States of an Ensemble of Two-Level Atoms with Reduced Quantum Uncertainty

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States of an Ensemble of Two-Level Atoms with Reduced Quantum Uncertainty

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We generate entangled states of an ensemble of $5 \times 10^4 \, ^{87}\text{Rb}$ atoms by optical quantum nondemolition measurement. The resonator-enhanced measurement leaves the atomic ensemble, prepared in a superposition of hyperfine clock levels, in a squeezed spin state. By comparing the resulting reduction of quantum projection noise [up to 8.8(8) dB] with the concomitant reduction of coherence, we demonstrate a clock input state with spectroscopic sensitivity 3.0(8) dB beyond the standard quantum limit.

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Atomic clocks [1–3] and atom interferometers [4] are reaching the standard quantum limit (SQL) of precision [1,5,6], set by the quantum projection noise inherent in measurements on a collection of uncorrelated particles. In the canonical Ramsey interferometer with $N_0$ particles, a quantum mechanical phase is converted into occupation probabilities for two states and read out as a population difference $N$ between them. Entanglement can reduce the projection noise $\Delta N$ by redistributing it to another variable that does not directly affect the experiment precision. The resulting “squeezed spin state” [5–16] can be used as an input state to an interferometer to overcome the SQL [5,6,8,9].

Formally, the system can be described by an ensemble spin vector $\mathbf{S} = \sum s_i \mathbf{s}_i$ that is the sum over the (pseudo-) spins $s_i$ of the individual (spin-1/2) particles [5–7]. The ensemble spin $\mathbf{S}$ with $(S^2) = S(S+1)$ can take on values in the range $0 \leq S \leq S_0$, where $S_0 = N_0/2$. For a given $S$, the minimum variance $\Delta S_z^2$ of $S_z = N/2$ for an unentangled state is realized by the coherent spin state (CSS), and is given by $\Delta S_z^{2,\text{CSS}} = S/2 = |S|/2$, where it is assumed that the mean ensemble spin vector $\langle \mathbf{S} \rangle$ lies in the $xy$ plane. A spin state can be defined as squeezed if it satisfies $\xi_S = 2\Delta S_z^2/|S| < 1$ (entanglement criterion [7,11]), or $\xi_m = 2\Delta S_z^2 S_m/|S|^2 < 1$ (criterion for metrological gain [5,6], where $S_m$ is the initial spin of the uncorrelated ensemble before the squeezing). $\xi_m^{-1}$ represents the increase in the squared signal-to-noise ratio $|\langle S \rangle|^2/\Delta S_z^2$ over the value $2S_m$ for the initial uncorrelated state. Since $|\langle S \rangle| \leq S_m$, we have $\xi_S \leq \xi_m$; i.e., metrological gain guarantees entanglement.

The process utilized for spin squeezing can reduce $|\langle S \rangle|$ below the initial spin $S_m$ before the squeezing, thereby reducing the minimum variance $\Delta S_z^2$ that is consistent with an unentangled state [11]. Therefore, measurements of both spin noise $\Delta S_z$ and average spin length after squeezing $|\langle S \rangle|$ are necessary to verify spin squeezing or quantify metrological gain. While reduction of spin noise alone has sometimes been referred to as “spin squeezing” [17,18] or “number squeezing” [19,20], we take spin squeezing to require at least demonstrated entanglement, $\xi_S < 1$, although we are primarily interested in metrological gain, $\xi_m < 1$.

Spin noise has been modified by atomic collisions [19–21] and by absorption of squeezed light [15]. In dilute atomic systems, quantum nondemolition (QND) measurements with light [10–13,17,18,22] have reduced the projection noise of rotating [17] and stationary [18] spins. Spin squeezing has been achieved with two ions [8], and spectroscopic sensitivity further improved with a maximally entangled state of three ions [9]. Recently, spin squeezing with a Bose-Einstein condensate (BEC) in a multiple-well potential has been reported [23]. Demonstrated metrological gains over the SQL include $\xi_m^{-1} = 3.2(1)$ dB in the three-ion system [9]; $\xi_m^{-1} \approx 4$ dB by light-induced squeezing within individual atoms of large spin $s = 3$ [24]; and $\xi_m^{-1} = 3.8(4)$ dB for the BEC [23].

In this Letter, we demonstrate the generation of squeezed spin states of $5 \times 10^4$ trapped $^{87}\text{Rb}$ atoms on an atomic-clock transition by resonator-aided QND measurement with a far-detuned light field, as proposed by Kuzmich, Bigelow, and Mandel [10]. We verify the entanglement by comparing the observed reduction in projection noise below that of a coherent spin state [up to 8.8(8) dB] with the accompanying reduction in clock signal, and achieve a 3.0(8) dB improvement in precision over the SQL.

The light-induced spin squeezing presented here requires strong ensemble-light coupling [10,12–14] (large collective cooperativity [25]). This is achieved by means of a near-confocal optical resonator with, at the $2\pi/k = 780$ nm wavelength of the probe light, a finesse $F = 5.6(2) \times 10^3$, a linewidth $\kappa = 2\pi \times 1.01(3)$ MHz, and a mode waist $w = 56.9(4)$ $\mu$m at the atoms’ position, corresponding to a maximal single-atom cooperativity $\eta_0 = 24F/(\pi k^2 w^2) = 0.203(7)$ [25]. Our experiments are performed on an ensemble containing up to $N_a = 5 \times 10^4$ laser-cooled $^{87}\text{Rb}$ atoms optically trapped inside the resonator in a standing wave of 851-nm light (Fig. 1).

One resonator mode is tuned $3.57(1)$ GHz to the blue of the $|S^2S_{1/2}, F = 2 \rightarrow |S^2 P_{3/2}, F' = 3\rangle$ transition in $^{87}\text{Rb}$, such that the atomic index of refraction results in a
mode frequency shift $\omega$ that is proportional to the population difference $N = N_2 - N_1$ between the hyperfine clock states $|1\rangle = |S_1/2, F = 1, m_F = 0\rangle$ and $|2\rangle = |S_1/2, F = 2, m_F = 0\rangle$. The transmission of a probe laser tuned to the slope of this mode thus directly measures $S_e = N/2$, and is insensitive to total atom number (Fig. 1). The atom-resonator coupling also gives rise to a differential light shift between the clock states, which we use to verify experimentally the coupling strength calculated from first principles using spectroscopically determined resonator parameters. We measure a phase shift of 250(20) $\mu$rad per transmitted photon for a maximally coupled atom (on the resonator axis at an antinode of the probe standing wave), in excellent agreement with the calculated value 253(8) $\mu$rad [25].

To account for the spatial variation in coupling between standing-wave probe light and atoms, we define an effective atom number units, $N_e = (\langle \eta \rangle / \langle \eta^2 \rangle) N_0 = 0.66 N_0$, where the single-atom cooperativity $\eta$, proportional to the local intensity of probe light, is averaged over the ensemble containing $N_0$ atoms [25]. The definition is chosen so that the projection noise variance of the effective atom number measured via the mode shift $\omega \propto N_e \langle \eta \rangle$ satisfies the usual relation $\Delta N^2_e = N_e$. This avoids carrying near-unity factors through the equations and allows direct comparison to a spatially uniform system of collective cooperativity $N_0 \eta_{eff}$, where $\eta_{eff} = (2/3) \langle \eta \rangle^2 / \langle \eta \rangle = 0.47(1) \eta$, taking into account the oscillator strength 2/3 of the D2 line and the measured rms transverse cloud radius of 8.1(8) $\mu$m $\ll w$. The mode frequency shift per effective atom of population difference $N$ between the clock states is $d\omega / dN = 4.5(2) \times 10^{-5} \kappa$ [25].

To quantify spin squeezing, we need to measure $\Delta S^2_e$ and $\langle S \rangle$. The latter can be obtained from the observed contrast $C$ of Rabi oscillations as $\langle S \rangle = CS_0$, where the maximum spin $S_0 = N_0/2$ is measured by optically pumping the atoms between the two hyperfine states $F = 1, 2$. For large $S_0$, the cavity shift $\omega$ exceeds $\kappa (\omega \leq 1.8 \kappa)$, which we take into account by correcting for the (accurately measured) Lorentzian line shape of the resonator. To verify the atom numbers $25\rho_j$ thus obtained, we have also directly measured the cavity mode frequency shift $\omega \propto S_e$, finding agreement to within 2(4)% [25]. $\Delta S^2_e$ is obtained from transmission measurements that always remain in the linear regime, with $2\Delta S^2_e d\omega / dN = 0.01 \kappa$.

The probe laser is frequency-stabilized to a far-detuned, negligibly shifted mode [25]. Each measurement of $S_e$ employs two probe light pulses of duration $T = 50 \mu$s $\gg \kappa^{-1} = 158$ ns separated by a 280 $\mu$s delay, during which we apply a microwave $\pi$ pulse sequence [25] to suppress inhomogeneous light shifts (spin-echo sequence). Each probe light pulse contains $10^8$ to $10^9$ photons which, after traversing the resonator, are detected with an overall quantum efficiency $Q = 0.43(4)$. From the detected photon numbers in the two pulses, we deduce two cavity shifts $\omega_\pm$ whose difference constitutes a single measurement $M$ of $S_e = (\omega_+ - \omega_-)/(4d\omega / dN)$. In a typical experiment [Fig. 1(c)], after initializing the ensemble spin state by optical pumping into $|1\rangle$ (A) and applying a $\pi/2$ microwave pulse to rotate the CSS into an equal superposition of $|1\rangle$ and $|2\rangle$ (B), we perform two measurements $M_1$ and $M_2$ to induce and verify conditional spin squeezing. We quantify spin noise $\Delta S_e$ by extracting variances from 100 repetitions of such a sequence.

We determine the CSS projection noise level $\Delta S^2_{e, CSS} = N_0/4$ from the measured atom number $N_0$ and verify it [5,6,15,17] either by evaluating the variance $\text{Var}(M_1)$ of the set of single measurements $M_1$, or by inserting between two measurements $M_1$ and $M_2$ a second CSS preparation, consisting of optical pumping into state $|1\rangle$ and a $\pi/2$ pulse, and evaluating $\text{Var}(M_1 - M_2)/2$. Figure 2 shows the dependence of the corresponding quantities in atom number units, $y_1 = 4 \text{Var}(M_1)$ (open triangles) or $y_2 = 2 \text{Var}(M_1 - M_2)$ (open circles), on $N_0$. The contribution of CSS projection noise scales as $\Delta S^2_{e, CSS} \propto N_0$, while atom-number-dependent technical noise, e.g., due to microwave power fluctuations or any sensitivity to atom number fluctuations, generically scales as $\Delta S^2_{\text{tech}} \propto N_0^2$.

A quadratic fit $y_{1,2} = a_0 + a_1N_0 + a_2N_0^2$ yields $a_1 = 1.3(1)$ and $a_2 = 1(2) \times 10^{-6}$ (not shown in Fig. 2), but the data are also well fit by setting $a_1 = 1$, as required by independently measured cavity and atomic properties with no free parameters [25], and allowing a small technical noise contribution $a_2 N_0^2 < N_0$ with $a_2 = 9(3) \times 10^{-6}$ (solid curve). Slow drifts in microwave power of 0.4% over the set of measurements could account for the technical noise of $y_1$, which vanishes if the data are analyzed by comparing only adjacent cycles of the experiment [25]. Our ability to prepare an unentangled state close to a CSS—with $S_e$ variance $\Delta S^2_{e, prep} \sim 1.3 S_0/2$ for our largest atom number—is not a prerequisite for spin squeezing but does provide independent confirmation of the CSS refer-
The reduction of $|\Delta S_z^2|_{M_1}$ below the CSS value $\Delta S_z^{2\text{CSS}}$ is accompanied by a substantial increase in $\Delta S_z^2$ because the differential light shift of the atomic levels, corresponding to a rotation of the Bloch vector about the $z$ axis, depends on the intracavity intensity, which in turn depends on $S_z$. To observe the antisqueezing, we apply a microwave pulse after the squeezing measurement [at $X$ in Fig. 1(c)] to rotate the spin state by a variable angle $\alpha$ about ($S_z$) before reading out $S_z$. The variance $\Delta S_z^{2\text{rot}}$ of $S_z$ in the rotated state, displayed in the inset to Fig. 2, is a sinusoid that is well described with no free parameters by our model of the ensemble-cavity interaction [25].

To verify spin squeezing, we also need to measure $|\langle S_z\rangle|$, observable as the interference contrast $C = |\langle S_z\rangle|/S_0$ of Rabi oscillations induced between measurements $M_1$ and $M_2$. Figure 3 shows $C$ as a function of photon number $p$ used in the state-preparation measurement at $N_0 = 4.0(1) \times 10^3$, and we have verified that the contrast $C$ is independent of atom number [25]. Both normalized spin noise $\sigma^2$ and $C$ can be fit by simple models (dashed and dotted curves) [25]. From these two measurements, we deduce the metrological squeezing parameter $\xi_{z_m}$ (solid triangles and solid curve). For $p = 3 \times 10^4$, we achieve $\xi_{z_m}^{-1} = C^2/\sigma_{z_m}^2 = 3.0(8)$ dB of metrological gain [and an inverse entanglement parameter $\xi_{z_m}^{-1} = C/\sigma^2 = 4.2(8)$ dB, not shown]. The finite initial contrast $\sigma_{z_0}^2 = S_0/S_0 = 0.7$ in the ensemble without squeezing is due to the resonator locking light, and can be improved by detuning this light further from atomic resonance. The probe-induced contrast reduction probably arises from differential light shifts between the clock states that are imperfectly canceled by the spin-echo technique because of atomic motion. In the absence of any technical noise, a fundamental limit to the spin squeezing, associated with photon scattering into free space, would be $\xi_{z_m}^{-1} = \sqrt{3/2}N_0\eta_{eff} \sim 18$ dB in our system with cooperativity $N_0\eta_{eff} \sim 3100$ [13,14,25].

For the data presented above, the readout quantifying the entanglement was completed 500 $\mu$s after preparation of the squeezed state. We have further verified that the squeezing remains after a Ramsey clock sequence, in which two $\pi/2$ pulses about the $x$ axis, separated by a short (70 $\mu$s) precession time, are inserted at $X$ in Fig. 1(c). Such a clock can achieve precision below the SQL because the first of these $\pi/2$ rotations initiates it with a phase that
is known, from the squeezing measurement, to better than the CSS uncertainty.

The phase coherence time of the unsqueezed CSS in our current trap is 10(2) ms. Both microwave and optical clocks with $\sim 1$ s coherence times have already been demonstrated with trapped atoms [2,3,26–28]. Whether and to what degree the squeezing technique demonstrated here could benefit such clocks and other precision experiments [4] will depend on the clock characteristics, noise sources [16], and lifetime of the squeezed state. These questions, as well as possible systematic effects, need to be investigated in the future.

The group of E. Polzik independently and simultaneously achieved results similar to ours in a Mach-Zehnder interferometer [29]. We have recently demonstrated a new squeezing method using cavity feedback [30].

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