where we include the $D^{(*)}$ decay modes $D^{(*)} \rightarrow D^0 \pi^+$ and $D^{(*)} \rightarrow D^+ \pi^0$ and $D$ decay modes $D^0 \rightarrow K^- \pi^+$, $D^0 \rightarrow K^- \pi^+ \pi^0$, $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$, $D^0 \rightarrow K^0_S \pi^+ \pi^-$, and $D^+ \rightarrow K^- \pi^+ \pi^+$. In the $B^0 \rightarrow D^{(*)} \rightarrow D^{(*)} \rightarrow D^{(*)} \rightarrow D_{\pi}^{(*)}$ mode, we reject $B^0$ candidates where both $D^*$ mesons decay to $D_{\pi}^{(*)}$ because of its smaller branching fraction and larger backgrounds. Reference [18] contains the details of the reconstruction procedure, outlined here, used to select signal candidates. Charged kaon candidates must be identified as such using a likelihood technique based on the opening angle of the Cherenkov light measured in the DIRC and the ionization energy loss measured in the SVT and DCH [16]. We reconstruct $K^0_S$ candidates from two oppositely charged tracks, geometrically constrained to a common vertex and with an invariant mass within 20 MeV of the nominal value [19]. We also require that the $\chi^2$ probability of the vertex fit of the $K^0_S$ be greater than 0.1%. We form $\pi^0$ candidates from a pair of photons detected in the calorimeter, each with energy greater than 40 MeV. The invariant mass of the two photons must be less than 30 MeV/c$^2$ from the nominal $\pi^0$ mass, and their summed energy must be greater than 200 MeV. In addition, we apply a mass constraint to the $\pi^0$ candidates. We require the reconstructed $D$ meson candidate mass to be within 20 MeV/c$^2$ of the nominal value, except for the $D^0 \rightarrow K^- \pi^+ \pi^0$ decays where we use a looser requirement of 40 MeV/c$^2$. The daughters of each $D$ candidate are fit to a common vertex with their combined mass constrained to that of the $D$ meson. We use $D$ candidates combined with a pion track with momentum less than 450 MeV/c in the CM frame to form $D^{(*)}$ candidates. We fit the $B^0$ decay with a vertex constraint.

Since the time of our previous publications [11,12,18], the BABAR reconstruction routines have been extensively revised, leading to significant improvements in localizing and reconstructing tracks, particularly for low momentum charged particles. These improvements have increased the reconstruction efficiency for final states with multiple slow particles, such as the $B^+ \rightarrow D^{(*)} \rightarrow D^{(*)} \rightarrow D^{(*)} \rightarrow D^{(*)}$ channel which has a better than 20% improvement. As a result, the statistical sensitivity of the measurements in this paper has increased more than would be expected by just the increment in luminosity.

To suppress $e^+ e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) continuum background, we exploit the spherical shape of $B\bar{B}$ events by requiring the ratio of second to zeroth order Fox-Wolfram moments [20] to be less than 0.6. We select the $B^0$ candidates based on four variables: $\Delta E \equiv E_{B^0} - \sqrt{s}/2$, where $E_{B^0}$ is the energy of the $B$ meson in the CM frame, the $D$ candidate flight length significance, defined as the sum of the two $D$ candidate flight lengths divided by the error on the sum, a Fisher discriminant [21], and a mass likelihood of the $D^{(*)}$ mesons. The Fisher discriminant is a linear combination of 11 variables: the momentum flow in nine concentric cones around the thrust axis of the $B^0$ candidate, the angle between the thrust axis and the beam axis, and the angle between the line-of-flight of the $B^0$ candidate and the beam axis. The mass likelihood is formed from Gaussian functions,

$$L_{\text{mass}} = G(m_D; m_{D_{\text{ECMC}}, \sigma_{m_D}}) \times G(m_{D^*}; m_{D^*_{\text{ECMC}}, \sigma_{m_{D^*}}}) \times \left[ f_{\text{core}} G(\Delta m_{D^{(*)}}; m_{D^{(*)}_{\text{ECMC}}, \sigma_{\Delta m_{D^{(*)}}}}) + (1 - f_{\text{core}}) G(\Delta m_{D^{(*)}}; m_{D^{(*)}_{\text{ECMC}}, \sigma_{m_{D^{(*)}}}}) \right] \times \left[ f_{\text{core}} G(\Delta m_{D^{(*)}}; m_{D^{(*)}_{\text{ECMC}}, \sigma_{\Delta m_{D^{(*)}}}}) + (1 - f_{\text{core}}) G(\Delta m_{D^{(*)}}; m_{D^{(*)}_{\text{ECMC}}, \sigma_{m_{D^{(*)}}}}) \right],$$

where the PDG subscript refers to the nominal value [22]. The reconstructed masses and uncertainties $\sigma_{m_{D^{(*)}}}$ for the $D$ mesons prior to the mass constraint are used in the likelihood. The $D^*$ portion of the likelihood is the sum of two Gaussian functions, a central core and a wider tail. The value of $f_{\text{core}}$ and the widths of the $D^*$ Gaussian functions are taken from detailed signal MC studies, which show good agreement between data and MC samples. The selection criteria are optimized for each $D$ decay channel to maximize the total signal significance $S/\sqrt{S + B}$ for each $B^0$ decay mode, where $S$ and $B$ are the signal and background yields, respectively. The optimized selections are specified in [18]. We keep candidates with $m_{\text{ES}} \equiv \sqrt{s}/4 - p_{l_B}^2 > 5.23$ GeV/c$^2$, where $p_{l_B}^2$ is the momentum of the $B$ candidate in the CM frame. On average 1.1–1.8 candidates per event satisfy all of the selection criteria.
depending on the process. When more than one $B^0$ candidate meets the selection criteria, the one with the best $L_{\text{mass}}$ is kept. We find from MC that this procedure retains the correct candidate more than 95% of the time.

To determine the signal yields of the data sample, we use unbinned maximum likelihood (ML) fits to the $m_{ES}$ distributions. The signal is described by a Gaussian function and the combinatorial background by a threshold function [23]. In detailed MC studies of the background, we find that there is a background contribution that exceeds the threshold function in the region $m_{ES} > 5.27$ GeV/$c^2$, where most of the signal events lie. We describe this component with a Gaussian function having the same mean and width as the signal and refer to it as peaking background because if neglected, it would lead to an overestimate of the signal yields. In the $B^0 \rightarrow D^{*+}D^{*-}$ channel, the peaking background arises primarily from misreconstructed $B^+ \rightarrow D^{*+}\bar{D}^{0}$ events where the slow $\pi^0$ from the $\bar{D}^{*0} \rightarrow \bar{D}^0\pi^0$ decay is replaced by a $\pi^-$ to form a $D^{*-}$ candidate. For the other three processes, our studies of the composition of the peaking background show it to be consistent with that of the combinatorial background in the region $m_{ES} < 5.27$ GeV/$c^2$. We treat the peaking background component as an extension of the combinatorial background. The peaking background yields relative to the signal are fixed from MC to $(1.6 \pm 1.9)$%, $(7.1 \pm 5.9)$%, and $(7.4 \pm 2.9)$% for the $B^0 \rightarrow D^{*+}D^{*-}$, $B^0 \rightarrow D^{*}D^-$, and $B^0 \rightarrow D^{*}D^-$ modes, respectively, where the errors are due primarily to the size of the MC sample available for background studies. The signal mean and background shape are free parameters in the fits. We fix the width of the signal Gaussian shape for $B^0 \rightarrow D^{*}D^+$ and $B^0 \rightarrow D^{*}D^-$ to 2.46 MeV/$c^2$ and 2.55 MeV/$c^2$, respectively, determined from MC, while the width of the $B^0 \rightarrow D^{*+}D^{*-}$ signal is allowed to float because of its much higher purity. The signal yields are $934 \pm 40 B^0 \rightarrow D^{*+}D^{*-}$ events, $152 \pm 17 B^0 \rightarrow D^{*}D^+$ events, $365 \pm 26 B^0 \rightarrow D^{*}D^-$ events, and $359 \pm 26 B^0 \rightarrow D^{*}D^-$ events, where the uncertainty is statistical only. The signal yields are consistent with previously measured $B^0 \rightarrow D^{(*)+}D^{(*)-}$ decay branching fractions from BABAR [18] and Belle [15,24]. When compared with past BABAR measurements.

![Projected results](image.png)

**FIG. 2** (color online). Projections of the $m_{ES}$ fit results. The solid line represents the total fit PDF and the dashed line is the background contribution.
[18,25,26], the low $B^0 \to D^+ D^{*-}$ yield in Ref. [11] is consistent with a statistical fluctuation. The fit projections for each mode onto $m_{ES}$ are shown in Fig. 2.

III. TIME-INTEGRATED MEASUREMENT OF THE CP-ODD FRACTION

The $B^0 \to D^{*+} D^{*-}$ process has two vector mesons in the final state and is an admixture of CP-even and CP-odd states depending on the orbital angular momentum of the decay products. We measure the CP-odd fraction $R_\perp$ using

\[
\frac{1}{\Gamma} \frac{d^4 \Gamma}{d \cos \theta_\perp d \cos \theta_\parallel d \phi_\perp d \phi_\parallel} = \frac{9}{16 \pi} \frac{1}{|A_0|^2 + |A_\perp|^2 + |A_{\parallel 1}|^2} \left\{ 2 \cos^2 \theta_\parallel \sin^2 \theta_\perp \sin^2 \theta_\parallel |\mathbf{A}_0|^2 + \sin^2 \theta_\parallel \sin^2 \theta_\perp \sin^2 \phi_\parallel |\mathbf{A}_\perp|^2 \right.
\]

\[
+ \sin^2 \theta_\parallel \cos^2 \theta_\perp |\mathbf{A}_{\parallel 1}|^2 - \sin^2 \theta_\parallel \sin^2 \theta_\perp \sin \phi_\parallel \text{Im}(A_\parallel^* A_{\parallel 1}) + \frac{1}{\sqrt{2}} \sin 2 \theta_\parallel \sin 2 \theta_\perp \sin 2 \phi_\parallel \text{Re}(A_\parallel^* A_{\parallel 1})
\]

\[
- \frac{1}{\sqrt{2}} \sin 2 \theta_\perp \sin 2 \phi_\parallel \text{Re}(A_0 A_{\parallel 1}) \bigg\}.
\]

where $A_k$, with $k = \parallel, 0, \perp$, represent time-dependent amplitudes given by

\[
A_k(t) = \sqrt{2} A_k(0) e^{-i \eta_{CP} t} e^{-t/2 \tau_B} \left( \cos \frac{\Delta m_d t}{2} + i \eta_{CP} \lambda_k \sin \frac{\Delta m_d t}{2} \right).
\]

Here, $\eta_{CP}$ is the CP eigenvalue, $+1$ for $A_{\parallel 0}, -1$ for $A_{\perp}$; $\lambda_k$ is the CP parameter defined in Sec. IV; $\Delta m_d$ is the $B^0$ mixing frequency, $(0.507 \pm 0.005)$ ps$^{-1}$; and $\tau_B$ is the $B^0$ lifetime, $(1.530 \pm 0.009)$ ps [19]. Expressions similar to Eq. (2) hold for $B^0$ decays where each $A_k$ is replaced by the appropriate $A_j$, including $A_{\perp} \to -A_{\perp}$. Integrating Eq. (2) over $t$, $\phi_\parallel$, $\cos \theta_\parallel$ and averaging over $B$ flavor while taking into account detector efficiency yields

\[
\frac{1}{\Gamma} \frac{d^4 \Gamma}{d \cos \theta_\perp d \cos \theta_\parallel d \phi_\perp d \phi_\parallel} = \frac{9}{32 \pi} (1 - R_\perp) \sin^2 \theta_\parallel \left\{ \frac{1 + \alpha}{2} I_0(\cos \theta_\parallel) + \frac{1 - \alpha}{2} I_\parallel(\cos \theta_\parallel) \right\} + \frac{3}{2} R_\perp \cos^2 \theta_\parallel \times I_\perp(\cos \theta_\parallel),
\]

where we define

\[
R_\perp = \frac{|A_{\parallel 1}^0|^2}{|A_{\parallel 0}^0|^2 + |A_{\parallel 1}^0|^2 + |A_{\parallel 1}^0|^2}, \quad \alpha = \frac{|A_{\parallel 0}^0|^2 - |A_{\parallel 1}^0|^2}{|A_{\parallel 0}^0|^2 + |A_{\parallel 1}^0|^2},
\]

\[
I_k(\cos \theta_\parallel) = \int d \cos \theta_1 d \phi_\parallel g_k(\theta_1, \phi_\parallel) \varepsilon(\theta_1, \theta_\parallel, \phi_\parallel),
\]

and $A_k^0 = A_k(0)$. The three efficiency moments $I_k(\cos \theta_\parallel)$ are defined as

\[
I_k(\cos \theta_\parallel) = \int d \cos \theta_1 d \phi_\parallel g_k(\theta_1, \phi_\parallel) \varepsilon(\theta_1, \theta_\parallel, \phi_\parallel),
\]

where $g_0 = 4 \cos^2 \theta_1 \cos^2 \phi_\parallel$, $g_\parallel = 2 \sin^2 \theta_1 \sin^2 \phi_\parallel$, $g_\perp = \sin^2 \theta_1$, and $\varepsilon$ is the detector efficiency. The moments $I_k$ are parameterized as second-order even polynomials in $\cos \theta_\parallel$ whose parameters are determined from signal MC simulation and fixed in the fit. The three $I_k$ functions deviate only slightly from the same constant, making Eq. (4) nearly insensitive to $\alpha$, which we fix to zero in the fit.

Because $\cos \theta_\parallel$ is defined with respect to the slow pion from the $D^{*+}$ decay, the measurement resolution smears its distribution. We convolve the function from Eq. (4) with a resolution function $R(\Delta \theta_\parallel)$ which is modeled as the sum of three Gaussian functions. In addition, we include an uncorrelated Gaussian shape centered at $\pi/2$ and normalized in $0 < \theta_\parallel < \pi$ to describe decays where the slow pion is poorly reconstructed leading to a loss of angular information. The uncorrelated term represents 3% of the signal events where both slow pions are charged and around 16%
in the modes where one of the slow pions is neutral. We determine the parameters of the resolution model and of the uncorrelated term from signal MC simulation and fix them in the ML fit. Small differences observed in the angular distributions based on the charge of the slow pions lead us to divide the efficiency moment and resolution parameters into three categories, $\pi^0\pi^-$, $\pi^+\pi^0$, and $\pi^+\pi^-$. We determine $R_\perp$ in a simultaneous unbinned ML fit to the $m_{ES}$ and $\cos\theta_w$ distributions for the three slow-pion modes. The $m_{ES}$ probability density function (PDF) was described in Sec. II B. The signal $\cos\theta_w$ distribution is given by Eq. (4) convolved with the resolution model. The background $\cos\theta_w$ distribution is modeled as a second-order even polynomial $f_{bg}(\cos\theta_w) = 1 + b_2\cos^2\theta_w$, where $b_2$, common to the three slow-pion modes, is allowed to float. The yield for each of the three slow-pion modes is determined by the fit. We validate the fitting procedure using high-statistics MC samples divided into data-sized subsets and find no significant bias. Fitting the fit result. The dashed line is the background component.

Figure 4 shows the projection of the fit result onto $\cos\theta_w$ for events with $m_{ES} > 5.27\text{GeV}/c^2$. The solid line is the projected fit result. The dashed line is the background component.

\begin{equation}
R_\perp = 0.158 \pm 0.028(\text{stat}) \pm 0.006(\text{syst}).
\end{equation}

\begin{table}[h]
\centering
\caption{Summary of systematic uncertainties on the measurement of $R_\perp$.}
\begin{tabular}{l|c}
\hline
Angular efficiency moments & 0.0024 \\
Angular measurement resolution & 0.0036 \\
$\alpha$ parameter uncertainty & 0.0026 \\
Peaking background & 0.0014 \\
$\cos\theta_w$ background shape & 0.0002 \\
Potential fit bias & 0.0017 \\
Total & 0.0055 \\
\hline
\end{tabular}
\end{table}

IV. TIME-DEPENDENT CP MEASUREMENT

The decay rate $f_+ (f_-)$ of the neutral $B$ meson to a common final state accompanied by a $B^0 (\bar{B}^0)$ tag is

\begin{equation}
f_{\pm}(\Delta t) \propto e^{-|\Delta t|/\tau_{\text{bkg}}} [(1 \mp \Delta w) \pm (1 - 2w)] \times [S \sin(\Delta m_d\Delta t) - C \cos(\Delta m_d\Delta t)],
\end{equation}

with CP asymmetry parameters $S = 2 \text{Im}(\Lambda)/(1 + |\Lambda|^2)$, $C = (1 - |\Lambda|^2)/(1 + |\Lambda|^2)$, and $\lambda = (q/p)(\bar{\Lambda}/\Lambda)$, where $\Lambda$ ($\bar{\Lambda}$) is the decay amplitude for $B^0$ ($\bar{B}^0$) and $q/p$ is the ratio of the flavor contributions to the mass eigenstates [28]. The parameter $w$ is the average mistag probability, and $\Delta w$ is the difference between the mistag probabilities for $B^0$ and $\bar{B}^0$. Here, $\Delta t \equiv t_{\text{reco}} - t_{\text{tag}}$ is the proper time difference between the $B$ reconstructed as $B^0 \rightarrow D^{(*)+}D^{(*)-}$ (rec) and the $B$ used to tag the flavor ($B_{\text{tag}}$). In the case of $B^0 \rightarrow D^{++}D^{*-}$, we obtain an expression similar to Eq. (7) from Eqs. (2) and (3),

\begin{equation}
f_{\pm}(\Delta t, \cos\theta_w) \propto e^{-|\Delta t|/\tau_{\text{bkg}}} [F(1 \mp \Delta w) \pm (1 - 2w)] \times [G \sin(\Delta m_d\Delta t) - H \cos(\Delta m_d\Delta t)].
\end{equation}

The $F, G$, and $H$ coefficients [29] are

\begin{equation}
\begin{aligned}
F &= (1 - R_{\perp})\sin^2\theta_{w} + 2R_{\perp}\cos^2\theta_{w}, \\
G &= (1 - R_{\perp})S_{+}\sin^2\theta_{w} - 2R_{\perp}S_{-}\cos^2\theta_{w}, \\
H &= (1 - R_{\perp})C_{+}\sin^2\theta_{w} + 2R_{\perp}C_{-}\cos^2\theta_{w}.
\end{aligned}
\end{equation}

The $\lambda_k$ parameters in Eq. (3) need not be the same because
of possible differences in the relative contribution of penguin and tree amplitudes, therefore the $S$ and $C$ parameters for each of the three ($0, \|, \perp$) amplitudes can also differ. Note that the minus sign before $S_\perp$ in the expression for $G$ absorbs $\eta_{CP}$. We then define

$$S_+ = S_\| A_\|^2 + S_0 A_0^2 A_\|^2, \quad C_+ = C_\|^2 A_\|^2 A_0^2 + A_0^2 = C_0 A_\|^2 + C_0 A_\|^2,$$

(10)

where $A_0^2 = A_\|(0)$ from Eq. (3).

In the absence of penguin contributions, $S_{D^+D^-} = S_\perp = -\sin 2\beta$, and $C_{D^+D^-} = C_\perp = C_\parallel = 0$. Because $B^0 \rightarrow D^{\pm}\bar{D}^{\mp}$ is not a $CP$ eigenstate, the expressions for $S$ and $C$ are related, $S_{D^+D^-} = -\sqrt{1-C_{D^+D^-}^2 \sin (2\beta_{\rm eff} \pm \delta)}$, where $\delta$ is the strong phase difference between $B^0 \rightarrow D^{*+}D^-$ and $B^0 \rightarrow D^+D^{*-}$ decays [30]. Neglecting the penguin contributions, $\beta_{\rm eff} = \beta$, and $C_{D^+D^-} = -C_{D^+D^-}^*$. The technique used to measure the time-dependent $CP$ asymmetry is discussed in detail in Ref. [31]. We calculate $\Delta t$ between the two $B$ decays from the measured separation $\Delta z$ of their decay vertices along the $z$ axis. The $B_{\rm rec}$ decay vertex is determined from the daughter tracks of the $B^0 \rightarrow D^{(*)+}D^{(*)-}$ decay. The $B_{\rm tag}$ decay vertex is determined in a fit of the charged tracks not belonging to $B_{\rm rec}$ to a common vertex with a constraint on the beam spot location and the $B_{\rm rec}$ momentum. Events that do not satisfy $|\Delta t| < 20$ ps and $\sigma_{\Delta t} < 2.5$ ps are considered untagged in the time-dependent fit.

The flavor of the $B_{\rm tag}$ meson is determined using a multivariate analysis of its decay products [31]. The tagging algorithm classifies the $B$ flavor and assigns the candidate to one of six mutually exclusive tagging categories based on the output. A seventh untagged category is for events where the flavor could not be determined. The performance of the tagging algorithm, its efficiency and mistag rates, is evaluated using the time-dependent evolution of a high-statistics data sample of $Y(4S) \rightarrow B_{\rm tag} B_{\rm flav}$, where the $B_{\rm flav}$ meson decays to a flavor eigenstate $D^{(*)+}h^+$ and $h^+$ may be a $\pi^+$, $\rho^+$, or $\phi^+$. The tagging algorithm has an efficiency $e_{\rm tag} = (74.4 \pm 0.1\%)$ and an effective tagging power $Q \equiv e_{\rm tag}(1-w)^2 = (31.2 \pm 0.3\%)$. The finite resolution of the $B$ vertex reconstruction smears the distributions described in Eqs. (7) and (8). This measurement resolution is modeled as the sum of three Gaussian functions described in Ref. [31], the parameters of which are also determined from the $B_{\rm flav}$ sample.

We determine the $CP$ asymmetry parameters in unbinned ML fits to the $m_{ES}$, $\Delta t$, and in the case of $B^0 \rightarrow D^{*+}D^{-}$, $\cos \theta_{\ell\ell}$ distributions. The $\Delta t$ signal distributions are given in Eqs. (7) and (8) convolved with the experimental resolution. The $\Delta t$ background distribution has both zero and nonzero lifetime components which are convolved with the experimental resolution. The lifetime component is allowed to have effective $CP$ parameters and lifetime, which are determined in the fits. The angular measurement resolution, determined for the $CP$-odd fraction measurement, is convolved with the signal angular distribution. The efficiency moments are not modeled but rather absorbed into an effective $R_\perp$, which is determined in the fit. This procedure simplifies the $\cos \theta_{\ell\ell}$ distribution and does not introduce a bias. The peaking background for the $B^0 \rightarrow D^{(*)+}D^{-}$ channels shares the $\Delta t$ background distributions with the combinatorial background because it originates from similar sources. The $B^0 \rightarrow D^{*+}D^{-}$ peaking background has only a lifetime component, since it originates from a specific $B^+$ decay. Untagged events are also included in the fits to constrain the $m_{ES}$ and $\cos \theta_{\ell\ell}$ shapes but do not contribute to the determination of the $CP$ parameters. We also allow the signal yield, the $m_{ES}$ background shape, and the $\cos \theta_{\ell\ell}$ background shape to vary in the fits. Again we use high-statistics MC samples divided into data-sized subsets to validate the fitting procedure and find no significant bias.

The statistical uncertainties of the $CP$ measurements below are consistent with the expected uncertainties obtained from MC studies that include the signal and background yields observed in data. The statistical uncertainty for the $B^0 \rightarrow D^{(*)+}D^{-}$ channels is essentially unchanged or even slightly worse than our previous measurement [11]. We interpret this as a downward fluctuation in the statistical uncertainty of the previous measurement. Using MC data, we estimate the probability of observing such a fluctuation at about $20\%$. For each measurement that follows, the first uncertainty is statistical and the second is systematic.

From the fit to the $B^0 \rightarrow D^{*+}D^{-}$ data, we find

$$S_+ = -0.76 \pm 0.16 \pm 0.04$$

$$C_+ = +0.00 \pm 0.12 \pm 0.02$$

(11)

with an effective $R_\perp = 0.155 \pm 0.030$. If we perform the fit with the additional constraints that $S_+ = S_\perp = S_{D^+D^-}$ and $C_+ = C_\perp = C_{D^+D^-}$, we obtain

$$S_{D^+D^-} = -0.70 \pm 0.16 \pm 0.03$$

$$C_{D^+D^-} = +0.05 \pm 0.09 \pm 0.02,$$

(12)

having an effective $R_\perp = 0.171 \pm 0.028$. Fitting the $B^0 \rightarrow D^+D^-$ data yields

$$S_{D^+D^-} = -0.63 \pm 0.36 \pm 0.05$$

$$C_{D^+D^-} = -0.07 \pm 0.23 \pm 0.03,$$

(13)

and fitting the $B^0 \rightarrow D^{*\pm}D^\mp$ data yields
Projections of the fit results onto $\Delta t$ for events in the region $m_{ES} > 5.27$ GeV/$c^2$, and their flavor asymmetry, can be seen in Fig. 5. To enhance the visibility of the signal in these projections, we use three of the six tagging categories with the highest purity, which account for 80% of the total effective tagging power $Q$. The correlations among the CP parameters are given in the appendix.

We evaluate systematic uncertainties in the CP asymmetries for each mode by varying the fixed parameters for the mistag quantities and $\Delta t$ resolution model within their uncertainties while accounting for correlations among the parameters. For the $B^0 \to D^{*+}D^-$ and $B^0 \to D^{*0}D^+$ modes, we change the fixed $m_{ES}$ signal width by $\pm 0.2$ MeV/$c^2$, an amount determined from a comparison of data and MC event samples in modes with high purity, and take the difference in fitted results as a systematic uncertainty. Additionally, we vary the fraction and shape of the peaking background component. We also include systematics for possible detector misalignment and the presence of doubly-Cabibbo suppressed decays of the $B$ tag meson [32]. We assign a systematic uncertainty equal to the statistical uncertainty of the MC sample used to validate the fit. Other sources of systematic uncertainty include: the $B_0^+$ meson properties ($m_{d}$ and $\tau_{B^0}$), which we vary to $\pm 1\sigma$ of their world averages, and uncertainty in the boost; the corresponding changes in the CP asymmetries are taken as the estimate of the systematic uncertainties.

For the $B^0 \to D^{*+}D^-$ mode, we vary the $\cos\theta_w$ resolution parameters and background shape in the manner described for the evaluation of systematic uncertainties on $R_{L}$ and take the effects on the CP parameters as the associated systematic uncertainty. A summary of the systematic uncertainties for the CP parameters is given in Tables II and III. As before, the total systematic uncertainty is the sum in quadrature of the individual contributions.

\[
\begin{align*}
S_{D^{*+}D^-} &= -0.62 \pm 0.21 \pm 0.03 \\
S_{D^{*+}D^0} &= -0.73 \pm 0.23 \pm 0.05 \\
C_{D^{*+}D^-} &= +0.08 \pm 0.17 \pm 0.04 \\
C_{D^{*+}D^0} &= +0.00 \pm 0.17 \pm 0.03.
\end{align*}
\tag{14}
\]
Because $B^0 \rightarrow D^{*\pm} D^\mp$ decays are not $CP$ eigenstates, it is illustrative to express the $CP$ asymmetry parameters $S$ and $C$ in a slightly different parametrization [33]
\[
S_{D^*D} = \frac{1}{2} (S_{D^+D^-} + S_{D^0D^0}) \\
\Delta S_{D^*D} = \frac{1}{2} (S_{D^+D^-} - S_{D^0D^0}) \\
C_{D^*D} = \frac{1}{2} (C_{D^+D^-} + C_{D^0D^0}) \\
\Delta C_{D^*D} = \frac{1}{2} (C_{D^+D^-} - C_{D^0D^0}) .
\]

The $S_{D^*D}$ and $C_{D^*D}$ parameters characterize mixing-induced $CP$ violation related to the angle $\beta$ and flavor-dependent direct $CP$ violation, respectively. $\Delta S_{D^*D}$ is insensitive to $CP$ violation but is related to the strong phase difference $\delta$, $\Delta C_{D^*D}$ describes the asymmetry between the rates $\Gamma(B^0 \rightarrow D^{*+}D^-) + \Gamma(B^0 \rightarrow D^{*0}D^{*-})$ and $\Gamma(B^0 \rightarrow D^{*0}D^+ + \Gamma(B^0 \rightarrow D^{*+}D^-)$. Using the results from Eq. (14) and taking into account correlations among the variables, we find
\[
S_{D^*D} = -0.68 \pm 0.15 \pm 0.04 \\
\Delta S_{D^*D} = +0.05 \pm 0.15 \pm 0.02 \\
C_{D^*D} = +0.04 \pm 0.12 \pm 0.03 \\
\Delta C_{D^*D} = +0.04 \pm 0.12 \pm 0.03 .
\]

From the signal yields $N_{D^*D^-}$ and $N_{D^0D^-}$ determined in the time-dependent fit described above, we also measure the time-integrated $CP$ asymmetry in $B^0 \rightarrow D^{*\pm} D^\mp$ decays, defined as
\[
\mathcal{A} = \frac{N_{D^*D^-} - N_{D^0D^-}}{N_{D^*D^-} + N_{D^0D^-}} .
\]
We find
\[
\mathcal{A} = +0.008 \pm 0.048 {\rm(stat)} \pm 0.013 {\rm(syst)} ,
\]
where the systematic uncertainty is dominated by track reconstruction efficiency differences for positive and negative tracks (0.013). There is also a small contribution from the $m_{ES}$ signal width, peaking background, and MC statistics (0.002).

### V. CONCLUSION

We have measured the $CP$ asymmetry parameters for $B^0 \rightarrow D^{(*)\pm} D^{(*)\mp}$ decays, including the $CP$-odd fraction in the $B^0 \rightarrow D^{*+} D^{*-}$ channel, using the final BABAR data sample. All of the $S$ parameters are consistent with the value of $\sin2\beta$ measured in $b \rightarrow (c\bar{c})s$ transitions [34] and with the expectation from the standard model for small penguin contributions. The $C$ parameters are consistent with zero in all modes. In particular, we see no evidence of the large direct $CP$ violation reported by the Belle Collaboration in the $B^0 \rightarrow D^* D^-$ channel [15]. This measurement supersedes the previous BABAR measurements [11,12] of $CP$ asymmetries in these decays.
We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the U.S. Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), the Commissariat à l’Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung and Deutsche Forschungsgemeinschaft (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Foundation for Fundamental Research on Matter (The Netherlands), the Research Council of Norway, the Ministry of Education and Science of the Russian Federation, Ministerio de Educación y Ciencia (Spain), and the Science and Technology Facilities Council (United Kingdom). Individuals have received support from the Marie-Curie IEF program (European Union) and the A. P. Sloan Foundation.

APPENDIX: CORRELATIONS AMONG THE CP PARAMETERS

To allow detailed use of these results, we include the correlation matrices for the CP parameters. Table IV contains correlations among the fit parameters in the $B^0 \rightarrow D^{*+} D^{-}$ channel with separate CP-even and CP-odd asymmetries, and in the combined case, the correlation between $S_{D^{+} D^{-}}$ and $C_{D^{+} D^{-}}$ is 0.8% with correlations to the effective $R_{\perp}$ the same as the CP-even parameters. Table V contains the correlations among the $B^0 \rightarrow D^{*+} D^{-}$ asymmetries. The correlation of the time-integrated CP asymmetry $A$ with any of the CP parameters is less than 0.1%. The correlation between $S_{D^{+} D^{-}}$ and $C_{D^{+} D^{-}}$ is $-1.2\%$.

<table>
<thead>
<tr>
<th>$S_{D^{+} D^{-}}$</th>
<th>$C_{D^{+} D^{-}}$</th>
<th>$S_{D^{+} D^{-}}$</th>
<th>$C_{D^{+} D^{-}}$</th>
</tr>
</thead>
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<td>0.002</td>
</tr>
<tr>
<td>$C_{D^{+} D^{-}}$</td>
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<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>$C_{D^{+} D^{-}}$</td>
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<td>-0.009</td>
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</table>

<table>
<thead>
<tr>
<th>$S_{D^{+} D^{-}}$</th>
<th>$C_{D^{+} D^{-}}$</th>
<th>$S_{D^{+} D^{-}}$</th>
<th>$C_{D^{+} D^{-}}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.008</td>
<td>0.376</td>
</tr>
<tr>
<td>$C_{D^{+} D^{-}}$</td>
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<td>-0.465</td>
</tr>
<tr>
<td>$C_{D^{+} D^{-}}$</td>
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<td>0.471</td>
</tr>
<tr>
<td>$R_{\perp}$</td>
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</tr>
</tbody>
</table>


[29] The order of the definition of these coefficients has changed since our previous publication [12].