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Range Resolution of Unequal Strength Targets

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Abstract—This paper examines the problem of resolving targets whose amplitudes may differ. A common metric used for resolution that assumes targets of equal strength is modified for the subject scenario. An expression for range resolution that accounts for the difference in target strength is also developed. Computational results are presented to depict how resolution performance is affected by various parameters.

Keywords—radar, range resolution

I. INTRODUCTION

A key performance parameter of a radar is its ability to resolve two targets that are closely spaced in range. Practical examples where this ability is important include maintaining an accurate track of a target trajectory and detection of a missile immediately after being launched from an aircraft.

In general, the range resolution of a radar is described using rules of thumb that are derived for various practical assumptions. For example, it is often stated that the range resolution is given by $c/2B$, where $c$ is the speed of light and $B$ is the effective waveform bandwidth. In [1], the problem of target resolution was analyzed using a resolution metric motivated by prior work on the resolution of direction of arrival estimation algorithms (see [2],[3] and the references therein). Using the pulse-compressed return spectrum, the metric in [1] essentially looks at spectral height at the midpoint of the target ranges subtracted from the average value of the target spectral peaks. If this quantity is positive, the targets are said to be resolved, whereas if it is negative, the targets are not resolved.

It is often the case that the strength/radar cross section (RCS) of the two targets to be resolved may differ. In such a case, it may be more appropriate to modify the resolution metric used in [1]. This paper examines how such a metric may be modified and develops an expression for range resolution that accounts for the difference in target strength. These results are used to develop a basic understanding of how various parameters affect resolution performance (see [4]--[6] for related work on radar target resolution).

This paper is organized as follows. Section 2 formulates the problem and develops a modified resolution criterion along with an expression for range resolution that accounts for the difference in target strength. Section 3 presents computational examples of the results developed in Section 2.

II. DEVELOPMENT

Consider two point scatterers with respective strength and range locations of $\sigma_k$ and $R_k$, $k = 1,2$. It will be assumed in this paper that the pulse compressed return from a single point target follows a Gaussian profile, and may thus be described as

$$ f_k(r) = \sigma_k e^{-\beta(r-R_k)^2} \quad (1) $$

where $r$ is the range variable and $\beta$ is related to the radar waveform bandwidth $B$ through the expression

$$ B = \frac{c}{4 \sqrt{\ln 2}} \quad (2) $$

and (2) is derived by assuming that the half-power bandwidth of (1) is equal to $c/2B$. The assumption of the Gaussian profile is selected for several reasons. The choice of a Gaussian model often simplifies the analysis of many problems. While a parabolic shape could have been selected, it is observed that a parabola is simply the second-order approximation of the Gaussian profile.

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Additionally, the Gaussian profile not only obeys the requisite properties of an ambiguity function (even and non-negative with maximum at target location), but also serves as a limiting case of a polynomial expansion.

The pulse compressed spectrum for two point targets is developed as follows. The return signal after matched filtering can be expressed as

\[ B(r) = \sum_{k=1}^{2} p_k(r)e^{-j\phi_k} \]  

(3)

where \( p_k(r) = \sqrt{f_k(r)} \) and \( \phi_k \) is a phase offset. Taking the magnitude-squared of (3) yields the pulse compressed spectrum

\[ \Im(r) = B(r)^2 \]

\[ = \sum_{k=1}^{2} p_k^2(r) + 2\cos(\phi) p_1(r) p_2(r) \]  

(4)

In (4), the value of \( \phi = \phi_2 - \phi_1 \) is affected by a number of parameters, such as center frequency, target separation, and various radar system effects. In order to assess the average behavior of the matched filter output, it is assumed that \( \phi \) is a random variable uniformly distributed on \([-\pi, +\pi]\]. Thus, taking the statistical expectation of (4) yields

\[ F(r) = \mathbb{E}\{\Im(r)\} \]

\[ = \sum_{k=1}^{2} f_k(r) \]  

(5)

Figures 1 and 2 depict two resolution scenarios. In Figure 1, two equal amplitude scatterers are located at a range of \( R_1 \) and \( R_2 \). Using the example in Figure 1, an appropriate metric for resolution as developed in [2],[3] is given by the expression

\[ \varepsilon = \frac{1}{2}(F(R_1) + F(R_2)) - F(R_m) \]  

(6)

The expression in (6) essentially checks for the existence of a "dip" between the peaks of the pulse compressed spectrum, which occurs when \( \varepsilon > 0 \). Figure 2 depicts what happens when the two targets are of different strength. Note that in comparison to Figure 1, the location of the "dip" in Figure 2 deviates from \( \frac{1}{2}(R_1 + R_2) \). As such, the definition of \( R_m \) in (7) should be accordingly modified. Additionally, in this paper, the resolution requirement will be made more conservative by requiring not only that \( \varepsilon > 0 \), but also that a local minimum exist between the two peaks of the pulse compressed spectrum.
A. Resolution Criterion

By examining Figure 2, it is observed that not only has the local minimum shifted from $\frac{1}{2}(R_1 + R_2)$, but the spectral peaks also deviate slightly from $R_1$ and $R_2$. In order to characterize the critical points of the pulse compressed spectrum, denote the actual location of the spectral peaks and local minimum as $\hat{R}_1$, $\hat{R}_2$, and $\hat{R}_m$, respectively. A more appropriate resolution metric than (6) is one that compares the value at $\hat{R}_m$ of the line connecting the spectral peaks to the amplitude of the pulse compressed waveform at $\hat{R}_m$. This can be expressed as

$$\hat{e} = \frac{F(\hat{R}_2) - F(\hat{R}_1)(\hat{R}_m - \hat{R}_1) + F(\hat{R}_1) - F(\hat{R}_m)}{R_2 - R_1}$$

(8)

Though analytical expressions for $\hat{R}_1$, $\hat{R}_2$, and $\hat{R}_m$ cannot be obtained, it is observed that because the behavior of $F(r)$ about the critical points is somewhat parabolic, the slope of $F(r)$ about these points changes in an approximately linear manner. Thus, $dF(r)/dr$ can be expanded as a first-order Taylor series about the nominal location of the critical points and then set to zero to obtain the location of actual critical points.

Towards this end, consider an expansion point $R$ in the neighborhood of the critical point $\hat{R}$. Approximating $dF(\hat{R})/dR$ about $R$ and equating to zero yields

$$\frac{dF(\hat{R})}{dR} \approx \sum_{k=1}^{2} -2\beta(R - R_k)f_k(R)$$

$$+ (\hat{R} - R)\sum_{k=1}^{2} \left(4\beta^2(R - R_k)^2 - 2\beta\right)f_k(R)$$

$$= 0$$

Thus

$$\hat{R} = R + \frac{\sum_{k=1}^{2}(R - R_k)f_k(R)}{\sum_{k=1}^{2}[2\beta(R - R_k)^2 - 1]f_k(R)}$$

(9)

To obtain $\hat{R}_1$, $\hat{R}_2$, and $\hat{R}_m$, the variable $R$ in (9) should be replaced with $R_1$, $R_2$, $\frac{1}{2}(R_1 + R_2)$, respectively.

As stated earlier, in order for two targets to be considered resolved, it is required that a local minimum exist between the two peaks of the pulse compressed spectrum. The existence of this local minimum requires that there exist a range $r$ between $R_1$ and $R_2$ such that $dF(r)/dr = 0$ and $d^2F(r)/dr^2 > 0$, so that

$$\sum_{k=1}^{2} -2\beta(r - R_k)f'_k(r) = 0$$

(10)

$$\sum_{k=1}^{2}(4\beta^2(r - R_k)^2 - 2\beta)f'_k(r) > 0$$

(11)

Using (10) and (11) yield the following inequality:

$$2\beta\left[(r - R_2)^2 - (r - R_1)(r - R_2)\right] > \frac{R_2 - R_1}{r - R_1}$$

(12)

Now assume that $R_2 > R_1$, and define the range difference as $R_d = R_2 - R_1$. Furthermore, assume without loss of generality that $R_1 = 0$. Then (12) can be simplified as

$$2\beta r(R_d - r) > 1$$

(13)

The value of $r$ in (13) can now be replaced by $\hat{R}_m$ from (9), so that

$$\hat{R}_m = \frac{R_d}{2} + \frac{R_d}{\beta R_d^2} - 2\sigma_1 + \sigma_2$$

(14)

Again, without loss of generality, assume that $\sigma_1 > \sigma_2$, and that

$$\sigma_2 = \alpha \sigma_1$$

(15)

for some $\alpha < 1$. Then (14) may now be written as

$$\hat{R}_m = \frac{R_d}{2} + \frac{R_d}{\beta R_d^2} \frac{1 - \alpha}{1 + \alpha}$$

(16)

Substituting (16) for $r$ in (13) yields

$$\frac{1}{2}\sqrt{\frac{\beta R_d^2 - 2}{\beta R_d^2}} > \frac{1 - \alpha}{1 + \alpha}$$

(17)
Using the first-order approximation of the left hand side of (17) with respect to the quantity $\beta R_d^2$ results in the following inequality

$$\beta R_d^2 > \frac{3\sqrt{3}}{2} \left( \frac{\sqrt{3}}{2} + \frac{1-\alpha}{1+\alpha} \right)$$  \hspace{1cm} (18)

By substituting (2) into (18), the following expression results:

$$R_d > \frac{c}{4B} \sqrt{\frac{3\sqrt{3}}{2 \ln 2} \left( \frac{\sqrt{3}}{2} + \frac{1-\alpha}{1+\alpha} \right)}$$  \hspace{1cm} (19)

The expression in (19) captures a constraint on the minimum resolvable target range that accounts for both waveform bandwidth and the difference in target strength. It is observed that when $\alpha = 1$ (equal strength targets), (19) becomes $R_d > c/2.2B$, whereas for a smaller value of $\alpha$ such as $\alpha = 0.2$, (19) becomes $R_d > c/1.7B$, indicating that the size of a resolution cell increases by 30% when the strength of the targets differ by 14 dB. Note that due to the use of approximations and the assumption in this paper of the presence of two targets, (19) is not valid for values of $\alpha$ approaching zero.

Figure 3. Contour of range resolution as a function of ratio of target strength and waveform bandwidth.

Figure 3 shows a contour plot of (19), where the contour represents the range resolution as a function of the ratio of the strength of the targets $\sigma_2/\sigma_1 = \alpha$ and the waveform bandwidth.

III. COMPUTATIONAL EXAMPLES

Figures 4 and 7 depict contour plots of the resolution metric $\hat{\delta}$ in (8) as a function of various parameters. Lower contour values correspond to poor resolution, while higher contour values correspond to better resolution. In these Figures, contour
values less than zero correspond to the white (unresolved) regions. It is seen in both Figures 4 and 7 that (19) accurately predicts the resolution boundary.

In Figure 4, the contour of $\hat{\epsilon}$ is shown as a function of the ratio of the target strength and the target range separation when the waveform bandwidth is fixed at 50 MHz. It can be seen that for a fixed range separation, resolution performance worsens as the difference in target strength increases. Figures 5 and 6 show cuts through $\alpha = 5$ m and $\alpha = 0.4$, respectively, of the contour in Figure 4. It can be seen from Figure 5 that an approximately linear relationship exists between $\alpha$ and $\hat{\epsilon}$.

Figure 7 shows the contour of $\hat{\epsilon}$ as a function of the ratio of the target strength and the waveform bandwidth when the target range separation is fixed at 5 m. As in Figure 4, it can be seen that for a fixed bandwidth in Figure 7, the resolution performance worsens as the difference in target strength increases. Increasing range separation or waveform bandwidth (which decreases the width of the pulse compressed waveform) improves resolution performance, since there is less interference between the pulse compressed spectrums of the two targets. Figure 8 shows a cut through $\alpha = 0.4$ of the contour in Figure 7.

IV. CONCLUSION

A common metric used for assessing resolution performance for equal strength targets was modified for targets of unequal strength. This modification included the effects of the shifting of the critical points of the pulse compressed spectrum from their expected location. Conditions for the existence of a local minimum between the two peaks of the pulse compressed spectrum, which is required for resolution to occur, were used to develop an expression for the range resolution that includes as a parameter the difference in target strength. Computational examples using the results were carried out, and the effects of various parameters were investigated to study their impact on resolution performance.

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