**MIMO enabled multipath clutter rank estimation**

The MIT Faculty has made this article openly available. *Please share* how this access benefits you. Your story matters.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>As Published</td>
<td><a href="http://dx.doi.org/10.1109/RADAR.2009.4977137">http://dx.doi.org/10.1109/RADAR.2009.4977137</a></td>
</tr>
<tr>
<td>Publisher</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Accessed</td>
<td>Mon Feb 04 02:12:09 EST 2019</td>
</tr>
<tr>
<td>Citable Link</td>
<td><a href="http://hdl.handle.net/1721.1/59987">http://hdl.handle.net/1721.1/59987</a></td>
</tr>
<tr>
<td>Terms of Use</td>
<td>Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.</td>
</tr>
<tr>
<td>Detailed Terms</td>
<td></td>
</tr>
</tbody>
</table>
MIMO Enabled Multipath Clutter Rank Estimation

Vito F. Mecca and Jeffrey L. Krolik
Department of Electrical and Computer Engineering
Duke University, Durham, NC 27708

Abstract—Multiple-input multiple-output (MIMO) radar is an emerging technology that has the capability of providing range dependent transmit-domain degrees of freedom via receiver processing. When providing these additional degrees of freedom for target tracking, MIMO radar exhibits a lower signal-to-noise ratio (SNR) when compared to that of traditional single-input multiple-output (SIMO) phased array radar. Previous work has indicated the efficacy of combining MIMO operation with space-time adaptive processing (STAP) techniques in the presence of multipath clutter to improve the signal-to-clutter-plus-noise ratio (SCNR). The tradeoff between target SNR and SCNR in multipath propagation environments is a crucial consideration in MIMO radar. In this paper, a transmit-receive directionality spectrum (TRDS) is used to examine the clutter characteristics at a range-Doppler bin of interest, most notably in multipath situations where MIMO operation is advantageous. In situations where ground clutter is spread in Doppler frequency and azimuth by motion in the propagation environment, the clutter rank can be significantly higher than a Brennan’s rule estimate. However, the transmit observability within the MIMO data vector allows for a low rank representation of the clutter when compared to the total available degrees of freedom. A TRDS-based method based on the resolution limits of uniformly spaced linear transmit and receive arrays is presented which furnishes an estimate of the transmit-receive clutter rank in scenarios where Brennans rule provides a significantly underestimated measure. The proposed TRDS-based clutter rank estimation method is applied to both numerical simulations and experimental data.

I. INTRODUCTION

Recently, multiple-input multiple-output (MIMO) radar techniques have been proposed [1]–[8] to enhance radar performance and increase target parameter identifiability by emitting a set of waveforms from the transmit elements. This method of operation is in contrast that of to single-input multiple-output (SIMO) radars that emit a single waveform across all transmit elements. Of interest in this paper are the scenarios where target detection is limited by multipath clutter. In particular, when Doppler-spread multipath clutter returns arrive in the same receive azimuth as a target of interest, traditional SIMO space-time adaptive processing (STAP) methods are precluded.

STAP methods are multidimensional filtering techniques that jointly operate on the pulsed-Doppler and receive element dimensions in radar returns. These adaptive methods can efficiently allocate degrees of freedom when clutter has a low rank representation based on the physical nature of the propagation environment. Brennan’s rule [9], [10] provides an approximation for the radar clutter rank in monostatic radars. An extension of Brennan’s rule appears in [11] for distorted linear arrays and bistatic geometries. Additionally, the work of [8] has developed a Brennan’s rule clutter rank estimate for monostatic MIMO radars; however, this estimate may not be reasonably close to the true clutter rank for scenarios that are not limited by Doppler-spread or multipath clutter.

In situations where ground clutter is spread in Doppler frequency and azimuth by motion in the propagation environment, the clutter rank can be significantly higher than the estimate provided by Brennan’s rule. However, the higher dimensionality of the MIMO data vector allows for a low rank representation of the clutter when compared to the total available degrees of freedom. In this paper, an interpretation in [11] is extended to the MIMO case where the radar clutter rank is measured as the total number spatial frequency resolution cells spanned by the unambiguous clutter’s spatial spectrum. A transmit-receive directionality spectrum is introduced that allows for a characterization of the multipath nature of radar returns. In addition, a partially adaptive spectral estimate is explored that enables a reduction in the MIMO data vector’s dimension while maintaining information about clutter rank. Numerical simulations and laboratory experiments are provided to illustrate the theoretical techniques presented herein.

II. MIMO RADAR DATA MODEL

A modification to commonly accepted radar models [9], [10], [12] is presented as the framework by which MIMO operation is enabled. Consider concentric $L$ element transmit and $N$ element receive arrays with uniform interelement spacings $d_{tx}$ and $d_{rx}$, respectively. To allow for Doppler frequency discrimination, transmitted waveform is repeated at each a total of $M$ times with a pulse repetition frequency (PRF) of $f_r$. The MIMO radar operates at a center frequency $f_0$, or equivalently at a center wavelength $\lambda_0$. The propagation speed in the medium is $c$.

For a far-field target at a transmit angle $\theta_t$, a receive angle $\phi_r$ and a Doppler shift $f_d$, direction and Doppler vectors are defined as follows:

\[
\mathbf{t}(\theta) = \left[1, \ldots, e^{+j(2\pi f_0/c)(L-1)d_{tx}\sin(\theta)}\right]^T,
\]

\[
\mathbf{r}(\phi) = \left[1, \ldots, e^{-j(2\pi f_0/c)(N-1)d_{rx}\sin(\phi)}\right]^T,
\]

\[
\mathbf{d}(f) = \left[1, \ldots, e^{j2\pi(M-1)f/f_r}\right]^T.
\]

The phase of the elements of $\mathbf{t}$ and $\mathbf{r}$ is different because of the 180° direction difference in the outgoing transmit direction.

\* Vito Mecca is currently employed at Massachusetts Institute of Technology’s Lincoln Laboratory in Lexington, MA.

This work was supported by ONR Code 313, grant #N000140610003.
and the incoming receive direction. A shorthand notation for target terms will be employed such that \( t_1 = t(\theta_t) \), \( r_1 = r(\phi_t) \) and \( d_1 = d(f_t) \). A complex zero-mean random variable \( \alpha_t \) accounts for the effects of target scattering and has variance \( \sigma_t^2 \).

Radar returns are collected into a vector \( \chi \) that consists of several components

\[
\chi = \chi_t + \chi_c + \chi_n, \tag{1}
\]

where the target, clutter and noise responses are \( \chi_t \), \( \chi_c \), and \( \chi_n \), respectively. The effect of jamming will not be explored in this paper, but (1) can be extended to include jamming following the approaches [6], [8] in the MIMO case.

Without loss of generality, consider an element-space MIMO radar where a series of \( L \) orthogonal waveforms are emitted from the \( L \) transmitters. Each waveform is uniquely associated with a transmit element, and thus, a unique phase response, which in turn will lead to an multiplicative increase in the slow-time MIMO STAP degrees of freedom that can be allocated within receive processing. These additional degrees of freedom have the capability of affecting the transmit domain of freedom have the capability of affecting the transmit domain.

After pulse-compression for ranging and the matched filtering for separation of the orthogonal transmitted waveforms, the received data at each range can be reshaped into a data vector \( \chi \), with elements corresponding to each transmit element, receive element and Doppler pulse. The far-field point data vector \( \chi_c \) consists of several components

\[
\chi_c = \sum_{k=1}^{K} \alpha_k \nu(\theta_k, \phi_k, f_k), \tag{4}
\]

and the clutter covariance matrix \( \mathbf{R}_c \) is

\[
\mathbf{R}_c = \sum_{k=1}^{K} \mathbf{R}(\theta_k, \phi_k, f_k) \mathbf{R}(\theta_k, \phi_k, f_k) \tag{5}
\]

The white Gaussian noise clutter covariance matrix is

\[
\mathbf{R}_n = \sigma_n^2 \mathbf{I}_{C,M,N}, \tag{6}
\]

an identity matrix with a larger dimension than that of its SIMO counterpart in [9].

### III. MIMO Transmit-Receive Directionality Spectrum

A spatial mapping technique is presented here that applies well-known beamforming and spectral estimation techniques to the elemental received MIMO space-time waveform of (3). A joint transmit-receive directionality spectrum will be useful to identify and characterize multipath clutter limiting environments. Because the MIMO elemental waveform includes phase information associated with transmitted directions, both transmit and receive directions can be estimated at all Doppler frequencies. This allows for a directionality spectrum that can distinguish multipath clutter from other direct path returns.

#### A. Fully Adaptive TRDS

In a MIMO radar, consider the covariance matrix at a single range gate. After the transmit channels have been baseband and separated in the Doppler frequency domain, the data vector for a single target takes the form of (3) with covariance \( \mathbf{R} \) as

\[
\mathbf{R} = \sigma_t^2 \nu(\theta, \phi, f) \mathbf{R}(\theta, \phi, f). \tag{7}
\]

The conventional output power spectrum of a joint transmit-receive beamformer and Doppler processor via the weight vector \( \mathbf{w}(\theta, \phi, f) \) is

\[
S_{\text{CONV}}(\theta, \phi, f) = \mathbf{w}^H(\theta, \phi, f) \mathbf{R}(\theta, \phi, f) \mathbf{w}(\theta, \phi, f). \tag{8}
\]

Any number of spectral estimation techniques can be readily applied to the covariance matrix of (7). In this paper, the MVDR technique [13] will be examined further within the MIMO radar framework. MVDR spectral estimation is a data dependent method seeking minimization of the output power spectrum given a filter weight vector \( \mathbf{w} \). The MVDR spectral estimate of the \( L,M,N \times L,M,N \) matrix \( \mathbf{R} \) is the solution to the linearly constrained problem

\[
\min \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{s.t. } \mathbf{w}^H \mathbf{v}(\theta, \phi, f) = 1.
\]

The well-known solution to this problem [13] is achieved when

\[
\mathbf{w} = \frac{\mathbf{R}^{-1} \mathbf{v}(\theta, \phi, f)}{\mathbf{v}^H(\theta, \phi, f) \mathbf{R}^{-1} \mathbf{v}(\theta, \phi, f)}, \tag{9}
\]

giving a spectral estimate \( S_{\text{MVDR}}(\theta, \phi, f) \) of

\[
S_{\text{MVDR}}(\theta, \phi, f) = \frac{1}{\mathbf{v}^H(\theta, \phi, f) \mathbf{R}^{-1} \mathbf{v}(\theta, \phi, f)}. \tag{10}
\]

The 3D surface obtained via (10) characterizes the nature of the multipath propagation. Consider a slice of the surface at a Doppler frequency \( f_k \) which will be referred to as a transmit-receive directionality spectrum (TRDS). For co-located arrays, direct path clutter appears where \( \theta = \phi \). All other points where \( \theta \neq \phi \) account for multipath propagation. An example of this surface appears as figure 1. This surface is of interest in array
processing applications when there is a relationship between doppler frequency shift and receiver azimuth angles such as the airborne STAP scenario for monostatic [9] and bistatic [11] geometries.

In practical MIMO radars, the fully adaptive solution achieved in (10) may be computationally intractable or otherwise unfeasible for several reasons. Having to estimate \( \mathbf{R} \) from the data is an issue in real systems. Snapshots of the received data \( \chi \) are used to form an estimate \( \hat{\mathbf{R}} \):

\[
\hat{\mathbf{R}} = \frac{1}{N_s} \sum_{n=0}^{N_s-1} \chi(n)\chi_H(n) \tag{11}
\]

where \( \chi(n) \) denotes the \( n \)th data snapshot and \( N_s \) is the total number of snapshots used in the estimate. Reference [9] mentions that between two and five times \( LMN \) independent samples are necessary for covariance estimation when the data is statistically stationary. Depending on the values of \( L \), \( M \), and \( N \), there may not be enough data samples to drive down the bias of \( \hat{\mathbf{R}} \). Compounding this estimation issue is the computational complexity of the matrix inversion operation. The number of operations necessary to invert the Hermetian symmetric \( \mathbf{R} \) is on the order of \( (LMN)^3 \) [14]. In radars with large receive arrays or with high PRFs and long CPIs, this can be a prohibitive operation. These problems with covariance estimation have motivated reduced-rank processing techniques.

### B. Post-Doppler TRDS

Physically, the TRDS has a useful interpretation at a single Doppler frequency; therefore, a decreased dimension processing technique is presented here to exploit this natural reduction. Alternatively, the terms in the \( LMN \times 1 \) elemental MIMO data vector of (3) can be reordered and reshaped into a \( M \times LN \) matrix \( \mathbf{V} \) to allow for conventional Doppler processing as

\[
\mathbf{V}(\theta, \phi, f) = \alpha \cdot \mathbf{d}(f) \cdot (\mathbf{t}(\theta) \otimes \mathbf{r}(\phi))^T. \tag{12}
\]

Given a Doppler weight vector \( \mathbf{w}_d \), denote the output to the linear Doppler processing operation as

\[
y(\theta, \phi; \mathbf{w}_d) = \text{vec}\left\{\mathbf{w}_d^H \mathbf{V}(\theta, \phi, f)^T\right\}. \tag{13}
\]

When the weight vector is matched to the far-field scatterer’s Doppler frequency \( \mathbf{w}_d = \mathbf{d}(f) \), the result is

\[
y(\theta, \phi; \mathbf{d}(f)) = \alpha \cdot M \cdot \mathbf{t}(\theta) \otimes \mathbf{r}(\phi). \tag{14}
\]

When pre-processed to the target Doppler frequency, the clutter response in (4) may be of a smaller rank. Let \( K_d \) represent the number of clutter responses that lie within the target’s Doppler bin where \( K_d \leq K \). In this post-Doppler processing scheme, the summation carried out in (5) is added over \( K_d \) terms.

Now, Doppler processed data in (14) can serve as snapshots to estimate a reduced dimension post-Doppler covariance matrix \( \hat{\mathbf{R}}_p \):

\[
\hat{\mathbf{R}}_p = \frac{1}{N_s} \sum_{n=0}^{N_s-1} \mathbf{y}(n)\mathbf{y}_H(n), \tag{15}
\]

where the subscript ‘p’ indicates post-Doppler. The matrix \( \hat{\mathbf{R}}_p \) can now be used in place of \( \mathbf{R} \) in beamformer-based spectral estimate of (8) where \( \mathbf{w}(\theta, \phi, f) \) is replaced with \( \mathbf{w}_p(\theta, \phi) \).

\[
S_{\text{CONV}}(\theta, \phi) = \mathbf{w}_p^H(\theta, \phi) \hat{\mathbf{R}}_p \mathbf{w}_p(\theta, \phi). \tag{16}
\]

The same substitution can be made in the MVDR calculation of (10) with a reduced size \( \mathbf{v}_p(\theta, \phi) = \mathbf{t}(\theta) \otimes \mathbf{r}(\phi) \) in place of \( \mathbf{v}(\theta, \phi, f) \). The resulting reduced dimension MVDR spectral estimate is calculated as

\[
S_{\text{MVDR}_p}(\theta, \phi) = \frac{1}{\mathbf{v}_p^H(\theta, \phi) \hat{\mathbf{R}}_p^{-1} \mathbf{v}_p(\theta, \phi)}. \tag{17}
\]

The 2D spectrum of (17) is in the form of the TRDS that appears in figure 1 and similar interpretations of direct path and multipath clutter can be made.

Because the dimension of \( \mathbf{R}_p \) has been reduced by a factor of \( M \) to \( LN \times LN \), the necessary number of statistically identical and independent data snapshots required for estimation has been reduced by a factor of two to five times \( M \). In addition, matrix inverse operations on \( \mathbf{R}_p \) require a factor of \( M^2 \) fewer calculations.

### IV. CLUTTER RANK ESTIMATION

The transmit-receive directionality spectrum also can lead to an estimate of the clutter rank at a particular Doppler frequency bin. Because this spectrum has a resolution defined by the transmit and receive apertures, an estimate of the clutter rank can via the geometrical interpretation in [11] where the clutter rank represents the number of clutter occupied rectangular resolution cells. The wavenumber resolutions bin widths \( \gamma_{tx} \) and \( \gamma_{rx} \) of the transmit and receive arrays are

\[
\gamma_{tx} = \frac{2\pi}{Ld_{tx}}, \quad \gamma_{rx} = \frac{2\pi}{Nd_{rx}}. \tag{18}
\]

Note that in (18) the products \( Ld_{tx} \) and \( Nd_{rx} \) are related to the length of the transmit and receive apertures in meters.
Increasing an array’s aperture has the effect of decreasing a resolution cell and increasing the array’s resolving power.

The 2D TRDS surface as a function of transmit angle $\theta$ and receive angle $\phi$ is depicted in figure 1 for direct path propagation at a single Doppler frequency. Without loss of generality, assume this TRDS represents the zero Doppler bin so the dashed line return can be interpreted as direct path ground clutter. Resolution bins as given in (18) for the stationary uniformly spaced transmit and receive linear arrays are marked.

For a monostatic pulsed Doppler radar with $N$ receivers uniformly spaced at $d_{rx}$ that are aligned with the platform’s velocity $v_r$ and with $M$ pulses emitted at a pulse repetition frequency $f_r$, the Brennan’s rule estimate [9] for the rank of the clutter $\rho$ is

$$\rho = [N + (M - 1)\beta]$$  \hspace{1cm} (19)

where $\lfloor \cdot \rfloor$ represents a ‘floor’ rounding to the lowest integer less than or equal to the argument and $\beta = (2v_r)/(f_r d_{rx})$ represents the number of half-wavelength spacings the array traverses over a coherent processing interval (CPI). The well-known form of Brennan’s rule in (19) has been modified to account for distorted monostatic array geometries as well as for bistatic radar systems [11]. In a similar manner, the work of [8] has extended Brennan’s rule to MIMO radars with $L$ channels and uniform transmit array spacing $d_{tx}$ to

$$\rho_{MIMO} = [N + \eta(L - 1) + (M - 1)\beta]$$  \hspace{1cm} (20)

where $\eta = d_{tx}/d_{rx}$. The discussion leading to (20) assumes a direct path clutter where $\theta = \phi$. When the ground clutter is not spread into other Doppler bins, the number of resolution cells $K$ that contains clutter is $\rho_{MIMO}$ and (20) provides an accurate estimate of the clutter rank. However, in situations dominated by multipath or spread-Doppler clutter this is not the case. Examining the number of clutter occupied resolution cells in the TRDS surface yields a more reasonable estimate of the clutter rank at each Doppler frequency.

The clutter rank estimate of the resolution cell counting method will be denoted as $\rho_{RCC}$. Consider the case where $d_{tx} = d_{rx} = \lambda_0/2$. The notation $G(t_1, t_2, r_1, r_2)$ will be used to represent the rectangular region in the TRDS that is bounded by the transmit wavenumbers $t_1$ and $t_2$ and the receive wavenumbers $r_1$ and $r_2$. The estimate $\rho_{RCC}$ can be calculated from a TRDS surface as

$$\rho_{RCC} = \sum_{\ell=0}^{L-1} \sum_{n=0}^{N-1} F\left\{ G\left( (-\frac{N}{2} + \ell) \gamma_{tx}, (-\frac{N}{2} + \ell + 1) \gamma_{tx}, \ldots \left( -\frac{N}{2} + n \right) \gamma_{rx}, \left( -\frac{N}{2} + n + 1 \right) \gamma_{rx} \right) \right\}$$  \hspace{1cm} (21)

where $F$ is an indicator function defined as

$$F\{G\} = \begin{cases} 1 & \text{if } \exists g \in G \geq T \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (22)

and $T$ represents a threshold value. For cases where $d_{tx}, d_{rx} \neq \lambda_0/2$, the summation is carried out over the rectangular resolution bins that contain the unambiguous clutter response. An adjustment of the indices in the summation of (21) is necessary to limit the operation over the appropriate bins.

Consider the zero Doppler TRDS for $L = 6$, $N = 9$, $\eta = 1$ and $\beta = 0$ in figure 1 for direct path ground clutter. The MIMO modified Brennan’s rule of [8] in (20) estimates a rank of $\rho_{MIMO} = [9 + (1)(6 - 1)] = 14$. The $\rho_{RCC} = 14$ resolution cells containing the direct path ground clutter in figure 1 are hi-lighted, as in the interpretation of [11]. However, if any portion of this ground clutter is spread to an additional Doppler frequency bin via multipath propagation, (20) can severely underestimate the clutter rank. Also, the clutter rank can be much higher in situations where ground clutter returns experience higher order scattering via multipath propagation and are subsequently spread across receiver azimuth. In this sense, a TRDS-based approach is advantageous because the spectrum accounts for multipath and spread-Doppler clutter.

V. EXPERIMENTAL RESULTS

A. Numerical Simulation

In this section, the degradation of the Brennan’s rule estimate on the clutter covariance will be explored for the MIMO radar. Consider concentric uniformly spaced transmit and receive arrays where $L = 11$ and $N = 15$ and $d_{tx} = d_{rx} = \lambda/2$. The velocity of both arrays will be $v_r = 0$. Thus $\eta = 1$ and $\beta = 0$. Direct-path ground clutter exists at zero Doppler frequency and at all azimuth angles with a clutter-to-noise ratio (CNR) of 40 dB relative to the receive elements’ white noise level in a single range bin of interest. A multipath scenario is considered that complicates the clutter covariance matrix structure from that of figure 1. Multipath clutter is simulated to exist in the range bin of interest spread evenly across $\theta \in [-20, 20]^{\circ}$ and $\phi \in [30, 30]^{\circ}$.

A conventional TRDS estimate is obtained using the true value for $R_c$ in (16), and the result appears as figure 2. Angular resolution cells obtained via (18) are marked as dashed lines. Note that the resolution cells grow wider near the endfire directions of the array as expected. For the uniform linear arrays considered the clutter response at endfire directions is ambiguous, which is manifest in figure 2. The MVDR TRDS calculated via (17) with the true $R_c$ and appears in figure 3. A plot of the eigenvalue spectrum is given in figure 4. Brennan’s rule estimates $\rho = 15$ which only captures 47.3% of the total energy in $R_c$ and $\rho_{MIMO} = 25$ captures 67.4% of the total energy. However, 99.5% of the total energy is represented by the first $\rho_{RCC} = 61$ eigenvalues. In this multipath situation, $\rho_{RCC}$ provides a much better estimate of the clutter covariance because situations where $\theta \neq \phi$ are accounted for whereas they are otherwise neglected in $\rho$ and $\rho_{MIMO}$.

B. Laboratory Experiment

The beamspace slow-time MIMO radar technique of [15] was implemented in an acoustic laboratory experiment where $c = 344$ m/s. The TRDS will be employed in this section to aid in the visualization of the multipath in the laboratory environment.
A total of $L = 5$ speakers are used as transmit elements to excite 3 discrete prolate spheroidal sequence (DPSS) shaded MIMO transmit channels. The DPSS weights are designed to spread transmit energy maximally between $\theta = \pm 45^\circ$. The uniformly spaced linear array of these speakers operates at a center frequency of $f_0 = 2$ kHz ($\lambda_0 \approx 17$ cm) with inter-element spacing $d_{tx} = 10.5$ cm. Each transmitter is driven by MATLAB and emits a hyperbolic frequency modulated (HFM) chirp waveform with a 2 kHz bandwidth in this non-dispersive environment. The HFM chirp is used in this application because it has the property that the Doppler shifts remain insensitive to frequency [16]. Because the bandwidth is large relative to the center frequency (in order to achieve higher range resolution), the use of an HFM chirp is crucial in maintaining a constant, linear Doppler frequency shift. The NIST Mark-III Microphone Array is used on receive. A total of $N = 16$ elements are used at a spacing $d_{rx} = 4em \approx 0.23\lambda_0$ and a total aperture of approximately $3.5\lambda_0$. Both arrays have a total aperture on the order of 3-4 times $\lambda_0$. The receive array has a phase center positioned approximately 10 cm below the transmit array's phase center. A total of $M = 420$ pulses were emitted at a $f_r = 21$ Hz giving a CPI of 20 seconds.

Covariance estimation for the TRDS surface was performed using neighboring range bins between within the 0.5 m of data centered at the range of interest while only considering the Doppler bin of interest. An example of the conventional TRDS appears in figure 5, which is taken from a zero Doppler bin in the far-field. Note that the response of the strong direct path clutter ridge along the indicated line where $\theta = \phi$ is limited to $\pm 45^\circ$ in both transmit and receive angular extents due to the DPSS beams. This direct path ridge was previously explored in figure 1. A plot of the eigenspectrum appears as figure 6. A threshold value was set to be 15 dB above the noise floor, giving an estimate of $\rho_{RCC} = 2$ which captures over 99.8% of total energy in the eigenvalues in this low-resolution array (the two peaks in figure 5). The off-diagonal energy that appears in figure 5 is caused by naturally occurring multipath propagation in the laboratory environment.

**VI. CONCLUSION**

Multipath propagation environments can cause higher clutter ranks than those predicted by Brennan’s rule. Forming the MIMO transmit-receive directionality spectrum is useful in such situations for several reasons. Because the post-Doppler TRDS is of a reduced dimension when compared to the full MIMO covariance matrix, fewer independent snapshots are required to reduce the bias in the covariance estimate. The rank of the clutter covariance matrix can be more appropriately measured by examining the number of rectangular resolution cells that contain unambiguous clutter responses in the TRDS. In multipath situations where Brennan’s rule may not be appropriate, simulations indicated a TRDS-based method gave
Fig. 5. Conventional MIMO transmit-receive directionality spectrum for far-field direct path clutter (in dB).

Fig. 6. Eigenspectrum of Clutter Covariance Matrix

![Eigenspectrum of Clutter Covariance Matrix](image)

a rank estimate that captured over 30% more energy in the clutter eigenvalues than that of a MIMO radar Brennan rule. Finally the TRDS can be used to visualize the nature of multipath propagation at any range and Doppler in otherwise complicated propagation environments.

**REFERENCES**


