Breakdown Voltage for Superjunction Power Devices With Charge Imbalance: An Analytical Model Valid for Both Punch Through and Non Punch Through Devices

The MIT Faculty has made this article openly available. Please share how this access benefits you. Your story matters.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>As Published</td>
<td><a href="http://dx.doi.org/10.1109/ted.2009.2032595">http://dx.doi.org/10.1109/ted.2009.2032595</a></td>
</tr>
<tr>
<td>Publisher</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Accessed</td>
<td>Fri Nov 23 01:58:58 EST 2018</td>
</tr>
<tr>
<td>Citable Link</td>
<td><a href="http://hdl.handle.net/1721.1/54726">http://hdl.handle.net/1721.1/54726</a></td>
</tr>
<tr>
<td>Terms of Use</td>
<td>Article is made available in accordance with the publisher’s policy and may be subject to US copyright law. Please refer to the publisher’s site for terms of use.</td>
</tr>
<tr>
<td>Detailed Terms</td>
<td></td>
</tr>
</tbody>
</table>
Breakdown Voltage for Superjunction Power Devices With Charge Imbalance: An Analytical Model Valid for Both Punch Through and Non Punch Through Devices

Han Wang, Ettore Napoli, and Florin Udrea

Abstract—An analytical model for the electric field and the breakdown voltage (BV) of an unbalanced superjunction (SJ) device is presented in this paper. The analytical technique uses a superposition approach treating the asymmetric charge in the pillars as an excess charge component superimposed on a balanced charge component. The proposed double-exponential model is able to accurately predict the electric field and the BV for unbalanced SJ devices in both punch through and non punch through conditions. The model is also reasonably accurate at extremely high levels of charge imbalance when the devices behave similarly to a PIN diode or to a high-conductance layer. The analytical model is compared against numerical simulations of charge unbalanced SJ devices and against experimental results.

Index Terms—Analytical model, charge imbalance (C.I.), power semiconductor devices, semiconductor device modeling, superjunction (SJ).

I. INTRODUCTION

SUPERJUNCTION (SJ) is a power device concept that allows a favorable tradeoff between breakdown voltage (BV) and on-state loss for power MOSFETs [1]–[10]. In SJ MOSFETs, the drift region is replaced by alternatively stacked heavily doped N and P regions (pillars). Unlike in conventional power MOSFETs, where the specific on-state resistance is proportional to BV^2, the SJ MOSFET specific on-state resistance is virtually linear with the BV, allowing a cut in the specific on-state resistance of 5X to 20X limited by the technology of forming thin and deep N and P pillars. The superior static and dynamic performance is demonstrated by Infineon in their CoolMOS series and by STMicroelectronics in their MDmesh power MOSFETs [11], [12]. Several analytical models for the BV of SJ have been proposed [13]–[17]. These models are of utmost importance since they provide a fast and reliable way to devise the optimal performance of an SJ structure. However, most of the previous work assumes an ideal charge balance between P and N pillars, while it is well known that a tiny imbalance in the charge significantly affects the BV [14], [18], [19]. Since charge imbalance (C.I.) is unavoidable in a real device, a physical insight into the degradation mechanisms and an analytical solution for it are highly desirable.

In [13], a 1-D model that only works for balanced SJ devices with very narrow pillars is presented. Reference [14] presents a simple model of the BV that is suitable for balanced pillars. Reference [15] provides a rigorous treatment of the interaction between the depletion regions in the two directions, but only the balanced case is considered, and the model is complex. A simple and accurate model for the 2-D potential distribution of an SJ is proposed in [16] and [17], but the general solution is solely used to derive two BV models for the balanced case. A novel approach to model an unbalanced SJ is introduced in [18], which gives an analytic BV model using particularly simple and intuitive equations. A limitation of this model, however, is that it can only predict the BV up to a relatively small level of C.I. since the electric field and BV equations do not account for the non punch through (NPT) case. On the other hand, a model that is also valid at high levels of C.I. is very useful in offering insights into the physics of power device behavior when the C.I. also causes the device to move from a punch through (PT) state to an NPT one. In this paper, an extended formulation of the models proposed in [16]–[18] is proposed. The model uses a superposition approach treating the asymmetric charge in the pillars as an excess charge component superimposed on a balanced charge component. The electric field of the unbalanced SJ is modeled using a pair of exponential equations. Furthermore, this paper derives an analytical equation for the BV based on this double-exponential formulation. This new model improves the prediction of the BV provided in [18] for the PT case and makes it also valid for the NPT case. Extensive 2-D numerical simulations confirm the validity of the proposed model. The analytical results are also compared with the experimental data from [19] to verify its accuracy.

This paper is organized as follows. Section II recalls the analytical model for the balanced SJ proposed in [16] and [17]. Section III presents the superposition principle and recalls the linear model for the unbalanced SJ proposed in [18]. Section IV presents the double-exponential model, which leads to an analytical solution for both the electric field and the BV.
of symmetric unbalanced SJ devices. The proposed model is the only proposed one to date that is valid for both PT and NPT SJ devices. Section V presents the BV results of the double-exponential model compared against both numerical simulations and experimental results. Furthermore, the model gives physical insights into the behavior of the balanced and unbalanced SJ, as well as the extreme cases when the SJ effectively becomes a 1-D PiN diode.

II. ELECTRIC FIELD MODEL FOR BALANCED CHARGE SJ DEVICES

The basis of the proposed model for the unbalanced SJ is described in [16] and [17]. In [16] and [17], the Poisson equation is solved for a one unit cell of the SJ layer [Fig. 1(a)] with boundary conditions specified in Fig. 1(b). This leads to an analytical expression for the 2-D electrostatic potential and the electric field. In the balanced symmetrical case in which both pillar widths and pillar doping are equal ($Y_N = Y_P$ and $N_D = N_A = N$), the peak electric field occurs both at $(x = 0, y = Y_N)$ and at $(x = W, y = -Y_P)$. This is shown in Fig. 2(a) in which the electric field distribution for a balanced device with $W = 30 \, \mu m$, $Y_N = Y_P = 2.5 \, \mu m$, and $N_D = N_A = 4 \times 10^{15} \, cm^{-3}$ is presented. The applied reverse voltage is 500 V. Note the symmetric behavior of the electric field and the presence of two equivalent points of the maximum electric field.

The BV is obtained by analyzing the section $y = Y_N$ in which the maximum electric field is located at $x = 0$ and the $y$ component of the electric field is zero. The electric field along this path, for a balanced symmetric device, as demonstrated in [16] and [17], is

$$E_{bal}(x)|_{y=Y_N} = -\frac{V_D}{Y_N} + \frac{V_R + V_D}{W} + \sum_{n=1}^{\infty} \frac{K_n \gamma_n}{2} \cos(K_n x) \cosh(K_n Y_N)$$

where $V_D = \frac{qN}{2\varepsilon_s} W^2$, $W$ and $Y_N$ are the length and width of the pillars, $V_R$ is the applied reverse voltage, $N$ is the doping in each pillar, $K_n = n\pi/W$, and $\gamma_n = 8V_D/(n\pi)^3((-1)^n - 1)$.

Equation (1) is exploited in [16] and [17] and provides an analytic model for the balanced SJ case that allows the calculation of the BV as a function of pillar doping and dimension and also provides guidelines for the minimum ON-state resistance design of an SJ device.

III. SUPERPOSITION PRINCIPLE AND ANALYTICAL MODEL FOR CHARGE IMBALANCED SJ DEVICES

A. Superposition Concept

In this paper, it is assumed that the P and N pillars have symmetrical geometry ($Y_N = Y_P$) and uniform doping. At this stage, it is also assumed that the pillars are fully depleted at the onset of breakdown, and, hence, the SJ device is in a PT mode. The PT assumption is justified as, in balanced SJ devices, full depletion normally occurs at a much lower voltage than the BV. The PT hypothesis will however be removed in order to account for the NPT case in Section IV of this paper. With respect to [16] and [17], the hypothesis of equal doping in the N and in
that the P pillars is removed, and, hence, in Fig. 1, it is assumed that \( N_D \neq N_A \). The resulting SJ sustaining layer is, therefore, unbalanced. It is well known that an unbalanced charge reduces the voltage sustaining capability of SJ devices [14], [18], [19]. In fact, the unbalanced charge breaks the symmetry of the electric field in the device and exalts one peak on the electric field with respect to the other. This is shown in Fig. 2(b) in which the electric field distribution for an unbalanced device with \( W = 30 \mu m, Y_N = Y_P = 2.5 \mu m, N_D = 4 \times 10^{15} \text{ cm}^{-3} \), and \( N_A = 3.0 \times 10^{15} \text{ cm}^{-3} \) is presented. The applied reverse voltage is 220 V. Note the asymmetric behavior of the electric field and the dominant peak electric field located at \((x = 0, y = Y_N)\).

The unbalanced charge is considered as a balanced charge component superimposed to a differential charge component. We suppose that \( N_D > N_A \) without loss of generality. As shown in Fig. 3, the balanced component has a p-type doping of \((N_D + N_A)/2\) in the P pillar and an equal n-type doping in the N pillar. The differential component consists of a negative amount of p-type doping whose concentration is \((N_D - N_A)/2\) in the P pillar and a positive amount of n-type doping whose concentration is \((N_D - N_A)/2\) in the N pillar. From the potential point of view, the negative amount of p-type doping is equivalent to a n-type doping at the same magnitude. Combining the contribution of the P pillar and of the N pillar, the differential component in the entire drift region has a uniform n-type doping that is equal to \( N^* = (N_D - N_A)/2 \). Note that the structure formed by the differential component is equivalent to a PiN diode. In conclusion, the unbalanced SJ sustaining layer considered in this paper is studied as the superposition of a balanced SJ sustaining layer with \( N = (N_D + N_A)/2 \) and a PiN diode with \( N^* = (N_D - N_A)/2 \) n-type doping.

**B. Electric Field Distribution**

The potential distribution due to the balanced and differential charge components is separately calculated with the boundary conditions in Fig. 1. The electric field due to the differential component is calculated assuming that it is at the edge of the PT condition (that is, with the electric field equal to zero for \( x = W \)). The resulting equation of the electric field has the well-known triangular shape, typical of the PiN diode, in which \( V_u \triangleq qN^*W^2/2\varepsilon_s \) is the voltage supported by the differential charge component

\[
E_{\text{diff}}(x, y) = \frac{2V_u}{W} \left(1 - \frac{x}{W}\right). \tag{2}
\]

The electric field due to the balanced SJ charge component \( E_{\text{bal}} \), reported in [16] and [17], is given by (1). The electric field equation along the section \( y = Y_N \), where \( V_u \) and \( V_B \) are the voltage supported by the differential charge and balanced charge components, is the sum of \( E_{\text{bal}} \) and \( E_{\text{diff}} \)

\[
E_{\text{SJ}}(x)|_{y=Y_N} = E_{\text{bal}}(x)|_{y=Y_N} + E_{\text{diff}}(x)|_{y=Y_N}
= \frac{2V_u}{W} \left(1 - \frac{x}{W}\right) + \frac{V_B}{W} + \frac{V_D}{W} \left(1 - \frac{2x}{W}\right) + \sum_{n=1}^{\infty} \frac{K_n \gamma_n}{2} \frac{\cos(K_n x) \cosh(K_n Y_n)}{\cosh(K_n Y_n)}. \tag{3}
\]
It is worth noting that the two solutions \( E_{\text{bal}} \) and \( E_{\text{diff}} \) are both an exact analytical solution of the particular Poisson problem. Furthermore, if the PT condition holds, the sum of the two solutions is also an exact analytical solution of the complete unbalanced SJ electrostatic problem.

C. Previously Proposed BV Model

Since we assumed, without loss of generality, that \( N_D > N_A \), the unbalanced SJ sustaining layer will have the peak electric field and, hence, the breakdown at the corner on the PT side (\( y = Y_N, x = 0 \)). The applied reverse voltage is the integral of the electric field in the \( x \)-direction at \( y = Y_N \) given by (3) and is \( V_R = V_B + V_u \). The calculation of the BV is carried out using the critical electric field \( E_C \) approximation, that is, assuming that the device breaks when the peak electric field is equal to \( E_C \). The BV is therefore obtained solving the following equation:

\[
E_{\text{SJ}}(0)|_{y=Y_N} = \frac{2V_u}{W} + \frac{V_B}{W} + \frac{V_D}{W} + \sum_{n=1}^{\infty} \frac{K_n \gamma_n \cos(K_n \cdot 0)}{2 \cosh(K_n Y_N)} = E_C. \tag{4}
\]

The previous equation can be simplified by substituting in (4), as shown in [16] and [17], the \( L(\cdot) \) function defined as

\[
L(t) = \frac{1}{2} - \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{(n \pi)^2 \cosh(n \pi t)}. \tag{5}
\]

The result is

\[
\frac{2V_u}{W} + \frac{V_B}{W} + \frac{V_D}{W} L(Y_N/W) = E_C. \tag{6}
\]

The BV is the applied voltage \( V_R = V_B + V_u \) when (6) is verified

\[
V_{BV} = V_u + V_B = W \cdot E_C - V_u - V_D L(Y_N/W). \tag{7}
\]

Equation (7) has been reported in [18] and is a particularly meaningful equation since it states that the actual BV for an unbalanced SJ sustaining layer is equal to the ideal BV, given by \( W \cdot E_C \) diminished by the effect of the balanced SJ component \( (V_D L(\cdot)) \) and by the effect of the unbalanced SJ component \( (V_u) \). Equation (7) is, however, only valid for SJ sustaining layers in which the breakdown arises in PT conditions.

IV. DOUBLE-EXPONENTIAL BV MODEL

In this section, a new method for calculating the BV of an unbalanced SJ device is proposed. The new method, compared to the model proposed in [18], is also accurate at high levels of C.I. and for both PT and NPT cases.

A. Double-Exponential Electric Field Model

In [17], in order to allow the analytical solution of the ionization integral, (1) is approximated by an exponential function \( E^*(x) \) that satisfies the following boundary conditions:

\[
E^*(0) = E_{\text{bal}}(0) = E_{\text{max}} = \frac{V_R}{W} + \frac{V_D}{W} L \left( \frac{Y_N}{W} \right) \tag{8}
\]

\[
\frac{dE^*(x)}{dx} \bigg|_{x=0} = \frac{dE_{\text{bal}}(x)}{dx} \bigg|_{x=0} = -\frac{2V_D}{W^2} \tag{9}
\]

\[
\lim_{x \to \infty} E^*(x) = \frac{V_R}{W}. \tag{10}
\]

Equations (8) and (9) guarantee that \( E^* \) and \( E_{\text{bal}} \) have equal value and derivative at \( (x = 0, y = Y_N) \). Equation (10) states that \( E^* \) behaves similarly to the electric field of an ideal intrinsic sustaining layer as \( x \to \infty \).

The resulting approximation of the electric field along the \( y = Y_N \) section is [17]

\[
E^*(x)|_{y=Y_N} = \frac{V_R}{W} + \frac{V_D}{W} L \left( \frac{Y_N}{W} \right) \exp \left[ -\frac{2x}{L(Y_N/W)W} \right]. \tag{11}
\]

This approximation is remarkably good near \( x = 0 \) but becomes increasingly worse as \( x \) is close to \( W \). We can significantly improve the estimation of (1) by adding a further exponential term to (11). The additional exponential term is negligible near \( x = 0 \) and is meaningful when \( x \) is close to \( W \). The constants that characterize the additional exponential term are defined imposing proper boundary conditions. The new approximation is the double-exponential model that, using the simplified notation \( L(\cdot) = L(Y_N/W) \) and \( V_{\text{SLJ}} = V_D L(Y_N/W) \), provides the following estimation for the electric field:

\[
E^*_{\text{bal}}(x)|_{y=Y_N} = \frac{V_B}{W} + \frac{V_{\text{SLJ}}}{W} \exp \left( -\frac{2x}{L(\cdot)W} \right) - \frac{V_{\text{SLJ}}}{W} \exp \left( \frac{2(x-W)}{L(\cdot)W} \right). \tag{12}
\]

The conditions that have been imposed on \( E^*_{\text{bal}} \), in the limit \( (Y_N/W) \ll 1 \), in order to properly fit the \( E_{\text{bal}} \) distribution are

\[
E^*_{\text{bal}}(0)|_{y=Y_N} = E_{\text{bal}}(0)|_{y=Y_N} = \frac{V_B}{W} + \frac{V_D}{W} L \left( \frac{Y_N}{W} \right) \tag{13}
\]

\[
E^*_{\text{bal}}(W)|_{y=Y_N} = E_{\text{bal}}(W)|_{y=Y_N} = \frac{V_B}{W} - \frac{V_D}{W} L \left( \frac{Y_N}{W} \right) \tag{14}
\]

\[
\frac{\partial E^*_{\text{bal}}(x,y)}{\partial x} \bigg|_{x=0,y=Y_N} = \frac{\partial E_{\text{bal}}(x,y)}{\partial x} \bigg|_{x=0,y=Y_N} = -\frac{2V_D}{W^2} \tag{15}
\]

\[
\frac{\partial E^*_{\text{bal}}(x,y)}{\partial x} \bigg|_{x=W,y=Y_N} = \frac{\partial E_{\text{bal}}(x,y)}{\partial x} \bigg|_{x=W,y=Y_N} = -\frac{2V_D}{W^2}. \tag{16}
\]
B. Double-Exponential BV Model for the PT Case

Equation (17) can conveniently be used for the calculation of the BV. After imposing that the maximum electric field \( E(x = 0, y = Y_N) = E^*_{SJ}(0) \) is equal to \( E_C \), the BV is accurately obtained by integrating (17) with respect to \( x \) along the \( y = Y_N \) section

\[
E^*_{SJ}(0)|_{y=Y_N} = \frac{2V_a}{W} + \frac{V_B}{W} + \frac{V_{SJ}}{W} \exp\left(-\frac{2x}{L(\cdot)}\right) - \frac{V_{SJ}}{W} \exp\left(-\frac{2}{L(\cdot)}\right) = E_C
\]

\[
BV_{PT} = \int_0^W E^*_{SJ}(x) dx = E_C W - V_a + V_{SJ} \left[ \exp\left(-\frac{2}{L(\cdot)}\right) - 1 \right] = V_{BV} + V_{SJ} \exp\left(-\frac{2}{L(\cdot)}\right) \approx V_{BV}.
\]

Equation (20) shows that the proposed double-exponential model provides, in the PT case, a BV prediction that is, as should be, very similar to the prediction of the model proposed in [18].

C. Double-Exponential BV Model for the NPT Case

So far, the model assumes full depletion of the device at the onset of breakdown, i.e., PT devices. However, an optimized device, as shown in [17], is more often designed at the edge of the PT-NPT condition or even slightly in the NPT region. The NPT condition is also enhanced by the amount of unbalanced charge in the device. An SJ device working in the NPT region or an SJ device with a certain amount of unbalanced charge is characterized by the presence of epitlayer undepleted regions when the device is at the edge of the breakdown condition. The undepleted regions appear near \( (x = W, y = Y_N) \) and \( (x = 0, y = -Y_P) \).

As shown in Fig. 4 for the C.I. = -30% case, the double-exponential model in (17) accurately predicts the electric field in the SJ device also for the NPT case. The relevant difference of the NPT case is the presence of an unphysical negative electric field region that is close to \( x = W \). In Fig. 4, the electric field in the undepleted region is therefore set to zero.

Our target is using the double-exponential model for the electric field (17) to calculate the BV also for the NPT devices. Using (17) for the calculation of the BV requires two steps. The first one is determining the NPT condition. The second one is the integration of (17) from \( x = 0 \) to the edge of the undepleted region, identified with \( x^* \) in Fig. 4.

In this way, while keeping the \( N_D > N_A \) hypothesis, it is possible to explore the whole SJ design space. The -10% charge imbalanced device is characterized by \( N_D = 4 \times 10^{15} \) cm\(^{-3}\) and \( N_A = 3.6 \times 10^{15} \) cm\(^{-3}\). The -30% charge imbalanced device is characterized by \( N_D = 4 \times 10^{15} \) cm\(^{-3}\) and \( N_A = 2.8 \times 10^{15} \) cm\(^{-3}\). In every case, the single-exponential model accurately fits the maximum electric field and the central portion of the electric field. It fails to fit in the region of the device in which the electric field is minimum, particularly in the C.I. = -10% and C.I. = 0% cases. The double-exponential model in (17), shown with solid lines in Fig. 4, is instead able to accurately fit the electric field in every portion of the device and for different amounts of C.I.
Identifying the NPT Condition: The boundary between the NPT and the PT regions occurs when the electric field reaches zero at \( x = W \) with the device on the verge of breakdown. An example is the device with C.I. \( \approx -10\% \) shown in Fig. 4. The NPT–PT boundary is, therefore, \( E_{NPT}^c(0) = E_C \) with \( E_{PT}^c(W) = 0 \). From (17), we have

\[
\begin{cases}
2V_u + V_B + V_{SJ} - V_{SJ} \exp\left(-\frac{2}{L(\cdot)}\right) = E_C W \\
V_B + V_{SJ} \exp\left(-\frac{2}{L(\cdot)}\right) - V_{SJ} = 0.
\end{cases}
\]

From (21), subtracting the second equation from the first, it is possible to define the NPT–PT boundary as

\[ V_u + V_{SJ} - V_{SJ} \exp\left(-\frac{2}{L(\cdot)}\right) \approx V_u + V_{SJ} = E_C W/2. \] (22)

In this particular condition, summing the first and the second equations in (21), we obtain an important result that is the analytical equation for the BV of the unbalanced SJ device as a function of the unbalanced charge when the device is at the NPT–PT boundary. The result is

\[ BV_{NPT-PT} = E_C W/2 - V_u. \] (23)

Equation (23) is the extension to the unbalanced case of the BV equation for the balanced SJ provided in [16].

If the SJ device is in the NPT condition, the following condition holds:

\[ V_u + V_{SJ} > E_C W/2. \]

Let us define a dimensionless measure of the amount of NPT condition. Such measure will be indicated as \( NP \)

\[ NP = 1 - \frac{E_C W/2}{V_{SJ} + V_u}. \] (24)

Integrate the Electric Field: It is first necessary to determine the \( x^* \) value for which \( E_{SJ}(x^*) = 0 \) while keeping the condition that \( E_{SJ}^c(0) = E_C \). The two conditions bring to the solution of the following equation for \( x^* \):

\[
E_C + \frac{V_{SJ}}{W} \exp\left(-\frac{2}{L(\cdot)}\right) - \frac{V_{SJ}}{W} - \frac{2V_u x^*}{W} + \frac{V_{SJ}^*}{W} \\
\times \exp\left(-\frac{2x^*}{L(\cdot)W}\right) - \frac{V_{SJ}}{W} \exp\left(\frac{2(x^* - W)}{L(\cdot)W}\right) = 0.
\]

Since (25) has no closed-form solution for \( x^* \), an approximate solution has been determined. The mathematical details of the calculations are provided in the Appendix. The approximate solution for \( x^* \) is then

\[
\begin{cases}
x^* = W - W \frac{L(\cdot)^2 (V_u + V_{SJ}/L(\cdot)) (\beta + \beta^2)}{V_{SJ} (V_u + V_{SJ}/L(\cdot))^2} \\
\beta = NP \frac{L(\cdot)^2 (V_u + V_{SJ}/L(\cdot))}{V_{SJ} (V_u + V_{SJ}/L(\cdot))^2}.
\end{cases}
\] (26)

The integration of the electric field provides

\[
\begin{align*}
V_{NPT} = \int_{0}^{x^*} E_{SJ}(x) dx &= \frac{2V_u x^*}{W} \left[ 1 - \frac{x^*}{2W} \right] + \frac{V_B}{W} x^* \\
+ & \frac{V_{SJ} L(\cdot)}{2} \left[ 1 - \exp\left(-\frac{2x^*}{L(\cdot)W}\right) \right] \left[ 1 - \exp\left(-\frac{2}{L(\cdot)}\right) \right].
\end{align*}
\] (27)

The BV is obtained by substituting the breakdown condition given by (19) in (27). This means eliminating \( V_B \) from (27) using

\[ V_B = E_C W - 2V_u - V_{SJ} \left[ 1 - \exp\left(-\frac{2}{L(\cdot)}\right) \right]. \] (28)

The resulting BV for the NPT condition is

\[
BV_{NPT} = BV_{PT} - \frac{BV_{PT} - \frac{V_u x^*}{W} \left[ 1 - \frac{x^*}{W} \right]}{2} \\
+ \frac{V_{SJ} L(\cdot)}{2} \left[ 1 - \exp\left(-\frac{2x^*}{L(\cdot)W}\right) \right] \left[ 1 - \exp\left(-\frac{2}{L(\cdot)}\right) \right] \\
\approx BV_{PT} - \frac{BV_{PT} - \frac{V_u x^*}{W} \left[ 1 - \frac{x^*}{W} \right]}{2}.
\] (29)

Note that \( BV_{NPT} \) is lower than \( BV_{PT} \). Furthermore, for \( x^* = W \), we have \( BV_{NPT} \approx BV_{PT} \) confirming the continuity of the BV prediction ranging from the PT to the NPT conditions.

V. PERFORMANCE OF THE DOUBLE-EXPONENTIAL MODEL

With reference to the electric field, the double-exponential model shown in (17), as previously reported commenting Fig. 4, accurately predicts the electric field throughout the device and provides a much better prediction of the electric field than the model proposed in [17].

The first results regarding the double-exponential model prediction for the BV are reported in Fig. 5 that shows the BV for two SJ structures with a doping of \( 4 \times 10^{15} \text{ cm}^{-3} \) and \( 2 \times 10^{15} \text{ cm}^{-3} \) in the balanced case. The dimensions of both SJ devices are \( W = 30 \text{ µm} \) and \( Y_N = Y_P = 2.5 \text{ µm} \). The bullets and stars in Fig. 5 are the results from the 2-D numerical simulations. The solid and the dashed lines show the BV predicted by the double-exponential model (29) when the value of \( x^* \) is...
The doping concentration of the SJ in the balanced case is 3 \times 10^{15} \text{ cm}^{-3}. The solid line in Fig. 6 refers to the double-exponential model (29) with \( x^* \) solved numerically; the squares refer to the double-exponential model (29) with \( x^* \) given by (26); the dashed line refers to the linear model (7), whereas the bullets are the experimental results from [19]. The considered \( E_C \) is 3.2 \times 10^5 \text{ V/cm}. The experimental results are in good agreement with the double-exponential analytical model. It is worth highlighting that Saito et al. [19] state that the termination edge effect is responsible for the experimental BV results. Our model is instead able to properly devise the trend of the BV as a function of the C.I. without including any edge termination effect. This hypothesis is confirmed by the subsequent results proposed by Ono et al. [20]. Note also that both the experimental results in [19] and [20] confirm that the impact of positive C.I. on the BV is higher than the effect of the negative C.I.

VI. Conclusion

An extended analytical model for the electric field and the BV of SJ structures with C.I. has been presented. The model uses a superposition approach treating the asymmetric charge in the pillars as an excess charge component superimposed on a balanced charge component. A pair of exponential equations is used to model the electric field due to the balanced charge component, while the unbalanced charge component is modeled as a PiN diode. This paper presents analytical relations for the BV of balanced and unbalanced SJ devices. The presented analytical relations are, for the first time, valid for SJ operating in both PT and NPT modes. The model accuracy is demonstrated against the 2-D numerical simulations and experimental results from [19].

APPENDIX

APPROXIMATE ANALYTICAL SOLUTION FOR THE \( x^* \) VALUE IN THE NPT CONDITION

This Appendix reports the approximate calculation of the \( x^* \) value that satisfies (25), which is reported in the following for clarity:

\[
E_C + \frac{V_{SJ}}{W} \exp \left( \frac{-2}{L(\eta)} \right) - \frac{2V_u}{W} x^* + \frac{V_{SJ}}{W} \exp \left( \frac{2(x^* - W)}{L_{(*)} W} \right) = 0. \quad (30)
\]
Since $L(\cdot) \ll 1$, it is also true that $\exp(-2/L(\cdot)) \ll 1$ and $\exp(-2x/L(\cdot)W) \ll 1$. Equation (30) is then approximated as

$$E_C - \frac{V_{SJ}}{W} - 2\frac{V_u (x^* - W)}{W} - 2\frac{V_a}{W} - \frac{V_{SJ}}{W} \exp\left(\frac{2(x^* - W)}{L(\cdot)W}\right) = 0. \quad (31)$$

Since $(2(x^* - W)/L(\cdot)W) \ll 1$ when $x^* \approx W$, it is reasonable to expand the exponential term in Taylor series. The expansion is up to the second order

$$E_C - \frac{V_{SJ}}{W} - 2\frac{V_u (x^* - W)}{W} - 2\frac{V_a}{W} - \frac{V_{SJ}}{W} \times \left[1 + \frac{2}{L(\cdot)} \frac{(x^* - W)}{W} + \frac{1}{2} \left(\frac{2}{L(\cdot)} \frac{(x^* - W)}{W}\right)^2\right] = 0. \quad (32)$$

The second-order equation is solved with respect to the variable $\alpha = (x^* - W)/W$, obtaining

$$\alpha = \frac{L(\cdot)^2}{2V_{SJ}} \left[-V_u - V_{SJ}/L(\cdot) + (V_a + V_{SJ}/L(\cdot)) \sqrt{1 - \frac{4V_{SJ}NP(V_{SJ} + V_a)}{L(\cdot)^2(V_u + V_{SJ}/L(\cdot))^2}}\right]. \quad (33)$$

Since the required solutions are close to the balanced condition for which $NP = 0$, the square root is also conveniently Taylor expanded up to the second order. With the position

$$\beta = NP \frac{V_{SJ}(V_u + V_{SJ}/L(\cdot))}{L(\cdot)^2(V_u + V_{SJ}/L(\cdot))^2}, \quad (34)$$

we have

$$\alpha = \frac{L(\cdot)^2}{2V_{SJ}} \left[-V_u - V_{SJ}/L(\cdot) + (V_u + V_{SJ}/L(\cdot)) \sqrt{1 - 4\beta}\right]$$

$$\simeq \frac{L(\cdot)^2}{2V_{SJ}} \left[-V_u - V_{SJ}/L(\cdot) + (V_u + V_{SJ}/L(\cdot)) (1 - 2\beta - 2\beta^2)\right]$$

$$= -\frac{L(\cdot)^2(V_u + V_{SJ}/L(\cdot))}{V_{SJ}}(\beta + \beta^2). \quad (35)$$

Equation (35) results in (26) reported in the following for clarity:

$$x^* = W - W \sqrt{\frac{L(\cdot)^2(V_u + V_{SJ}/L(\cdot))}{V_{SJ}}} (\beta + \beta^2). \quad (36)$$

**REFERENCES**


Han Wang received the B.A. and M.Eng. degrees (with highest honors) in electrical and information science from the University of Cambridge, Cambridge, U.K., in 2006 and 2007, respectively. He is currently working toward the Ph.D. degree in electrical engineering and computer science at the Massachusetts Institute of Technology (MIT), Cambridge, MA. During the summer of 2005, he was a Student Intern with the Mobile Device Research Group, British Telecom, Ipswich, U.K. In the summer of 2006, he also worked as a UROP Student with the Communications and Signal Processing Group, Imperial College, London, U.K. From 2006 to 2007, he was with the University of Cambridge, where he worked on the modeling and simulation of power electronic devices. Since 2008 with MIT, he has been working on GaN- and graphene-based devices. His current research interest focuses on the development of GaN-based transistors for millimeter-wave applications and the search of novel graphene-based unipolar devices for applications in nonlinear electronics.
Ettore Napoli was born in Italy in 1971. He received the M.Sc. degree in electronic engineering (with honors) in 1995, the Ph.D. degree in electronic engineering in 1999, and the B.Sc. degree in physics (with honors) in 2009, all from the University of Napoli Federico II, Napoli, Italy.

In 2004, he was a Research Associate with the Engineering Department, University of Cambridge, Cambridge, U.K. Since 2005, he has been an Associate Professor with the University of Napoli Federico II, Napoli, Italy. His scientific interests include the modeling and design of power semiconductor devices and VLSI circuit design. In the power devices field, his main interests are PiN diodes, vertical IGBTs, superjunction devices, and lateral IGBTs. In the VLSI field, his interests are high-speed arithmetic subsystems and advanced flip-flops. He is the author or coauthor of more than 80 papers published in international journals and conferences.

Florin Udrea received the Ph.D. degree from the Engineering Department, University of Cambridge, Cambridge, U.K., in 1995.

He was an Advanced EPSRC Research Fellow from August 1998 to July 2003 and, prior to this, a College Research Fellow with Girton College, University of Cambridge. Since October 1998, he has been a Reader with the Engineering Department, University of Cambridge. He has published over 250 papers in journals and international conferences and is the holder of over 50 patents in power semiconductor devices and sensors. He is currently leading a research group in power semiconductor devices and solid-state sensors. In August 2000, he cofounded with Prof. Amaratunga Cambridge Semiconductor (also called CamSemi), a start-up company in the field of power integrated circuits. In 2008, he cofounded Cambridge CMOS Sensors with Prof. Gardner and Prof. Milne.