A Low-Complexity Receive-Antenna-Selection Algorithm for MIMO–OFDM Wireless Systems

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Abstract—In this paper, a novel low-complexity antenna-selection algorithm based on a constrained adaptive Markov chain Monte Carlo (CAMCMC) optimization method is proposed to approach the maximum capacity or minimum bit error rate (BER) of receive-antenna-selection multiple-input multiple-output (MIMO)–orthogonal frequency division multiplexing (OFDM) systems. We analyze the performance of the proposed system as the control parameters are varied and show that both the channel capacity and the system BER achieved by the proposed CAMCMC selection algorithm are close to the optimal results obtained by the exhaustive search (ES) method. We further demonstrate that this performance can be achieved with less than 1% of the computational complexity of the ES rule and is independent of the antenna-selection criteria, outage rate requirements, antenna array configuration, and channel frequency selectivity. Similar to the existing antenna-selection algorithms, both channel capacity and system BER improvements achieved by the proposed CAMCMC method are reduced as the channel frequency selectivity increases. Therefore, we conclude that, whether it is designed to maximize the channel capacity or minimize the system BER, the CAMCMC-optimization-method-based antenna-selection technique is appropriate for a MIMO–OFDM system with low frequency selectivity.

Index Terms—Antenna selection, bit error rate (BER), capacity, complexity, constrained adaptive Markov chain Monte Carlo (CAMCMC), frequency selectivity, multiple-input multiple-output (MIMO)–orthogonal frequency division multiplexing (OFDM), zero forcing (ZF).

I. INTRODUCTION

Future wireless communication systems are expected to provide higher data rates to meet the requirements of increasing multimedia services. There has been considerable interest in multiple-input multiple-output (MIMO) systems that employ multiple antennas at each terminal. The channel capacity can substantially be increased through the extra degrees of freedom available with multiple antenna systems. Theoretically, the capacity of MIMO systems linearly increases with the minimum number of transmit and receive antennas [1]. In practice, the major impediment in MIMO-based systems is the cost of the hardware, because every antenna element requires a complete radio frequency (RF) chain that is composed of mixers, amplifiers, and analog-to-digital conversion. A promising technique named antenna-subset selection has been proposed to reduce the hardware complexity, i.e., save on RF chains, while retaining many diversity benefits [2]. An antenna-subset-selection-based MIMO system employs a number of RF chains, each of which is switched to serve multiple antennas. In addition to reducing the system cost, antenna selection can also improve the throughput/reliability trade-off [3].

In recent years, there has been increasing interest in applying antenna-subset selection to MIMO systems over frequency-selective channels [4]. The introduction of the orthogonal frequency division multiplexing (OFDM) is expected to offer improved performance in combating adverse frequency-selective fading encountered in wideband wireless communication systems [5]. Combined with multiple antennas as MIMO–OFDM systems, a data stream at each transmit antenna is sent over a number of narrowband orthogonal subcarriers [6]. However, the antenna-subset-selection problem in MIMO–OFDM systems is more complex than that in single-carrier MIMO systems. In order to realize the maximum channel capacity, each subcarrier of an OFDM symbol should be considered if signals are sent through frequency-selective channels. Several algorithms have been developed for selecting the optimal antenna subset. For example, an exhaustive search (ES) method is exploited to maximize the MIMO–OFDM capacity in [7]. It involves searching all possible antenna sets and results in high computational complexity particularly for large array systems. Therefore, this method is not attractive for practical MIMO–OFDM systems, even though it offers optimal capacity performance. The Frobenius-norm-based selection rule is usually adopted. This method has been presented for narrowband MIMO systems and can be extended to address MIMO–OFDM systems [7]. It reduces the computational complexity but with a considerable capacity loss in the high-signal-to-noise-ratio (SNR) region.

Whether for single-carrier or multicarrier MIMO systems, the aforementioned methods mostly address the problem of antenna selection for Shannon capacity maximization. However, these antenna-selection criteria cannot minimize the system bit error rate (BER) unless the detector at the receiver has infinite complexity [8]. In practical spatial multiplexing systems, the receiver employs a finite complexity decoder, and the minimum BER performance cannot be achieved by these maximum-capacity-based antenna-selection criteria. In fact, the
minimum-BER antenna-selection criteria should be tailored to the specific receiver design [8]. It is well known that the maximum likelihood receiver has better BER performance than linear receivers, such as the zero forcing (ZF) and minimum mean square error (MMSE) receivers [9]. Nevertheless, it requires high complexity. On the other hand, as the simplest receivers, these linear receivers are more practical in spatial multiplexing systems, particularly for large antenna array systems. Therefore, several researchers have investigated how to adapt antenna-selection techniques to minimize the system BER based on linear receivers. For example, the largest minimum postprocessing SNR antenna-selection criterion is proposed for both ZF and MMSE receiver-based spatial multiplexing systems [10].

In this paper, our main contributions are described as follows.

1) A significantly low complexity receive-antenna-selection algorithm based on a constrained adaptive Markov chain Monte Carlo (CAMCMC) optimization method is proposed for MIMO–OFDM systems.

2) The selection criteria of maximum ergodic capacity and minimum BER are considered, and it is shown that the CAMCMC algorithm performs extremely closely to ES for both criteria.

3) Regardless of the selection criterion, the agreement between the CAMCMC and ES results is independent of SNR, outage rate, number of selected antennas, or channel delay spread.

The rest of this paper is organized as follows. In Section II, the receive-antenna-selection MIMO–OFDM system model is presented. In Section III, we formulate the receive-antenna-selection problem as a combinatorial problem and propose the CAMCMC selection algorithm based on various selection criteria followed by the simulation results and performance evaluation in Section IV. Finally, Section V discusses the advantages and disadvantages of the approach and concludes this paper.

II. SYSTEM MODEL

Fig. 1 shows a receive-antenna-selection MIMO–OFDM system with \( M_T \) transmit and \( M_R \) receive antennas.\(^1\) We let \( \mathbf{H}_l \) denote the channel response matrix for the \( l \)th tap, where \( l = \{0, \ldots, L - 1\} \), and \( L \) is the number of channel taps. We assume that \( \mathbf{H}_l \) is uncorrelated and \( h_l(m_r, m_t) \) follows the zero-mean complex Gaussian distribution with variances \( \sigma^2_l \), where \( \sum_{l=0}^{L-1} \sigma^2_l = 1 \), \( m_r = 1, \ldots, M_R \), and \( m_t = 1, \ldots, M_T \). The impulse response for this frequency-selective channel can be written as

\[
\mathbf{H}_l = \sum_{l=0}^{L-1} \mathbf{H}_l \delta(\tau - l) \quad (1)
\]

where \( \delta(\cdot) \) is the Kronecker delta function. If the cyclic prefix is designed to be of at least length \( L - 1 \), the channel frequency response matrix of the \( n \)th subcarrier for an \( N \)-tone MIMO–OFDM system can be described using another \( M_R \times M_T \) matrix \( \mathbf{H}_n \)

\[
\mathbf{H}_n = \sum_{l=0}^{L-1} \mathbf{H}_l e^{-j2\pi nl/N}. \quad (2)
\]

In this system, only a subset of receive antennas \( M_r \) (\( M_r \leq M_R \)) is used at each time slot. We define the indicator function of the selected receive-antenna subset as \( \omega_q \)

\[
\omega_q = \{I_i\}_{i=1}^{M_R}, \quad \{I_i\} \in \{0, 1\}; \quad q = 1, 2, \ldots, Q \quad (3)
\]

where \( i \) is the index of the rows of \( \mathbf{H}_n \), and the indicator function \( I_i \) indicates whether the \( i \)th row of \( \mathbf{H}_n \) (the \( i \)th receive antenna) is selected. If the \( i \)th row of \( \mathbf{H}_n \) is selected, \( I_i \) will be set to 1. In (3), \( Q \) is the number of all possible selected receive-antenna subsets, and \( Q = \binom{M_R}{M_r} \). Here, \( \cdot \) stands for

\(^1\)Forward error-correction coding and interleaving are not considered in this paper.
the binomial coefficient. After selection, the received signal for the nth subcarrier at the receiver is

\[ [r_n]_{\omega_q} = [H_n]_{\omega_q} s_n + [v_n]_{\omega_q} \]

where \([r_n]_{\omega_q} \in \mathbb{C}^{M_T \times 1}\) and \([H_n]_{\omega_q} \in \mathbb{C}^{M_r \times M_T}\) denote the received data and the channel frequency response matrix for the nth subcarrier associated with the selection, respectively, and \([s_n]_{\omega_q} \in \mathbb{C}^{M_T \times 1}\) is the transmitted data for the nth subcarrier. We assume that \(s_n\) is modulated from the same unit-energy constellation and is uncorrelated. Therefore, these transmitted data satisfy \(\varepsilon(s_n, s_n^H) = I_{M_T}\). Here, \(\varepsilon(\cdot)\) and \(\cdot^H\) denote the statistical expectation and the Hermitian operation, respectively. \([v_n]_{\omega_q} \in \mathbb{C}^{M_r \times 1} \sim C_N(0, N_0 I_{M_r})\) is the complex additive white Gaussian noise, and \(I_{M_r}\) is an \(M_r \times M_r\) identity matrix similar to \(I_{M_T} \in \mathbb{C}^{M_T \times M_T}\). We further assume that perfect channel state information is available at the receiver but not at the transmitter. The total available power is assumed to be uniformly allocated across all space-frequency subchannels [11]. Therefore, the mutual information of the \(N\)-tone receive-antenna-selection MIMO–OFDM system is

\[
c_{sel}(\omega_q) = \frac{1}{N} \sum_{n=1}^{N} \log \left[ \det \left( I_{M_r} + \frac{\rho}{M_T} [H_n]_{\omega_q} [H_n]^H_{\omega_q} \right) \right]
\]

where \(N\) is the total number of OFDM subcarriers, \(\rho\) is the SNR per subcarrier, and \(\det(\cdot)\) stands for the determinant operation. The ergodic capacity of this system is [11]

\[
C_{sel}(\omega_q) = \varepsilon \{ c_{sel}(\omega_q) \} .
\]

III. RECEIVE-ANTENNA-SELECTION ALGORITHM

In this section, three antenna-selection criteria are presented before a discussion of the proposed CAMCMC optimization method.

A. Selection Criteria

1) Channel-Capacity-Based Selection Criterion: This criterion is proposed to maximize the channel capacity of the MIMO–OFDM system. We formulate the antenna-selection problem as a combinatorial optimization problem

\[
\omega^*_C = \arg \max_{\omega_q \in \Omega} C_{sel}(\omega_q)
\]

where \(\omega^*_C\) is the receive-antenna-subset-selection indicator that achieves the global optimum of the objective function \(C_{sel}(\omega_q)\), and \(\Omega\) is the set of all possible receive-antenna-subset-selection indicators \(\{\omega_1, \ldots, \omega_q\}\). However, because of singular value decomposition computations, the capacity-based selection criterion results in high complexity, given by \(O(NM_r^2M_T)\), particularly for large antenna array MIMO–OFDM systems.

2) Frobenius-Norm-Based Selection Criterion: The norm-based selection criterion maximizes \(\| [H_n]_{\omega_q} \|_F\), which indicates the power of the selected channel frequency response matrix of the nth subcarrier, where \(\| \cdot \|_F\) denotes the Frobenius norm function. Let \(C_{sel}^{Norm}(\omega_q) = (1/N) \sum_{n=1}^{N} \| [H_n]_{\omega_q} \|_F\).

The norm-based antenna-selection problem can be formulated as

\[
\omega^*_N = \arg \max_{\omega_q \in \Omega} C_{sel}^{Norm}(\omega_q)
\]

where \(\omega^*_N\) is the selection indicator. Compared with the channel-capacity-based selection criterion, the norm-based criterion has a lower complexity, given by \(O(NM_r^2M_T)\). That significantly reduces the computational complexity but with a considerable capacity loss at the high-SNR region.

3) MSE of the ZF-Receiver-Based Selection Criterion: In a linear ZF-detector-based receive-antenna-selection MIMO–OFDM system, the MSE between the soft detecting output and the transmitted data for each subcarrier is expressed as

\[
\text{MSE}_{\omega_q, n}^{ZF} = \left( \frac{1}{N} [H_n]^H [H_n]_{\omega_q} \right)^{-1}
\]

Each element on the main diagonal of \(\text{MSE}_{\omega_q, n}^{ZF}\) is the MSE of the corresponding detected data at the nth subcarrier. In order to minimize the system BER, we define \(\text{MSE}_{sel}^{ZF}(\omega_q) = (1/N) \sum_{n=1}^{N} \text{tr}(\text{MSE}_{\omega_q, n}^{ZF})\) and formulate the selection problem as

\[
\omega^*_\text{ZF} = \arg \min_{\omega_q \in \Omega} \text{MSE}_{sel}^{ZF}(\omega_q)
\]

where \(\text{tr}(\cdot)\) denotes the matrix trace, and \(\omega^*_\text{ZF}\) stands for the receive-antenna-subset-selection indicator that achieves the global optimum of the objective function \(\text{MSE}_{sel}^{ZF}(\omega_q)\). The linear ZF receiver has a low complexity, given by \(O(NM_r^2M_T)\).

B. CAMCMC Optimization Method

The MCMC algorithm [12] is a stochastic simulation technique designed to explore a probability distribution of interest. It has drawn considerable attention in the last few decades over a wide range of fields, such as engineering, statistics, and biology [13]. Aside from the ability to sample from a distribution, the MCMC algorithm is also a powerful tool for stochastic optimization. Given that one can appropriately represent the subspace of interest by a probability distribution [14], MCMC can explore the “promising” subspaces only instead of exhaustively searching the whole solution space. The samples \(\{\omega_q^{(n)}\}_{n=1}^{N_{MCMC}}\) from an MCMC algorithm can be adopted to estimate the maximum of the reformed objective function \(C_{sel}(\omega_q)\)

\[
\tilde{2} \omega^*_C = \arg \max_{\omega_q \in \Omega} \max_{n=1, \ldots, N_{MCMC}} C_{sel}(\omega_q)
\]

where \(N_{MCMC}\) is the number of samples used to estimate \(\tilde{2} \omega^*_C\) and \(\omega^*_C\) denotes the estimated value of \(\omega^*_C = \arg \max_{\omega_q \in \Omega} C_{sel}(\omega_q)\).

In order to represent the feasible solution space appropriately by a probability distribution, we use the Boltzmann distribution of the objective function \(C_{sel}(\omega_q)\) with a suitable temperature \(\tau : \pi(\omega_q) = \exp\{C_{sel}(\omega_q) / \tau\} / \Gamma\), where \(\Gamma = \sum_{\omega_q \in \Omega} \exp\{C_{sel}(\omega_q) / \tau\}\) is a normalization constant in
the MCMC algorithm that can be ignored. Thus, maximizing $C_{sel}(\omega_q)$ is equivalent to maximizing $\pi(\omega_q)$, i.e.,

$$\omega^*_C = \arg \max_{\omega_q \in \Omega} C_{sel}(\omega_q) = \arg \max_{\omega_q \in \Omega} \pi(\omega_q)$$  \hspace{1cm} (12)$$

which means that $\omega^*_C$ from the MCMC algorithm is also the estimate of the maximum of $C_{sel}(\omega_q)$.

To demonstrate the MCMC algorithm for exploring the distribution $\pi(\omega_q)$, we take a Metropolized independence sampler (MIS) [15], which is a generic MCMC algorithm, as an example. An initial value $\omega_q^{(0)}$ is chosen randomly or according to a certain rule. Given the current sample $\omega_q^{(i)}$, a candidate sample $\omega_q^{(new)}$ is drawn from the proposal distribution $q(\omega_q; \mathbf{p})$. According to the accepting probability $\min\{1, \frac{\pi(\omega_q^{(new)})/\pi(\omega_q^{(i)})}{q(\omega_q^{(i)})/q(\omega_q^{(new)})}\}$, the new sample will be $\omega_q^{(i+1)} = \omega_q^{(new)}$ if $\omega_q^{(new)}$ is accepted; otherwise, $\omega_q^{(i+1)} = \omega_q^{(i)}$. After $N_{MCMC}$ iterations, we can obtain a set of samples $\{\omega_q^{(0)}, \omega_q^{(1)}, \omega_q^{(2)}, \ldots, \omega_q^{(N_{MCMC})}\}$, which is subjected to distribution $\pi(\omega_q)$.

In traditional MCMC algorithms, such as the aforementioned MIS algorithm, a high convergence rate can be obtained by adjusting the associated parameters $\mathbf{p}$ of the proposal distribution $q(\omega_q; \mathbf{p})$, where, specifically, $\mathbf{p} = (p_1, \ldots, p_{M_{R\Omega}})$ denotes the probability vector to indicate the probability of the receive antennas to be chosen in this paper. In recent years, adaptive MCMC algorithms have been proved to improve the performance of MCMC in terms of both convergence and efficiency by automatically adjusting the proposal distribution according to previous sampled points [16], [17]. In this paper, we propose a novel CAMCMC algorithm for antenna selection to reduce the computational complexity with neglectable system performance loss, where the proposal distribution $q(\omega_q; \mathbf{p})$ is updated by minimizing the Kullback–Leibler divergence [18] between the target distribution, given by $\pi(\omega_q)$, and the proposal distribution $q(\omega_q; \mathbf{p})$.

1) Derivation of Updating Rule for the CAMCMC Method:

In the CAMCMC algorithm, the adaptive proposal distribution is proportional to the product of Bernoulli distributions, namely

$$q(\omega_q; \mathbf{p}) = \frac{\prod_{i=1}^{M_{R\Omega}} p_i^{[\omega_q]_i} \cdot (1 - p_i)^{1-[\omega_q]_i}}{\Gamma'} \propto \prod_{i=1}^{M_{R\Omega}} p_i^{[\omega_q]_i} \cdot (1 - p_i)^{1-[\omega_q]_i},$$  \hspace{1cm} (13)$$

where $p_i$ denotes the probability of the $i$th receive antenna to be chosen, $[\omega_q]_i$ represents the $i$th dimension of $\omega_q$ and indicates whether the $i$th receive antenna is selected, and $\Gamma' = \sum_{Q=1}^{Q} \prod_{i=1}^{M_{R\Omega}} p_i^{[\omega_q]_i} \cdot (1 - p_i)^{1-[\omega_q]_i}$ is a normalization constant that can be neglected in the CAMCMC algorithm. The adaptation strategy is used to adjust the parameterized proposal distribution $q(\omega_q; \mathbf{p})$ and minimize the Kullback–Leibler divergence [18] between the distribution $\pi(\omega_q)$ and the proposal distribution $q(\omega_q; \mathbf{p})$, namely

$$D[\pi(\omega_q)||q(\omega_q; \mathbf{p})] = \sum_{q=1}^{Q} \pi(\omega_q) \times \log \left( \frac{\pi(\omega_q)}{q(\omega_q; \mathbf{p})} \right).$$  \hspace{1cm} (14)$$

It has been proved that $D = \pi(\omega_q) \times \log \pi(\omega_q) - D[\pi(\omega_q)||q(\omega_q; \mathbf{p})]$ is a convex function [19]. Therefore, the minimization of the Kullback–Leibler divergence $D[\pi(\omega_q)||q(\omega_q; \mathbf{p})]$ w.r.t. $\mathbf{p}$ can be obtained when $\partial D/\partial \mathbf{p} = 0$. Then, we have

$$\frac{\partial D}{\partial p_j} = \frac{\partial}{\partial p_j} \sum_{q=1}^{Q} \pi(\omega_q) \times \log \left( \prod_{i=1}^{M_{R\Omega}} p_i^{[\omega_q]_i} \cdot (1 - p_i)^{1-[\omega_q]_i} \right) = \frac{1}{p_j(1 - p_j)} \sum_{q=1}^{Q} \pi([\omega_q]_j) \cdot (|\omega_q]_j - p_j).$$  \hspace{1cm} (15)$$

Given a number of samples $\{\omega_q^{(n)}\}_{n=1}^{N_{MCMC}}$ drawn from $\pi(\omega_q)$, the Monte Carlo estimate of $\partial D/\partial \mathbf{p}$ is

$$\frac{1}{N_{MCMC}} \left[ \sum_{n=1}^{N_{MCMC}} \frac{1}{p_j(1 - p_j)} \sum_{q=1}^{Q} \left( [\omega_q^{(n)}]_j - p_j \right) \right].$$  \hspace{1cm} (16)$$

Employing the Robbins–Monro stochastic approximation algorithm [19], we can obtain the recursive update equation to approach the root of $\partial D/\partial \mathbf{p} = 0$, namely

$$p_j^{(t+1)} = p_j^{(t)} + \frac{r^{(t+1)}}{p_j^{(t)}(1 - p_j^{(t)})} \times \left( \frac{1}{N_{MCMC}} \sum_{n=1}^{N_{MCMC}} [\omega_q^{(n)}]_j - p_j^{(t)} \right).$$  \hspace{1cm} (17)$$

where $r^{(t)}$ is a sequence of decreasing step sizes, e.g., satisfying the conditions $\sum_{t=0}^{\infty} r^{(t)} = \infty$ and $\sum_{t=0}^{\infty} r^{(t)^2} < \infty$ [20]. Moreover, (17) can be simplified as the following because $p_j(1 - p_j)$ has no significant influence on the convergence of (17). Therefore

$$p_j^{(t+1)} = p_j^{(t)} + r^{(t+1)} \left( \frac{1}{N_{MCMC}} \sum_{n=1}^{N_{MCMC}} [\omega_q^{(n)}]_j - p_j^{(t)} \right).$$  \hspace{1cm} (18)$$

The updated proposal distribution (18) is iteratively used with the aim to approach the target distribution.

2) Constraint for the Receive-Antenna-Selection Problem:

For the receive-antenna-selection problem, the constraint is $|\omega_q| = M_r$, where $|\omega_q| = \sum_{i=1}^{M_{R\Omega}} I_i$, and $M_r \leq M_{R\Omega}$. However, applying (18) with MIS to generate samples, it is impossible to guarantee that all samples meet the requirement of this constraint. In order to change the infeasible samples into feasible ones, we use the following projection strategies.
**Definition:** Run MIS and obtain a sample $\omega_q \sim \text{Ber}(p)$. The feasible subspace is defined as $\Omega = \{\omega_q \sim \text{Ber}(p) : |\omega_q| = \sum_{i=1}^{M_R} I_i = M_r\}$. Therefore, the projection of $\omega_q$ on $\Omega$ can be denoted by $\omega_q^{\text{proj}} \in P_\Omega^+(\omega_q)$, when $|\omega_q| > M_r$, and $\omega_q^{\text{proj}} \in P_\Omega^-(\omega_q)$, when $|\omega_q| < M_r$. Herein, $P_\Omega^+(\omega_q)$ and $P_\Omega^-(\omega_q)$ are

$$P_\Omega^+(\omega_q) = \{\omega_q^{\text{proj}} \in \Omega : |\omega_q^{\text{proj}}|_i > 0 \}
$$

$$P_\Omega^-(\omega_q) = \{\omega_q^{\text{proj}} \in \Omega : |\omega_q^{\text{proj}}|_j < 0 \}
$$

$$i = 1, \ldots, M_R \} \quad (19)
$$

$$j = 1, \ldots, M_R \} \quad (20)
$$

Given the aforementioned definition, the proposed projection strategy can be described as follows: If $\omega_q^{\text{proj}} \in P_\Omega^+(\omega_q)$, sort $p_i$ from largest to smallest according to the probability value, for example, $p_1 > p_2 > \cdots > p_{M_R}$, and $\omega_q^{\text{proj}}$ will be $\{I_{p_1}; I_{p_2}; \ldots; I_{p_{M_R}}\}$; if $\omega_q^{\text{proj}} \in P_\Omega^-(\omega_q)$, find zeros in $\omega_q$, namely, obtaining the indexes of nonselection receive antennas. After randomly selecting $M_r - \sum_{j=1} I_j$ ones from these zeros, $\omega_q^{\text{proj}}$ can be obtained.

3) **Convergence Analysis of the CAMCMC Method:** Here, we present an intuitive analysis to show that the proposed CAMCMC method has a fast convergence rate in exploring the solution space $\Omega$. We also take the MIS algorithm as an example. It is known that the performance of MIS is strongly dependent on the selection of the proposal density function. Its convergence rate is bounded by [15]

$$||K_p^\ast(\omega_q, \cdot) - \pi(\cdot)|| = \sup_{Z \in \sigma(\Omega)} |K_p^\ast(\omega_q, Z) - \pi(Z)| \leq 2 \left(1 - \frac{1}{W^n}\right)^n \quad (21)
$$

where $K_p^\ast(\omega_q, \cdot)$ denotes the $n$-step transition probabilities with initial state $\omega_q$ [15], $\sigma(\omega)$ is the $\sigma$-field of the solution space $\omega$, $Z$ is a subset of $\sigma(\omega)$, and $W \equiv \sup_{\omega_q \in \sigma(\Omega)} (\pi(\omega_q)/q(\omega_q; p))$. The adaption strategy in our CAMCMC algorithm is to minimize the Kullback–Leibler divergence [18] between the distribution $\pi(\omega_q)$ and the proposal distribution, given by $q(\omega_q; p)$. Intuitively, as the updated proposal distribution $\{\pi(\omega_q)|q(\omega_q; p)\}$ becomes smaller, so does $\sup(\pi(\omega_q)/q(\omega_q; p))$, and the upper bound of the convergence rate becomes smaller. In the ideal case, the adaptation enables $D[\pi(\omega_q)|q(\omega_q; p)] \rightarrow 0$, and then, the upper bound of convergence rate will also approach zero even with a small value of $n$. This implies that, starting with any initial value $\omega_q$, the chain has the chance to jump into any space $Z \in \sigma(\omega)$ with probability $\pi(Z)$ in a few steps. With the careful design of $\pi(\omega_q)$ described previously, the neighborhood of $\omega_q^\ast$ has a large chance to mass. It offers that the proposed CAMCMC algorithm has a large chance to visit the neighborhood of $\omega_q^\ast$ and, at least, find a close-to-optimal solution $\omega_q^\ast$ in a reasonable duration. More convergence analysis can be found in [21]. In [13], it is shown that the complexity of the MCMC algorithm is dimension independent and only related to the sample size $N_{\text{MCMC}}$. The convergence rate of the MCMC algorithm can significantly be improved via the proposed adaptation strategy that requires a small sample size and number of iterations. Therefore, the proposed CAMCMC algorithm for antenna selection can be seen as an efficient stochastic optimization approach compared with the existing selection schemes.

Generally, the proposed CAMCMC algorithm can be described by the following iterative steps. At iteration $t$, $N_{\text{MCMC}}$ samples $\{\omega_q^{(n)}\}_{n=1}^{N_{\text{MCMC}}}$ are generated by using the MIS according to a certain known proposal distribution $q(\omega_q; p^{(t)})$. Then, the new proposal $q(\omega_q; p^{(t+1)})$ will be updated to close the target distribution $\pi(\omega_q)$ by the Kullback–Leibler divergence. After a number of iterations, the generated samples will be equal to $\omega_q^\ast$ or within the neighborhood of $\omega_q^\ast$. The detailed CAMCMC algorithm is described as follows.

**Step 1)** Initialize $\omega_q^{(0)}$ according to the power of receive antennas, namely, $\omega_q^{(0)} = \max_{\omega_q \in \sigma(\Omega)} C_{\text{norm}}(\omega_q, 2)^2$. After that, let $\omega_q^\ast = \omega_q^{(0)}$, and set $p_j^{(0)} = \{p_j^{(0)}\}_{j=1}^{M_R}$, where $p_j^{(0)} = (1/2)^3$ for the proposal density function $q(\omega_q, p^{(0)})$. Set the iteration counter $t := 1$.

**Step 2)** Run the MIS. Draw a small set of samples $\{\omega_q^{(n)}\}_{n=1}^{N_{\text{MCMC}}}$ from the objective function $\pi(\omega_q)$ using the proposal $q(\omega_q, p^{(t-1)})$. If $\omega_q$ is not feasible, then change $\omega_q$ into $\omega_q^{\text{proj}}$ or $\omega_q^{\text{proj}^{-1}}$ according to the projection strategy in Section III-B.2.

**Step 3)** Update the parameter $p_j^{(t)}$ via

$$p_j^{(t+1)} = p_j^{(t)} + \epsilon^{(t+1)} \left(\frac{1}{N_{\text{MCMC}}} \sum_{n=1}^{N_{\text{MCMC}}} |\omega_q^{(n)}|_j - p_j^{(t)}\right) \quad (22)
$$

where the probability entries $p_j$, $j = 1, \ldots, M_R$, represent the probability of the $j$th receive antenna to be chosen, $\epsilon^{(t)}$ is the sequence of decreasing step sizes, and $|\omega_q^{(n)}|_j$ represents the $j$th dimension of $\omega_q^{(n)}$.

**Step 4)** If $\pi(\omega_q^{(n)}) > \pi(\omega_q^\ast)$ for $n = 1, \ldots, N$, then $\omega_q^\ast = \omega_q^{(n)}$.

**Step 5)** If the stopping criterion is satisfied, then stop; otherwise, set $t := t + 1$, and go back to Step 2). Here, the stopping criterion is a predefined number of iterations.

**IV. SIMULATION RESULTS**

We consider a 64-tone ($N = 64$) MIMO–OFDM system with $M_T(M_T = 4)$ transmit antennas, and $M_r (M_r = 4$, selected from $M_R = 16$) receive antennas are employed with $\omega_q^{(0)}$ can be obtained by sorting the power of receive antennas.

The algorithm converges without the constraint of a starting point, but for simplicity, we set $p_j^{(0)} = (1/2)$.

To simplify the notation, $\omega_q$ is used to denote $\omega_q^{\text{proj}^+}$ or $\omega_q^{\text{proj}^-}$ in the following steps.
Fig. 2. Ergodic capacity versus SNR with $M_r = 4$, $M_R = 16$, $M_T = 4$, and $L = 2$.

$L$-tap ($L = 1, 2, \ldots, 8$) frequency-selective channels. In addition to the CAMCMC algorithm, the selection algorithm given by Gorokhov [4], [22], the adapted Frobenius norm [7], and ES methods are applied. The capacity enhancement introduced by these antenna-selection algorithms over frequency-selective channels is first analyzed. Then, we focus on the system BER improvements. In this section, the receive-antenna-selection algorithms based on various criteria are described as follows:

1) channel-capacity-criterion-based CAMCMC selection algorithm (C-CAMCMC);
2) MSE of the ZF-receiver-criterion-based CAMCMC selection algorithm (ZF-CAMCMC);
3) channel-capacity-criterion-based ES selection algorithm (C-ES);
4) MSE of the ZF-receiver-criterion-based ES selection algorithm (ZF-ES);
5) Gorokhov selection algorithm (Gorokhov);
6) Frobenius norm selection algorithm (NBS).

A. Capacity Performance

The ergodic capacity (50% outage rate) performance of the receive-antenna-selection MIMO–OFDM system is shown in Fig. 2 when $L = 2$. We find that the capacity of the C-CAMCMC algorithm is almost identical to the maximum result achieved by the C-ES algorithm for a wide range of SNRs. For example, when the SNR is 20 dB, they approximately obtain 25.46- and 25.42-b/s/Hz capacity values. Within 99% of the optimal capacity is achieved by the proposed C-CAMCMC algorithm. On the other hand, it is seen that the capacity obtained by either the Gorokhov or the NBS selection algorithm is close to optimal in the low-SNR region, although the former method has better performance. It can be observed that the gap between the C-ES and Gorokhov (or NBS) selection results is significant as the SNR increases. Compared with the C-ES method, there are approximately 0.7- and 1.1-b/s/Hz capacity offsets for the Gorokhov and NBS algorithms, respectively, when the SNR is 20 dB. However, similar to the C-ES or C-CAMCMC algorithms, both Gorokhov and NBS rules offer superior capacity performances relative to the nonselection (Nosel) algorithm. In this figure, when the SNR is 25 dB, approximately 3.6-, 3.6-, 2.9-, and 2.3-b/s/Hz capacity improvements can be achieved by the C-ES, C-CAMCMC, Gorokhov, and NBS selection algorithms, respectively.

We now extend the study to outage rates. Fig. 3 shows the different outage capacities when the SNR is 20 dB. These curves indicate that the capacity obtained by the C-CAMCMC algorithm coincides with the capacity of the C-ES method for the whole outage rate range. It implies that the capacity coincidence is independent of the outage rate. As a consequence, the proposed C-CAMCMC algorithm can be employed for arbitrary outage capacity requirement instead of the C-ES method. Moreover, Fig. 3 also shows that, similar to the NBS algorithm, the capacity performance of the Gorokhov algorithm is sensitive to the outage rate at the high-SNR region. It can be observed that the difference between the curves of the Gorokhov (or NBS) and C-ES algorithms is more significant at the 10% outage rate compared with the result at the 90% outage rate.

We offer more insight into the antenna array configuration. In Fig. 4, we find that the channel capacity of each selection algorithm grows as the number of selected receive antenna ($M_r$) increases. Compared with the Nosel result, the capacity improvements introduced by the C-ES, C-CAMCMC, Gorokhov, and NBS selection algorithms are more significant as $M_r$ increases up to $M_r = 4$. When $M_r > 4$, the superior capacity performance achieved by the aforementioned algorithms is degraded. For example, the maximum capacity improvement, which is approximately 3 b/s/Hz at SNR = 20 dB, is obtained when four receive antennas are selected. This bottleneck is created by the characteristic of the MIMO system whose capacity linearly increases with the minimum number of transmit and receive antennas [1]. Referring to Fig. 4, we conclude that the most significant capacity improvement is achieved when an identical number of transmit and selected receive antennas are adopted.
Fig. 4. Ergodic capacity versus selected receive antennas with $M_R = 16$, $M_T = 4$, $L = 2$, and SNR = 20 dB.

We now consider the impact of channel frequency selectivity, quantified by $L$, on capacity, as shown in Fig. 5. This figure shows that the capacity of the C-CAMCMC selection algorithm closing to the C-ES result is not affected by the channel frequency selectivity. We observe that, compared with the Nosel result, the capacity improvement achieved by the C-CAMCMC algorithm gradually degrades as the channel frequency selectivity increases. The capacity performance mitigation accompanying increasing channel frequency selectivity is more significant at the high-SNR region. The gap among the C-CAMCMC, Gorokhov, and NBS algorithms is reduced. All of these are caused by the channel frequency selectivity that counteracts the benefit of antenna selection. Therefore, antenna selection is more effective in MIMO–OFDM systems with low frequency selectivity.

At the end of this section, we compare the computational complexity in terms of the number of function evaluations for C-ES, C-CAMCMC, Gorokhov, and NBS antenna-selection methods whose complexity orders can be presented as $O(NM_T^2M_r), O(NM_T^2M_r)(M_{NMCMC}^{-1})$, $O(M_T^2M_R^2L)$, and $O(NM_TM_r)(M_R \log(M_R))$, respectively. Here, $M_R \log(M_R)$ is the complexity order of the sorting operation, with the value of 19 for $M_R = 16$. Referring to Table I, when choosing $M_r = 2$ out of $M_R = 16$ receive antennas, the computational complexity orders of C-ES and C-CAMCMC algorithms are 245 760 and 30 720, respectively. It means that the C-CAMCMC algorithm is about eight times faster than the ES method, but when $M_r = 8$ and $M_R = 16$, the C-CAMCMC algorithm is approximately 200 times faster than the ES method. This demonstrates that the rapid convergence performance introduced by the C-CAMCMC algorithm is more significant when the solution space is very large. Compared with the C-CAMCMC rule, the Gorokhov algorithm requires a lower computational complexity, since it concerns the antenna-selection problem in time domain [4]. For example, the complexity of the Gorokhov method is 8192 when selecting $M_r = 2$ out of $M_R = 16$ receive antennas for $L = 2$. However, the complexity of this algorithm linearly grows with the number of channel taps. When $L = 8$, the complexity of this algorithm is 32 768, which is four times than the previous result. In this case, the Gorokhov rule requires less than 10% of the computational complexity of the C-CAMCMC algorithm, but its capacity performance is suboptimal. The capacity losses of the Gorokhov and NBS methods are obvious relative to C-CAMCMC, particularly for the high-SNR and low-frequency-selectivity cases, which are shown in Figs. 2–5.

B. BER Performance

In this paper, the system BER for the receive-antenna-selection MIMO–OFDM system is considered to investigate whether the BER result of the proposed CAMCMC algorithm is close to that of the ES such as the mentioned capacity performance. A 16-quadrature amplitude modulation scheme is employed with a ZF receiver.

Fig. 6 shows the BER performance of the system with a linear ZF receiver. In contrast with the Nosel result, some system BER improvements are introduced by the C-ES, C-CAMCMC, Gorokhov, and NBS selection algorithms. Similar to the case of the capacity performance, the system BER of the C-CAMCMC algorithm is nearly identical to that of the C-ES algorithm. We also find that C-ES and C-CAMCMC have superior BER performances to the Gorokhov and NBS selection algorithms. This superiority is more obvious as the SNR increases. When the SNR is 20 dB, the BER performances of the C-CAMCMC, Gorokhov, and NBS selection algorithms are approximately $4 \times 10^{-3}$, $2 \times 10^{-2}$, and $3 \times 10^{-2}$, respectively.

To prove the CAMCMC algorithm performance not depending on selection criteria, Fig. 6 shows the BER performance of both ZF-ES and ZF-CAMCMC selection algorithms. It can be seen that the BER obtained by ZF-CAMCMC is quite close to that of ZF-ES. For example, the BER of the ZF-CAMCMC selection algorithm is approximately $1 \times 10^{-3.1}$ while that of the ZF-ES method is about $1 \times 10^{-3.2}$ at the SNR = 20 dB. By comparing the BER result of ZF-CAMCMC with C-CAMCMC, further BER improvements offered by the former can be realized. The BER performance gap between them is obvious in the high-SNR region.

Finally, the various frequency-selective channels are again introduced to examine the BER performance of antenna-selection techniques. In contrast to Fig. 6, Fig. 7 shows the BER performance of ZF-based antenna-selection MIMO–OFDM systems with highly frequency-selective channels (L = 8). Here, channel parameter $L$ is equal to 8. We find that the BER improvement of the ZF-CAMCMC selection algorithm compared to the Nosel, C-CAMCMC, Gorokhov, and NBS algorithms is no longer significant. Similar to previous work [4], the antenna-selection benefit of BER performance is undermined by increasing the channel frequency selectivity. In Fig. 6, when $L = 2$, the ZF-CAMCMC selection algorithm leads to approximately 2.5-, 6.5-, 7.5-, and 10-dB SNR gains at $10^{-2}$ BER compared with C-CAMCMC, Gorokhov, NBS, and Nosel performances, respectively. However, in Fig. 7, when $L = 8$, only about 1.8-, 5-, 5.5-, and 6.5-dB SNR gains can
be achieved by ZF-CAMCMC. Hence, in order to efficiently minimize the system BER, antenna-selection techniques are suggested to be utilized in an MIMO–OFDM system with low frequency selectivity.

V. CONCLUSION

In this paper, we have presented a novel low-complexity receive-antenna-selection algorithm based on the CAMCMC optimization method for receive-antenna-selection MIMO–OFDM wireless systems. Compared with the ES method, less
than 1% of computational complexity is required by the proposed CAMCMC algorithm. Both the capacity and system BER achieved by this scheme are nearly identical to the optimal results from ES. It has been shown that this performance is independent of the antenna-selection criteria, outage rate requirements, antenna array configuration, and channel frequency selectivity. Similar to other existing antenna-selection algorithms, whether it is designed to maximize the channel capacity or minimize the system BER, the proposed CAMCMC selection algorithm is more effective for MIMO–OFDM systems under low- to moderate-frequency-selective fading.

REFERENCES


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