Angular distributions in the decay $B\rightarrow K^{*}\pi^{+}\pi^{-}$

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Angular distributions in the decay $B \to K\ell^+\ell^-$

We use a sample of $384 \times 10^6 \ BB$ events collected with the BABAR detector at the PEP-II $e^+ e^-$ collider to study angular distributions in the rare decays $B \rightarrow K^* \ell^+ \ell^-$, where $\ell^+ \ell^-$ is either $e^+ e^-$ or $\mu^+ \mu^-$. For low dilepton invariant masses, $m_{\ell \ell} < 2.5 \text{ GeV}/c^2$, we measure a lepton forward-backward asymmetry $A_{FB} = 0.24^{+0.18}_{-0.13} \pm 0.05$ and $K^*$ longitudinal polarization $F_L = 0.35 \pm 0.16 \pm 0.04$. For $m_{\ell \ell} > 3.2 \text{ GeV}/c^2$, we measure $A_{FB} = 0.76^{+0.52}_{-0.32} \pm 0.027$ and $F_L = 0.71^{+0.22}_{-0.00}$.

DOI: 10.1103/PhysRevD.79.031102 PACS numbers: 13.20.He
The decays \( B \rightarrow K^+ \ell^+ \ell^- \), where \( K^+ \rightarrow K \pi \) and \( \ell^+ \ell^- \) is either an \( e^+e^- \) or \( \mu^+\mu^- \) pair, arise from flavor-changing neutral currents (FCNC), which are forbidden at tree level in the standard model (SM). The lowest-order SM processes contributing to these decays are the photon or \( Z \) penguin and the \( W^+W^- \) box diagrams shown in Fig. 1. The amplitudes can be expressed in terms of effective Wilson coefficients for the electromagnetic penguin, \( C_{\text{eff}} \), and the vector and axial-vector electroweak contributions, \( C_{\gamma}^{\text{eff}} \) and \( C_{10}^{\text{eff}} \), respectively, arising from the interference of the \( Z \) penguin and \( W^+W^- \) box diagrams [1]. The angular distributions in these decays as a function of dilepton mass squared \( q^2 = m_{\ell^+\ell^-}^2 \) are sensitive to many possible new physics contributions [2].

We describe measurements of the distribution of the angle \( \theta_k \), between the \( K^+ \) and the \( B \) directions in the \( K^+ \) rest frame. A fit to \( \cos \theta_k \) of the form [3]

\[
\frac{3}{2} F_L \cos^2 \theta_k + \frac{3}{2}(1 - F_L)(1 - \cos^2 \theta_k)
\]

determines \( F_L \), the \( K^+ \) longitudinal polarization fraction. We also describe measurements of the distribution of the angle \( \theta_\ell \), between the \( \ell^+ \) ( \( \ell^- \) ) and the \( B(\bar{B}) \) direction in the \( \ell^+ \ell^- \) rest frame. A fit to \( \cos \theta_\ell \) of the form [3]

\[
\frac{3}{2} F_L (1 - \cos^2 \theta_\ell) + \frac{3}{2}(1 - F_L)(1 + \cos^2 \theta_\ell) + \mathcal{A}_{\text{FB}} \cos \theta_\ell
\]

determines \( \mathcal{A}_{\text{FB}} \), the lepton forward-backward asymmetry. These measurements are done in a low \( q^2 \) region 0.1 < \( q^2 < 6.25 \) GeV\(^2\)/c\(^4\), and in a high \( q^2 \) region above 10.24 GeV\(^2\)/c\(^4\). We remove the \( J/\psi \) and \( \psi(2S) \) resonances by vetoing events in the regions \( q^2 = 6.25 - 10.24 \) GeV\(^2\)/c\(^4\) and \( q^2 = 12.96 - 14.06 \) GeV\(^2\)/c\(^4\), respectively.

The SM predicts a distinctive variation of \( \mathcal{A}_{\text{FB}} \), arising from the interference between the different amplitudes. The expected SM dependence of \( \mathcal{A}_{\text{FB}} \) and \( F_L \) on \( q^2 \) along with variations due to opposite-sign Wilson coefficients are shown in Fig. 3. At low \( q^2 \), where \( C_{\gamma}^{\text{eff}} \) dominates, \( \mathcal{A}_{\text{FB}} \) is expected to be small with a zero-crossing point at \( q^2 \sim 4 \) GeV\(^2\)/c\(^4\) [4–6]. There is an experimental constraint on the magnitude of \( C_{\gamma}^{\text{eff}} \) coming from the branching fraction for \( b \rightarrow s \gamma \) [6,7], which corresponds to the limit \( q^2 \rightarrow 0 \). However, a reversal of the sign of \( C_{\gamma}^{\text{eff}} \) is allowed. At high \( q^2 \), the product of \( C_{\gamma}^{\text{eff}} \) and \( C_{10}^{\text{eff}} \) is expected to give a large positive asymmetry. Right-handed weak currents have an opposite-sign \( C_{\gamma}^{\text{eff}}C_{10}^{\text{eff}} \) which would give a negative \( \mathcal{A}_{\text{FB}} \) at high \( q^2 \). Contributions from non-SM processes can change the magnitudes and relative signs of \( C_{\gamma}^{\text{eff}} \), \( C_{\text{eff}} \) and \( C_{10}^{\text{eff}} \), and may introduce complex phases between them [3,8]. An experimental determination of \( F_L \) is required to obtain a model-independent \( \mathcal{A}_{\text{FB}} \) result, and thus avoid drawing possibly incorrect inferences about new physics from our observations.

We reconstruct signal events in six separate flavor-specific final states containing an \( e^+e^- \) or \( \mu^+\mu^- \) pair, and a \( K^*(892) \) candidate reconstructed as \( K^+ \pi^- \), \( K^+ \pi^0 \) or \( K^0 \pi^+ \) (or their charge conjugates). To understand combinatorial backgrounds we also reconstruct samples containing the same hadronic final states and \( e^-\mu^\mp \) pairs, where no signal is expected because of lepton-flavor conservation. To understand backgrounds from hadrons (\( h \)) misidentified as muons, we similarly reconstruct samples containing \( h^\pm\mu^\mp \) pairs with no particle identification requirement for the \( h^\pm \).

We use a data set of \( 384 \times 10^6 B\bar{B} \) pairs collected at the Y(4S) resonance with the BABAR detector [9] at the PEP-II asymmetric-energy \( e^+e^- \) collider. Tracking is provided by a five-layer silicon vertex tracker and a 40-layer drift chamber in a 1.5 T magnetic field. We identify electrons with a CsI(Tl) electromagnetic calorimeter, muons with an instrumented magnetic flux return, and \( K^+ \) using a detector of internally reflected Cherenkov light as well as ionization energy loss information. Charged tracks other than identified \( e, \mu \) and \( K \) candidates are treated as pions. Electrons (muons) are required to have momenta \( p > 0.3(0.7) \) GeV/c in the laboratory frame. We add photons to electrons when they are consistent with bremsstrahlung, and do not use electrons that arise from photon conversions to low-mass \( e^+e^- \) pairs. Neutral \( K^0 \) and \( \pi^- \) candidates are required to have an invariant mass consistent with the nominal \( K^0 \) mass [10], and a flight distance from the \( e^+e^- \) interaction point which is more than 3 times its uncertainty. Neutral pion candidates are formed from two photons with \( E_\gamma > 50 \) MeV, and an invariant mass between 115 and 155 MeV/c\(^2\). We require \( K^*(892) \) candidates to have an invariant mass 0.82 < \( M(K\pi) < 0.97 \) GeV/c\(^2\).

\( B \rightarrow K^+ \ell^+ \ell^- \) decays are characterized by the kinematic variables \( m_{\text{ES}} = \sqrt{s}/4 - p_B^2 \) and \( \Delta E = E_B^* - \sqrt{s}/2 \), where \( p_B^* \) and \( E_B \) are the reconstructed \( B \) momentum and energy in the center-of-mass (CM) frame, and \( \sqrt{s} \) is the total CM energy. We define a fit region \( m_{\text{ES}} > 5.2 \) GeV/c\(^2\), with \( -0.07 < \Delta E < 0.04 \) (\( -0.04 < \Delta E < 0.04 \) GeV) for \( e^+e^- \) (\( \mu^+\mu^- \) ) final states in the low \( q^2 \) region, and \( -0.08 < \Delta E < 0.05 \) (\( -0.05 < \Delta E < 0.05 \) GeV) for high \( q^2 \). We use the wider (narrower) \( \Delta E \) windows to select the \( e^+\mu^- \) (\( h^\pm\mu^\mp \)) background samples.

The most significant background arises from random combinations of leptons from semileptonic \( B \) and \( D \) de-
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cays. In $B \bar{B}$ events the leptons are kinematically correlated if they come from $B \rightarrow D^{(*)} l \nu$, $D \rightarrow K^{(*)} l \nu$. Uncorrelated backgrounds combine leptons from separate $B$ decays or from continuum $e^+ e^- \rightarrow c\bar{c}$ events. We suppress these types of combinatorial background through the use of neural networks (NN). For each final state we use four separate NN designed to suppress either continuum or $B \bar{B}$ backgrounds in either the low or high $q^2$ regions, and different selections of NN inputs are used depending on $q^2$ bin (low, high), the identity of the leptons in the final state ($e$, $\mu$), and the type of background ($B \bar{B}$, continuum). Inputs include:

(i) event thrust;
(ii) ratio of second-to-zeroth Fox-Wolfram moments $^{[11]}$;
(iii) $m_{ES}$ and $\Delta E$ of the rest of the event (ROE), comprising all charged tracks and neutral energy deposits not used to reconstruct the signal candidate;
(iv) the magnitude of the total event transverse momentum, which is correlated with missing energy due to unreconstructed neutrinos in background semileptonic decays;
(v) dilepton system’s distance of closest approach along the beam axis, and separately in the plane perpendicular to the beam axis, to the primary interaction point;
(vi) vertex probability of the signal candidate and, separately, of the dilepton system;
(vii) the cosines in the CM frame of the angle between the $B$ candidate’s momentum and the beam axis, the angle between the event thrust axis and the beam axis ($\theta_{thrust}$), the angle between the ROE thrust axis and the beam axis ($\theta_{ROE}$), and the angle between $\theta_{thrust}$ and $\theta_{ROE}$.

There is also a background contribution in the signal region from $B \rightarrow D(K^+)\pi$ decays, where both pions are misidentified. The misidentification rates for muons and electrons are $\sim 2\%$ and $\sim 0.1\%$, respectively, so this background is only significant in the $\mu^+\mu^-$ final states. These events are vetoed if the invariant mass of the $K^+$ signal is in the range 1.84–1.90 GeV/$c^2$.

We optimize the NN and $\Delta E$ selections for each final state in each $q^2$ bin to give the best combined statistical signal significance in the $m_{ES}$ signal region $m_{ES} > 5.27$ GeV/$c^2$ for the sum of all six final states. After all these selections have been applied, the final reconstruction efficiencies and expected yields for signal events (calculated using world average branching fractions $^{[7]}$), as well as expected yields for background events in the signal region, are shown in Table I.

For each $q^2$ region, we combine events from all six final states and perform three successive unbinned maximum likelihood fits. Because of the relatively small number of signal candidates in each $q^2$ region, a simultaneous fit over $m_{ES}$, $\cos \theta_K$, and $\cos \theta_\ell$ is unlikely to converge and a sequential fitting procedure is required. We initially fit the $m_{ES}$ distribution using events with $m_{ES} > 5.2$ GeV/$c^2$ to obtain the signal and background yields, $N_S$ and $N_B$, respectively. We use an ARGUS shape $^{[12]}$ with a free shape parameter to describe the combinatorial background in this fit. For the signal, we use a Gaussian shape with a mean $m_{ES} = 5.2791 \pm 0.0001$ GeV/$c^2$ and $\sigma = 2.60 \pm 0.03$ MeV/$c^2$, which are determined from a fit to the vetoed charmonium samples. In this and subsequent fits we account for a small contribution from misidentified hadrons by subtracting the $K^+h^\pm\mu^\mp$ events, weighted by the probability for the $h^\pm$ to be misidentified as a muon. We also account in all fits for charmonium events that escape the veto, and for misreconstructed signal events. We estimate contributions from nonresonant $K\pi$ decays by fitting events outside the $K^+$ mass window in the range 0.7–1.1 GeV/$c^2$. We find no signal-like events that are not accounted for by the tails of the resonant mass distribution, and thus do not expect any significant contribution from nonresonant events within the mass window.

The second fit is to the cosine of the helicity angle of the $K^+$ decay, $\cos \theta_K$, for events with $m_{ES} > 5.27$ GeV/$c^2$. In this fit, the only free parameter is $F_L$, with the normalizations for signal and combinatorial background events taken from the initial $m_{ES}$ fit. The background normalization is obtained by integrating, for $m_{ES} > 5.27$ GeV/$c^2$, the ARGUS shape resulting from the $m_{ES}$ fit. We model the $\cos \theta_K$ shape of the combinatorial background using $e^+e^-$ and $\mu^+\mu^-$ events, as well as lepton-flavor violating $e^+\mu^-$ and $\mu^+e^-$ events, in the $5.20 < m_{ES} < 5.27$ GeV/$c^2$ sideband. The signal distribution given in Eq. (1) is folded with the detector acceptance as a function of $\cos \theta_K$, which is obtained from simulated signal events.

The final fit is to the cosine of the lepton helicity angle, $\cos \theta_\ell$, for events with $m_{ES} > 5.27$ GeV/$c^2$. The only free parameter in this fit is $A_{FB}$, with the signal distribution given in Eq. (2) folded with the detector acceptance as a function of $\cos \theta_\ell$. In this fit, the value of $F_L$ is fixed from the result of the second fit, and normalizations for signal and combinatorial background events are identical to those used in the second fit. We constrain the $\cos \theta_\ell$ shape of the

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**TABLE I.** Signal efficiencies (%), and expected signal and background yields for $m_{ES} > 5.27$ GeV/$c^2$, for low and high $q^2$ regions.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Signal Eff.</th>
<th>Signal Yield</th>
<th>Bkgd. Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>low</td>
<td>high</td>
<td>low</td>
</tr>
<tr>
<td>$K^+\pi^0\mu^+\mu^-$</td>
<td>1.6</td>
<td>3.1</td>
<td>1.0</td>
</tr>
<tr>
<td>$K^0_\Sigma\pi^+\mu^+\mu^-$</td>
<td>3.6</td>
<td>5.5</td>
<td>3.0</td>
</tr>
<tr>
<td>$K^0_\Sigma\pi^+\pi^+\pi^-$</td>
<td>4.5</td>
<td>8.1</td>
<td>5.5</td>
</tr>
<tr>
<td>$K^+\pi^0e^+e^-$</td>
<td>4.6</td>
<td>5.3</td>
<td>2.8</td>
</tr>
<tr>
<td>$K^0_\Sigma\pi^+e^+e^-$</td>
<td>7.0</td>
<td>5.4</td>
<td>5.9</td>
</tr>
<tr>
<td>$K^+\pi^-e^+e^-$</td>
<td>8.6</td>
<td>10.3</td>
<td>10.5</td>
</tr>
<tr>
<td><strong>Total Yield</strong></td>
<td>28.6</td>
<td>35.8</td>
<td>35.8</td>
</tr>
</tbody>
</table>
TABLE II. Results for the $B \to J/\psi K^*$ control samples. $\Delta BF$ are the differences between the measured branching fractions and the world average [10]. The previously measured $F_L = 0.56 \pm 0.01$ [13], and the expected $A_{FB} = 0$.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\Delta BF$ (10$^{-3}$)</th>
<th>$F_L$</th>
<th>$A_{FB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+ \pi^0 \mu^+ \mu^-$</td>
<td>$+0.09 \pm 0.12$</td>
<td>$0.54 \pm 0.03$</td>
<td>$-0.04 \pm 0.05$</td>
</tr>
<tr>
<td>$K^0_S \pi^+ \mu^+ \mu^-$</td>
<td>$+0.02 \pm 0.11$</td>
<td>$0.55 \pm 0.02$</td>
<td>$+0.00 \pm 0.05$</td>
</tr>
<tr>
<td>$K^+ \pi^+ \mu^+ \mu^-$</td>
<td>$-0.03 \pm 0.07$</td>
<td>$0.56 \pm 0.02$</td>
<td>$-0.02 \pm 0.02$</td>
</tr>
<tr>
<td>$K^0_S \pi^+ \mu^+ \mu^-$</td>
<td>$+0.16 \pm 0.10$</td>
<td>$0.54 \pm 0.03$</td>
<td>$+0.02 \pm 0.03$</td>
</tr>
<tr>
<td>$K^0_S \pi^- \mu^- \mu^-$</td>
<td>$+0.07 \pm 0.10$</td>
<td>$0.55 \pm 0.02$</td>
<td>$-0.02 \pm 0.04$</td>
</tr>
<tr>
<td>$K^0_S \pi^- \mu^- \mu^-$</td>
<td>$+0.02 \pm 0.07$</td>
<td>$0.56 \pm 0.02$</td>
<td>$+0.01 \pm 0.02$</td>
</tr>
</tbody>
</table>

Combinatorial background using the same sideband samples as for the $\cos \theta_K$ fit. The correlated leptons from $B \to D^{(*)} \ell \nu$, $D \to K^{(*)} \ell \nu$ give rise to an $m_{ES}$-dependent peak in the combinatorial background at $\cos \theta_K > 0.7$, and we consider this correlation in our study of systematic errors. No such correlation is observed for $\cos \theta_K$.

We test our fits using the large sample of vetoed charmonium events. The branching fractions (BF) and $K^0$ polarization for $B \to J/\psi K^*$ are well known [10,13], and $A_{FB}$ is expected to be zero. The results of the fits to the six final states are all consistent with expected values (see Table II). We further test our methodology by performing the $m_{ES}$ and $\cos \theta_K$ fits on a sample of $B^+ \to K^+ \ell^+ \ell^-$ decays. The results are given in Table III and are consistent with negligible forward-backward asymmetry, as expected in the SM and most new physics models [14].

TABLE III. Results for the fits to the $K^+ \ell^+ \ell^-$ and $K^0_S \ell^+ \ell^-$ samples. $N_S$ is the number of signal events in the $m_{ES}$ fit. The quoted errors are statistical only.

<table>
<thead>
<tr>
<th>Decay</th>
<th>$q^2$</th>
<th>$N_S$</th>
<th>$F_L$</th>
<th>$A_{FB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+ \ell^+ \ell^-$</td>
<td>low</td>
<td>$26.0 \pm 5.7$</td>
<td>$+0.04^{+0.16}_{-0.12}$</td>
<td>$0.35 \pm 0.16$</td>
</tr>
<tr>
<td></td>
<td>high</td>
<td>$26.5 \pm 6.7$</td>
<td>$+0.20^{+0.14}_{-0.22}$</td>
<td>$0.35 \pm 0.16$</td>
</tr>
<tr>
<td>$K^0_S \ell^+ \ell^-$</td>
<td>low</td>
<td>$27.2 \pm 6.3$</td>
<td>$+0.24^{+0.18}_{-0.22}$</td>
<td>$0.35 \pm 0.16$</td>
</tr>
<tr>
<td></td>
<td>high</td>
<td>$36.6 \pm 9.6$</td>
<td>$+0.76^{+0.32}_{-0.23}$</td>
<td>$0.35 \pm 0.16$</td>
</tr>
</tbody>
</table>

The systematic errors on the fitted values of $F_L$ and $A_{FB}$ are summarized in Table IV. The uncertainties in the fitted signal yields $N_S$, due to variations in the ARGUS shape in the $m_{ES}$ fits, are propagated into the angular fits. The errors on the fitted $F_L$ values are propagated into the $A_{FB}$ fits. We vary the combinatorial background shapes by dividing the sideband sample into two disjoint regions in $m_{ES}$. We try the signal model using simulated events generated with different form factors [5,15], and with a range of values of $C_7^{\text{eff}}$, $C_9^{\text{eff}}$, and $C_{10}^{\text{eff}}$, to determine an average fit bias. Finally, the modeling of misreconstructed signal events is constrained by the fits to the charmonium samples (Table II), where it is the largest systematic uncertainty.

The final fits to the $K^+ \ell^+ \ell^-$ samples are shown in Fig. 2. The results for $F_L$ and $A_{FB}$ are given in Table III and are shown in Fig. 3. In the low $q^2$ region, where we expect

![FIG. 2 (color online). $K^+ \ell^+ \ell^-$ fits: (a) low $q^2$ $m_{ES}$, (b) high $q^2$ $m_{ES}$, (c) low $q^2 \cos \theta_K$, (d) high $q^2 \cos \theta_K$, (e) low $q^2 \cos \theta_K$, and (f) high $q^2 \cos \theta_K$, with combinatorial (dotted line) and peaking (long dashed line) background, signal (short dashed line) and total (solid line) fit distributions superimposed on the data points.](image-url)
where the first error is statistical and the second is systematic.

\[ A_{FB} = 0.24^{+0.18}_{-0.15} \pm 0.05 \] and \( F_L = 0.35 \pm 0.16 \pm 0.04 \), where the first error is statistical and the second is systematic. In the high \( q^2 \) region, the SM expectation is \( A_{FB} \sim 0.38 \) and \( F_L \sim 0.40 \), and we measure \( A_{FB} = 0.76^{+0.52}_{-0.32} \pm 0.07 \) and \( F_L = 0.71^{+0.20}_{-0.22} \pm 0.04 \), with a signal yield of 36.6 \( \pm 9.6 \) events. Theoretical uncertainties on the expected SM \( F_L \) and \( A_{FB} \) values are generally difficult to characterize in the high \( q^2 \) region, and although under better control for \( 1 < q^2 < 6 \) GeV\(^2/\)c\(^4\), the extension of our low \( q^2 \) region below 1 GeV\(^2/\)c\(^4\) makes estimates of uncertainties there difficult also. The quoted values are obtained using our implementation of the physics models described in [4,15], corresponding to the SM curves in Fig. 3.

The expected SM value of \( C_{10}^{\text{eff}} \) at next-to-next-to-leading logarithmic (NNLL) order is \( C_{10}^{\text{eff}} = -4.43 \) [16]. A more recent NNLL calculation which evaluates contributions from the full set of seven form factors gives \( C_{10}^{\text{eff}} = -4.13 \) [17]. The magnitude of possible contributions from new physics to \( C_{10} \) can be constrained if \( A_{FB} > 0 \) at high \( q^2 \). By combining such a constraint on \( A_{FB} \) with inclusive \( b \rightarrow s \ell^+ \ell^- \) branching fraction results, an upper bound of \( |C_{10}^{\text{eff}}| \leq 7 \) can be obtained, improving on an upper bound derived solely from branching fraction results of \( |C_{10}^{\text{eff}}| \leq 10 \) [18]. Our results are consistent with measurements by Belle [19], and replace the earlier BABAR results in which only a lower limit on \( A_{FB} \) was set in the low \( q^2 \) region [20].

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MES (Russia), MEC (Spain), and STFC (United Kingdom). Individuals have received support from the Marie Curie EIF (European Union) and the A. P. Sloan Foundation.

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