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Mode Retrieval from Intensity Profile Measurements using Irradiant Waveguide-modes

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Abstract—The fields radiated from the waveguide were used to analyze propagating mode content in an oversized circular corrugated waveguide. The radiated fields were calculated by the Kirchhoff integral in the paraxial approximation. The irradiant waveguide modes of the radiated field are orthogonal within the paraxial approximation. The complex fractions in the irradiant mode expansion were deduced from an iteration process in which the reconstructed amplitude profile from the complex fractions coincided with the measured amplitude profile.

I. INTRODUCTION

Electron Cyclotron Heating (ECH) using high power millimeter waves is an attractive method for plasma production, auxiliary heating, and current drive in a nuclear fusion research. A Gaussian beam from a high power gyrotron oscillator is coupled into oversized circular corrugated waveguides in the ECH system, which is transmitted to the launcher using the HE\textsubscript{11} mode of the waveguide. To obtain high transmission efficiency in the waveguide, the beam center-position and tilt in the coupling into the waveguide line should be aligned within tolerable limits to provide coupling into the HE\textsubscript{11} mode from the Gaussian beam. The offset or tilted injection excites unwanted coupling higher modes, which cause high transmission losses and arcing events in the high power transmission. The analysis of the mode content transmitted in the waveguide is a key issue to study how to attain the high coupling and transmission efficiencies.

In order to convert the output beam from the gyrotron into a Gaussian beam, the phase-correcting mirror system has been developed. The phase-correcting mirror has been designed using the phase profiles retrieved from the intensity profiles, when precise phase measurements are unavailable in the high power application. In the retrieval process, the field is evaluated at a few propagating position. This retrieval process was experimentally verified by direct phase measurements at a low power level [1]. Similar technique to the phase retrieval were used to analyze the mode content in the waveguide by using the radiated field from the waveguide [2]. The phase profiles were retrieved from the intensity profiles. The field at the waveguide aperture was calculated from the retrieved phase and intensity profiles, and the mode content was analyzed at the waveguide aperture. A new approach to analyze the mode content in the waveguide was proposed for the low power experiments where precise direct phase measurements were available [3]. The mode content of the waveguide modes was evaluated using the irradiant waveguide modes in the radiated field. In this method, the phase retrieval process was not required.

In this paper, the new approach using the irradiant waveguide modes is extended to use in the high power application where the phase measurements are unavailable. The mode content is directly retrieved from evolution of the amplitude profiles along the propagation, without the phase measurements and the phase retrieval process.

II. EXPERIMENTAL SETUP AND RESULTS

A Gaussian beam was prepared to inject to a 1m long corrugated waveguide with a diameter of 88.9mm. This injected beam was tilted by 1 degree with respect to the waveguide z-axis in the x-direction. The electric field was in y-direction, in perpendicular to the tilted x-z plane. Here, the wave frequency of the beam was 84 GHz. In the tilted injection, the unwanted modes were excited. The distorted intensity profiles were measured at the several propagation position along the z-axis. Figure 1 shows the intensity x-y profiles measured at z=150 mm. The intensity pattern of the output beam was off-center in the x-direction. The phase profiles were retrieved from the measured intensity profiles by the iteration method in ref.3.

III. IRRADIANT WAVEGUIDE MODE EXPANSION

The field radiated from the waveguide was used to analyze the mode content transmitted in the waveguide. The radiated fields were calculated by the Kirchhoff Integral in the paraxial approximation. The irradiant waveguide modes $\Phi_{mn}$ which are the radiated waveguide eigen modes $\phi_{mn}$ at the z position were expressed as

$$\Phi_{mn}(x, y) = \frac{i k}{2\pi z} \int d\xi d\eta \phi_{mn}(\xi', \eta') \exp\left[-ik[(\xi' - x)^2 + (\eta' - y)^2]/2z\right]$$

where the coordinates of $(\xi', \eta')$ and $(x, y)$ were at the waveguide aperture and at the radiated positions, respectively.

Figure 1: Intensity x-y profiles measured at z=150 mm in the tilted injection.

Figure 2: Intensity and phase profiles of an irradiant $\Phi_{21}$ waveguide mode in x-direction.
The orthogonality of the propagating modes in a free space comes from the parabolic equation. Since the irradiant waveguide modes $\Phi_{mn}(x,y)$ meet the parabolic equation, the functions $\Phi_{mn}$ were orthogonal bases in the free-space propagation within the paraxial approximation. The radiated field $E(x,y)$ was expressed with the orthogonal bases as,

$$E(x,y) = \sum_{m,n} C_{mn} \phi_{mn}(x,y)$$

where $C_{mn}$ is a complex fraction of the $(m,n)$ mode.

IV. MODE CONTENT ANALYSIS

The irradiant waveguide modes $\Phi_{mn}$ were calculated by the Kirchhoff integral in the paraxial approximation. Figure 2 shows the intensity and phase profiles of the $\Phi_{21}$ mode in $x$-direction at $y = 0$ cut. There was the phase jump in the $+/x$ direction. The beam center offset, shown in Fig.1, mainly came from the interference between the $\Phi_{11}$ and $\Phi_{21}$ modes. The phase profile of the $\Phi_{11}$ mode was isotropically distributed and approximately flat. The asymmetric phase profile of the $\Phi_{21}$ mode caused the asymmetric interference with the $\Phi_{11}$ mode in the $x$-direction, and induced the intensity profile with the beam-offset in the $x$-direction. On the interference point of view, the beam size was also affected by mixing between the $\Phi_{11}$ and $\Phi_{1x}$ modes.

The amplitude profile radiated from the waveguide can be expressed with mode expansion by the irradiant waveguide modes. The mode content analysis resulted in a root finding of the complex fractions that can explain the measured amplitude profile. The iteration process was applied to find the complex fractions in the mode expansion for the intensity profile at $z =150 \text{mm}$ in Fig.1. The complex fractions $C_{11}$ and $C_{21}$ were initially defined in the iteration process, assuming that the observed beam-offset was explained only by the interference between the $\Phi_{11}$ and $\Phi_{21}$ modes. The complex fractions of the other irradiant waveguide modes were assumed to be zero. The complex fractions $C_{11}$, $C_{21}$ and $C_{12}$ were retrieved in the iteration process on assumption of the initial complex fractions. Figure 3 shows an evolution of a difference $\varepsilon$ between the measured and reconstructed amplitude profiles in the iteration process. The difference $\varepsilon$ was defined to be integrated value in the $x$-$y$ plane. The difference $\varepsilon$ was reduced and converged in the iteration process well. The mode content of the beam was evaluated as 0.69, 0.13 and 0.08 for the $\Phi_{11}$, $\Phi_{21}$ and $\Phi_{12}$ modes in the previous mode content analysis using the phase measurements and the phase retrieval process [3]. The main modes were the $\Phi_{11}$ and $\Phi_{21}$ mode in the beam. The rather smaller difference $\varepsilon$, resulting from good assumption of the main complex fractions $C_{11}$ and $C_{21}$, may be easily converged in the iteration. In order to check availability of this approach, the iteration process starting from only $C_{11}$ (i.e. $C_{21} = 0$) was tested. Figure 4 shows the evolution of the difference $\varepsilon$ in the iteration on initial assumption of only $C_{11}$. Although the initial difference $\varepsilon$ was large, the difference $\varepsilon$ was well reduced and converged in the iteration process. Finally, the same complex fractions were retrieved as those under the main mode assumption. This iteration process to find the complex fractions was confirmed to be an appropriate approach.

Figure 5 shows the measured and reconstructed amplitude profile at the propagating $z=150 \text{mm}$ position. The profile reconstructed from the evaluated complex fractions was coincided with the measured profile. The mode content analyzed from the obtained complex fractions were 0.67, 0.13 and 0.05 for the $\Phi_{11}$, $\Phi_{21}$ and $\Phi_{12}$ modes, indicating a good agreement with the previous mode content analysis. Figure 6 shows the measured and reconstructed amplitude profile at the propagating $z=200 \text{mm}$ position. The measured amplitude profiles at $z=150$ and $200 \text{mm}$ may not perfectly explained by the evaluated complex fractions $C_{11}$, $C_{21}$ and $C_{12}$. More various modes in a few amplitude profiles along the propagation should be included in the analysis.