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Detailed Terms
ON THE ROLE OF FINANCIAL FRictions AND THE SAVING RATE DURING TRADE LIBERALIZATIONS

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Abstract
We study how financial frictions and the saving rate shape the long-run effects of trade liberalization on income, consumption, and the distribution of wealth in financially underdeveloped economies. In our model, regardless of whether the capital account is open or not, trade liberalization reduces the share of wealth in the hands of entrepreneurs and may well reduce steady-state consumption and income. Furthermore, trade opening is more likely to reduce steady-state consumption and output, the higher is the level of financial development. For economies with an open capital account, a higher saving rate also increases the likelihood that a trade liberalization leads to a reduction in steady-state consumption and output. (JEL: E2, F1, F2, F3, F4)

1. Introduction
In this paper, we study how financial frictions and the saving rate shape the long-run effects of trade liberalization on income, consumption, and the distribution of wealth in financially underdeveloped economies. We build on our previous work in Antràs and Caballero (2009; AC hereafter) where we developed a dynamic $2 \times 2$ general equilibrium model of international trade featuring heterogeneous financial frictions across countries and sectors. In AC, we focused on the effect of trade liberalization on the steady-state rental rate of capital and highlighted the result that in a world with heterogeneous financial development, trade and capital mobility are complements in financially underdeveloped economies.

The goal of this paper is to describe in more detail the dynamics of the model and to derive new results concerning the role of certain financial and macroeconomic factors in shaping these dynamics. Our first key result is that when financial frictions are important, the standard static gains from trade liberalization...
can be severely diluted over time in financially underdeveloped economies. The reason for this is that trade integration erodes the return to entrepreneurial capital due to competition from more developed economies, which are better able to channel funds to their entrepreneurs. These induced changes in the distribution of wealth lead to an endogenous tightening of credit conditions and may well result in steady-state consumption and income levels that are lower than those that would be attained without the trade liberalization.

Somewhat paradoxically, we find that trade opening is more likely to reduce steady-state consumption and output, the higher is the level of financial development (provided that this level is below the average one in the world). Furthermore, for economies with an open capital account, a higher saving rate also increases the likelihood that trade liberalization leads to a reduction in steady-state consumption and output.

Our work is related to the vast literature introducing financial frictions in international finance and international trade models (see AC and the references therein). A particularly related paper is Chesnokova (2009), who argues that opening an economy to trade can result in welfare reducing deindustrialization when agents are subject to credit constraints. More broadly, our work is related to the second-best literature in international trade (see Bhagwati and Ramaswami 1963).

The rest of the paper is organized as follows. In Section 2, we develop our small open-economy model and characterize its equilibrium. In Section 3, we study the effects of a partial trade liberalization for the case in which the capital account is closed, and in Section 4 we repeat the exercise for an economy with an open account. We offer some concluding remarks in Section 5.

2. A Small Open-Economy with Financial Frictions

Time evolves continuously. Infinitesimal agents are born at a rate $\varphi$ per unit of time and die at the same rate; population mass is constant and equal to $L$. All agents are endowed with one unit of labor services which they supply inelastically to the market. Agents save all their income and consume only when they (are about to) die. Thus, if $W_t$ denotes aggregate savings accumulated up to date $t$, then aggregate consumption at time $t$ is $\varphi W_t$, and $\varphi$ is inversely related to the aggregate propensity to save of this economy. The economy produces two goods (1 and 2) and agents consuming at time $t$ allocate their spending between these two consumptions in a way that maximizes the following instantaneous utility function

$$U = \left( \frac{C_1}{\eta} \right)^\eta \left( \frac{C_2}{1 - \eta} \right)^{1-\eta}.$$  
(1)
Physical capital is the only store of value in the economy and is freely tradable within borders. We will later consider the case in which it is also tradable across borders. We assume that the initial stock of capital is equal to $K_0$, that there is no depreciation, and that new physical capital can be produced by combining goods 1 and 2 according to the same utility aggregator in (1). As a result, the relative price of capital $q_t$ is equal to the ideal price index, $q_t = (p_1)^\eta (p_2)^{1-\eta}$, which we choose as our numéraire. We thus have $q_t = 1$ and $W_t = K_t$ at all times.

Production in both sectors combines physical capital and labor according to

$$Y_i = Z(K_i)^\alpha (L_i)^{1-\alpha}, \quad i = 1, 2,$$

where $K_i$ and $L_i$ are the amounts of capital and labor employed in sector $i$, and $Z$ is a Hicks-neutral productivity parameter. Although technology is identical in both sectors, we think of production in sector 1 as being relatively more complex, in the sense that at any point in time $t$, only a fraction $\mu$ of the population knows how to operate that production technology. We refer to these agents as entrepreneurs. In equilibrium, these agents will rent capital from the rest of the population, so we refer to these other agents as rentiers.

Goods and labor markets are perfectly competitive and factors of production are freely mobile across sectors. The distinctive feature of our model is that the capital market has a friction and that the financial friction has an asymmetric effect in the two sectors. In particular, we assume that financial contracting in sector 2 is perfect in the sense that any agent in the economy can hire any desired amount of capital at the equilibrium rental rate $\delta$ provided that capital is used in sector 2.

Conversely, there is a financial friction in sector 1. Because the production process is particularly complex, a problem of asymmetric information arises and rentiers are willing to lend to entrepreneurs only an amount proportional to the wealth of entrepreneurs (rather than an unlimited amount). Provided that financial constraints bind, in equilibrium entrepreneurs always invest their capital in sector 1, so the allocation of capital to sector 1 is given by

$$K_{1,t} = \theta s_t K_t,$$

where $\theta > 1$, and $s_t$ is the share of wealth (and thus of the physical capital stock) in the hands of entrepreneurs. In AC, we developed microfoundations for this constraint and we mapped the parameter $\theta$ to institutional features of the economy. We interpret a larger value of $\theta$ as reflecting a more developed financial system. In AC we also showed that a sufficient condition for financial constraints to bind (even in the steady state) is $\eta > \mu \theta$, so we make this parametric assumption throughout the text.

For simplicity, we assume that the economy is small in world markets and faces exogenously given prices of goods 1 and 2 ($p_1, p_2$). Furthermore, as shown
in AC, in our setup, as long as financial constraints bind in world markets, we have $\pi \equiv p_2/p_1 < 1$. Intuitively, financial constraints depress the relative supply of good 1 in world markets and this leads to a depressed relative price for good 2 (in the absence of financial frictions, we naturally have $\pi = 1$).

We are now ready to characterize the equilibrium of this dynamic economy. There are two state variables in the model: the stock of physical capital $K_t$ and the share of wealth $s_t$ in the hands of entrepreneurs. We will also be concerned with the determination of four prices: the wage rate ($w_t$), the rental rate of capital ($\delta_t$), the return to entrepreneurial capital ($R_t$), and the interest rate ($r_t$).

Although physical capital is tradable, we will assume that entrepreneurial ability is inalienable, and thus entrepreneurial rents are not capitalizable (i.e., only entrepreneurs can enjoy them). Given that the price of capital is always equal to 1, the financial return on holding a unit of capital (or interest rate) is simply equal to the rental rate of capital $\delta_t$, so $r_t = \delta_t$. Note that surviving agents spend all their income in buying capital and rentiers are always the marginal buyers.

In order to characterize the dynamic path of this economy, note that aggregate savings of each group (entrepreneurs $e$ and rentiers $r$) decrease with consumption, and increase with labor income, entrepreneurial rents (if any) and the return on accumulated savings:

$$\dot{K}_t^e = -\varphi s_t K_t + \mu w_t L + s_t R_t K_t,$$  \hspace{1cm} (4)

$$\dot{K}_t^r = -\varphi (1 - s_t) K_t + (1 - \mu) w_t L + \delta_t (1 - s_t) K_t.$$ \hspace{1cm} (5)

Manipulating these expressions we obtain a law of motion for capital,

$$\dot{K}_t = \delta_t K_t + w_t L + (R_t - \delta_t) s_t K_t - \varphi K_t,$$  \hspace{1cm} (6)

and a law of motion for the share of wealth in the hands of entrepreneurs,

$$\dot{s}_t = \frac{(1 - s_t)(R_t - \delta_t) s_t K_t - (s_t - \mu) w_t L}{K_t},$$ \hspace{1cm} (7)

both in terms of factor prices.

Equilibrium factor prices can in turn be obtained by (a) equating the wage rate to the value of the marginal product of labor in each sector, (b) imposing factor market clearing, and (c) equating the value of the marginal product of capital in sector 1 and 2 to $\delta_t (\theta - 1)/\theta + R_t/\theta$ and $\delta_t$, respectively (see AC for details). Defining

$$\rho(s_t, \pi) \equiv (1 - \theta s_t) \pi^{1/\alpha} + \theta s_t < 1,$$ \hspace{1cm} (8)
these steps yield

\[ w_t = \frac{(1 - \alpha)Z}{\pi^{1-\eta}} \left( \frac{\rho(s_t, \pi)}{L} \right)^{\alpha}, \]  

(9)

\[ \delta_t = \alpha Z \pi^{1/\alpha + \eta - 1} \left( \frac{\rho(s_t, \pi)}{L} \right)^{\alpha - 1}, \]

(10)

\[ R_t = (1 + \theta(\pi^{-1/\alpha} - 1))\delta_t. \]

Plugging these expressions back in equations (6) and (7), we can finally express the dynamic path of \( K_t \) and \( s_t \) in terms of these two state variables and exogenous parameters:

\[ \dot{K}_t = \frac{Z}{\pi^{1-\eta}}(\rho(s_t, \pi)K_t)\alpha L^{1-\alpha} - \varphi K_t \]

and

\[ \dot{s}_t = \left[ \alpha(1 - s_t)(1 - \pi^{1/\alpha})\theta s_t - (s_t - \mu)(1 - \alpha)\rho(s_t, \pi) \right] \frac{Z}{\pi^{1-\eta}} \left( \frac{\rho(s_t, \pi)}{L} \right)^{\alpha - 1}. \]

As shown in AC, this system is stable and, regardless of the initial values \( K_0 \) and \( s_0 \), the economy converges to a steady state implicitly defined by the following expressions:

\[ K^* = \left( \frac{Z}{\varphi} \frac{(\rho(s^*, \pi))^{\alpha}}{\pi^{1-\eta}} \right)^{1/(1-\alpha)} L, \]

(10)

\[ \frac{\alpha(1 - s^*)(1 - \pi^{1/\alpha})\theta s^*}{\rho(s^*, \pi)} = (s^* - \mu)(1 - \alpha); \]

(11)

and with associated factor prices

\[ w^* = (1 - \alpha)\varphi \left( \frac{Z}{\varphi} \frac{\rho(s^*, \pi)}{\pi^{1-\eta}} \right)^{1/(1-\alpha)}, \]

(12)

\[ \delta^* = \alpha \varphi \frac{\pi^{1/\alpha}}{\rho(s^*, \pi)}, \]

(13)

\[ R^* = (1 + \theta(\pi^{-1/\alpha} - 1))\delta^*. \]

(14)

3. A Trade Liberalization with a Closed Capital Account

Suppose now that at some time \( T > 0 \), this economy experiences an unexpected trade liberalization. We think of this economy as being relatively financially underdeveloped and so, as argued in AC, a reduction in trade barriers brings about an
increase in the relative price of this economy’s export sector, which is the less financially dependent sector 2. More formally, in AC we showed that as long as \( \theta \) is lower than the rest of the world’s average level of financial development, a fall in trade barriers increases the relative price \( \pi \) faced by that country.

In AC, we focused on a comparison of steady states for different values of \( \pi \) and emphasized the fact that trade liberalization increases the steady-state value of the rental rate of capital \( \delta^* \). This can be easily verified by combining equations (8), (11), and (13) and is the result of two effects. First, \( \delta^* \) increases with \( \pi \) holding constant \( s^* \). This is the impact effect of trade liberalization and is the effect emphasized in AC. The result is intuitive. The increased trade integration allows this economy to further specialize in its comparative advantage sector, which is the sector without financial frictions. The resulting shift of labor to sector 2 increases the return to capital in that sector (i.e., the rental rate of capital). The second channel through which trade liberalization affects the rental rate of capital is through its effect on the distribution of wealth in the economy. This is not an impact effect but rather a dynamic effect. Because of the higher rental rate and the reduced entrepreneurial return, the share of capital in the hands of entrepreneurs gradually falls through time and settles at a steady-state level that is decreasing in \( \pi \). Because the rental rate is decreasing in \( s^* \) (see equation (13)), we have that this endogenous fall in \( s^* \) leads to further increases in the rental rate along the transition path.

The dynamic effect has important consequences for the effects of trade liberalization on the remaining factor prices as well as on some of the aggregates of the economy. The reason is that the economy is inefficient because it is unable to allocate enough resources to the complex sector 1. By reducing the share of capital in the hands of entrepreneurs, trade liberalization aggravates this problem and qualifies the standard arguments in favor of trade liberalization.

A first illustration of the importance of the endogenous decline in entrepreneurial rents (and hence in their share of capital) comes from the behavior of wages following the trade liberalization period. Plugging expression (8) into equation (9) and differentiating, it is straightforward to verify that wages increase on impact following the reduction in trade barriers. Nevertheless, the subsequent fall in \( s^* \) leads to a gradual fall of the wage rate from its higher level achieved on impact. Hence, the gradual tightening of credit conditions erodes the static wage gains from trade liberalization. Whether wages settle at a level that is higher or lower than before the trade liberalization depends crucially on how the term

\[
\Lambda(\pi) = \frac{((1 - \theta s^*(\pi))\pi^{1/\alpha} + \theta s^*(\pi))^{\alpha}}{\pi^{1-\eta}}
\]

varies with \( \pi \) (see equation (12)). The total derivative of \( \Lambda(\pi) \) with respect to \( \pi \) is also crucial for signing the long-run effects of trade liberalization on the steady-state levels of the economy’s aggregate capital-labor ratio, aggregate
output and aggregate consumption. Differentiation then allows us to conclude as follows:

**Proposition 1.** Consider an economy with a closed capital account and a level of financial development below the average world level. Then, other things equal, a trade liberalization is more likely to reduce steady-state wages, consumption and output the higher is the level of financial development $\theta$.

See Appendix A for the proof.

To understand this result, remember that trade liberalization has two effects in the model. On the one hand, it generates the standard static gains from trade resulting from an improvement in the economy’s terms of trade. This gain is naturally higher for economies with financial development levels farther away from the average world one. This explains why the positive impact effect of trade liberalization on wages, consumption, and output is lower the higher is $\theta$. On the other hand, trade liberalization leads to a compression in the economy’s wealth distribution and this makes financial frictions more binding. This negative effect is more pronounced in economies that had less binding financial constraints to begin with (see Appendix A for details). Overall, the long-run effects of trade liberalization on wages, consumption and output may well be negative, and this is more likely to be so for economies with larger levels of $\theta$ (with $\theta$ being lower than the average level of financial development in the world).

Figure 1 illustrates the result in Proposition 1 for an economy with the following parameter values: $\alpha = 1/3$, $\mu = 0.2$, $\eta = 0.75$; $\varphi = 0.1$; $L = 1$; $Z = 1$. The experiment is an increase of the relative price $\pi$ from 0.7 to 0.8 in period 10. The main difference between panels A and B is that the level of financial development is lower in panel A ($\theta = 1.1$) than in panel B ($\theta = 1.4$). Figure 1 then confirms that the gains from trade liberalization are much more nuanced for economies with higher levels of $\theta$ (provided that $\theta$ is below the world average level of financial development). In particular, trade liberalization generates a jump in income and capital growth, but the subsequent income and capital growth paths are below those of an economy not experiencing the trade liberalization episode.

4. Trade Liberalization with an Open Capital Account

So far we have been assuming that the country is linked to the world economy only through the goods market. Consider now the case in which the country

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1. See equation (10) for the aggregate capital–labor ratio. Aggregate consumption is simply equal to $\varphi K_t$ at any point in time. Finally, aggregate output equals factor payments and thus $Y_t = \omega t L_t + \delta t K_t + s_t (\delta_t - \delta_t) K_t = Z (\rho(s_t, \pi) K_t) a L^{1-\alpha} / \pi^{1-\eta}$, where remember that $\rho(s_t, \pi) \equiv (1-\theta s_t) \pi^{1/\alpha} + \theta s_t$.

2. It can be verified that our choice of parameters implies that the world economy’s level of $\theta$ is larger than 1.4. These parameter choices also ensure that the autarky relative prices of the economies being studied are below 0.7.
Figure 1. Trade liberalization and financial development.

A. Trade Liberalization for $\theta = 1.1$

B. Trade Liberalization for $\theta = 1.4$
undergoes trade liberalization while having an open capital account. In such a case, the dynamics of the domestically owned capital stock \( K_t \) and the share \( s_t \) of this capital in the hands of entrepreneurs continue to be characterized by equations (6) and (7), but the determination of factor prices is now quite different.

First, note that the real rental rate of capital of this small open economy will be pinned down by the world rental rate. More precisely, rentier capital will move across borders to ensure that its return is equated worldwide. Denoting the net capital position of the country by \( F_t \) (where \( F_t \) can be negative), we have that at any point in time,

\[
\delta_t = \alpha Z \pi^\eta \left( \frac{(1 - \theta s_t) K_t + F_t}{L - L_{1t}} \right)^{\alpha - 1} = \delta^W,
\]

where \( L - L_{1t} \) is the economy’s allocation of labor to sector 2. For simplicity, we assume that the world rental rate is time-invariant. This expression indicates that the capital-labor ratio in sector 2 is pinned down by the world rental rate and time-invariant parameters, which in turn implies that the wage rate is also independent of local conditions (and time invariant)

\[
w_t = \frac{1 - \alpha}{\alpha} \left( \frac{\alpha Z \pi^\eta}{\delta^W} \right)^{1/(1-\alpha)} \delta^W.
\]

Finally, the return to entrepreneurial capital is also time-invariant, and as before is given by

\[
R_t = (1 + \theta (\pi^{-1/\alpha} - 1)) \delta^W.
\]

Given these expressions for factor prices, it is apparent that a process of trade liberalization raises the wage rate, reduces the return to entrepreneurial capital, and leaves the rental rate of capital unchanged. Furthermore, factor prices jump to their new level on impact and remain at that level thereafter. How is aggregate income affected by the trade liberalization? As in the case with a closed capital account, one can show that an increase in \( \pi \) always increases aggregate income in the economy with an autarky relative price lower than the world one.

3. A small complication arises from the fact that it is now theoretically possible that an economy with a level of financial development below the world average level of \( \theta \) may feature an autarky relative price \( \pi \) above the world one. The reason for this is that if the economy faces a sufficiently high world rental rate, rentier capital will to a large extent be employed abroad and domestic production in sector 2 will be relatively low (hence putting upward pressure on \( \pi \)). Still, such an economy will also benefit from trade liberalization because trade opening would then lead to a decline in \( \pi \) and such a decline is always welfare enhancing for an economy with an autarky relative price above the world one.
The fact that factor prices remain constant after the trade liberalization episode does not imply that the economy does not feature interesting dynamics after the shock. In particular, the impact changes on factor prices will affect aggregate income, the incentives of the economy to invest as well as the wealth accumulation paths of entrepreneurs and rentiers. Plugging the above expressions for factor prices into equations (6) and (7), and solving for the steady state, we obtain

\[ K^* = \frac{1 - \alpha}{\alpha} \left( \frac{\alpha Z \pi^\eta}{\delta W} \right)^{1/(1-\alpha)} \delta W \left( \varphi - \delta W \right) \left( 1 - \frac{\mu \theta (\pi^{-1/\alpha} - 1) \delta W}{\varphi - \delta W - \theta \delta W (1 - \mu) (\pi^{-1/\alpha} - 1)} \right) L \]

and

\[ s^* = \frac{(\varphi - \delta W) \mu}{\varphi - \delta W - \theta \delta W (1 - \mu) (\pi^{-1/\alpha} - 1)}. \]

As in the closed capital account case, it is clear that a process of trade liberalization will reduce \( s^* \). The effect on the steady-state capital stock (and thus on the steady-state consumption and income levels) is more complicated and crucially depends on the term

\[ \Gamma(\pi) \equiv \frac{(\pi^\eta)^{1/(1-\alpha)}}{\left( 1 - \frac{\mu \theta (\pi^{-1/\alpha} - 1) \delta W}{\varphi - \delta W - \theta \delta W (1 - \mu) (\pi^{-1/\alpha} - 1)} \right)}. \]

Straightforward differentiation yields the following result.

**Proposition 2.** Consider an economy with an open capital account and an autarky relative price of good 2 below the world relative price. Then, other things equal, a trade liberalization is more likely to reduce steady-state consumption and output, the higher is the level of financial development \( \theta \) and the propensity to save \( 1 - \varphi \).

See Appendix B for the proof.

The intuition behind the result regarding the level of financial development is similar to the one explained above for the case of a closed capital account. The main novelty of Proposition 2 is that the propensity of the economy to save is now also an important determinant of whether trade liberalization increases or decreases steady-state consumption and income. The reason for this is related to the negative effect of trade opening on the share of wealth in the hands of entrepreneurs. Economies with high saving rates (low levels of \( \varphi \)) tend to accumulate higher levels of entrepreneurial capital income relative to labor income,
and thus the negative effect of trade on the share $s^*$ is particularly harmful for those economies. Why does the saving rate matter with an open capital account but not with a closed capital account? In the latter case, factor prices are a function of the capital stock in the economy and this leads to a ratio of entrepreneurial capital income to labor income that is independent of the savings rate (see equation (11)). Although this feature of our closed capital account model seems related to our Cobb–Douglas assumptions on production, the general point is that the distribution of wealth will be much more responsive to the savings rate in economies where factor prices are pinned down by international markets (as is the case of our open capital account variant of the model).

As our discussion suggests, the key behind the interaction between the saving rate and the sign of the gains from trade relates to the effect of savings behavior on the share of wealth in the hands of entrepreneurs. It is naturally the case that the more entrepreneurs save, the more wealth they will accumulate relative to the other agents in the economy and the larger is the welfare loss associated with the erosion of these agents’ rents. Nevertheless, a higher propensity to save by rentiers should have the opposite effect on the share $s^*$. A simple extension of our model that incorporates distinct savings parameters $\varphi_e$ and $\varphi_r$ for entrepreneurs and rentiers confirms this intuition (details available upon request). In our model, the effect of the parameter $\varphi_e$ dominates that of $\varphi_r$, but it is important to bear in mind these offsetting effects in empirical exercises that are able to identify them separately.

Figure 2 illustrates the result in Proposition 2 for an economy with the same parameters as in Figure 1 ($\alpha = 1/3, \mu = 0.2, \eta = 0.75, L = 1, Z = 1$), while we set $\theta = 1.25$ and $\delta W = 0.014$. The experiment is an increase of the relative price $\pi$ from 0.7 to 0.75, and panels A and B correspond to the cases in which $\varphi = 0.1$ and $\varphi = 0.075$, respectively. As is clear from the figure and consistent with Proposition 2, the effect of trade liberalization on steady-state income and consumption is negative for a sufficiently low value of $\varphi$ (high savings rate).

5. Concluding Remarks

Our model illustrates that the long-run effects of trade liberalization crucially depend on the level of financial frictions and the saving rate. Our model is highly stylized so it is important to bear in mind some of the limitations in our analysis. First, our result regarding the role of the level of financial development in affecting the outcome of the trade liberalization naturally depends on the way we have modeled financial constraints. For instance, in our model trade opening tightens credit constraints by reducing wealth inequality, but alternative frameworks might predict a negative link between wealth inequality and financial frictions (see Banerjee and Duflo (2003) for more on this). Second, our result regarding the role of the saving rate is derived from a particularly stylized modeling of
Figure 2. Trade liberalization and the saving rate.

A. Trade Liberalization for $\phi = 0.1$

B. Trade Liberalization for $\phi = 0.075$
intertemporal substitution in consumption, and also seems to be particularly tied to the propensity to save of entrepreneurs. Future research should shed light on the robustness of our results in richer and more realistic frameworks.

Appendix A: Proof of Proposition 1

It proves simpler to redefine the function $\Lambda(\pi)$ as

$$
\Lambda(\pi) = \frac{((1 - x(\pi))\pi^{1/\alpha} + x(\pi))^{\alpha}}{\pi^{1-\eta}},
$$

where $x(\pi) = \theta s^*(\pi)$. Note then that

$$
\frac{\partial \Lambda(\pi)}{\partial \pi} = \frac{\eta(1 - x(\pi))\pi^{1/\alpha} + x(\pi) - (1 - \eta(1 - \pi^{1/\alpha}))x(\pi)}{\pi^{2-\eta}(1 - x(\pi))\pi^{1/\alpha} + x(\pi))^{1-\alpha}}
$$

and hence it is positive if and only if

$$
\eta(1 - x(\pi))\pi^{1/\alpha} + x(\pi) - (1 - \eta(1 - \pi^{1/\alpha}))x(\pi) > 0. \quad (A.1)
$$

But it is straightforward to show that the left-hand-side of this inequality is decreasing in $\theta$. Solving for $s$ in equation (11) and multiplying by $\theta$, we have

$$
x(q, \theta) = \frac{1}{2(1-q)} \left( \theta \alpha (1-q) - (1-\alpha)(q - \mu (1-q)\theta) 
+ \sqrt{(\theta \alpha (1-q) - (1-\alpha)(q - \mu (1-q)\theta))^2 + 4\theta (1-q)q \mu (1-\alpha)} \right),
$$

where $q \equiv \pi^{1/\alpha}$. In AC we showed that $x(q, \theta)$ is increasing in $\theta$ and we next note that $\partial x(q)/\partial q$ is decreasing in $\theta$:

$$
\frac{\partial^2 x}{\partial q \partial \theta} = \frac{2(1-\mu)(1-\alpha)^2 q \theta \alpha \mu}{((\theta \alpha (1-q) - (1-\alpha)(q - \mu (1-q)\theta))^2 + 4\theta (1-q)q \mu (1-\alpha))^{3/2}} < 0.
$$

It follows then that the larger $\theta$ is, the harder it is that inequality (A.1) holds.

Appendix B: Proof of Proposition 2

Simple differentiation indicates that $\Gamma'(\pi) > 0$ if and only if

$$
\frac{\eta}{1-\alpha} \left( \frac{\varphi - \delta^W}{(\pi^{-1/\alpha} - 1)} - \theta \delta^W \right) - \frac{1}{\alpha} \frac{(\varphi - \delta^W)\mu \theta \delta^W}{(\pi^{-1/\alpha} - 1)} - \theta \delta^W (1 - \mu) \left( \frac{\pi^{-1/\alpha}}{(\pi^{-1/\alpha} - 1)^2} > 0.
$$
Notice that the left-hand-side is decreasing in $\theta$ and increasing in $\varphi$. Hence, the inequality is more likely to hold the lower is $\theta$ and the larger is $\varphi$.

References


