Preferential Mode of gas invasion in sediments: Grain-scale mechanistic model of coupled multiphase fluid flow and sediment mechanics

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Preferential Mode of gas invasion in sediments: 
Grain-scale mechanistic model of coupled multiphase fluid flow and sediment mechanics

A. K. Jain¹ and R. Juanes¹

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[1] We present a discrete element model for simulating, at the grain scale, gas migration in brine-saturated deformable media. We rigorously account for the presence of two fluids in the pore space by incorporating forces on grains due to pore fluid pressures and surface tension between fluids. This model, which couples multiphase fluid flow with sediment mechanics, permits investigation of the upward migration of gas through a brine-filled sediment column. We elucidate the ways in which gas migration may take place: (1) by capillary invasion in a rigid-like medium and (2) by initiation and propagation of a fracture. We find that grain size is the main factor controlling the mode of gas transport in the sediment, and we show that coarse-grain sediments favor capillary invasion, whereas fracturing dominates in fine-grain media. The results have important implications for understanding vent sites and pockmarks in the ocean floor, deep subsediment storage of carbon dioxide, and gas hydrate accumulations in ocean sediments and permafrost regions. Our results predict that in fine sediments, hydrate will likely form in veins following a fracture network pattern, and the hydrate concentration will likely be quite low. In coarse sediments, the buoyant methane gas is likely to invade the pore space more uniformly, in a process akin to invasion percolation, and the overall pore occupancy is likely to be much higher than for a fracture-dominated regime. These implications are consistent with laboratory experiments and field observations of methane hydrates in natural systems.


1. Introduction

[2] Gas migration through water-filled soft sediment is an essential component of seafloor dynamics. It governs, for instance, the spatiotemporal characteristics of natural gas seeps and vent sites [Judd et al., 2002; Heeschen et al., 2003; Best et al., 2006], the biochemical processes in the shallow subsurface as well as the ocean floor [Suess et al., 1999], the mechanical and acoustic properties of submarine sediments [Anderson and Hampton, 1980; Anderson et al., 1998; Waite et al., 2008], the creation of pockmarks in the ocean floor [Hovland et al., 2002; Säling et al., 2008], and the accumulation of gas hydrate (notably methane) in ocean sediments. Understanding gas transport in soft sediments is also key to assessing the viability of carbon dioxide sequestration in the subsurface, either by hydrate formation [Koide et al., 1995, 1997b] or gravitational trapping [Koide et al., 1997a; House et al., 2006; Levine et al., 2007; Goldberg et al., 2008].

[3] Methane hydrates (crystalline ice-like compounds composed of methane molecules caged in a lattice of water molecules [Sloan, 1998]) form naturally at high pressures and low temperatures, like those typical of most of the ocean floor. It is believed that an enormous pool of carbon exists in the form of methane gas and methane hydrate in oceanic sediment along the continental margins [Kvenvolden, 1988; Sloan, 2003], although the global estimates of the energy resource are highly uncertain. It also seems likely that this carbon pool plays an important role in massive submarine landslides [Paull et al., 2003] and in the global carbon cycle [Dickens, 2003]; its sudden or gradual release has been hypothesized to be the cause of past and future climate change [Dickens et al., 1995; Buffett and Archer, 2004; Archer et al., 2009].

[4] Methane hydrate systems in ocean sediments have been the subject of intense research in recent years. A significant component of that effort is directed toward gaining a better conceptual picture of the hydrogeological environment of gas hydrate systems. Particular attention has been devoted to the two end-members [Trehu et al., 2006b]: (1) The hydrogeologically more active, dynamic end-member, exemplified by Hydrate Ridge, offshore Oregon [Suess et al., 1999; Tryon et al., 2002; Heeschen et al., 2003; Trehu et al., 2004b; Weinberger et al., 2005] and

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2. Theory, Formulation, and Methods

[10] The discrete element method (DEM) [Cundall and Strack, 1979] has proved a valuable tool to study the mechanisms for deformation and failure of granular materials with variable degree of cementation [Bruno and Nelson, 1991]. Moreover, based on simple geometric arguments, stress variations (and subsequent deformation) have been shown to affect flow properties such as porosity and permeability [Bruno, 1994].

[11] Each element or grain is identified separately by its own mass, moment of inertia and contact properties. For each grain, its translational and rotational movements are described by solving Newton’s second law of motion. The mechanical behavior at the deformation region of each grain contact is approximated by introducing a grain contact model, such as a system of a spring, dashpot and slider (Figure 1).

2.1. Micromechanics of “Dry” Media

[12] The movement of a grain is dictated by the net force and moment acting on it. For a dry model, that is, one in which pore pressures are negligible, the forces for each
contact bonds. Inelasticity is reflected by the constraints

\[ F_n = k_n U_n, \quad \Delta F^s = -k_s \Delta U_s, \]  

where \( U_n \) is the overlap, \( \Delta U_s \) is the tangential displacement, and \( k_n \) and \( k_s \) are the normal and shear stiffness at the contact, respectively [Itasca Consulting Group, Inc. (Itasca), 2004; Potyondy and Cundall, 2004]. Inelastic behavior emerges due to either slip between grains, or breakage of contact bonds. Inelasticity is reflected by the constraints

\[ F^s \leq \mu F_n, \quad F^s \leq \varphi_n, \quad F^s \leq \varphi_s, \]  

where \( \mu \) is the contact friction coefficient and \( \varphi_n \) and \( \varphi_s \) are the normal and shear contact strengths (in force units) of the contact.

[14] Bulk behavior of a granular system is a collective response determined by all the individual grain-grain interactions. For dry sample analyses, the interparticle interactions can be associated with a network of grain-grain contact forces connecting the centroids of grains that are in contact.

[15] Given the set of forces \( \mathbf{F}_j \) and moments \( \mathbf{M}_j \) acting on the \( i \)th particle, its motion is described by the following:

\[ m_i \ddot{x}_i = \sum_j \mathbf{F}_j \]  

\[ I_i \ddot{\theta}_i = \sum_j \mathbf{M}_j. \]  

Here, \( \mathbf{x}_i \) and \( \mathbf{\theta}_i \) are the position vector of the grain centroid and the angle vector of rotation about the centroid; the double dots denote second time derivatives of the position and rotation angle; \( m_i \) is the mass; and \( I_i \) is the tensor of moments of inertia, respectively. The equations of motion (3–4) must be solved simultaneously for all grains in the system via a numerical integration scheme. A commercial three-dimensional DEM code, PFC2D [Itasca, 2004], was used to integrate these equations in time.

### 2.1.1. Micromechanical Versus Macroscopic Parameters

[16] The parameters that need to be defined at the grain-scale level are \( \rho_i \) (grain density), \( \mu, k_n, k_s, \varphi_n \) and \( \varphi_s \), as well as the grain size distribution, which we shall characterize simply by the grain radius interval \( r_{\text{min}} \leq r_g \leq r_{\text{max}} \).

[17] From DEM simulations of biaxial tests, the linear elastic macroscopic parameters (Young modulus \( \varepsilon \) and Poisson ratio \( \nu \), as well as strength properties (yield stress \( \sigma_y \), friction angle \( \varphi \), cohesion \( c \), etc.) may be computed. In order to obtain macroscopic parameters that are independent (or only slightly dependent) on the grain size, the contact strengths must scale with the grain size [Potyondy and Cundall, 2004]:

\[ \varphi_n = \sigma_c 2 \varphi w, \quad \varphi_s = \sigma_s 2 \varphi w, \]  

where \( \sigma_c \) and \( \sigma_s \) are the normal and shear contact strengths (in stress units), assumed to be independent of grain size, and \( w \) is the width of the 2-D assembly in the third dimension.

### 2.1.2. Time Step Selection for Mechanics Simulation

[18] Since explicit time integration is used, the time step is bounded by stability considerations. The characteristic time required to capture the dynamics is [Itasca, 2004]

\[ \delta t \sim \sqrt{m/k_n}, \]  

where \( m \) is the mass of the particle. In PFC2D, grains are assumed to be disks of unit width \( (w = 1 \text{ m}) \), so \( m = 2\pi r_g^2 \rho_g \) and, therefore, the critical time step for mechanical stability scales as follows:

\[ \delta t_{\text{crit}} \sim r_g \sqrt{\rho_g/k_n}. \]  

### 2.2. Microporomechanics of Single-Fluid Systems

[19] From the theory of poromechanics [Biot, 1941], it is well known that pore pressure will influence mechanical behavior. Essentially, compressive stresses in granular media are transmitted both through a solid skeleton and the pore fluids. Recently, models have been developed to incorporate this effect in DEM with a single-phase pore fluid [Shimizu, 2004; Cook et al., 2004; Li and Holt, 2001, 2004].

[20] When the pore space is filled with a single fluid phase at nonnegligible pressure, the associated forces must be incorporated in the model. A conceptual view of the new set of forces is shown in Figure 2. Computationally, the model then consists of two overlapping and interacting networks: the grain network and the fluid network. A particular instance is shown in Figure 3. The force fluid in a given domain exerts on a neighboring grain is obtained by integrating the pressure along the pore-grain contact area. In our implementation, a pressure force is directed...
from the center of the fluid domain to the grain center. Therefore, pressure forces do not induce rotation.

Consider one particular fluid domain, as sketched in Figure 4. The micromechanical equations can be summarized as follows. The flow rate out of the fluid domain through a pore throat is

\[ q_j = C_j \frac{p_j - p}{L_j}, \quad (8) \]

where \( C_j \) is the throat conductance, \( L_j \) is an effective distance between pore centers, and \( p, p_j \) represent the pressures in the fluid domain and its neighbor, respectively. The conductance is inversely proportional to the fluid viscosity \( \mu \), and proportional to the square of the effective throat area \( A_j \):

\[ C_j = \tilde{C}_j \frac{A_j^2}{2\kappa} \mu, \quad (9) \]

where \( \tilde{C}_j \) is a dimensionless throat conductance. The derivation of this equation from the solution of a Stokes flow problem, and the expressions for \( C \) and \( A \), are given in Appendix A.

The grains have a certain compressibility, and the radius of a spherical grain varies according to

\[ r_g = r_{g,0} \left(1 - \frac{p}{3K_s}\right), \quad (10) \]

where \( r_{g,0} \) is the initial radius (at zero fluid pressure), \( K_s \) is the bulk modulus of the solid grain, and \( p \) is the average of the pore pressures around the grain. Finally, mass balance over a fluid domain gives the following pressure evolution equation for a pore volume \( V_p \):

\[ \delta p = \frac{K_f}{V_p} \left(-\delta V_p - \sum_j q_j \delta t\right), \quad (11) \]

where \( K_f \) is the fluid bulk modulus, and \( \delta p \) is the pressure variation after a time step \( \delta t \). The main feature of our model is the term \(-\delta V_p\), which accounts for the change in volume of each pore caused by changes in grain locations. This term has been neglected in previous investigations of pore-scale poromechanical models but is essential, for example, to reproduce pressurization of the fluid upon fast compaction. It also reflects the reverse coupling present in Biot’s self-consistent theory of poroelasticity.

There is a formal analogy between the microporomechanical equations (8)–(11) and Biot’s self-consistent theory of poroelasticity [Biot, 1941; Wang, 2000]. We expect the DEM formulation will reproduce the linear theory of poroelasticity only in the range of small deformations and small pressure changes. Under such conditions, the poroelastic parameters can then be determined from DEM simulations. When these conditions are not met, nonlinear/irreversible behavior is expected to emerge in the DEM model, driven by contact slip, bond breaking and grain rearrangement.

2.2.1. Time Step Selection for Fluid Flow Simulation

The grain-scale fluid flow equations (11) are solved using an explicit time integration scheme. The time step must be restricted for the scheme to be stable. The charac-
The characteristic time associated with the microscopic fluid flow dynamics is

\[ \delta t = \frac{V_p}{K_f} \frac{\delta p}{\sum_j q_j} \]  

(12)

Introducing equations (8) and (9), we express the characteristic time as

\[ \delta t = \frac{V_p}{K_f} \frac{2 \rho g \mu}{w} \sum_j \frac{L_j}{C_j A_j^2}. \]  

(13)

Using the scaling \( V_p \sim r_g w \) and \( A \sim r_g^2 \), the critical time step for fluid flow stability scales as follows:

\[ \delta t_{\text{crit}} \sim \frac{\mu}{C K_f}. \]  

(14)

In a coupled poromechanics simulation, the time step must be smaller than the minimum of the critical values in equations (7) and (14).

### 2.3. Microporomechanics of Two-Fluid Systems

[25] In the environments of interest for methane hydrates, in particular, at the base of the hydrate stability zone, two fluid phases exist: a liquid brine phase, and methane gas. A key difference between single-fluid and two-fluid systems is the presence of a fluid-fluid interface. Because of surface tension effects, the pressures on either side of the interface (that is, the pressure in the brine and the pressure in the methane gas) can be very different.

[26] The key question is: what is the preferential mode of gas invasion? Two different fundamental mechanisms are at play (Figure 5): (1) capillary-dominated invasion of a rigid solid skeleton, and (2) fracturing of the sediment. While capillarity governs invasion of gas through the porous medium, mechanical effects may lead to deformation and fracturing of the sediment skeleton, thereby triggering invasion when it would otherwise not occur. Preferential fracturing of the sediment requires differences in pressure between neighboring pores. While this is

![Figure 5. Schematic diagram of the two modes of methane gas invading a sediment. (top) Before invasion, the gas-water interface of a buoyant gas plume underlies water-filled sediment. (middle) Invasion will occur if the capillary pressure (the difference between gas pressure and water pressure) exceeds the capillary entry pressure, which is inversely proportional to the pore diameter. (bottom) Invasion by fracture opening; if the exerted pressure is sufficient to overcome compression and friction at grain contacts, a fracture will form. In a multiphase environment, due to surface tension effects, the pressure difference between water and gas will not dissipate quickly through the porous medium, and water at grain contacts will increase cohesion.](image-url)
2.3.1. Capillary Invasion

The pressure difference between gas and water pressures, as in hydraulic fracturing operations, it is typically not a favored scenario in single-fluid systems (unless a fluid is injected at very fast flow rates and pressures, as in hydraulic fracturing operations), it is natural in two-fluid systems because the two fluids have different pressures. Before a water-saturated pore is invaded by gas, it is the water pressure. Once the pore is invaded by gas, it is the gas pressure that exerts a net force onto the surrounding grains. Since the two fluids do not mix, a pressure difference does not dissipate. This pressure difference may lead to preferential fracturing of the sediment. These processes clearly couple flow and deformation, at both the grain scale and the macroscopic scale.

2.3.2. Fracture Opening

Clearly, if the grain size is large, the process of capillary invasion is favored and gas invasion can occur even if the porous medium is rigid. In this case, the gas invasion pressure into the sediment is given by equation (15). On the other hand, for small grain size (high capillary entry pressures), gas invasion will not occur until the grains are pushed apart (Figure 5, bottom).

For an idealized scenario of cohesionless material under undrained plane-strain conditions, a fracture will propagate when the gas pressure exceeds the minimum compressive stress (assumed horizontal):

\[ p_g - \sigma_H \geq 0. \tag{16} \]

This fracture opening condition must be extended to the case when cohesion \( \bar{\sigma}_c \) exists. In porous media filled with a single fluid, the source of cohesion (tensile strength) is grain cementation and consolidation. When the pore space is occupied by two fluids of different wettability, capillary forces induce additional adhesion between particles (Figure 6) [Orr et al., 1975; Lian et al., 1993; Cho and Santamarina, 2001].

Because there is stress concentration at the fracture tip, the fracturing pressure depends not only on the earth stresses and the cohesive stress, but also the fracture’s length. In the realm of linear elastic fracture mechanics (LEFM), the fracturing pressure is

\[ p_g - \sigma_H \geq C_{\text{LEFM}} \frac{K_{Ic}}{\sqrt{\pi a}}, \tag{17} \]

where \( K_{Ic} \) is the fracture toughness, \( a \) is the length of the fracture, and \( C_{\text{LEFM}} \) is a coefficient that depends on the geometry, the ratio of horizontal to vertical stresses, and loading conditions [Anderson, 1991]. In the context of particle assemblies, the “measurable” fracture toughness depends on the cohesive strength (and, therefore, on the interfacial tension between fluids) and on the grain size. This last dependency emanates from the observation that the internal length scale in the fracture toughness is determined by the grain size [Potyondy and Cundall, 2004]. Therefore, under the assumptions of LEFM, the gas pressure for fracture opening takes the form

\[ p_g - \sigma_H \geq C_{\text{LEFM}} \frac{\gamma \sqrt{r_g}}{r_g \sqrt{\pi a}}. \tag{18} \]

Even though LEFM conditions do not always apply to natural sediments, equation (18) indicates that asymptotically, \( p_g^{\text{frac}} - \sigma_H \sim r_g^{-1/2} \). Equation (15) says that the gas pressure for capillary invasion scales like \( p_g^{\text{cap}} - p_w \sim r_g^{-1} \). Both invasion pressures increase as the grain size decreases, but the capillary invasion pressure increases faster. This
analysis suggests that fracturing is favored over capillary invasion for fine-grained sediments.

[31] In any case, we do not use LEFM. It is the nonlinear evolution of the DEM microporomechanical model that determines when bonds break, and when the gap between grains is large enough for the gas interface to advance, according to equation (15). In this fashion, the gas/water interface advances and a new pore is loaded with a higher pressure. The implementation of multifluid poromechanics is therefore very similar to that of single-fluid systems, except that the key hydraulic property (the conductance between pore bodies) is set to zero until condition (15) is satisfied.

[32] Capillary invasion and fracture opening are the two end-member mechanisms for methane transport in its own gas phase, and our coupled grain-scale model allows us to investigate the competition between the two as a function of grain size, earth stresses, and sediment cohesion.

3. Results

3.1. Micromechanics of “Dry” Media

3.1.1. Sediment Model Generation and Initialization

[33] A model sediment is generated by first choosing the number of particles, and reproducing the desired grain size distribution. As we show in section 3.1.2, several macroscopic properties (both mechanical and fluid flow properties) are dependent on the grain size. Therefore, it is important that the sediment model either reproduces the desired grain size distribution, or that the assigned microproperties (e.g., bond strength) reflect the disparity in grain size [Potyondy and Cundall, 2004]. In most of the examples shown in this paper, we have chosen a relatively narrow, uniform grain radius distribution \([r_{\text{min}}, r_{\text{max}}]\), with \(r_{\text{max}} = (5/3)r_{\text{min}}\).

[34] The particles are randomly placed in a box and allowed to fall under gravity, simulating sedimentation (Figure 7). The settling process has two differentiating stages: (1) free fall under gravity, with limited grain-grain interaction, and (2) settling and grain rearrangement until static conditions are reached.

[35] The time step is larger initially, during the “free fall” stage, and quickly converges to the value required for stability of the dynamical system dominated by grain-grain interactions. The time step is proportional to the grain radius, and inversely proportional to the square root of the grain stiffness, confirming the stability condition of equation (7).

3.1.2. Uniaxial Compaction for “Dry” Media

[36] Here we show that DEM simulations of “dry” media (infinitely compressible pore fluid) are able to capture the mechanical behavior of real sediments. In Figure 8 we plot experimental stress-strain curves of sediment samples from Hydrate Ridge [Tan et al., 2006] along with curves from DEM simulations. The data come from triaxial tests that reproduce constant rate of strain (CRS) conditions, in which the lateral stress was also measured. The DEM model was created by sedimenting particles, and applying an isotropic confining stress of 20 kPa, to start the uniaxial vertical compaction test at the same stress level as the experiments [Tan, 2004]. The only parameter that we varied to reproduce measured stress-strain behavior was the grain stiffness \(k_v\). The rest of the micromechanical parameters are as follows: \(k_r = k_v, \mu = 0.5, \sigma_c = \tau_c = 0\).

[37] The DEM simulations match the stress-strain behavior measured in the lab even for very high deformations (up to 16% strain), capturing the material nonlinearity. For an undisturbed ideal clay sample, the initial portion of the strain-log stress curve would be a straight line with slope equal to the unload/reload loop slope until reaching its preconsolidation stress. After this point, the slope would change to the virgin consolidation line slope. The Hydrate Ridge samples exhibit a large “rollover” at small strains (in the plot of vertical strain versus effective vertical stress on a logarithmic scale) because they are highly disturbed [Tan, 2004]. For the purpose of validation of the DEM model, the
relevant portion of the curve is that above the preconsolidation stress.

[38] Although we do not show it here, the DEM model also displays irreversible behavior in that loading/unloading cycles show hysteresis. However, it is unable to reflect the dramatic increase in stiffness upon unloading that the data show [Jain and Juanes, 2008].

[39] We also compared the lateral-to-vertical stress ratio, $K_0$, predicted by the DEM model with the values measured for Hydrate Ridge sediment samples [Tan et al., 2006]. The stress ratio is measured during 1-D consolidation in triaxial cells. In preparation for the test, and to make sure the sample is fully saturated, initial vertical and horizontal effective stresses were brought to about 20 kPa (which is small enough that there is not significant strain) to remove any bubbles from the sample. To reproduce these conditions, we isotropically consolidated the DEM samples to 20 kPa as well prior to performing the 1-D consolidation. Therefore, the initial $K_0$ value is 1. We used the same grain stiffness parameters employed to match the lab stress-strain behavior (Figure 8), and we let the DEM model predict the evolution of the $K_0$ ratio for those experimental conditions (Figure 9). While the agreement between model and experiments is generally good, the experimentally measured values drop down to about 0.5, while those predicted by the DEM model drop to about 0.6.

3.2. Microporomechanics of Single-Fluid Systems

3.2.1. Uniaxial Fluid Flow

[40] In this section, we evaluate the fluid flow capabilities of the grain-scale model. We do so by simulating a one-dimensional fluid flow problem in a cell with drained top and bottom boundaries, and impervious fixed lateral boundaries. The initial pressure is constant, and equal to the boundary pressures. Suddenly, a pressure change is applied to the top boundary, and we simulate the evolution of the pressure and fluid inflow/outflow until a new steady state is reached. The continuum problem is described mathematically by the partial differential equation:

$$c_v \frac{\partial p}{\partial t} + k \frac{\partial^2 p}{\partial x^2} = 0, \quad 0 < x < H,$$

where $x$ is elevation, $H$ is the height of the cell, $k$ is the intrinsic permeability, $\mu$ is the fluid viscosity, and $c_v$ is the consolidation coefficient [Wang, 2000]. The initial condition is given by

$$p(x, 0) = 0, \quad 0 \leq x \leq H,$$

and the boundary conditions are

$$p(0, t) = 0, \quad p(H, t) = \Delta p, \quad t > 0.$$

The problem can be expressed in dimensionless form by defining the following dimensionless quantities:

- Distance $\xi = \frac{x}{H}$
- Time $\tau = \frac{t}{T_c}$
- Pressure $\bar{p} = \frac{p}{p_0}$
- Flow rate $\bar{Q}_D = \frac{Q}{Q_c}$, $\bar{Q}_i = \frac{k}{\mu} \frac{\Delta p}{H} W w$

where $W$ is the width and $w$ is the thickness of the cell (that is, the dimensions of the cell in the directions perpendicular to the flow). The analytical solution to the problem can be found by the method of separation of variables [Crank, 1975]. The dimensionless pressure field is given by

$$p_D(\xi, \tau) = \xi + 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(n\pi\xi) \exp\left(-(n\pi)^2\tau\right). \quad (22)$$

By differentiating the expression above, we find the expression for the dimensionless flow rate in and out of the cell:

$$Q_D^{\text{in}} = 1 + 2 \sum_{n=1}^{\infty} (-1)^n \cos(n\pi) \exp\left(-(n\pi)^2\tau\right), \quad (23)$$

$$Q_D^{\text{out}} = 1 + 2 \sum_{n=1}^{\infty} (-1)^n \exp\left(-(n\pi)^2\tau\right). \quad (24)$$

[41] The objective is to determine whether the grain-scale model reproduces the macroscopic behavior. The relevant macroscopic parameters are the intrinsic permeability $k$ and the consolidation coefficient $c_v$. The intrinsic permeability is obtained by matching the flow rate at steady state. The consolidation coefficient is determined by matching the dimensionless inflow and outflow curves.

[42] We generated an assembly with 1000 grains, and a minimum radius $r_{\text{min}} = 1$ cm. The vertical and horizontal
dimensions of the cell are, approximately, \( H = 1 \text{ m} \) and \( W = 0.7 \text{ m} \). We set the pressure increment \( \Delta p \) to a small value, so that the effects of pore pressure on the mechanical deformation are minimal. Once the flow stabilizes, inflow and outflow rates are equal to \( Q_c \), and the intrinsic permeability of the medium can be computed as

\[
k = \frac{\mu H Q_c}{W w \Delta p}.
\]

The characteristic time \( T_c \) is then obtained by matching the numerical inflow and outflow curves, from which the macroscopic consolidation coefficient is computed as

\[
c_v = \frac{k T_c}{\mu H^2}.
\]

[43] In Figure 10 we plot the dimensionless inflow and outflow rates as a function of dimensionless time. The agreement between the DEM results and the analytical solution is excellent, indicating the flow formulation accurately captures the macroscopic behavior (Darcy flow in porous media). As a further validation of the model, we compare in Figure 11 the evolution of dimensionless pore pressure within the sample. By plotting the pressure values from the DEM simulation at individual pores, we obtain a scattered profile of the average pressure as a function of depth. We compare these results with the analytical solution at different dimensionless times. Again, the agreement is excellent.

[44] By repeating the fluid flow simulations with different values of \( r_{\text{min}} \), we determine the dependence of the hydraulic and poromechanical parameters on grain size. The results are compiled in Table 1, where we confirm that the intrinsic permeability scales with the square of the grain size (as expected from Stokes theory, and the Kozeny-Carman relation for granular materials). Moreover, if the fluid is significantly more compressible than the skeleton, the DEM simulations also reflect that the effective consolidation coefficient \( c_v \) is inversely proportional to \( K_f \), with the constant of proportionality being approximately equal to the porosity [Wang, 2000].

3.2.2. Uniaxial Undrained Compaction

[45] A sensitive test of the DEM coupled model’s validity is fluid-solid behavior during undrained consolidation tests. A sediment model is initialized by gravitational settling. Then the walls are adjusted to achieve an isotropic confining stress state of 0.1 MPa (above atmospheric pressure). Until that point, the fluid is allowed to drain and the pore pressure is atmospheric (\( p = 0 \)). Thus, the initial effective stress is 0.1 MPa. After that, the sample is sealed so that no fluid is allowed to drain, and it is subjected to uniaxial compaction. During the undrained compaction process, the vertical strain \( \varepsilon \), total vertical stress \( \sigma \), and average pore pressure \( p \) are recorded. In view of the effective stress concept [Terzaghi, 1943; Biot, 1941], the total stress required to achieve a given deformation in a fluid-saturated medium is larger than for a dry medium. In the realm of the linear theory of poroelasticity, the effective stress is given by

\[
\sigma' = \sigma - bp,
\]

where \( b \) is the Biot coefficient. The dependence of the Biot coefficient on the solid and fluid properties of the

<p>| Table 1. Macroscopic Hydraulic and Poromechanical Parameters for Different Grain Size Distributions |
|---------------------------------|-----------------|-----------------|</p>
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<th>( r_{\text{min}} ) (m)</th>
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<th>( c_v/K_f )</th>
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<tr>
<td>0.01</td>
<td>( 0.289 \times 10^{-6} )</td>
<td>0.161</td>
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<tr>
<td>0.001</td>
<td>( 0.289 \times 10^{-8} )</td>
<td>0.156</td>
</tr>
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</table>
constituents is reasonably well understood \[ Coussy, 1995; Wang, 2000 \]. The Biot coefficient approaches a value of one only in the limit of incompressible grains and point grain-grain contacts. If the grain and fluid compressibilities are comparable, the Biot coefficient is less than one.

\[ 46 \] We used an assembly with 1000 grains, \( r_{\text{min}} = 0.01 \text{ m} \), \( k_n = 10^7 \text{ N/m} \), \( k_n/k_s = 2.5 \), and \( K_f = 10^7 \text{ Pa} \). In Figure 12 we show the stress-strain curves for a cemented/cohesive sample (bond strength \( \bar{\tau}_c = \tau_c = 10^6 \text{ Pa} \), Figure 12, left), and for an unconsolidated/cohesionless sample (bond strength \( \bar{\tau}_c = \tau_c = 10^2 \text{ Pa} \), Figure 12, right). In both cases, we plot the stress-strain curves for the fluid-saturated medium (total stress), and for a dry medium. We confirm that the dry stress curve can be interpreted as the effective stress, and recovered by subtracting the pore pressure times the Biot coefficient from the total stress. We infer the Biot coefficient in this way, and the values obtained agree well with experimental values \[ Wang, 2000, \text{ Table C.1} \].

3.3. Microporomechanics of Two-Fluid Systems

\[ 47 \] Migration of a gas phase through a deformable medium may occur by two end-member mechanisms: (1) capillary invasion through a rigid medium and (2) fracture opening. Our DEM model is capable of reproducing both mechanisms, and can therefore predict the conditions under which one is favored over the other, and predict gas migration as a result of their combined effect.

3.3.1. Capturing the Fracturing Phenomenon

\[ 48 \] We first illustrate that our DEM model of coupled two-phase fluid flow and grain mechanics can reproduce fracture initiation and propagation upon invasion of an immiscible gas phase.

\[ 49 \] In many (passive) depositional environments, the horizontal stress is lower than the vertical stress. In such scenarios, one expects the development of vertical fractures that open up the sediment in the direction of minimum compressive stress. In Figure 13 we show that fracturing of the sediment is not necessarily restricted to anisotropic earth stresses. Even when horizontal and vertical stresses are equal, the medium tends to fracture in a set of radial, geometrically complex fractures if gas is injected into a brine-saturated sediment.

3.3.2. Fracturing Versus Capillary Invasion: Influence of Grain Size

\[ 50 \] We find the most sensitive factor in determining the mode of methane gas transport (sediment fracturing or capillary invasion) is the grain size: fracturing is favored for fine-grained sediments, while capillary invasion is favored for coarse-grained sediments. Here we illustrate these two end-members.

\[ 51 \] The simulation is set up as follows. A sample of 300 grains of grain size \( [r_{\text{min}}, 2r_{\text{min}}] \) is generated by gravitational settling. Since the sample size is much smaller than a representative elementary volume of sediment, we only simulate a narrow range of grain sizes at a time. The lateral boundaries are fixed. The sediment is then compacted vertically under constant pore pressure until a vertical effective stress of 3 MPa is achieved. This level of vertical effective stress corresponds to a depth of about 300 m below seafloor. During this vertical compaction, the
horizontal effective stress increases to a value of about 1.6 MPa, that is, $K_h \approx 0.53$. Inasmuch as gravitational effects are negligible in a grain-scale model, the results are independent of water depth; they depend on the relative magnitudes of capillary pressure (the difference between gas pressure and water pressure) and effective stress (the difference between the total stress and the pore pressure).

[52] The interfacial tension is $\gamma = 50 \times 10^{-3}$ N/m. We assume the cohesion is inversely proportional to grain radius. This is phenomenologically adequate (fine-grained material like clays are cohesive) and is also consistent with the adhesive forces that result from the presence of a gas-water interface [Kato et al., 2004]. For simplicity, we take $\sigma_c = 2\gamma/r_c$. The only parameter that is left free is the grain size $r_{\text{min}}$.

[53] During the simulation, we inject gas at the bottom center pore. We incrementally increase the gas pressure. Between each increment, we allow sufficient time to pass for fluid flow and granular displacements to stabilize, so mechanical equilibrium is reached.

[54] In Figure 14 we show two snapshots of the evolution of the methane-water interface for a coarse-grain sediment of characteristic size $r_{\text{min}} = 50 \mu m$. It is apparent that during the invasion of methane gas, there is virtually no movement of the solid grains: the sediment acts like a rigid skeleton. Indeed, the network of grain contact compressive forces remains the same during the process. Invasion of gas from pore to pore occurs when the gas pressure (minus the water pressure) exceeds the capillary entry pressure of the throat (equation (15)). In this case, the capillary entry pressure is much lower than the fracturing pressure (Figure 14 (left) corresponds to $P_c \approx 5$ kPa), and fluid transport is well described by invasion percolation [Wilkinson and Willemsen, 1983; Lenormand et al., 1988]. Ultimately, if the gas pressure is sufficiently high, almost all the pores have been invaded by methane gas. In this case, this occurs at a slightly higher capillary entry pressure of $P_c \approx 6$ kPa.

[55] The behavior is completely different when a much smaller grain size is used. The evolution of the methane gas migration for $r_{\text{min}} = 0.1 \mu m$ is shown in Figure 15. The range of capillary entry pressures for the initial configuration is now on the order of 3 MPa. However, at this pressure, mechanical effects become dominant, and the solid skeleton no longer behaves like a rigid medium. At $P_c \approx 2.5$ MPa, the invading gas starts to initiate a fracture, with its characteristic stress concentration at the fracture tip captured by the DEM model [Potyondy and Cundall, 2004]. The fracture propagates vertically. Animations S1 and S2 are movies of the developing fracture and are available in the auxiliary material.1 The value of the capillary pressure needed to open the fracture corresponds to a gas column thickness of about 300 m below the base of the HSZ. Gas column thicknesses of this magnitude have been observed in similar geologic environments [Holbrook et al., 1996; Hornbach et al., 2004], and interpreted as the cause of critical pressures for gas migration through faults [Flemings et al., 2003; Hornbach et al., 2004; Trehu et al., 2004a].

[56] Our grain-scale model explains why focused gas flow can occur by means of fracture opening, even in the absence of preexisting faults and fractures. It is likely, however, that our model overestimates the invasion capillary pressure required for fracturing, due to boundary effects. This can be seen from the grain forces on the lateral boundaries in Figure 15, which change significantly as the fracture propagates. This is confirmed by the increase of the vertical and, especially, horizontal mean effective stress

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1Auxiliary materials are available in the HTML. doi:10.1029/2008JB006002.
Figure 14. Snapshots of the evolution of the methane gas-water interface for the case $r_{\text{min}} = 50 \mu m$. The dominant mechanism is capillary invasion. The pores occupied fully by gas are represented with blue dots at the pore centers. The maroon lines indicate compression at grain-grain contacts. The green lines represent tension, which is supported by cohesion between grains. (left) $P_c = 5$ kPa. (right) $P_c = 6$ kPa.

Figure 15. Snapshots of the evolution of the methane gas-water interface for an assembly with $r_{\text{min}} = 0.1 \mu m$. The dominant mechanism is fracture opening. The pores occupied fully by gas are represented with blue dots at the pore centers. The maroon lines indicate compression at grain-grain contacts. The green lines represent tension, which is supported by cohesion between grains. The pink lines show where cohesive bonds were broken. (left) $P_c = 2.5$ MPa. (right) $P_c = 2.55$ MPa. Movies of the developing fracture are available in the auxiliary material as Animation S2.
during fracture growth (Figure 16). The presence of computational boundaries near the propagating fracture introduces an artificial stiffness in the problem that results in an overestimation of the gas invasion pressure.

4. Discussion and Conclusions

[57] We have presented a discrete element model for simulating, at the grain scale, gas migration in brine-saturated deformable media. The model has been validated for many processes, including (1) generating sediment models by gravitational settling and compaction; (2) stress-strain behavior of ocean sediments; (3) transient single-phase flow for determining hydraulic parameters; and (4) undrained compaction tests for determining poromechanical parameters.

[58] The coupled model permits investigating an essential process that takes place at the base of the hydrate stability zone: the upward migration of methane in its own free gas phase. We elucidate the two ways in which gas migration may take place: (1) by capillary invasion in a rigid-like medium and (2) by initiation and propagation of a fracture.

[59] Each end-member can be analyzed separately, and conditions for gas invasion can be found for the capillary-dominated and fracture-dominated regimes. We find the main factors controlling the mode of gas transport in the sediment are the grain size and the effective confining stress. We have shown that coarse-grain sediments favor capillary invasion, whereas fracturing dominates in fine-grain media. Recent laboratory experiments of gas invasion and bubble growth in soft, fine-grained sediments provide convincing evidence that fracturing is a relevant gas transport mechanism [Boudreau et al., 2005; Best et al., 2006]. The cornflake-shaped, subvertical fractures observed in those experiments are strikingly similar to those simulated with the mechanistic grain-scale model presented here.

[60] The significant contribution of our coupled model is that it captures both phenomena and, as a result, allows us to study the transition between the two regimes. We synthesize the transition from capillary invasion to fracture opening in Figure 17. We plot the gas pressure required for invasion into a sediment at an effective confining stress of 3 MPa (typical of a sediment column depth of about 300 m) as a function of grain size. For sufficiently coarse grain size, gas invades by capillarity. According to equation (15), the invasion capillary pressure for this regime decreases with increasing grain size and has the following scaling:

\[ p_{\text{cap}} \sim r_g^{-1}. \] (28)

Figure 16. Evolution of the vertical and horizontal mean effective stress during fracture propagation for the simulation shown in Figure 15. The invading gas pressure remains constant throughout, but the vertical and, especially, the horizontal stress at the boundaries increase during fracture growth. The boundaries introduce an artificial stiffness in the problem that results in an overestimation of the gas invasion pressure.

Figure 17. Plot of invasion pressure versus grain size (open red circles) for a sediment under 3 MPa vertical effective stress (sediment column depth of about 300 m). The solid red circle denotes the critical grain size at which the transition in the mode of gas invasion occurs. For larger grain size, gas invades by capillarity. For smaller grain size, it invades by opening a fracture. In the capillarity-dominated regime, the invasion capillary pressure decreases with increasing grain size with a slope of \(-1\) (black solid line), consistent with the theory (equation (28)). In the fracture-dominated regime, the invasion capillary pressure increases with decreasing grain size, though the predictions of linear elastic fracture mechanics (black dashed line with slope \(-1/2\)) are not in agreement with our coupled grain-scale model. The deviation from linear-elastic fracture mechanics suggests that the effects of capillarity and inelastic material behavior are important but it may also be due to boundary effects in the grain-scale simulations.
This scaling is clearly confirmed by our DEM simulations, which collapse onto a straight line of slope $-1$ on the log-log plot of $P_c$ versus $r_g$.

[61] There exists a critical grain size, in this case $r_g \approx 0.1$ $\mu$m, at which the transition from capillary invasion to fracturing occurs. For grain sizes below this critical value, invasion is always by fracture opening. The pressure required for opening a fracture, however, does depend on grain size. Under LEFM conditions, the expected scaling from equation (18) is

$$P_{c}^{\text{frac}} - \sigma_{H}^{\prime} \sim r_{g}^{-1/2},$$

which corresponds to a straight line of slope $-1/2$. This behavior is not confirmed by our grain-scale model, which, unlike LEFM, accounts for the coupling between two-phase flow and inelastic grain-scale mechanics. This suggests that inelastic and capillary effects are essential in the fracturing process.

[62] These emergent phenomena have important implications for understanding hydrates in natural systems (either ocean sediments and permafrost regions). Our model predicts that in fine sediments, hydrate will likely form in veins that follow a fracture-network pattern. Since the mechanism of fracture propagation is self-reinforcing, our results indicate that it is possible, and even likely, that methane gas will penetrate deeply into the HSZ (and maybe all the way to the ground surface).

[63] Our model supports the view that in coarse sediments, the buoyant methane gas is likely to invade the pore space more uniformly, in a process akin to invasion percolation. While this is definitely affected by heterogeneity in grain size distribution, the overall pore occupancy is likely to be higher than for a fracture-dominated regime, leading to larger timescales for transport. The predictions from our model are consistent with field observations of hydrates in natural systems [Suess et al., 1999; Sassen et al., 2001; Flemings et al., 2003; Trehu et al., 2004a; Hornbach et al., 2004; Trehu et al., 2006a; Riedel et al., 2006; Weinberger and Brown, 2006; Liu and Flemings, 2006, 2007; Collett et al., 2008].

[64] We have not undertaken here an analysis on the relative importance of transport in solution versus as a separate free gas phase flowing through fractures. However, the importance of free gas fluxes through fractures is demonstrated by the massive free methane gas venting rates observed intermittently at northern Hydrate Ridge [Torres et al., 2002]. Tryon et al. [1999, 2002] hypothesized that fracture networks rapidly transport the gas from beneath the HSZ to the vents, a distance of 70–100 m. The total methane flux coming out of 10 discrete vents (~1 cm diameter each), clustered in a depression several meters in diameter, constitutes two thirds of the total flux, calculated from seawater methane concentrations in the overlying water column [Heeschen et al., 2005]. Acoustic imaging shows massive bubble plumes at two other sites in the region [Heeschen et al., 2005], which likely account for the remaining one third of the flux to the water column. Free gas ebullition through vents fed by fractures releases methane from the seafloor at rates many orders of magnitude faster than transport by advection of dissolved methane in solution through sediments [Torres et al., 2002]. In terms of velocities, the gas bubbles exit conduits episodically at about 1 m/s [Torres et al., 2002], whereas pore water advection occurs at 0.5–1 m/a, calculated from dissolved calcium profiles in sediment pore water, or 0.3–1 m/a, calculated from seafloor methane seepage rates [Torres et al., 2002], so free gas exits conduits more than $10^7$ times faster than the speed of fluid advection in the sediments. At less active regions, like Blake Ridge, advective fluxes are 2 to 3 orders of magnitude slower than those at Hydrate Ridge [Nimblett and Ruppel, 2003].

[65] The results from this work also have important implications for carbon dioxide storage in the deep subsea-floor, where sequestration is possible by hydrate formation [Koide et al., 1995, 1997b] and gravitational trapping [Koide et al., 1997a; House et al., 2006; Levine et al., 2007; Goldberg et al., 2008]. Whether the migration of supercritical CO$_2$ is dominated by capillary invasion or fracture opening may determine the viability of this sequestration concept in ocean sediments. In these systems, fractures will have a tendency to propagate vertically [Nunn and Meulbroek, 2002; Boudreau et al., 2005], which could provide fast pathways for the escape of injected CO$_2$ by pressure gradients. In order to avoid the fracturing regime, it is conceivable that one should inject in coarse-grained, high-energy sediments such as turbidites.

Appendix A: Throat Conductance

[66] In this section we derive equation (9) for the throat conductance. Our formulation resolves one fundamental problem of two-dimensional grain-scale models: the fact that when grains are in contact, the aperture of the throat between pores is zero. Three-dimensional models do not suffer from this problem, because the throat can be associated with the section of minimum cross-sectional area between two pore bodies.

[67] Before discussing our formulation for 2-D models, consider an individual pore throat in three dimensions, as shown in Figure A1. A throat can be considered a micro-fluidics pipe, with a certain angular cross section. For creeping flow in a small channel, the Navier-Stokes equations reduce to the elliptic Poisson equation. For a coordinate system in which one of the axes (say, the $z$ axis) is parallel to the channel, the equation and boundary conditions describing the flow read [Bird et al., 1960]

$$\nabla^2 \nu = -\frac{\Xi}{\mu} \quad \text{in} \quad \Omega, \quad \text{(A1)}$$

$$\Xi = -\frac{\partial p}{\partial z}, \quad \text{(A2)}$$

$$\nu(x,y) = 0 \quad \text{on} \quad \partial \Omega, \quad \text{(A3)}$$

where $\mu$ is the fluid viscosity, $\Xi$ is the (negative) pressure gradient, and $\partial \Omega$ is the boundary of the two-dimensional cross section $\Omega$.

[68] Many solutions to this equation exist for simplified geometries, and useful parameterizations have been devel-
The total flow rate \( q \) through the throat cross section can be expressed in the following form:

\[
q / \mathcal{C} = Z W v(x, y) dW = \mathcal{X} \quad \text{(A4)}
\]

where \( \mathcal{C} \) is the throat conductance. Dimensional analysis dictates that the conductance can, in turn, be expressed as follows:

\[
\mathcal{C} = \frac{1}{\mu A^2 \tilde{C}} \quad \text{(A5)}
\]

where \( A \) is the cross-sectional area of the throat, and \( \tilde{C} \) is a dimensionless conductance. It has been shown that for a very wide range of throat shapes, the dimensionless conductance is simply a function of the shape factor,

\[
\tilde{C} = \frac{3}{5} G \quad \text{(A6)}
\]

where \( P \) is the perimeter of the throat. In fact, for triangular cross sections, the dimensionless conductance can be approximated by the simple expression \([\text{Patzek and Silin, 2001}]\)

\[
\tilde{C} = \frac{3}{5} G \quad \text{(A7)}
\]

With precise knowledge of the geometry of the grain assembly, one could compute the area and shape factor for each throat, and evaluate the throat conductance using expressions (A5)–(A7). However, since these equations themselves rely on the assumption of creeping flow in cylindrical channels, it is sufficient to consider a “master” geometry (like the one shown in Figure A1). In any case, it is important to note that the throat conductance scales with the fourth power of the grain size:

\[
\mathcal{C} \sim \frac{\mathcal{C}}{\mu \gamma_s} \quad \text{(A8)}
\]

The question is How do we apply this conductance formulation to two-dimensional grain-scale models? In 2-D models, the cross section (or aperture) of a pore throat is zero if grains are in contact. This would lead to a model that does not conduct fluid. To address this issue, previous investigations typically resort to defining an artificial throat aperture using heuristic arguments \([\text{Bruno and Nelson, 1991; Li and Holt, 2001, 2004; Itasca, 2004}]\).

We resolve this problem by understanding a two-dimensional model as a collapsed three-dimensional model. We must make some approximations with respect to the grain arrangement in the third dimension. In particular, we assume cubic packing of the 3-D assembly. Consider two grains of the same size that are in contact in the 2-D model (Figure A2). For cubic packing in the third dimension, the geometry of the throats is well defined, and the flow rate through an individual throat can be computed with equations (A4)–(A5).

Importantly, this model leads to physically realistic throat geometries (and, therefore, throat conductances) regardless of whether the two grains defining a throat are just in contact (gap \( d = 0 \)), whether there is a gap between them \( (d > 0) \), or whether there is overlap between them \( (d < 0) \). For each configuration, the shape factor can be computed using elementary geometry, and the dimensionless conductance evaluated therefrom. In Figure A3, we summarize the throat conductance formulation for 2-D grain assemblies.

In a 2-D model, we must collapse the third dimension, and compute the flow rate between pore bodies as

\[
q_{2D} = q_{w} \frac{w}{2\gamma_s} \quad \text{(A9)}
\]

where if \( w = 1 \), \( Q \) is the flow rate per unit width. Combining equation (A9) with (A4) and (A5) leads to equations (8) and (9), as desired.

Appendix B: Capillary Entry Pressure

The conceptual model presented in Appendix A also provides a framework for determining capillary entry pressures in two-dimensional models, which is required to simulate two-phase flow.

Consider, first, invasion of gas through a throat in a three-dimensional setting (Figure A1). The surface tension between gas and brine is \( \gamma \), and the contact angle between the gas-water interface and the solid surface is \( \theta \). Gas will penetrate through the throat cross section when the capillary pressure (the gas pressure minus the water pressure) exceeds...
the threshold capillary pressure. The Mayer-Stowe-Princen (MSP) [Mayer and Stowe, 1965; Princen, 1969a, 1969, 1970] method for calculating the threshold pressure relies on equating the curvature of the corner arc menisci to the curvature of the invading interface. Expressions for the drainage capillary entry pressure have been derived for a variety of cross sections [Mason and Morrow, 1991; Øren et al., 1998; Patzek, 2001]. They take the form

\[ P_c = \frac{2\gamma}{r_{th}} F(\theta, G, D), \]  \hspace{1cm} (B1)

where \( r_{th} \) is the radius of the inscribed circle and \( F \) is a function of the receding contact angle and the geometry of the throat, through the throat shape factor \( G \) and a function \( D \) of the throat corner angles. It turns out that if the contact angle is small (as it normally is for gas invasion into a natural sediment), \( F \approx 1 \) [Lenormand et al., 1983; Patzek, 2001]. Therefore, for zero contact angle, the capillary entry pressure can be approximated by

\[ P_c = \frac{2\gamma}{r_{th}}. \]  \hspace{1cm} (B2)

\[ P_c = \frac{2\gamma}{r_{th}}. \]  \hspace{1cm} (B2)
For a throat formed in the space between three spherical grains of equal radius $r_g$ that are in contact, elementary geometry leads to the expression

$$r_{th} = r_g \left( \frac{2}{\sqrt{3}} - 1 \right), \quad (B3)$$

and, therefore, we obtain the simple and useful estimate:

$$P_e \approx 10 \frac{2}{r_g}. \quad (B4)$$

Once again, we apply this concept to two-dimensional grain-scale models, by assuming simple cubic packing in the third dimension. In this way, we can rigorously define a throat radius (and, from it, a capillary entry pressure) even when the 2-D throat aperture is zero (because the 2-D grains defining a pore throat are in contact). If the two grains are exactly in contact (see Figure B1), one can immediately obtain the relation

$$r_{th} = r_g \left( \sqrt{2} - 1 \right). \quad (B5)$$

When the two grains of equal size are separated by a gap ($d > 0$), or when they overlap ($d < 0$), relation (B5) can be extended as follows:

$$r_{th} = r_g \left( \sqrt{1 + \left(1 + \frac{d}{2r_g} \right)^2} - 1 \right). \quad (B6)$$

Substituting equation (B6) into (B2) leads to equation (15), as desired.

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