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Properties and Performance of the IEEE 802.11b Complementary-Code-Key Signal Sets

Michael B. Pursley, Fellow, IEEE, and Thomas C. Royster IV, Member, IEEE

Abstract—We describe similarities and differences between complementary-code-key (CCK) modulation and modulation that is derived from biorthogonal signals, and we present performance results and other information that may be useful to those who have applications for CCK modulation that do not require IEEE 802.11b compliance. The properties and performance of the high-rate IEEE CCK 802.11b modulation formats are investigated and compared with the properties and performance of alternative modulation formats that are based on biorthogonal signals. Several complementary properties are derived for the full-rate (11 Mb/s) CCK signal set, the half-rate (5.5 Mb/s) CCK signal set, a full-rate signal set obtained from biorthogonal signals, and a half-rate biorthogonal signal set. Each signal set is a complementary set, but each also has stronger complementary properties. We evaluate the performance of IEEE 802.11b standard CCK modulation, CCK with certain modifications that depart from the IEEE standard, and modulation that is derived from biorthogonal signals. Performance comparisons are presented for additive white Gaussian noise (AWGN) channels and for channels with specular multipath. In particular, for AWGN channels, we provide an accurate analytical approximation for the frame error probability for full-rate CCK modulation.

Index Terms—Modulation, wireless LAN, channel coding.

I. INTRODUCTION

C OMPLEMENTARY-CODE-KEY (CCK) modulation and modulation that is derived from biorthogonal signals were proposed as IEEE 802.11b standards for high-rate (11 Mb/s) and 5.5 Mb/s) communication in wireless local area networks. CCK was selected, in part because the signal set has a number of complementary properties. We consider two alternatives to CCK that we refer to as biorthogonal-key (BOK) modulation, each of which uses a biorthogonal signal set of size 16. The half-rate (5.5 Mb/s) BOK signal set is a true biorthogonal set that we denote by 16-B, whereas the full-rate (11 Mb/s) BOK modulation, which we denote by 256-IQB, employs a set of 16 biorthogonal signals for the inphase modulation and another such set for the quadrature modulation to give a set of 256 signals. If they are viewed as complex signals, the signals in the 256-IQB set have nonzero real and imaginary parts, but the signals in the 16-B set are real. We define specific complementary properties and show which properties are satisfied by the two CCK signal sets in the IEEE 802.11b standard and the two BOK signal sets. Some of the complementary properties can be deduced from the construction procedure that was used to obtain the signal set, but our results are derived from the mathematical expressions that define the signals.

In the IEEE 802.11b standard [1], the CCK modulation formats are not employed with error-control coding or a pseudo-random signature sequence. This is in contrast to some previous systems that use orthogonal or quasi-orthogonal signals but apply a pseudo-random signature sequence to the modulated waveforms and use error-control coding. Examples are the TIA-95 mobile cellular CDMA system (e.g., see [2] or [3]) or the much earlier Joint Tactical Information Distribution System (JTIDS) (e.g., see [4] or [5]). Although the original applications that were envisioned for IEEE 802.11b involve indoor short-range communication, the IEEE 802.11b CCK modulation formats have been proposed and tested for use on longer-range outdoor communication channels [6]–[11] for which the need for error-control coding is greater, the number of multipath components is often smaller, and the excess delays for the multipath components are larger. We do not examine the complete IEEE 802.11b standard, which includes specifications on the media-access protocol, frame structure, etc., nor is it our intent to propose modifications to the standard. Instead, our purpose is to present information that may be useful to those who might wish to use CCK for applications that do not require compliance with the IEEE 802.11b standard.

II. COMPLEMENTARY PROPERTIES

Let \( \mathcal{A} \) be an arbitary set of unit-magnitude complex numbers that represent the complex modulation chips for digital modulation waveforms. A modulation symbol is a sequence \( y = (y_1, y_2, \ldots, y_N) \) of modulation chips, so \( y_k \in \mathcal{A} \) for each \( k \). Consider a set \( \mathcal{X} = \{x_i : 1 \leq i \leq M\} \) of distinct modulation symbols. The aperiodic autocorrelation function [12] for \( x_i = (x_{i1}, x_{i2}, \ldots, x_{iN}) \) is defined by

\[
C_i(\ell) = \begin{cases} 
\sum_{k=1}^{N-\ell} x_{ik}^* x_{i,k+\ell} & 0 \leq \ell \leq N-1, \\
\sum_{k=1}^{N+\ell} x_{i,k-\ell} x_{i,k}^* & -(N-1) \leq \ell \leq -1, \\
0 & \text{otherwise},
\end{cases}
\]

where \( z^* \) denotes the complex conjugate of \( z \). The set \( \mathcal{X} \) is a complementary set if

\[
\sum_{j=1}^{M} C_j(\ell) = \begin{cases} 
MN & \ell = 0, \\
0 & \ell \neq 0.
\end{cases}
\]

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The condition in (2) for $\ell = 0$ is satisfied by any set of sequences of unit-magnitude complex numbers. It is well known [12] that $C_\ell(\ell) = [C_\ell(\ell)]^*$, so the sum in (2) is zero for all negative values of $\ell$ if and only if it is zero for all positive values of $\ell$. As a special case of the definition for sets of size two, the pair $x_1$ and $x_2$ from $X$ is said to be a complementary pair if $C_\ell(0) + C_\ell(0) = 2N$ and $C_\ell(\ell) + C_\ell(0) = 0$ for all $\ell \neq 0$. It follows easily from (2) that a sufficient condition for $X$ to be a complementary set is the existence of a partition of $X$ into two disjoint ordered subsets $U = \{u_i : 1 \leq i \leq M_2\}$ and $V = \{v_i : 1 \leq i \leq M_2\}$, each of size $M_2 = M/2$, for which $\{u_i, v_i\}$ is a complementary pair for each $i$. We refer to sets with the latter property as pairwise complementary. Complementary sets with more than two sequences need not be pairwise complementary.

For an arbitrary set $X = \{x_i : 1 \leq i \leq M\}$ of sequences of length $N$, let $X$ be a corresponding matrix whose rows are the sequences in $X$. For each integer $k$ in the range $1 \leq k \leq N$, define $w_k = (x_{1,k}, x_{2,k}, \ldots, x_{M,k})$. The elements of $w_k$ are from the $k$th column of $X$. For each $m$ in the range $1 \leq m \leq N-1$, define

$$
\Gamma_m = \sum_{k=1}^{N-m} \langle w_k, w_{k+m} \rangle,
$$

where $\langle u, v \rangle$ denotes the inner product of vectors $u$ and $v$. It follows from (3) that

$$
\Gamma_m = \sum_{k=1}^{N-m} \sum_{i=1}^{M} x_{i,k}^* x_{i,k+m} = \sum_{i=1}^{M} \sum_{k=1}^{N-m} x_{i,k}^* x_{i,k+m} = \sum_{i=1}^{M} C_i(m);
$$

therefore,

$$
\Gamma_m = 0, \quad 1 \leq m \leq N-1, \tag{5}
$$

is equivalent to (2) for each set of modulation symbols. We refer to (5) as the necessary and sufficient condition (NSC) for $X$ to be a complementary set. Because (5) is satisfied if

$$
\langle w_k, w_j \rangle = 0, \quad 1 \leq k < j \leq N, \tag{6}
$$

orthogonality of the columns of $X$ is a sufficient condition for $X$ to be a complementary set. Any complementary pair of length $N > 2$ demonstrates that (6) is not a necessary condition for a set to be complementary. We refer to the condition in (6) as the orthogonal-column condition (OCC).

Suppose the signal set $H_2$ consists of the modulation symbols $x_0 = (1, 1)$ and $x_1 = (1, -1)$, which are the rows of the Hadamard matrix $H_2$ of order 2 [13]. The columns of this matrix are orthogonal, so $H_2$ is a complementary set. Because there are only two signals, $H_2$ is pairwise complementary. The set $H_N$ that consists of all rows of a real Hadamard matrix $H_N$ of order $N$ with elements from the set $\{+1, -1\}$ is a set of $N$ orthogonal signals. Because the columns of a Hadamard matrix are also orthogonal [14], the set $H_N$ satisfies (6) and is therefore a complementary set. It follows that the matrix $-H_N$ also satisfies (6), so the set $H_N$ consists of the rows of $-H_N$ is a complementary set. This proves that the biorthogonal set $B_N = H_N \cup \overline{H_N}$ is a complementary set. For some sets of sequences, it is convenient to apply (5) or (6) to subsets of the original set. If $X$ is a set of sequences with matrix representation $X_m$, then let $\{X_m : 0 \leq m \leq J-1\}$ denote a partition of $X$ into $J$ disjoint subsets with corresponding matrix representations $X_m$, $0 \leq m \leq J-1$. Each matrix $X_m$ is a submatrix of the full matrix $X$. If each submatrix satisfies (5), then the full matrix satisfies (5), and the same is true for (6). As a simple example, consider the biorthogonal set $B_4 = H_2 \cup -H_2$. The two rows of $H_2$ are complementary, so the two rows of $-H_2$ are complementary, and therefore the biorthogonal set $B_4$ is pairwise complementary.

In general, we cannot say that $H_N$ is pairwise complementary, even though it is always a complementary set. For example, if a Hadamard matrix $H_N$ is in normal form [13], then each element of the first row is $+1$. If $N \geq 4$, then the first row cannot form a complementary pair with any other row. However, the Hadamard matrix

$$
H_4 = \begin{bmatrix}
-1 & +1 & +1 & +1 \\
+1 & -1 & +1 & +1 \\
+1 & +1 & -1 & +1 \\
+1 & +1 & +1 & -1
\end{bmatrix}, \tag{7}
$$

which is not in normal form, provides a pairwise-complementary signal set. If we partition $H_4$ into two submatrices, one of which consists of the first and third rows and the other consists of the second and fourth rows, then we see that each submatrix satisfies (5), the NSC, and therefore $\{x_1, x_3\}$ and $\{x_2, x_4\}$ give a partition into complementary pairs. Two applications of (5) show the sets $\{x_1, x_2\}$ and $\{x_3, x_4\}$ also consist of complementary pairs.

### III. BACKGROUND: COMPLEMENTARY SEQUENCES AND CCK MODULATION

The notion of a pair of complementary binary sequences was introduced by Golay [15]. Extensions to complementary sets with more than two binary sequences were provided by Tseng and Liu [16]. Complementary sets of nonbinary sequences were derived by Sivaswamy [17] and Frank [18]. For a review of the results obtained prior to 1980 on nonbinary complementary sequences, see [18]. Other sets and properties, including (5), the NSC, and (6), the OCC, are discussed in [19], but (5) is referred to as a sufficient condition.

The application of complementary sets of nonbinary sequences to CCK modulation for use in IEEE 802.11b is the subject of several articles including [20]–[27]. In particular, the description of the complementary set on which IEEE 802.11b CCK modulation is based is given in [20], discussions of the complementary properties of the IEEE 802.11b CCK signals are provided in [21] and [22], partitions of the IEEE 802.11b CCK signal set into sets of orthogonal signals are given in [26], and the weight distributions of the IEEE 802.11b CCK signal sets viewed as a linear block codes are presented in [25]. Some of the previous results on IEEE 802.11b CCK signal sets and their properties are presented without derivations. In the sections that follow, we present derivations of some key results on full-rate and half-rate CCK and full-rate and half-rate BOK signal sets. For other key results on IEEE 802.11b CCK, we provide alternative derivations to ones that are available in the literature.
The two high-rate signal sets for IEEE 802.11b [1] are the full-rate set (256-CCK), which has 256 modulation symbols of length eight and provides a data rate of 11 Mb/s, and the half-rate set (16-CCK), which has 16 modulation symbols of length eight and provides 5.5 Mb/s. Descriptions of these signals are given in [20]–[27]. If \( X \) denotes the set of 256 modulation symbols in the IEEE 802.11b full-rate signal set, then each \( x \in X \) is a vector of length eight from the complex alphabet \( A = \{ \exp(j\theta) : \theta \in \Theta \} \), where \( \Theta = \{0, \pi/2, \pi, 3\pi/2\} \). Each modulation symbol represents a unique sequence \((d_0, d_1, \ldots, d_7)\) of eight bits. The \( k \)th component of \( x \) is \( x_k = \exp(j \theta_k) \), where \( \theta_0 = \phi_1 + \phi_2 + \phi_3 + \phi_4, \theta_2 = \phi_1 + \phi_3 + \phi_4, \theta_3 = \phi_1 + \phi_2 + \phi_4, \theta_4 = \phi_1 + \phi_4 + \pi, \theta_5 = \phi_1 + \phi_2 + \phi_3, \theta_6 = \phi_1 + \phi_3 + \phi_4, \theta_7 = \phi_1 + \phi_2 + \pi, \) and \( \pi_0 = \phi_1 \). Note especially that the term \( \pi_0 \) is present in the expression for \( \theta_k \) for each \( k \). For each \( k \), \( 1 \leq k \leq 4 \), the IEEE 802.11b specification for the full-rate signal set requires that the phase \( \theta_k \) must depend only on bits \( d_{2k-2} \) and \( d_{2k-1} \), so the phases \( \phi_1, \phi_2, \phi_3, \) and \( \phi_4 \) are independent of each other. Unlike for the TIA-95 cellular CDMA modulation formats [2] or the JTIDS modulation format [4], a pseudo-random sequence is not applied to the data waveform in high-rate IEEE 802.11b systems, so \( x \) is transmitted without modification.

For each integer \( m \) in the range \( 0 \leq m \leq 3 \), let \( X_m \) be the set of all 64 modulation symbols for which \( \phi_1 = j m \pi / 2 \) and let \( Y_m \) be a 64 \( \times \) 8 matrix whose rows are the vectors in \( X_m \).

If \( X_m = \exp(j m \pi / 2) X_0 \) for \( 1 \leq m \leq 3 \), then the rows of \( X_m \) are the sequences from \( X_0 \). The proof that the columns of \( Y_m \) are orthogonal is given in [28]. It follows that the columns of \( X_m \) are orthogonal for each \( m \), which in turn implies orthogonality of the columns of the full 256 \( \times \) 8 signal matrix \( X \) and establishes that the signals in the full-rate set satisfy (6), the OCC. An alternative proof is given in the Appendix.

The four rows of \( Y_0 \) correspond to the four unique pairs \( \phi = (\phi_2, \phi_3) \), which represent the four different two-bit data vectors \( d = (d_2, d_3) \). Each row of \( Y_0 \) is of the form \( a = [\exp(j \varphi_1 \pi / 2), \exp(j \varphi_2 \pi / 2), \ldots, \exp(j \varphi_8 \pi / 2)] \), where \( \varphi_1 = \phi_2 + \phi_4, \varphi_2 = \phi_1 + \phi_4, \varphi_3 = \phi_3 + \phi_4, \varphi_4 = \phi_2 + \phi_4, \varphi_5 = \phi_2 + \phi_3, \varphi_6 = \phi_2 + \phi_3, \varphi_7 = \phi_2 + \phi_3, \pi_0 = \phi_2 + \pi, \) and \( \pi_0 = 0 \). If \( y_{1, k} \) denotes the element in the \( k \)th row and \( k \)th column of the matrix \( Y_0 \), then \( c_k = (y_{1, k}, y_{2, k}, y_{3, k}, y_{4, k}) \) is the \( k \)th column of \( Y_0 \). The arguments employed in [28] show that \( (c_k, c_n) = 0 \) for all \( k \) and \( (k \neq n) \) except when \( n = k + 2 \) and \( k \in \{1, 2, 5, 6\} \). This proves that \( \Gamma_m = 0 \) for \( m \neq 2 \). For \( m = 2 \) we observe that the phase differences for \( c_k \) and \( c_{k+2} \) with \( k \in \{1, 2, 5, 6\} \) are \( \psi_1 - \psi_3 = 0, \psi_2 - \psi_4 = \pi, \psi_5 - \psi_7 = \pi, \) and \( \psi_6 - \psi_8 = 0 \). It follows that \( (c_1, c_4) = 4, (c_2, c_4) = -4, (c_3, c_7) = -4, \) and \( (c_6, c_8) = 4 \). Therefore, the columns of \( A \) follow the columns of \( B \). For the full-rate CCK signal set, let \( G_0(\phi_2, \phi_3) = X_{0}(\phi_2, \phi_3, 0, 0) \) and \( X_{0}(0, \phi_2, \phi_3, 0) \) and \( G_0(\phi_2, \phi_3) \) be the corresponding matrix.

If we define the matrices \( A(\phi_2, \phi_3) \) and \( B(\phi_2, \phi_3) \) by

\[
A(\phi_2, \phi_3) = \begin{bmatrix}
e^{j(\phi_2+\phi_3)} & e^{j\phi_3} & e^{j\phi_2} & -1 \\
e^{j(\phi_2+\phi_3)} & -e^{j\phi_3} & -e^{j\phi_2} & 1 \\
\end{bmatrix}
\]

and

\[
B(\phi_2, \phi_3) = \begin{bmatrix}
e^{j(\phi_2+\phi_3)} & e^{j\phi_3} & e^{j\phi_2} & 1 \\
e^{j(\phi_2+\phi_3)} & -e^{j\phi_3} & -e^{j\phi_2} & 1 \\
\end{bmatrix}
\]

then we find that

\[
G_0(\phi_2, \phi_3) = [A(\phi_2, \phi_3) | B(\phi_2, \phi_3)].
\]

Let the eight column vectors of \( G_0(\phi_2, \phi_3) \) be denoted by \( g_k \) for \( 1 \leq k \leq 8 \) and observe that \( (g_k, g_n) = 0 \) for \( n \geq k + 4 \), which implies that \( \Gamma_m = 0 \) for \( m \geq 4 \). Simple calculations show that \( \Gamma_m = 0 \) for \( 1 \leq m \leq 3 \), so \( G_0(\phi_2, \phi_3) \) satisfies the NSC for each choice of \( \phi_2 \) and \( \phi_3 \). Similarly, if \( G_1(\phi_2, \phi_3) \) is the matrix that corresponds to \( G_1(\phi_2, \phi_3) = X_{0}(0, \phi_2, \phi_3, 0) \) and \( A_{0}(0, \phi_2, \phi_3, 0) \), then

\[
G_1(\phi_2, \phi_3) = [A(\phi_2, \phi_3) | B(\phi_2, \phi_3)].
\]

Again we observe that the columns satisfy \( (g_k, g_n) = 0 \) for \( n \geq k + 4 \) and straightforward computations show that \( \Gamma_m = 0 \) for \( 1 \leq m \leq 3 \), so \( G_1(\phi_2, \phi_3) \) also satisfies the NSC for each choice of \( \phi_2 \) and \( \phi_3 \). It follows that each of the matrices \( \exp(j\pi_0) G_0(\phi_2, \phi_3) \) and \( \exp(j\pi_0) G_1(\phi_2, \phi_3) \) satisfies the NSC for each
choice of \( \varphi_1, \varphi_2, \) and \( \varphi_3. \) Notice that the rows of \( \exp(j\varphi_1)G_0(\varphi_2, \varphi_3) \) are \( x(\varphi_1, \varphi_2, \varphi_3, 0) \) and \( x(\varphi_1, \varphi_2, \varphi_3, \pi), \) and the rows of \( \exp(j\varphi_1)G_1(\varphi_2, \varphi_3) \) are \( x(\varphi_1, \varphi_2, \varphi_3, \pi/2) \) and \( x(\varphi_1, \varphi_2, \varphi_3, 3\pi/2). \) Therefore, we have shown that for each choice of \( \varphi_1, \varphi_2, \) and \( \varphi_3, \) the modulation symbols \( x(\varphi_1, \varphi_2, \varphi_3, 0) \) and \( x(\varphi_1, \varphi_2, \varphi_3, \pi) \) are complementary pairs and \( x(\varphi_1, \varphi_2, \varphi_3, \pi/2) \) and \( x(\varphi_1, \varphi_2, \varphi_3, 3\pi/2) \) are complementary pairs. The sets

\[
U = \bigcup_{\varphi_1, \varphi_2, \varphi_3} X'(\varphi_1, \varphi_2, \varphi_3, 0) \cup X'(\varphi_1, \varphi_2, \varphi_3, \pi/2)
\]

and

\[
V = \bigcup_{\varphi_1, \varphi_2, \varphi_3} X'(\varphi_1, \varphi_2, \varphi_3, \pi) \cup X'(\varphi_1, \varphi_2, \varphi_3, 3\pi/2)
\]

give a partition that demonstrates the full-rate CCK signal set is pairwise complementary. In (12) and (13), the range for each of the variables \( \varphi_1, \varphi_2, \) and \( \varphi_3 \) is \( \Theta = \{0, \pi/2, \pi, 3\pi/2\}. \)

The corresponding result for the half-rate CCK signal set is a special case of the partition used for the full-rate CCK set that is obtained by setting \( \varphi_3 = 0 \) and allowing \( \varphi_1 \) to take only two values, 0 and \( \pi. \) The details are given in [28]. The pairwise-complementary orthogonal signal set given in Table I corresponds to a Hadamard matrix of order 8. The signal set can be partitioned into pairs of complementary signals in several ways, including \( \{x_1, x_2\}, \{x_3, x_4\}, \{x_5, x_6\}, \) and \( \{x_7, x_8\}. \) A half-rate pairwise-complementary biorthogonal signal set is obtained by combining the signals from Table I with the negatives of these signals.

VII. DISTRIBUTIONS OF INNER PRODUCTS

The distance distribution for a set \( \mathcal{Y} \) of equal-energy complex modulation symbols is

\[
\mathcal{S} = \{d_1, d_2, \ldots \}
\]

where the parameters \( d_1, d_2, \ldots \) are the distinct Euclidean distances in increasing order between pairs of modulation symbols in \( \mathcal{Y} \) and the integer \( n_i \) is the number of pairs of modulation symbols in \( \mathcal{Y} \) that are separated by distance \( d_i. \) Many signal sets, including all signal sets that we consider, have the property that the distribution of distances from one signal in the set is the same as the distribution of distances from any other signal in the set. For such a set, we consider the distances from an arbitrary signal in the set, and this gives \( n_1 + n_2 + \ldots = M - 1 \) for a set of \( M \) signals. The normalized real part of the inner product for modulation symbols \( x \) and \( y, \) each of which has energy \( \mathcal{E}, \) is \( r(x,y) = \text{Re}(\langle x, y \rangle) / \mathcal{E}, \) which is related to the Euclidean distance \( d(x,y) \) by \( d^2(x,y) = 2\mathcal{E}[1 - r(x,y)]. \) If the distance between two modulation symbols is large, then the real part of their inner product is small. If we replace \( d_i \) by \( r_i = 1 - |d_i^2/(2\mathcal{E})| \) for each \( i, \) then we obtain the distribution of the normalized inner products, which is written as \( \mathcal{I} = \{(r_1, n_1), (r_2, n_2), \ldots\}, \) with the normalized inner products \( r_i \) in decreasing order.

If maximum-likelihood coherent demodulation is employed and the only disturbance in the channel is thermal noise, then an important performance measure for signal set \( \mathcal{Y} \) is the minimum distance

\[
d_1 = \min \{d(x,y) : x \in \mathcal{Y}, y \in \mathcal{Y}, x \neq y\},
\]

which, for a fixed energy \( \mathcal{E}, \) should be as large as possible for good performance. It follows that

\[
r_1 = \max \{r(x,y) : x \in \mathcal{Y}, y \in \mathcal{Y}, x \neq y\}
\]

should be as small as possible for good performance. It is well known and easy to prove that

\[
r_1 \geq -\frac{1}{M-1}
\]

for any complex signal set of size \( M. \) Furthermore, the bound is tight, because the inner product for any pair of signals in a real simplex set of size \( M \) is \(-1/(M-1).\) For large signal sets, orthogonal signals are almost as good as simplex signals, achieving \( r_1 = 0 \) rather than \( r_1 = -1/(M-1).\) Biorthogonal signals also achieve \( r_1 = 0, \) and they have slightly better bandwidth efficiency and normalized-inner-product distribution than orthogonal signals.

A. Inner Products for Full-Rate Signals

A construction method and the inner-product distribution for \( M\text{-IQB} \) signal sets are given here for \( M = 2^{2m-1} \) for any positive integer \( m, \) which includes \( M = 256 \) as a special case. Begin with a signal set \( \mathcal{O}_0 \) that consists of the rows of a \( 2^m \times 2^m \) Hadamard matrix and then form \( \mathcal{O}_1 = \{-w : w \in \mathcal{O}_0\}. \) The set \( \mathcal{B} = \mathcal{O}_0 \cup \mathcal{O}_1 \) is a biorthogonal signal set with \( K = 2^{m+1} \) modulation symbols, each of which has \( K = 2^m \) chips per symbol. For \( m = 3, \) the set \( \mathcal{B} \) is the 16-B signal set.

If \( \mathcal{B} \) is used for both inphase and quadrature modulation, then the resulting set of \( M = 2^K \) equal-energy modulation symbols is \( \mathcal{Y} = \{u + jv : u \in \mathcal{B}, v \in \mathcal{B}\}. \) If \( \mathcal{E}_0 = ||w||^2 \) is the energy in each \( v \in \mathcal{O}_0, \) then the energy in each \( x \in \mathcal{Y} \) is \( \mathcal{E} = ||u + jv||^2 = ||u||^2 + ||v||^2 = 2\mathcal{E}_0. \) If \( x = u + jv \) and \( y = w + jz \) are two arbitrary vectors in \( \mathcal{Y}, \) then \( \text{Re}(\langle x, y \rangle) = \langle u, w \rangle + \langle v, z \rangle. \) If \( x = u + jv \) is an arbitrary modulation symbol in \( \mathcal{Y}, \) then the distribution of \( \langle x, y \rangle = \text{Re}(\langle x, y \rangle) \) for \( y \in \mathcal{Y} \) and \( y \neq x \) is given in Table II, where \( v \perp z \) means that \( v \) and \( z \) are orthogonal and \( n(y) \) is the number of vectors \( y \) in \( \mathcal{Y} \) that satisfy the conditions on \( y = w + jz \) listed in the first column.

### Table I

**A Pairwise-Complementary Orthogonal Signal Set**

| \( x_1 = (1, +1, +1, +1, +1, +1, +1, 1) \) | \( x_2 = (1, +1, +1, +1, +1, +1, +1, 1) \) | \( x_3 = (1, +1, +1, +1, +1, +1, +1, 1) \) | \( x_4 = (1, +1, +1, +1, +1, +1, +1, 1) \) | \( x_5 = (1, +1, +1, +1, +1, +1, +1, 1) \) | \( x_6 = (1, +1, +1, +1, +1, +1, +1, 1) \) | \( x_7 = (1, +1, +1, +1, +1, +1, +1, 1) \) | \( x_8 = (1, +1, +1, +1, +1, +1, +1, 1) \) |

### Table II

**Inner Products \( \rho(x,y) \) for a Fixed \( x = u + jv, K = \sqrt{M} \)**

<table>
<thead>
<tr>
<th>( y = w + jz )</th>
<th>( \rho(x,y) )</th>
<th>( n(y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( {w = u, z = -v} )</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( {w = u, z = -v} )</td>
<td>2(\mathcal{E}_0 )</td>
<td>0</td>
</tr>
<tr>
<td>( {w = u, z = -v} )</td>
<td>2(\mathcal{E}_0 )</td>
<td>0</td>
</tr>
</tbody>
</table>
If the inner products in Table II are specialized to $M = 256$, normalized, grouped, and listed in decreasing order, then we obtain the results listed in the first two rows of Table III. The third row gives the corresponding results for 256-CCK, which are obtained from the known weight distributions [25]. The entries for $n_B$ and $n_c$ are the numbers of modulation symbols $y$ for which $r(x, y) = \zeta$ for the 256-IQB set and the 256-CCK set, respectively. From Table III alone, it is difficult to say which signal set is superior, even if we restrict attention to channels in which the only disturbance is thermal noise. The maximum inner product is the same for the two sets, hence they also have the same minimum distance. Although approximately 11.0% of the pairs of 256-IQB signals have the maximum inner product compared to approximately 9.4% of the 256-CCK pairs, approximately 89.0% of the pairs of 256-IQB signals are orthogonal or better whereas only approximately 84.3% of the 256-CCK pairs are orthogonal or better. Because of the similarities in the distributions of their inner products, the symbol and frame error probabilities should be approximately the same for the two signal sets if they are used on an AWGN channel, which is verified Section VIII. However, this does not imply that their bit error probabilities are approximately the same or, if they are used with error-control coding, that their frame error probabilities are approximately the same.

### B. Inner Products for Half-Rate Signals

For the half-rate CCK signal set $\mathcal{Y}$, recall from Section V that $\phi_0 = 0$ for each signal and $\mathcal{Y}_0$ is the set of all four modulation symbols in $\mathcal{Y}$ for which $\phi_0 = 0$. Let $x$ be the modulation symbol in $\mathcal{Y}_0$ that corresponds to $\phi_0 = \pi/2$ and $\phi_0 = 0$, and let $y \neq x$ be the modulation symbol in $\mathcal{Y}$ that corresponds to $\phi_1 = \zeta_1$, $\phi_2 = \zeta_2$, and $\phi_4 = \zeta_4$. Because $y$ is a half-rate CCK signal, $\zeta_2 \in \{\pi/2, 3\pi/2\}$ and $\zeta_4 \in \{0, \pi\}$. First observe that if $y = x \exp(j\zeta_1)$, then $r(x, y) = 0$ for $\zeta_1 = \pi/2$ and $\zeta_1 = 3\pi/2$ and $r(x, y) = -1$ for $\zeta_1 = \pi$. Thus, contrary to a statement in [24], the half-rate CCK signals are not orthogonal to each other.

Next we use the fact that if $y \neq x \exp(j\zeta_1)$, then there exists a unique modulation symbol $u \in \mathcal{Y}_0$ for which $u \neq x$ and $y = u \exp(j\zeta_1)$. We do not exclude the possibility that $y = u$, which corresponds to $\zeta_1 = 0$. Next, we use the fact that $x \in \mathcal{Y}_0$, $u \in \mathcal{Y}_0$, and $u \neq x$ imply that $\zeta_2, \zeta_4 \neq (\pi/2, 0)$. Clearly, $(x, y) = (x, u) \exp(-j\zeta_1)$. It is easy to show that $\langle x, u \rangle = 2[1+\cos(\zeta_4)+\cos(\zeta_2-\pi/2)+\cos(\zeta_4+\zeta_2-\pi/2)]$, from which it follows that $\langle x, u \rangle = 0$, because $(\zeta_2, \zeta_4) \notin \{(\pi/2, \pi), (3\pi/2, 0), (3\pi/2, \pi)\}$. This shows that $r(x, y) = 0$ if $y \neq x \exp(j\zeta_1)$. Hence, only one choice of $y$ gives $r(x, y) = -1$ and all others give $r(x, y) = 0$.

Because of symmetry, we conclude that for an arbitrary half-rate CCK modulation symbol $x$, the distribution of the normalized inner-products $r(x, y)$ is $\mathcal{I} = \{(0, 1), [1, -1]\}$, which agrees with the weight distribution given in [25]. This is the same as the normalized inner-product distribution for the standard binary biorthogonal signal set with 16 signals and 8 chips per signal. Thus, for both the half-rate CCK set and the 16-B set, any two signals in the set are either antipodal or orthogonal, and there are 14 times as many orthogonal pairs as antipodal pairs.

### VIII. ERROR PROBABILITIES FOR AWGN CHANNELS

Although all our channel models include thermal noise, which is modeled as additive white Gaussian noise, we reserve the phrase AWGN channel for a channel in which thermal noise is the only disturbance. For an AWGN channel and a system with BOK modulation and no error-control coding, the exact probability of frame error can be computed from the expression for the BOK symbol error probability. No analytical method is available for IEEE 802.11b CCK modulation, so simulation is employed. However, we show that the analytical result for BOK modulation is an accurate approximation for the frame error probability for full-rate IEEE 802.11b CCK transmission on the AWGN channel and it is exact for half-rate IEEE 802.11b CCK transmission. For channels with multipath or systems with error-control coding (especially those with iterative decoding), the available analytical approximations and bounds are not sufficiently accurate for our purposes, so simulations are required.

Let $\mathcal{B}$ be a set of $K = 2N$ biorthogonal signals, each of which has energy $E_r = E_1/2$. Let $\mathcal{Y}$ consist of all signals whose inphase and quadrature components are from $\mathcal{B}$, so $\mathcal{Y}$ is an $M$-IQB signal set with $M = K^2 = 4N^2$ signals, each of which has energy $E_1$. For transmission over an AWGN channel, let $P_{e,1}(E_1)$ be the probability of error for an $M$-ary modulation symbol from set $\mathcal{Y}$ and let $P_{e,2}(E_2)$ be the probability of error for a $K$-ary modulation symbol from set $\mathcal{B}$. Because an error occurs for an IQB symbol if and only if an error occurs for at least one of the biorthogonal component symbols, the error probabilities are related by

$$P_{e,1}(E_1) = 1 - |1 - P_{e,2}(E_2)|^2. \quad (17)$$

The symbol error probability for the $K$-biorthogonal signal set $\mathcal{B}$ is

$$P_{e,2}(E_2) = 1 - \int_{-\beta}^{\beta} [2\Phi(x+\beta) - 1]^{N-1} \frac{\exp(-x^2/2)}{\sqrt{2\pi}} \, dx, \quad (18)$$

where $N = K/2$, $\beta = \sqrt{2\Sigma_2/N_0}$, $N_0$ is the one-sided power spectral density for the thermal noise, and $\Phi$ is the standard Gaussian distribution function. For the $M$-IQB set and the $K$-biorthogonal set, the bit error probability is

$$P_b = \frac{1}{2} P_{e,2}(E_2) + \frac{1}{2} \int_{-\beta}^{\beta} [1 - 2\Phi(x+\beta)]^{N-1} \frac{\exp(-x^2/2)}{\sqrt{2\pi}} \, dx. \quad (19)$$

Equations (18) and (19) can be written in terms of $E_b$, the energy per bit, by use of $E_b = E_r / (\log_2 K) = E_1 / (\log_2 M)$.

If a frame comprises a sequence of $L_2$ BOK modulation symbols, then the probability of frame error when $K$-biorthogonal modulation is used on the AWGN channel is

$$P_{f,2}(E_2) = 1 - |1 - P_{e,2}(E_2)|^{L_2}. \quad (20)$$

Similarly, if a frame consists of $L_1$ modulation symbols from the $M$-IQB set, then the probability of frame error for the AWGN channel is

### TABLE III

INNER PRODUCT DISTRIBUTIONS FOR 256-IQB ($n_B$) AND 256-CCK ($n_c$)

<table>
<thead>
<tr>
<th>$\zeta$</th>
<th>$1/2$</th>
<th>$1/4$</th>
<th>$-1/4$</th>
<th>$-1/2$</th>
<th>$-1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_B$</td>
<td>28</td>
<td>0</td>
<td>198</td>
<td>0</td>
<td>28</td>
</tr>
<tr>
<td>$n_c$</td>
<td>24</td>
<td>16</td>
<td>174</td>
<td>16</td>
<td>24</td>
</tr>
</tbody>
</table>
As shown in Section VII-B, 16-CCK and 16-B have the same normalized inner-product distribution; therefore, for the same energy per symbol, the two signal sets have the same symbol error probability when used with coherent maximum-likelihood demodulation on the same AWGN channel. Thus, for the AWGN channel, the exact symbol and frame error probabilities for 16-CCK are obtained from (18) and (20) if \( N = 8 \) in (18).

Without error-control coding, the energy per information bit for an equal-energy signal set of size \( M \) is \( E_b = E / \log_2(M) \), where \( E \) is the energy per modulation symbol. The ratio of the energy per information bit to the noise density in decibels (dB) is \( \text{ENR} = 10 \log_{10}(E_b/N_0) \). Equations (17), (18), (20) and (21) provide analytical expressions for the probability of frame error as a function of the energy per modulation symbol, but it is convenient to present the performance results in terms of ENR. In Table IV, we present the values of ENR that are required to provide the specified values of the probability of frame error \( P_f \) for an AWGN channel. For the results in Table IV, the modulation formats 256-IQB and 256-CCK are employed in systems with no error-control coding and each frame consists of 4096 information bits. As expected, the simulation results show that 256-IQB and 256-CCK have nearly the same frame error probabilities. The difference in their required values of ENR is less than 0.05 dB for each of the three values of the frame error probability. The accuracy of our simulations is verified by comparing the results for 256-IQB in the last two columns of Table IV, where we see that the analytical and simulation results differ by less than 0.02 dB for each frame error probability.

For long-range outdoor communications, error-control coding may be required for satisfactory performance. To see how much can be gained and how CCK compares with BOK when they are used with binary error-control coding, we evaluate the performance of bit-interleaved coded modulation. At the transmitter, the information bits are encoded with a binary code and the binary code symbols are interleaved, grouped into sets of size \( m \), and mapped into modulation symbols from a set of size \( M = 2^m \). At the receiver, the \( M \)-ary modulation symbols are demodulated, and the resulting binary symbols are deinterleaved and decoded. As before, each frame represents 4096 binary symbols; however, the codes have rate approximately 1/2, so encoded frames carry approximately half as many information bits as the frames with no error-control coding. For full-rate modulation, each encoded frame is a sequence 512 modulation symbols, and each encoded frame has 1024 modulation symbols for half-rate modulation.

Performance results are given for the NASA-standard convolutional code of constraint length \( K = 7 \) and a turbo product code (TPC). The NASA-standard convolutional code is also in the IEEE 802.11a standard [1]. Its generator polynomials in octal are \((171, 133)\), six tail bits are used, and its actual rate is 0.4985. The TPC is the three-dimensional product code \((32,26) \times (32,26) \times (4,3)\) whose constituent codes are two Hamming codes and a parity-check code, so its block length is 4096 and its rate is approximately 0.495. Soft-decision Viterbi decoding is used for the convolutional code, and soft-decision iterative (turbo) decoding with available hardware [29] is employed for the product code. The decoder for each code uses the dual-max metric, which is similar to a nonparametric metric that was originally proposed for cellular CDMA [30]. If the demodulator output vector is \( z = (z_0, z_1, \ldots, z_{m-1}) \), then the dual-max metric for \( b_k \), the \( k \)-th bit represented by the \( M \)-ary symbol, is \( D(b_k) = c(v_{0,k} - v_{1,k}) \), where \( v_{j,k} = \max\{z_m : m \in S_j,k\} \) and \( S_j,k \) is the set of \( M/2 \) symbols for which the \( k \)-th bit is \( j \). The parameter \( c \) is a scale factor that is chosen to match the modulation format and the soft-decision decoder that are used for the packet being received. For the Viterbi decoder, we use \( c = 1 \). For the TPC decoder, we use \( c = 2.5 \) for CCK modulation and \( c = 1 \) for BOK modulation. For BOK modulation and frame error probabilities down to \( 10^{-3} \), we compared the dual-max metric with the log-likelihood ratio (LLR) metric and found the performance difference to be less than 0.2 dB for the AWGN channel. This difference is too small to justify the additional complexity of the LLR metric; furthermore, it appears that the LLR metric is not known for IEEE 802.11b 256-CCK.

The performance results for 256-IQB and 256-CCK when used with error-control coding on an AWGN channel are summarized in Table V. For each value of the frame error probability, 256-IQB modulation has a performance advantage of 2.2 dB over 256-CCK if the convolutional code is used, and the performance advantage of 256-IQB over 256-CCK is 1.7 dB for the TPC. We found that for each code, 16-biorthogonal modulation gives even larger performance gains over 16-CCK, so both forms of BOK are superior to IEEE 802.11b CCK when either of the two binary codes is used on an AWGN channel.

Some insight into the advantages of BOK over IEEE 802.11b CCK for binary coding can be obtained by considering the bit error probabilities for the two modulation formats when error-control coding is not used. The values of ENR required for a frame error probability of \( 10^{-1} \) when the modulation formats are used with the TPC are roughly the same as the values of ENR required for a bit error probability of \( 10^{-1} \) if there is no error-control coding. In this range, the value of ENR required by IEEE 802.11b 256-CCK is approximately 1.1 dB larger than the value of ENR required by 256-IQB. There are two reasons that uncoded 256-CCK has a larger bit error probability than uncoded 256-IQB, which we observed to be true for each value of ENR.

### Table IV

<table>
<thead>
<tr>
<th>ENR Requirements for Systems with No Error-Control Coding</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_f )</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>( 10^{-1} )</td>
</tr>
<tr>
<td>( 10^{-2} )</td>
</tr>
<tr>
<td>( 10^{-3} )</td>
</tr>
</tbody>
</table>

### Table V

<table>
<thead>
<tr>
<th>ENR Requirements for Systems with Error-Control Coding (AWGN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_f )</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>( 10^{-1} )</td>
</tr>
<tr>
<td>( 10^{-2} )</td>
</tr>
<tr>
<td>( 10^{-3} )</td>
</tr>
</tbody>
</table>
that we checked. First, for each pair of antipodal symbols in the biorthogonal set, the assigned bit sequences disagree in each position [31], which is not true for IEEE 802.11b CCK. As described in Section IV, antipodal CCK symbols differ only through the phase $\varphi_C$. As described in Section IV, antipodal CCK symbols differ only through the phase $\varphi_C$, so their bit assignments differ in only two bit positions, $d_0$ and $d_1$. Thus, for IEEE 802.11b CCK, the least likely symbol errors do not correspond to the largest number of bit errors. On the other hand, the pairs of signals with the highest correlations (i.e., largest inner products) should disagree in as few bit positions as possible, which is also not true for IEEE 802.11b CCK. For example, among the pairs of symbols with a positive inner product, the average number of bit disagreements is 3.75 for the 256-CCK symbols compared with only 2 for the 256-IQB symbols. Second, the differential encoding of $\varphi_1$ in IEEE 802.11b CCK causes additional bit errors beyond those that are a direct consequence of an erroneous decision for a modulation symbol. An erroneous symbol decision may lead to an erroneous value for $\varphi_1$, which produces one or two additional bit errors. The combination of the direct and indirect consequences of a single erroneous symbol decision may give several bit errors. For example, even though there are four bits per 16-CCK symbol, one 16-CCK symbol error can cause up to six bit errors as a result of differential decoding between the adjacent symbols. For binary codes, we believe that the poorer performance of IEEE 802.11b CCK is at least partly a consequence of the bit assignments and differential encoding that are required by IEEE 802.11b. For applications of CCK that do not require compliance with IEEE 802.11b, other bit assignments are preferable.

IX. ERROR PROBABILITIES FOR SPECULAR MULTIPATH CHANNELS

To simplify the presentation of the results for multipath channels, we focus on a two-path specular multipath channel and a receiver that demodulates only the component that arrives first, which we refer to as the desired component. The other component, the interference component, arrives $\tau$ chips later and with a phase difference $\xi$ relative to the desired component. The time $\tau$ between the arrivals of the two components is the excess delay for the multipath channel. For the modulation formats and receivers that we are considering, we believe that two-path specular multipath represents at least approximately the worst multipath profile for a given total multipath interference power, provided that the excess delay and relative phase of the interference component are chosen to give the maximum performance degradation. In this approach, a channel with multiple multipath interference components is replaced by a two-path specular multipath channel by allocating the total interference power in the original channel to a single interference component with the worst excess delay and relative phase. We also give performance results in which the error probabilities are averaged over ranges of excess delays and phase angles. The suggestion that the two-path model may be the worst case in most instances is also made in [32], and the two-path specular multipath model is used in [10] to obtain experimental results for outdoor use of IEEE 802.11b systems in suburban and rural areas.

For $0 < \tau < 8$, there are two components of the multipath interference. One is intrasymbol interference that depends on the aperiodic autocorrelation functions for the modulation symbols, and the other is intersymbol interference that depends on the aperiodic crosscorrelation functions [12] for adjacent modulation symbols in the transmitted signal. The maximum interference occurs for excess delays that are multiples of the chip duration (i.e., positive integer values for $\tau$). For $\tau = 1$ or $\tau = 2$, it can be argued that the influence of the autocorrelation functions dominates the performance. As described in Sections II–VI, the IEEE 802.11b CCK signal set has strong complementary properties that are derived from the aperiodic autocorrelation functions, so CCK performs well for $\tau \leq 2$. For larger values of $\tau$, the performance is not determined primarily by the complementary properties of the signal set. In fact, for $\tau \geq 8$ the multipath interference is determined completely by the aperiodic crosscorrelation functions for pairs of modulation symbols.

Various forms of equalization have been proposed for IEEE 802.11b systems (e.g., [9], [26], [27], and [33]–[35]). For long-range communications, the signal-to-noise ratio may be inadequate for some types of equalizers, such as a decision-feedback equalizer (DFE). If error-control coding is used to increase the communications range, then soft-decision decoding should be employed, but a DFE does not work well with a separate soft-decision decoder [27]. In addition, although equalization can be effective in mitigating intersymbol interference caused by multipath with small to moderate excess delays, the complexity required to handle multipath with large excess delays may be prohibitive. If combined decoding and equalization is required, the complexity is even greater. The modulation chip duration for IEEE 802.11b CCK is approximately 0.091 $\mu$s, and the modulation symbol duration is approximately 0.73 $\mu$s. For indoor communications in very large factories or for outdoor communications, the excess delay is likely to be greater than the duration of two chips and it may exceed the CCK symbol duration ($\tau > 8$). For outdoor applications in urban areas, excess delays of 1–2 $\mu$s are common and they can be in the range 10–20 $\mu$s or more in some areas (e.g., [36]). Excess delays up to 25 $\mu$s were observed in cellular CDMA tests in Geneva, Switzerland [37]. For IEEE 802.11b CCK, the range 10–25 $\mu$s corresponds to approximately 14–34 CCK symbols (110–275 chips), which exceeds the limits of effective operation for many equalizers. Even if an equalizer is used, we believe that a signature sequence should be employed to mitigate the effects of multipath components with large excess delays, and only negligible complexity is added by doing so. In fact, the additional complexity required to incorporate a signature sequence is also considerably less than the complexity required to include a rake receiver in the system.

For our performance results, each BOK signal set employs a random signature sequence with one sequence chip per modulation chip as described in [38]. For CCK modulation, the receiver’s performance is sensitive to the excess delay. The application of a pseudo-random sequence reduces this sensitivity, and an ideal random sequence makes the performance the same for all excess delays that are not less than the chip duration (i.e., for $\tau \geq 1$). The chip rate for the
signature sequence is identical to the chip rate of the modulation waveform and the chips are aligned, so the application of the signature sequence does not increase the bandwidth. The ratio of the bit energy to the thermal noise density is $\text{ENR} = 13 \text{ dB}$ for all performance results for the multipath channel, and the soft-decision metric is the dual-max metric defined in Section VIII. A log-likelihood ratio (LLR) metric for a multipath channel is difficult to derive (e.g., the multipath interference is not Gaussian) and too complex to implement in a practical system because the signal strengths, arrival times, and phase angles of the multipath components would have to be known accurately. An LLR metric that is designed for an AWGN channel can be used for the multipath channel; however, the dual-max metric is much easier to implement and it performs nearly as well. For orthogonal modulation and a multiple-access channel, it is shown in [38] that the performance of the dual-max metric is within 0.2 dB of the performance of the LLR metric designed for an AWGN channel. Specular multipath interference is nearly the same as multiple-access interference, so we believe the maximum difference of 0.2 dB is also accurate for orthogonal modulation and a multipath channel. Our experience suggests that the performance difference is about the same if the orthogonal modulation is replaced with BOK modulation or with quasi-orthogonal modulation such as CCK or cyclic code-shift key (CCSK) modulation [38].

If the power in the desired component is $P$ and the power in the interference component is $I$, then the signal-to-interference ratio in dB is $\text{SIR} = 10\log_{10}(P/I)$. For two-path specular multipath channels, we define maxSIR to be the maximum value of SIR required to achieve a given frame error probability, where the maximum is over all relative phases $\xi$ and all excess delays $\tau \geq 1$. Results on maxSIR are given for the convolutional code and the TPC in Fig. 1 for IEEE 802.11b 256-CCK; DS 256-CCK, which is 256-CCK with a random signature sequence; and DS 256-IQB, which is 256-IQB with a random signature sequence. For the same codes and modulation formats, the average frame error probability is given as a function of SIR for random relative phases and excess delays in Fig. 2. For these results, $\xi$ is uniformly distributed on $[0, 2\pi]$ and $\tau$ is uniformly distributed on the integers from $8n$ to $8(n+m)$ for arbitrary positive integers $n$ and $m$. Although Fig. 1 illustrates performance that is guaranteed for all $\xi$ and all $\tau \geq 1$, Fig. 2 gives frame error probabilities that are averaged over the values of $\xi$ and $\tau$. If a 256-CCK system with binary error-control coding may encounter multipath channels with large excess delays, then the results in Fig. 1 and Fig. 2 imply that it is advantageous to apply a signature sequence to the modulation.

The addition of a signature sequence to 256-CCK improves performance for either form of error-control coding. For example, for a frame error probability of $10^{-2}$, the improvement in maxSIR due to the signature sequence is more than 0.7 dB for the TPC and more than 1.1 dB for the convolutional code. The amount of each improvement is approximately the same for a frame error probability of $10^{-1}$ as well as for a frame error probability of $10^{-3}$. Even larger performance improvements are obtained by changing to DS 256-IQB modulation, which, for a frame error probability of $10^{-2}$, is almost 2.5 dB better than IEEE 802.11b 256-CCK for the TPC and more than 3.7 dB better than IEEE 802.11b 256-CCK for the convolutional code. The amount of each improvement is approximately the same if the desired frame error probability is $10^{-1}$, and it is slightly larger if the desired frame error probability is $10^{-3}$. The performance comparisons among the corresponding half-rate modulation formats are approximately the same as for full-rate modulation, except that the performance improvement due to the addition of a signature sequence to 16-CCK is approximately 3.7 dB for both codes, which is even larger than the improvement for 256-CCK.

X. Conclusion

We have shown that the signal sets for IEEE 802.11b full-rate and half-rate CCK modulation are not only complementary sets, but they each satisfy the NSC, which is given by (5), and they are pairwise complementary. In addition, the full-rate set satisfies the OCC, which is given by (6). Our
results on the inner product distributions and our analytical and simulation results are in agreement: The frame error probabilities for uncoded 256-QOB modulation and uncoded 256-CCK are nearly the same, even though we found that 256-QOB modulation gives smaller bit error probabilities. When binary error-control codes are employed on an AWGN channel, the two forms of BOK modulation are superior to their CCK counterparts. If binary error-control coding is employed on a two-component specular multipath channel, the addition of a signature sequence to CCK modulation improves the protection against multipath interference, even for very large excess delays that cannot be handled by equalizers of reasonable complexity.

APPENDIX

An alternative proof that the full-rate CCK signal set satisfies (6), the OCC, is obtained by showing that the full-rate CCK signal set can be partitioned into 32 subsets, each of which has eight mutually orthogonal signals. In [24] it is stated that there are eight sets of 32 mutually orthogonal signals, which is incorrect. Let $\Theta_0 = \{0, \pi\}$ and $\Theta_1 = \{\pi/2, 3\pi/2\}$, so that $\Theta = \Theta_0 \cup \Theta_1$. For $0 \leq n \leq 7$, we define

$$S_n = \bigcup_{\Omega_1 \in \Theta_1} \bigcup_{\Omega_0 \in \Theta_0} \bigcup_{\varphi \in \varphi_n} \mathcal{X}(0, \varphi_2, \varphi_3, \varphi_4),$$

(22)

where $\Omega_1 \Omega_0 \varphi$ is the binary representation of the integer $n$; that is, $i_2$, $i_1$, and $i_0$ are elements of $\{0, 1\}$ that satisfy $n = 4i_2 + 2i_1 + i_0$. The union of $S_n$ over $0 \leq n \leq 7$ is the set of all full-rate CCK signals for which $\varphi_1 = 0$. From (22), we see that the set $S_0 = \{s_i : 0 \leq i \leq 7\}$ of the signals listed in Table VI, which it is easy to verify, consists of $S_0$ by multiplying the appropriate columns by $exp(\pi j/2)$. Multiplication of each element of the $k$th column by $exp(\theta)$ results in multiplication of the $k$th term in the expression for the inner product of any pair of rows by $exp(\theta) exp(-\theta) = 1$, so the inner products are not changed and the orthogonality is preserved. The definition of the partition $\{S_i : 0 \leq i \leq 31\}$ is completed by defining $S_{31+3m} = exp(\pi m/2)S_m$, for $1 \leq m \leq 3$ and $0 \leq n \leq 7$, to account for the four possible values of $\varphi_1$. Let $S$ be the matrix corresponding to the set

$$S = \bigcup_{i=0}^{31} S_i,$$

(23)

which is the entire set of full-rate CCK signals. Each $S_i$ is a square matrix with orthogonal rows, so $S_i$ has orthogonal columns and the full matrix $S$ also has orthogonal columns. Other partitions into sets of eight orthogonal signals can be found in [26].

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