Further results on MIMO networks based on the distribution of the eigenvalues of arbitrarily correlated Wishart matrices
Further Results on MIMO Networks Based on the Distribution of the Eigenvalues of Arbitrarily Correlated Wishart Matrices

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Abstract—We review some recent results on the distribution of the eigenvalues of Gaussian quadratic forms and Wishart matrices with arbitrary correlation, in particular including the case where the covariance matrix has eigenvalues of arbitrary multiplicity. Then, we apply these recent results to produce further numerical results on the performance of Multiple-input/multiple-output (MIMO) communication systems in the presence of multiple MIMO co-channel interferers and noise. We consider the situation in which transmitters have no information about the channel and all links undergo Rayleigh fading.

We show that, in a network of MIMO($n$, $n$) systems, the larger $n$, the better, irrespectively on the signal-to-interference ratio and signal-to-noise ratio. On the contrary, if the number of transmitting antenna is fixed, it is be better to use a small number of transmitting antennas if the signal-to-interference ratio is below some threshold.

I. INTRODUCTION

Multiple transmitting and receiving antennas can provide high spectral efficiency and link reliability for point-to-point communication in fading environments [1]. The analysis of capacity for MIMO channels in [2] suggested practical receiver structures to obtain such spectral efficiency. Since then, many studies have been devoted to the analysis of MIMO systems, starting from the ergodic [3] and outage [4] capacity for uncorrelated fading, to the case where correlation is present on one of the two sides (at the transmitter or at the receiver) or at both [5]–[7].

Only a few papers, by using simulation or approximations, have studied the capacity of MIMO systems in the presence of cochannel interference [9]–[12]. In particular, a simulation study is presented in [9] for cellular systems, assuming up to 3 transmit and 3 receive antennas. The simulations showed that cochannel interference can seriously degrade the overall capacity when MIMO links are used in cellular networks. In [10], [11] it is studied whether, in a MIMO multiuser scenario, it is always convenient to use all transmitting antennas. It was found that for some values of signal-to-noise ratio (SNR) and signal-to-interference ratio (SIR), allocating all power into a single transmitting antenna, rather than divide power equally between independent streams from the different antennas, would lead to a higher overall system mutual information. So, even if from an individual capacity user’s perspective it is always convenient to use all transmitting antennas, in principle there is an advantage in terms of overall system mutual information in coordinating the number of antenna elements used by each transmitter [10], [11]. The study in [10], [11] adopts simulation to evaluate the capacity of MIMO systems in the presence of cochannel interference, and the difficulties in the evaluations limited the results to a scenario with two MIMO users employing at most two antenna elements. Other approximate analyses can be found in the recent literature (see references in [13]).

In this paper, by using some recent results on the distribution of the eigenvalues for Wishart matrices [13], [14], we produce further numerical results on the effects of interference caused by MIMO links on the ergodic capacity of MIMO systems in the presence of multiple MIMO cochannel interferers and additive white Gaussian noise (AWGN).

Throughout the paper vectors and matrices are indicated by bold, $|A|$ and $\det A$ denote the determinant of matrix $A$, and $a_{i,j}$ is the $(i,j)$th element of $A$. Expectation operator is denoted by $\mathbb{E}\{\cdot\}$, and in particular $\mathbb{E}_X\{\cdot\}$ denotes expectation with respect to the random variable $X$. The superscript $\dagger$ denotes conjugation and transposition, $I$ is the identity matrix, $\text{tr}\{A\}$ is the trace of $A$ and $\oplus$ is used for the direct sum of matrices defined as $A \oplus B = \text{diag}(A, B)$.

II. SYSTEM MODELS

We consider a network scenario as shown in Fig. 1, where a MIMO-($N_T$, $N_R$) link, with $N_T$ and $N_R$ denoting the numbers of transmitting and receiving antennas, respectively, is subject to $N_I$ MIMO co-channel interferers from other links, each with arbitrary number of antennas. The $N_R$-dimensional equivalent lowpass signal $y$, after matched filtering and sampling, at the output of the receiving antennas can be written as

$$y = H_0 x_0 + \sum_{k=1}^{N_I} H_k x_k + n$$

(1)
strategy is that each user transmits circularly symmetric Gaussian vector signals with zero mean and i.i.d. elements. Thus, the transmit power per antenna element of the $k^{\text{th}}$ user is $P_k/N_{Rk}$.

Hence, conditioned on all channel matrices $\{H_k\}_{k=0}^{N_t}$ in (1), both $y$ and $x_0$ are circularly Gaussian. The conditional mutual information of a MIMO link in the presence of multiple MIMO interferers with CSI at the receiver only is [13]:

$$C_{\text{MU}} \left( \{H_k\}_{k=0}^{N_t} \right) = \log \frac{\det(I_{N_R} + \tilde{H}^\dagger \tilde{H})}{\det(I_{N_R} + H^\dagger H)}$$

where $H = [H_1|H_2|\cdots|H_{N_t}]$ is an $N_R \times (\sum_{i=1}^{N_t} N_{T_i})$ matrix, $\tilde{H} = [H_0|H]$ is an $N_R \times N_{T_1}$ matrix, the covariance matrices $\Psi, \tilde{\Psi}$ are

$$\Psi = \varrho_1 I_{N_{T_1}} \oplus \cdots \oplus \varrho_{N_t} I_{N_{T_t} N_t}$$

and

$$\tilde{\Psi} = \varrho_0 I_{N_{T_0}} \oplus \cdots \oplus \varrho_{N_t} I_{N_{T_t} N_t}$$

with

$$\varrho_i = \frac{P_i}{N_{T_i} \sigma_z^2}.$$  

With random channel matrices the mutual information in (2) is the difference between random variables of the form

$$\log \det \left( I + \mathbf{H}^\dagger \mathbf{H} \right)$$

where the elements of $\mathbf{H}$ are i.i.d. complex Gaussian and $\mathbf{H}$ is a covariance matrix. The statistics of such random variables has been investigated in [5]–[7], assuming that the eigenvalues of $\mathbf{H}$ were distinct. However, in the scenario under analysis these results cannot be used directly, since in (2) each eigenvalue $\varrho_i$ of $\Psi, \tilde{\Psi}$ has multiplicity $N_{T_i}$.

More specifically, let us use as performance measure the ergodic mutual information: taking the expectation of (2) with respect to the distribution of $\{H_k\}_{k=0}^{N_t}$, we get

$$C_{\text{MU}} \triangleq \mathbb{E} \left\{ C_{\text{MU}} \left( \{H_k\}_{k=0}^{N_t} \right) \right\}$$

$$= C_{\text{SU}} \left( \sum_{i=1}^{N_t} N_{T_i}, N_{R}, \Psi \right) - C_{\text{SU}} \left( \sum_{i=1}^{N_t} N_{T_i}, N_{R}, \tilde{\Psi} \right)$$

where $C_{\text{SU}} (n_{T}, n_{R}, \Phi) \triangleq \mathbb{E}_H \left( \log \det \left( I_{n_{T}} + \mathbf{H}^\dagger \mathbf{H} \right) \right)$ denotes the ergodic mutual information of a single-user MIMO-($n_{T}, n_{R}$) Rayleigh fading channel with unit noise variance per receiving antenna and channel covariance matrix at the transmitter side $\Phi$.

Note that the “building block” $\mathbb{E}_H \left( \log \det \left( I + \mathbf{H}^\dagger \mathbf{H} \right) \right)$ is simple to evaluate when the covariance matrix $\mathbf{H}$ is proportional to an identity matrix, which corresponds to a typical interference-free case with equal transmit power among all transmitting antennas. Unfortunately, in the presence of interference the covariance matrix is of the type indicated in (3) and (4), where the power levels of the different users are in general different. Note that even when the power for the $i^{\text{th}}$ user is equally spread over the $N_{R}$ antennas, the matrices $\Psi, \tilde{\Psi}$ in (3), (4) and (6) are in general not proportional to identity matrices and their eigenvalues have multiplicities.

Fig. 1. MIMO with MIMO interferers.

where $x_0, x_1, \ldots, x_{N_t}$ denote the complex transmitted vectors with dimensions $N_{T_0}, N_{T_1}, \ldots, N_{T_{N_t}}$, respectively. Subscript 0 is used for the desired signal, while subscripts 1,\ldots,$N_t$ are for the interferers. The additive noise $n$ is an $N_R$-dimensional random vector with zero-mean independent, identically distributed (i.i.d.) circularly symmetric complex Gaussian entries, each with independent real and imaginary parts having variance $\sigma_z^2/2$, so that $\mathbb{E} \left\{ n n^\dagger \right\} = \sigma_z^2 I$. The power transmitted from the $k^{\text{th}}$ user is $P_k$.

The matrices $H_k$ in (1) denote the channel matrices of size $(N_R \times N_{T_k})$ with complex elements $h_{i,j}^{(k)}$ describing the gain of the radio channel between the $j^{\text{th}}$ transmitting antenna of the $k^{\text{th}}$ MIMO interferers and the $i^{\text{th}}$ receiving antenna of the desired link. In particular, $H_0$ is the matrix describing the channel of the desired link (see Fig. 1).

When considering the statistical variations of the channel, the channel gains must be described as random variables (r.v.s). In particular, we assume uncorrelated MIMO Rayleigh fading channels for which the entries of $H_k$ are i.i.d. circularly symmetric complex Gaussian r.v.s with zero-mean and variance one, i.e. $\mathbb{E} \left\{ |h_{i,j}^{(k)}|^2 \right\} = 1$. With this normalization, $P_k$ represents the short-term average received power per antenna element from user $k$, which depends on the transmit power, path-loss and shadowing level between transmitter $k$ and the (interfered) receiver. Thus, the $P_k$ are in general different.

Here we consider the scenario in which the receiver has perfect channel state information (CSI), and all the transmitters have no CSI. In this case, since the users do not know what is the interference seen at the receiver (if any), a reasonable
greater than one. Therefore, studying MIMO systems in the presence of multiple MIMO cochannel interferers requires the characterization of $C_{SU}(n_T, n_R, \Phi)$ in the most general case, i.e., considering a covariance matrix $\Phi$ with eigenvalues of arbitrary multiplicities.

To this aim, we derived in [13], [14] simple expressions for the hypergeometric functions of matrix arguments with not necessarily distinct eigenvalues; then, we obtained the joint probability distribution function (p.d.f.) of the eigenvalues of central Wishart matrices and Gaussian quadratic forms with arbitrary covariance matrix.

Some of the next derivations can be obtained from the following result for the function $\hat{F}_q$ [13], [14].

Lemma 1: Let $A = \text{diag} (\lambda_1, \ldots, \lambda_m)$ and $W = \text{diag} (w_1, \ldots, w_m)$ with $\lambda_1 > \cdots > \lambda_m$ and $w_1 > \cdots > w_k = w_{k+1} = \cdots = w_{k+L-1} > w_{k+L} > \cdots > w_m$. This means that $W$ has $L$ coincident eigenvalues. Then we have

$$p\hat{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; |A, W|) = \Xi \prod_{i<j} (\lambda_i - \lambda_j)^{i} \prod_{1<j, w_i \neq w_j} (w_i - w_j)$$

where $i = (m \times n)$ matrix $C$ has elements as follows

$$c_{i,j} = \lambda_i^{-1} \cdot \hat{F}_q(a_1 - m + l + k - j, \ldots, b_q - m + l + k - j; i, w_j)$$

for $j = k, \ldots, k + L - 1$, and

$$c_{i,j} = p\hat{F}_q(\tilde{a}_1, \ldots, \tilde{a}_p; \tilde{b}_1, \ldots, \tilde{b}_q; \lambda_i w_j)$$

elsewhere. In (7) the constant $\Xi$

$$\Xi = \frac{\Gamma(m_i)}{\Gamma(L)} \prod_{1}^{L-1} \prod_{i=1}^{m_i} \prod_{b_i}^{a_i} \prod_{a_i}^{b_i}$$

Proof: See [13], [14].

III. GAUSSIAN QUADRATIC FORMS WITH COVARIANCE MATRIX HAVING EIGENVALUES OF ARBITRARY MULTIPlicity

We now give the joint p.d.f. of the eigenvalues for Gaussian quadratic forms and central Wishart matrices with arbitrary one-sided covariance matrix.

Lemma 2: Let $H$ be a complex Gaussian $(p \times n)$ random matrix with zero-mean, unit variance, i.i.d. entries and let $\Phi$ be an $(n \times n)$ positive definite matrix. The joint p.d.f. of the (real) non-zero ordered eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{n_{\text{min}}} \geq 0$ of the $(p \times p)$ quadratic form $W = H^H \Phi H$ is given by

$$f(x_1, \ldots, x_{n_{\text{min}}}) = K \left| V(x) \right| \prod_{i=1}^{n_{\text{min}}} x_i^{\mu_i - 1}$$

where $n_{\text{min}} = \min(n, p)$, $V(x)$ is the $(n_{\text{min}} \times n_{\text{min}})$ Vandermonde matrix with elements $v_{ij} = x_j^{i-1}$.

$$K = \frac{\Gamma(n_{\text{min}})}{\Gamma(p)} \prod_{i=1}^{L} \frac{\mu_i^{m_i}}{\Gamma(m_i)} \prod_{i<j} \frac{\mu_i - \mu_j}{m_i m_j}$$

and $\mu_1 > \mu_2 > \ldots > \mu_L$ are the $L$ distinct eigenvalues of $\Phi^{-1}$, with corresponding multiplicities $m_1, \ldots, m_L$ such that $\sum_{i=1}^{L} m_i = n$.

The $(n \times n)$ matrix $G(x, \mu)$ has elements

$$G_{ij} = \begin{cases} (\sigma_j)^{i} e^{-(\mu_i) x_j} & j = 1, \ldots, n_{\text{min}} \\ [n - i]_{\mu_j}^{j} \mu^{n - j} & j = n_{\text{min}} + 1, \ldots, n \\ \end{cases}$$

where $[a]_b = a(a-1) \cdots (a-k+1)$, $[a]_b = 1$, $\nu_i$ denotes the unique integer such that

$$m_1 + \ldots + m_{\nu_i} < i \leq m_1 + \ldots + m_{\nu_i} + 1$$

and $d_i = \sum_{k=1}^{\nu_i} m_k - m_i$.

Proof: See [13].

Note that Lemma 2 gives, in a compact form, the general joint distribution for the eigenvalues of complex central Wishart matrices ($p \geq n$), and central pseudo-Wishart matrices or quadratic form ($n \geq p$), with arbitrary one-sided covariance matrix with not-necessarily distinct eigenvalues.

In fact, Lemma 2 can be used for both $p \geq n$ and $n \geq p$; note in particular that for $n \geq p$ the last product in the RHS of (9) is $\prod_{i=1}^{n_{\text{min}}} x_i^{\mu_i - n_{\text{min}}} = 1$, while for $p \geq n$ the second row in (11) is not used and $(-1)^{p(n-n_{\text{min}})} = 1$.

Note also that, given the expression for the eigenvalues joint p.d.f. the marginal distribution of individual eigenvalues is not of an arbitrary subset of the eigenvalues is known by using the approaches in [15], [16].

IV. ERGODIC MUTUAL INFORMATION OF A SINGLE-USER MIMO SYSTEM

We here provide a unified analysis of the ergodic mutual information of a single-user MIMO system with arbitrary power levels/correlation among the transmitting antenna elements or arbitrary correlation at the receiver, admitting covariance matrices with not-necessarily distinct eigenvalues.

Let us consider the function

$$C_{SU}(n, p, \Phi) = E[H \{ \log \det (I_p + H^H H^H) \}$$

where $\Phi$ is a generic $(n \times n)$ positive definite matrix and $H$ is a $(p \times n)$ random matrix with zero-mean, unit variance complex Gaussian i.i.d. entries.

Now, consider a single-user MIMO-$n_T n_R$ Rayleigh fading channel with $\Psi_T, \Psi_R$ denoting the $(n_T \times n_T)$ transmit and $(n_R \times n_R)$ receive covariance matrices, respectively, having diagonal elements equal to one. Assume the transmit vector $x$ is zero-mean complex Gaussian, with arbitrary (but fixed) $(n_T \times n_T)$ covariance matrix $Q = E\{xx^H\}$ so that $\text{tr}\{Q\} = P$. Then, the function (12) can be used to express the ergodic mutual information in the following cases [5]-[7]:

1) the MIMO-$n_T n_R$ channel with no correlation at the receiver ($\Psi_R = I$), covariance matrix at the transmitter side $\Psi_T$, and transmit covariance matrix $Q$. In this case the mutual information is $C_{SU}(n_T, n_R, \Phi)$ with $\Phi = (1/\sigma^2) \Psi_T^H Q$. If also $\Psi_T = I$, we have $\Phi = (1/\sigma^2) Q$ and therefore $\text{tr}\{\Phi\} = P/\sigma^2$. 
2) the MIMO-(n_T, n_R) channel with no correlation at the transmitter (Ψ_T = I), covariance matrix at the receiver side Ψ_R and equal power allocation \( Q = P/n_T I \).

In this case the capacity is \( C_{SU}(n_R, n_T, \Phi) \) with \( \Phi = (P/n_T \sigma^2) \Psi_R \), giving \( \text{tr}\{\Phi\} = (P/\sigma^2)(n_R/n_T) \), in accordance to [5, Theorem 1]. In both cases \( P/\sigma^2 \) represents the SNR per receiving antenna.

We then have the following Theorem [13].

**Theorem 1:** The ergodic mutual information of a MIMO Rayleigh fading channel with CSI at the receiver only and one-sided covariance matrix \( \Phi \) having eigenvalues of arbitrary multiplicities is given by

\[
C_{SU}(n, p, \Phi) = K \prod_{k=1}^{n_{\text{min}}} \det \left( R^{(k)} \right). \tag{13}
\]

In the previous equation \( n_{\text{min}} = \min(n, p) \), the matrix \( R^{(k)} \) has elements:

\[
\begin{align*}
    r_{i,j}^{(k)} &= (-1)^{d_i} \int_0^\infty x^{p-n_{\text{min}}+j-1+d_i} e^{-x \mu_{(i)}} dx \\
    &\text{for } j = 1, \ldots, n_{\text{min}}, j \neq k; \\
    r_{i,j}^{(k)} &= (-1)^{d_i} \int_0^\infty x^{p-n_{\text{min}}+j-1+d_i} e^{-x \mu_{(i)}} \log(1 + x) dx \\
    &\text{for } j = 1, \ldots, n_{\text{min}}, j = k; \text{ and} \\
    r_{i,j} &= [n-j]d_i \mu_{(e_i)}^{n-d_i-j} \\
    &\text{for } j = n_{\text{min}} + 1, \ldots, n.
\end{align*}
\]

Here, \( d[i], e[i], d_i, K \) are defined as in Lemma 2, and \( \mu(1) > \mu(2) \ldots > \mu(L) \) are the \( L \) distinct eigenvalues of \( \Phi^{-1} \), with associated multiplicities \( m_1, \ldots, m_L \).

**Proof:**

See [13].

Theorem 1 gives, in a unified way, the exact mutual information for MIMO systems, encompassing the cases of \( n_R \geq n_T \) and \( n_T \geq n_R \) with arbitrary correlation at the transmitter or at the receiver, avoiding the need for Monte Carlo evaluation. The application of the results in Sections III and IV enables a unified analysis for MIMO systems, which allow the generalization for ergodic and outage capacity [5][7][17], for optimum combining multiple antenna systems [18], for MIMO-MMSE systems [19], for MIMO relay networks, as well as for multiuser MIMO systems and for distributed MIMO systems, accounting arbitrary covariance matrices.

**V. NUMERICAL RESULTS**

Using (6) together with Theorem 1, we have the exact expression of the ergodic capacity of MIMO systems in the presence of multiple MIMO interferers in Rayleigh fading. In particular, the eigenvalues to be used in Theorem 1 are given by \( \mu(1) = 1/\gamma_1 = \sigma^2 N_{T1}/P_1 \), allowing an easy analysis for several scenarios.

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In fact, with our normalization on the channel gains the mean received power from user \( i \) is \( P_i \).

With Gaussian approximation the performance is evaluated as if interference were absent, except the overall noise power is set to \( \sigma^2 + \sum_{i \geq 1} P_i \), giving a signal to interference plus noise ratio \( \text{SINR} = (\text{mean} + \text{sum})^{-1} \).

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**Fig. 2.** Mean capacity for MIMO-(6,6) as a function of SIR in the presence of one MIMO cochannel interferer with \( N_{T1} = 1, 2, 4, 6, 10 \). The signal-to-noise ratio is SNR= 20 dB. The Gaussian approximation of the interference is also shown. Diamond: capacity of a single-user MIMO-(6,6). Circles: capacity of a single-user MIMO-(6,6-\( N_{T1} \)) (only for \( N_{T1} = 1, 2, 4 \).**
the minimum number of transmitting antennas (i.e., MIMO\textsuperscript{-}$(N_{T_0}, N_R)$) for two different regions: for small SIR the dominating effect is the interference-free channels, which represent the asymptotes of the capacity of single-user MIMO-(${N_{T_0}, N_R}$). Scenario with one and two interferers, each with the same number of transmitting antennas as the desired user. Cases of 3, 4, 5 and 6 transmitting antennas.

$N_{T_1}$ interfering antenna elements approaches asymptotically, for large interference power (small SIR), to a floor given by the capacity of a single-user MIMO-(${N_{T_0}, N_R}$) system. This behavior can be thought of as using $N_{T_1}$ degrees of freedom (DoF) at the receiver to null the interference, in small SIR regime. On the other hand, when $N_R \leq N_{T_1}$ the capacity approaches to zero for small SIR. This is due to the limited DoF at the receiver (related to the number of receiving antenna elements) that prevents mitigating all interfering signals (one from each antenna element) while processing the $N_{T_0}$ useful parallel streams [18].

In general, given the MIMO interferers, the capacity of the desired link increases with the number of its transmitting antennas. However, an uncoordinated increase in the number of transmitting antenna elements of the desired user will decrease the capacity for the other MIMO links in the network. It is therefore important to understand to what extent, in a network of MIMO users, it is useful to increase the number of transmitting antenna elements per user.

In Fig. 3 we assume a MIMO-(${N_{T_0}, 6}$) system in the presence of one and two MIMO interferers in the network, each equipped with the same number of antennas as for the desired user, i.e., $N_{T_0} = N_{T_1} = N_{T_2}$. The curves are obtained for $N_{T_1} = 3, 4, 5, 6$ and SNR = 10 dB. We clearly see here two different regions: for small SIR the dominating effect is that of interference, and it is better for all users to employ the minimum number of transmitting antennas (i.e., MIMO-(3,6) for all users), so as to allow the receiver to mitigate the interfering signals. On the contrary, for larger SIR the channel tends to that of a single-user MIMO system and it is better to employ the maximum number of transmitting antennas. In the same figure we also report the capacity for interference-free channels, which represent the asymptotes of the four curves. The switching points between the interference dominated and the thermal noise dominated regions are at SIR around 3 and 2 dB for one and two interferers, respectively.\footnote{In general if we allow $N_{T_1}$ to be in a range $\{N_{T_{min}}, \ldots, N_{T_{max}}\}$ the switching point for the two extreme curves $N_{T_1} = N_{T_{min}}$ and $N_{T_1} = N_{T_{max}}$ can be easily evaluated by finding the SIR value that equates the capacities in the two cases.}

Note also that the capacity in the presence of two interferers is always lower compared to that of one MIMO interfering user, due to the larger number of interfering signals. For the same setting, in the figure we also show the performance obtained by using the Gaussian approximation. Clearly the Gaussian approximation can incorrectly suggest that it is always better to use the largest possible number of transmitting antennas. In Fig. 4 we consider a MIMO-(${N_{T_0}, 6}$) system in the presence of two MIMO interferers in the network, each equipped with the same number of antennas as for the desired user, i.e., $N_{T_0} = N_{T_1} = N_{T_2}$. The curves are obtained for $N_{T_1} = 2, 3, 4, 5$ and SNR = 5 dB. We observe that a decrease in the SNR has a twofold effect: first, a decrease in the capacity values occurs; second, the switching operation point moves to lower values of the SIR.

Finally, we investigate a scenario where all users employ the same number of transmitting and receiving antennas, i.e., MIMO-(${n, n}$). Figure 5 shows the ergodic capacity as a function of SIR in the presence of one and two MIMO interferers. Also shown is the limiting case of an infinite number of users in the network, for which the Gaussian approximation is valid. We see that there is no switching point in this scenario, regardless of the number of transmitting antennas.

This behavior is confirmed for different values of the SNR in Fig. 6, where we can see that the network of MIMO-(6,6) systems has higher capacity than that of MIMO-(4,4) systems, and the network of MIMO-(4,6) systems has higher capacity than that of MIMO-(2,2) systems. Thus, in a network where all users are using the same MIMO-(${n, n}$) systems, larger values of $n$ achieve higher capacity, for all values of SIR and SNR.

In general if we allow $N_{T_1}$ to be in a range $\{N_{T_{min}}, \ldots, N_{T_{max}}\}$ the switching point for the two extreme curves $N_{T_1} = N_{T_{min}}$ and $N_{T_1} = N_{T_{max}}$ can be easily evaluated by finding the SIR value that equates the capacities in the two cases.
achieve higher mutual information for all users, regardless of the maximum number of transmitting elements. If all links are MIMO-(nt, nt), we have shown, for instance, that in a network of MIMO-(N₁, N₁), N₁=2 MIMO(2,2), N₁=4 MIMO(4,4), N₁=6 MIMO(6,6), NT₁=2 MIMO Rayleigh fading channels, with SNR of 10 dB. Curves for one interferer, two interferers and Gaussian approximation. Circles: capacity of single-user MIMO-N₁=1, N₁=2, N₁=3, N₁=4, N₁=6, N₁=8, N₁=10, N₁=12, N₁=14, N₁=16, N₁=18, N₁=20, N₁=22E[C].

VI. CONCLUSION

By using some recent results on the distribution of the eigenvalues of Wishart matrices we have studied MIMO communication systems in the presence of multiple MIMO interferers and noise. With reference to a multiuser scenario we have shown, for instance, that in a network of MIMO-(nt, nt) users with fixed receiving antenna elements N_R, it is better to employ the minimum number of transmitting antennas when SIR is below some values; on the contrary, when the interference is weak, i.e., SIR is large, the channel behaves like that of a single-user and thus it is better to employ the maximum number of transmitting elements. If all links are MIMO-(nt, nt), we have shown that the larger values of nt achieve higher mutual information for all users, regardless of the SIR and of the SNR values.

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